Mecânica do corpo Rígido

Senha do moodle: torque3

Exercício 2.5

$\vec{F_1} = 150 \sin 30° \cos 60° \vec{i} + 150 \sin 30° \sin 60° \vec{j} + 150 \cos 30° \vec{k}$ (N)

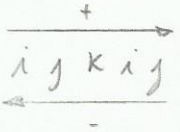
$= F_{1x} = 13{,}02 \vec{i}$ $\quad = F_{1y} = 73{,}86 \vec{j}$ $\quad = F_{1z} = 129{,}90 \vec{k}$

$\vec{F_2} = 100 \cos 40° \vec{i} + 100 \cos 65° \vec{j} + 100 \cos 119° \vec{k}$

$= 76{,}6 \vec{i} + 42{,}26 \vec{j} - 46{,}18$

* Para conferir se as componentes estão corretos, devemos apenas calcular o módulo.

O que são ângulos diretores?



Exercício 2.12

$$\boxed{\vec{\tau}_O^{\vec{F}} = \vec{r} \wedge \vec{F}}$$

$\vec{r} = P - O = (4,9,7) - (0,0,0)$ (P, O) $\quad \vec{r} = 4\vec{i} + 9\vec{j} + 7\vec{k}$

$\vec{F} = F\hat{\lambda} = F \cdot \frac{A-P}{|A-P|} = 240 \cdot \frac{(-\vec{i} + 2\vec{j} - 7\vec{k})}{7{,}35} = -32{,}65\vec{i} + 65{,}31\vec{j} - 228{,}57\vec{k}$ (N)

$A = (3,11,0) \rightarrow A - P = -1\vec{i} + 2\vec{j} - 7\vec{k}$ (m)

$|A-P| = 7{,}35$

$\vec{\tau}_O^{\vec{F}} = \vec{r} \wedge \vec{F} = (4\vec{i} + 9\vec{j} + 7\vec{k}) \wedge (-32{,}65\vec{i} + 65{,}31\vec{j} - 228{,}57\vec{k})$

$= 4 \cdot 65{,}31 \vec{k} + 4 \cdot 228{,}57 \vec{j} + 9 \cdot 32{,}65 \vec{k} - 9 \cdot 228{,}57 \vec{i} - 7 \cdot 32{,}65 \vec{j} - 7 \cdot 65{,}31 \vec{i}$

$= \underline{-2514{,}3\vec{i} + 685{,}73\vec{j} + 555{,}09\vec{k}}$

3.12

$m_1 = 10\ kg$; $\vec{C}_1 = (-16,0,10)\ m$

$m_2 = 100\ kg$; $\vec{C}_1 = (16,0,30)\ m$

$m_3 = 20\ kg$; $\vec{C}_3 = (32,0,20)\ m$

$m_4 = 50\ kg$; $\vec{C}_4 = (8,0,-10)\ m$

$m_5 = 200\ kg$; $\vec{C}_5 = (32,0,-30)\ m$

$$\bar{x} = \frac{\Sigma x_i m_i}{\Sigma m_i} = \frac{-16.10 + 16.100 + 32.20 + 8.50 + 32.200}{360} = 23,37$$

$$\bar{y} = \frac{\Sigma \bar{y}_i m_i}{\Sigma m_i} = 0$$

$$\bar{z} = \frac{\Sigma \bar{z}_i m_i}{\Sigma m_i} = \frac{10.10 + 30.100 + 20.20 - 20.50 - 30.200}{360} = -9,21$$

$$\vec{r}_{cm} = 23,368\,\vec{i} - 9,21\,\vec{k}\ \ (m)$$

Exercício 3.24

$\lambda = cte$; $A_{seção} = cte$; $C_L = C_M$

- transv.

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} \quad ; \quad \bar{y} = \frac{\int \tilde{y}\, dL}{\int dL}$$

$d\theta\ (rad), \frac{arco}{raio} = \frac{dL}{R} \Rightarrow dL = R.d\theta$

$$L = \int dL = \int R d\theta = R\int_0^{\pi/2} d\theta = R\frac{\pi}{2}$$

$$\int \tilde{x}\, dL = \int (R\cos\theta)(R d\theta) = R^2 \int_0^{\pi/2} \cos\theta\, d\theta = R^2 (\sin\theta)\Big|_0^{\pi/2} = R^2\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$= R^2$$

$$\int \tilde{y}\, dL = \int (R\sin\theta)(R d\theta) = R^2 \int_0^{\pi/2} \sin\theta\, d\theta = R^2\left(-\cos\frac{\pi}{2} + \cos 0\right) = R^2$$

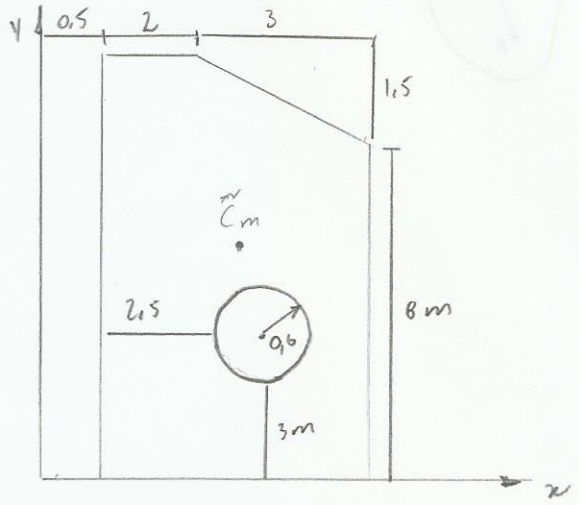
$$\bar{x} = \frac{R^2}{\left(\pi\frac{R}{2}\right)} = \frac{2R}{\pi} \qquad \vec{r}_m = \frac{2R}{\pi}\,\vec{i} + \frac{2R}{\pi}\,\vec{j}$$

$$\bar{y} = \frac{R^2}{\left(\pi\frac{R}{2}\right)} = \frac{2R}{\pi}$$

Teorema de Pappus

$$\begin{cases} A = \theta \bar{r} L \\ V = \theta \vec{r} A \end{cases}$$

3.76



$A_{\square} = 5 \cdot 9{,}5 = 47{,}5\ m^2$, $\bar{C}_1 = (3; 4{,}75)$

$A_{\triangle} = \frac{(3 \cdot 1{,}5)}{2} = 2{,}25\ m^2$, $\bar{C}_2 = (4{,}5 ; 9)$

$A_3 = \pi\, 0{,}6^2 = 1{,}13\ m^2$, $\bar{C}_3 = (3{,}6 ; 3{,}6)$

$$\bar{x} = \frac{\sum \bar{x}_i \cdot A_i}{\sum A_i} = \frac{47{,}5 \cdot 3 - 2{,}25 \cdot 4{,}5 - 1{,}13 \cdot 3{,}6}{44{,}12} = 2{,}91$$

$$\bar{y} = \frac{\sum \bar{y}_i \cdot A_i}{\sum i} = \frac{47{,}5 \cdot 4{,}75 - 2{,}25 \cdot 9 - 1{,}13 \cdot 3{,}6}{44{,}12} = 4{,}56$$

$$\vec{r}_C = 2{,}91\,\vec{i} + 4{,}56\,\vec{j}\ (m)$$

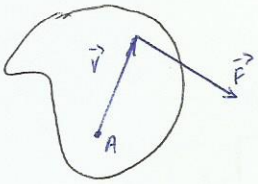
$V = \theta \cdot \vec{r} \cdot A$

$V = (2\pi) \cdot 2{,}91 \cdot 44{,}12 = 806{,}7\ m^3$

No moodle

3ª lista de exercícios com a capa, entrega entre 12 e 16/09

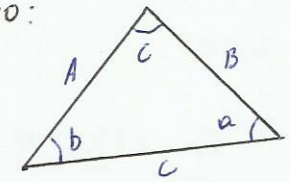
Torque Polar



$$\vec{\tau}_A^F = \vec{r} \wedge \vec{F}$$

Material de estudo (Mecanica do corpo Rigido)

Revisão:



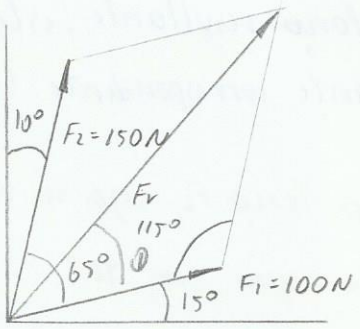
Lei dos cossenos:

$$C = \sqrt{A^2 + B^2 - 2AB\cos C}$$

Lei dos senos:

$$\frac{A}{\operatorname{sen} a} = \frac{B}{\operatorname{sen} b} = \frac{C}{\operatorname{sen} c}$$

Exemplo 1: Determine a intensidade e a direção da força resultante.



Aplicando lei dos cossenos

$$\vec{F_r}^2 = 100^2 + 150^2 - 2.100.150.\cos 115$$

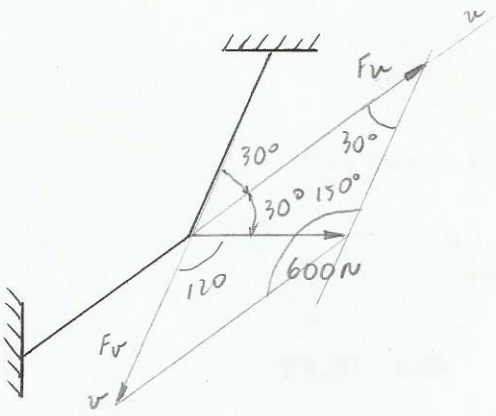
$$\therefore \vec{F_r} = \underline{212,55\ N}$$

Aplicando lei dos senos

$$\frac{212,55}{\operatorname{sen} 115} = \frac{150}{\operatorname{sen}\theta} \quad \therefore \quad \theta = 39,76°$$

$\therefore$ Direção: $39,76 + 15 = \underline{54,76°}$

Exemplo 2: Decomponha a força horizontal de 600N nos eixos u e v e determine as intensidades dessas componentes.

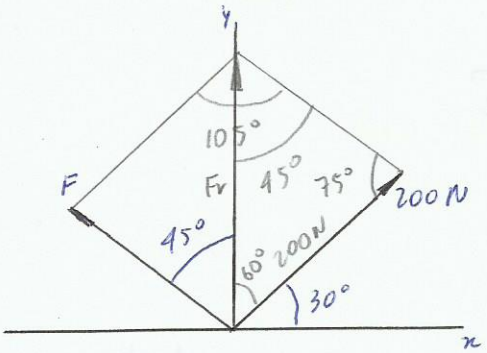


Aplicando Lei dos senos

$$\frac{600}{\operatorname{sen} 30} = \frac{Fu}{\operatorname{sen} 120} \quad \therefore \quad Fu = \underline{1039,23\ N}$$

$$\frac{600}{\operatorname{sen} 30} = \frac{Fv}{\operatorname{sen} 30} \quad \therefore \quad Fv = \underline{600N}$$

Exemplo 3 Determine a intensidade da força componente F e a intensidade da força resultante se Fr estiver direcionado ao longo do eixo y positivo.

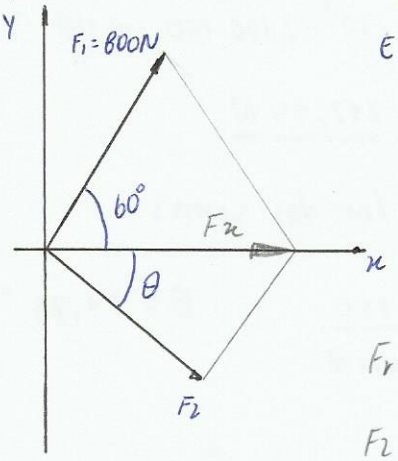


Aplicando Lei dos senos

$$\frac{Fr}{\sin 75} = \frac{200}{\sin 45} \quad \therefore \quad Fr = 273,21\,N$$

$$\frac{F}{\sin 60} = \frac{273,21}{\sin 75} \quad \therefore \quad F = 244,95\,N$$

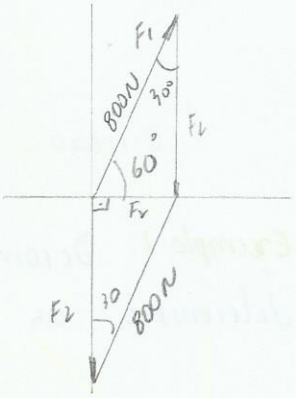
Exemplo 4 Determine θ para que a força resultante, esteja no eixo x. E a força resultante correspondente. (e F_2 mínima)



Para que a força F_2 seja mínima, o ângulo θ, tem que ser 90°.

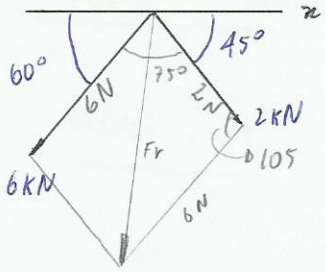
$$F_x = 800 \cos 60° = 400\,N$$

$$F_2 = 800 \cos 30° = 693\,N$$



Problemas Fundamentais

2.1 -) Determine a força resultante

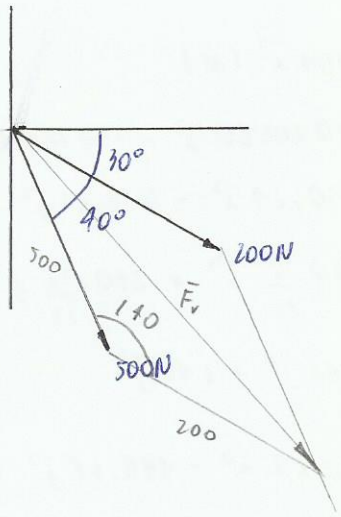


$$Fr^2 = 6^2 + 2^2 + 2.6.2.\cos 75°$$

$$\therefore Fr = 6,8\ kN$$

$$\frac{6,8}{\sin 105} = \frac{6}{\sin\theta} \quad \therefore \quad \theta = 58,49$$

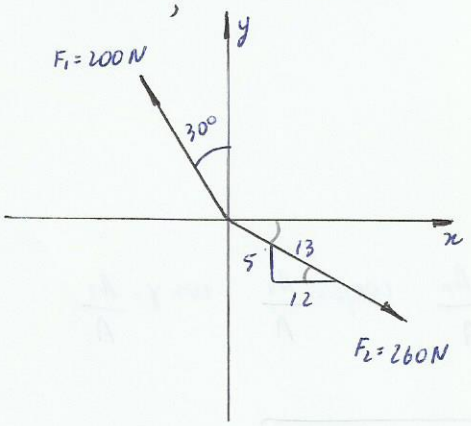
2.2-) Determine a intensidade da força



$$\bar{F}_v^{\,2} = 500^2 + 200^2 + 2.500.200.\cos 140$$

$$\therefore \bar{F}_v = 369{,}85\ N$$

Forças resultantes



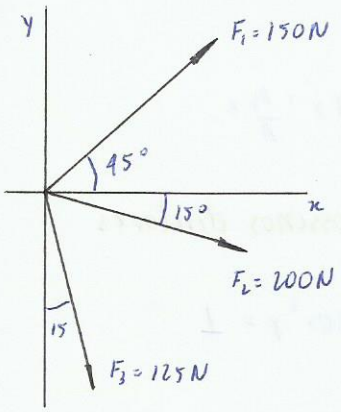
$$\vec{F_1} = -200 \operatorname{sen} 30°\ \vec{i} + 200 \cos 30°$$

$$= -100\ \vec{i} + 173{,}21\ \vec{j}$$

$$\vec{F_2} = 260.\frac{5}{13}\ \vec{i} - 260.\frac{12}{13}\ \vec{j}$$

$$= 100\ \vec{i} - 240\ \vec{j}$$

2.32. Determine a intensidade da força resultante, direção e sentido



$$\vec{F_1} = 150.\cos 45\ \vec{i} + 150 \operatorname{sen} 45\ \vec{j}$$

$$= 106{,}07\ \vec{i} + 106{,}07\ \vec{j}\ (N)$$

$$\vec{F_2} = 200 \cos 15°\ \vec{i} - 200 \operatorname{sen} 15°\ \vec{j}$$

$$= 193{,}19\ \vec{i} - 51{,}76\ \vec{j}\ (N)$$

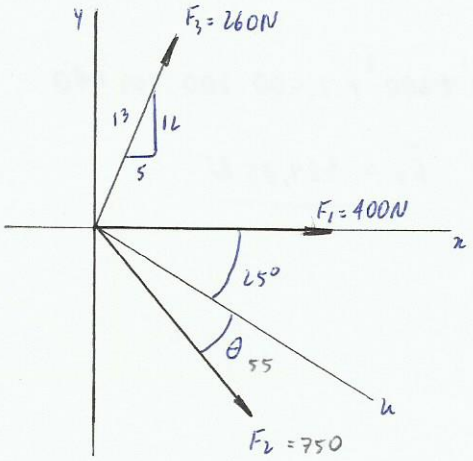
$$\vec{F_3} = 125 \operatorname{sen} 15°\ \vec{i} - 125 \cos 15°\ \vec{j}$$

$$= 32{,}35\ \vec{i} - 120{,}74\ \vec{j}\ (N)$$

$$\therefore \vec{F_r} = 331{,}61\ \vec{i} - 66{,}43\ \vec{j}\ (N) \quad |\vec{F_r}| = 338{,}20\ N$$

$$\theta = \operatorname{arctg}\left(\frac{-66{,}43}{331{,}61}\right) \quad \therefore \theta = -11{,}32°$$

2.55. $F_2 = 750N$ e $\theta = 55°$, determine a intensidade e a direção θ de F_R



$\vec{F_1} = 400\,\vec{i}\ (N)$

$\vec{F_2} = 750\cos 80\,\vec{i} - 750\,\text{sen}\,80\,\vec{j}$

$= 130{,}24\,\vec{i} - 738{,}61\,\vec{j}$

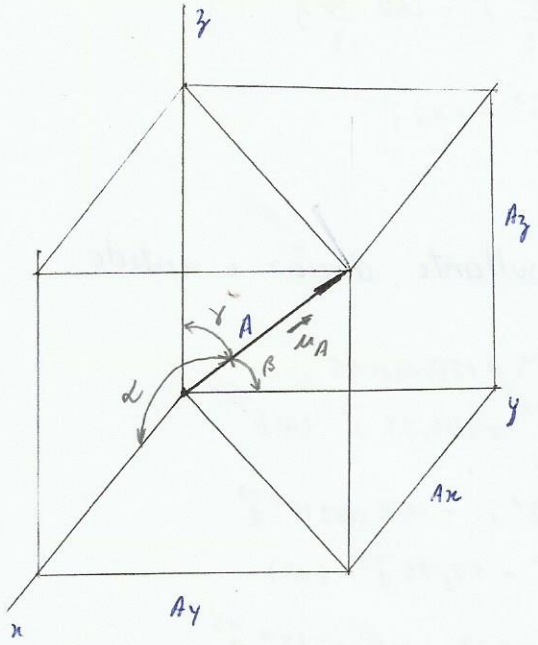
$\vec{F_3} = 260.\frac{5}{13}\,\vec{i} + 260.\frac{12}{13}\,\vec{j}$

$= 100\,\vec{i} + 240\,\vec{j}$

$\vec{F_r} = 630{,}24\,\vec{i} - 498{,}61\,\vec{j} \Rightarrow |\vec{F_r}| = 803{,}63\,N$

$\theta = \text{arc tg}\left(\frac{-498{,}61}{630{,}24}\right) \therefore \theta = -38{,}35°$

Vetores cartesianos



$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Vetor unitário

$\mu_A = \frac{A}{A} = \frac{A_x}{A}\,i + \frac{A_y}{A}\,j + \frac{A_z}{A}\,k$

relação entre os cossenos diretores

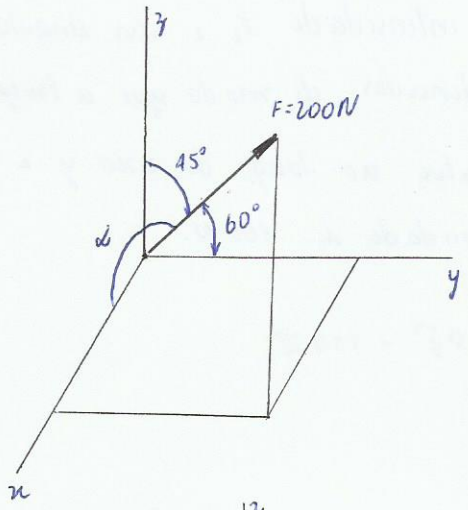
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

- Outra relação importante:

$A = A\cos\alpha\,\vec{i} + A\cos\beta\,\vec{j} + A\cos\gamma\,\vec{k}$

* Para realizar adição de vetores cartesianos, é realizar a soma das componentes.

Exemplos:



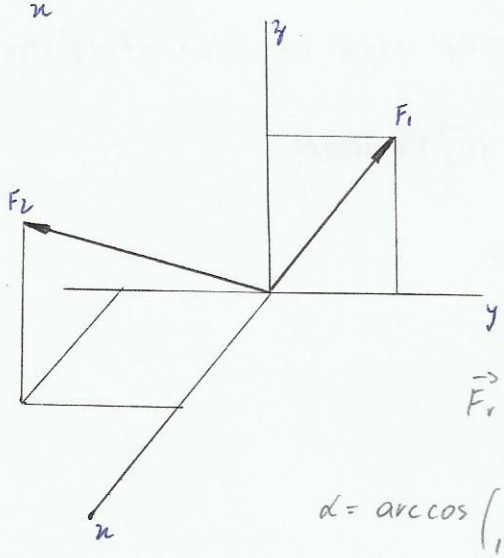
Expresse a força F, como um vetor cartesiano

$$\cos^2 45 + \cos^2 60 + \cos^2 \alpha = 1$$

$$\therefore \alpha = 60°$$

$$F = 200 \cdot \cos 60° \vec{i} + 200 \cos 60° \vec{j} + 200 \cdot \cos 45 \vec{k}$$

$$\therefore F = 100\vec{i} + 100\vec{j} + 141,42\vec{k} \ (N)$$



Determine a intensidade e os ângulos de direção coordenados da força resultante. Dados:

$$F_1 = \{ 50\vec{i} - 100\vec{j} + 100\vec{k} \} kN$$

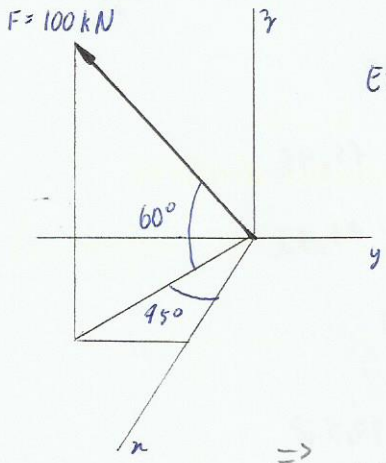
$$F_2 = \{ 60\vec{j} + 80\vec{k} \} kN$$

$$\vec{F_r} = \{ 50\vec{i} - 40\vec{j} + 180\vec{k} \} kN \quad |\vec{F_r}| = 191,05 \ kN$$

$$\alpha = \arccos\left(\frac{50}{191,05}\right) = 74,83°$$

$$\beta = \arccos\left(\frac{-40}{191,05}\right) = 102,09°$$

$$\gamma = \arccos\left(\frac{180}{191,05}\right) = 19,58°$$



Expresse a força F, como um vetor cartesiano

$$\vec{F_z} = 100 \operatorname{sen} 60 = 86,6 \ kN$$

$$\vec{F_x}' = 100 \cos 60° = 50 \ kN$$

$$\vec{F_x} = 50 \cos 45° = 35,36 \ kN$$

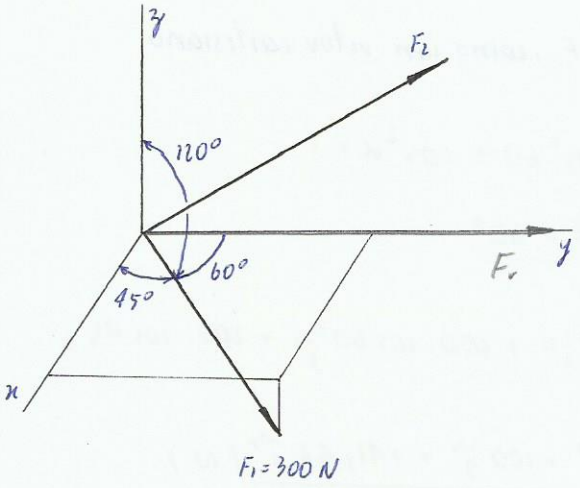
$$\vec{F_y} = -50 \operatorname{sen} 45° = -35,36 \ kN$$

$$\vec{F} = 35,36\vec{i} - 35,36\vec{j} + 86,6\vec{k} \ \{kN\}$$

$$\alpha = \cos^{-1}\left(\frac{35,36}{100}\right) = 69,30°$$

$$\beta = \cos^{-1}\left(\frac{-35,36}{100}\right) = 110,70°$$

$$\gamma = \cos^{-1}\left(\frac{86,6}{100}\right) = 29,93°$$



Especifique a intensidade F_2 e seus ângulos de direção coordenados, de modo que a força resultante F_R atue ao longo do eixo y e tenha intensidade de 800 N.

$$\vec{F_1} = 212{,}13\,\vec{i} + 150\,\vec{j} - 150\,\vec{k}$$

$$\vec{F_R} = \vec{F_1} + \vec{F_2}$$

$$800\,\vec{j} = (212{,}13\,\vec{i} + 150\,\vec{j} - 150\,\vec{k}) + F_{2x}\,\vec{i} + F_{2y}\,\vec{j} + F_{2z}\,\vec{k}$$

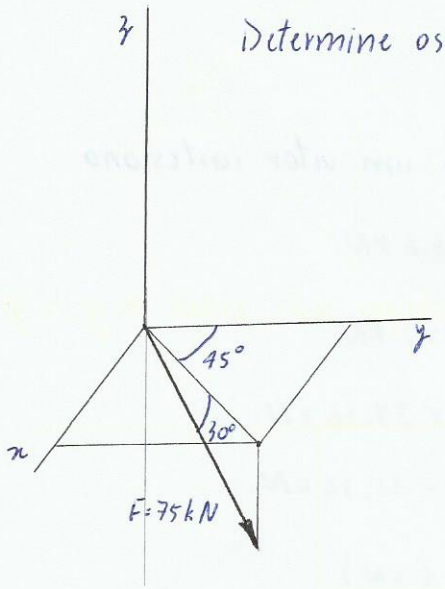
$$\vec{F_2} = -212{,}13\,\vec{i} + 650\,\vec{j} + 150\,\vec{k} \quad \Rightarrow \quad |\vec{F_2}| = 700\,N$$

$$\alpha = \cos^{-1}\left(\frac{-212{,}13}{700}\right) = 107{,}64°$$

$$\beta = \cos^{-1}\left(\frac{650}{700}\right) = 21{,}79°$$

$$\gamma = \cos^{-1}\left(\frac{150}{700}\right) = 77{,}63°$$

Problemas Fundamentais



Determine os ângulos de direção coordenados da força

$$F' = 75\cos 30 = 64{,}95$$

$$\vec{F_x} = 64{,}95 \operatorname{sen} 45 = 45{,}93$$

$$\vec{F_y} = 64{,}95 \cos 45 = 45{,}93$$

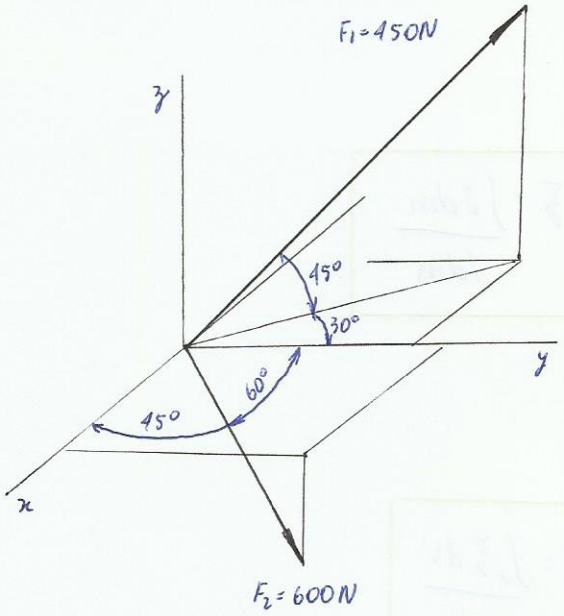
$$\vec{F_z} = 75 \operatorname{sen} 30 = 37{,}5$$

$$F = 45{,}93\,\vec{i} + 45{,}93\,\vec{j} - 37{,}5\,\vec{k}$$

$$\alpha = \cos^{-1}\left(\frac{45{,}93}{75}\right) = 52{,}24°$$

$$\beta = \cos^{-1}\left(\frac{45{,}93}{75}\right) = 52{,}24°$$

$$\gamma = \cos^{-1}\left(\frac{-37{,}5}{75}\right) = 120°$$



Determine a intensidade e os ângulos de direção coordenados da força resultante.

$\bar{F}_2 = 600\cos 45\,\vec{i} + 600\cos 60\,\vec{j} + 600\cos\gamma$

$600 = 424{,}26\,\vec{i} + 300\,\vec{j} + F_{2z}$

$\therefore F_{2z} = 300\,\vec{k}$

$\therefore \vec{F_2} = 424{,}26\,\vec{i} + 300\,\vec{j} - 300\,\vec{k}$

$F' = 450\cos 45 = 318{,}20$

$\vec{F_1} = -318{,}20\sin 30 + 318{,}20\cos 30 + 450\sin 45$

$\therefore \vec{F_1} = -159{,}1\,\vec{i} + 275{,}57\,\vec{j} + 318{,}20\,\vec{k}$

$\vec{F_r} = 265{,}16\,\vec{i} + 575{,}57\,\vec{j} + 618{,}2 \iff |\vec{F_r}| = 885{,}3\ N$

$\alpha = \cos^{-1}\left(\frac{265{,}16}{885{,}3}\right) = 72{,}57°$

$\beta = \cos^{-1}\left(\frac{575{,}57}{885{,}3}\right) = 49{,}45°$

$\gamma = \cos^{-1}\left(\frac{618{,}2}{885{,}3}\right) = 45{,}71°$

Centro de gravidade e centroide

Localização do centro de gravidade G com relação aos eixos x, y, e z.

$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\,dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\,dW}{\int dW}$$

$\bar{x}, \bar{y}, \bar{z}$ são coordenadas do centro de gravidade.

$\tilde{x}, \tilde{y}, \tilde{z}$ são as coordenadas de cada partícula no corpo.

