FORMULÁRIO BÁSICO DE RESISTÊNCIA DOS MATERIAIS
NOTAÇÃO BEER–JOHNSTON (B.J.) E APOSTILA RENATO MIRANDA (AP.)

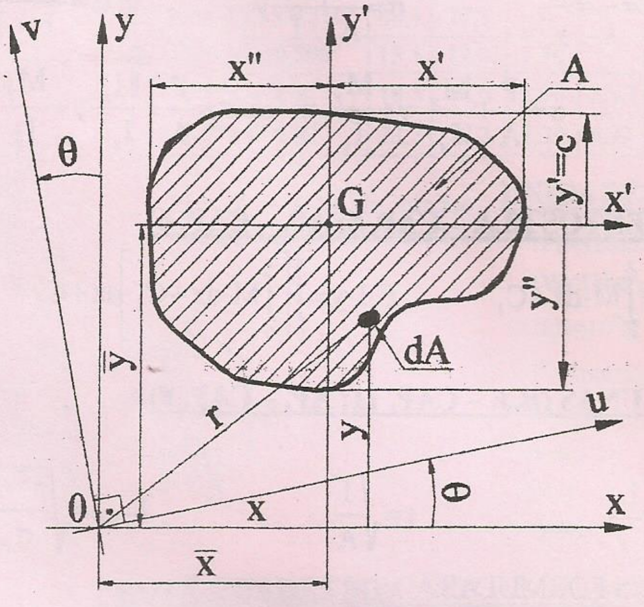
EQUILÍBRIO – ESFORÇOS EXTERNOS E INTERNOS – REAÇÕES – DIAGRAMAS (B.J. – CAP. 7 + ESTÁTICA; AP. – CAP. 2)

$$\sum F_x = 0 \qquad R = W = \int w \cdot dx \qquad \frac{dV}{dx} = -w \qquad V = -\int w \cdot dx$$

$$\sum F_y = 0 \qquad \bar{x} = \frac{\int x \cdot w \cdot dx}{W} \qquad \frac{dM}{dx} = V \qquad M = \int V \cdot dx$$

$$\sum M = 0 \qquad \frac{d^2M}{dx^2} = \frac{dV}{dx} = -w$$

FIGURAS PLANAS (B.J. – APÊNDICE A + ESTÁTICA; AP. – CAP. 3)



$$Q_x = M_x = \int_A y\,dA = \sum y_i A_i$$

$$Q_y = M_y = \int_A x\,dA = \sum x_i A_i$$

$$\bar{x} = \frac{Q_y}{A} = \frac{\sum x_i \cdot A_i}{\sum A_i}$$

$$\bar{y} = \frac{Q_x}{A} = \frac{\sum y_i \cdot A_i}{\sum A_i}$$

$$I_x = \int_A y^2 \cdot dA$$

$$I_y = \int_A x^2 \cdot dA$$

$$I_{xy} = P_{xy} = \int_A x \cdot y \cdot dA$$

$$J_0 = \int_A r^2 \cdot dA = I_x + I_y$$

$$I_x = I_{x'} + A \cdot \bar{y}^2 \qquad I_y = I_{y'} + A \cdot \bar{x}^2 \qquad I_{xy} = I_{x'y'} + A \cdot \bar{x} \cdot \bar{y} \qquad I_x = I_{x'} + A \cdot d^2$$

$$I_{x'} = I_x - A \cdot \bar{y}^2 \qquad I_{y'} = I_y - A \cdot \bar{x}^2 \qquad I_{x'y'} = I_{xy} - A \cdot \bar{x} \cdot \bar{y} \qquad W'_{x'} = \frac{I_{x'}}{y'} \qquad W''_{x'} = \frac{I_{x'}}{y''}$$

$$r_x = \sqrt{\frac{I_x}{A}} \qquad r_y = \sqrt{\frac{I_y}{A}} \qquad r_0 = \sqrt{\frac{J_0}{A}} \qquad W'_{y'} = \frac{I_{y'}}{x'} \qquad W''_{y'} = \frac{I_{y'}}{x''}$$

$$I_u = I_x \cdot \cos^2\theta + I_y \cdot \operatorname{sen}^2\theta - 2 \cdot I_{xy} \cdot \operatorname{sen}\theta \cdot \cos\theta$$

$$I_v = I_x \cdot \operatorname{sen}^2\theta + I_y \cdot \cos^2\theta + 2 \cdot I_{xy} \cdot \operatorname{sen}\theta \cdot \cos\theta \qquad I_u + I_v = I_x + I_y = I_1 + I_2$$

$$I_{uv} = I_x \cdot \operatorname{sen}\theta \cdot \cos\theta - I_y \cdot \operatorname{sen}\theta \cdot \cos\theta + I_{xy} \cdot (\cos^2\theta - \operatorname{sen}^2\theta)$$

$$I_{1,2} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad \operatorname{tg}2\theta = -\frac{2 \cdot I_{xy}}{I_x - I_y} \qquad \operatorname{tg}\theta_1 = \frac{I_x - I_1}{I_{xy}} \qquad \operatorname{tg}\theta_2 = \frac{I_x - I_2}{I_{xy}}$$

TRAÇÃO – COMPRESSÃO – CISALHAMENTO (B.J. – CAP. 1 E 2; AP. – CAP. 4)

$$\sigma=\frac{P}{A} \qquad \tau=\frac{V}{A} \qquad cs=s=\frac{P_{lim}}{\overline{P}} \qquad s=\frac{\sigma_{lim}}{\overline{\sigma}} \qquad s=\frac{\tau_{lim}}{\overline{\tau}} \qquad \varepsilon=\frac{\delta}{L}=\frac{\Delta L}{L} \qquad \sigma=E\cdot\varepsilon$$

$$\delta=\Delta L=\int_0^L\frac{P\cdot dx}{E\cdot A}=\frac{P\cdot L}{E\cdot A}=\sum\frac{P_i\cdot L_i}{E_i\cdot A_i} \qquad \delta_T=\alpha\cdot\Delta T\cdot L \qquad \varepsilon_T=\frac{\Delta d}{d} \qquad \upsilon=-\frac{\varepsilon_T}{\varepsilon}=-\frac{\varepsilon_y}{\varepsilon_x}=-\frac{\varepsilon_z}{\varepsilon_x} \qquad \sigma_{CIR}=\frac{p\cdot d}{2\cdot e}=\frac{p\cdot r}{t}$$

$$\sigma_{LON}=\frac{p\cdot d}{4\cdot e}=\frac{p\cdot r}{2\cdot t} \qquad \gamma=\frac{\Delta v}{L} \qquad \tau=G\cdot\gamma \qquad G=\frac{E}{2(1+\upsilon)} \qquad \Delta v=\int_0^L\frac{V\cdot dx}{A\cdot G}=\frac{V\cdot L}{A\cdot G}$$

TORÇÃO (B.J. – CAP. 3; AP. – CAP. 7)

$$\tau=\frac{T}{J}\cdot\rho \qquad \tau_{MAX}=\frac{T}{J}\cdot c=\frac{T}{J}\cdot R=\frac{T}{W_T} \qquad W_T=\frac{J}{R} \qquad \phi=\int_0^L\frac{T\cdot dx}{G\cdot J}=\frac{T\cdot L}{G\cdot J}=\sum\frac{T_i\cdot L_i}{G_i\cdot J_i} \qquad f=\frac{n}{60}$$

$$\tau=G\cdot\gamma \qquad P=T\cdot\omega=T\cdot 2\cdot\pi\cdot f \qquad T=71620\cdot\frac{P}{n}\rightarrow T\ [Kgf\cdot cm];\ n\ [rpm];\ P\ [cv]$$

FLEXÃO - TENSÕES (B.J. – CAP. 4 E 7; AP. – CAP. 5)

$$\sigma=\frac{M}{I}\cdot y \qquad \sigma_{MAX}=\frac{M}{I}\cdot c=\frac{M}{I}\cdot y'=\frac{M}{W} \qquad W=\frac{I}{c}=\frac{I}{y'} \qquad \sigma=\frac{P}{A}+\frac{M}{I}\cdot y \qquad tg\,\alpha=\frac{-I_x}{I_y}\cdot\frac{M_y}{M_x}$$

$$\sigma=\frac{M_x}{I_x}\cdot y+\frac{M_y}{I_y}\cdot x \text{ ou } \sigma=\frac{M_z}{I_z}\cdot y+\frac{M_y}{I_y}\cdot z \qquad \sigma=\frac{P}{A}+\frac{M_x}{I_x}\cdot y+\frac{M_y}{I_y}\cdot x \text{ ou } \sigma=\frac{P}{A}+\frac{M_z}{I_z}\cdot y+\frac{M_y}{I_y}\cdot z$$

FLEXÃO - DESLOCAMENTOS (B.J. – CAP. 8; AP. – CAP. 6)

$$\frac{1}{\rho}=\frac{M}{E\cdot I} \qquad y''=\frac{d^2y}{dx^2}=\frac{d\theta}{dx}=\frac{M_{(x)}}{E\cdot I} \qquad E\cdot I\cdot\theta=\int_0^x M\cdot dx+C_1 \qquad E\cdot I\cdot y=\int_0^x\left[\int_0^x M\cdot dx+C_1\right]dx+C_2$$

FLAMBAGEM DE COLUNAS (B.J. – CAP. 11; AP. – CAP. 8)

REGIME ELÁSTICO – $\lambda \geq \lambda_{LIM}$ EULER:

$$P_{cr}=P_{fl}=\frac{\pi^2\cdot E\cdot I}{L_e^2} \qquad \sigma_{cr}=\frac{\pi^2\cdot E}{\lambda^2} \qquad \lambda=\frac{L_e}{r} \qquad r=\sqrt{\frac{I}{A}} \qquad \lambda_{LIM}=\sqrt{\frac{\pi^2\cdot E}{\sigma_P}}$$

REGIME PLÁSTICO – $\lambda < \lambda_{LIM}$ DIVERSAS FÓRMULAS:

$$\sigma_{cr}=a-b\cdot\lambda \qquad \sigma_{cr}=a-b\cdot\lambda^2 \qquad \sigma_{cr}=a-b\cdot\lambda+c\cdot\lambda^2$$

PROPRIEDADES MECÂNICAS MÉDIAS DE ALGUNS MATERIAIS (B.J. – APÊNDICE B; AP. – ANEXO I)

Material	E (GPa)	G (GPa)	ν	σ_{ET} (MPa)	σ_{RT} (MPa)
Aço segundo ASTM A36	207	80	0,30	250	400
Aço ARBL segundo ASTM A808	207	80	0,30	345	450
Aço SAE 4340 temp. rev. 315°C por 1 hora	207	80	0,30	1586	1724
Ferro Fundido segundo ASTM A48	67	27	0,28	-	240
Liga de alumínio AA 2024-T3	69	26	0,33	345	483
Liga Ti-6%Al-4%V	110	41	0,34	1103	1172
Nylon 6,6	3	1,1	0,39	65	94,5
Policarbonato (PC)	2,38	0,9	0,36	62	68
Vidro comum	69	28	0,23	-	69
Alumina	393	161	0,22	-	275-550

Tabela elaborada pelo Prof. Rodrigo Magnabosco

(continuação na folha 3)

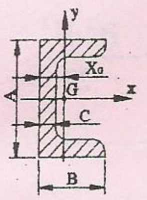
PERFIS INDUSTRIAIS (B.J. – APÊNDICE C; AP. – ANEXO III)

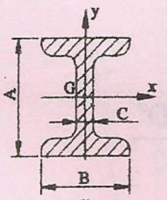
Dimensão Nominal	Área mm^2	Altura mm	Abas		Espessura Alma mm	Eixo X-X			Eixo Y-Y			
			Largura mm	Espessura mm		I_x 10^6 mm^4	W_x 10^3 mm^3	r_x mm	I_y 10^6 mm^4	W_y 10^3 mm^3	r_y mm	$\bar{X}$ mm
W530x150	19200	543	312	20,3	12,7	1007	3710	229	103,2	662	73,4	
S200x34	4368	203	106	10,8	11,2	27,0	266	78,7	1,794	33,8	20,3	
C380x74	9480	381	94	16,5	18,2	168,2	883	133,1	4,58	62,1	22,0	20,3

Lado e Espessura mm	Massa por Metro kg/m	Área mm^2	Eixo X-X				Eixo Y-Y				Eixo Z-Z	
			I_x 10^6 mm^4	W_x 10^3 mm^3	r_x mm	y mm	I_y 10^6 mm^4	W_y 10^3 mm^3	r_y mm	x mm	r_z mm	$tg\alpha$
L203x152x25,4	65,5	8390	33,6	247	63,3	67,3	16,15	146,2	43,9	41,9	32,5	0,543

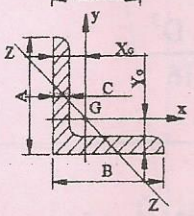
Lado e Espessura mm	Massa por Metro kg/m	Área mm^2	Eixo X-X e Eixo Y-Y				Eixo Z-Z
			I 10^6 mm^4	W 10^3 mm^3	r mm	x ou y mm	r mm
L203x203x25,4	75,9	9680	37,0	259	61,8	60,2	39,6

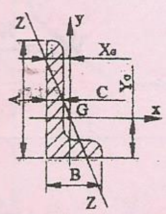
PERFIS INDUSTRIAIS (CATÁLOGO C.S.N; AP. – ANEXO III)





Perfil AxBx C	A cm^2	p kg/m	X_G cm	Y_G cm	I_x cm^4	I_y cm^4	W_x cm^3	W_y cm^3	r_x cm	r_y cm	r_z cm
I 10" x 4⅝" x 0,310"	48,1	37,7	-	-	5140	282	405	47,7	10,3	2,42	2,42
U 6" x 2" x 0,200"	15,5	12,2	1,30	-	546	28,8	71,7	8,20	5,94	1,36	1,36
L 4" x 4" x ⅜"	18,5	14,6	2,90	2,90	183	183	24,6	24,6	3,12	3,12	2,00
L 6" x 4" x ⅜"	23,3	18,3	2,39	4,93	562	204	54	26	4,90	2,97	2,24





Eixo Z = Eixo de mínimo momento de inércia (Eixo 2)

UNIDADES E DIVERSAS NOMENCLATURAS

$1Pa=1N/m^2$	$1MPa=10^6Pa=N/mm^2$	$1GPa=10^9Pa$
$1W=1Nm/s$	$1hp=746W$	$1rad=180/\pi°$
$1kN=10^3N$	$1Kgf\approx10N$	$1tf=10^3Kgf$
1"=1pol=2,54cm	$1m=10^2cm=10^3mm$	
$T=M_T$	$w=p=q$	$A=S$
$\theta=\varphi$	$L_e=L_{fl}$	$I=J$
$P=N$	$V=Q$	$\phi=\Delta\theta$
$Q=M_s$	$r=i=k$	