Centro de massa de um corpo

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} \qquad \bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

Centroide de um volume

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$

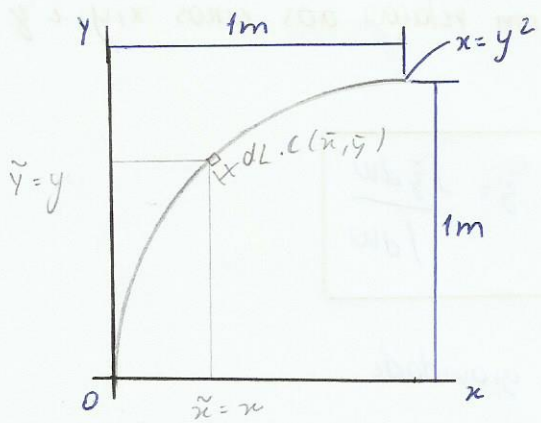
Centroide de uma área

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

Centróide de uma linha

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL}$$

Exemplos: 9.1



$$dL = \sqrt{(dx)^2 + (dy)^2}$$

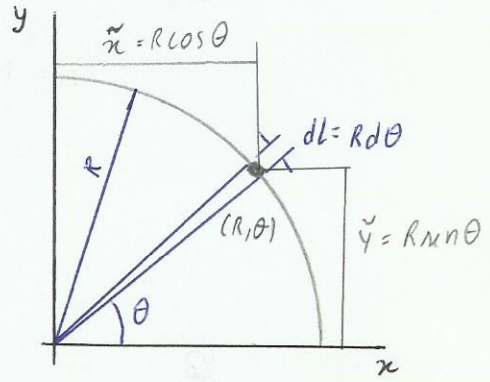
$$dL = \sqrt{\frac{[(dx)^2 + (dy)^2](dy)^2}{(dy)^2}}$$

$$dL = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$$

$$\frac{dx}{dy} = 2y \quad \therefore \quad dL = \sqrt{(2y)^2 + 1}\, dy$$

$$\bar{x} = \frac{\int_0^1 x\sqrt{4y^2+1}\, dy}{\int_0^1 \sqrt{4y^2+1}\, dy} = \frac{\int_0^1 y^2\sqrt{4y^2+1}\, dy}{\int_0^1 \sqrt{4y^2+1}\, dy}$$

Exemplo 9.2



$$d\theta = \frac{dL}{R} \therefore dL = R\,d\theta$$

Fórmula

$$\bar{x} = \frac{\int \tilde{x}\,dL}{\int dL} \qquad \bar{y} = \frac{\int \tilde{y}\,dL}{\int dL}$$

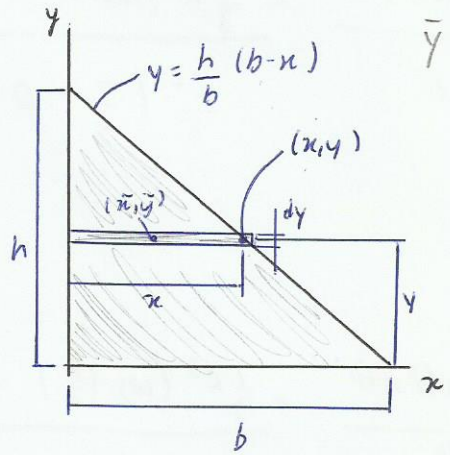
$$\bar{x} = \frac{\int R\cos\theta \cdot R\,d\theta}{\int R\,d\theta} = \frac{R^2\int_0^{\pi/2}\cos\theta\,d\theta}{R\int_0^{\pi/2}d\theta} = \frac{R^2\cdot(\sin(\pi/2)-\sin(0))}{R\cdot\frac{\pi}{2}}$$

$$\therefore \bar{x} = \frac{2R}{\pi}$$

$$\bar{y} = \frac{\int R\sin\theta \cdot R\,d\theta}{\int R\,d\theta} = \frac{R^2\int_0^{\pi/2}\sin\theta\,d\theta}{R\int_0^{\pi/2}d\theta} = \frac{R^2\cdot(\cos(\pi/2)-\cos 0)}{R\cdot\frac{\pi}{2}}$$

$$\bar{y} = \frac{2R}{\pi}$$

Exemplo 9.3



$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA}$$

$$dA = x\,dy$$

$$y = \frac{h}{b}(b-x)$$

$$\frac{by}{h} - b = -x$$

$$x = b - \frac{by}{h}$$

$$x = \frac{bh-by}{h} \qquad x = \frac{b(h-y)}{h}$$

$$dA = \frac{b}{h}(h-y)\,dy$$

$$\bar{y} = \frac{\int y\cdot\frac{b}{h}(h-y)\,dy}{\int \frac{b}{h}(h-y)\,dy}$$

$$\int y\cdot\frac{b}{h}(h-y)\,dy$$

$$\frac{b}{h}\int_0^h hy - y^2\,dy = \frac{b}{h}\left(\frac{h}{2}y^2 - \frac{y^3}{3}\right)\Big|_0^h$$

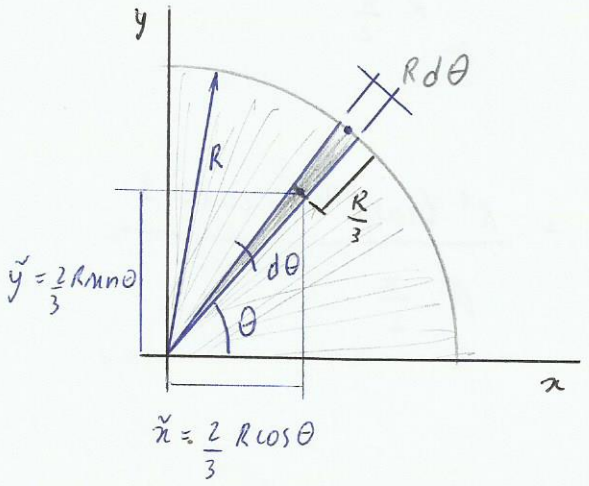
$$= \frac{b}{h}\left(\frac{h^3}{2} - \frac{h^3}{3}\right) = b\left(\frac{3h^2-2h^2}{6}\right) = \frac{bh^2}{6}$$

$$\int_0^h \frac{b}{h}(h-y)\,dy \sim \frac{b}{h}\int(h-y)\,dy = \frac{b}{h}\left(hy-\frac{y^2}{2}\right)\Big|_0^h =$$

$$\frac{b}{h}\left(h^2-\frac{h^2}{2}\right) = \frac{bh}{2}$$

$$\therefore \bar{y} = \frac{bh^2}{6}\cdot\frac{bh}{2} = \frac{h}{3}$$

Exemplo 9.4



$$dA = \frac{1}{2}\cdot R\,d\theta\cdot R = \frac{R^2}{2}d\theta$$

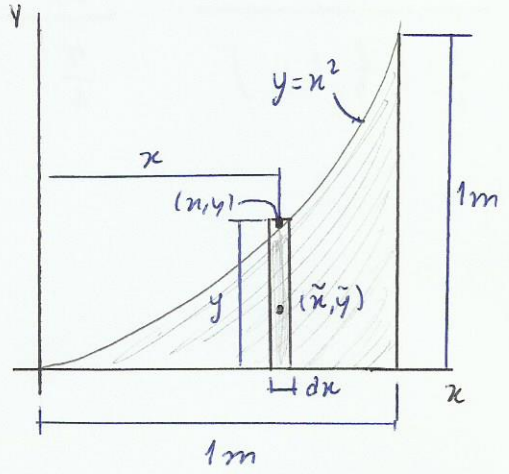
$$\bar{x} = \frac{\int \frac{2}{3}R\cos\theta\cdot\frac{R^2}{2}\,d\theta}{\int\frac{R^2}{2}\,d\theta} = \frac{\frac{2}{6}\cdot R^3\int_0^{\pi/2}\cos\theta\,d\theta}{\frac{R^2}{2}\int_0^{\pi/2}d\theta} = \frac{\frac{1}{3}R^3(\operatorname{sen}(\pi/2)-\operatorname{sen}0)}{\frac{R^2}{2}(\frac{\pi}{2}-0)}$$

$$= \frac{R^3}{3}\div\frac{\pi R^2}{4} = \frac{4R}{3\pi}$$

$$\bar{y} = \frac{\int \frac{2}{3}R\operatorname{sen}\theta\,\frac{R^2}{2}\,d\theta}{\int\frac{R^2}{2}\,d\theta} = \frac{\frac{1}{3}R^3\int_0^{\pi/2}\operatorname{sen}\theta\,d\theta}{\frac{R^2}{2}\int_0^{\pi/2}d\theta} = \frac{\frac{1}{3}R^3(\cos(\frac{\pi}{2})\cdot\cos 0)}{\frac{R^2}{2}(\frac{\pi}{2}-0)}$$

$$= \frac{R^3}{3}\div\frac{\pi R^2}{4} = \frac{4R}{3\pi}$$

Exemplo 9.5



$dA = y\,dx$

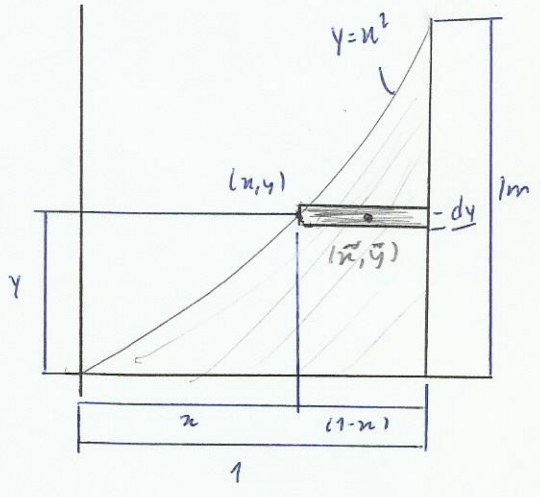
$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$$\bar{x} = \frac{\int_0^1 x\,y\,dx}{\int y\,dx} = \frac{\int_0^1 x^3\,dx}{\int_0^1 x^2\,dx} = \frac{\frac{1}{4}\cdot x^4\Big|_0^1}{\frac{1}{3}\cdot x^3\Big|_0^1} = \frac{\frac{1}{4}(1-0)}{\frac{1}{3}(1-0)} = \frac{3}{4}\,m$$

$$\bar{y} = \frac{\int_0^1 \frac{y}{2}\cdot y\,dx}{\int_0^1 y\,dx} = \frac{\int_0^1 \frac{y^2}{2}\,dx}{\int_0^1 y\,dx} = \frac{\int_0^1 \frac{x^4}{2}\,dx}{\int_0^1 x^2\,dx} = \frac{\frac{1}{10}x^5\Big|_0^1}{\frac{1}{3}x^3\Big|_0^1}$$

$$= \frac{1}{10} \div \frac{1}{3} = \frac{3}{10}\,m$$



$\tilde{x} = x + \frac{(1-x)}{2} = \frac{2x+1-x}{2} = \frac{x+1}{2}$

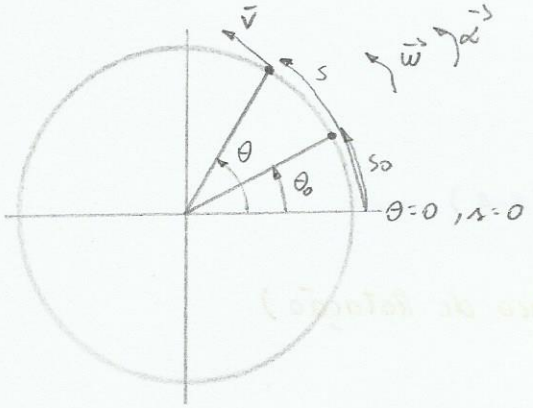
$\tilde{y} = y$

$dA = \left(\frac{1-x}{2}\right) dy$

$$\bar{x} = \frac{\int_0^1 \left(\frac{x+1}{2}\right)\left(\frac{x+1}{2}\right) dy}{\int_0^1 \left(\frac{x+1}{2}\right) dy} = \frac{\int_0^1 \frac{x^2+2x+1}{4}\,dy}{\frac{1}{2}\int_0^1 \sqrt{y}+1\,dy} = \frac{\frac{1}{4}\int_0^1 y+2\sqrt{y}+1\,dy}{\frac{1}{2}\int_0^1 \sqrt{y}+1\,dy}$$

$$= \frac{\frac{1}{4}\left(\frac{1}{2}y^2 + \frac{4}{3}y^{3/2} + y\right)\Big|_0^1}{\frac{1}{2}\left(\frac{2y^{3/2}}{3} + y\right)\Big|_0^1} = \frac{\frac{1}{4}\left(\frac{1}{2} + \frac{4}{3} + 1\right)}{\frac{1}{2}\left(\frac{2}{3} + 1\right)} = \frac{\frac{17}{24}}{\frac{5}{6}}$$

Capítulo 8 - Cinemática Rotacional



$$v = \frac{ds}{dt} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r}$$

$$a = \frac{dv}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{a}{r}$$

Obs.: A vantagem de trabalhar com as expressões angulares é que elas independem do raio, elas valem para todos os pontos

Movimento Circular e Uniforme

$$\begin{cases} a_t = 0 \\ v = \text{const} \\ s = s_0 + vt \end{cases} \sim \begin{cases} \frac{a_t}{r} = 0 = \alpha = \text{aceleração angular} \\ \frac{v}{r} = \text{const} = \omega = \text{velocidade angular} \\ \frac{s}{r} = \frac{s_0}{r} + \frac{v}{r}t = \theta = \text{espaço angular} \end{cases} \sim \begin{cases} \alpha = 0 \\ \omega = \text{const} \\ \theta = \theta_0 + \omega t \end{cases}$$

Movimento Circular Uniformemente Variado (MCUV)

$$\begin{cases} a_t = \text{const} \\ v = v_0 + a_t t \\ s = s_0 + v_0 t + \frac{1}{2} a_t t^2 \\ v^2 = v_0^2 + 2 a_t \Delta s \end{cases} \Rightarrow \begin{cases} \alpha = \text{const} \\ \omega = \omega_0 + \alpha_t t \\ \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta \end{cases}$$

$$\vec{\tau} = I\vec{\alpha}$$

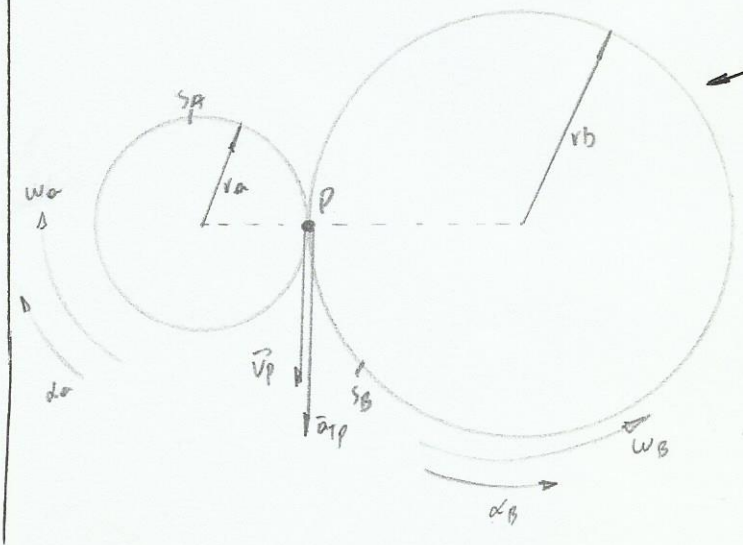
$$I = m \cdot k^2$$

- Para expressões genéricas do Movimento Circular

$\alpha = f(t)$

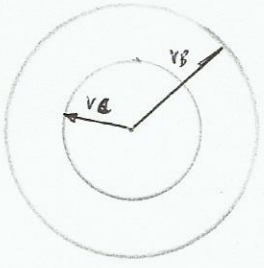
$$\alpha = \frac{d\omega}{dt} \qquad \alpha\, dt = d\omega \qquad \omega = \alpha \int dt + \text{const} \qquad \ldots$$

Princípio Invariante entre duas engrenagens



$$\begin{cases} s_A = s_B \Rightarrow \theta_A r_A = \theta_B r_B \\ v_{AP} = v_{BP} \Rightarrow \omega_A r_A = \omega_B r_B \\ a_{TAP} = a_{TBP} \Rightarrow \alpha_A r_A = \alpha_B r_B \end{cases}$$

$$\begin{cases} \theta_B = \theta_C \\ \omega_B = \omega_C \\ \alpha_B = \alpha_C \end{cases} \rightarrow$$



Velocidade Relativa

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{BA} \wedge (B-A)$$

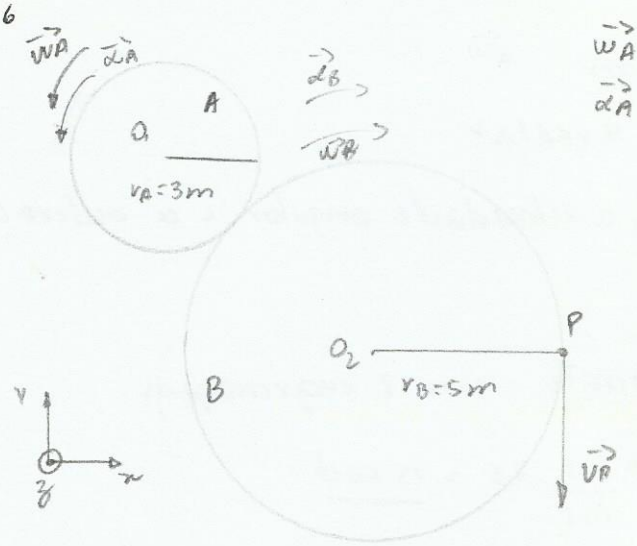
Aceleração Relativa

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{BA} \wedge (B-A) - \omega_{BA}^2 (B-A)$$

Equação do CIR (Centro Instantâneo de Rotação)

$$\omega = \frac{v_A}{IA} = \frac{v_B}{IB} = \dots = \frac{v_P}{IP}$$

8.6



$\vec{\omega_A} = 7\,\vec{k}$ rad/s

$\vec{\alpha_A} = 4\,\vec{k}$ rad/s²

Determine o vetor velocidade e o vetor aceleração total do ponto P no disco B da figura.

Através do Princípio Invariante entre duas engrenagens (PIE)

a) $\vec{\omega_A} r_A = \vec{\omega_B} r_B$

$\vec{\omega_B} = \frac{r_A}{r_B} \cdot \vec{\omega_A} = \frac{3}{5} \cdot 7 \therefore \vec{\omega_B} = -4{,}2\,\vec{k}\ \left(\frac{rad}{s}\right)$

$\vec{\omega_B} = \frac{v_B}{r_B} \therefore v_B = -4{,}2 \cdot 5 \therefore \vec{v_B} = \underline{-21\,\vec{j}\ (m/s)}$

b) $\vec{\alpha_A} r_A = \vec{\alpha_B} r_B$

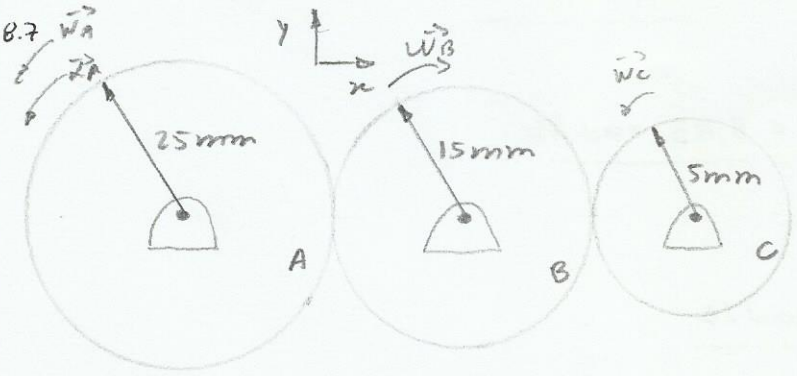
$\alpha_B = \frac{r_A}{r_B} \cdot \vec{\alpha_A} = \frac{3}{5} \cdot 4 \therefore \vec{\alpha_B} = -2{,}4\,\vec{k}\ \left(\frac{rad}{s^2}\right)$

$a = \omega^2 r = \frac{v^2}{r} = \frac{(-21)^2}{5} = 82{,}2 \therefore a_n = -82{,}2\,\vec{i}\ \left(\frac{m}{s^2}\right)$

$\alpha = \frac{a}{r} \therefore a = -2{,}4\,\vec{k} \cdot 5\,\vec{i} = -12\,\vec{j} \therefore a_t = -12\,\vec{j}\ \left(\frac{m}{s^2}\right)$

$\vec{a_p} = \underline{-82{,}2\,\vec{i} - 12\,\vec{j}\ \left(\frac{m}{s^2}\right)}$

8.7



$\vec{\omega_A} = 25$ rad/s

$\vec{\alpha_A} = 4$ rad/s²

Determine as velocidades angulares e acelerações angulares das engrenagens B e C.

$\vec{\omega_A} r_A = \vec{\omega_B} r_B$

$\omega_B = \frac{r_A}{r_B} \cdot \vec{\omega_A}$

$= \frac{25}{15} \cdot 25 \therefore \omega_B = -41{,}67\vec{k}\ \frac{rad}{s}$

$\vec{\omega_B} r_B = \vec{\omega_C} r_C$

$\vec{\omega_B} = \frac{15}{5} 41{,}67 \therefore \vec{\omega_C} = 125\,\vec{k}$ rad/s

$\alpha_B = \frac{15}{15} \cdot 4 = -6{,}67\vec{k}\ \frac{rad}{s^2}$ $\alpha_C = 20\vec{k}\ \frac{rad}{s^2}$

8.8

$r_A = 0{,}2\ m \quad r_B = 0{,}4\ m \quad r_C = 0{,}6\ m$

$\theta_C = 5\ rad \quad w_C = 12\ rad/s \quad \alpha_C = 9\ rad/s^2$

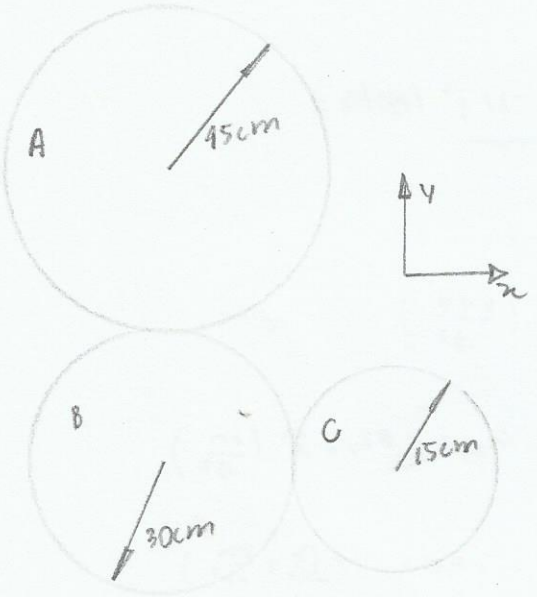
Determine o espaço angular, a velocidade angular e a aceleração das engrenagens A e B.

- Através do princípio Invariante entre 2 engrenagens.

$\theta_B = \frac{0{,}6}{0{,}4} \cdot 5 = \underline{7{,}5\ rad} \qquad \theta_A = \frac{0{,}4}{0{,}2}\ 7{,}5 = \underline{15\ rad}$

$\vec{w_B} = \frac{0{,}6}{0{,}4} \cdot 12 = \underline{18\ \frac{rad}{s}} \qquad \vec{w_A} = \frac{0{,}4}{0{,}2}\ 18 = \underline{36\ \frac{rad}{s}}$

8.9) $w_{C0} = 6\ rad/s \quad \alpha_C = 1{,}5\ rad/s^2$



(a) Qual deve ser as velocidades angulares das 3 engrenagens ao final $\theta_A = 5$ voltas

1 volta = 2π

5 voltas = 10π

$\theta_A = 10\pi$ Através do PIE

$\theta_A r_A = \theta_B r_B$

$\theta_B = \frac{45}{30} \cdot 10\pi = 15\pi$

$\theta_B r_B = \theta_C r_C$

$\theta_C = \frac{30}{15} \cdot 15 = 30\pi$

$w^2 = w_0^2 + 2\alpha \cdot \Delta\theta$

$w^2 = 6^2 + 2 \cdot 1{,}5 \cdot 30\pi \quad \therefore \quad w_C = \underline{17{,}85\ rad/s}$

$w_C r_C = w_B r_B$

$w_B = \frac{15}{30} \cdot 17{,}85 \quad \therefore \quad w_B = \underline{8{,}93\ rad/s}$

$w_A r_A = w_B r_B$

$w_A = \frac{30}{45} \cdot 8{,}93 = \underline{5{,}95\ rad/s}$

8.10) $r_A = 50\,mm \quad r_B = 20\,mm$

$\alpha_A = 7.\theta_A^2 \ \frac{rad}{s^2}$

(a) Determine α_B em função de θ_B

$\theta_a r_A = \theta_b r_B$

$50\,\theta_a = 20\,\theta_b$

$\theta_a = \frac{2}{5}\theta_b$

$\alpha_A r_A = \alpha_B r_B$

$\alpha_B = \frac{5}{2} \cdot 7 \cdot \left(\frac{2}{5}\theta_b\right)^2$

$\therefore \alpha_B = \underline{2,8\,\theta_b^2} \ \frac{rad}{s^2}$

(b) $w_A = ? \ ; \ w_{A0} = 0$

$w^2 = w_0^2 + 2\alpha\Delta\theta$

$w_A^2 = 2,7\,\theta_A^2 \cdot \theta_A$

$= 14\,\theta_A^3$

$w_A^2 = 14 \cdot \left[\frac{2}{5}\cdot\theta_b\right]^3$

$w_a = \underline{0,95 \cdot \theta_b^{3/2}} \ \frac{rad}{s}$

$w_a r_a = w_b r_b$

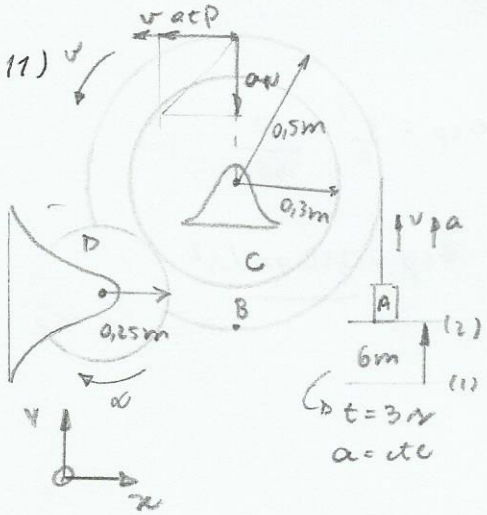
$w_b = \frac{5}{2} \cdot 0,95\,\theta_b^{3/2} \quad \therefore w_b =$

(c) $v_L(\theta_B) = ? \ ; \ |P| = 15\,cm$

Admitindo $w_B = 1,366 \ \frac{rad}{s}$

$\therefore w = \frac{v}{r} \quad \therefore v = 1,366 \cdot 15\,10^{-1} \quad \therefore v = \underline{0,205\ m/s^2}$

8.11)



Na posição (2), determine:

(a) $\vec{v}_B = ? \quad \vec{a}_B = ?$ (No ponto P)

$s = s_0 + v_0 t + \frac{at^2}{2}$

$s - s_0 = \frac{at^2}{2}$

$6 \cdot 2 = a \cdot 3^2 \quad \therefore a = 1,33\ m/s^2$

$\therefore \underset{tP}{a} = \underline{-1,33\,\vec{i}} \left(\frac{m}{s^2}\right)$

$v = \vec{v_0}^{\,0} + at$

$v = 1,33 \cdot 3 = 4\,m/s$

$a_{cp} = \frac{4^2}{0,5} = -32\,\vec{j} \left(\frac{m}{s^2}\right)$

$\therefore v_B = -4,00\,\vec{i} \left(\frac{m}{s}\right)$

$a_t = -1,33\,\vec{i} - 32\,\vec{j} \left(\frac{m}{s^2}\right)$

b) $\omega_D = ?$ $\alpha_D = ?$

$\omega_C = \omega_B$ $\quad \omega_D r_D = \omega_C r_C$

$\alpha_C = \alpha_B$ $\quad \omega_D = \frac{0,3}{0,25} \cdot \omega_C$ $\quad \omega_D = \frac{0,3 \cdot 8}{0,25} \therefore \omega_D = -9,6\,k \left(\frac{rad}{s}\right)$

$\omega_B = \frac{v_B}{r} = \frac{4}{0,5} = 8\,\vec{k}\ \frac{rad}{s}$

$\alpha_B = \frac{a_B}{r} = \frac{1,33}{0,5} = 2,66\ \frac{rad}{s}$ $\quad \alpha_D = \frac{0,3}{0,25} 2,66 \therefore \alpha_D = -3,2\,k \left(\frac{rad}{s^2}\right)$

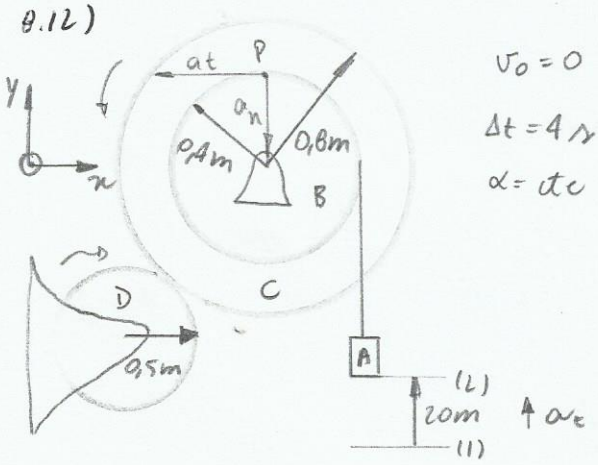
c) $\vec{\tau} = I \cdot \vec{\alpha}$ $\quad ; m = 2\,kg \quad e \quad K = 0,6m$

$I = m \cdot K^2$

$I = 2 \cdot 0,6^2 = 0,72$

$\vec{\tau} = 3,2 \cdot 0,72 = -2,30\,\vec{k}\,(N\,m)$

8.12)



$v_0 = 0$

$\Delta t = 4\,s$

$\alpha = cte$

Na posição que se entra o corpo A, determine:

a-) v, a_t, a_n da polia B

b-) ω, α da engrenagem C

c-) ω_D, α_D

d-) τ_D, sabendo $m_D = 2kg$ $K = 0,6m$

a-) $s = \cancel{s_0}^{0} + \cancel{v_0 t}^{0} + \frac{1}{2} a t^2$

$a = \frac{2 \cdot 20}{4^2} \therefore a_t = 2,5\ m/s^2$

$v = v_0 + at$

$v = 2,5 \cdot 4 \therefore v = 10\ m/s$

$a_{cp} = \frac{v^2}{K} = \frac{10^2}{0,4}$

$\therefore a_{cp} = 250\ m/s^2$

b-) $\omega_C = \omega_B = \frac{v}{R} = \frac{10}{0,4} = 25\ rad/s$

$\alpha_C = \alpha_B = \frac{a}{r} = \frac{2,5}{0,4} = 6,25\ rad/s^2$

c-) $\omega_D r_D = \omega_C r_C$

$\omega_D = \frac{0,8}{0,5} \cdot 25 \therefore \omega_D = 40\ rad/s$

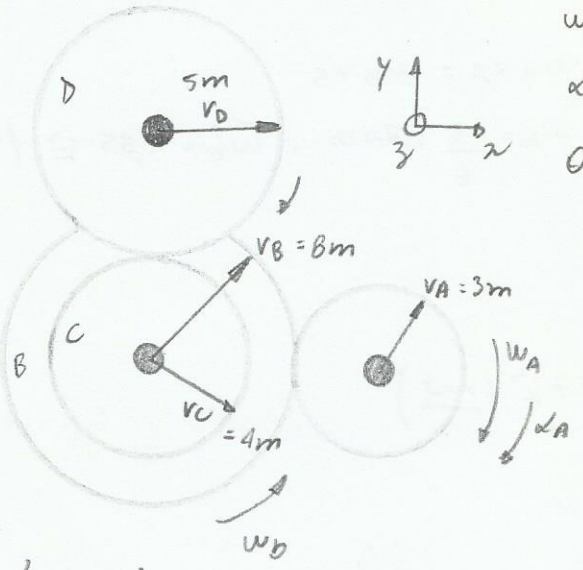
$\alpha_D r_D = \alpha_C r_C$

$\alpha_D = \frac{0,8}{0,5} \cdot 6,25 \therefore \alpha_D = \underline{10 \frac{rad}{s^2}}$

d) $\tau = I \cdot \alpha$ $\quad I = 2 \cdot 0,6^2 = 0,72$

$I = m \cdot k^2$ $\quad \tau = 0,72 \cdot 10 = \underline{7,2\ N.m}$

8.13-)



$w_{A_0} = 3\ rad/s$

$\alpha_A = 2\ rad/s^2$ (ctte)

θ = 2 voltas da engrenagem A

1 volta $= 2\pi$

2 volta $= 4\pi$ $\therefore \theta_A = 4\pi$

- Calcule todas velocidades angulares

$w_A^2 = w_{A0}^2 + 2 \cdot \alpha_A \cdot \Delta\theta$

$w_A^2 = 3^2 + 2 \cdot 2 \cdot 4\pi \therefore w_A = \underline{7,70\ rad/s} \Rightarrow \underline{\vec{w_A} = -7,70\vec{k}} \left(\frac{rad}{s}\right)$

$w_a r_a = w_b r_b$

$w_b = \frac{3}{8} \cdot 7,70 \therefore \underline{\vec{w_b} = 2,89\vec{k}} \left(\frac{rad}{s}\right)$

$w_b = w_c \therefore w_c = 2,89\vec{k} \left(\frac{rad}{s}\right)$

$w_c r_c = w_D r_D$

$w_D = \frac{4}{5} \cdot 2,89 \therefore w_D = 2,31 \frac{rad}{s} \Rightarrow \underline{\vec{w_D} = -2,31\vec{k}} \left(\frac{rad}{s}\right)$

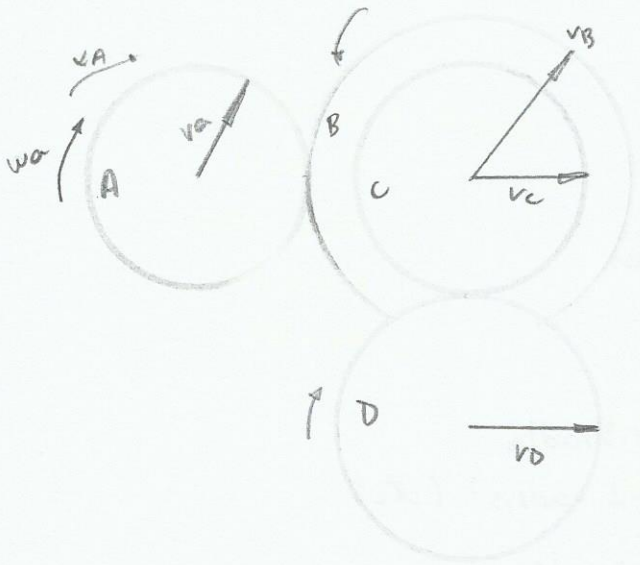
R: $\vec{w_a} = -7,70\vec{k} \left(\frac{rad}{s}\right)$

$\vec{w_b} = 2,89\vec{k} \left(\frac{rad}{s}\right)$

$\vec{w_c} = 2,89\vec{k} \left(\frac{rad}{s}\right)$

$\vec{w_D} = -2,31\vec{k} \left(\frac{rad}{s}\right)$

8.14)



$\omega_{A_0} = 3\ rad/s$ $\quad r_A = 3m$

$\alpha_A = 2\ rad/s^2$ $\quad r_B = 6m$ $\quad \theta_A = 4\pi$

$r_C = 4m$

$r_D = 6m$

$\omega^2 = 3^2 + 2.2.4\pi$

$\therefore \vec{\omega}_A = -7{,}70\,\vec{k}\ \left(\frac{rad}{s}\right)$

$\omega_A r_A = \omega_B v_B$

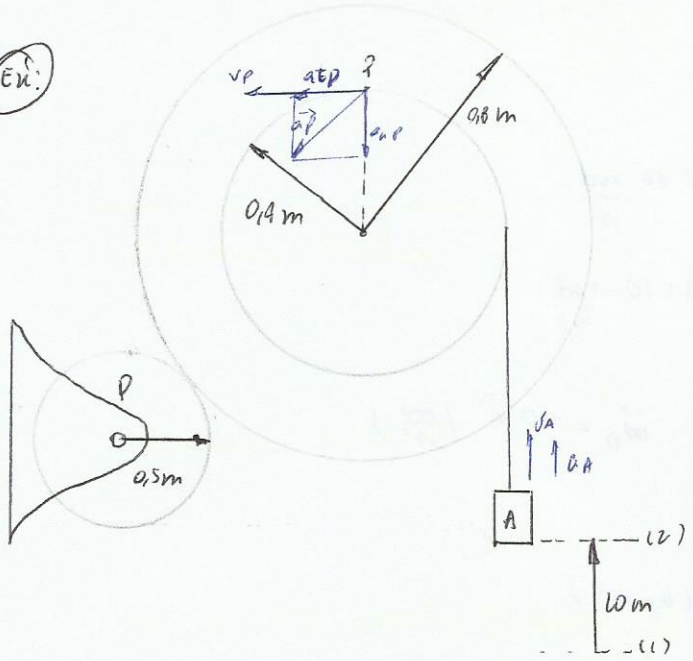
$\omega_B = \frac{3}{6} \cdot 7{,}70 \quad \therefore \vec{\omega}_B = 3{,}85\,\vec{k}\ \left(\frac{rad}{s}\right)$

$\vec{\omega}_B = \vec{\omega}_C = 3{,}85\,\vec{k}\ \left(\frac{rad}{s}\right)$

$\omega_C r_C = \omega_D r_D$

$\omega_D = \frac{4}{6} \cdot 3{,}85 \quad \therefore \vec{\omega}_D = -2{,}57\,\vec{k}\ \left(\frac{rad}{s}\right)$

Ex:



Corpo A
- $v_{A0} = 0$
- (1) → (2)
- $s_{A(2)} = 20\,m$
- $t_{(2)} = 4\,s$
- $a_A = cte$

Calcule na posição (2) de A:

(a) $\vec{v}_p = ?$ $\vec{a}_p = ?$ polia B

(b) $\vec{\omega}_C = ?$ $\vec{\alpha}_C = ?$ (c) $\vec{\omega}_D = ?$ $\vec{\alpha}_D = ?$ (d) $\vec{a}_d = ?$ $m_D = 2\,kg$ $R_D = 0,6\,m$

Solução:

$$\begin{cases} a_A = cte & I \\ v_A = v_{A0}^{\,0} + a_a t & II \\ s_A = s_{A0}^{\,0} + v_{a0}^{\,0} t + \frac{a_a}{2} t^2 & III \\ v_a^2 = v_{a0}^{2\,0} + 2 a_a \Delta s_a & IV \end{cases}$$

Usando a III

$$s_A = \frac{a_A}{2} t^2 \rightarrow 20 = \frac{a_A}{2} \cdot 4^2$$

$$\therefore a_A = 2,5\ m/s^2$$

Usando a II

$$v_A = a_A t$$

$$\therefore v_A = 2,5 \cdot 4 = 10\ m/s \quad \therefore \vec{v}_p = -10\,\vec{i}\ m/s$$

$$a_{tp} = -2,5\,\vec{i}\ (m/s^2)$$

$$a_{np} = \frac{v_p^2}{r_p} = \frac{10^2}{0,4} = 250\ \frac{m}{s^2} \quad ; \quad \vec{a}_{np} = -250\,\vec{j}\ \frac{m}{s^2}$$

$$\vec{a}_p = -2,5\,\vec{i} - 250\,\vec{j}\ \left(\frac{m}{s^2}\right)$$

b) $\vec{\omega}_B = \vec{\omega}_C$ $\quad \vec{\alpha}_B = \vec{\alpha}_C$

$$\omega_B = \frac{v_p}{r_p} = \frac{10}{0,4} = 25\ \frac{rad}{s} \quad \therefore \vec{\omega}_C = 25\,\vec{k}\ (rad/s)$$

$$\alpha_B = \frac{a_{tp}}{r_p} = \frac{2,5}{0,4} = 6,25\ \frac{rad}{s^2} \quad \vec{\alpha}_C = 6,25\,\vec{k}\ \left(\frac{rad}{s^2}\right)$$