PROPRIEDADES MECÂNICAS MÉDIAS DE ALGUNS MATERIAIS

Material	Alongamento total em 50 mm (%)	Tenacidade a fratura K_{IC} $(MPa.m^{1/2})$	Coef. expansão térmica α $(10^{-6}/°C)$	ρ (KN/m^3)	Custo estimado (US$/Kg)
Aço segundo ASTM A36	23	80	12	78	0,60
Aço ARBL segundo ASTM A808	22	120	12	78	0,65
Aço SAE 4340 temp. rev. 315°C por 1 hora	10	58	12	78	1,10
Ferro Fundido segundo ASTM A48	0,5	-	12	72	0,80
Liga de alumínio AA 2024-T3	17	44	23	28	3,6
Liga Ti-6%Al-4%V	25	55	8	44	16,25
Nylon 6,6	60	2,5	144	11,5	4,30
Policarbonato (PC)	120	2,2	122	12	4,90
Vidro comum	0	0,75	9	25	1,39
Alumina	0	3,1	7	40	18,10

Tabela elaborada pelo Prof. Rodrigo Magnabosco

(continuação da folha 2)

Referências bibliográficas da tabela de materiais:

-HERTZBERG, R. W. Deformation and fracture mechanics of engineering materials. John Wiley : NY 1996 4. ed.
-CALLISTER Jr, W. D. Materials sci. eng. John Wiley : NY 1997 4. ed.
-MANGONON, P. L. The principles of materials selection for engineering design. Prentice Hall : NJ 1999.
-ASM handbook 10.ed. 1990 v. 1

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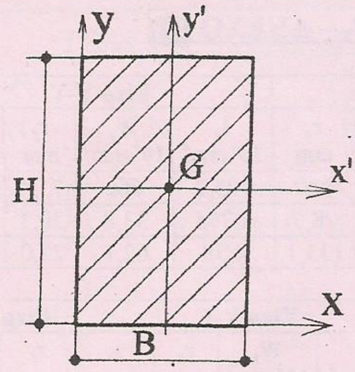
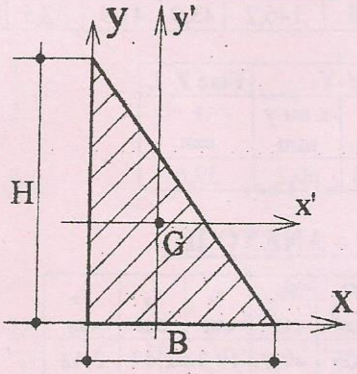
Responsáveis pela elaboração do formulário: Prof. Werner Mangold, Eng. Eduardo T. Tanaka e Eng. Leonardo M. Zamboni



$$\bar{x}=\frac{B}{2} \qquad \bar{y}=\frac{H}{2} \qquad A=B\cdot H$$

$$I_x=\frac{B\cdot H^3}{3} \qquad I_y=\frac{H\cdot B^3}{3} \qquad I_{xy}=\frac{B^2\cdot H^2}{4}$$

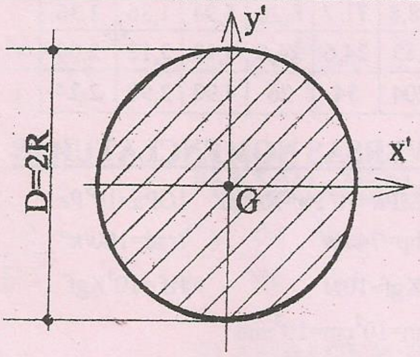
$$I_{x'}=\frac{B\cdot H^3}{12} \qquad I_{y'}=\frac{H\cdot B^3}{12} \qquad I_{x'y'}=0$$



$$\bar{x}=\frac{B}{3} \qquad \bar{y}=\frac{H}{3} \qquad A=\frac{B\cdot H}{2}$$

$$I_x=\frac{B\cdot H^3}{12} \qquad I_y=\frac{H\cdot B^3}{12} \qquad I_{xy}=\frac{B^2\cdot H^2}{24}$$

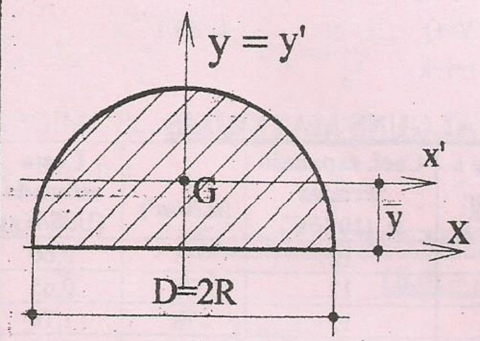
$$I_{x'}=\frac{B\cdot H^3}{36} \qquad I_{y'}=\frac{H\cdot B^3}{36} \qquad I_{x'y'}=-\frac{B^2\cdot H^2}{72}$$



$$A=\pi\cdot R^2=\frac{\pi\cdot D^2}{4} \qquad I_{x'y'}=0$$

$$I_{x'}=I_{y'}=\frac{\pi\cdot R^4}{4}=\frac{\pi\cdot D^4}{64} \qquad W_{x'}=\frac{\pi\cdot D^3}{32}$$

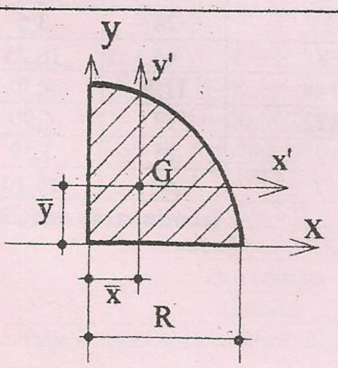
$$J=\frac{\pi\cdot R^4}{2}=\frac{\pi\cdot D^4}{32} \qquad W_T=\frac{\pi\cdot D^3}{16}$$



$$\bar{y}=\frac{4}{3}\cdot\frac{R}{\pi} \qquad A=\frac{\pi\cdot R^2}{2}=\frac{\pi\cdot D^2}{8}$$

$$I_x=I_y=\frac{\pi\cdot R^4}{8}=\frac{\pi\cdot D^4}{128} \qquad I_{xy}=I_{x'y'}=0$$

$$I_{x'}=0{,}1098\cdot R^4 + A d_y^2$$



$$\bar{x}=\bar{y}=\frac{4}{3}\cdot\frac{R}{\pi} \qquad A=\frac{\pi\cdot R^2}{4}=\frac{\pi\cdot D^2}{16}$$

$$I_x=I_y=\frac{\pi\cdot R^4}{16}=\frac{\pi\cdot D^4}{256} \qquad I_{xy}=\frac{R^4}{8}$$

$$I_{x'}=I_{y'}=0{,}05488\cdot R^4 \qquad I_{x'y'}=-0{,}01647\cdot R^4$$

Resistência dos Materiais - Parte B

Tensão Normal (σ - Sigma)

$$\boxed{\sigma = \frac{F}{A}} \quad \frac{N}{m^2} = Pa \quad \Big| \quad \frac{N}{mm^2} = 1MPa$$

Convensão de sinais

$\sigma > 0$ tração

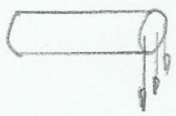
$\sigma < 0$ compressão

$\tau > 0$ Giro horário

$\tau < 0$ Giro anti-horário

Tensão Cisalhante (τ - Tau)

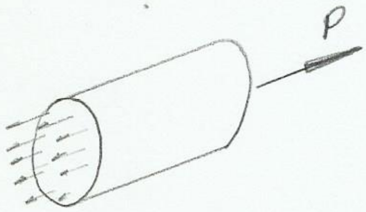
$$\boxed{\tau = \frac{F}{A}}$$



× Força Normal (P, N)

$$\boxed{P = \int_A \sigma_x \, dA}$$

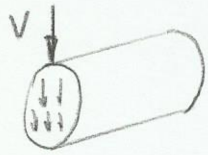
(Tração ou compressão)

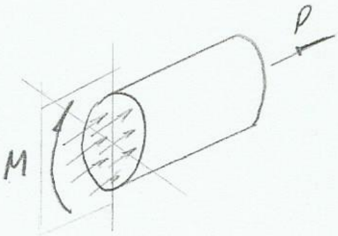


× Força Cortante (V, Q)

$$\boxed{V = \int_A \tau \, dA}$$

(Cisalhamento Puro)



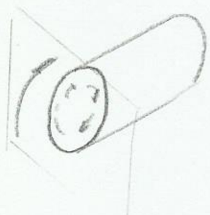


× Momento Fletor (M)

$$\boxed{M = \int \sigma . y \, dA}$$

(Flexão)

× Momento Torçor (T, M_t)



$$\boxed{T = \int \tau . r \, dA}$$

(Torção)

× Tensão admissível

$$\boxed{\text{Tensão}_{\text{Admissível}} = \frac{\text{Tensão}_{\text{Limite}}}{\text{Coeficiente de segurança (C.S.)}}}$$

De acordo com o material

$\boxed{C.S. >>> 1}$ (SEMPRE)

Portanto, temos: $\boxed{\bar{\sigma} = \frac{\sigma_{lim}}{C.S.}}$ $\boxed{\bar{\tau} = \frac{\tau_{lim}}{C.S}}$

Deformação $\left[\varepsilon = \frac{\Delta L}{L} \right]$

Deformação específica longitudinal $\boxed{\varepsilon = \frac{\delta}{L}}$

Deformação específica transversal $\boxed{\varepsilon_t = \frac{\Delta d}{d}}$

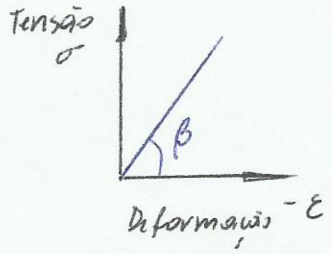
Relação

$$\boxed{\varepsilon_T = -\nu . \varepsilon}$$

(ν - coef. de Poisson)

$0 \leq \nu \leq 0{,}5$)

Lei de Hooke



$$tg(\beta) = \frac{\sigma}{\varepsilon} = cte = E$$

E: Módulo elasticidade longitudinal

$$\boxed{\sigma = E . \varepsilon}$$

$\sigma = \frac{P}{A}$ $\quad \sigma = E . \varepsilon$

$\frac{P}{A} = E . \varepsilon$; $\varepsilon = \frac{\delta}{L}$

$\frac{P}{A} = E . \frac{\delta}{L}$ $\therefore$ $\boxed{\delta = \frac{P . L}{E . A}}$

Tensões no Cisalhamento

Lei de Hooke $\boxed{\tau = G \cdot \gamma}$

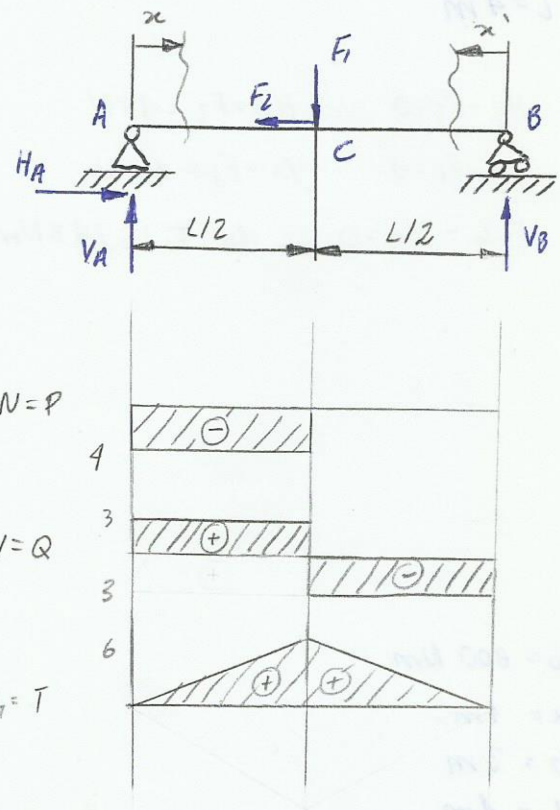
G: Módulo de elasticidade transversal

γ: ângulo de distorção. $\Rightarrow \gamma = \frac{\Delta v}{L}$

$$\boxed{\Delta v = \frac{V \cdot L}{G \cdot A}}$$

Resistência dos Materiais

2.1



$F_1 = 6\ kN$

$F_2 = 4\ kN$

$L = 4m$

$\Sigma Fx = 0 \rightarrow H_A - F_2 = 0 \therefore H_A = F_2$

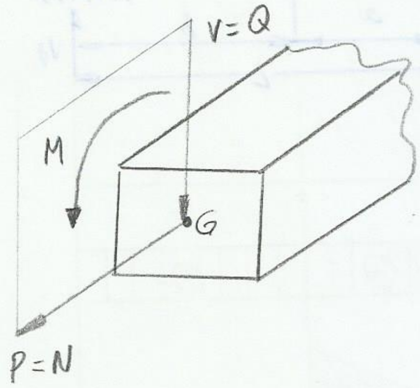
$\Sigma Fy = 0 \rightarrow F_1 - V_A - V_B = 0$

$\Sigma M = 0 \rightarrow \frac{LF_1}{2} - LV_B = 0$

$F_2 = H_A = 4\ kN$

$V_B = \frac{\cancel{L}F_1}{2} \div \cancel{L} = \frac{6}{2} = 3\ kN$

$V_A = F_1 - V_B = 6 - 3 = 3\ kN$



P=N = Esforço Normal - tração ou compressão

- tração > 0
- compressão < 0

V=Q = Esforço Cortante (Cisalhamento)