(c) PIE: $w_C R_C = w_D r_D$

$\alpha_C r_C = \alpha_D r_D$

$w_D = \frac{R_C}{R_D} \cdot w_C = \frac{0,8}{0,4} \cdot 25 = 40 \ \frac{rad}{s}$

$\alpha_D = \frac{r_C}{r_D} \cdot \alpha_C = \frac{0,8}{0,15} \cdot 6,25 = 10 \ \frac{rad}{s^2}$

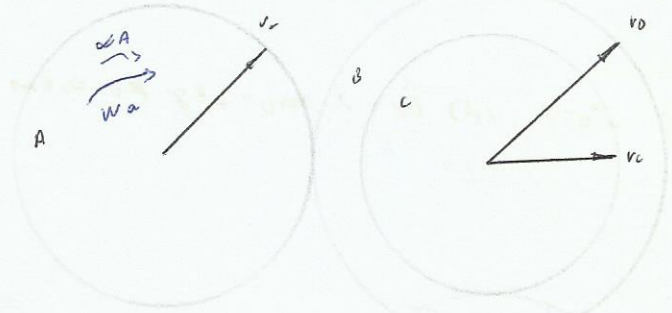
$\vec{w}_D = -40\vec{k} \ \left(\frac{rad}{s}\right)$ $\qquad \vec{\alpha}_D = -10\vec{k} \ \left(\frac{rad}{s^2}\right)$

(d) $\vec{\tau}_D = I_D \vec{\alpha}_D$

$I_D = m_D k_D^2 = 2 \cdot 0,6^2 = 0,72 \ (kg\,m^2)$

$\vec{\tau}_D = 0,72 (-10\vec{k}) = -7,2\vec{k} \ (Nm)$

(Ex)



Dados

engr. A $\begin{cases} w_{A_0} = 3 \ \frac{rad}{s} \\ \alpha_A = 2 \ \frac{rad}{s^2} = cte \end{cases}$

$r_a = 3 \ m$
$r_b = 6 \ m$
$r_c = 4 \ m$
$r_d = 6 m$

Calcular para $\theta_A = 2$ revoluções

(a) $\vec{w}_A = ?$

(b) $\vec{w}_B = ?$

(c) $\vec{w}_C = ?$

(d) $\vec{w}_D = ?$

Solução

$\begin{cases} \alpha_A = 2 \ \frac{rad}{s^2} = cte \ (MCUV) \\ w_A = w_{A_0} + \alpha_A t \\ \theta_A = \cancel{\theta_{A_0}}^{\,0} + w_{A_0} t + \frac{\alpha_A}{2} t^2 \\ w_A^2 = w_{A_0}^2 + 2 \alpha_A \Delta\theta_A \end{cases}$

Usando a IV

$w_A^2 = w_{A_0}^2 + 2 \cdot \alpha_A \cdot \Delta\theta_A$

$\Delta\theta_A = 2 rev = 4\pi \ rad$

$w_A^2 = 3^2 + 2 \cdot 2 \cdot 4\pi \ \therefore \ \vec{w}_A = -7,70\vec{k} \ \left(\frac{rad}{s}\right)$

(b) PIE: $w_A r_A = w_B r_B$

$\alpha_A r_A = \alpha_B r_B$

$w_B = \frac{r_A}{r_B} \cdot w_A = \frac{3}{6} \cdot 7,70 \ \therefore \ \vec{w}_B = 3,85\vec{k} \ \left(\frac{rad}{s}\right)$

(c) $\vec{w}_B = \vec{w}_C \ \therefore \ \vec{w}_C = 3,85\vec{k} \ \left(\frac{rad}{s}\right)$

(d) PIE: $w_C r_C = w_D r_D$

$w_D = \frac{r_C}{r_D} \cdot w_C = \frac{4}{6} \cdot 3,85 \quad \therefore \vec{w}_D = -2,56\,\vec{k}\ \left(\frac{rad}{s}\right)$

Velocidade Relativa

$\vec{v}_B = \vec{v}_A + \vec{w}_{BA} \wedge (B-A)$

Aceleração Relativa

$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{BA} \wedge (B-A) - w_{BA}^2 (B-A)$

Eq. do CIR (Centro Instantâneo de Rotação)

$w = \frac{v_A}{IA} = \frac{v_B}{IB} = \dots = \frac{v_P}{IP}$

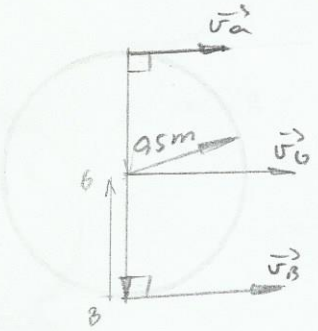
Exercícios:

B.19) $v_a = 5\vec{i}\ (m/s)$

$v_b = 8\vec{i}\ (m/s)$

a) $\vec{w} = ?$ b) $IA = ?$ c) velocidade do cilindo $(\vec{v}_G) = ?$

1º método analítico (veloc rel)



(a) $\vec{v}_B = \vec{v}_A + \vec{w}_{BA} \wedge (B-A)$

$8\vec{i} = 5\vec{i} + (w\vec{k}) \wedge (-1\vec{j})$

$3\vec{i} = w\vec{i} \quad \therefore w = 3\,\frac{rad}{s} \Rightarrow \vec{w} = 3\vec{k}\ \left(\frac{rad}{s}\right)$

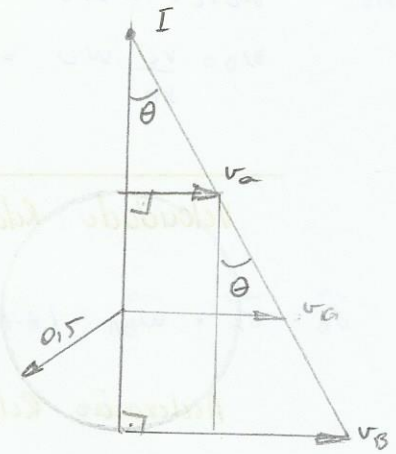
(b) $w = \frac{v_A}{IA} \quad \therefore IA = \frac{5}{3} = 1,667\,m$

(c) $\vec{v}_G = \vec{v}_B + w_{GB} \wedge (G-B)$

$\vec{v}_G = 8\vec{i} + 3\vec{k} \wedge 0,5\vec{j}$

$\vec{v}_G = 6,5\,\vec{i}\ \left(\frac{m}{s}\right)$

2.º método: método geométrico (CIR)



Eq. CIR: $\omega = \frac{v_A}{IA} = \frac{v_B}{IB} = \frac{v_G}{IG}$

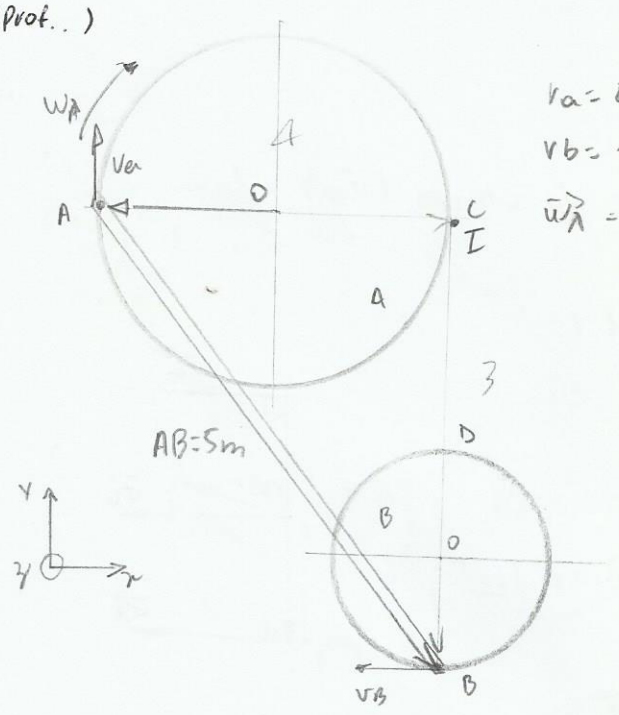
(a) $tg\theta = \frac{v_A}{IA} = \omega$

$tg\theta = \frac{v_B - v_A}{AB} = \frac{8-5}{1} = 3$ $\therefore \omega = 3 \frac{rad}{s} \Rightarrow \vec{\omega} = 3\vec{k} \left(\frac{rad}{s}\right)$

(b) $\omega = \frac{v_A}{IA}$ $\therefore IA = \frac{5}{3} = 1{,}667\ m$

(c) $\omega = \frac{v_G}{IG}$ $\therefore v_G = 3.(1{,}667 + 0{,}5) = 6{,}5\ \vec{i}\ \left(\frac{m}{s}\right)$

Ex (Prof.)



$r_a = 2m$

$r_b = 1{,}2\ m$

$\vec{\omega}_\lambda = -6\vec{k}\ \left(\frac{rad}{s}\right)$

Questões

a) $\vec{v}_a = ?$ b) $I_{AB} = ?$

c) $\vec{\omega}_{AB} = ?$ d) $\vec{\omega}_B = ?$

$v_B = ?$

1: Método (Método analítico)

(a) disco λ: $\vec{v}_a = \vec{v}_O^{\,0} + \omega_\lambda \wedge (A - O)$

$\vec{v}_a = (-6\vec{k}) \wedge (-2\vec{i})$ $\vec{v}_a = 16\vec{j}\ \left(\frac{m}{s}\right)$

c) barra AB: $\vec{v}_B = \vec{v}_A + \omega_{BA} \wedge (B - A)$

$-\vec{v}_B\vec{i} = 16\vec{j} + (\omega_b\vec{k}) \wedge (4\vec{i} - 3\vec{j})$

$-\vec{v}_B\vec{i} = 16\vec{j} + 4\omega_b\vec{j} + 3\omega_b\vec{i}$

$-\vec{v}_B\vec{i} = 3\omega_b\vec{i} + (16 + 4\omega_b)\vec{j}$

$-v_B = 3\omega_b$

$0 = 16 + 4\omega_b \therefore \omega_b = -4 \frac{rad}{s} \Rightarrow \vec{\omega}_b = \underline{-4k} \left(\frac{rad}{s}\right)$

$\therefore v_b = 12 m/s \Rightarrow \vec{v}_b = \underline{-12\vec{i}} \left(\frac{m}{s}\right)$

(b) $\omega_{barra} = \frac{v_A}{I_A} \quad I_A = \frac{16}{4} = 4m \Rightarrow I_{AB} \equiv C$

(d)

$\vec{v}_B = \cancel{\vec{v}_O}^{0} + \omega_{BO} \wedge (B-O)$

$-12\vec{i} = (\omega_B \vec{k}) \wedge (-1,2\vec{j})$

$-12\vec{i} = 1,2\,\omega_b \vec{i} \quad \therefore \omega_b = 10 \frac{rad}{s} \quad \vec{\omega}_B = -10\vec{k} \left(\frac{rad}{s}\right)$

2º Método (Método Geométrico (CIR))

(a) disco λ: $I_\lambda = O \quad \omega_\lambda = \frac{v_A}{I_\lambda A} = 8 = \frac{v_A}{2} \therefore v_a = 16 \frac{m}{s} \quad \vec{v}_a = \underline{16\vec{j}} \left(\frac{m}{s}\right)$

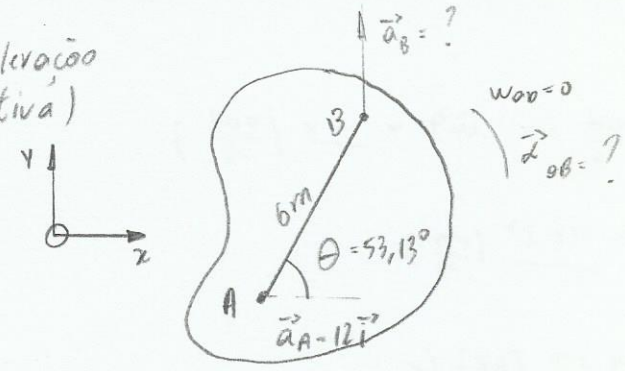
(b) $\underline{I_{AB} \equiv C}$

(c) $\omega_{AB} = \frac{v_A}{I_A} = \frac{v_B}{I_C B} = \frac{16}{4} = 4 \frac{rad}{s} \Rightarrow \vec{\omega}_{ab} = \underline{-4\vec{k}} \left(\frac{rad}{s}\right)$

$v_B = 4.3 = 12 \frac{m}{s} \Rightarrow \vec{v}_B = -12\vec{i} \left(\frac{m}{s}\right)$

(d) $\omega_B = \frac{v_B}{I_p B} = \frac{12}{1,2} = 10 \frac{rad}{s}$

*Ex (Aularação relativa)



Dados

$w_{AB} = 0$

$\bar{AB} = 6\,m$

$\vec{a}_A = 12\vec{i}\ (m/s^2)$

$\vec{a}_B = a_B\vec{j}\ (m/s^2)$

$\theta = 53,13°$

$\vec{\alpha} = ?$

$\vec{a}_B = ?$

Solução

$$\vec{a}_b = \vec{a}_A + \vec{\alpha}_{BA} \wedge (B-A) - w_{BA}^{2\,\to 0}(B-A)$$

$$\vec{ab}\,\vec{j} = 12\vec{i} + (\vec{\alpha}_{BA}\,\vec{k}) \wedge (6\cos\theta\,\vec{i} + 6\,\mathrm{sen}\,\theta\,\vec{j})$$

$$\vec{ab}\,\vec{j} = 12\vec{i} + (\vec{\alpha}_{BA}\,\vec{k}) \wedge (3,6\vec{i} + 4,8\vec{j})$$

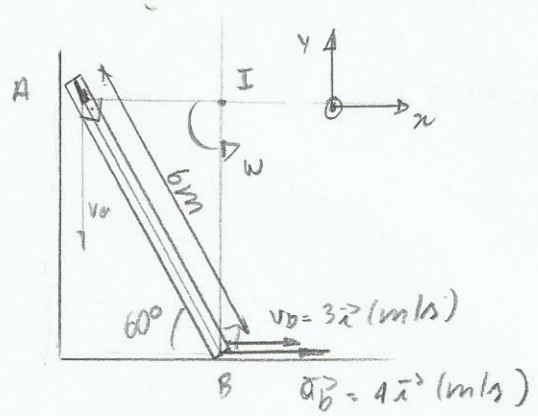
$$\vec{ab}\,\vec{j} = 12\vec{i} + 3,6\,\vec{\alpha}\,\vec{j} - 4,8\,\vec{\alpha}\,\vec{i}$$

$$\vec{ab}\,\vec{j} = (12 - 4,8\alpha)\vec{i} + 3,6\,\alpha\,\vec{j}$$

$12 - 4,8\alpha = 0 \quad \therefore \quad \alpha = \underline{2,5\ rad/s^2}$

$\vec{a}b = 3,6\,\alpha \quad \therefore \quad \vec{a}b = \underline{9\ m/s^2}$

*



Questões:

a) $\vec{w}_{AB} = ?$

$\vec{v}_A = ?$

b) $\vec{\alpha}_{AB} = ?$

$\vec{a}_A = ?$

1º) Método (Analítico - velocidade relativa)

$$\vec{v}_a = \vec{v}_b + \vec{w}_{AB} \wedge (A-B)$$

$$-v_a\,\vec{j} = 3\vec{i} + (w\,\vec{k}) \wedge (-6\cos 60\,\vec{i} + 6\,\mathrm{sen}\,60\,\vec{j})$$

$$-v_a\,\vec{j} = 3\vec{i} + (w\vec{k}) \wedge (-3\vec{i} + 5,20\vec{j})$$

$$-v_a\,\vec{j} = 3\vec{i} - 3w\,\vec{j} - 5,20w\,\vec{i}$$

$$-v_a\,\vec{j} = (3 - 5,20w)\vec{i} - 3w\,\vec{j}$$

$3 - 5,20w = 0 \quad \therefore \quad w = \underline{0,58\ rad/s}$

$-3w = -v_a \quad v_a = \underline{1,73\ m/s}$

Método geométrico

$$\omega = \frac{v_a}{IA} = \frac{v_b}{IB} = \dots$$

$$\omega = \frac{v_a}{3} = \frac{3}{5{,}20} \quad \therefore \omega = 0{,}58 \frac{rad}{s} \quad e \quad v_a = 1{,}73 \frac{m}{s}$$

b) 1º Método (Analítico - aceleração relativa)

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \wedge (A-B) - \omega_{AB}^2 (A-B)$$

$$-a_A \vec{j} = 4\vec{i} + (\alpha \vec{k}) \wedge (-6\cos 60\,\vec{i} + 6\sin 60\,\vec{j}) - (0{,}58)^2 (-6\cos 60\,\vec{i} + 6\sin 60\,\vec{j})$$

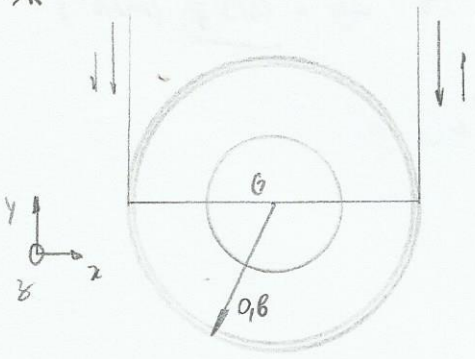
$$-a_A \vec{j} = 4\vec{i} - 3\alpha \vec{j} - 5{,}2\alpha \vec{i} + \vec{i} - 1{,}73\vec{j}$$

$$-a_A \vec{j} = (4 - 5{,}2\alpha + 1)\vec{i} + (-3\alpha - 1{,}73)\vec{j}$$

$$\begin{cases} 5 - 5{,}2\alpha = 0 \\ -1{,}73 - 3\alpha = -a_A \end{cases} \quad \therefore \alpha = 0{,}96 \ rad/s^2$$

$$\therefore \vec{a}_A = -4{,}61 \ (m/s^2)$$

*



Dados:

$\vec{v}_A = -2\vec{j}$ (m/s)

$\vec{v}_B = -3\vec{j}$ (m/s)

$\vec{a}_A = -0{,}8\vec{j}$ (m/s²)

$\vec{a}_B = 0{,}8\vec{j}$ (m/s²)

Questões:

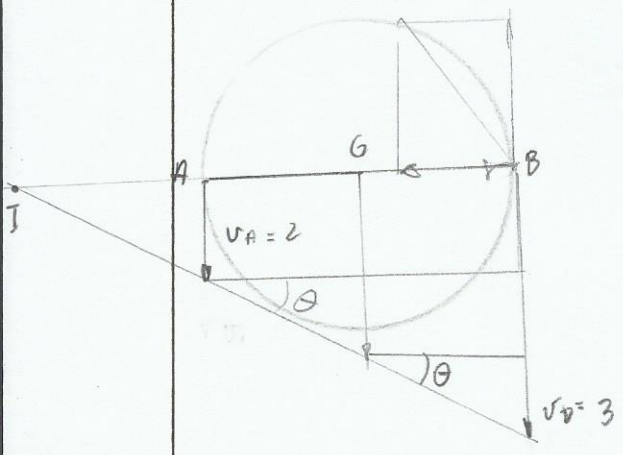
(a) $\vec{\omega} = ?$

$\vec{v}_G = ?$

(b) $\vec{\alpha} = ?$

$\vec{a}_G = ?$

Solução



$$\omega = \frac{v_a}{IA} = \frac{v_B}{IB} = \frac{v_G}{G} = \frac{v_B - v_A}{AB} = \frac{1}{1{,}6} = 0{,}625 \frac{rad}{s}$$

$$\therefore \vec{\omega} = -0{,}625\vec{k} \ \left(\frac{rad}{s}\right)$$

$$v_G = \omega \cdot IG = \omega(IA + AG)$$

$$IA = \frac{v_A}{\omega} = \frac{2}{0{,}625} = 3{,}2$$

$$v_G = 0{,}625(3{,}2 + 0{,}8) \quad \therefore v_G = 2{,}5 \ m/s$$

$$\therefore \vec{v}_B = -2{,}5\vec{j} \left(\frac{m}{s}\right)$$

$$\vec{a_B} = \vec{a_{tB}} + \vec{a_{nB}} = 0{,}8\,\vec{j} + a_{nB}\vec{i}$$

$$\vec{a_A} = \vec{a_{tA}} + \vec{a_{nA}} = -0{,}5\,\vec{j} + a_{nA}\vec{i}$$

$$\vec{a_B} = \vec{a_A} + \vec{\alpha}_{BA} \wedge (B-A) - \omega_{BA}^2 (B-A)$$

$$0{,}8\,\vec{j} + a_{nB}\vec{i} = -0{,}5\,\vec{j} + a_{nA}\vec{i} + (\alpha\vec{k}) \wedge (1{,}6\,\vec{i}) - 0{,}625^2 (1{,}6\,\vec{i})$$

$$0{,}8\,\vec{j} + a_{nB}\vec{i} = -0{,}5\,\vec{j} + a_{nA}\vec{i} + 1{,}6\alpha\,\vec{j} - 0{,}625\,\vec{i}$$

$$0{,}8\,\vec{j} + a_{nB}\vec{i} = (a_{nA} - 0{,}625)\vec{i} + (-0{,}5 + 1{,}6\alpha)\vec{j}$$

$$\begin{cases} a_{nB} = a_{nA} - 0{,}625 \\ 0{,}8 = -0{,}5 + 1{,}6\alpha \end{cases} \quad \therefore \alpha = 0{,}8125 \ \frac{rad}{s^2} \Rightarrow \vec{\alpha} = 0{,}8125\,\vec{k} \ \frac{rad}{s^2}$$

$$\vec{a_G} = \vec{a_A} + \vec{\alpha}_{GA} \wedge (G-A) - \omega_{GA}^2 (G-A)$$

$$a_G\,\vec{j} = -0{,}5\,\vec{j} + a_{nA}\vec{i} + 0{,}8125\,\vec{k} \wedge 0{,}8\,\vec{i} - 0{,}625^2 \cdot (0{,}8\,\vec{i})$$

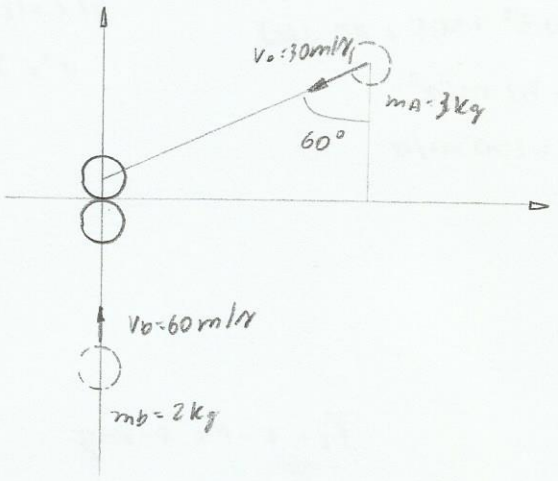
$$a_G\,\vec{j} = -0{,}5\,\vec{j} + a_{nA}\vec{i} + 0{,}65\,\vec{j} - 0{,}313\,\vec{i}$$

$$a_G\,\vec{j} = (-0{,}5 + 0{,}65)\,\vec{j} + (a_{nA} - 0{,}313)\,\vec{i}$$

$$a_G = -0{,}5 + 0{,}65 = 0{,}15 \ m/s^2 \Rightarrow \vec{a_g} = 0{,}15\,\vec{j} \ (m/s^2)$$

$$0 = a_{nA} - 0{,}313 \quad \therefore a_{nA} = 0{,}313 \ \frac{m}{s^2}$$

7.15)



Dados:

$v_0 = 30 m/s$

$\theta_A = 60°$

$m_A = 3kg$

$v_b = 60 m/s$

$m = 2kg$

$e = 0,6$

Questões

$\vec{v_2}$? v_b ?

Antes da colisão

$\vec{v_A} = 30 \sin 60° (-\vec{i}) + 30 \cos 60° (-\vec{j}) \ (\frac{m}{s})$

$$\begin{cases} \vec{v_A} = -26\vec{i} - 15\vec{j} \\ v_b = 60\vec{j} \end{cases}$$

Na direção ⊥ à colisão (x)

$$\begin{cases} v'_{Ax} = -26 \ m/s \\ v'_{Bx} = 0 \end{cases}$$

Na direção da colisão (y)

1º) PCM $m_A v_{Ay} + m_B v_{By} = m_A v'_{Ay} + m_B v'_{By}$

$3.(-15) + 2.60 = 3v'_{Ay} + 2v'_{By}$

$3v'_{Ay} + 2v'_{By} = 75$

2º) Coef. Rest.

$$e = \frac{v'_{By} - v'_{Ay}}{v_{Ay} - v_{By}} = 0,6 \quad \therefore \quad \frac{v'_{By} - v'_{Ay}}{-15 - 60} = 0,6$$

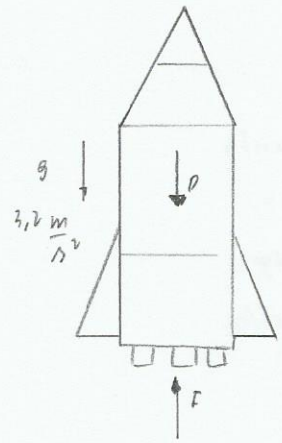
$v'_{By} - v'_{Ay} = -45$

$$\begin{cases} 3v'_{Ay} + 2v'_{By} = 75 \\ -v'_{Ay} + v'_{By} = -45 \end{cases} \sim \begin{cases} 3v'_{Ay} + 2v'_{By} = 75 \\ 5v'_{By} = -60 \end{cases} \quad \therefore v'_{By} = -12 \ m/s$$

$\therefore v'_{Ay} = 33 \ m/s$

$\vec{v_A}' = -26\vec{i} + 33\vec{j} \ (\frac{m}{s}) \quad \vec{v_B}' = -12\vec{j} \ (\frac{m}{s})$

Ex (Prof) 8.22



Dados:

$m = 2t = 2000\ kg$

$F = 3000t^2 + 50t + 80\ (N)$

$g = 3{,}2\ m/s^2$

$v_0 = 250\ m/s$

Questão:

p/ $t = 12s$

$v' = ?$

Solução

$\vec{F}_r = F - P = F - mg$

PIM: * $m\vec{v}_{cm} + \int \vec{F}_r\,dt = m\vec{v}'_{cm}$

$2000 \cdot 250 + \int_0^{12} (3000t^2 + 50t - 6320)\,dt = 2000\,\vec{v}'_{cm}$

$500000 + \left(\frac{3000t^3}{3} + \frac{50t^2}{2} - 6320t \right)\Big|_0^{12} = 2000\,\vec{v}'_{cm}$

$\therefore \vec{v}_{cm} = 1077{,}68\,\vec{j}\left(\frac{m}{s}\right)$

Ex (Prof) 8.23

Dados: $m = 2t = 2000\ kg$

$T = 32000 - 2000\,e^{-t/5}\ (N)$

$v_0 = 180\ \left(\frac{m}{s}\right)$

Questão:

p/ $t = 10s$

$\vec{v}' = ?$

Solução:

PIM: $m\vec{v}_{cm} + \int \vec{F}_r\,dt = mv'_{cm}$

$v'_{cm} = \vec{v}_{cm} + \int \vec{F}_r\,dt \cdot \frac{1}{m}$

$\vec{v}_{cm} = 180 + \int_0^{10} 32000 - 2000\,e^{-t/5}\,dt \cdot \frac{1}{2000}$

$\int e^{-t/5}\,dt = \int e^{u} \cdot (-5)\,du = -5e^{-t/5}$

$-\frac{t}{5} = u$

$-\frac{1}{5}\,dt = du$

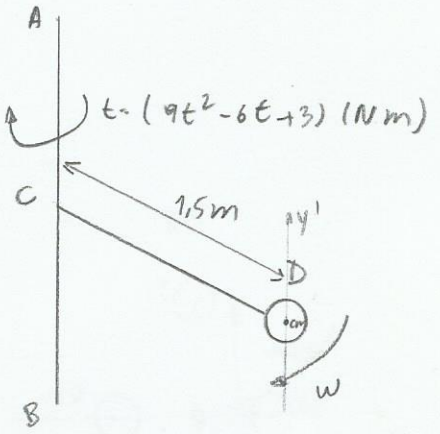
$dt = -5\,du$

$\vec{v}_{cm} = 180 + \left(32000t - 2000 \cdot (-5)\,e^{-t/5} \right)\Big|_0^{10} \cdot \frac{1}{2000}$

$\therefore \vec{v}_{cm} = 335{,}68\,\vec{i}\left(\frac{m}{s}\right)$

Princípio Im da Conservação Momento Angular

Ex: 8.26



Dados:

esfera $\begin{cases} m = 3kg \\ r = 0,5m \end{cases}$

$\overline{CD} = 1,5m$

$\vec{\tau}_y = (9t^2 - 6t + 3)$ (Nm)

$t = 0 \quad v_0 = 5 \frac{m}{s}$

Questão

$p/\ t = 4s: \quad v' = ?$

Solução

PIMA: $\vec{L} + \int \vec{\tau}_r \, dt = \vec{L}'$

$\vec{L} = I_y \vec{\omega}_0 \; ; \; \vec{L}' = I_y \vec{\omega}' \quad \vec{\tau}_r = \vec{\tau}_y$

$I_{\substack{esfera \\ y'}} = \frac{2}{5} m r^2$ (formulário)

$I_{\substack{esfera \\ y}} = \bar{I}_{y'} + m \cdot d^2_{yy'}$ (Teorema dos eixos paralelos)

$= \frac{2}{5} m r^2 + m(CD)^2 = \frac{2}{5} \cdot 3 \cdot 0,5^2 + 3 \cdot 1,5^2 = 7,05 \left(\frac{kg}{m^2}\right)$

$\omega = \frac{v_0}{CD} = \frac{5}{1,5} = 3,33 \left(\frac{rad}{s}\right) \Rightarrow \vec{\omega}' = -3,33 \vec{j} \left(\frac{rad}{s}\right)$

PIMA: $7,05 \cdot (-3,33) \vec{j} + \int_0^4 (9t^2 - 6t + 3) dt \, \vec{j} = 7,05 \cdot \vec{\omega}'$

$-23,5 \vec{j} + \left(3t^3 - 3t^2 + 3t \Big|_0^4\right) \vec{j} = 7,05 \vec{\omega}'$

$-23,5 \vec{j} - 156 \vec{j} = 7,05 \vec{\omega}'$

$\vec{\omega}' = -25,46 \vec{j} \left(\frac{rad}{s}\right)$

$\vec{v}' = \vec{\omega}' \cdot CD$

$= -25,46 \cdot 1,5 \quad \therefore \quad \vec{v}' = 38,19 \frac{m}{s}$

$\vec{p} = m\vec{v}$ $\quad L = I\vec{\omega} = \vec{r} \wedge \vec{p}$

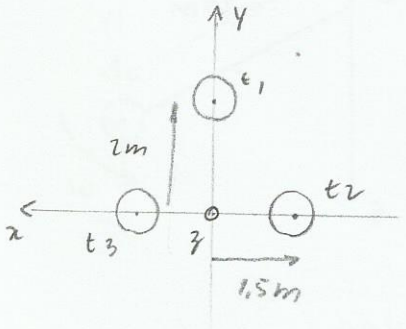
$\vec{F} = m.\vec{a}$ $\quad \vec{G} = I\vec{\alpha} = \vec{r} \wedge \vec{F}$

$\vec{I} = \int F dt$ $\quad \vec{I}^0 = \int G dt = \vec{r} \wedge \vec{I}$

PIM: $\vec{p} + \int \vec{F}_R dt = \vec{p}'$

PIMA: $\vec{L_A} + \int \vec{G}_{r_A} dt = \vec{L_A}'$

Ex: 8.32 pg 167



Dados:

$m = 10\,000$ kg

$K_{Gx} = 15$ m

$K_{Gy} = 11$ m

$\vec{v_G} = 1000(-\vec{k})\left(\frac{m}{s}\right)$

$\vec{F_1} = 1960000\, e^{-t}(-\vec{k})$ (N)

$T_2 = 2420000\, e^{-t}(-\vec{k})$ (N)

Questões:

p/ $t = 10s$:

a) $\vec{v_G}' = ?$

b) $w_x = ?$

c) $w_y = ?$

$\vec{\omega} = ?$

Solução:

PIM: $\underbrace{\vec{p}}_{m\vec{v}_{G_0}} + \underbrace{\int \vec{F_r} dt}_{T_1+T_2} = \underbrace{\vec{p}'}_{m\vec{v}_G'}$

$10000 \cdot 1000(-\vec{k}) + (1960000 + 2420000)(-\vec{k}) \int e^{-t} dt = 10000\, \vec{v_G}'$

$\int e^{-t} dt \sim -\int_0^{10} e^{-t} d(-t) = -e^{-t} = (-e^{-10} + e^0) = 1$

$\vec{v_G}' = 1436(-\vec{k})\left(\frac{m}{s}\right)$

$I = \begin{bmatrix} mk_{Gx}^2 & 0 & 0 \\ 0 & mk_{Gy}^2 & 0 \\ 0 & 0 & I_z \end{bmatrix}$

b) PIMA: $\vec{L_G}^{\,0} + \int \vec{G}_{r_G} dt = \vec{L_G}'$

em x: $+ \int_0^{10} (t_1 \times 2)\, dt = I_x w_x' = (mk_{Gx}^2) w_x'$

$2 \cdot 1960000 \int_0^{10} e^{-t} dt = 10000 \cdot 15^2 \cdot w_x'$

$\therefore w_x' = 1{,}74$ rad/s

em y: $+ \int_0^{10} (t_2 \times 2)\, dt = I_y w_y' = (mk_{Gy}^2) w_y'$

$1{,}8 \cdot 2420000 \int_0^{10} e^{-t} dt = 10000 \cdot 11^2 w_y'$

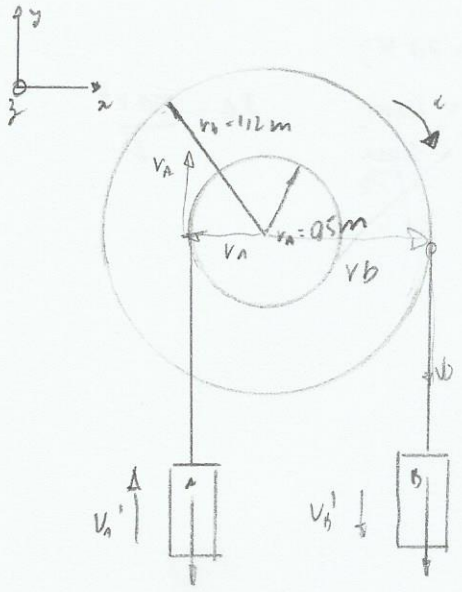
$w_y' = 3$

$\vec{\omega} = 1{,}74\vec{i} + 3\vec{j}\left(\frac{rad}{s}\right)$

8.41

PIM $\vec{p} + \int \vec{F}_r \, dt = \vec{p}'$

PIMA $\vec{L}_A + \int \vec{\tau}_{rA} \, dt = \vec{L}_A'$



Dados:

$\begin{cases} m_{polia} = 50\,kg \\ k_G = 0,6\,m \end{cases}$ (raio de giração)

$\begin{cases} m_A = 30\,kg \\ r_A = 0,5\,m \end{cases}$

$\begin{cases} m_b = 20\,kg \\ r_b = 1,2\,m \end{cases}$

$t = 0 \begin{cases} \omega_0 = 0 \\ v_A = 0 \\ \omega_B = 0 \end{cases}$

Questões

(a) $\vec{I}^{\,0}_{G} = ?$

b) $P/\ t = 5s$

$\vec{\omega}'_{polia} = ?$

Solução

$P_A = m_a g = 30 \cdot 9,8 = 294\ N$

$P_B = m_b \cdot g = 20 \cdot 9,8 = 196\ N$

(Ela gira no sentido horário, porque o I é o sentido $-k$)

(a) $\vec{I}^{\,0}_{G} = \int_0^5 \vec{\tau}_{r_G} \, dt = \int_0^5 (P_A r_A \vec{k} - P_b r_b \vec{k}) \, dt =$

$= (294 \cdot 0,5 - 196 \cdot 1,2) \int_0^5 dt = -441\,\vec{k}\ (N m s)$

(b) PIMA $\cancel{\vec{L}_r^{\,0}} + \int_0^5 \vec{\tau}_{rA} \, dt = \vec{L}_r'$

$-441\,\vec{k} = \vec{L}'_{polia_G} + \vec{L}'_{A_G} + \vec{L}'_{B_G}$

(momento angular)

$\vec{L}_{polia_G} = I_G \vec{\omega}' = (m k_g^2) \vec{\omega}' = (50 \cdot 0,6^2)\,\vec{\omega}' = -18\,\vec{\omega}'\,\vec{k}$

$\vec{L}_{A_G} = \vec{r}_A \wedge \vec{p}_A = (m_A v_A)\, r_A (-\vec{k}) = (30 \cdot \vec{\omega}' \cdot 0,5) \cdot 0,5\,(-\vec{k}) = 7,5\,\vec{\omega}'\,(\vec{k})$

$\vec{L}_{B_G} = \vec{r}_B \wedge \vec{p}_A = (1,2\,\vec{i}) \wedge (m_B v_B (-\vec{j})) = 1,2 \cdot 20 \cdot 1,2\,\vec{\omega}'\,(-\vec{k}) = -28,8\,\vec{\omega}'\,(\vec{k})$

$-441\,\vec{k} = -54,3\,\vec{\omega}'\,\vec{k} \quad \therefore \vec{\omega}' = -8,12\,\vec{k}\ \left(\frac{rad}{s}\right)$

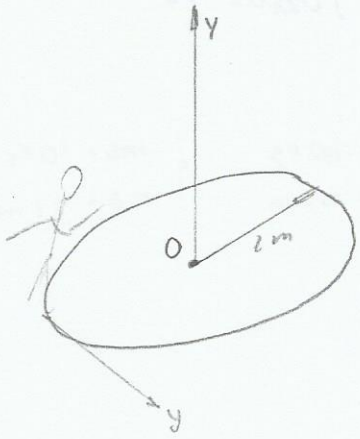
8.42

Dados:

$R = 3m$ $\quad m_H = 70\ kg$

$M = 50\ kg$ $\quad r_H = 2m$ $\quad I_G = \frac{mr^2}{2}$

$t = 0 \quad v_0 = 0 \quad v_H = 3\ m/s$



Questões:

(a) $\vec{L}_{H_O} = ?$

(b) $\vec{\omega}' = ?$

Solução

(a) $\vec{L}_{H_O} = \vec{r}_A \wedge \vec{p}_H = |r_A|\,|p|\,\frac{\sin 90(\vec{k})}{1} = 2.70.3 = 420\,\vec{k}\ (Nms)$

$m_H v_H$

(b) PCMA: $\vec{L}_G^{\,O} + \int \vec{G}_i^{\,O}\,dt = \vec{L}_G^{\,O'}$ (Pois não existe nenhuma força externa)

$\therefore$ constante $= 0$

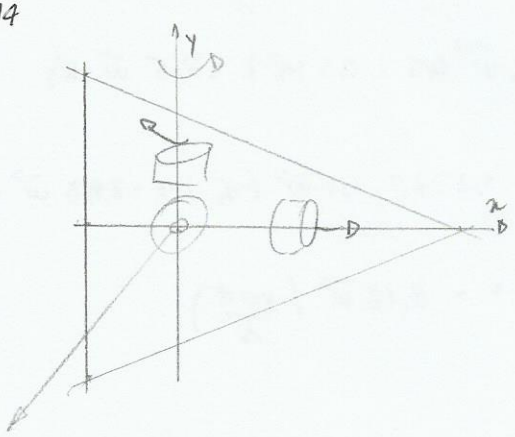
$\vec{L}_{H_i} + \vec{L}_D = 0$

$+420\vec{k} \quad -420\vec{k}$

$\vec{L}_{D_O} = -420\,\vec{k} = I_{G_{disco}}\,\vec{\omega} = \left(\frac{mr^2}{2}\right)\vec{\omega} = \left(\frac{50.3^2}{2}\right)\vec{\omega} = 225$

$\vec{\omega} = \frac{420}{225} = 1,867\ \left(\frac{rad}{s}\right)$

8.44



Dados:

$m_n = 2000\ kg$ $\quad m_v = 50\ kg$

$k_g = 2m$ $\quad r = 0,8m$

$I_{Gy} = \frac{mr^2}{2}$

$\vec{\alpha} = \begin{cases} -100\,\vec{j} \rightarrow 0 < t < 3\,s \\ +100\,\vec{j} \rightarrow 3 < t < 6\,s \end{cases}$

Determinar:

(a) $\vec{\omega}_{motor} = ?$

(b) $\theta_v = ?$

(c) $\theta_N = ?$

Solução

$0 < t < 3s \quad \alpha_V = 100 \frac{vad}{s^2} = cte \quad (MCUV)$

$\omega_V = \omega_{V0} + \alpha_V t$

$\theta_V = \theta_{V0} + \omega_{V0} t + \frac{1}{2} \alpha_V t^2$

Para $t = 3s \rightarrow \omega_{Vmax}$

$\therefore \omega_{Vmáx} = 100 . 3 = 300 \frac{vad}{s} \rightarrow \vec{\omega}_{Vmáx} = -300 \vec{j} \left(\frac{rad}{s}\right)$

PIMA: $\vec{L}_0 + \int \vec{\tau}_V dt = \vec{L}' = \vec{\tau}_c = 0 = \vec{L}_V + \vec{L}_N$

$\vec{L_V} + \vec{L_N} = I_V \vec{\omega}_{V_{MAX}} + I_N \vec{\omega}_{N_{MAX}}$

$I_y^V = \frac{mr^2}{2} = \frac{50 . 0{,}8^2}{2} = 16 \ (kg\,m^2)$

$I_y^N = m k_y^2 = 2000 . 2^2 = 8000 \ (kg\,m^2)$

$\vec{L_V} + \vec{L_N} = 16 . (-300) \vec{j} + 8000 \ \vec{\omega}_N = 0 \quad \therefore \vec{\omega}_{N_{MAX}} = 0{,}6 \vec{j} \left(\frac{vad}{s}\right)$

(b) $\theta_V = \frac{1}{2} \alpha_V t^2$

$\theta_V = \frac{1}{2} . 100 . 3^2 \quad \therefore \quad \theta_V = 450$ } Para apenas 0 a 3s

Para o θ_V total $= 2 . \theta_{V(1/2)}$

$\theta_V = 900 \ vad$

$1 vad = \frac{1}{2\pi}$ volta $\quad \therefore \quad \theta_V = 143{,}24$ voltas

ou $\theta_V = 51\,566{,}20^\circ$

(c) Nave $0 < t < 3s$: MCUV

$\begin{cases} \alpha_A = const \\ \omega_N = \alpha_N t \\ \theta_V = \frac{1}{2} \alpha_N t^2 \end{cases} \rightarrow \alpha_N = \frac{\omega_N}{t}$

Para $t = 3s$, temos $\omega_{N\,max}$

$\alpha_N = \frac{0{,}6}{3} = 0{,}2 \frac{vad}{s^2}$

$\theta_N = \frac{1}{2} . 0{,}2 . 3^2 = 0{,}9 \ vad \quad \therefore \quad \theta_{N_{Total}} = 1{,}8 \ vad$

$\theta_{N\,Total} = 0{,}286$ voltas $= 103{,}13^\circ$

Capítulo 9

Dinâmica de Forças e torques

$\Sigma F_x = m \cdot a_{G_x}$

$\Sigma F_y = m \cdot a_{G_y}$

$\Sigma \tau_{z_G} = I_G \cdot \alpha$

ou $\Sigma \tau_{z_A} = I_G \alpha + \tau_A^{m_{AG}} = I_{A_{fixo}} \alpha$

Ex 9.9 p. 176

Dados:

$m = 1200$ kg

tração dianteira

$\mu_e = 0,4$

$\mu_d = 0,1$

$m_{rodas} \cong 0$

$t = 0 \quad v_0 = 0$

sem escorregar

Questões

(a) Forças nofi

(b) $t_{min} = ?$ p/ $v = 1000 \frac{km}{h}$

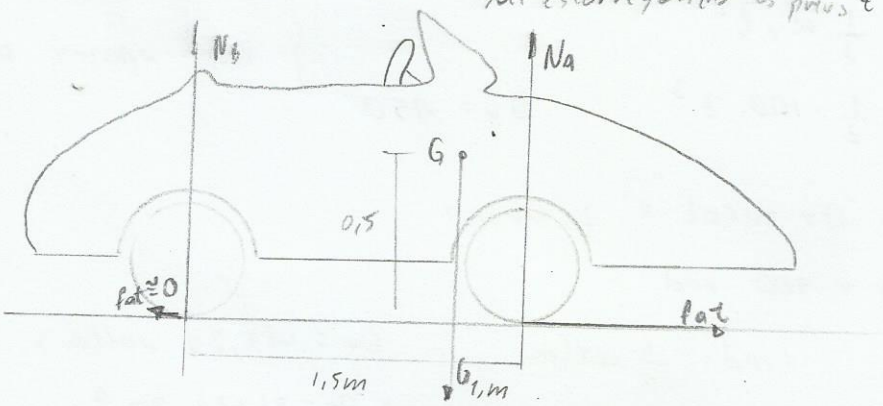
$a_{max} = ?$

(c) $N_A = ?$

$N_B = ?$

(d) tração nas 4 rodas $a = ?$ $v = 100 \frac{km}{h}$

sai escorregando os pneus $t = ?$



Solução

$\Sigma F_x = m \cdot a_{G_x}$ $\quad F_{at} = 1200\, a_{G_x}$

$\Sigma F_y = m \cdot a_{G_y}$ $\quad N_A + N_B - 11760 = 0$

$\Sigma \tau_{z_G} = I_G \alpha$ $\quad N_A \cdot 1 - N_B \cdot 1,5 + F_{at} \cdot 0,5 = 0$

(b) $F_{at(max)} = \mu \cdot N = 0,4\, N_A$

temos: $N_B = 11760 - N_A$

$N_A \cdot 1 - 1,5 \cdot 11760 + 1,5 N_A + 0,4 N_A = 0$

$N_A \quad \frac{1,5 \cdot 11760}{1 + 1,5 + 0,4 \cdot 0,5}$ $\therefore N_A = 6\,533,33$ N $\therefore N_B = 5227$ N

$$a = \frac{Fat}{1200} = \frac{0,4 \cdot 6533}{1200} = 2,178 \frac{m}{s^2}$$

$$v = \cancel{v_0}^{\,0} + at \Rightarrow v = 100 \frac{km}{h} = \frac{100 \cdot 10^3 m}{3600 s} = 27,77 \frac{m}{s}$$

$$t = \frac{v}{a} = \frac{27,77}{2,178} = 12,75 s$$

(d): $Fat_d = Fat_{dA} + Fat_{dB} = \mu_a N_A + \mu_b N_B = \mu_d P = 1576 = 1200\, a$

$$\therefore a = \frac{1576}{1200} = 0,98 \frac{m}{s} \Rightarrow t = \frac{v}{a} = 28,35 s$$

Ex: 9.23

Dados

$m = 120\ kg$

$\mu_e = 0,2$

$\mu_d = 0,15$

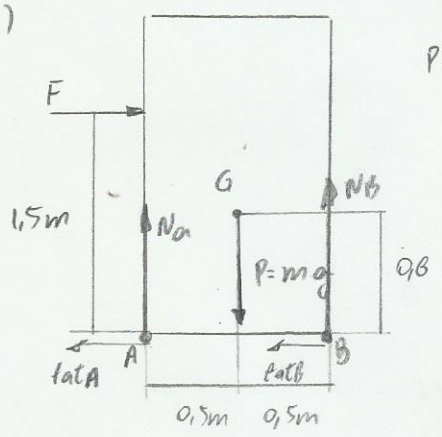
Questões:

(a) Diagrama do corpo livre

(b) tomba antes ou desliza antes $F = ?$

(c) Cai deslizando no limiar de tombar $F = ?$ $a = ?$

(a)



$P = mg = 120 \cdot 9,8 = 1176\ N$

(b)

① $\Sigma F_x = m \cdot a_{G_x} \therefore F - fatA - fatB = m \cdot a_{Gx}$

② $\Sigma F_y = m \cdot a_{Gy} \therefore N_A + N_B - 1176 = 0$

③ $\Sigma \tau_G = I_G \cdot \alpha \therefore 0,5 N_B - 0,5 N_A - 0,8 (fatA + fatB) - F \cdot 0,7 = 0$

(1°) Tomba antes de deslizar: No limiar de tombar: $N_A = 0$; $Fat_A = 0$; (Eq 2) $\Rightarrow N_B = 1176\ N$; $a_{Gx} = 0$; (Eq 1) $F = Fat_B$

(Eq 3) $0,5 \cdot 1176 - 0,8F - 0,7F = 0$

$$F_{\text{limiar de tombar}} = \frac{0,5 \cdot 1176}{0,8 + 0,7} = 392\ N$$

(2°) Desliza antes de tombar: No limiar de deslizar:

$Fat_{max} = Fat_{e\,max\,A} + Fat_{e\,max\,B}: \mu_e (N_A + N_B) = \mu_e B = 0,2 \cdot 1176 = 235,2\ N = F$ (Eq 1)

Como $F_{desliza} = 235,2 < F_{tombar} = 392\ N$ ela desliza antes de tombar

(c) Deslizando: $Fat_d = Fat_{d_A} + Fat_{d_B} = \mu_d (N_A + N_B) = \mu_d \cdot P$

$= 0,15 \cdot 1176 = 176,4\ N$

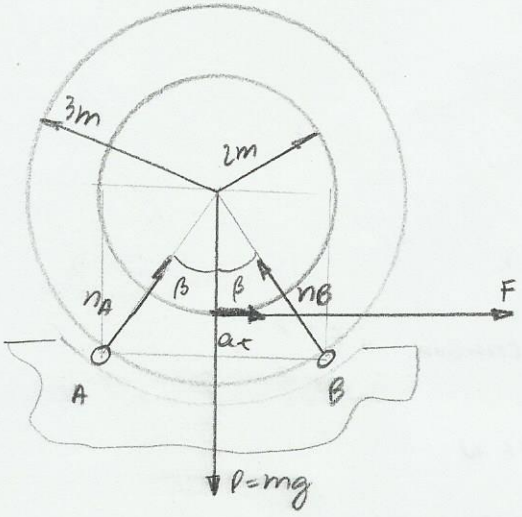
Quando ela estiver no limiar de tombar:

$N_A = 0$; $fat_A = 0$; $N_B = 1176\ N$

(Eq 3) $1176 \cdot 0,5 - 176,4 \cdot 0,8 - F \cdot 0,7 = 0 \quad \therefore F = \underline{638,4\ N}$

(Eq 1) $638,4 - 176,4 = 120 \cdot a \quad \therefore a = \underline{3,85\ \frac{m}{s^2}}$

Ex: 9.38



Dados:

$m = 120\ kg$

$K_G = 2,1\ m$

$\beta = 16°$ c/ vertical

$F = cont$
$s = 20m$
$t = 5\ s$

Questões

(a) $F = ?$

(b) $N_A = ?$

$N_B = ?$

Solução:

$\Sigma F_x = m a_{Gx} = F + n_A \sin\beta - n_B \sin\beta = 0$

$\Sigma F_y = m a_{GH} = n_A \cos\beta + n_B \cos\beta - 1176 = 0$

$\Sigma \tau_{G_z} = I_G \alpha \quad : \quad F \cdot 2 = (m k_G^2) \cdot (+\alpha)$

Como $F = const \therefore a = const$ (MUV)

$s = \cancel{s_0}^{\,0} + \cancel{v_0 t}^{\,0} + \frac{1}{2} a t^2$

$20 = \frac{1}{2} a \cdot 5^2 \quad \therefore a_t = 1,6\ m/s^2 = \alpha r$

$\therefore \alpha = \frac{1,6}{2} = 0,8\ \frac{rad}{s^2}$

$F = \frac{120 \cdot 2,1^2 \cdot 0,8}{2} = \underline{211,68\ N}$

(b)

$$N_A - N_B = \frac{-F}{\sin\beta} = \frac{-211{,}66}{\sin 16} = -767{,}97\ N$$

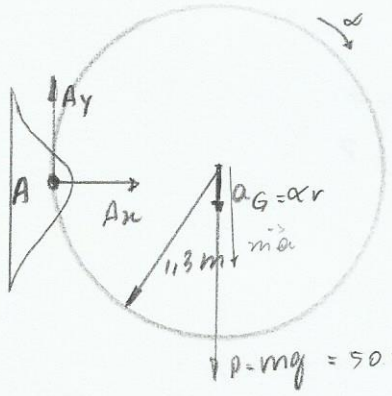
+

$$N_A + N_B = \frac{1176}{\cos 16} = 1223{,}39\ N$$

$$2N_A = 455{,}42 \quad \therefore N_A = \underline{227{,}7\ N}$$

$$N_B = 1223{,}39 - 227{,}7 = \underline{995{,}68\ N}$$

9.46-)



Dados:

$m = 50\ kg$

$t=0 \begin{cases} v=0 \\ \omega=0 \end{cases}$

Questões:

(a) Forças na figura PIT

$\vec{\alpha} = ?$

$\vec{a}_{Gy} = ?$

(b) $\Sigma \tau_{A_z} = I_G \alpha + \tau_{ma}$

(c) $\Sigma \tau_{A_z} = I_A \alpha$

Solução

PFT

$\Sigma F_x = m a_{Gx}$ $\quad A_x = 0$

$\Sigma F_y = m a_{Gy}$ $\quad A_y - 490 = 50.(-a)$ ($a = \alpha r$)

$\Sigma \tau_G = I_G \alpha$ $\quad -A_y . 1{,}3 = \left(\frac{1}{2} m r^2 .\right)(-\alpha)$

$$A_y = \frac{50 . 1{,}3^2}{2} \cdot \frac{\alpha}{1{,}3} = 32{,}5\alpha$$

Substituindo:

$32{,}5\alpha - 490 = -50\alpha r$

$32{,}5\alpha - 490 = -65\alpha \quad \therefore \alpha = \underline{-5{,}03\ rad/s^2} \Rightarrow \vec{\alpha} = \underline{-5{,}03\ \vec{k}}\ \left(\frac{rad}{s^2}\right)$

$a = -5{,}03 . 1{,}3 = -6{,}53\ \frac{m}{s^2} \quad \vec{a} = \underline{-6{,}53\ \vec{j}}\ \left(\frac{m}{s^2}\right)$

b) $\Sigma \tau_{AG} = I_G \alpha + \tau_{ma}$

$-490 . 1{,}3 = 42{,}25\,(-\alpha) - 50\, a_G . 1{,}3$ ($a_G = \alpha r$)

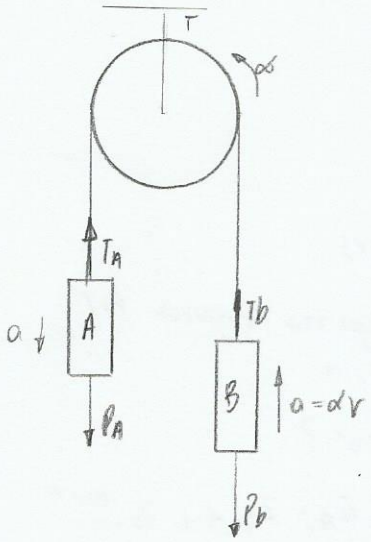
$-490 . 1{,}3 = -42{,}25\alpha - 84{,}5\alpha$

$\therefore \alpha = 5{,}026\ \frac{rad}{s^2}$

(c) $\Sigma \vec{\tau}_{A_G} \cdot I_A \alpha = (\vec{I}_G + m \cdot \vec{d}_{AG}^{\,2}) \cdot (-\alpha) = 42,25 + 50 \cdot 1,3^2 = -126,75\,\alpha$

$-490,13$ $\quad \therefore \alpha = 5,026 \text{ rad/s}^2$

9.48



Dados:

$r_p = 0,6\,m$

$m_p = 6\,kg$

$m_A = 4\,kg$

$m_b = 2\,kg$

Não escorrega $a = \alpha r$

Questões:

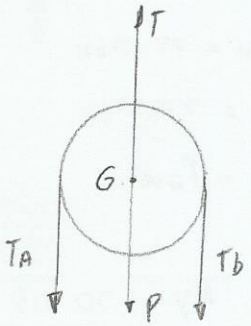
(a) $\vec{a}_A = ?$

(b) $t = 0 \quad v = 0$

$t = 3 \quad v = ?$

Solução

1º método : Isola corpo separadamente:



Corpo A $\quad \Sigma F_y = m a_y \quad T_A - P_A = m_A(-a) \quad (1)$

Corpo B $\quad \Sigma F_y = m a_y \quad T_B - P_B = m_b \cdot a \quad (2)$

Polia $\quad \Sigma \tau_G = I_G \alpha \quad T_A \cdot r - T_B \cdot r = \frac{m r^2}{2} \cdot \alpha \quad (3)$

Eq (3) : $T_A - T_B = \frac{6 \cdot 0,6}{2} \cdot \alpha = 3,2\alpha$ $\qquad P_A = 4 \cdot 9,8 = 39,2\,N$

Eq (1) : $T_A = 39,2 - 4a$ $\qquad P_b = 2 \cdot 9,8 = 19,6\,N$

Eq (2) : $T_B = 19,6 + 2a$ $\quad \Big) T_A - T_B$

$T_A - T_B = 19,6 - 6a = 3,2\alpha$

$19,6 - 0,6 \cdot 6 \cdot \alpha = 3,2\alpha \quad \therefore \alpha = 2,45 \text{ rad/s}^2$

$a = r\alpha = 2,45 \cdot 0,6 \quad \therefore \vec{a} = -1,96\,\vec{j}\ (\text{m/s}^2)$

(b) $\omega = \omega_0^{\,\nearrow 0} + \alpha t$

$\omega = 2,45 \cdot 3 = 7,35\ \text{rad/s}$

2º Método | Sistema como um todo: PFT

$\Sigma F_x = m \cdot a_{Gx}$ $\quad 0 = 0$

$\Sigma F_y = m \cdot a_{Gy}$ $\quad T - P_P - P_A - P_B = -m_a a + m_b \cdot a$

$\Sigma \tau_{Gx} = I_G \alpha$ $\quad P_a r - P_b r = \frac{mr^2}{2} \cdot \alpha + \underbrace{\tau_G^{m_a G}}_{\to m_a a r} + \underbrace{\tau_G^{m_b G}}_{m_b a r}$

$\downarrow \alpha r \qquad \downarrow \alpha r$

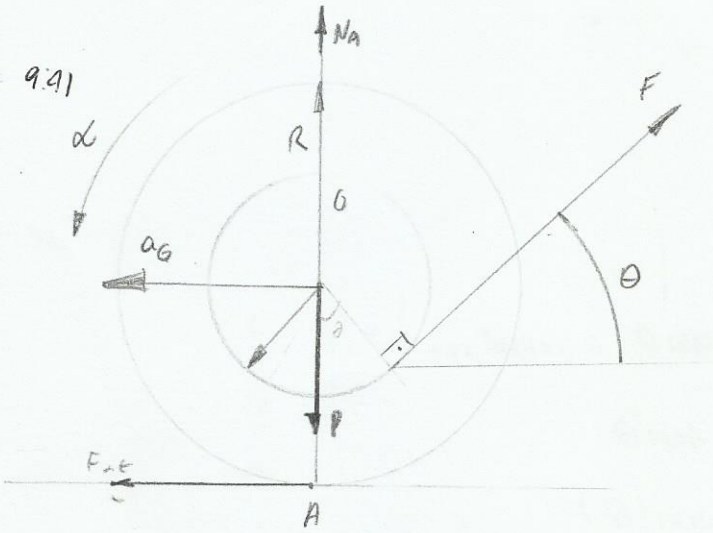
$m_a \alpha r^2 \qquad m_b \alpha r^2$

$I_G = \frac{mr^2}{2} = \frac{8 \cdot 0{,}8^2}{2} = 2{,}56$

$\to 39{,}2 \cdot 0{,}8 - 19{,}6 \cdot 0{,}8 = 2{,}56\,\alpha + \underbrace{4 \cdot 0{,}8^2 \cdot \alpha}_{2{,}56\,\alpha} + \underbrace{2 \cdot \alpha \cdot 0{,}8^2}_{1{,}28\,\alpha}$

$\alpha = \frac{15{,}68}{6{,}4} = 2{,}45 \ \frac{rad}{s}$

Ex. 9.41



Dados
m
R
r
k_G
F
θ

Questões
$\vec{\tau} = ?$
$\vec{F}_{at} = ?$

Observação:
- Quando gira sem escorregar
$v = \omega R$
$a = \alpha R$

$\Sigma F_x = m \cdot a_{Gx}$ $\quad \to F\cos\theta - F_{at} = -m \cdot a_{Gx}$

$\Sigma F_y = m\, a_{Gy}$ $\quad N_A + F\sin\theta - mg = 0$

$\Sigma \tau_{Gz} = I_G \alpha$ $\quad -F_{at} \cdot R + F r = mk^2 \alpha$

$\alpha = \frac{-F_{at} R + F r}{mk^2}$ $\quad F_{at} = F\cos\theta + m \cdot a \ ; a = \alpha r$

$F_{at} = F\cos\theta + m\alpha R$

$\alpha = \frac{-(F\cos\theta + m\alpha r) R + F r}{mk^2}$

$\begin{cases} \alpha > 0 & r > R\cos\theta \ (\text{Desenv.}) \\ \alpha = 0 & r = R\cos\theta \ \text{arrasta} \\ \alpha < 0 & r < R\cos\theta \ (\text{Envol.}) \end{cases}$

$mk^2 \alpha = -F\cos\theta \cdot R - m\alpha R^2 + F r$

$mk^2\alpha + m\alpha R^2 = -F\cos\theta\, R + F r$

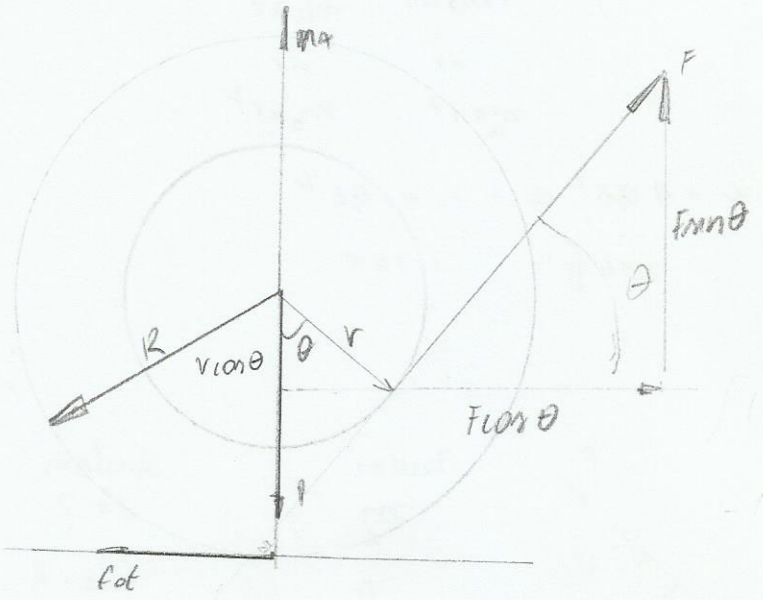
$\alpha = \frac{-F\cos\theta \cdot R + F}{mk^2 + mR^2} = \frac{F(-R\cos\theta + r)}{m(k^2 + R^2)}$ $\quad \therefore \vec{\alpha} = \frac{F}{m} \cdot \frac{(-R\cos\theta + r)}{(k^2 + R^2)} \vec{k}$

$F_{at} = \left[F\cos\theta - \frac{FR(r - \cos\theta)}{k_G^2 + R^2} \right] (-\vec{k}) \ (N)$

Exercícios

Dados: m kg R m F $\vec{\theta}$ roda sem escorregamento

Questão: $\vec{\omega_0} = 0$ $\vec{\omega} = ?$



PIMA: $\vec{L}_A + \int_0^t \vec{\tau}_A \, dt = \vec{L}'_A$

PIMA: $\cancel{\vec{L}_A}^{\,0} + \int \vec{\tau}_A \, dt = \underbrace{\vec{L}'_A}_{I.\omega}$

$$\vec{\tau}_A = \{-(R - r\cos\theta) F\cos\theta\,(\vec{k}) + F\sin\theta \cdot r \sin\theta\,(\vec{k})\} =$$

$$= -FR\cos\theta + Fr\cos^2\theta + Fr\sin^2\theta$$

$$= -FR\cos\theta + Fr(\cos^2\theta + \sin^2\theta)$$

$$= -FR\cos\theta + Fr$$

$\int_0^t \vec{\tau}_A \, dt = I_A \omega$ teorema dos eixos paralelo

$\int -FR\cos\theta + Fr = m(k_g^2 + R^2)\omega$ $\quad I_A = I_G + md_{AG}^2 = mk_G^2 + mR^2$

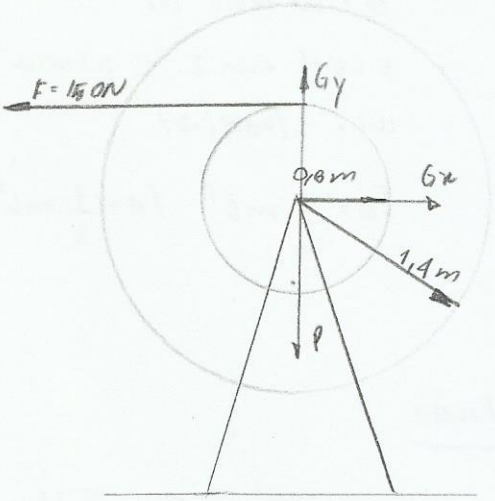
$(-FR\cos\theta + Fr)t = m(k_g^2 + R^2)\omega = m(k_G^2 + R^2)$

$$\vec{\omega} = \frac{(-FR\cos\theta + Fr)\,t}{m(k_g^2 + R^2)}\,\vec{k}$$

Princípio Energia Trabalho

$$T_1 + \Sigma U_{1\to 2} = T_2$$

Ex: 10.8



Dados

F=150N
m=100kg
r=0,8 m
R=1,4 m
k_G=0,9m

Questões:

p/θ=4voltas: $\vec{\omega}'$=?