- horário > 0
- antihorário < 0

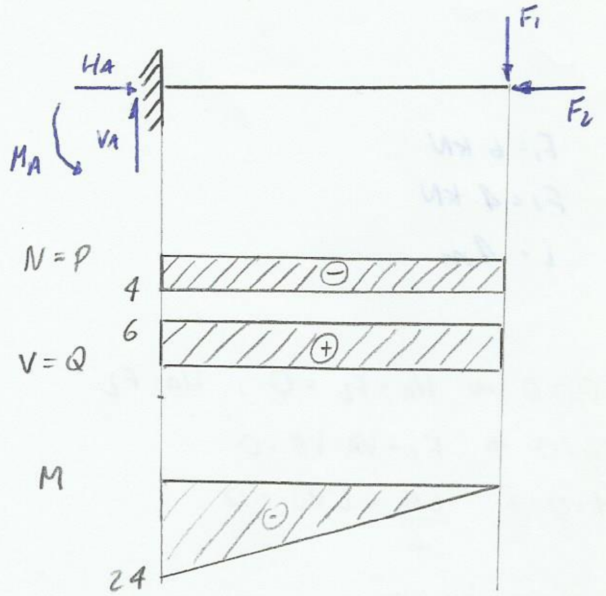
M = Momento Fletor

- Tração embaixo > 0
- tração em cima < 0

$M_T = T$ Momento torçor

- horário > 0
- antihorário < 0

2.2



$F_1 = 6\,KN$

$F_2 = 4\,KN$

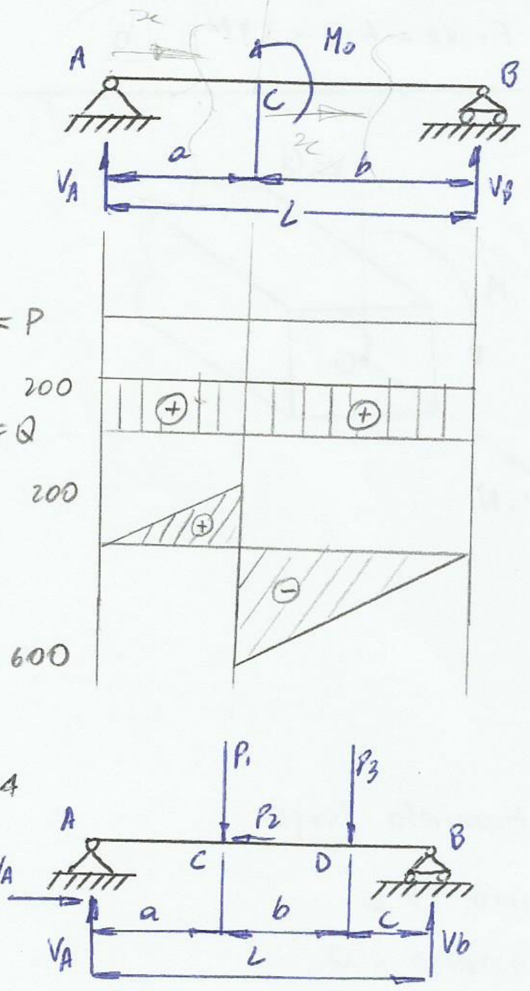
$L = 4\,m$

$H_A - F_2 = 0 \;\therefore\; H_A = F_2 = 4\,KN$

$V_A - F_1 = 0 \;\therefore\; V_A = F_1 = 6\,KN$

$F_1 . L - M_A = 0 \;\therefore\; M_A = F_1 L = 24\,KNm$

2.3



$M_o = 800\,Nm$

$a = 1m$

$b = 3m$

$L = 4m$

$$\begin{cases} V_A + V_B = 0 \\ V_A . a - V_B . b - M_o = 0 \end{cases}$$

$$\begin{cases} V_A + V_B = 0 \\ V_a - 3V_b = 800 \end{cases} \sim \begin{cases} V_A = -V_B \\ 4V_b = -800 \end{cases}$$

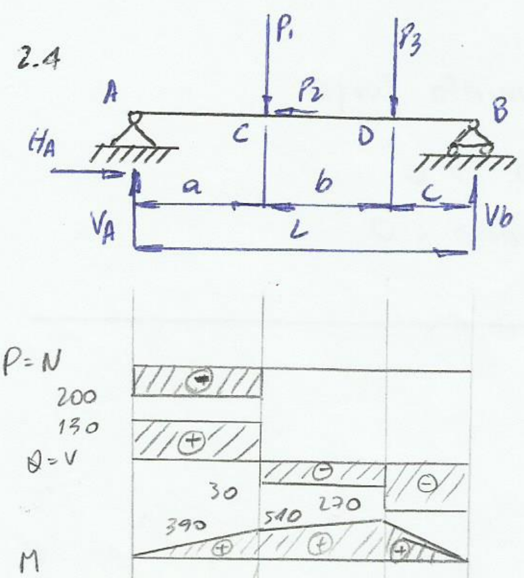
$\therefore\; V_b = -200 \;\; e \;\; V_a = 200$

$M_o = V_a x = 200x \;\therefore\; M(0) = 0 \quad M(1) = 200$

$V_a . (a + x) - M_o = 0$

$200 + 200x - 800 \;/\; 200x - 600$

2.4



$P_1 = 100\,N \quad P_2 = 200\,N \quad P_3 = 300\,N$

$a = 3m \quad b = 5m \quad c = 2m \quad L = 10m$

$H_A - P_2 = 0 \;\therefore\; H_A = P_2 = 200\,N$

$V_A + V_B - P_1 - P_3 = 0 \;\therefore\; V_A + V_B = 400$

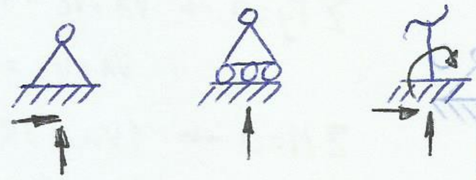
$P_1 . a + P_3 (a + b) - L V_b = 0$

$$\therefore\; V_b = \frac{100 . 3 + 300\,(3+5)}{10} = 270\,N$$

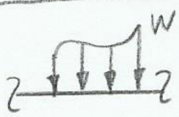
$V_A = 400 - 270 \;\therefore\; V_A = 130\,N$

Relações Fundamentais entre M, V, W e etc.

Reações de Apoio:



Carga distribuída

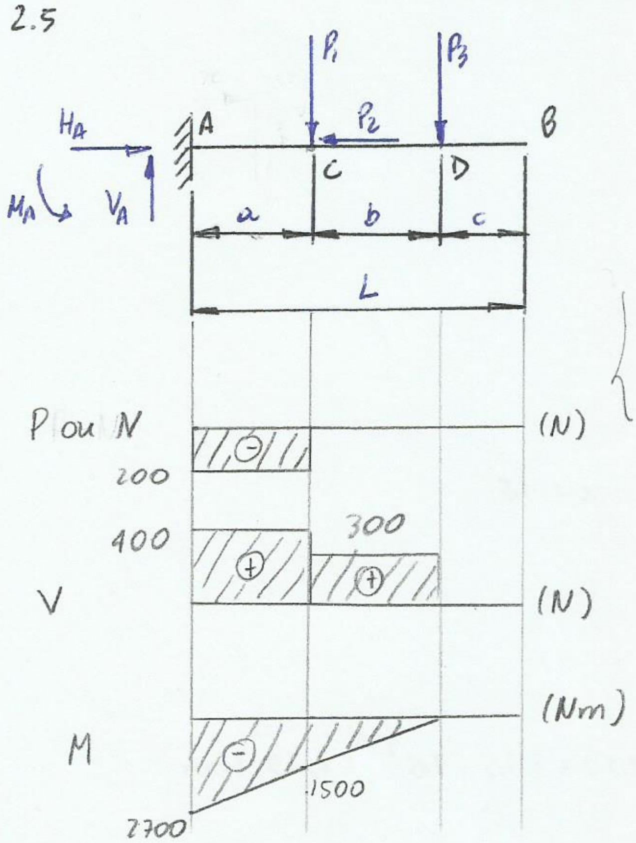


$$\boxed{\frac{dV}{dx} = -w \qquad V = -\int w\,dx \qquad \frac{dM}{dx} = V \qquad M = \int V\,dx}$$

Obs: A resultante das cargas distribuídas é dada por $R = \int w\,dx$

$$\boxed{\text{Portanto:} \quad \frac{d^2M}{dx^2} = \frac{dV}{dx} = -w}$$

2.5



$P_1 = 100\,N$ $\quad a = 3m$

$P_2 = 200\,N$ $\quad b = 5m$

$P_3 = 300\,N$ $\quad c = 2m$

$L = 10m$

$$\begin{cases} H_A - P_2 = 0 \\ V_A - P_1 - P_3 = 0 \\ -M_A + a.P_1 + (a+b)P_3 = 0 \end{cases}$$

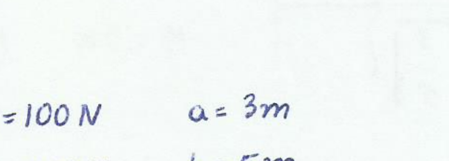
$H_A = 200\,N$

$V_A = P_1 + P_3 = 400\,N$

$M_A = a P_1 + (a+b) P_3$

$M_a = 3.100 + (3+5).300 = 2700\,Nm$

trecho AC:

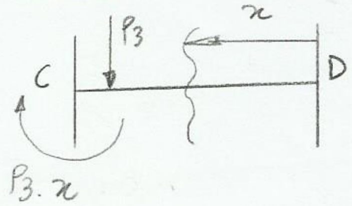


$V_a x$

$V_a x - M_a$

$400x - 2700$

trecho CD:

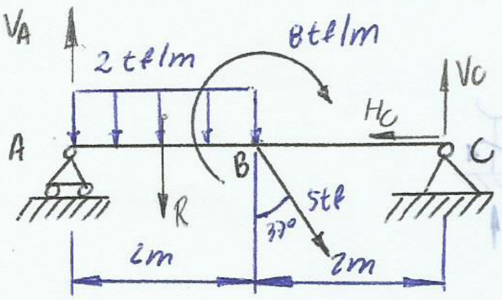


$P_3 . x$

$300x$

trecho DB:

2.6



Cálculo das reações:

$\sum F_x = 0 \rightarrow 5\sin 37 - H_C = 0 \therefore H_C = 3\,tf$

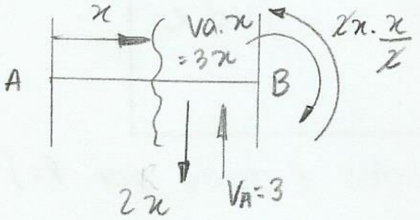
$\sum F_y = 0 \rightarrow V_A + V_C - 5\cos 37 - 2.2 = 0$

$\therefore V_A + V_C = 8 \quad \therefore V_C = 5$

$\sum M = 0 \rightarrow 4V_a - 3R - 4.2 + 8 = 0$

$V_a = \dfrac{3.\cancel{2.2} + \cancel{4.2} - \cancel{8}}{4} \therefore V_a = 3$

Cortes: Trecho AB

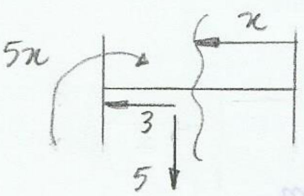


$N = 0$

$V = 3 - 2x \quad V(0) = 3 \quad V(2) = -1$

$M = 3x - x^2 \quad M(0) = 0 \quad M(2) = 2$

trecho BC

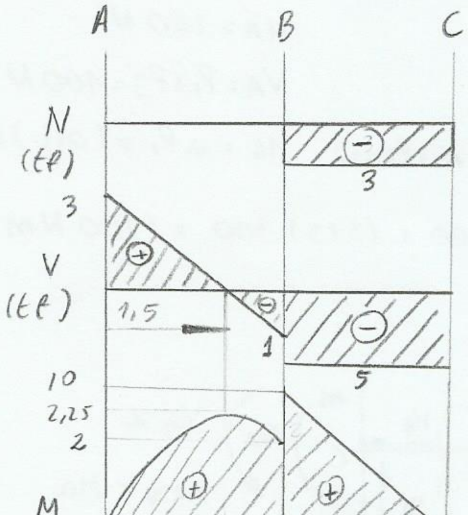


$N = -3$

$V = -5$

$M = 5x \quad M(0) = 0 \quad M(2) = 10$

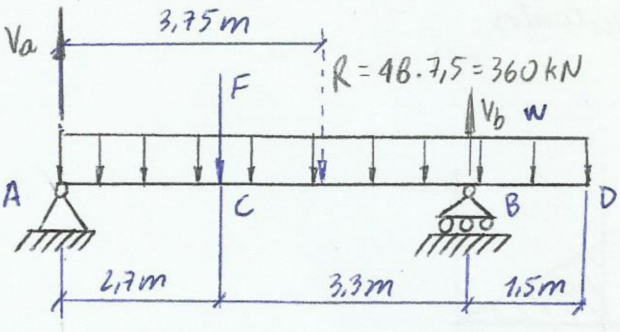
Diagrama de esforços internos solicitante



$3 - 2x = 0 \quad \therefore x = 1{,}5$

$M_{max} = M(1{,}5) = 3.1{,}5 - 1{,}5^2 = 2{,}25\ tfm$

2.7



$F = 100$ kN

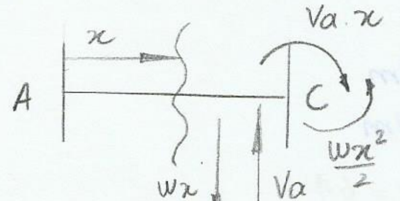
$w = 48$ kN/m

Cálculo das reações:

$Va + Vb - F - R = 0 \qquad Va + Vb = 360 + 100 \;\therefore\; Va + Vb = 460$

$2{,}7F + \frac{7{,}5}{2} . 360 - 6 . Vb = 0 \;\therefore\; Vb = 270$ e $Va = 190$

Cortes: trecho AC



$N = 0$

$V = Va - wx = 190 - 48x$

$V(0) = 190 \qquad V(2{,}7) = 60{,}4$

$M = Va . x - w . x . \frac{x}{2} = Va x - \frac{w . x^2}{2}$

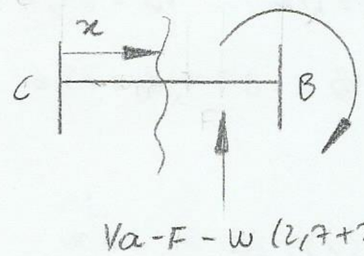
$M(0) = 0 \qquad M(2{,}7) = 338 \;;\; M = 190x - 24x^2$

$\frac{dM}{dx} = V = 190 - 48x = 0$

$\therefore x = 3{,}96 > 2{,}7$

$\therefore M_{max} = 338$

trecho CB



$Va . (2{,}7 + x) - F . x - \frac{w}{2}(2{,}7 + x)^2$

$N = 0$

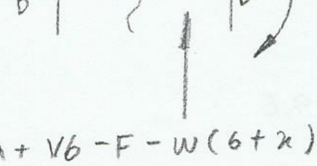
$V = 190 - 100 - 48(2{,}7 + x) \;\therefore\; V = -39{,}6 - 48x \qquad V(0) = -39{,}6 \quad V(3{,}3) = -198$

$M = 190(2{,}7 + x) - 100x - \frac{48}{2}(2{,}7 + x)^2 = 513 + 190x - 100x - 24(7{,}29 + 5{,}4x + x^2)$