(a) P.F.T
(b) P.E.T

Soluções:

(a) P.F.T

$\Sigma F_x = m a_{Gx}$: $G_x - 150 = 0$ ∴ $G_x = 150\ N$

$\Sigma F_y = m a_{Gy}$ $G_y - mg = 0$ $G_y = 100 \cdot 9{,}8$ ∴ $G_y = 98N$

$\Sigma \mathcal{G}_G = I_G \cdot \alpha_G$ $F \cdot 0{,}8 = (100 \cdot 0{,}9^2)\,\alpha$ ∴ $\alpha = 1{,}48 \frac{rad}{s^2}$ (MCUV)

$150 \cdot 0{,}8$

$\omega = \omega_0^{\nearrow 0} + \alpha t$

$\theta = \theta_0 + \omega_0^{\nearrow 0} t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^{2\ \nearrow 0} + 2\alpha\Delta\theta$

1 volta = 2π
4 voltas = 8π

$\omega^2 = 2 \cdot 1{,}48 \cdot 8\pi$ ∴ $\omega = 8{,}63 \frac{rad}{s}$ => $\vec{\omega} = 8{,}63\,\vec{k}\ (\frac{rad}{s})$

(b) P.E.T

$T_1^{\nearrow 0} + \Sigma U_{1\to 2} = T_2$

$\Sigma U_{1\to 2} = U^F = F \cdot s = 100 \cdot 4 \cdot 2\pi \cdot 0{,}8 = 3015{,}9\ J$

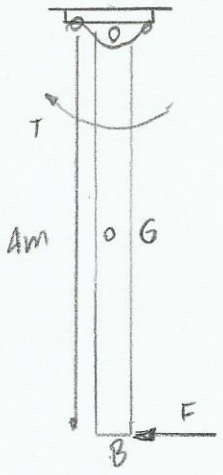
ou

$\Sigma U_{1\to 2} = U_G^F = \mathcal{G}^F \cdot \Delta\theta = (150 \cdot 0{,}8) \cdot 4 \cdot 2\pi = 3015{,}9\ J$

P.E.T $3015{,}9 = \frac{1}{2} I_G \omega^2 = \frac{1}{2}(m k_G^2) \cdot \omega^2 = \frac{1}{2} \cdot (100 \cdot 0{,}9^2) \cdot \omega^2$

∴ $\omega = 8{,}629 \frac{rad}{s}$ => $\vec{\omega} = 8{,}629\,\vec{k}\ (\frac{rad}{s})$

Ex 10.14



Dados:

$\lambda = 2{,}25 \ \frac{Kg}{m}$

$\tau = 20 \ Nm$ cte

$F = 8N$ em B $\perp$ a barra (cte)

$\omega_i = 3 \ (rad/s)$

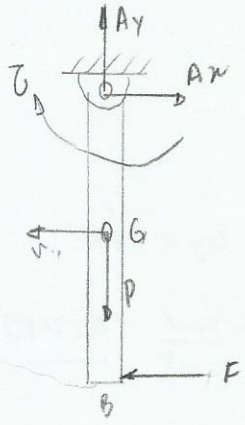
$I_G = \frac{1}{12} mL^2 \quad I_A = \frac{1}{3} mL^2$

Questões:

(a) p/ $\theta = 90°$ $\vec{\omega} = ?$

(b) p/ $\theta = 360°$ $\vec{\omega}' = ?$

Solução



PET: $T_1 + \Sigma U_{1\to 2} = T_2$ — Energia cinética final e inicial

$T_1 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 = \frac{1}{2} I_A \omega^2$

$m = \lambda L = 2{,}25 \cdot 4 = 9 \ Kg$

$P = 9 \cdot 9{,}8 = 88{,}2 \ N$

$T_1 = \frac{1}{2} \left(\frac{9 \cdot 4^2}{3}\right) \cdot 3^2 \quad \therefore \ T_1 = 216 J$

$I_A = 48$

$\Sigma U_{1\to 2} = U^F + U^\tau + U^P$

$U^F = F \cdot s = 8 \left(2\pi \cdot \frac{1}{4} \cdot 4\right) = 50{,}27 \ J$

$U^\tau = \tau \cdot \Delta\theta = 20 \cdot \frac{\pi}{2} = 31{,}4 \ J$

$U^P = -P \cdot 2 = -88{,}2 \cdot 2 = -176{,}4 \ J$

$\Sigma U_{1\to 2} = 50{,}27 + 31{,}4 - 176{,}4 = -94{,}76 J$

PET $\quad 216 - 94{,}76 = \frac{1}{2} I_A \omega'^2 = \frac{1}{2} \cdot 48 \vec{\omega}'^2 \quad \therefore \ \vec{\omega} = -2{,}25 \vec{k} \ \left(\frac{rad}{s}\right)$

(b) p/ $\theta = 360°$

$\Sigma U_{1\to 2} = U^F + U^\tau + U^P = 8 \cdot 2\pi \cdot 4 + 20 \cdot 2\pi + 0 = 326{,}73 \ N$

$216 + 326{,}73 = \frac{1}{2} 48 \ \vec{\omega}'^2 \quad \therefore \ \vec{\omega}' = -4{,}76 \vec{k} \ \left(\frac{rad}{s}\right)$

10.16

Dados:

$m = 15\ kg$

$\overline{AG} = 3,4\ m$

$\vec{C} = -70\vec{k}\ N.m$

$\omega_0 = 8\ rad/s$

$k_G = 2,8\ m$

Determine

(a) $I_A = ?$

(b) T_1

(c) $\sum U_{1\to 2} = ?$ (90°)

(d) $\vec{\omega}' = ?$

(e) $v'_G = ?$

Solução

(a) $I_A = I_G + m.d_{AG}^2 = m k_G^2 + m d_{AG}^2 =$

$= 15(2,8^2 + 3,4^2) = \underline{291\ kg/m^2}$

(b) $T_1 = ?$

$T_1 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 = \frac{1}{2} I_m \vec{\omega}_0^2 = \frac{1}{2} \cdot 291 \cdot 8^2 = \underline{9312\ J}$

(c) $\sum U_{1\to 2} = U^P + U^C = P \times \overline{AG} + C \cdot \frac{\pi}{2} = 15.9,8.3,4 + 70.\frac{\pi}{2} = \underline{609,76\ J}$

(d) $\vec{\omega}' = ?$

PET: $T_1 + \sum U_{1\to 2} = T_2 \Rightarrow 9312 + 609,76 = \frac{1}{2} 291.\omega^2 \therefore \vec{\omega}' = \underline{-8,26\ \vec{k}\ (rad/s)}$

(e) $\vec{v}'_G = \omega \times \vec{AG} = 8,26.3,4 \therefore \vec{v}'_G = \underline{-280,7\ \vec{i}\ (m/s)}$

Integral de linha

$I_1 = \int \vec{v}.d\vec{r}$

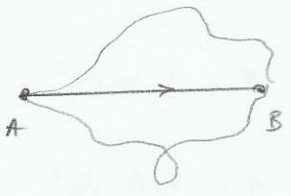
$C_1 = \oint \vec{v}\, d\vec{r} = 0$

$\vec{v}$ é conservativo

I_1 não depende do caminho

∴ depende apenas dos pontos finais e iniciais

Podemos definir para este vetor uma função do ponto escalar.



Função potencial ↳ quando a integral é em função do deslocamento.

↳ $\begin{cases} dU_P = -\vec{v}\, dr \\ v = -\text{grad}\, U_P \\ \text{rot}\, \vec{v} = 0 \end{cases}$

$\tau^F = \int \vec{F}\, d\vec{r} = -\Delta U_p$

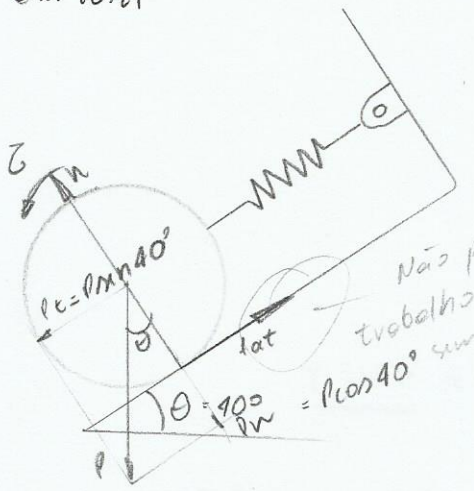
U_p - energia potencial

$\int \vec{E}\, d\vec{r} = -\Delta V$

V = potencial elétrico

$\int \vec{g}\, d\vec{r} = -\Delta V_{grav}$

Ex: 10.21



Dados:

$m = 8\,kg$

$R = 0,3\,m$

$k = 200\,N/m$

$\theta = 40°$

Questão

(a) p/ $s = 0,2\,m$; $v_G = ?$

(b) $v_g = 0$ $s_{max} = ?$

mola e esfera em equilíbrio inicial

$\tau = 12\,Nm = cte$

$I_g = \frac{2}{5} m R^2$

disco sem escorregar

Forças perpendicular ao movimento não realiza trabalho

Solução

(a) $P = mg = 8 \cdot 9,8 = 78,4$ (Equilíbrio)

$P_t = P \sin 40 = 50,39\,N = F_e = kx = 200\,x \quad \therefore \underline{x = 0,252\,m}$

PET: $T_1^{\to 0} + \Sigma U_{1\to 2} = T_2$

$\Sigma U_{1\to 2} = U^{Pt} + U^{\tau} + U^{Fe} = P \times s + \tau \cdot \theta + \left(-\frac{1}{2} k (x_1^2 - x_i^2)\right)$

$= 50,39 \cdot 0,2 + 12 \cdot \frac{0,2}{0,3} - \frac{1}{2} 200 (0,452^2 - 0,252^2)$

$= 10,08\,J + 8\,J - 14,08\,J = 4\,J$

$\tau^{Fel} = -\frac{1}{2} k (x^2 - x_0^2)$

$0,252 + 0,2$

$\theta_{rad} = \frac{arco}{raio} = \frac{s}{R} = \frac{0,2}{0,3}$

PET: $4 = \frac{1}{2} m v_0^2 + \frac{1}{2} I_G w^2 = \frac{1}{2} 8 v_0^2 + \frac{1}{2} \cdot \frac{2}{5} \cdot 8 \cdot 0,3^2 \cdot \left(\frac{v_0}{0,3}\right)^2$

$4 = 5,6\, v_0^2 \quad \therefore \underline{v_0 = 0,845\ \frac{m}{s}}$

10.16

Dados:

$m = 15\ kg$

$\overline{AG} = 3,4\ m$

$\vec{\tau} = -70\ N.m$

$w_0 = 8\ rad/s$

$k_G = 2,8\ m$

Determine

(a) $I_a = ?$

(b) T_1

(c) $\sum U_{1\to 2} = ?$ (90°)

(d) $\vec{w}' = ?$

(e) $v'_G = ?$

Solução

(a) $I_A = I_G + m.d_{Ag}^2 = m k_g^2 + m d_{Ag}^2 =$

$= 15(2,8^2 + 3,4^2) = \underline{291\ kg/m^2}$

(b) $T_1 = ?$

$T_1 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G w_0^2 = \frac{1}{2} I_{A_{fixo}} \vec{w}_0^2 = \frac{1}{2} \cdot 291 . 8^2 = \underline{9312\ J}$

(c) $\sum U_{1\to 2} = U^P + U^\tau = P \times \overline{AG} + \tau . \frac{\pi}{2} = 15 . 9,8 . 3,4 + 70 . \frac{\pi}{2} = \underline{609,76\ J}$

(d) $\vec{w} = ?$

PET: $T_1 + \sum U_{1\to 2} = T_2 \Rightarrow 9312 + 609,76 = \frac{1}{2} 291 . w^2 \therefore \underline{\vec{w} = -8,26\ \vec{k}}\ (\frac{rad}{s})$

(e) $\vec{v}_G = w \times \vec{AG} = 8,26 . 3,4 \therefore \underline{\vec{v}_G = -280,7\ \vec{i}}\ (\frac{m}{s})$

Integral de linha

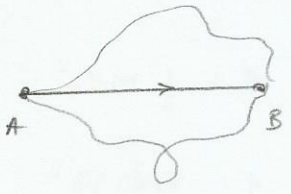
$I_1 = \int \vec{v} . d\vec{r}$

$C_1 = \oint \vec{v}\, d\vec{r} = 0$

$\vec{v}$ é conservativo

I_1 não depende do caminho

∴ depende apenas dos pontos finais e iniciais



Podemos definir para este vetor uma função do ponto escalar.

Função potencial ↳ quando a integral é em função do deslocamento.

↳ $\begin{cases} dU_p = -\vec{v}\, dr \\ v = -grad\ U_p \\ rot\ \vec{v} = 0 \end{cases}$

$\tau^F = \int \vec{F}\, d\vec{r} = -\Delta U_p$

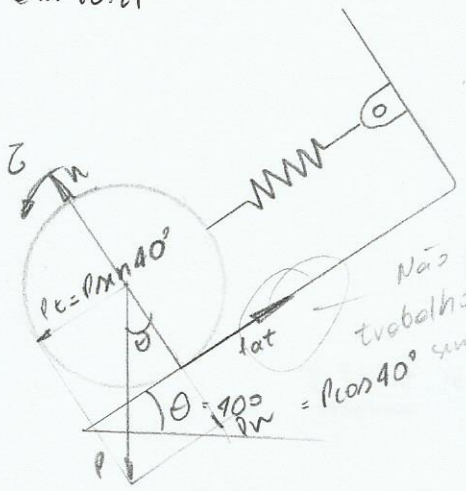
U_p - energia potencial

$\int \vec{E}\, d\vec{r} = -\Delta V$

V = potencial elétrico

$\int \vec{g}\, d\vec{r} = -\Delta V_{grav}$

Ex: 10.21



Não posso trabalho, pois vale sem escorregar

Dados:

$m = 8\ kg$

$R = 0{,}3\ m$

$k = 200\ N/m$

$\Theta = 40°$

Questão

(a) p/ $s = 0{,}2\ m$; $v_G = ?$

(b) $v_g = 0$ $s_{max} = d = ?$

mola e esfera em equilíbrio inicial

$\tau = 12\ Nm = ct.$

$I_g = \frac{2}{5} m R^2$

desce sem escorregar

Forças perpendicular ao movimento não realiza trabalho

Solução

(a) $P = mg = 8 \cdot 9{,}8 = 78{,}4$ (Equilíbrio)

$P_t = P \operatorname{sen} 40 = 50{,}39\ N = F_e = kx = 200\,x \quad \therefore \underline{x = 0{,}252\ m}$

PET: $T_1^{\to O} + \Sigma U_{1\to 2} = T_2$

$\Sigma U_{1\to 2} = U^{Pt} + U^{\tau} + U^{Fe} = P \times s + \tau \cdot \theta + \left(-\frac{1}{2} k (x_1^2 - x_i^2)\right)$

$= 50{,}39 \cdot 0{,}2 + 12 \cdot \frac{0{,}2}{0{,}3} - \frac{1}{2} 200 \left(0{,}452^2 - 0{,}252^2\right)$

$= 10{,}08\ J + 8\ J - 14{,}08\ J = 4\ J$

$\tau^{Fel} = -\frac{1}{2} k (x^2 - x_0^2)$

$0{,}252 + 0{,}2$

$\theta_{rad} = \frac{arco}{raio} = \frac{s}{R} = \frac{0{,}2}{0{,}3}$

PET: $4 = \frac{1}{2} m v_0^2 + \frac{1}{2} I_G w^2 = \frac{1}{2} 8 v_0^2 + \frac{1}{2} \cdot \frac{2}{5} \cdot 8 \cdot 0{,}3^2 \cdot \left(\frac{v_0}{0{,}3}\right)^2$

$4 = 5{,}6\ v_0^2 \quad \therefore \underline{v_0 = 0{,}845\ \frac{m}{s}}$

(b)

PET: $\cancel{T_1}^{\,?} + \sum U_{1\to 2} = \cancel{T_2}^{\,\to 0} = U^{Pt} + U^{Ce} + U^{Fe}$

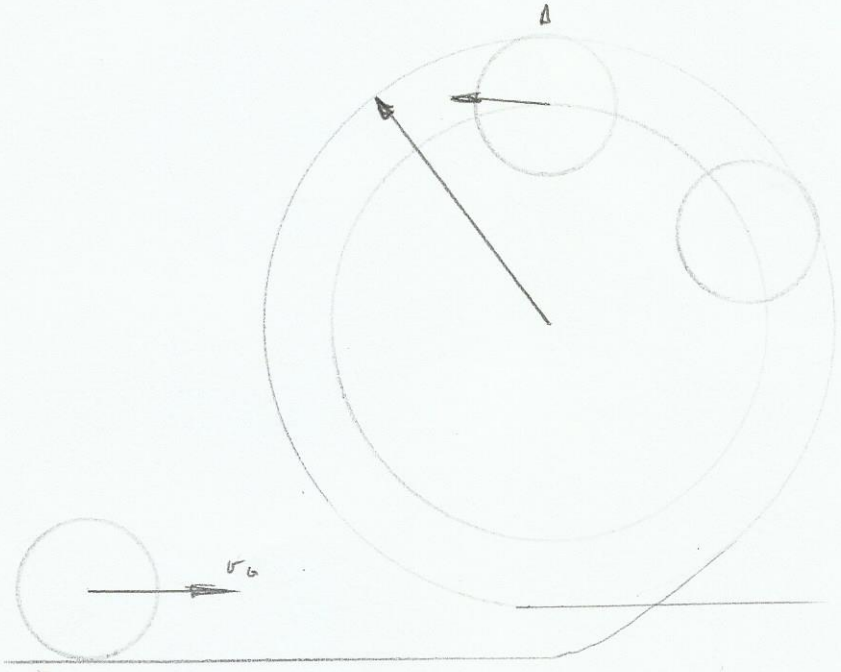
$$= 50{,}39\, d + 12\, \frac{d}{0{,}3} - \frac{1}{2}\, 200 \left[(d+0{,}252)^2 - 0{,}252^2 \right] = 0$$

$$d^2 + 0{,}504 d + \cancel{0{,}252^2} - \cancel{0{,}252^2}$$

$$= -100\, d^2 - 50{,}4\, d + 40 d + 50{,}39\, d = 0$$

$$= -100\, d - 50{,}4 + 40 + 50{,}39 = 0 \quad \therefore \quad \underline{d = 0{,}4\ m}$$

10:27



Dados:

$m = 2\,kg$

$r = 449\,mm = 0{,}449\,m$

rola sem escorregar:

$v_G = \omega r$

$R_{pist.} = 3\,m$

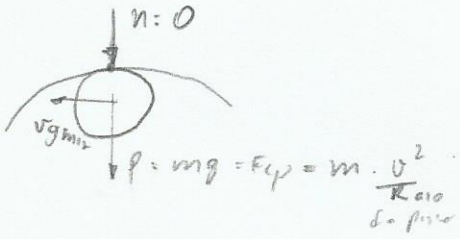
Questões

(a) $v_{G\,min\,topo\,A} = ?$ (não cair antes)

(b) $v_{G\,min}$ na base $= ?$

Solução

(a)



$$P = \cancel{m}g = \cancel{m}\frac{v_G^2}{R} \Rightarrow v_{G\,min} = \sqrt{g \cdot (R - r)} = \sqrt{9{,}6(3 - 0{,}449)} = \underline{5\ m/s}$$

(b) PET: $T_1 + \sum U_{1\to 2} = T_2$

$$T_1 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_g \omega^2 = \frac{1}{\cancel{2}} \cancel{2}\, v_G^2 + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5} \cdot 2 \cdot \cancel{0{,}449^2} \cdot \left(\frac{v_G}{\cancel{0{,}449}}\right)^2 = 1{,}4\, v_G^2$$

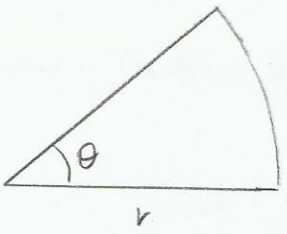
$$T_2 = \frac{1}{\cancel{2}} \cancel{2} \cdot 5^2 + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5} \cdot 2 \cdot \cancel{0{,}449^2} \left(\frac{5}{\cancel{0{,}449}}\right)^2 = 35\,J$$

$$\sum U_{1\to 2} = U^P = -P\Delta y = -2 \cdot 9{,}6(2R - 2 \cdot r) = -19{,}6(2 \cdot 3 - 2 \cdot 0{,}449) = -100\,J$$

PET: $1{,}4 v_G^2 - 100 = 35 \quad \therefore \quad \underline{v_G = 9{,}82\ \frac{m}{s}}$

Resumo

$$v = \frac{ds}{dt} \quad ; \quad a_t = \frac{dv}{dt}$$



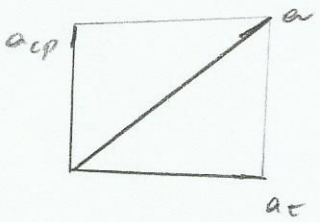
$$\theta = \frac{s}{r} = \frac{\text{arco}}{\text{raio}}$$

Obs.: 1 ciclo = 2π

As leis do movimento uniforme, também são válidas

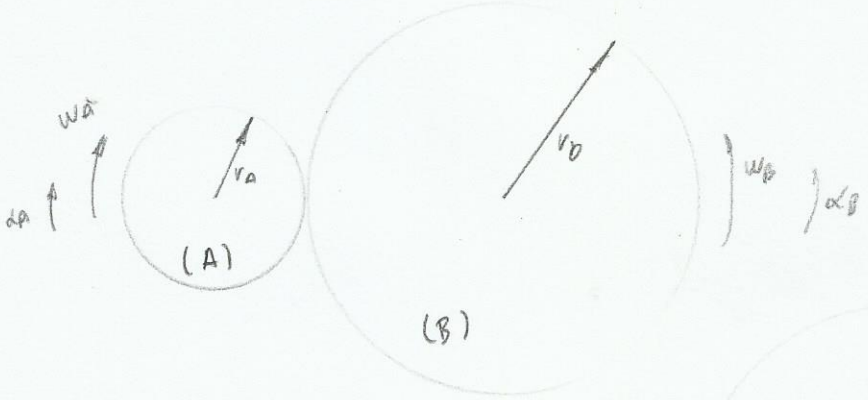
$$\alpha = \frac{a_t}{r} \qquad \omega = \frac{v}{r} \qquad \theta = \frac{s}{r}$$

$$a_{cp} = a_n = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$



$$\alpha = \frac{d\omega}{dt} \qquad \omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt} \omega \quad \therefore \quad \alpha = \omega \frac{d\omega}{d\theta}$$

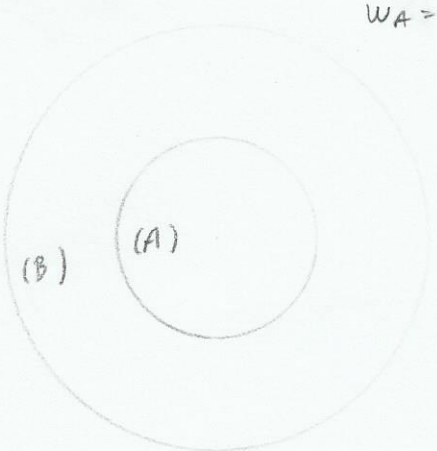


$\theta_A = \theta_B$
$\alpha_A = \alpha_B$
$\omega_A = \omega_B$

$$\theta_a r_a = \theta_b r_b$$

$$\omega_A r_A = \omega_B r_B$$

$$\alpha_A r_A = \alpha_B r_B$$



$$\begin{cases} \vec{v_B} = \vec{v_A} + \vec{\omega}_{BA} \wedge (B-A) \\ \vec{a_B} = \vec{a_A} + \vec{\alpha}_{BA} \wedge (B-A) - \omega_{BA}^2 \, (B-A) \end{cases}$$

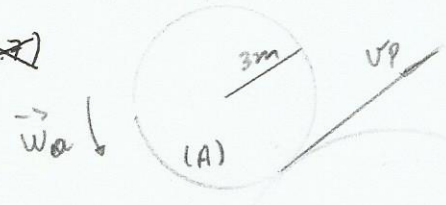
$$\omega = \frac{v_A}{IA} = \frac{v_B}{IB} = \frac{v_C}{IC} = \dots$$

$$\vec{a_B} = \vec{a_A} + \vec{a_{tB}} + \vec{a_{cpB}}$$

Exercícios Propostos

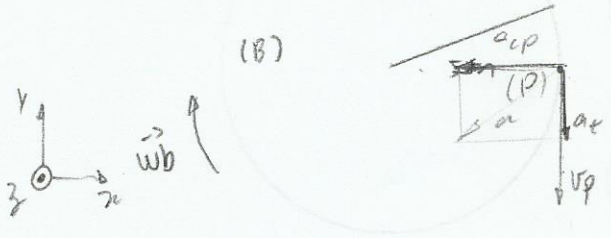
Cinemática Rotacional Escalar (Rotação em torno de um eixo fixo)

(7.6)



$\vec{\omega}_A = 7\vec{k}\ (\frac{rad}{s})$ $\vec{\alpha}_A = 1\vec{k}$

Determine: $\vec{v}$? a_t?



$\omega_A r_A = \omega_B r_B$

$\omega_B = \frac{7.3}{5} = 4,2$

$\omega = \frac{v}{r}$ $\therefore v = \omega r$

$v = 4,2.5 = 21\ m/s$

$\therefore \vec{v}_p = -21\vec{j}\ (\frac{m}{s})$

$a_T = a_n + a_t$

$\alpha_A r_A = \alpha_B r_B$

$4.3 = \alpha_B 5$ $\therefore \alpha_B = 2,4$

$\alpha_B = \frac{a_t}{r}$ $\therefore a_t = 2,4.5 = 12\ \frac{m}{s^2}$ (Acelr. tang)

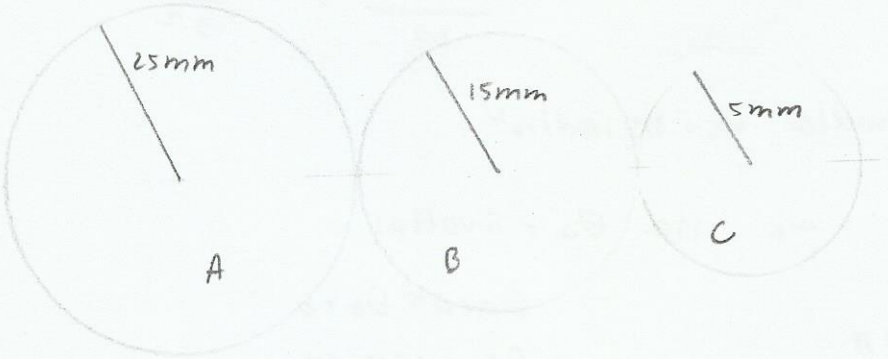
$a_{cp} = \frac{v^2}{r} = \frac{\omega^2 r^2}{r}$

$\therefore a_{cp} = \omega^2 r$

$a_t = 4,2^2.5 = 88,2$

$\vec{a}_p = -88,2\vec{i} - 12\vec{j}\ (\frac{m}{s^2})$

7.7



$\omega_A = 25\ \frac{rad}{s}$ $\alpha_A = 4\ \frac{rad}{s^2}$

Determine:

$\omega_B = ?$ $\omega_C = ?$

$\alpha_B = ?$ $\alpha_C = ?$

$\vec{w_a} r_A = \vec{w_b} r_b$

$\vec{w_b} = \frac{25.25}{15} \therefore \vec{w_b} = -41,67\ \vec{k}\ (\frac{rad}{s})$

$\vec{w_b} r_b = \vec{w_c} r_c$

$w_c = \frac{41,67.15}{5} \therefore \vec{w_c} = 125\ \vec{k}\ (\frac{rad}{s})$

$\vec{\alpha_a} r_a = \alpha_b r_b$

$\alpha_b = \frac{4.25}{15} \therefore \vec{\alpha_b} = -6,67\ \vec{k}\ (\frac{rad}{s^2})$

$\vec{\alpha_b} r_b = \alpha_c r_b$

$\alpha_c = \frac{6,67.15}{5} \therefore \vec{\alpha_c} = 20\ \vec{k}\ (\frac{rad}{s^2})$

(7.8) Dados: $r_A = 20cm$ $r_b = 40cm$ $r_c = 60cm$

$\theta_c = 5 rad$ $w_c = 12 \frac{rad}{s}$ $\alpha_c = 9 \frac{rad}{s^2}$

Determine:
θ_A e $\theta_b = ?$
w_A $w_b = ?$
α_A $\alpha_b = ?$

$\theta_c r_c = \theta_b r_b$

$\theta_b = \frac{5.60}{40} \Rightarrow \vec{\theta_b} = 7,5 rad$

$w_b r_b = w_c r_c$

$w_b = \frac{12.60}{40} = 18 \frac{rad}{s}$

$\theta_b r_b = \theta_a r_a$

$\theta_a = \frac{7,5.40}{20} = 15 rad$

$w_a r_a = w_b r_b$

$w_a = \frac{18.40}{20} = 36 \frac{rad}{s}$

$\alpha_b r_b = \alpha_c r_c$

$\alpha_b = \frac{9.60}{40} = 13,5 \frac{rad}{s^2}$

$\alpha_a r_a = \alpha_b r_b$

$\alpha_a = \frac{13,5.40}{20} = 27 \frac{rad}{s^2}$

(7,9) Dados: $w_{c0} = 6 rad/s$ $\alpha_c = 1,5 rad/s^2$

Determine w_A w_b e w_c no $\theta_a = 5$ voltas

1 volta $= 2\pi$

5 voltas $= 10\pi$

$\theta_a r_a = \theta_b r_b$

$\theta_b = \frac{10\pi.45}{30} = 47,12$

$w_c^2 = w_{c0}^2 + 2 \alpha_c \Delta\theta_c$

$\theta_c r_c = \theta_b r_b$

$w_c^2 = 6^2 + 2.1,5.94,25$

$\theta_c = \frac{47,12.30}{15} = 94,25$

$\therefore w_c = 17,85 rad/s$

$w_c r_c = w_b r_b$

$w_b = \frac{17,85 \cdot 15}{30} \therefore w_b = 8,93 \frac{rad}{s}$

$w_a r_a = w_b r_b$

$w_a = \frac{8,93 \cdot 30}{45} \therefore w_a = 5,95 \frac{rad}{s}$

7.10)

Dados: $r_A = 50mm \quad r_b = 20mm$

$\alpha_A = 7 \cdot \theta_A^2$

Determine.

(a) $\alpha_B = ?$

$\alpha_A r_A = \alpha_B r_b$

$\alpha_b = \frac{7 \cdot \theta_A^2 \cdot 50}{20} \therefore \alpha_b = 17,5 \cdot \theta_a^2$

$\theta_a r_a = \theta_b r_b$

$\theta_a = \frac{\theta_b \cdot 20}{50} = 0,4\,\theta_b \Rightarrow \theta_a = 0,4\,\theta_b$

$\alpha_b = 17,5 \cdot (0,4\,\theta_b)^2$

$\alpha_b = \underline{2,8\,\theta_b^2}$

(b) $w_b(\theta_b) = ?$

$\alpha = \frac{dw}{dt} = \frac{dw}{d\theta} \cdot \underbrace{\frac{d\theta}{dt}}_{w} \Rightarrow \alpha = w \frac{dw}{d\theta}$

$w_b dw_b = \alpha_b d\theta$

$\int w\,dw = \int 2,8\,\theta_b^2 d\theta$

$\frac{w^2}{2} = \frac{2,8\,\theta_b^3}{3}$

$w_b = \underline{1,37 \cdot \theta_b^{3/2}}$

(c) Determine $v_p = ?$ se $r_p = 15cm$

$w = \frac{v}{R} \therefore v = wR$

$v_p(\theta) = 1,37 \cdot \theta_b^{3/2} \cdot 0,15$

$v_p(\theta) = \underline{0,205\,\theta_b^{3/2}}$

7.11 Dados:

$r_b = 0{,}5\,m \quad r_c = 0{,}3\,m \quad r_d = 0{,}25\,m \quad v_{0A} = 0$

Corpo A sobe 6m em 3s a-cte

(a) $\vec{v} = ? \quad a_t = ? \quad a_{cp} = ?$ no ponto B

$v = v_0 + at$

$s = s_0 + v_0 t + \frac{1}{2} a t^2 \qquad 6 = \frac{1}{2} \cdot a \cdot 3^2 \quad \therefore a_t = 1{,}33\ m/s^2$

$v = v_0 + at$

$v = 1{,}33 \cdot 3 = 4\ m/s \quad \therefore \vec{v} = -4\,\vec{i}\ (m/s)$

$a_{cp} = \frac{v^2}{R} = \frac{4^2}{0{,}5} = 32 \quad \therefore \vec{a}_{cp} = -32\,\vec{j}\ (m/s^2)$ e $a_t = -1{,}33\,\vec{i}\ \left(\frac{m}{s^2}\right)$

(b) $\omega_d = ? \quad \alpha_d = ?$

$\omega_c r_c = \omega_d r_d$

$\omega_c = \omega_b = \frac{4}{0{,}5} = 8 \qquad \omega_d = \frac{8 \cdot 0{,}3}{0{,}25} \quad \therefore \omega_d = 9{,}6 \quad \therefore \vec{\omega}_d = -9{,}6\,\vec{k}\ \left(\frac{rad}{s}\right)$

$\alpha_b = \frac{1{,}33}{0{,}5} = 2{,}666$

$\alpha_c r_c = \alpha_d r_d$

$\alpha_d = \frac{2{,}66 \cdot 0{,}3}{0{,}25} \quad \therefore \alpha_d = 3{,}2 \Rightarrow \vec{\alpha}_d = -3{,}2\,\vec{k}\ \frac{rad}{s^2}$

(c) $\vec{\tau}_d = ? \quad m = 2\,kg \quad k = 0{,}6\,m$

$\vec{\tau}_D = I_D \vec{\alpha} \quad ; \quad I = mk$

$\vec{\tau} = 2 \cdot 0{,}6^2 \cdot 3{,}2 \quad \therefore \tau = 2{,}304\ Nm \rightarrow \vec{\tau} = -2{,}304\,\vec{k}\ N.m$

7.12 -) Dados:

$r_b = 0{,}4\,m \quad r_c = 0{,}8\,m \quad r_d = 0{,}5\,m$

Corpo A sobe 20m em 4s, com a=cte

$s = \cancel{s_0}^{0} + \cancel{v_0 t}^{0} + \frac{1}{2} a t^2$

$20 = \frac{1}{2} a 4^2 \therefore a = 2{,}5 \ m/s^2 \therefore \vec{a_t} = -2{,}5 \vec{i} \ (\frac{m}{s^2})$

$v = v_0 + at$

$v = 2{,}5 . 4 \therefore v = 10 \ m/s \therefore \vec{v} = -10 \vec{i} \ (\frac{m}{s})$

$a_{cp} = \frac{v^2}{r} = \frac{10^2}{0{,}4} = 250 \rightarrow \vec{a}_{cp} = -250 \vec{j} \ \frac{m}{s^2}$

(b) $\omega_c = ?$ $\alpha_c = ?$

$\omega_c = \omega_B = \frac{10}{0{,}4} \therefore \omega_c = 25 \quad \vec{\omega} = 25 \vec{k} \ |\frac{rad}{s}|$

$\alpha_c = \alpha_B = \frac{2{,}5}{0{,}4} = 6{,}25 \therefore \vec{\alpha}_c = 6{,}25 \vec{k} \ (\frac{rad}{s^2})$

(c) $\omega_d = ?$ $\alpha_d = ?$

$\omega_c = \omega_b = 25$

$25 . 0{,}8 = \omega_d . 0{,}5 \therefore \omega_d = 40 \rightarrow \vec{\omega}_d = -40 \vec{k} \ (\frac{rad}{s})$

$\alpha_c = \alpha_d = 6{,}25$

$6{,}25 . 0{,}8 = \alpha_d . 0{,}5 \therefore \alpha_d = 10 \rightarrow \vec{\alpha}_d = -10 \vec{k} \ (\frac{rad}{s^2})$

7.13

$\omega_{A0} = 3 \ rad/s$ $\quad r_A = 3m \ r_B = 8m \ r_C = 4m \ r_d = 5m$

$\alpha_A = 2 \ rad/s^2$ $\quad A$: duas voltas $\therefore \theta_A = 4\pi$

(a) $\vec{\omega_A} = ?$

$\vec{\omega_A}^2 = \omega_{A0}^2 + 2 \alpha_A \theta_A$

$\vec{\omega_A}^2 = 3^2 + 2 . 4\pi . 2 \therefore \vec{\omega_A} = 7{,}70 \quad \vec{\omega_A} = -7{,}70 \vec{k} \ (\frac{rad}{s})$

(b) $\vec{\omega_B} = ?$

$\omega_a r_a = \omega_b r_b$

$\omega_b = \frac{7{,}70 . 3}{8} \therefore \vec{\omega}_b = 2{,}89 \therefore \vec{\omega_b} = 2{,}89 \vec{k} \ \frac{rad}{s}$

(c) = $w_C = w_D = 2{,}69\ \vec{k}\ \frac{rad}{s}$

(d) = $w_C r_C = w_d r_d$

$w_d = \frac{2{,}69 \cdot 4}{5} \therefore \vec{w_d} = 2{,}31\ \vec{k}\ (\frac{rad}{s})$

7.14-)

Dados: $w_{A0} = 3\ \frac{rad}{s}$ $\alpha_A = 2\ \frac{rad}{s^2}$ 2 voltas de A $\therefore \theta_A = 4\pi$

$r_A = 3m$ $r_b = 6m$ $r_c = 4m$ $r_d = 6m$

(a) $w_A^2 = 3^2 + 2 \cdot 2 \cdot 4\pi \therefore \vec{w_A} = -7{,}70\ \vec{k}\ (\frac{rad}{s})$

7.15-)

$\alpha_A = 2\ \frac{rad}{s^2}$ $r_A = 25{,}6\ cm$ $r_b = 20cm$ $r_c = 7{,}0\ cm$ $r_d = 2{,}8cm$

$w_{A0} = 0$

(a) $w_B = ?$ no $t = 4s$

$w_A = w_{A0} + \alpha_A t$

$w_A = 0 + 2 \cdot 4 \therefore \vec{w_A} = 8\ \frac{rad}{s}$

$w_A r_A = w_b r_b$

$w_b = \frac{8 \cdot 25{,}6}{20} \therefore \vec{w_b} = 10{,}24\ \vec{k}\ (\frac{rad}{s})$

(b) Quando $\vec{w_c} = 20\ \frac{rad}{s}$, Qual é o nº de voltas

$\alpha_A r_A = \alpha_b r_b$

$\alpha_b = \frac{2 \cdot 25{,}6}{20} \therefore \alpha_b = 2{,}56$

$w^2 = 2 \alpha_c \theta_c \quad 20^2 = 2 \cdot 2{,}56\ \theta_c \therefore \theta_c = 78{,}125$

$\theta_c = 78{,}125$ $1\ vol = 2\pi$

$n = 78{,}125 \therefore n = 12{,}43$ voltas

(c) $t_d = ?$ $\theta_d = 2\pi.100 = 200\pi$

$\alpha_C r_C = \alpha_D r_D$

$\alpha_D = \frac{2,15 \cdot 6.7}{216} = 6,4$

$\theta = \cancel{\theta_0}^{0} + \cancel{\omega_0}^{0} t + \frac{1}{2}\alpha t^2$

$200\pi = \frac{1}{2} \cdot 6,4 \cdot t^2 \quad \therefore \quad t = 14,01\,s$

7.16

Dados: $r_A = 10\,mm \quad r_b = 15mm \quad r_C = 20mm$

$\theta_A = 3\,rad \quad \vec{\omega}_b = 9\vec{i}\ (rad/s) \quad \vec{\alpha}_C = -2\vec{j}\ (rad/s^2) \quad t = ?$

(a) θ_b e θ_c ?

$\theta_A r_A = \theta_b r_b$

$\theta_b = \frac{3.10}{15} \quad \therefore \quad \theta_b = 2\,rad$

$\theta_b r_b = \theta_C r_C$

$\theta_C = \frac{2.15}{20} = 1,5\,rad$

(b) $\vec{\omega}_a = ? \quad \vec{\omega}_b = ?$

$\omega_C r_C = \omega_b r_b$

$\omega_C = \frac{9.15}{20} = 6,75$

7.17.) $\alpha_A = 0,4\,\theta_A^2\ (rad/s^2)$

$\omega_{A0} = 8\ (rad/s) \quad \theta_{A0} = 0$

(a) $\omega_A = ?$

$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt} \quad \therefore \quad \alpha = \omega \frac{d\omega}{d\theta}$

$\omega\, d\omega = 0,4\,\theta_A^2\, d\theta$

$\frac{\omega^2}{2} = \frac{0,4}{3}\theta_A^3 + C$, para $\omega_{A0} = 8$ e $\theta_A = 0 \quad \therefore \quad C = 32$

$$\frac{w^2}{2} = \frac{0,4}{3}\theta_A^3 + 32$$

$$w^2 = 2\left(\frac{0,4}{3}\theta_A^3 + 32\right)$$

$$w^2 = 0,267\,\theta_A^3 + 64 \qquad w_A = (0,267\,\theta_A^3 + 64)^{1/2}$$

(b) $\vec{w_b} = ?$

$$\vec{w_A} v_A = \vec{w_b} v_b \qquad \theta_A v_A = \theta_b v_b$$

$$w_A^2 v_A^2 = w_B^2 v_b^2 \qquad \theta_A = \frac{4}{6}\theta_B$$

$$w_b^2 = \frac{6^2}{4^2}(0,267\,\theta_A^3 + 64) \qquad \theta_A = 0,66\,\theta_B$$

$$w_b = (0,6\,\theta_A^3 + 144)^{1/2}$$

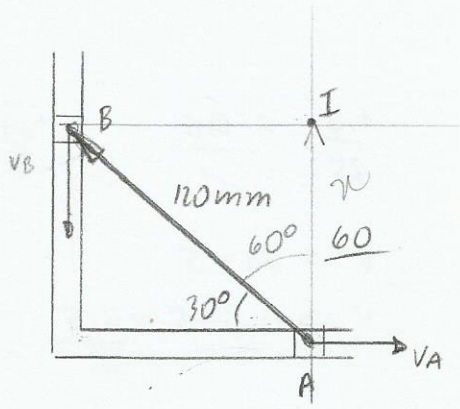
$$w_b = (0,6 \cdot 10,66 \cdot \theta_b)^3 + 144)^{1/2}$$

$$w_b = (0,176\,\theta_b^3 + 144)^{1/2}$$

(c) $w_b(8.2\pi) = (0,176 \cdot (8.2\pi)^3 + 144)^{1/2}$

$$w_b = 150,8 \ \frac{rad}{s}$$

(7.21)



$\vec{v}_a = 3\vec{i}$ (m/s)

$\vec{\omega}_{AB} = ?$

$\cos 60 = \frac{IA}{120} \therefore IA = 60\,mm$

$\omega_{AB} = \frac{3}{IA} = \frac{3}{60.10^{-3}} \therefore \vec{\omega}_{AB} = 50\vec{k}\ \frac{rad}{s}$

$\sin 60 = \frac{IB}{120} \therefore IB = 103{,}92.10^{-3}\,m$

$\omega_{AB} = \frac{v_B}{IB} \therefore v_B = 50.103{,}92\,10^{-3} \therefore \vec{v}_B = -5{,}20\,\vec{j}\ (\frac{m}{s})$

$\vec{v}_B = \vec{v}_A + \omega_{BA} \wedge (B-A)$

$-\vec{v}_B\vec{j} = 3\vec{i} + \omega_K \wedge (-120\cos 30\,\vec{i} + 120\sin 30\,\vec{j})$

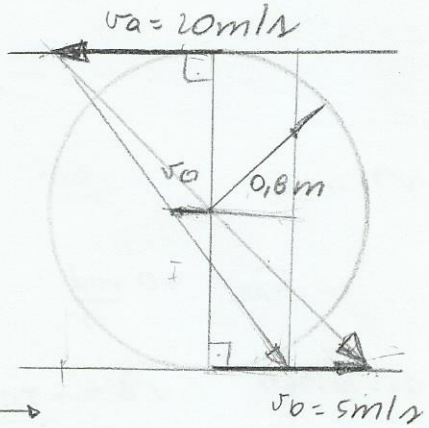
$-\vec{v}_B\vec{j} = 3\vec{i} - 0{,}104\,\omega\vec{j} - 0{,}06\,\omega\vec{i}$

$-v_B = -0{,}104\,\omega$

$0 = 3 - 0{,}06\,\omega \therefore \omega = 50 \therefore \vec{\omega} = 50\vec{k}\ (\frac{rad}{s})$

$v_B = 0{,}104.50 \therefore v_B = 5{,}2 \therefore \vec{v}_B = -5{,}2\,\vec{j}\ (\frac{rad}{s})$

7.22



Determine: $\omega = ?$ $v_b = ?$

$v_b = v_a + \omega_{BA} \wedge (B-A)$

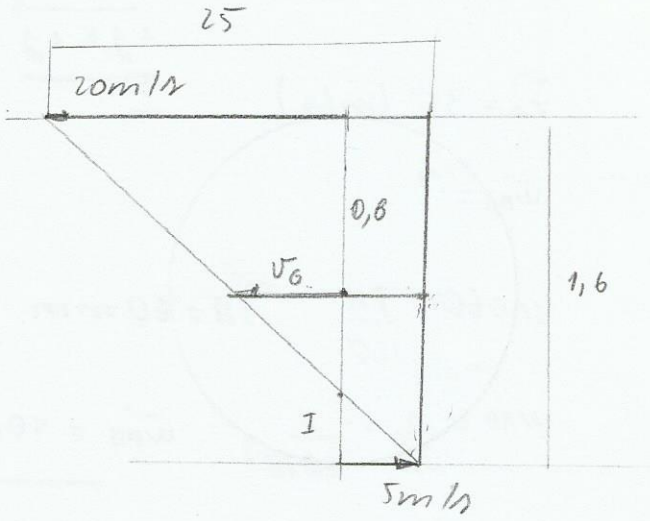
$5\vec{i} = -20\vec{i} + \omega_K \wedge (25\vec{i} - 1{,}6\vec{j})$

$5\vec{i} = -20\vec{i} + 25\omega\vec{j} + 1{,}6\,\omega\vec{i}$

$5 = -20 + 1{,}6\,\omega$

$\vec{\omega} = 15{,}625\ \frac{rad}{s}$

$\frac{1{,}6}{25} = \frac{0{,}8}{v'} \therefore v' = 12{,}5 \therefore \vec{v}_b = -7{,}5\,\vec{i}\ \frac{m}{s}$



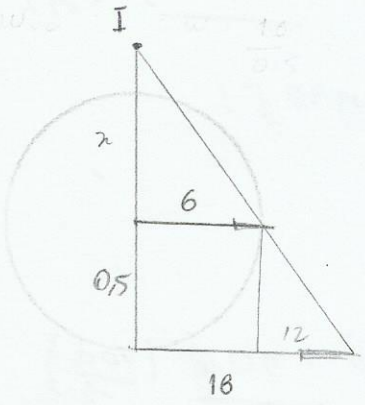
$$\frac{1,6}{25} = \frac{0,8}{v'} \quad \therefore v' = 12,5$$

$$v' = v_0 + 5$$

$$v_0 = 12,5 - 5 = \underline{7,5}$$

7.26

(a) $\omega_t = ?$; $r = 0,5$ m



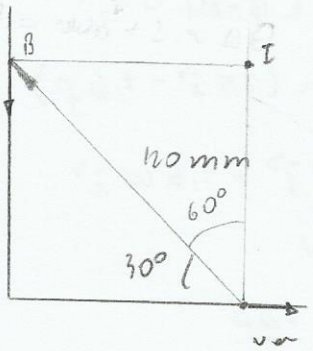
$$\frac{x+0,5}{18} = \frac{0,5}{12} \quad \therefore x = 0,25$$

$$\omega = \frac{18}{0,75} = 24 \quad \therefore \vec{\omega} = \underline{\quad} 24 \frac{rad}{s}$$

Princípio da Instantaneidade da Rotação e Velocidade Relativa

7.21

$\vec{v}_a = 3\vec{i}\ \frac{m}{s}$ $\quad \omega_{AB} = ?$ e $v_0 = ?$



$$IA = 120 \cos 60 = 60 \cdot 10^{-3} = 0,06$$

$$\omega_{AB} = \frac{3}{0,06} = 50 \frac{rad}{s} \quad \therefore \vec{\omega}_{AB} = 50 \frac{rad}{s}$$

$$\omega = \frac{v_B}{I_b} \quad \therefore v_b = 50 \cdot 0,12 \sin 60 \quad \therefore \vec{v}_B = -5,20\vec{j}\ \frac{m}{s}$$

Através da velocidade Relativa

$$v_B = v_A + \omega_{BA} \wedge (B-A)$$

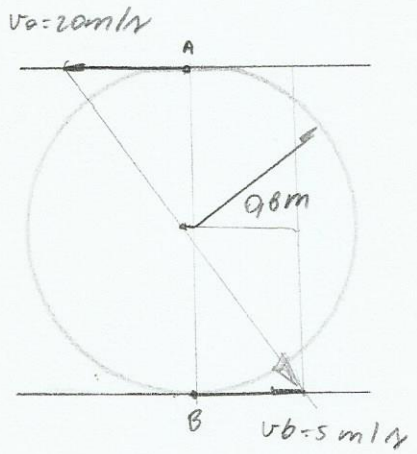
$$-v_B \vec{j} = 3\vec{i} + 50\vec{k} \wedge (-0,12 \sin 60\,\vec{i} + 0,12 \sin 30\,\vec{j})$$

$$-\vec{v_B}\vec{j} = 3\vec{i} - 5,2\vec{j} - 3\vec{i}$$

$$v_B = 5,2\vec{j}$$

7.22

$\vec{w} = ?$ $v_G = ?$



$v_B = v_A + w_{BA} \wedge (B-A)$

$5\vec{i} = -20\vec{i} + w \wedge (25\vec{i} - 1{,}6\vec{j})$

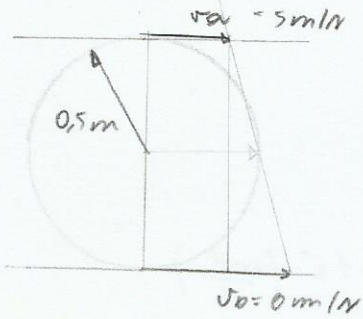
$5\vec{i} = -20\vec{i} + 25w\vec{j} + 1{,}6w\vec{i}$

$5 = -20 + 1{,}6w \quad \therefore w = 15{,}63$

$\therefore \vec{w} = 15{,}63\vec{k}\ (\frac{rad}{s})$

$\frac{25}{1{,}6} = \frac{v_G + 5}{0{,}8} \quad \therefore v_G = 7{,}5 \quad \therefore \vec{v}_G = -7{,}5\vec{i}\ \frac{m}{s}$

7.23



(a)

$w = ?$

$v_B = v_A + w_{BA} \wedge (B-A)$

$8\vec{i} = 5\vec{i} + w_K \wedge (3\vec{i} - 1\vec{j})$

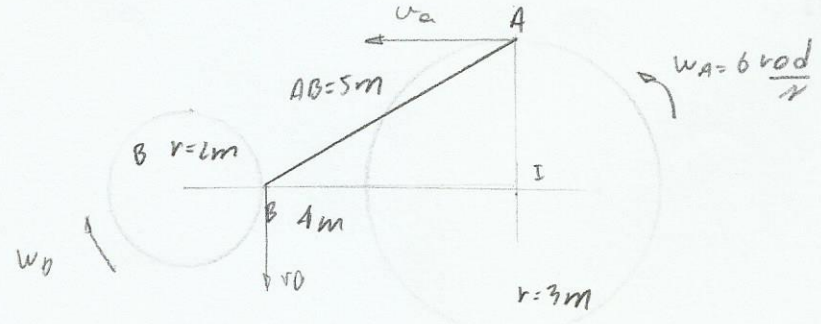
$8\vec{i} = 5\vec{i} + 3w\vec{j} + w\vec{i}$

$8 = 5 + w \quad \therefore w = 3 \quad \therefore \vec{w} = 3 rad/s$

$w = \frac{v_A}{IA} \quad \therefore \quad IA = \frac{5}{3} = 1{,}67\ m$

$w = \frac{v_G}{IG} \quad \therefore \quad v_G = 3.(1{,}67 + 0{,}5) \quad \therefore \vec{v_G} = 6{,}5\vec{i}\ \frac{m}{s}$

7.24



(a) $v_A = ?$

$w_A = \frac{v_A}{r} \quad \therefore v_A = 6.3 = 18\ \frac{m}{s} \quad \therefore \vec{v_A} = -18\vec{i}\ \frac{m}{s}$

$\vec{\omega}_{OB} = ?$

$\vec{\omega}_{OB} = \frac{18}{3} = 6\vec{k}\ \frac{rad}{s}$

(c) $\vec{v}_B = ?$

$v_B = 6 \cdot 4 = 24 \quad \therefore \vec{v}_B = -24\vec{j}\ (\frac{m}{s})$

(d)

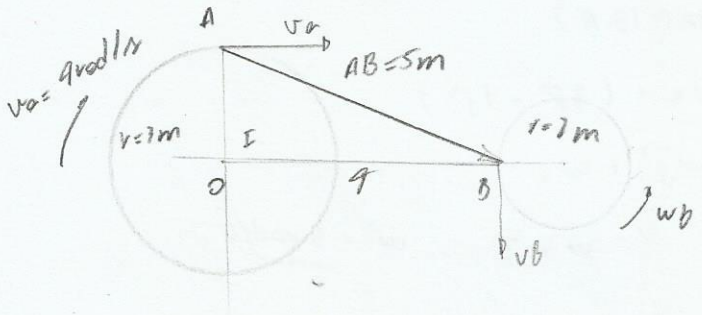
$\omega_b = \frac{v_b}{r_b} = \frac{24}{2} \quad \therefore \vec{\omega}_b = -12\vec{k}\ \frac{rad}{s}$

7.25

(a) $v_a = ?$

$v_a = 4 \cdot 3 = 12 \quad \therefore \vec{v}_a = 12\vec{i}\ m/s$

(b) $\omega_{ab} = ?$



$\omega_{ab} = \frac{v_a}{r_a} = \frac{12}{3}$

$\therefore \vec{\omega}_{ab} = -4\vec{k}\ (\frac{rad}{s})$

(c) $v_b = ?$

$\omega_{ab} = \frac{v_b}{IB} = 4 = \frac{v_b}{4} \quad \therefore v_b = 16$

$v_b = v_a + \omega_{BA} \wedge (B - A)$

$v_b\vec{j} = 12\vec{i} - 4\vec{k} \wedge (4\vec{i} - 3\vec{j})$

$v_b\vec{j} = 12\vec{i} - 16\vec{j} - 12\vec{i}$

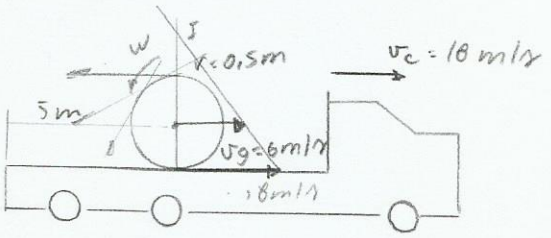
$\vec{v}_b = -16\vec{j}\ (\frac{m}{s})$

$\therefore \vec{v}_b = -16\vec{j}\ (m/s)$

(d) $\omega_b = \frac{v_b}{r_b}$

$\omega_b = \frac{16}{2} = 8 \quad \therefore \vec{\omega}_b = 8\vec{k}\ \frac{rad}{s}$

7.26



(a) $w = ?$

$$\frac{0,5 + w}{18} = \frac{0,5}{12} \quad \therefore \quad w = 0,25$$

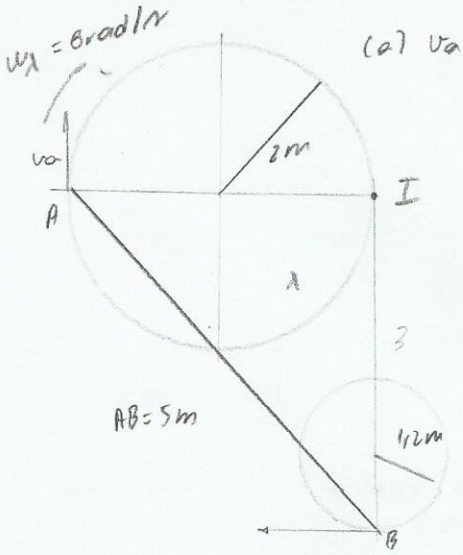
$$w = \frac{18}{0,75} \quad \therefore \quad w = 24 \quad \therefore \quad \vec{w} = 24\vec{k} \ \frac{rad}{s}$$

(b)

$$S = \cancel{S_0^{\rightarrow}} + vt$$

$$5 = 12t \quad \therefore \quad t = 0,417 \ s$$

7.27-)



(a) $v_a = ?$

$$w_1 = \frac{v_a}{a} \quad \therefore \quad v_a = 8.2 = 16$$

$$\therefore \quad \vec{v_a} = 16\vec{j} \ (m/s)$$

b) CIR = I em C

(c) $w_{AB} = ?$ $v_b = ?$

$$w_{AB} = \frac{16}{4} = 4 \quad \therefore \quad \vec{w} = -4\vec{k} \ \frac{rad}{s}$$

$$v_b = 4.3 = 12 \quad \therefore \quad \vec{v_b} = -12\vec{i} \ \left(\frac{rad}{s}\right)$$

(d) $w_b = \frac{12}{1,2} \quad \therefore \quad \vec{w_b} = -10\vec{k} \ \frac{rad}{s}$

7.28

(a) $\vec{w} = ?$

$$\frac{2,5}{x} = \frac{10}{4} \quad \therefore x = 1$$

$$\vec{w} = -\frac{12,5}{5} \quad \therefore \vec{w} = \underline{-2,5\,\vec{k}\ (\frac{rad}{s})}$$

(b) $\vec{v_g} = ?$

$$\frac{v_g}{3} = \frac{2,5}{1} \quad \therefore \vec{v_g} = 7,5\,\vec{i}\ (m/s)$$

7.29

7.30

Dados:

$\vec{w_A} = 4\,\vec{k}$ (rad/s) $\quad r_A = 0,5$ m

$\vec{w_E} = -4\,\vec{k}$ (rad/s) $\quad r_E = 0,9$ m

(a) $v_a = ?$

$$v_a = 4 \cdot 0,5 = 2 \quad \therefore \vec{v_a} = 2\,\vec{i}\ (\frac{m}{s})$$

(b) $v_b = ?$

$$v_b = 4 \cdot 0,9 \quad \therefore \vec{v_b} = -3,6\,\vec{i}\ (\frac{m}{s})$$

(c)

$$v_b = v_a + w_{BA} \wedge (B - A)$$

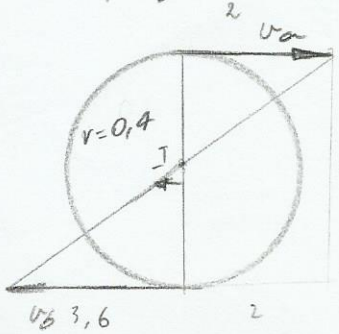
$$-3,6\,\vec{i} = 2\,\vec{i} + w_K \wedge (-5,6\,\vec{i} - 0,4\,\vec{j})$$

$$-3,6\,\vec{i} = 2\,\vec{i} - 5,6\,w\,\vec{j} + 0,4\,w\,\vec{i}$$

$$-5,6 = 0,4\,w \quad \therefore w = -14 \quad \vec{w} = \underline{-14\,\vec{k}\ rad/s}$$

$$w = \frac{v_b}{IB} \quad \therefore IB = \frac{3,6}{14} \quad \therefore \underline{\overline{IB} = 0,257\ m}$$

(d) $w_p = ?$



$$\frac{IB}{3,6} = \frac{0,4}{5,6} \quad \therefore IB = 0,257\ m$$

$$w_p = \frac{3,6}{0,257} = 14\ \frac{rad}{s}$$

$$\frac{0,4}{5,6} = \frac{0,257 - 0,2}{v_g}$$

$$\therefore \underline{v_g = -0,790\,\vec{i}\ (m/s)}$$

7.37.)

$\vec{v}_A = \vec{v}_b + \vec{\omega}_{AB} \wedge (A-B)$

$\vec{v}_A^{\,\to 0} = -v_b \vec{i} + 6\vec{k} \wedge (-3{,}5\vec{j})$

$v_{b_x} = 21$ $\quad \therefore \vec{v}_b = 21\vec{i}\ (m/s)$

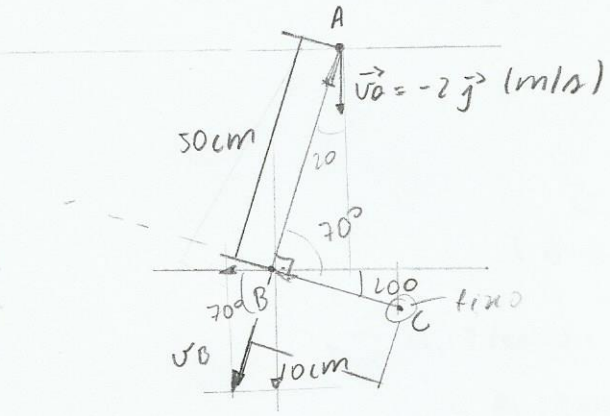
$\omega_p = \frac{v_b}{r_b} \quad \therefore \omega_p = \frac{21}{1{,}5} \quad \therefore \vec{\omega}_p = 14\vec{k}\ (rad/s)$

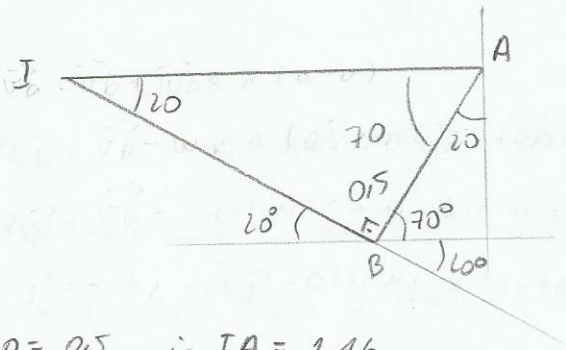
$14 \cdot 1{,}5 = \omega_a \cdot 2 = \omega_e \cdot 3{,}5 \quad \therefore \vec{\omega}_E = 6\vec{k}\ (rad/s)$

7.45.)

Determine:

$\vec{v}_B = ?\quad \vec{\omega}_{AB} = ?\quad \vec{\omega}_{BC} = ?$





$\sin 20 = \frac{0{,}5}{IA} \quad \therefore IA = 1{,}46$

$tg\ 20° = \frac{0{,}5}{IB} \quad \therefore IB = 1{,}37$

$\omega_{AB} = \frac{v_A}{IA} = \frac{2}{1{,}46}$

$\therefore \vec{\omega}_{AB} = -1{,}37\vec{k}\ \left(\frac{rad}{s}\right)$

$\omega_{AB} = \frac{v_B}{IB} \quad \therefore v_b = 1{,}37 \cdot 1{,}37 \quad \therefore v_b = 1{,}87\ m/s$

$\vec{v}_B = -1{,}87 \cdot \cos 70\,\vec{i} - 1{,}87 \cdot \sin 70\,\vec{j}$

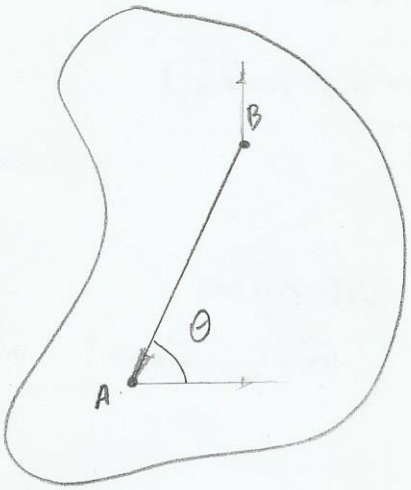
$\therefore \vec{v}_B = -0{,}640\vec{i} - 1{,}76\vec{j}\ (m/s)$

$\omega_{BC} = \frac{1{,}87}{0{,}1} \quad \therefore \vec{\omega}_{BC} = 18{,}7\vec{k}\ \left(\frac{rad}{s}\right)$

Aulvação vlativa

$$\vec{a}_B = \vec{a}_A + \alpha_{BA} \wedge (B-A) - \omega_{BA}^2 (B-A)$$

7.50



$\vec{\omega}_{AB} = 0$ Determine: $\vec{a}_B = ?$ $\vec{\alpha}_{AB} = ?$

$AB = 6m$

$\vec{a}_A = 12\,\vec{i}\ \frac{m}{s^2}$

$\vec{a}_B = a_B \vec{j}$

$\theta = 53{,}13°$

$$\vec{a}_A = \vec{a}_B + \alpha_{AB} \wedge (A-B) - \omega_{AB}^2 (A-B)$$

$$12\,\vec{i} = a_B \vec{j} + \alpha_K \wedge (-6.\cos 53{,}13\,\vec{i} - 6\sin 53{,}13\,\vec{j})$$

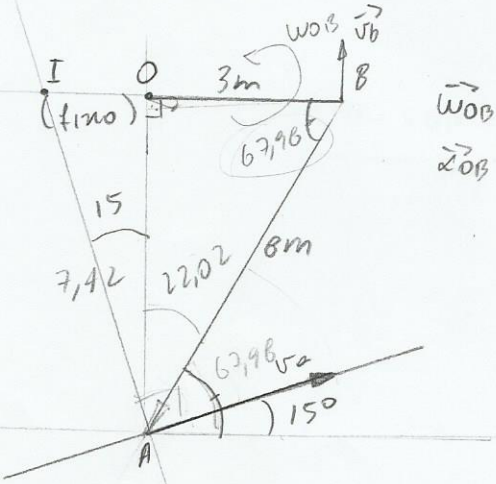
$$12\,\vec{i} = a_B \vec{j} - 6\cos 53{,}13\,\alpha\,\vec{j} + 6\sin 53{,}13\,\alpha\,\vec{i}$$

$12 = 4{,}8\,\alpha$

$0 = a_B - 3{,}6\,\alpha$ $\therefore \alpha = 2{,}5$ $\therefore \vec{\alpha}_{AB} = 2{,}5\,\vec{k}\ \left(\frac{rad}{s^2}\right)$

$\vec{a}_B = 9\,\vec{j}\ \left(\frac{m}{s^2}\right)$

7.53



$\vec{\omega}_{OB} = 2\,\vec{k}\ (rad/s)$

$\vec{\alpha}_{OB} = 0{,}5\,\vec{k}\ (rad/s^2)$

Determine:

(a) $v_b = ?$ (b) $a_A = ?$ $\alpha_{AB} = ?$

$\omega_{OB} = ?$

$\omega_{OB} = \frac{v_b}{BO}$ $\therefore \vec{v}_b = 6\,\vec{j}\ (m/s)$

$tg\,15 = \frac{IO}{AO}$ $\therefore IO = 7{,}42 . tg\,15$

$IO = 1{,}99$

$\therefore IB = 4{,}99$

$\omega_{AB} = \frac{6}{4{,}99}$ $\therefore \vec{\omega}_{AB} = 1{,}2\,\vec{k}\ \frac{rad}{s}$

$\vec{v_a} = \vec{v_b} + \omega_{OB} \wedge (a-B)$

$\vec{v_a} = 6\vec{j} + 1,2\vec{k} \wedge (-3\vec{i} - 7,42\vec{j})$

$\vec{v_a} = 6\vec{j} - 3,6\vec{j} + 8,9\vec{i}$

$\vec{v_a} = 8,9\vec{i} + 2,4\vec{j}$ $\therefore$ $v_a = 9,22\ m/s$

(b) $a_A = ?$

$\vec{a_o} = \vec{a_B} + \vec{\alpha}_{OB} \wedge (a-B) - \omega_{OB}^2 \wedge (a-B)$

$0 = \vec{a_B} + 0,5\vec{k} \wedge (-3\vec{i}) - (2k)^2 \wedge (-3\vec{i})$

$0 = \vec{a_B}\vec{j} + 1,5\vec{j} + 12\vec{i}$

$\therefore \vec{a_B} = 1,5\vec{j}$

$\vec{a_A} = \vec{a_B} + \vec{\alpha}_{AB} \wedge (A-B) + (\omega_{AB})^2 \wedge (A-B)$

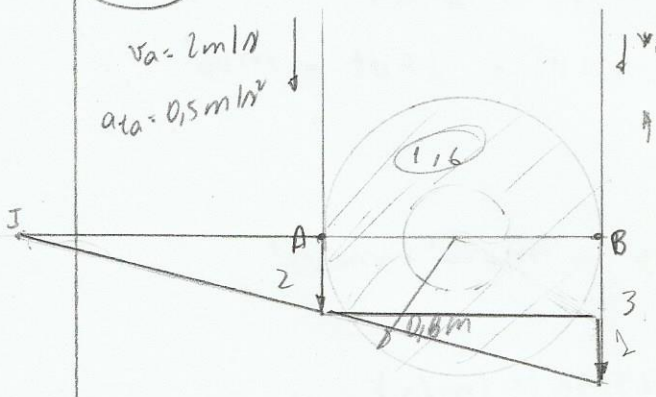
$\vec{a_A} = 1,5\vec{j} + \vec{\alpha}\vec{k} \wedge (-3\vec{i} - 7,42\vec{j}) - (1,2\vec{k})^2 \wedge (-3\vec{i} - 7,42\vec{j})$

$\vec{a_A} = 1,5\vec{j} - 3\alpha\vec{j} + 7,42\alpha i + 4,32\vec{i} + 10,68\vec{j}$

$\vec{a_A} = 7,42\alpha + 4,32$
$\vec{a_A} = 1,5 - 3\alpha + 10,68$ $\vee$ $\begin{cases} a_A - 7,42\alpha = 4,32 \\ a_A + 3\alpha = 12,18 \end{cases}$

$\begin{cases} a_A - 7,42\alpha = 4,32 \\ -10,42\alpha = -16,5 \end{cases}$ $\therefore \alpha = 1,58$ $\therefore \vec{\alpha} = 1,58\vec{k}\ (rad/s^2)$

7.56



Determine:

(a) $\omega_a = ?$ $v_G = ?$

(b) $\alpha = ?$

(1.º Método)

$\frac{1}{1,6} = \frac{2}{x}$ $\therefore x = 3,2$ $\omega_{AB} = \frac{2}{3,2} = 0,625$ $\therefore \vec{\omega} = -0,625\vec{k}\ (rad/s)$

$-2\vec{j} = -3\vec{j} + \omega_k \wedge (-1,6\vec{i})$ $-2j = -3\vec{j} - 1,6\omega\vec{j}$ $\vec{\omega} = -0,625\vec{k}\ (rad/s)$

$IG = IA + 0,8$

$= 3,2 + 0,8 \quad \therefore IG = 4,0$

$\overrightarrow{i\,j\,k\,i\,j}$ (+ / −)

$\omega_G = \frac{v_G}{IG} \quad \therefore v_G = 0,625 \cdot 4 = 2,5 \quad \therefore \vec{v_G} = -2,5\vec{j}\ (m/s)$

(b) $\vec{\alpha} = ?$

$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \wedge (A-B) + (\vec{\omega}_{AB})^2 \wedge (A-B)$

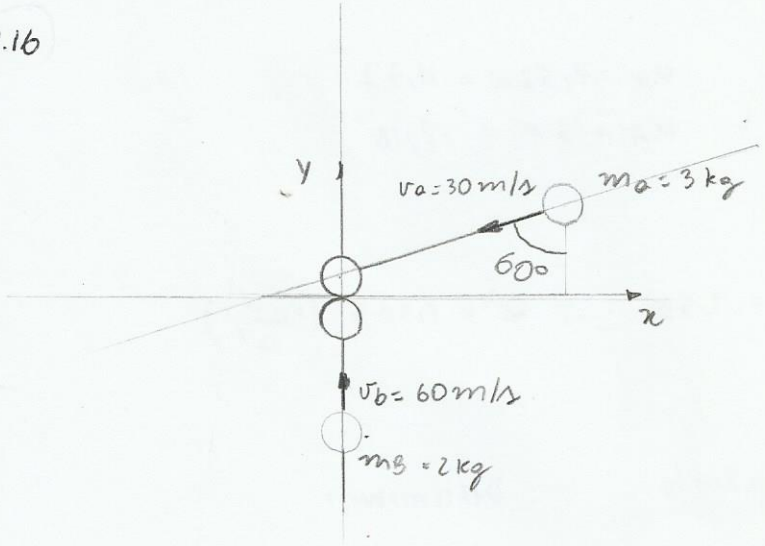
$-0,5\vec{j} = +0,8\vec{j} + \alpha k \wedge (-1,6\vec{i}) + (0,625)^2 \wedge (-1,6\vec{i})$

$-0,5\vec{j} = +0,8\vec{j} - 1,6\alpha\vec{j} - 0,625\vec{i}$

$\alpha = -\frac{(0,5+0,8)}{-1,6} \quad \therefore \vec{\alpha} = 0,813\vec{k}\ \left(\frac{rad}{s^2}\right)$