$M = -24x^2 - 39{,}6x + 338{,}04$

$M(0) = 338 \qquad M(3{,}3) = -54$

trecho BD



$Va(6 + x) + Vb . x - F . (3{,}3 + x) - \frac{w}{2}(6 + x)^2$

$190(6 + x) + 270 . x - 100(3{,}3 + x) - \frac{48}{2}(6 + x)^2$

$1140 + 190x + 270x - 330 - 100x - 24(36 + 12x + x^2)$

$Va + Vb - F - w(6 + x)$

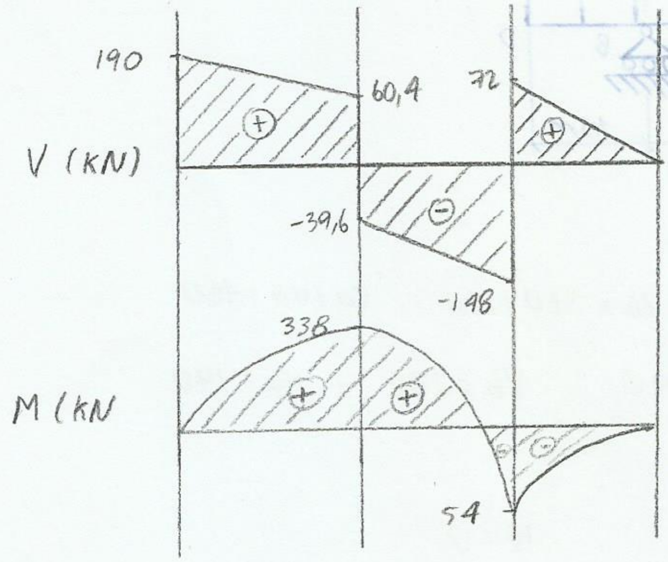
$190 + 270 - 100 - 48(6 + x)$

$N = 0$

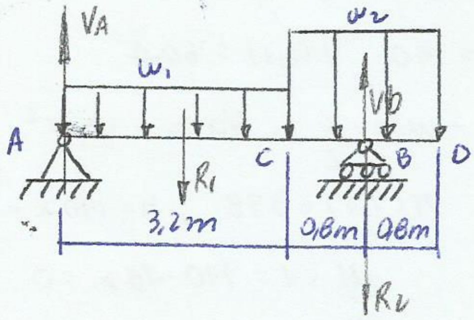
$V = 72 - 48x \qquad V(0) = 72 \quad V(1{,}5) = 0$

$M = -24x^2 + 72x - 54 \qquad M(0) = -54 \quad M(1{,}5) = 0$

Diagrama de esforços internos solicitantes



2.8



$w_1 = 20\ kN/m$

$w_2 = 30\ kN/m$

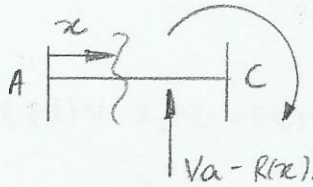
$R_1 = 20 . 3,2 = 64$

$R_2 = 30 . 1,6 = 48$

$\Sigma Fy = 0 \rightarrow Va + Vb - R_1 - R_2 = 0 \therefore Va + Vb = 112 \therefore Va = 38,4$

$R_1 . \frac{3,2}{2} - R_2 . 4 + Vb . 4 = 0 \rightarrow 64 . \frac{3,2}{2} + 48 . 4 - Vb . 4 = 0 \therefore Vb = 73,6$

Cortes   trecho AC

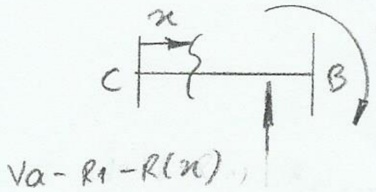


$Va . x - R(x) . \frac{x}{2}$; $R(x) = 20 . \frac{x}{2} = 10x$

$V = 38,4 - 20x \therefore V(0) = 38,4 \quad V(3,2) = -25,6$

$M = 38,4x - 10x^2 \therefore M(0) = 0 \quad M(3,2) = 20,48$

trechos CB:



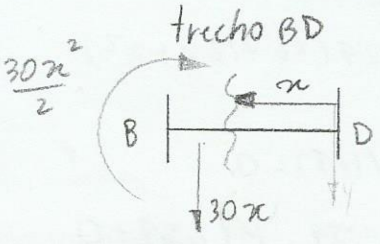
$Va(3,2 + x) - R_1 . (1,6 + x) - R(x) . \frac{x}{2}$; $R(x) = 30x$

$V = 38,4 - 64 - 30x = -25,6 - 30x \quad V(0) = -25,6 \quad V(0,8) = -49,6$

$M = 122,88 + 38,4x - 102,4 - 64x - 15x^2$

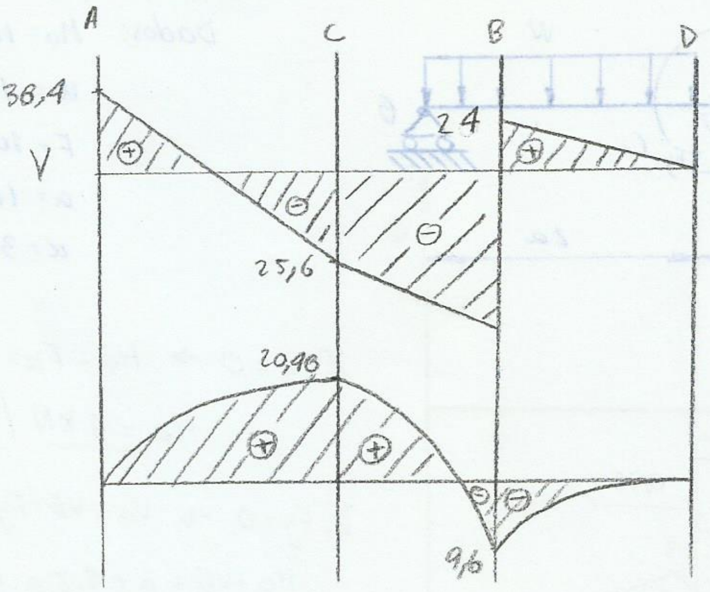
$M = -15x^2 - 25,6x + 20,48 \therefore M(0) = 20,48 \quad . M(0,8) = -9,6$

trecho BD

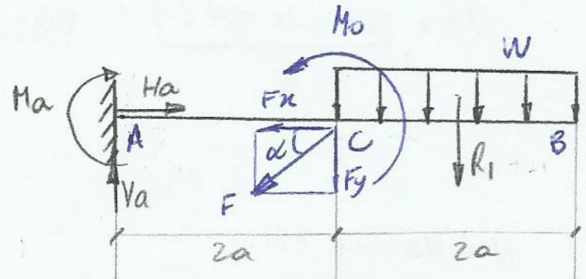


$V = 30x \quad V(0) = 0 \quad V(0,8) = 24$

$M = -15x^2 \quad M(0) = 0 \quad M(0,8) = -9,6$



2.9



Dados: $M_o = 16\,kN.m$

$w = 4\,kN/m$ $\qquad R_1 = 4.2 = 8$

$F = 10\,kN \rightarrow F_x = 8 \quad F_y = 6$

$a = 1\,m$

$\alpha = 37°$

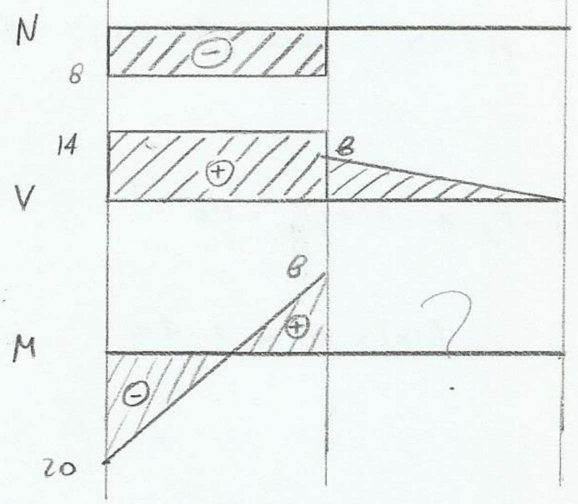
$\Sigma F_x = 0 \rightarrow H_a = F \cos\alpha = 10.\cos 37 \therefore \underline{H_a = 8\,kN}$

$\Sigma F_y = 0 \rightarrow V_a - F_y - R_1 = 0 \therefore V_a = F_y + R_1$

$V_a = 10 \sin 37 + 4.2 = 14\,kN \therefore \underline{V_a = 14\,kN}$

$\Sigma M_A = 0 \rightarrow M_a - M_o + F_y.2a + R_1.3a = 0$

$M_a = 16 - 10 \sin 37.2.1 - 8.3 = 0 \therefore M_a = -20\,kNm$



$N = 0$

$V = -R(x) = -4x \rightarrow V(0) = 0 \quad V(2) = -8$

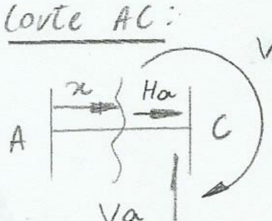
$M = R(x).\frac{x}{2} = \frac{4x^2}{2} = 2x^2$

$M(0) = 0 \quad M(2) = 8$

$\underline{R(x) = 4.x}$

Corte AC:

$V_a.x + M_a$



$N = H_a = -8\,kN$

$V = 14\,kN$

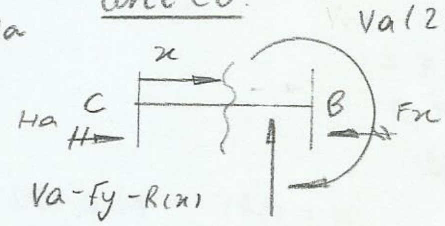
$M = 14x - 20$

$\therefore M(0) = -20\,kNm$

$M(2) = 8\,kNm$

Corte CB:

$V_a(2+x) - F_y.x - R(x).\frac{x}{2} + (-20)$



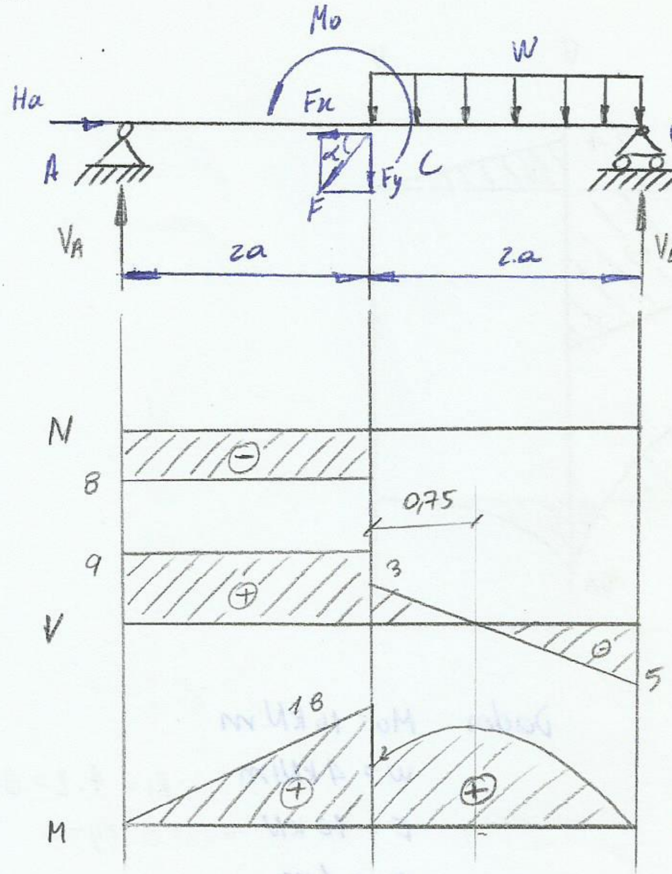
$N = 0 = H_a - F_x = 0$

$V = 14 - 6 - 4x = 8 - 4x \therefore V(0) = 8 \quad e \quad V(2) = 0$

$M = 14(2+x) - 6x - \frac{4x^2}{2} - 20 = 28 + 14x - 6x - 2x^2 - 20$

$M = -2x^2 + 8x + 8 \therefore M(0) = 8 \quad M(2) = 16$

2.10



Dados: $M_0 = 16$ kN.m

$w = 4$ kN/m

$F = 10$ kN $<$ $F_x = 8$, $F_y = 6$

$a = 1$ m

$\alpha = 37°$

$\Sigma F_x = 0 \rightarrow Ha = F_x = F\cos 37$

$\therefore \underline{Ha = 8 \text{ kN}}$

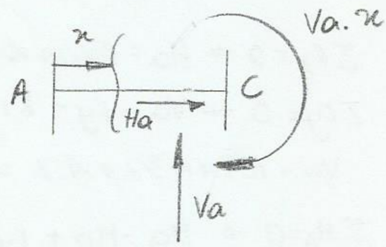
$\Sigma F_y = 0 \rightarrow Va + Vb - F_y - W.2a = 0$

$Va + Vb = 6 + 4.2 = 14 \quad \therefore \underline{Va = 9 \text{ KN}}$

$\Sigma M_A = 0 \rightarrow F_y.2 - 4Vb - M_0 + 4.2.3 = 0$

$Vb = \dfrac{6.2 - 16 + 4.2.3}{4} \quad \therefore \underline{Vb = 5 \text{ KN}}$

Trecho AC

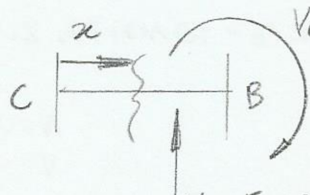


$N = Ha = -8$ KN

$V = Va = 9$ KN

$M = 9x \quad \therefore M(0) = 0 \quad M(2) = 18$

trecho BC



$Va.(2+x) - F_y.x - R(x)\dfrac{x}{2} - M_0$

$R(x) = W.x = 4x$

$N = 0$

$V = 9 - 6 - 4x = 3 - 4x$

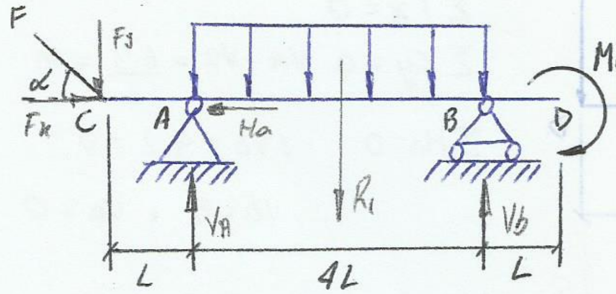
$M = 9(2+x) - 6.x - \dfrac{4x.x}{2} - M_0 = 18 + 9x - 6x - 2x^2 - 16 = -2x^2 + 3x + 2$

$x = 0 \rightarrow V = 3 \text{ e } M = 2$

$x = 2 \rightarrow V = -5 \text{ e } M = 0$

$\dfrac{dM}{dx} = V = 3 - 4x = 0 \quad \therefore x = 0{,}75 \therefore M(0{,}75) = 3{,}125$