Dinâmica de Momento e Impulso

8.16



; Sendo $e = 0,6$

Determine $v_a' = ?$ $v_b' = ?$

Princípio Impulso Momentum (PIM)

$\vec{P} + \vec{I} = \vec{P}'$

$m\vec{v_0} + \int F\,dt = mv_0'$

Através do PCM

$\Sigma \vec{p_i} = \Sigma \vec{p_i}' = const \quad \therefore \quad m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$

$\vec{v_a}_x = 30 \cdot \sin 60 = 25,98 \quad \therefore \vec{v_a}_x = -25,98\vec{i}\ (m/s)$

$\vec{v_a}_y = 30 \cos 60 = 15 \quad \therefore \vec{v_a}_y = -15\vec{j}\ (m/s)$

$\vec{v_a} = -25,98\vec{i} - 15\vec{j}$

$\vec{v_b}_y = 60\vec{j}\ (m/s)$

- Na direção y, temos

$3 \cdot (-15) \cdot 2 \cdot 60 = 3v_a'_y + 2v_b'_y \quad | \quad 3v_a'_y + 2v_b'_y = 75$

- Na direção x:

$25,98 \cdot 3 = 3v'_{ax} + 2v'_{bx}$

$3v'_{ay} + 2v'_{by} = 75$

$3v'_{ax} + 2v'_{bx} = 78$

$$e = \frac{v'_{ay} - v'_{by}}{v_{by} - v_{ay}}$$

$$0,6 = \frac{v'_{ay} - v'_{by}}{60 + 15} \quad \therefore \quad v'_{ay} - v'_{by} = 45$$

$$\begin{cases} 3v'_{ay} + 2v'_{by} = 75 \\ v'_{ay} - v'_{by} = 45 \end{cases} \sim \begin{cases} 3v'_a + 2v'_b = 75 \\ 2v'_a - 2v'_b = 90 \end{cases}$$

$5v_a = 165$

$v_a = 33$

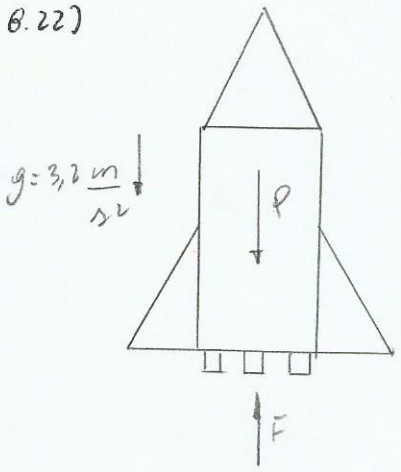
$\therefore \quad v'_{ay} = 33 \frac{m}{s} \quad v'_{by} = -12 \frac{m}{s}$

Sabendo que: $v'_{bx} = 0$ e $v'_{ax} = -26\vec{i}$ (m/s)

$\therefore \quad \vec{v}'_a = -26\vec{i} + 33\vec{j}$ (m/s)

$\vec{v}'_b = -12\vec{j}$ (m/s)

B.22)



$m = 2t = 2000$ kg

$F = 3000t^2 + 50t + 80$ no $t = 12s$

$v_0 = 250$ m/s

Determine a v final

Usando o PIM

$\vec{P}_0 + \int F dt = \vec{P}'_0$

$2000 \cdot 250 + \int_0^{12} (3000t^2 + 50t + 80 - 2000 \cdot 3,2)\, dt = 2000 \cdot v$

$5 \cdot 10^5 + (1000t^3 + 25t^2 - 6320t)\Big|_0^{12} = 2000\, v$

$\therefore v = 1077,68$ m/s

$\therefore \vec{v} = 1078\vec{j}$ (m/s)

B.23

$m = 2t = 2000\ kg$

$T = 32000 - 2000\, e^{-t/5}\ (N)$

$v_0 = 180\ m/s$

$\Delta t = 10s$

$$\int 32000 - 2000\, e^{-t/5}\, dt = 32000 \int dt - 2000 \int e^{-t/5}\, dt$$

$$\int e^{-t/5}\, dt \qquad u = -t/5 \qquad \int e^{u} \cdot (-5)\, du = -5 e^{-5/t}$$

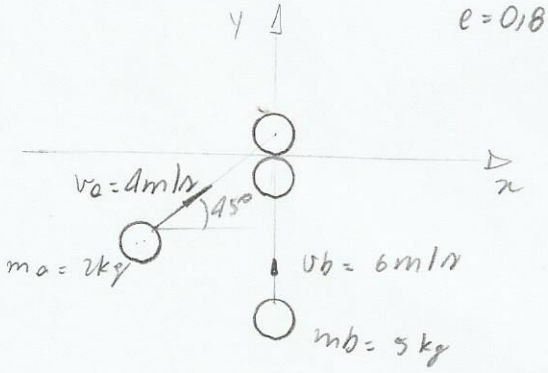
$$du = \frac{-1}{5} dt$$

$$dt = -5\, du$$

$$I = 32000 \cdot t + 2000 \cdot 5\, e^{-t/5} \Big|_0^{10} \qquad I = 3{,}2 \cdot 10^5$$

$$2000 \cdot 180 + 3{,}2\,10^5 = 2000 \cdot v \qquad \therefore \underline{v = 340\ m/s}$$

B.17-)



$e = 0{,}8$

$\vec{v_a} = 2{,}83\vec{i} + 2{,}83\vec{j}\ (m/s)$

$\vec{v_b} = 6\vec{j}\ (m/s)$

Na direção x, temos:

$v_b'x = 0 \qquad v_a'y = 2{,}83\vec{i}\ (m/s)$

Na direção y, temos:

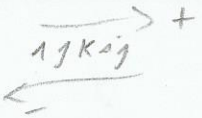
$$2{,}83 \cdot 2 + 5 \cdot 6 = v_a'y \cdot 2 + v_b'y \cdot 5 \quad \therefore \quad 2v_a'y + 5v_b'y = 35{,}66$$

$$0{,}8 = \frac{v_a'y - v_b'y}{6 - 2{,}83} \quad \therefore \quad v_a'y - v_b'y = 2{,}54$$

$$\begin{cases} 2v_a'y + 5v_b'y = 35{,}66 \\ v_a'y - v_b'y = 2{,}54 \end{cases} \sim \begin{cases} 2v_a'y + 5v_b'y = 35{,}66 \\ 7v_b'y = 30{,}58 \end{cases} \quad \therefore v_b'y = 4{,}37\ m/s \quad v_a'y = 6{,}91\ m/s$$

$\vec{v_a} = 2{,}83\,\vec{i} + 6{,}91\,\vec{j}$ (m/s)

$\vec{v_b} = 4{,}37\,\vec{j}$ (m/s)



8.21-)

$m = 5$ kg e $3m$

$I = \frac{m\ell^2}{3}$ e $\vec{\omega} = (2t^3 - 4t^2 + 6)\,\vec{k}$ $\left(\frac{rad}{s}\right)$

Determine para $t = 2s$, o momento angular $\vec{L}$ e o torque $\vec{\tau}$

Momento angular spin

$\vec{S} = I\,\vec{\omega}$

$S = \frac{m\ell^2}{3} \cdot (2t^3 - 4t^2 + 6)$

$= \frac{5 \cdot 3^2}{3} \cdot (2 \cdot 2^3 - 4 \cdot 2^2 + 6) = 90 \quad \therefore \quad \vec{S} = 90\,\vec{k}\ \left(\frac{kg \cdot m^2}{s}\right)$

~~B20~~ $\vec{\tau} = I \cdot \vec{\alpha}$ $\qquad \vec{\alpha} = \frac{d\omega}{dt} = 6t^2 - 6t$

$\tau = \frac{m\ell^2}{3} \cdot (6t^2 - 6t)$

$= \frac{5 \cdot 3^2}{3} \cdot (6 \cdot 2^2 - 6 \cdot 2) \quad \therefore \quad \vec{\tau} = 120\,\vec{k}\ \frac{kg \cdot m^2}{s}$

8.20

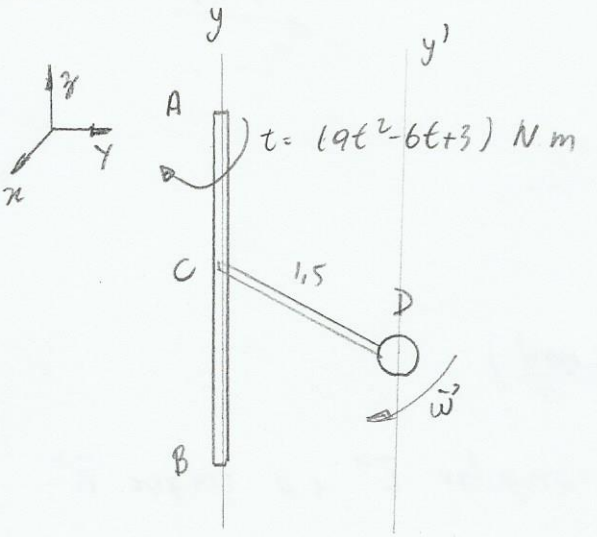
$L = \vec{r} \wedge \vec{p}$

$\vec{L_o^A} = 2\,\vec{j} \wedge (6\,\vec{i} \cdot 4) \quad \therefore \quad \vec{L_o^A} = 48\,\vec{k}\ \left(\frac{m^2 \cdot kg}{s}\right)$

$\vec{L_o^B} = (-5\,\vec{i} \wedge (10 \cdot 1{,}5\,\vec{k}))\,\vec{j} + (-4\,\vec{i} \wedge (10 \cdot 1{,}5\,\vec{k}))\,\vec{j}$

$\vec{L_o^B} = -75\,\vec{j} + 60\,\vec{i}$

8.26-)



$m = 3$ kg e $r = 0{,}5$ m

no $t = 0$ e $v_0 = 5$ m/s

Determine para $t = 4$ s a $v_{es} = ?$

PIMA

$$\vec{L} + \underbrace{\int_t^{t'} \vec{\tau}\, dt}_{I} = L'$$

$$\vec{L} = I_y \vec{\omega}_0 \qquad \vec{L}' = I_y \vec{\omega}' \qquad \vec{\tau}_y = \vec{\tau}_{y'}$$

$$I_{(esfera)} = \frac{2}{5} m r^2$$

$$I_y = I_{y\,(esfera)} + d_{CD}^2 \cdot m = \frac{2}{5} \cdot 3 \cdot 0{,}5^2 + 1{,}5^2 \cdot 3 = 7{,}05 \left(\frac{kg}{m^2}\right)$$

$$\vec{\omega}_0 = \frac{\vec{v}_0}{CD} = \frac{5}{1{,}5} \quad \therefore \quad \vec{\omega}_0 = -3{,}33\,(R)\,\frac{rad}{s}$$

$$\vec{L} + \vec{I}^{\theta} = \vec{L}'$$

$$I_y \omega_0 + \int_{t_1}^{t_2} 9t^2 - 6t + 3 \; dt = I_y \omega'$$

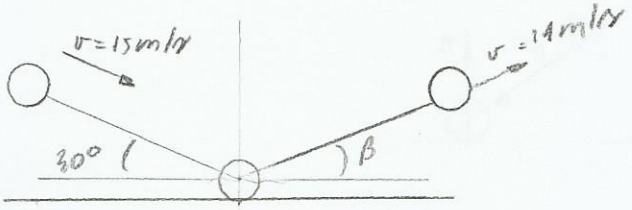
$$7{,}05 \cdot (-3{,}33\,\vec{k}) + -(3t^3 - 3t^2 + 3t)\Big|_0^4 \vec{k} = 7{,}05 \cdot \omega'$$

$$\therefore \quad \omega' = -25{,}46$$

$$\vec{\omega}' = \frac{\vec{v}'}{CD} \quad \therefore \quad \vec{v}' = \underline{38{,}19 \text{ m/s}}$$

8.15-)

$m = 0{,}2\ kg$



a) $\vec{v}$? e $\vec{p}$?

$$p = p'$$

$$m \cdot v = m' \vec{v}$$

$$0{,}2 \cdot (15\cos 30\,\vec{i} - 15\sin 30\,\vec{j}) = 0{,}2\,\vec{v}$$

$$\frac{2{,}598\,\vec{i} - 1{,}5\,\vec{j}}{0{,}2} = \vec{v} \quad \therefore \quad \vec{v} = 13\,\vec{i} - 7{,}5\,\vec{j}\ (m/s)$$

$$\vec{p} = m\vec{v}$$

$$= 0{,}2 \cdot (13\,\vec{i} - 7{,}5\,\vec{j}) \quad \therefore \quad \vec{p} = 2{,}6\,\vec{i} - 1{,}5\,\vec{j}\ (kg\,m/s)$$

(b) $\vec{v}'$ = ? $\vec{p}'$?

$$\vec{v}'_x = 13\,\vec{i}$$

$$v' = \sqrt{\vec{v}'_x{}^2 + \vec{v}'_y{}^2} \quad \therefore \quad 14 = \sqrt{13^2 + v_y} \quad \therefore v'_y = 5{,}20$$

$$\therefore \quad \vec{v}' = 13\,\vec{i} + 5{,}20\,\vec{j}$$

$$\beta = \operatorname{arctg}\left(\frac{5{,}20}{13}\right) \quad \therefore \beta = 21{,}790$$

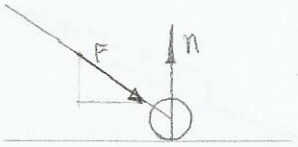
$\vec{p}$ = ?

$$\vec{p} = m\vec{v} = 0{,}2 \cdot (13\,\vec{i} + 5{,}20\,\vec{j})$$

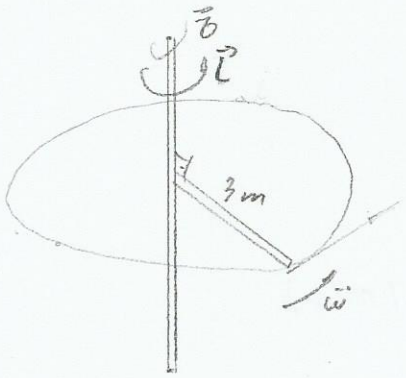
$$\therefore \quad \vec{p} = 2{,}6\,\vec{i} + 1{,}04\,\vec{j}\ (kg\,m/s)$$

(c)

$$e = \frac{v_{By} - v_{Ay}}{v_{Ay} - v_{By}} = \frac{5{,}2 - 0}{0 - (-7{,}5)} \quad \therefore e = 0{,}693$$



8.21



$\vec{L} + \vec{I}^{\theta} = L'$

$\vec{L} + \int_{t}^{t'} \vec{\tau}\, dt = \vec{L}'$

$m = 5kg$ e $3m$

$I = \frac{m\ell^2}{3}$ $\vec{\omega} = (2t^3 - 4t^2 + 6)\vec{k}$

Determine para $t = 2$ o $\vec{L}$ e $\vec{\tau}$.

$\vec{L} = \vec{r} \wedge \vec{p}$ ou $\vec{S} = I\vec{\omega}$

$\vec{L} = \frac{3^2 5}{3} \cdot 6 = 90$ $\therefore$ $\vec{L} = 90\vec{k}$ $\left(\frac{kgm^2}{s}\right)$

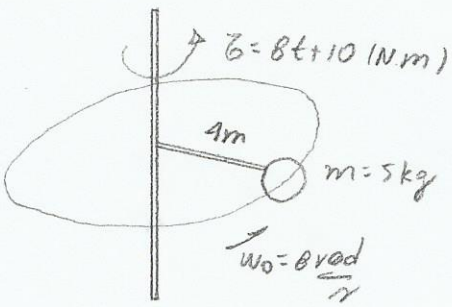
$I^{\theta} = \int_{t}^{t'} \vec{\tau}\, dt = \int_{t_1}^{t'} I\alpha\, dt = \int_{t_1}^{t_2} \frac{m\ell^2}{3} \cdot (6t^2 - 6t)\, dt$

$I^{\theta} = \frac{m\ell^2}{3} \cdot (2t^3 - 4t^2) = \frac{5.3^2}{3} \cdot (2.2^3 - 4\,2^2) = 0$

$\vec{\tau} = I\alpha$

$= \frac{m.\ell^2}{3} \cdot (6t^2 - 6t) = \frac{5.3^2}{3} \cdot (6.2^2 - 6.2)$ $\therefore$ $\vec{\tau} = 120\vec{k}$ (Nm)

8.14-)



Determinar L' para t e $t = 4$ (a)

(b) $\vec{\omega}' = ?$ Admitindo $k = 4$

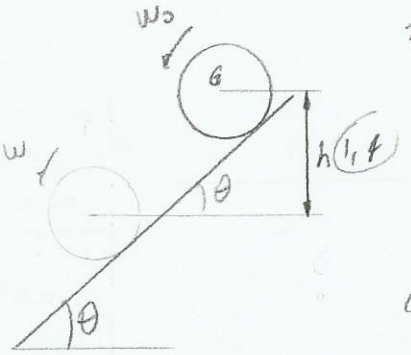
PIMA

$L + I^{\theta} = L'$

$I = mk^2 + m.d^2 = 80$

$L' = (4t^2 + 10t + 680)\vec{k}$

B.27-)



$m = 2kg \quad r = 1,2m$

$I_g = \frac{2mr^2}{5}$

$v_{G0} = 3 m/s$

(a) $w_0 = ?$ (b) $w = ?$ quando $h = 1,4$ m

(a) $w_0 = \frac{v_0}{r} = \frac{3}{1,2} \therefore w_0 = 2,5 \therefore \vec{w_0} = 2,5 \vec{k} \ (\frac{rad}{s})$

(b)

$\vec{L} = \frac{2.2.(1,2)^2}{5} . 2,5 \therefore \vec{L} = 2,88$

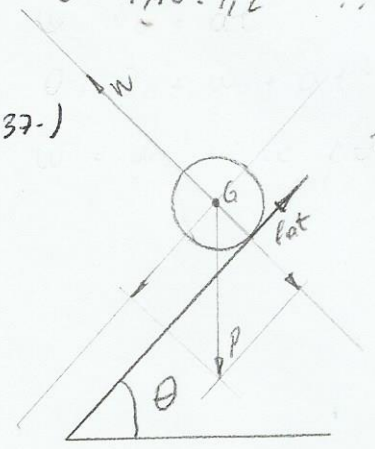
$K_1 = K_2$

$2.3^2 + \frac{4.1,2^2}{5} . 2,5^2 + 2.2.9,8.1,4 = 2.(1,2)^2 w^2 + \frac{4.1,2^2}{5} . w^2$

$\therefore w = 4,46 \therefore \vec{w} = 4,46 \vec{k} \ (\frac{rad}{s})$

$v = 4,46 . 1,2 \therefore v_G = 5,35 \ (m/s)$

B.37-)



$m = 90kg \quad r = 0,4m \quad \theta = 30° \quad \vec{w_0} = 5 \ (rad/s)$

$\mu_c = 0,4 \quad \mu_c = 0,5$ Determine $w = ?$ no $t = 3s$

$I_g = \frac{mr^2}{2}$

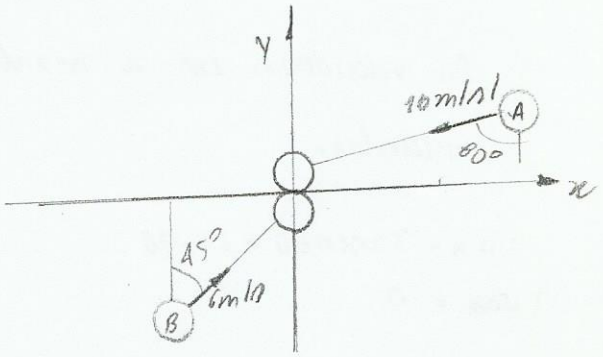
$I_g = \frac{90.0,4^2}{2} = 7,2$

$\vec{L} + \int \vec{G} dt = \vec{L'}$

$7,2.5 + 90.5.0,4 + \int_0^3 (90.9,8 \sin 30 - 90.9,8 \cos 30) . 0,4 \, dt = 7,2w + 90.0,4w$

$\therefore w =$

8.19-)



$m_A = 1\,kg \quad m_B = 2{,}5\,kg \quad e = 0{,}8$

Determinar v_a' e v_b'

$$\vec{p_0} + Ig = \vec{p_g}'$$

$$\Sigma mv + \int \vec{F}\,dt = \Sigma mv'$$

Para o eixo x, temos

$v_{ax} = v_a'_x \qquad v_a'_x = 16 \operatorname{sen} 60 \quad \therefore \quad \vec{v}_a'_x = -15{,}757\,\vec{i}\ (m/s)$

$v_{bx} = v_b'_x \qquad v_b'_x = 6 \operatorname{sen} 45 \quad \therefore \quad \vec{v}_b'_x = 4{,}24\,\vec{i}\ (m/s)$

$v_{ay} = 16 \cos 60 = 2{,}776 \quad \therefore \quad \vec{v}_{ay} = -2{,}776\,\vec{j}\ (m/s)$

$v_{by} = 6 \cos 45 = 4{,}24 \qquad \vec{v}_{by} = 4{,}24\,\vec{j}\ (m/s)$

$-1 \cdot 2{,}776 + 2{,}5 \cdot 4{,}24 = v_a'_y + 2{,}5\, v_b'_y$ (Para o eixo y)

$\therefore \quad v_a'_y + 2{,}5\, v_b'_y = 7{,}822$

$$e = \frac{v_a'_y - v_{by}'}{v_{by} - v_{ay}} \quad \therefore \quad v_a'_y - v_b'_y = 0{,}8\,(4{,}24 - (-2{,}776))$$

$$v_a'_y - v_b'_y = 5{,}6144$$

$$\begin{cases} v_a'_y + 2{,}5\, v_b'_y = 7{,}822 \\ -v_a'_y + v_b'_y = -5{,}6144 \end{cases}$$

$3{,}5\, v_b'_y = 2{,}2076 \quad \therefore \quad v_b'_y = 0{,}631$

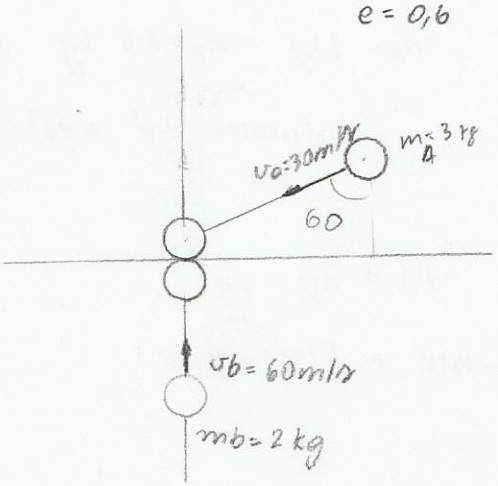
$v_a'_y = 5{,}6144 + v_b'_y \quad \therefore \quad v_a'_y = 6{,}245$

R: $\vec{v_a} = -16\,\vec{i} + 6{,}25\,\vec{j}\ (m/s)$

$\vec{v_b} = 4{,}24\,\vec{i} + 0{,}631\,\vec{j}\ (m/s)$

8.16)

$e = 0,6$



As velocidades em x mantêm constantes.

$\begin{cases} v_{ax} = 30 \times \cos 60 = 25,98 \\ v_{bx} = 0 \end{cases}$

$v_{ay} = 30 \cos 60 = 15$

$\vec{v_a} = -25,98\,\vec{i} - 15\,\vec{j}$

$\vec{v_b} = 60\,\vec{j}$

$2\,60 + 3.(-15) = 3v_a'y + 2v_b'y \quad \therefore \quad 3v_a'y + 2v_b'y = 75$

$e = \dfrac{v_a'y - v_b'y}{v_by - v_ay} \qquad v_a'y - v_b'y = 0,6\,(60 - (-15))$

$v_o'y - v_b'y = 15$

$\begin{cases} 3v_a'y + 2v_b'y = 75 \\ v_a'y - v_b'y = 15 \end{cases} \sim \begin{cases} 3v_o'y + 2v_b'y = 75 \\ 2v_a'y - 2v_b'y = 90 \end{cases} \qquad 5v_a'y = 165 \quad \therefore v_a'y = 33$

$v_b'y = v_o'y - 45 \quad \therefore v_b'y = -12$

R: $\vec{v_a} = -26\vec{i} + 33\vec{j}$ (m/s)

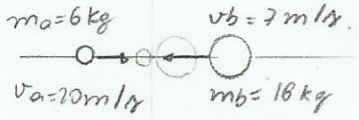
$\vec{v_b} = -12\vec{j}$ (m/s)

8.12:)

$m_a = 6$ kg $\quad v_a = 20$ m/s

$m_b = 16$ kg $\quad v_b = 7$ m/s

$e = 0,8$



* Avaloudade em y não altura

$6.20 - 7.16 = 6v_a' + 16v_b' \quad \therefore \quad 6v_a' + 16v_b' = -6 \sim v_a' + 3v_b' = -1$

$e = \dfrac{v_a' - v_b'}{v_b - v_a} \quad \therefore \quad v_o'y - v_b'y = 0,8\,(-7 - 20)$

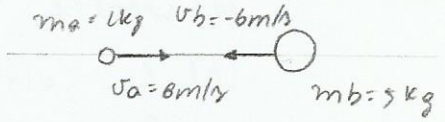
$v_o'y - v_b'y = -21,6$

$\begin{cases} v_a' + 3v_b' = -1 \\ v_a' - v_b' = -21,6 \end{cases} \sim \begin{cases} v_a' + 3v_b' = -1 \\ 4v_b' = 20,6 \end{cases} \qquad v_b' = 5,15 \Rightarrow \vec{v_b} = 5,15\,\vec{i}\ \left(\frac{m}{s}\right)$

$v_a = -1 - 3v_b = -1 - 3.5,15 \quad \therefore \quad \vec{v_a} = -16,45\,\vec{i}$ (m/s)

8.11

$e = 0,8$



$2.8 - 6.5 = 2va' + 5vb'$ $\therefore 2va' + 5vb' = -14$

$e = \frac{va' - vb'}{vb - va}$ $\therefore va' - vb' = 0,8(-6-8)$

$\therefore va' - vb' = -11,2$

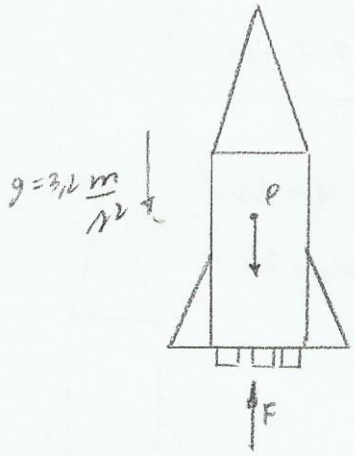
$\begin{cases} 2va' + 5vb' = -14 \\ -2va' + 2vb' = 22,4 \end{cases}$ $\sim$ $7vb' = 8,4$

$vb' = 1,2 \rightarrow \underline{\vec{vb}' = 1,2\,\vec{i}\ (m/s)}$

$va' = -11,2 + vb'$

$va' = -11,2 + 1,2$ $\therefore \underline{\vec{va}' = -10\,\vec{i}\ (m/s)}$

8.22



$m = 2t = 2000\ kg$

$F = 3000t^2 + 50t + 80\ (N)$ $\Delta t = 12s$

$v_0 = 250\ m/s$. Determine $v_f = ?$

$\vec{p} + I = \vec{p}'$

$2000 \cdot 250 + \int 3000t^2 + 50t + 80 - 2000 \cdot 3,2\ dt = 2000\,vf$

$2000\,250 + (1000t^3 + 25t^2 - 6320t)\Big|_0^{12} = 2000vf$

$\therefore \underline{vf = 1076\ \frac{m}{s}}$

8.23

$m = 2t = 2000\ kg$

$T = 32000 - 2000e^{-t/5}\ (N)$

$v_0 = 180\ m/s$

$v_f = ?$ no $t = 10s$

$I = \int 32000 - 2000e^{-t/5}\ dt$

$= 32000t - 2000 \int e^{-t/5}\ dt$

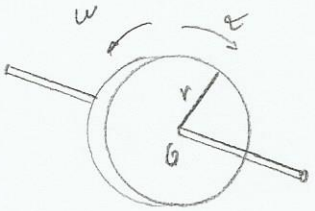
$\int e^{-t/5}\ dt \cdot \frac{(-5)}{(-5)} = -5 \cdot e^{-t/5}$

$I = 32000t + 10000e^{-t/5}$

$2000 \cdot 180 + (32000 \cdot t + 10000e^{-t/5})\Big|_0^{10} = vf \cdot 2000$

$\therefore \underline{v_f = 340,68\ \frac{m}{s}}$

8.25



$r = 1{,}2\ m$

$m = 150\ kg$

$\vec{\omega}_0 = 120\ \vec{k}\ (rad/s)$

$\vec{\sigma}_{at} = 30\vec{k}\ (N\,m)$

$I_g = \frac{mr^2}{2}$

Determine:

(a) $w(t) = ?$

(b) $t = ?$ até parar

(c) $\theta = ?$ " "

(a) $\vec{L} + I^{\theta} = \vec{L}'$

$$120 \cdot \frac{150 \cdot 1{,}2^2}{2} + \int 30\,dt = \frac{150 \cdot 1{,}2^2}{2} \cdot w$$

$$\therefore w = \frac{12960 - 30t}{108} \quad \therefore \underline{w(t) = 120 - 0{,}278t \ \left(\frac{rad}{s}\right)}$$

(b) $0 = 120 - 0{,}278t \quad \therefore \underline{t = 432\ s}$

(c) $\theta = \theta_0 + w_0 t + \frac{1}{2}\alpha t^2$, Sabendo que

$\theta = 120 \cdot 432 - \frac{1}{2} \cdot 0{,}278 \cdot 432^2 \quad \sigma = I \cdot \alpha \quad \therefore \alpha = \frac{30}{108} = 0{,}278$

$\therefore \theta = 25\,920 \qquad 1\ rev = 2\pi$

$x = 25\,920 \quad \therefore \underline{x = 4125{,}3\ rev}$

8.29

$m = 3000\ kg$

$kg = 2{,}5\ m$

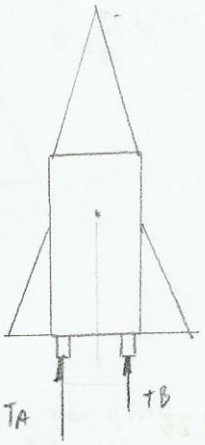
$T_A = T_B = 40000\ N$

Quando $V = 500\ m/s \rightarrow T'_B = 20000\ N$

$G = 3000 \cdot 9{,}8 = 29400\ N$

Determine após $t = 4\ s$

(a) $v_G = ?$ (b) $w_b = ?$



$$3000 \cdot 500 + \int 40000 + 20000 - 29400\,dt = 3000\,v_G$$

$$1500\,000 + 30600\,t\Big|_0^4 = 3000\,v_G \quad \therefore \underline{v_G = 540{,}8\ m/s}$$

$$0 + \int -(40000 \cdot 0{,}8) + (20000 \cdot 0{,}8)\,dt = 3000 \cdot 2{,}5^2\,w$$

$$-16000\,t\Big|_0^4 = 3000 \cdot 2{,}5^2 \cdot w \quad \therefore \underline{\vec{\omega} = -3{,}41\,\vec{k}\ \left(\frac{rad}{s}\right)}$$

9.9

$m = 1200$ kg (Obs.: tração dianteira)

$\mu_e = 0{,}4$ $\mu_d = 0{,}1$

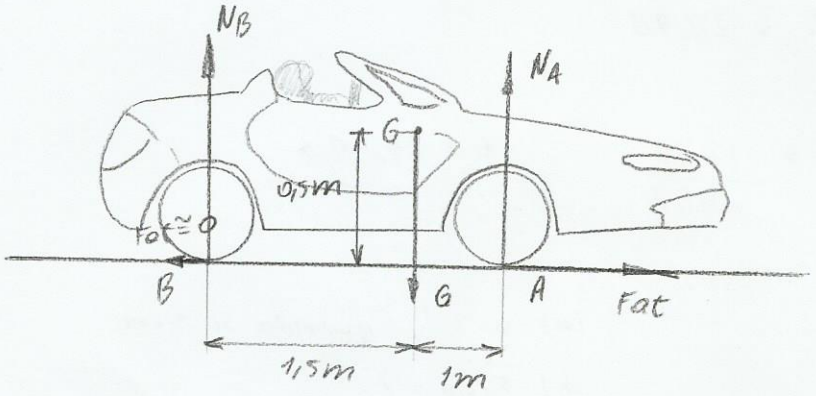
$v_0 = 0$ (vai começar $\therefore \mu_e$)

(a) Forças que atuam no carro

(b) $a = ?$ $t_{min} = ?$ para atingir 100 km/h

(c) Forças normais durante $a = ?$

(d) $a = ?$ t_{min} para atingir 100 km/h para μ_d



$\Sigma F_x = m a_{Gx}$: $Fat = m . a_{Gx}$

$\Sigma F_y = m . a_{Gy}$: $N_B + N_A - mg = 0$

$\Sigma \tau_{3a} = I \alpha$: $N_A - 1{,}5 N_B + 0{,}5 Fat = 0$

$mg = 1200 . 9{,}8 = 11760$ N $\therefore$ $N_B + N_A = 11760$

$N_B = 11760 - N_A$

$N_A - 1{,}5 N_B + 0{,}5 Fat = 0$

$N_A - 1{,}5 (11760 - N_A) + 0{,}5 . N_A . 0{,}4 = 0$

$N_A - 17640 + 1{,}5 N_A + 0{,}2 N_A = 0$ $\therefore$ $N_A = \underline{6533{,}33 \text{ N}}$

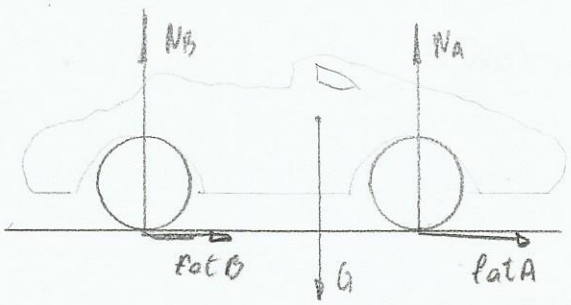
$\therefore$ $N_B = \underline{5226{,}67 \text{ N}}$

$a_{Gx} = \dfrac{0{,}4 . 6533{,}33}{1200}$ $\therefore$ $a_{Gx} = \underline{2{,}18 \text{ m/s}}$

$v = \cancel{v_0^0} + at$ $\qquad \dfrac{100 \text{ km}}{h} = \dfrac{100 . 10^3 . m}{3600 \, s} = 27{,}78 \text{ m/s}$

$t = \dfrac{27{,}78}{2{,}18}$

$\therefore t = \underline{12{,}74 \text{ s}}$



$\Sigma F_x = m a_{Gx}$: $fat_A + fat_B = m a_{Gx}$

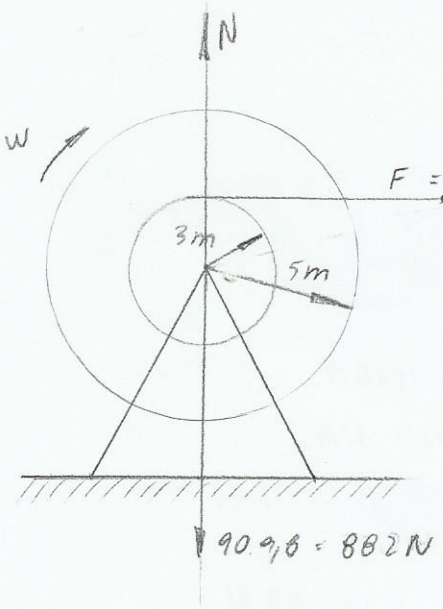
$6533{,}33 . 0{,}1 + 5276{,}67 . 0{,}1 = 1200 . a_{Gx}$

$\therefore a_{Gx} = 0{,}98\ m/s^2$

$100\ \frac{km}{h} = \frac{100.10^3}{3600}\ \frac{m}{s} = 27{,}76$

$v = \cancel{v_0}^{\,0} + at \qquad t = \frac{27{,}76}{0{,}98} \quad \therefore \quad t = 28{,}34\ s$

9.40



(a) $w = ?$ quando $x = 5m$

(b) $\vec{F}_{cinc} = ?$

$m = 90kg$

$k_G = 3{,}2$

$\Sigma F_x = m . a_{Gx} \qquad 200 = 90 . a$

$\Sigma \tau_{Gz} = I \alpha \quad : \quad 200 . 3 = 90 . 3{,}2^2 . \alpha$

$a = \frac{20}{9}\ m/s^2$

$\alpha = 0{,}65\ \frac{rad}{s^2}$

$2\pi . 3 = 2\pi$

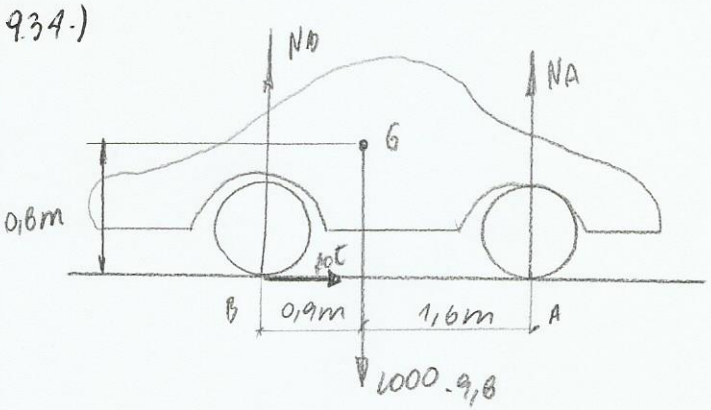
$5 = x \quad \therefore \quad x = \frac{5}{3}\ rad = \theta$

$w^2 = \cancel{w_0^2}^{\,0} + 2 . \alpha . \Delta\theta$

$w^2 = 2 . 0{,}65 . \frac{5}{3} \quad \therefore w = 1{,}47 \quad \therefore \quad \vec{w} = -1{,}47 \vec{k}\ \left(\frac{rad}{s}\right)$

$\vec{F}_{cinc} = 200 \vec{i} - 882 \vec{j}\ (N)$

9.34.)