2.11



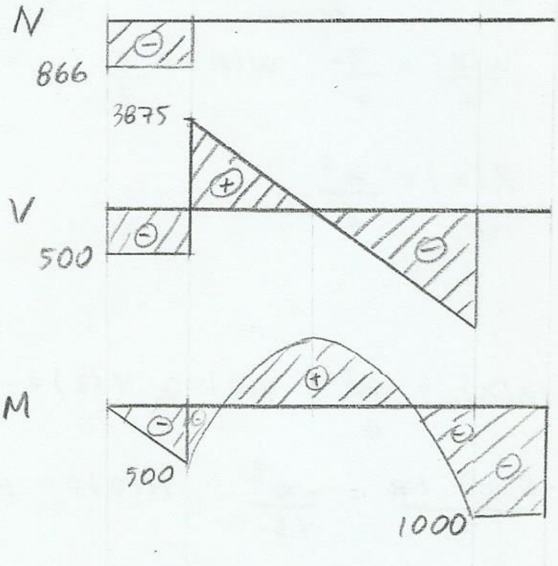
Dados:

$F = 1000$ kN.m

$M_0 = 1000$ kN.m

$w = 2000$ kN/m

$\alpha = 30°$ $L = 1m$



$F_x = 1000 \cos 30 = 866$ kN

$F_y = 1000 \sin 30 = 500$ kN

$R_1 = 2000 \cdot 4 = 8000$ kN

$R(x) = 2000x$

$\sum F_x = 0 \rightarrow F_x - H_a = 0 \therefore H_a = 866$ kN

$\sum F_y = 0 \rightarrow V_a + V_b = 500 + 8000 = 8500$

$\sum M_C = 0 \rightarrow -V_a \cdot L + R_1 \cdot 3L - V_b \cdot 5L + M_0 = 0$

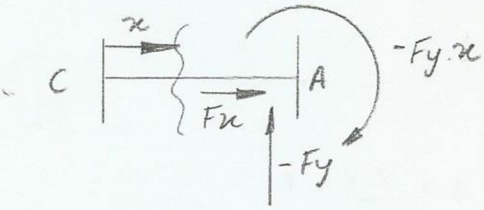
$-V_a + 24000 - 5V_b + 1000 = 0$

$$\begin{cases} -V_a - 5V_b = -25000 \\ V_a + V_b = 8500 \end{cases}$$

$-4V_b = -16500 \therefore V_b = 4125$ kN

$\therefore V_a = 4375$ kN

Trecho CA:

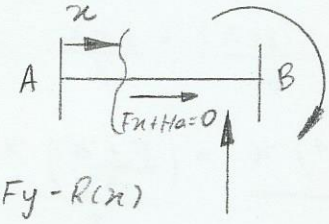


$N = F_x = -866$ kN

$V = -F_y = -500$ kN

$M = -500x \therefore M(0) = 0 \quad M(1) = -500$

Trecho AB:



$V_a \cdot x - F_y(1+x) - R(x) \cdot \frac{x}{2}$

$V_a - F_y - R(x)$

$N = 0$

$V = 4375 - 500 - 2000 \cdot x$

$V = 3875 - 2000x$

$V(0) = 3875$ e $V(4) = -4125$

$M = 4375x - 500(1+x) - 1000x^2$

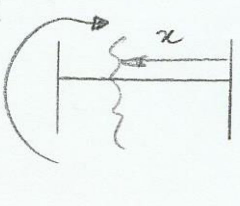
$M = -1000x^2 + 3875x - 500$

$M(0) = -500 \quad M(4) = -1000$

$\frac{dM}{dx} = V = 3875 - 2000x = 0$

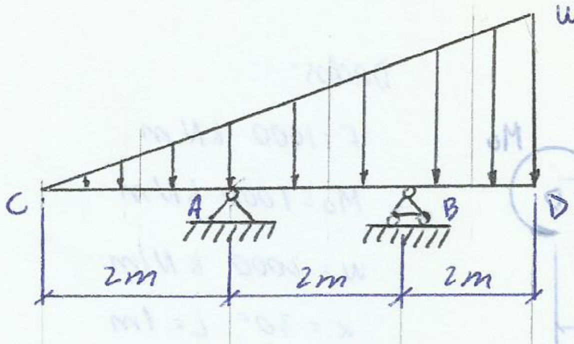
$\therefore x = 1{,}94$

Trecho BD:



$M = -1000$ kN.m

2.12



$R = \frac{3.6}{2} = 9$

$\Sigma F x = 0$

$\Sigma F y = 0 \quad V_A + V_B = \frac{6.3}{2} = 9$

$\Sigma M_A = 0 \quad -2 V_b + 9.2 = 0 \therefore$

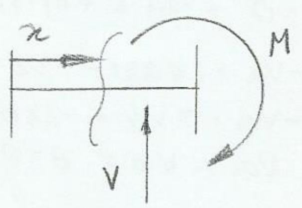
$\therefore V_b = 9$ e $V_a = 0$

$R(x) = \frac{w(x).x}{2}$

$\frac{w(x)}{x} = \frac{3}{6} \therefore w(x) = \frac{x}{2}$

$R(x) = \frac{x^2}{4}$

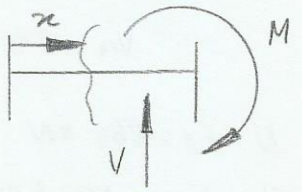
Trecho CA:

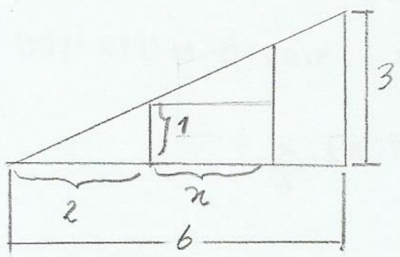


$V = -R(x). = \frac{-x^2}{4} \therefore V(0) = 0 \quad V(2) = -1$

$M = -\frac{R(x).1x}{3} = \frac{-x^3}{12} \therefore M(0) = \quad M(2) = -0{,}67$

Trecho AB:





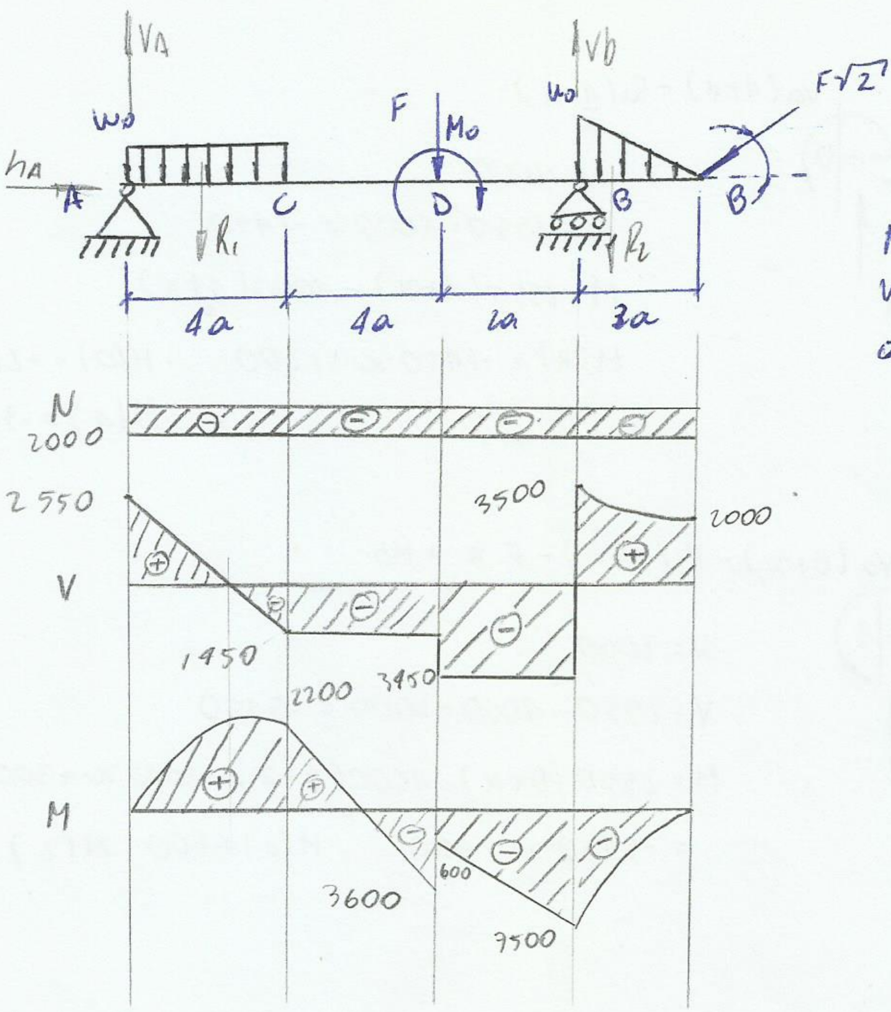
$\frac{6}{3} = \frac{2}{s} \therefore s = \frac{2.3}{6} = 1$

$\frac{3}{6} = \frac{B}{2+x} \therefore B = \frac{3(2+x)}{6} = \frac{2+x}{3} \quad b = 1$

$A = R(x) = \frac{\left(\frac{2+x}{3} + 1\right).x}{2} = \frac{\left(\frac{5+x}{3}\right).x}{2}$

$R(x) = \frac{5x + x^2}{6}$

2.15



$M_0 = 3000$ N.m

$W_0 = 1000$ N/m

$a = 1$ m

Calculo dos reações

$F_x = 2000 \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{2} = 2000 \quad \therefore \quad F_y = 2000\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 2000$

$R_1 = 4 \cdot 1000 = 4000$ N $\quad R_2 = \frac{3 \cdot 1000}{2} = 1500$

$\Sigma F_x = 0 \quad h_a = 2000$

$\Sigma F_y = 0 \quad V_a + V_b = 4000 + 2000 + 1500 + 2000 = 9500$

$\Sigma M_A = 0 \quad 4000 \cdot 2 + 2000 \cdot 8 + 1500 \cdot 11 + 2000 \cdot 13 + 3000 = 10 V_b$

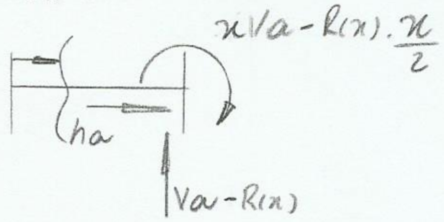
$\therefore V_b = 6950$ e $V_a = 2550$

Cortes

trecho AC

$R(x) = W(x) \cdot x$

$R(x) = 1000x$



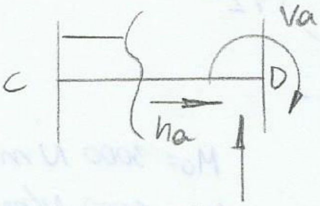
$N = -2000$

$V = 2550 - 1000x \quad \therefore V(0) = 2550 \quad V(4) = -1450$

$M = 2550x - 500x^2 \quad \therefore M(0) = 0 \quad M(4) = 2200$

trecho CD

$Va(4+x) - R_1\left(\frac{4+x}{2}\right)$



$Va - R_1$

$N = -2000$

$V = 2550 - 4000 = -1450$

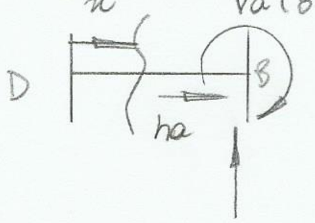
$M = 2550(4+x) - 4000\left(\frac{4+x}{2}\right)$

$M(x) = -1450x + 2200 \quad \therefore M(0) = +2200$

$M(4) = -3600$

Trecho DB

$Va(8+x) - R_1(6+x) - F.x + Mo$



$Va - R_1 - F$

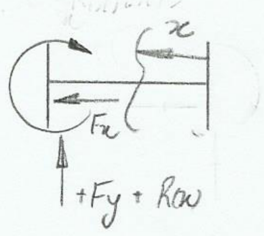
$N = 2000$

$V = 2550 - 4000 - 2000 = -3450$

$M = 2550(8+x) - 4000(6+x) - 2000x + 3000$

$= -3450x - 600 \quad \therefore M(0) = -600 \quad M(2) = -7500$

Trecho BE



$N = -2000$

$V = 2000 + \frac{500x^2}{3}$

$V(0) = 2000$

$V(3) = 3500$

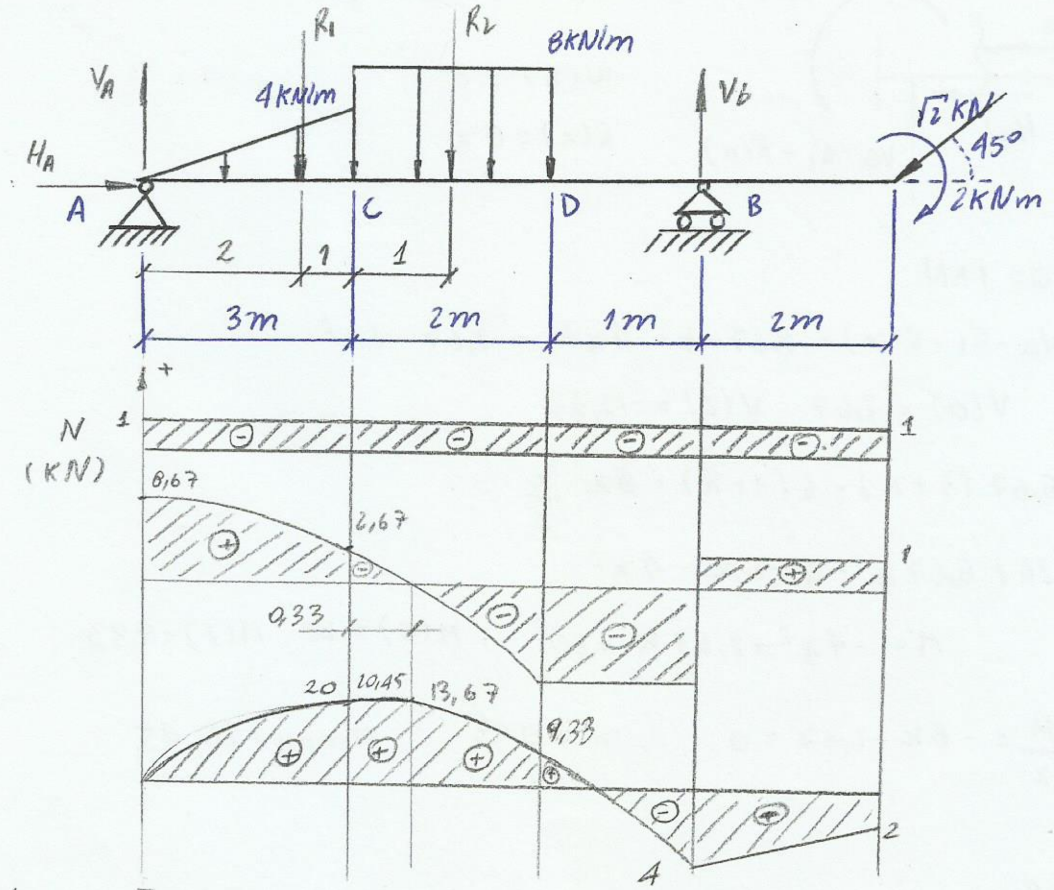
$\frac{w(x)}{x} = \frac{1000}{3} \quad \therefore w(x) = \frac{1000}{3}x$

$R(x) = \frac{w(x).x}{2} = \frac{1000x^2}{6}$

$M = -2000x - \frac{1000x^2}{6} \cdot \frac{x}{3} = -2000x - \frac{500}{9}x^3$

$M(0) = 0 \quad M(3) = -7500$

Exercício extra:



Cálculo das reações

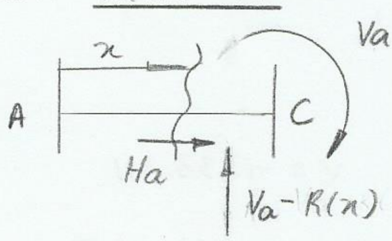
$R_1 = \frac{3.4}{2} = 6$ kN $\quad R_2 = 2.8 = 16$ kN

$\Sigma F_x = 0 \rightarrow Ha - \frac{\sqrt{2}.\sqrt{2}}{2} = 0 \quad \therefore Ha = 1$ kN

$\Sigma F_y = 0 \rightarrow Va + Vb - R_1 - R_2 - \frac{\sqrt{2}\sqrt{2}}{2} = 0 \quad \therefore Va + Vb = 6 + 16 + 1 = 23$ kN

$\Sigma M = 0 \rightarrow 2.6 + 4.16 - 6Vb + 1.8 + 2 = 0 \quad \therefore Vb = 14,33$ kN e $Va = 8,67$ kN

Cortes: trecho AC:



$Va.x - R(x).\frac{x}{3}$

$N = Ha = 1$ kN (−)

$V = Va - \frac{2x^2}{3} = 8,67 - \frac{2x^2}{3}$

$V(0) = 8,67 \quad V(3) = 2,67$

$\frac{dV}{dx} = \frac{-4x}{3} = 0 \quad \therefore x = 0 \quad V_{max} = 8,67$

$\frac{w(x)}{x} = \frac{4}{3} \quad \therefore w(x) = \frac{4x}{3}$

$R(x) = \frac{w(x).x}{2} = \frac{\frac{4x}{3}.x}{2}$

$R(x) = \frac{2x^2}{3}$

$M = Va.x - \frac{2}{3}.x^2.\frac{x}{3} = 8,67x - \frac{2}{9}x^3$

$M(0) = 0 \quad M(3) = 20$

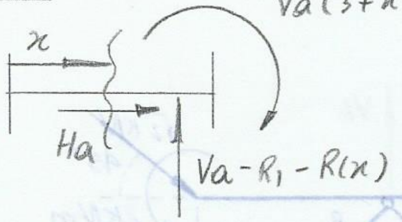
$\frac{dM}{dx} = 8,67 - \frac{2 \times 3}{9}.x^2 = 0 \quad \therefore x = \pm 3,61$

$\frac{d^2M}{dx^2} = \frac{-12}{9}x \quad M''(3,61) = -4,81$ (Max) 20,84

$M''(-3,61) = 4,81$ (Min) −20,84

trecho CD

$$V_a(3+x) - R_1 \cdot (3+x) - R(x) \cdot \frac{x}{2}$$



$w(x) = 8$

$R(x) = 8x$

$N = H_a = 1\,kN$

$V = V_a - R_1 - R(x) = 8{,}67 - 6 - 4x^2 = 2{,}67 - 4x^2$

$V(0) = 2{,}67 \quad V(2) = -13{,}33$

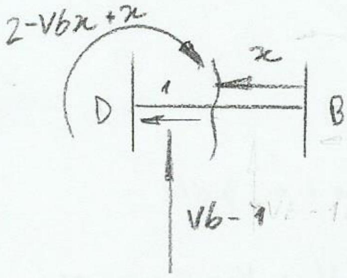
$M = 8{,}67\,(3+x) - 6\,(1+x) - 8x \cdot \frac{x}{2}$

$= 26 + 8{,}67x - 6 - 6x - 4x^2$

$\therefore M = -4x^2 + 2{,}67x + 20 \quad \therefore M(0) = 20 \quad M(2) = 9{,}33$

$\frac{dM}{dx} = -8x + 2{,}67 = 0 \quad \therefore x = 0{,}33 \quad \therefore M_{max} = 20{,}45$

trecho DB



~~$N = 1\,kN\ (-)$~~

~~$V = V_b - 1 = 14{,}33 - 1 \quad \therefore V = 13{,}33\,kN$~~

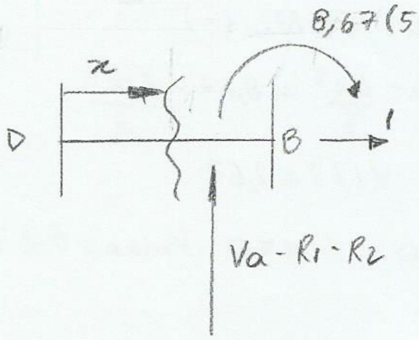
~~$M = 2 - V_b x + x = 2 - 14{,}33x + x = -13{,}33x + 2$~~

~~$M(0) = 2 \quad M(1) = -11{,}33$~~

?

Outra Maneira

$$8{,}67(5+x) - 6(3+x) - 16(1+x)$$



$N = 1\ (-)$

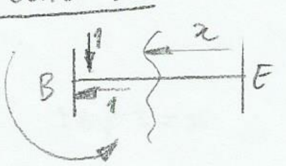
$V = 8{,}67 - 6 - 16 \quad \therefore V = -13{,}33\,kN$

$M = 43{,}33 + 8{,}67x - 18 - 6x - 16 - 16x$

$M = -13{,}33x + 9{,}33$

$M(0) = 9{,}33 \quad M(1) = -4$

trecho BE



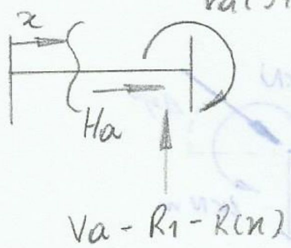
$N = 1\ (-)$

$V = 1\ (+)$

$M = x + 2 \quad M(0) = 2 \quad M(2) = 4\ (-)$

Trecho CD

$Va(3+x) - R_1(1+x) - R(x)\cdot\frac{x}{2}$



$R(x) = w_x \cdot x$

$R(x) = 8 \cdot x$

$N = Ha = -1$

$V = 8{,}67 - 6 - 8x = 2{,}67 - 8x \qquad V(0) = 2{,}67 \quad V(2) = -13{,}33$

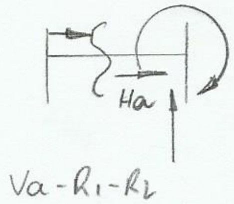
$M = 8{,}7(3+x) - 6(1+x) - \frac{8x^2}{2} = -4x^2 + 2{,}7x + 20{,}1$

$M(0) = 20{,}1 \qquad M(2) = 9{,}5$

$\frac{dM}{dx} = -8x + 2{,}7 = 0 \;\therefore\; x = 0{,}34 \;\therefore\; M(0{,}34) = 20{,}56$

Trecho DB

$Va(5+x) - R_1(3+x) - R_2(1+x)$



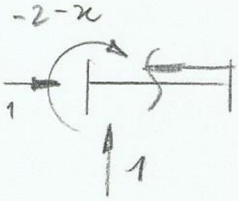
$N = -1$

$V = 8{,}67 - 6 - 16 = -13{,}33$

$M = 8{,}7(5+x) - 6\cdot(3+x) - 16(1+x)$

$= -13{,}3x + 9{,}5 \;\therefore\; M(0) = 9{,}5 \quad M(1) = -3{,}8$

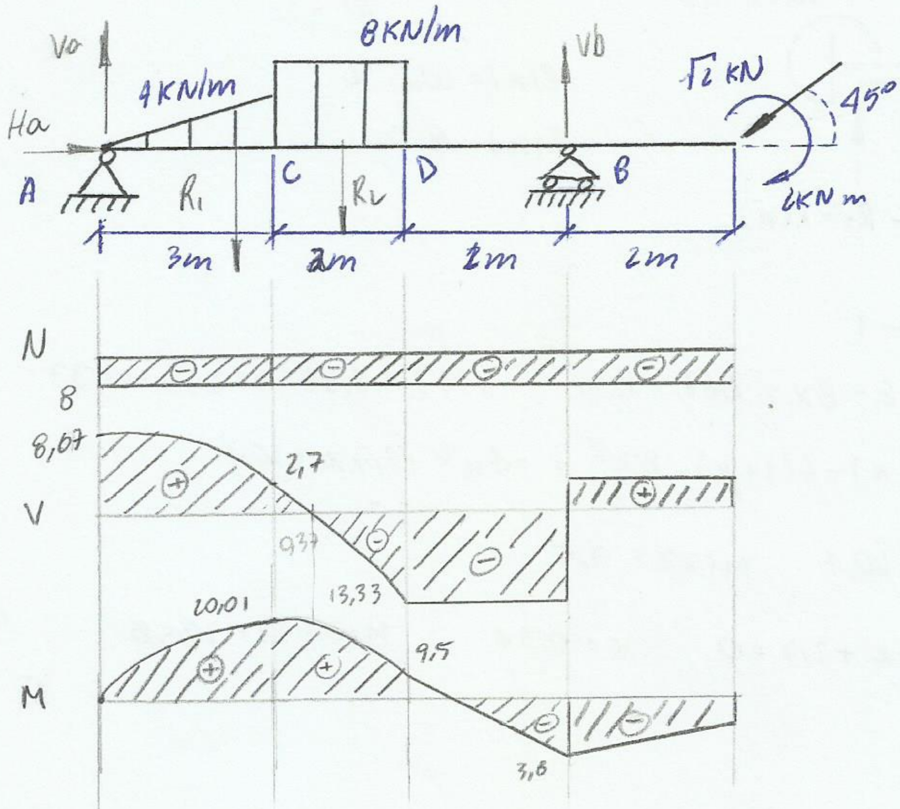
Trecho BE



$N = -1$

$V = 1$

$M = -2 - x \qquad M(0) = -2 \qquad M(2) = -4$



Cálculo das Reações

$\sqrt{2}\,sen\,45 = \sqrt{2}\cos 45 = 1\,KN$

$R_1 = \frac{4.3}{2} = 6\,KN \quad R_2 = 8.2 = 16\,KN$

$\Sigma F_x = 0 \therefore Ha = 8KN$

$\Sigma F_y = 0 \therefore Va + Vb = R_1 + R_2 + 1 \therefore Va + Vb = 23KN$

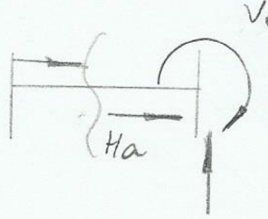
$\Sigma M_A = 0 \therefore 2R_1 + 4R_2 + 8.1 - 6Vb + 2 = 0 \therefore Vb = 14,33 \therefore Va = 8,67$

Análise dos cortes:

Trecho AC

$\frac{w(x)}{x} = \frac{4}{3} \therefore w(x) = \frac{4x}{3}$

$Vax - R(x).\frac{x}{3}$



$Va - R(x)$

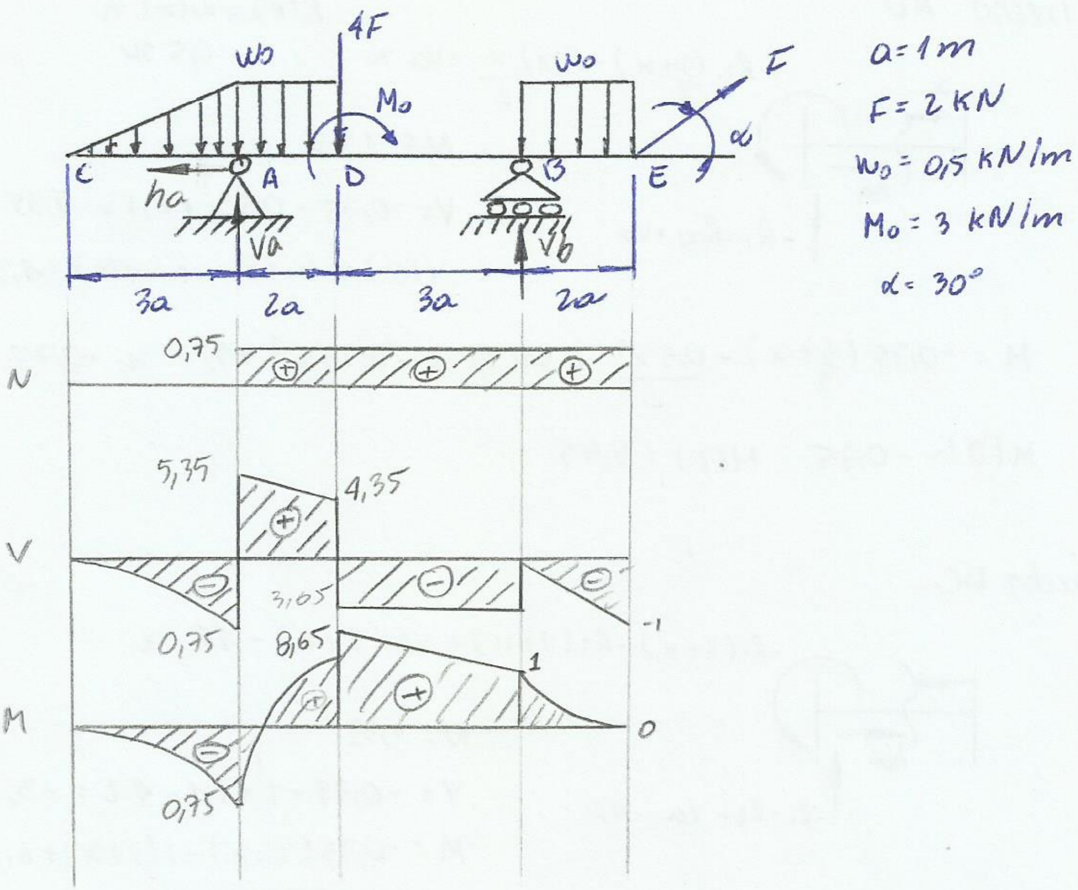
$R(x) = \frac{w(x).x}{2} = \frac{\frac{4x}{3}.x}{2} = \frac{2x^2}{3}$

$N = -8$

$V,Q = 8,7 - \frac{2x^2}{3} \therefore V(0) = 8,7 \quad V(3) = 2,7$

$M = 8,67x - \frac{2x^2}{3}.\frac{x}{3} = 8,67x - \frac{2x^3}{9}$

$M(0) = 0 \quad M(3) = 20,01$



Reações

$\Sigma Fx = 0 \quad ha = 2 \cos 30 = 1,732$

$\Sigma Fy = 0 \quad Va + Vb = 4.2 + \frac{3.0,5}{2} + 2.0,5 + 2.0,5 - 2 \sin 30$

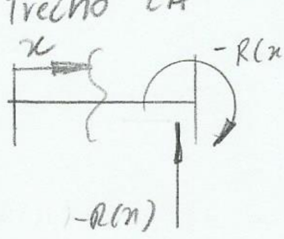
$Va + Vb = 9,75$

$\Sigma M_A = 0 \quad + \frac{3.0,5}{2}.1 - 0,5.2.1 - 4.2.2 - 0,5.2.6 + 5Vb + 7.2 \sin 30 - 3 = 0$

$\therefore Vb = \frac{1 + 16 + 6 - 7 - 0,75 + 3}{5} = 3,65 \text{ kN} \qquad Vb = 3,65 \quad Va = 6,1$

Diagramas

Trecho CA



$N = 0$

$V = -R(x) = \frac{-x^2}{12}$

$V(0) = 0 \quad V(3) = -0,75$

$M(x) = \frac{-x^3}{12} \div 3 = \frac{-x^3}{36}$

$M(0) = 0 \quad M(3) = -0,75$

$\frac{w(x)}{x} = \frac{0,5}{3} \therefore w(x) = \frac{1}{6}x$

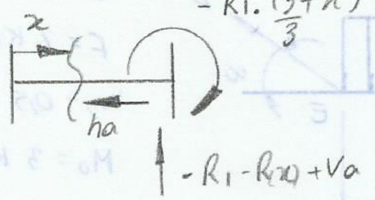
$R(x) = \frac{w(x).x}{2}$

$R(x) = \frac{x^2}{12}$

Trecho AD

$$R(x) = w(x).x = 0{,}5x$$

$$-R_1.\frac{(3+x)}{3} - R(x).\frac{x}{2} + V_a.x$$



$N = 1{,}73$

$V = -0{,}75 - 0{,}5x + 6{,}1 = 5{,}35 - 0{,}5x$

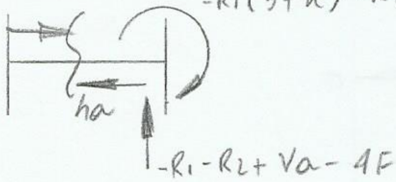
$V(0) = 5{,}35$ e $V(3) = 4{,}35$

$$M = -0{,}75\left(\frac{3}{3}+x\right) - \frac{0{,}5x^2}{2} + 6{,}1x = -0{,}25x^2 + 5{,}35x - 0{,}75$$

$M(0) = -0{,}75 \quad M(2) = 8{,}95$

Trecho DC

$$-R_1(3+x) - R_2(1+x) + V_a(2+x) - 4F.x$$



$N = 1{,}73$

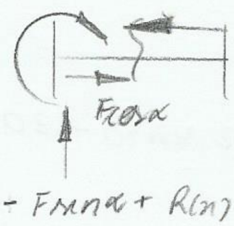
$V = -0{,}75 - 1 + 6{,}1 - 4.2 = -3{,}65$

$M = -0{,}75(3+x) - 1(1+x) + 6{,}1(2+x) - 8x + 3$

$M(x) = -3{,}65x + 11{,}95 \quad \therefore M(0) = 11{,}95 \quad M(3) = 1$

Trecho BE

$$R(x) = 0{,}5x$$



$N = 1{,}73$

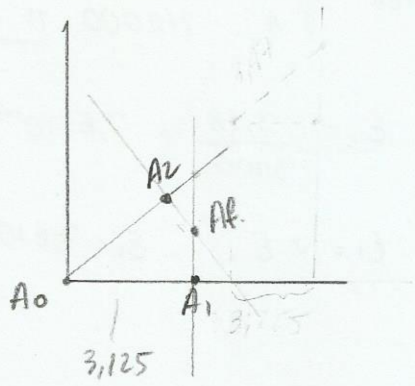
$V = -1 + 0{,}5x \quad \therefore V(0) = -1 \quad V(2) = 0$

$M = +1.x + 0{,}5x.\frac{x}{2} = -0{,}25x^2 + x$

$M(0) = 0 \quad M(2) = 1$

$\delta_{AB} = \dfrac{75000 \cdot 3000}{180000 \cdot 20^2} = 3{,}125 \text{ mm}$

$\delta_{AC} = \dfrac{125000 \cdot 5000}{180000 \cdot 20 \cdot 50} = 3{,}47 \text{ mm}$



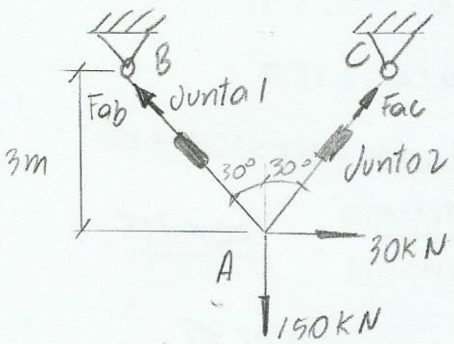
$\varepsilon = \dfrac{\delta_{AB}}{L} = \dfrac{3{,}125}{3000} = 1{,}04 \cdot 10^{-3}$

$\varepsilon_T = \nu \cdot \varepsilon = 1{,}04 \cdot 10^{-3} \cdot 0{,}35 = 3{,}65 \cdot 10^{-4}$

$\varepsilon_T = \dfrac{\Delta a}{a} \quad \therefore \quad \Delta a = 3{,}65 \cdot 10^{-4} \cdot 20 = 7{,}29 \cdot 10^{-3} \text{ mm}$

Exercícios - Cisalhamento

1-)



Dados:

$\bar{\sigma}_T = 140 \text{ MPa}$ $\quad E = 150 \text{ GPa}$

$\bar{\sigma}_{ESM} = 200 \text{ MPa}$ $\quad A_{AB} = A_{BC} = 1000 \text{ mm}^2$

$\bar{\tau} = 100 \text{ MPa}$

Reações de apoio

$\Sigma F_x = 0 \quad 30 + F_{ac} \operatorname{sen} 30 - F_{ab} \operatorname{sen} 30 = 0$

$\Sigma F_y = 0 \quad F_{ab} \cos 30 + F_{ac} \cos 30 - 150 = 0$

$\begin{cases} F_{ac} - F_{ab} = -60 \\ 0{,}866 F_{ac} + 0{,}866 F_{ab} = 150 \end{cases} \sim \begin{cases} 0{,}866 F_{ac} - 0{,}866 F_{ab} = -51{,}96 \\ 0{,}866 F_{ac} + 0{,}866 F_{ab} = 150 \end{cases}$

$1{,}732 F_{ac} = 98{,}04 \quad \therefore F_{ac} = 56{,}605 \text{ kN} \quad F_{ab} = 116{,}605 \text{ kN}$

$\sigma \leq \bar{\sigma}$

$\dfrac{116605}{40b} \leq 140 \quad \therefore \boxed{b \geq 20{,}82 \text{ mm}}$

$\tau < \bar{\tau}$

$\dfrac{116605}{2 \cdot 40a} \leq 100 \quad \therefore \boxed{a \geq 14{,}58 \text{ mm}}$

$$\delta_{ED} = \frac{P.L}{E.A} = \frac{33333.3000}{210000 . \frac{\pi . 16,29^2}{4}} = 2,28 \text{ mm}$$

$$\varepsilon_T = \frac{\Delta d}{L}$$

$$\varepsilon = \frac{2,28}{3000} = 7,6 . 10^{-4}$$

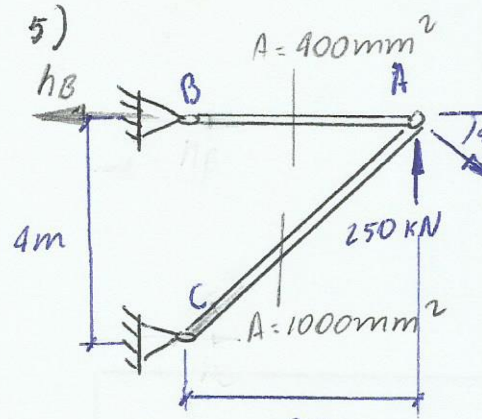
$$\therefore \Delta d = 1,9 . 10^{-4} . 11,52$$

$$\varepsilon_T = \nu . \varepsilon \quad \therefore \quad \varepsilon_T = 7,6 . 10^{-4} . 0,25 = 1,9 . 10^{-4}$$

$$\boxed{\therefore \Delta d = 2,2 . 10^{-3} \text{ mm}}$$

(diminui)

5)



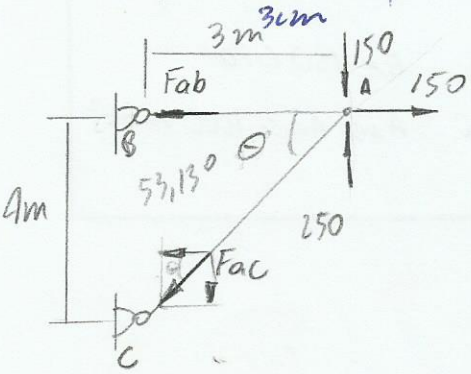
Dados

$\sigma_{LR-T} = 250 \text{ MPa}$

$\sigma_{LR-C} = 200 \text{ MPa}$

$E = 180 \text{ GPa}$

$\nu_{aço} = 0,35$



$\Sigma F_x = 0 \quad F_{ab} + F_{ac} . \cos 53,13 = 150$

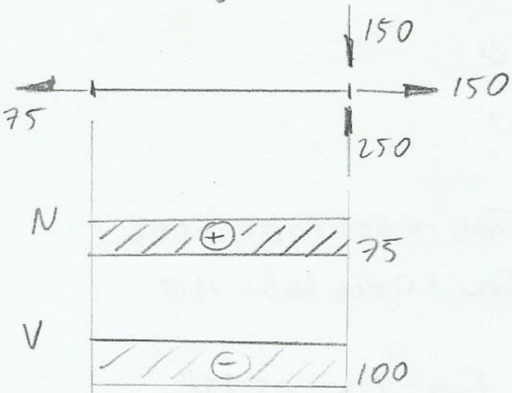
$F_{ab} + 0,6 F_{ac} = 150$

$\Sigma F_y = 0 \quad 250 - 150 - F_{ac} . \sin 53,13 = 0$

$F_{ac} = \frac{250 - 150}{\sin 53,13} \therefore F_{ac} = 125$

$F_{ab} = 150 - 0,6 . 125 \therefore F_{ab} = 75$

Diagramas



a)

$$\sigma_{AB} = \frac{75000}{20^2} = 187,5 \frac{N}{mm^2}$$

$$\sigma_{AC} = \frac{125000}{20.50} = 125 \frac{N}{mm^2}$$

b) $\bar{\sigma} = \frac{\sigma_{lim}}{CS} \qquad \sigma_{AB} \leq \bar{\sigma}$

Roteiro

- Reações de Apoio
- Diagramas
- ... O resto..

$187,5 \leq \frac{250}{CS} \therefore CS = 1,33$

$\sigma_{AC} \leq \bar{\sigma}$

$125 \leq \frac{250}{CS} \therefore CS = 2$

Exercício

$\Sigma Fx = 0 \quad Fac\,sen30 - Fab\,sen30 + 30 = 0$

$Fac - Fab = -60$

$\Sigma Fy = 0 \quad Fac\cos30 + Fab\cos30 = 150$

$Fac + Fab = 173{,}205$

$\begin{cases} Fac - Fab = -60 \\ Fac + Fab = 173{,}205 \end{cases}$

$2Fac = 113{,}205$

$\therefore Fac = 56{,}60 \text{ e } Fab = 116{,}6$

$\dfrac{116600}{40b} \leq 140 \therefore \boxed{b = 20{,}82\,mm}$

$\dfrac{116600}{2.40a} \leq 100 \therefore \boxed{a = 14{,}58\,mm}$ $\quad \dfrac{116600}{2.40c} \leq 200 \therefore \boxed{c = 7{,}29\,mm}$

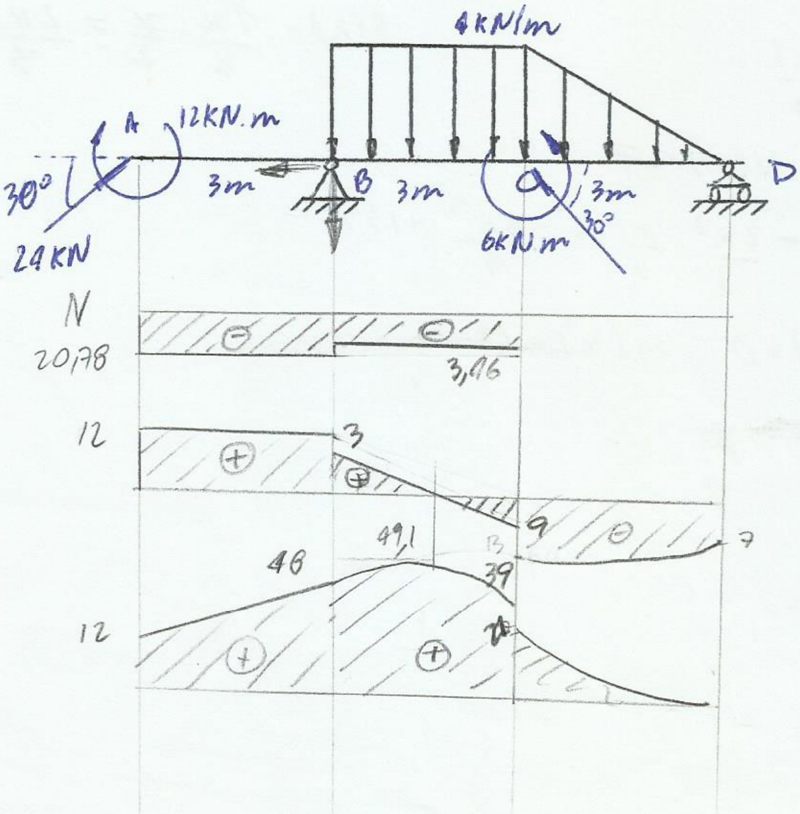
$\dfrac{116600}{2.40V} \leq 140 \therefore V = 10{,}41 \therefore e = 2.10{,}41 + 2.7{,}29 + 20{,}82$

$\boxed{\therefore e \geq 56{,}2\,mm}$

$\dfrac{56600/2}{(40-d).10} \leq 140 \therefore \boxed{d \leq 19{,}79\,mm}$ $\quad \boxed{13{,}42 \leq d \leq 19{,}79}$

$\dfrac{56600}{4\left(\frac{\pi.d^2}{4}\right)} \leq 100 \therefore \boxed{d \geq 13{,}42\,mm}$

$\dfrac{56600}{4.10\cos45L} \leq 100 \therefore \boxed{L \geq 20{,}01\,mm}$



$\Sigma F_x = 0 \quad 24\cos 30 - 4\cos 30 = h_B \therefore h_B = 17{,}32\ KN$

$\Sigma F_y = 0 \quad 24\,\text{sen}\,30 - 4.3 - \dfrac{4.3}{2} + 4\,\text{sen}\,30 + V_B + V_D = 0$

$\therefore V_B + V_D = 4 \rightarrow \text{New}: V_D - V_B = 4 \;;\; V_B = 9 \therefore V_d = 13$

$\Sigma M_D = 0 \quad 12 + 24\,\text{sen}\,30 .9 + 6\,V_B - 4.3.(3+1{,}5) + 4\,\text{sen}\,30.3 - \dfrac{4.3}{2} . \dfrac{2}{3}.3 - 6 = 0$

$\therefore V_B = \dfrac{-54}{6} = -9$

$\therefore \boxed{h_B = 17{,}32\ KN \quad V_B = 9\ KN \quad V_d = 13\ KN}$

Trecho AB

$N = -24\cos 30 = -20{,}78$

$V = 24\,\text{sen}\,30 = 12$

$M = 12x + 12 \therefore M(0) = 12 \quad M(3) = 48$

Trecho BC

$N = +17{,}32 - 20{,}78 = -3{,}46$

$R(x) = w(x).x$

$R(x) = 4x$

$V = 12 - 9 - R(x)$

$= 3 - 4x \therefore V(0) = 3 \quad V(3) = -9$

$M = 12 + 12(3+x) - 9x - 4x.\dfrac{x}{2}$

$= -2x^2 + 3x + 48 \therefore M(0) = 48 \quad M(3) = 39$

Trecho CD

$N = 0$

$\dfrac{w(x)}{x} = \dfrac{4}{3} \quad w(x) = \dfrac{4x}{3}$

$V = -13 - R(x)$

$R(x) = \dfrac{4x}{3} . \dfrac{x}{2} = \dfrac{2x^2}{3}$

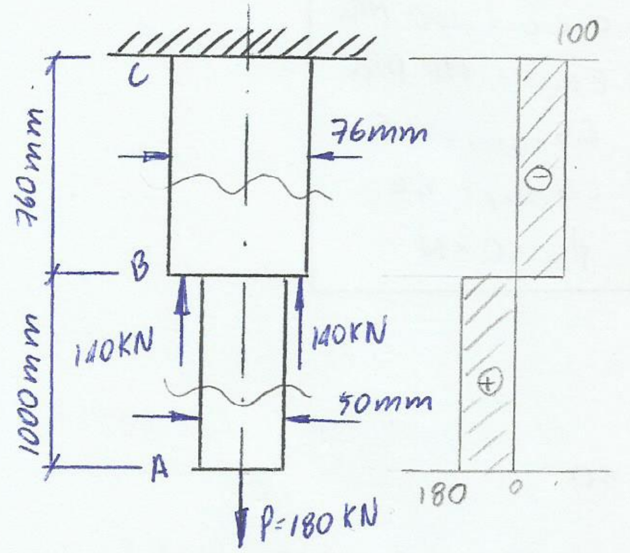
$= -13 + \dfrac{2x^2}{3}$

$V(0) = -13 \quad V(3) = -7$

$M = +13x - \dfrac{2x^2}{3} . \dfrac{x}{3} = -\dfrac{2x^3}{9} + 13x$

$M(0) = 0 \quad M(3) = 21$

2



Dados:

Barra AB: (Aço) E = 200 GPa

Barra BC: (latão) E = 105 GPa

a) $\sigma_{AB} = \frac{180 \cdot 10^3}{\pi \cdot 25^2} = \boxed{91,67\ MPa}$ (tração)

$\sigma_{BC} = \frac{180 - 2 \cdot 140}{\pi \cdot 38^2} = \boxed{22,04\ MPa}$ (compressão)

b) $\varepsilon = \frac{\delta}{L}$; $\varepsilon_T = \frac{\Delta d}{L}$

$\sigma = \frac{P}{A}$ $\sigma = E.\varepsilon$

$\frac{P}{A} = E.\varepsilon$; $\varepsilon = \frac{\delta}{L}$

$\therefore \boxed{\delta = \frac{P.L}{A\ E}}$

Trecho AB: $\delta = \Delta L = \frac{P.L}{A.E}$

$\delta = \frac{180\,\cancel{000}\,\cancel{N}}{200\,\cancel{000}\,\frac{\cancel{N}}{mm^2}} \cdot \frac{1000\ mm}{\pi \cdot 25^2\,\cancel{mm^2}}$

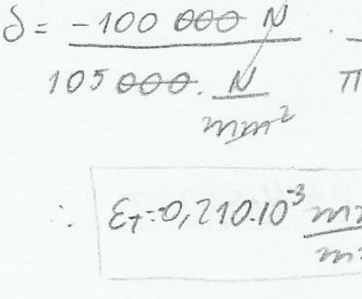
$\therefore \delta = 0,456\ mm$ $\therefore \varepsilon = \frac{0,458}{1000} = \boxed{0,458 \cdot 10^{-3}\ \frac{mm}{mm}}$ (alongou)

Trecho BC:

$\delta = \frac{-100\,\cancel{000}\,\cancel{N}}{105\,\cancel{000}\,\frac{\cancel{N}}{mm^2}} \cdot \frac{760 mm}{\pi \cdot 38^2\,\cancel{mm^2}}$ $\therefore$ $\varepsilon_T = \frac{\delta}{L} = \frac{-0,1596}{760}$

$\therefore \boxed{\varepsilon_T = 0,210 \cdot 10^{-3}\ \frac{mm}{mm}}$ (encurtou)

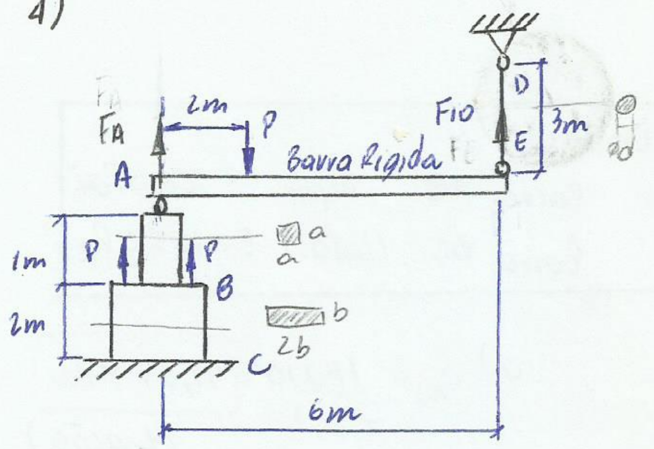
c)



$\therefore \delta_B = 0,16$ (Subiu)

$\delta_A = 0,16 - 0,46 = -0,30$ (Desceu)

4)



Dados

$\sigma_{LR-T} = 400\ MPa$

$\sigma_{LR-C} = 200\ MPa$

$E_{aço} = 210\ GPa$

$C.S_{tração} = 2,5$

$C.S_{comp} = 1,6$

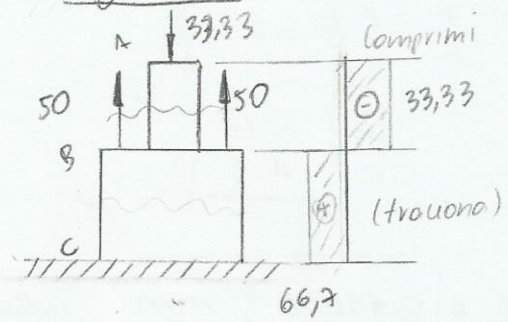
$P = 50\ kN$

Analizar as forças na barra

$\Sigma F_y = 0 \qquad F_a + F_e = P \quad \therefore \quad F_a + F_e = 50$

$\Sigma M_A = 0 \qquad 2P = 6F_e \quad \therefore \quad F_e = \dfrac{2.50}{6} = 16,67 \quad \therefore \quad F_a = 33,33$

Diagramas



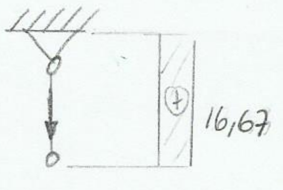
$\sigma_{AB} \leq \bar{\sigma}$

$-\dfrac{33333}{a^2}\dfrac{N}{mm^2} \leq \dfrac{200}{1,6}\ MPa$

$\therefore\ a \geq 16,33\ mm$

$\sigma_{BC} \leq \bar{\sigma}$

$\dfrac{66\,666}{2b^2} \leq \dfrac{400}{2,5} \quad \therefore \quad b \geq 14,43\ mm$

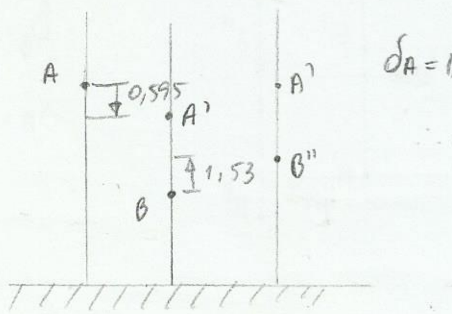


$\sigma_{DE} \leq \bar{\sigma}$

$\dfrac{16666}{\frac{\pi D^2}{4}} \leq \dfrac{400}{2,5} \quad \therefore \quad D \geq 11,52\ mm$

b) $\delta = \dfrac{PL}{E.A}$; $\delta_{AB} = \dfrac{33333.1000}{210.10^3.16,33^2} = 0,595\ mm$ (encurtou)

$\delta_{BC} = \dfrac{66666.2000}{210.10^3.2.14,43^2} = 1,525$



$\delta_A = 1,53 - 0,595$

$\delta_A = 0,935$

Dimensionamento da aresta DC :

$$\frac{36600}{a^2} \leq \frac{300}{2} \quad \therefore \quad a \geq 15,62\,mm$$

Dimensionamento do Ød

$$\frac{36600}{5 \cdot \pi \cdot \frac{d^2}{4}} \leq \frac{500}{3,5} \quad \therefore \quad Ød \geq 8,077\,mm$$

$$\therefore\ 8,08 \leq Ød \leq 8,86$$

Tração na chapa

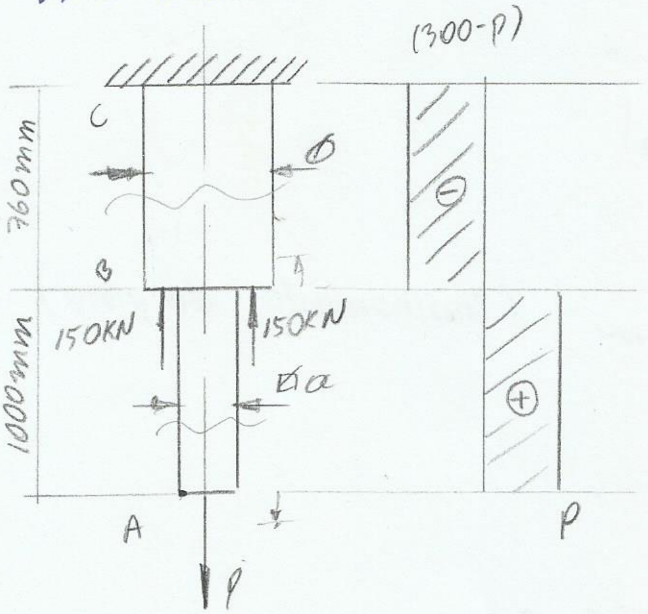
$$\frac{36600}{(70-2d) \cdot 10} \leq \frac{280}{4} \quad \therefore Ød \leq 8,86\,mm$$

$$\frac{36600}{2c \cdot 70} \leq \frac{350}{4} \quad \therefore \boxed{c = 2,99\,mm}$$

$\varepsilon_t = -\nu \cdot \varepsilon$

$\Delta d = -0,1 \times \varepsilon \times 75$

3) P1- 2°sem2011



$$\delta = \frac{P.L}{EA}$$

$$\delta_{AB} = \frac{P.1000}{180.000 \cdot 2500} = 2,22.10^{-6}\,P$$

$$\delta_{BC} = \frac{(300-P).760}{200000 \cdot \pi \frac{75^2}{4}} =$$

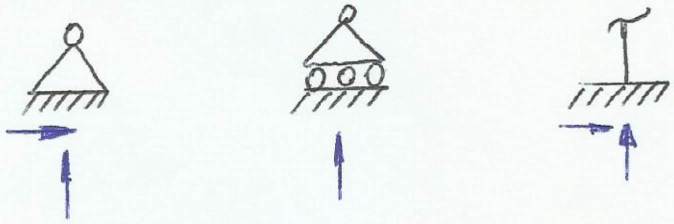
$$\delta_{BC} = 0,26.10^{-6} - 0,86.10^{-6}\,P$$

$$\delta_{AB} + \delta_{BC} = 0$$

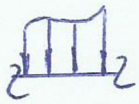
$$2,22\,\cancel{10^{-6}}\,P - (0,26.\cancel{10^{-6}} - 0,86.\cancel{10^{-6}}\,P) = 0 \quad \therefore \quad P = 3,7\,N$$

Resistência dos Materiais

Resumo de Diagrama de Esforços Internos



Carga distribuída



$$\boxed{\begin{array}{ll} \dfrac{dV}{dx} = -w \quad \therefore & V = -\int w\,dx \\[2ex] \dfrac{dM}{dx} = V \quad \therefore & M = \int V\,dx \end{array}}$$

$$\boxed{\frac{d^2M}{dx^2} = \frac{dV}{dx} = -w}$$

Simbolo	Esforço	Positivo	Negativo
N, P	Força Normal	$N>0$ tração	$N<0$ compressão
V, Q	Força Cortante	$V>0$ horário	$V<0$ Anti-horário
M	Momento Fletor	$M>0$ T. embaixo	$M<0$ T. em cima
T, M_T	Momento torçor	$T>0$ horário	$T<0$ Anti-horário