$m = 2000\ kg$

$v = 30\ m/s$

$Nd = 0,4$

Determine:

(a) $a = ?$

(b) $t = ?$ até $v = 0$

(c) $\Delta s = ?$

$\Sigma F_x = m a_{Gx}$: $fat = m a_{Gx}$

$\Sigma F_y = m a_{Gy}$: $N_B + N_A - 19600 = 0$

$\Sigma \tau_{Gz} = I \alpha$: $-0,9 N_B + 1,6 N_A + 0,8 fat = 0$

$N_B + N_A - 19600 = 0$ $\qquad fat = 0,4 . N_B$

$N_A = 19600 - N_B$

$-0,9 N_B + 1,6 N_A + 0,8 fat = 0$

$-0,9 N_B + 1,6 . (19600 - N_B) + 0,8 . 0,4 N_B = 0$

$-0,9 N_B + 31360 - 1,6 N_B + 0,32 N_B = 0$

$-2,18 N_B = -31360 \quad \therefore \quad N_B = 14385,32\ N$

$fat = m a_{Gx}$

$0,4 . 14385,32 = 2000 . a_{Gx} \quad \therefore \underline{a_{Gx} \simeq 2,877\ m/s^2}$

como $F_{at\,max} = 235{,}2$ e o meu $F_{tambor} = 392$, ele desliza antes do tambor.

(c)

$$-0{,}7F - 0{,}8(0{,}15 \cdot N_A + 0{,}15 N_B) - 0{,}5 N_A + 0{,}5 N_B = 0$$

$$-0{,}7F - 0{,}12 \cancel{N_A}^{\to 0} + 0{,}12 N_B - 0{,}5 \cancel{N_A}^{\to 0} + 0{,}5 N_B = 0$$

$$-0{,}7F - 0{,}62 N_A + 0{,}38 N_B = 0$$

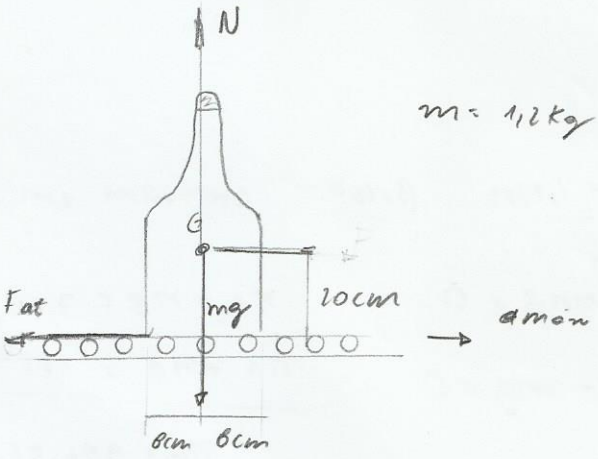
$$F - 0{,}15 \cancel{N_A}^{\to 0} - 0{,}15 N_B = 120 \cdot a$$

$$N_A + N_B = 1176$$

$$F = \frac{0{,}38 \cdot 1176}{0{,}7} \quad \therefore F = 636{,}9\,F$$

$$a = \frac{636{,}9 - 0{,}15 \cdot 1176}{120} \quad \therefore a = 3{,}65\ m/s^2$$

9.24



$m = 1{,}2\,kg \quad \mu_c = 0{,}5$

$F_{at} = m a_g$

$N = mg = 1{,}2 \cdot 9{,}8 = 11{,}76 \quad F_{at} = 11{,}76 \cdot 0{,}5 = 5{,}88$

$-0{,}2\,F_{at} = 0 \quad \therefore a = \frac{1{,}2}{5{,}88} = 0{,}2$

9.37

$m = 60\,kg$ $k_g = 1{,}7\,m$ $\beta = 12^\circ$ $\Delta x = 13{,}5\,m$ em $t = 3\,s$

$2n\cos\beta - F - 60 \cdot 9{,}8 = 0$

$2n\cos 12 - F - 784 = 0$

$-1{,}5F = 60 \cdot 1{,}7^2 \cdot \alpha$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ ($\theta_0 \to 0$, $\omega_0 \to 0$)

$\frac{13{,}5}{1{,}5} = \frac{1}{2}\alpha \cdot 3^2 \quad \therefore \alpha = 2$

$F = \frac{60 \cdot 1{,}7^2 \cdot 2}{1{,}5} \quad \therefore F = 308{,}27\,N$

$n = \frac{308{,}27 + 784}{2\cos 12} \quad \therefore n = 558\,N$

9.38

$m = 120\,kg$ $k_g = 2{,}1\,m$ $\beta = 16^\circ$ $\Delta x = 20\,m$ em $5\,s$

$F + n_A \sin\beta - n_B \sin\beta = 0$

$n_A \cos\beta + n_B \cos\beta - mg = 0$

$2F = 120 \cdot 2{,}1^2 \cdot \alpha$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ ($\omega_0 \to 0$)

$\frac{20}{2} = \frac{1}{2} \cdot \alpha \cdot 5^2 \quad \therefore \alpha = 0{,}8\ \frac{rad}{s}$

$F = \frac{120 \cdot 2{,}1^2 \cdot 0{,}8}{2} \quad \therefore F = 212\,N$

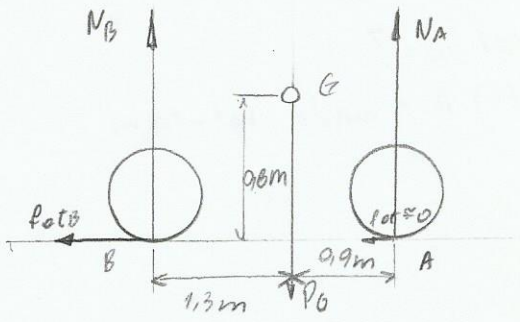
$n_A - n_B = 769{,}13$

$n_A + n_B = 1223{,}39$

$n_A = 996{,}26\,N$

$n_B = 227{,}13\,N$

9.13



$m = 900\,kg$

$\mu_e = 0,5 \quad \mu_d = 0,3$

(Há escorregamento)

Determine:

(a) N_A e N_B = ?

(b) a = ?

(c) t = ? para $v = 20\,m/s$, $v_0 = 6\,m/s$

(a)

$\Sigma F_x = m\,a_{cx}$: $f_{at} = m\,a_{cx}$

$\Sigma F_y = m\,a_{cy}$: $N_A + N_B = P_G$

$\Sigma G = I\,\alpha$: $0,9\,N_A - 0,8\,f_{atB} - 1,3\,N_B = 0$

$P_g = m\,9,8 = 900\,9,8 = 8820\,N$

$N_A + N_B = 8820 \quad N_A = 8820 - N_B$

$0,9(8820 - N_B) - 0,8 \cdot N_B \cdot 0,3 - 1,3\,N_B = 0$

$7938 - 0,9N_B - 0,24N_B - 1,3N_B = 0$

$2,44\,N_B = 7938 \quad \therefore N_B = 3253,28\,N$

$N_A = P_c - N_B \quad \therefore N_A = 5566,7\,N$

(b) $a_G = \dfrac{3253,28 \cdot 0,3}{900} \quad \therefore a_G = 1,08\,m/s^2$

(c) $v = v_0 + at$

$t = \dfrac{20 - 6}{1,08} \quad \therefore t = 11,07\,s$

9.36

$m = 50\,kg$

$R = 5m$ e $r = 3m$ $\quad \Delta x = 16m$

$F = 25N$

Determine:

(a) $\omega = ?$

(b) $A = ?$ mndo $kg^2 = 1,5m$

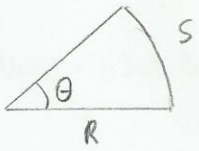
$N_A = mg \sim N_A = 50 \cdot 9,6 = 190N$ ✗

$\Sigma \tau = I\alpha \quad -25 \cdot 3 = 50 \cdot 1,5^2 \cdot \alpha \quad \therefore \alpha = 0,67 \frac{rad}{s^2}$

$\omega^2 = \omega_0^{\nearrow 0} + 2 \cdot \alpha \cdot \Delta\theta$

$\omega^2 = 2 \cdot 0,67 \cdot \frac{16}{3}$

$\therefore \omega = 2,67 \frac{rad}{s}$

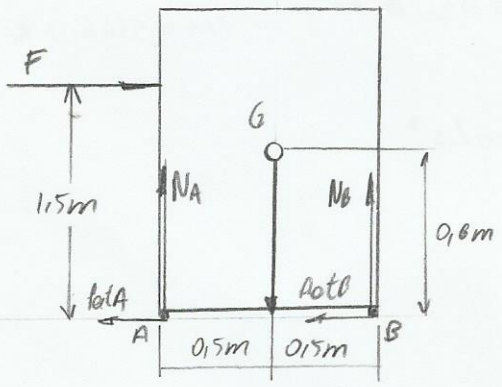


$\theta = \frac{16}{3}$

b) $\vec{A} = 25\vec{i} - 490\vec{j}\,(N)$

9.23

$m = 120\,kg \quad \mu_e = 0,2 \quad \mu_d = 0,15$



$\Sigma F_x = m a_{Gx}$

$\Sigma F_y = m a_{Gy}$

$\Sigma \tau = 0$

$mg = 120 \cdot 9,8$

$mg = 1176\,N$

$\Sigma F - fatA - fatB = m \cdot a_{Gx}$

$N_A + N_B = mg$

$-F \cdot 0,7 - 0,8\,(fatA + fatB) - 0,5 N_A + 0,5 N_B = 0$

Caso não escorregue, temos

$F = fatb \quad \therefore F = 0,2 \cdot 1176 \quad \therefore F = 235,2\,N$

$N_B = mg = 1176$

$-0,7F = 0,8 \cdot F + 0,5 \cdot 1176 = 0$

$\therefore F = 392\,N$

9.41

Dados: m, R, r, k_g, F, θ,

$$\begin{cases} F\cos\theta - fat = m.a_G \\ F\sin\theta + N_A - mg = 0 \\ Fr - FaR = mk^2.\alpha \end{cases} \qquad \therefore fat = F\cos\theta - m.a_g$$
$$= F\cos\theta - m.\alpha R$$

$$Fr - (F\cos\theta - m\alpha R).R = mk^2\alpha$$

$$Fr - F\cos\theta R + m\alpha R^2 = mk^2\alpha$$

$$m\alpha R^2 - mk^2\alpha = Fr - F\cos\theta R$$

$$\alpha(mR^2 - mk^2) = Fr - F\cos\theta R$$

$$\alpha = \frac{F(r - R\cos\theta)}{m(R^2 - k^2)}$$

9.44

$m = 120\,kg$ $\quad k_g = 2{,}1\,m$ $\quad F = 160\,N$

(a)

$$\begin{cases} Fat - F = m.a \\ N = mg \end{cases}$$

$$3Fat - 1{,}5F = mk^2.\alpha$$

$$3Fat - 1{,}5.160 = 120.2{,}1^2.\alpha$$

$$\begin{cases} 3Fat - 529{,}2\alpha = 270 \\ Fat - 160 = 120.(-\alpha)r \end{cases} \sim \begin{cases} Fat - 176{,}4\alpha = 90 \\ Fat + 360\alpha = 160 \end{cases}$$

$$\begin{cases} Fat - 176{,}4\alpha = 90 \\ -536{,}4\alpha = -90 \end{cases} \therefore \alpha = 0{,}168\ \frac{rad}{s^2}$$

(c) $w = ?$ $\quad no\ t = 7s$ $\quad w_0 = 2{,}5\ rad/s$

$$w = w_0 + \alpha t$$

$$w = 2{,}5 + 0{,}168.7 \therefore w = 3{,}67\ \frac{rad}{s}$$

10.5

$m = 50\,kg \quad \Delta x = 10\,m \quad F = 25\,N \quad k_g = 1{,}5\,m \quad \omega = ?$

$T_1^{\nearrow 0} + \sum U_{1\to 2} = T_2$

$\sum U_{1\to 2} = U^F = \int F\,dr = \int 25\,dr = 25r = 25 \cdot 16 = 400$

$T_2 = \frac{50 \cdot 1{,}5^2}{2}\,\omega^2 = 56{,}25\,\omega^2$

$400 = 56{,}25\,\omega^2 \quad \therefore \omega = 2{,}67\ \frac{rad}{s}$

10.7

$F = 200\,N \quad v_0 = 0 \quad \omega = ? \quad \Delta x = 5\,m \quad m = 90\,kg \quad k_g = 3{,}2\,m$

$T_1^{\nearrow 0} + U_{1\to 2} = T_2$

$U_{1\to 2} = U^F = 200 \cdot 5 = 1000$

$T_2 = \frac{90 \cdot 3{,}2^2}{2}\,\omega^2 = 460{,}8\,\omega^2$

$1000 = 460{,}8\,\omega^2 \quad \therefore \omega = 1{,}47\ \frac{rad}{s}$

10.13

$m = 4\,kg \quad r = 1{,}5\,m \quad k_g = 1{,}2\,m \quad G = 20\,N \quad \omega_0 = 6\,rad/s$

$m_A = 5\,kg \quad v_A = ?$ zpět 6 m

$T_1 + U_{1\to 2} = T_2$

$U_{1\to 2} = U^P + U^G = 5 \cdot 9{,}8 \cdot 6 + 20 \cdot \frac{8}{2\pi} = 417{,}46$

$T_1 = \frac{4 \cdot 1{,}2^2}{2} \cdot 6^2 = 103{,}68$

$T_2 = \frac{1}{2} \cdot 5 \cdot v^2$

$103{,}68 + 417{,}46 = \frac{5}{2}\,v^2 \quad \therefore v = 14{,}44\ m/s$

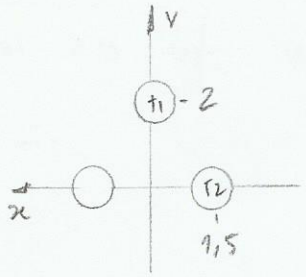
$\vec{L}^{\,0} + \int \vec{G}\,dt = \vec{L}^{\,1}$

$\int_0^4 0{,}6\,(40\,000 - 20\,000)\,dt = 3000.w$ $\therefore$ $\boxed{w = 3{,}41\ \frac{rad}{s}}$

8.32

$m = 10\,000\ kg$ $\quad kg_x = 15\,m$; $kg_y = 11\,m$ $\quad v_g = 1000\ m/s$

$T_1 = 1960\,e^{-t}\,(-k)\ kN$ $\quad T_2 = 2420\,e^{-t}\,(-\vec{k})\ kN$



$10\,000.\cancel{1000} + \int_0^{10} (1960+2420)\,e^{-t}\,\cancel{1000}\,dt = 10\,\cancel{000}\,v$

$\int e^{-t}dt = \int e^{-t}.dt.\frac{(-1)}{(-1)} = -\int e^{-t}.d(-t) = -e^{-t}$

$10\,000 + 4360\,.\,(-e^{-t})\Big|_0^{10} = 10v$ $\therefore$ $\boxed{\vec{v} = 1480_{\vec{k}}\ m/s}$

$\int 2.1960\,e^{-t}\,dt.\cancel{1000} = 10\,\cancel{000}.15^2\,w$

$\vec{w}_x = 1{,}74\,\vec{i}\ (rad/s)$

$\int 1{,}5.2420\,e^{-t}dt.\cancel{1000} = 10\,\cancel{000}.11^2\,w$

$\vec{w}_y = 3\,\vec{j}\ (rad/s)$

$\vec{w} = 1{,}74\,\vec{i} + 3\,\vec{j}\ (rad/s)$

8.33

$m = 8000\ kg$ $T_1 = 21430\ N$ $T_2 = 90000\ N$ $\Delta t = 6\ s$

$v_0 = 230\ m/s$ $kg = 1,2\ m$

(a) $8000 \cdot 230 + \int_0^6 21430 + 90000\ dt = 8000 \cdot v$

$8000 \cdot 230 + (21430 + 90000)\, t\,|_0^6 = 8000\, v \quad \therefore \boxed{v = 314\ m/s}$

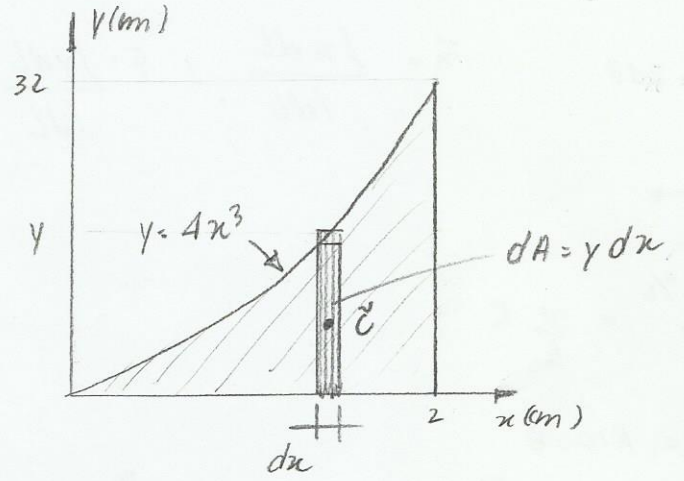
(b) $\int_0^6 1,4\,(90000 - 21430)\ dt = 8000 \cdot 1,2^2 \cdot w$

$1,4\,(90000 - 21430)\, t\,|_0^6 = 8000 \cdot 12^2 \cdot w \quad \therefore \boxed{\vec{w} = 0,5\ (rad/s)}$

(c) $w = \frac{V}{R} \Rightarrow R = \frac{314}{0,5} \Rightarrow \boxed{R = 628\ m}$

Mecânica do corpo rígido

(P1 1º07 Centro de Massa e Centróide) Determine a localização das coordenadas do centróide $(\bar{x}, \bar{y})$



$y = 4x^3$

$dy = 12x^2\,dx$

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} \quad ; \quad \int dA = \int y\,dx = \int_0^2 4x^3\,dx = \frac{4x^4}{4}\Big|_0^2 = 16\ cm$$

$$\int x\,dA = \int x \cdot y\,dx = \int x \cdot 4x^3\,dx = 4\int_0^2 x^4\,dx = \frac{4}{5}x^5\Big|_0^2 = 25{,}6$$

$$\therefore \bar{x} = \frac{25{,}6}{16} = 1{,}6$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} \quad ; \quad \int y\,dA = \int y \cdot y\,dx = \int_0^2 16x^6\,dx = \frac{16}{7}x^7\Big|_0^2 = 292{,}57$$

$$\bar{y} = \frac{292{,}57}{16} = 18{,}28 \quad \therefore \quad \boxed{C = (1{,}6;\ 9{,}14)\ cm}$$

Exercícios Resolvidos do Livro

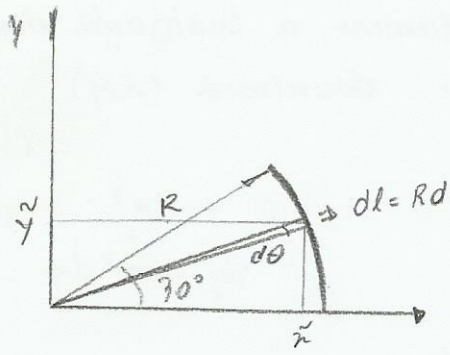
3.1*

$$\lambda = \frac{dm}{dl} \qquad dm = \lambda\,dl \rightarrow m = \int \lambda\,dl$$

Como $\lambda = ax + b$

$$m = \int (ax + b)\,dx = \frac{ax^2}{2} + bx = \frac{4 \cdot 5^2}{2} + 6 \cdot 5 = 80$$

3.1*



$\theta = \frac{L}{R} \qquad L = \theta R$

$dl = R d\theta$

$\bar{x} = \frac{\int x\,dL}{\int dL} \; ; \; \bar{y} = \frac{\int y\,dL}{\int dL}$

$\int dL = \int_0^{\pi/6} R\,d\theta = R.d\theta\Big|_0^{\pi/6} = \frac{\pi}{6} R$

$\int x\,dL = \int x.R\,d\theta \; ; \; x = R\cos\theta$

$= \int (R\cos\theta).R\,d\theta = R^2 \int_0^{\pi/6} \cos\theta\,d\theta = R^2 \sin\theta\Big|_0^{\pi/6} =$

$= R^2 \left(\sin\frac{\pi}{6} - \sin 0 \right) = \frac{1}{2} R^2$

$\therefore \bar{x} = \frac{R^2}{2} \div \frac{\pi}{6} R = \frac{3 R^2}{2\pi R} = \frac{3R}{\pi}$

$\bar{y} = \frac{\int y\,dL}{\int dL} \; ; \quad \int dL = \int R\,d\theta$

$= R\theta\Big|_0^{\frac{\pi}{6}} = \frac{\pi}{6} R$

$\int y\,dL = \int (R\sin\theta) R\,d\theta = R^2 \int \sin\theta\,d\theta = -R^2 \cos\theta\Big|_0^{\pi/6}$

$= -R^2 \left(\cos\frac{\pi}{6} - \cos 0 \right) = -R^2 \left(\frac{\sqrt{3}}{2} - 1 \right) = R^2 \left(1 - \frac{\sqrt{3}}{2} \right)$

$\bar{y} = R^2 \left(1 - \frac{\sqrt{3}}{2} \right) \div \frac{\pi}{6} R = \frac{6 R^2 \left(1 - \frac{\sqrt{3}}{2} \right)}{\pi R} = \frac{3R}{\pi} \left(2 - \sqrt{3} \right)$

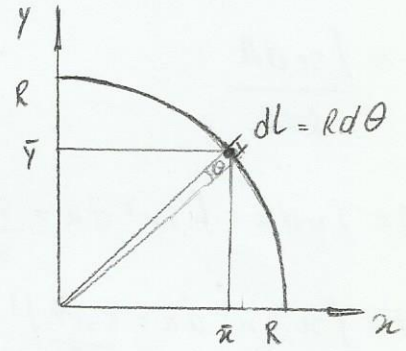
$\therefore C = \frac{3R}{\pi} \; ; \; \frac{3R}{\pi} \left(2 - \sqrt{3} \right)$

3.12

$$\bar{x} = \frac{\sum x_n \cdot m}{\sum m} = \frac{-16.10 + 6.50 + 100\,16 + 32(20+200)}{10+50+100+20+200} = \boxed{23,37\,m}$$

$$\bar{y} = \frac{\sum y m}{\sum m} = \frac{10.10 + 30.100 + 20.20 - 20.50 - 30.200}{10+50+100+20+200} = \boxed{-9,21\,m}$$

3.14



$$\bar{x} = \frac{\int x\,dL}{\int dL} \qquad d\theta = \frac{dl}{R}$$

$$\int dl = \int R\,d\theta = R\theta\Big|_0^{\pi/2} = \frac{\pi R}{2}$$

$$\int x\,dl = \int (R\cos\theta)R\,d\theta = R^2\int \cos\theta\,d\theta$$

$$= +R^2 \operatorname{sen}\theta\Big|_0^{\pi/2} = +R^2\left(\operatorname{sen}\frac{\pi}{2} - \operatorname{sen}0\right)$$

$$\bar{x} = R^2 \div \frac{\pi R}{2} = \frac{2R^2}{\pi R} = \frac{2R}{\pi}$$

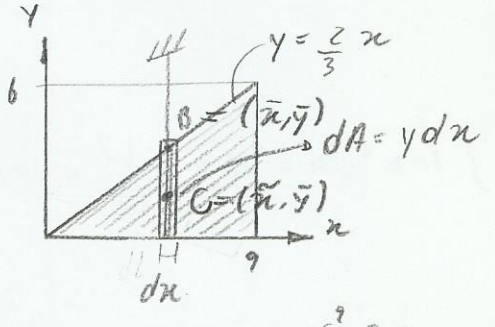
$$\bar{y} = \frac{\int y\,dL}{\int dL}$$

$$\int dl = \int R\,d\theta = R\theta\Big|_0^{\pi/2} = \frac{\pi}{2}R$$

$$\int y\,dl = \int (R\operatorname{sen}\theta)R\,d\theta = -R^2\cos\theta\Big|_0^{\pi/2} = -R^2\left(\cos\frac{\pi}{2} - \cos 0\right) = R^2$$

$$\bar{y} = \frac{R^2}{\frac{\pi R}{2}} = \frac{2R}{\pi} \qquad \vec{r}_{cm} = \left(\frac{2R}{\pi}, \frac{2R}{\pi}\right) \rightarrow \vec{r}_{cm} = \frac{2R}{\pi}\vec{i} + \frac{2R}{\pi}\vec{j}$$

3.16



$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

$$\int dA = \int y\,dx = \frac{2}{3}\int_0^9 x\,dx = \frac{2}{3}\cdot\frac{x^2}{2}\Big|_0^9$$

$$= \frac{9^2}{3} = 27$$

$$\int x\,dA = \int x\cdot\frac{2}{3}x\,dx = \frac{2}{3}\int_0^9 x^2\,dx = \frac{2}{3}\cdot\frac{x^3}{3}\Big|_0^9 = \frac{2}{9}\cdot 9^3 = 162$$

$$\bar{x} = \frac{162}{27} = 6$$

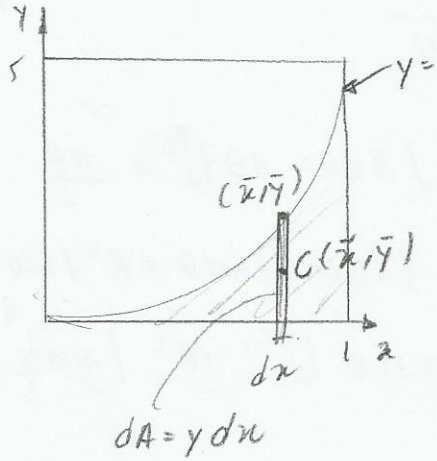
$$\bar{y} = \frac{\int y\,dA}{\int dA}$$

$$\int y\,dA = \int y \cdot y\,dx = \int \left(\frac{2}{3}x\right)^2 dx = \frac{4}{9} \cdot \frac{x^3}{3}\Big|_0^9 = \frac{4}{27} \cdot 9^3 = 108$$

$$\bar{y} = \frac{108}{27} = 4$$

$$\therefore \boxed{\begin{aligned} B &= (\bar{x}, \bar{y}) = (6,4)\ m \\ C &= (\bar{x}, \bar{y}) = (6,2)\ m \end{aligned}}$$

3.17



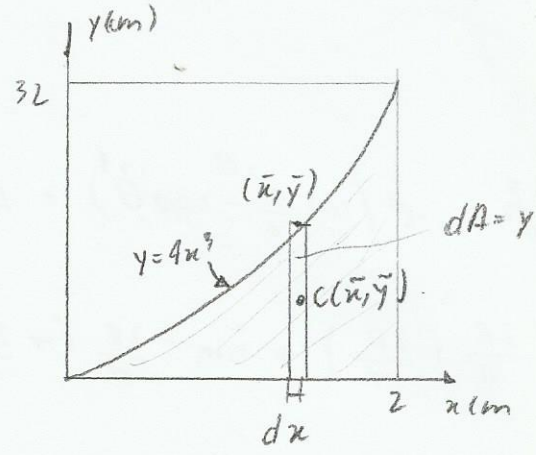
$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

$$\int dA = \int y\,dx = \int 5x^2\,dx = \frac{5x^3}{3}\Big|_0^1 = \frac{5}{3}$$

$$\int x\,dA = \int x \cdot 5x^2\,dx = \frac{5x^4}{4}\Big|_0^1 = \frac{5}{4}$$

$$\bar{x} = \frac{5}{4} \div \frac{5}{3} = \frac{3}{4} = \boxed{0{,}75\ m}$$

3.18



$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

$$\int dA = \int y\,dx = \int 4x^3\,dx = x^4\Big|_0^2$$

$$\int dA = 16$$

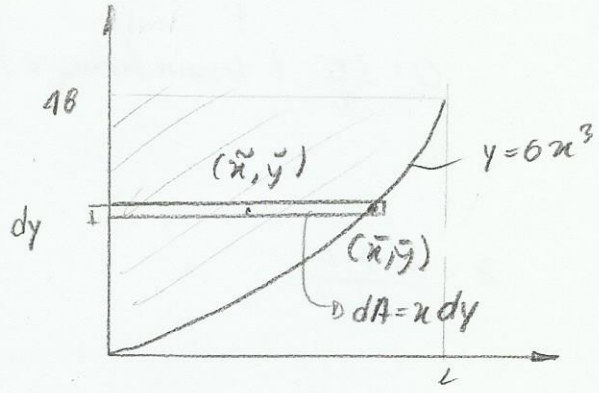
$$\int x\,dA = \int x \cdot 4x^3\,dx = \frac{4x^5}{5}\Big|_0^2 = 25{,}6$$

$$\bar{x} = \frac{25{,}6}{16} = \boxed{1{,}6\ cm}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} \quad ; \quad \int y\,dA = \int y^2\,dx = \int 16x^6\,dx = \frac{16x^7}{7}\Big|_0^2 = 292{,}57$$

$$\bar{y} = \frac{292{,}57}{16} = \boxed{18{,}29\ cm}$$

3.19



$\int dA = \int x\,dy$;

$y = 6x^3$

$dy = 18x^2\,dx$

$\int 18x^3\,dx = \frac{18}{4}x^4\Big|_0^2 = 72$

$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

$$\int x\,dA = \int x \cdot x\,dy = \int x^2\,18x^2\,dx = 18\int x^4\,dx = \frac{18x^5}{5}\Big|_0^2 = 115{,}2$$

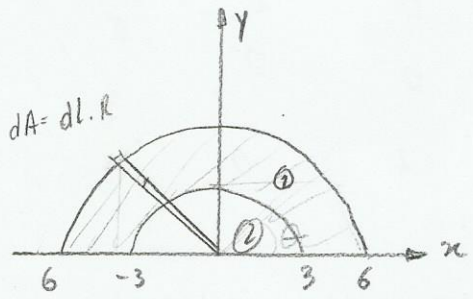
$$\bar{x} = \frac{115{,}2}{72} = \frac{1{,}6}{2} = 0{,}8$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} \quad ; \quad \int y\,dA = \int y \cdot x\,dy = \int 6x^3 \cdot x \cdot 18x^2\,dx = 108\int x^6\,dx$$

$$= \frac{108x^7}{7}\Big|_0^2 = 1974{,}86$$

$$\bar{y} = \frac{1974{,}86}{72} = 27{,}43 \quad \therefore \quad \boxed{(\bar{x},\bar{y}) = (0{,}8\ ;\ 27{,}4)\ (cm)}$$

3.20



① $\bar{x} = \frac{\int x\,dA}{\int dA}$ $\quad \bar{y} = \frac{\int y\,dA}{\int dA}$

$\int dA = \int R^2\,d\theta = R^2\theta\Big|_0^\pi = \pi R^2$

$\int x\,dA = \int R\cos\theta \cdot R^2\,d\theta = R^3\int \cos\theta\,d\theta$

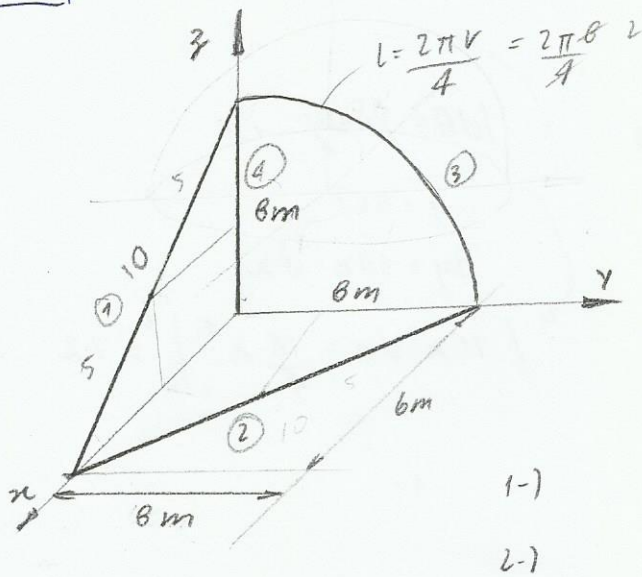
$= R^3 \cdot \sin\theta\Big|_0^\pi = R^3(\sin\pi - \sin 0) = 0$

$d\theta = \frac{dL}{R}$ $\quad dL = R\,d\theta$ $\quad \therefore \quad dA = R^2\,d\theta$

$\int y\,dA = \int R\sin\theta \cdot R^2\,d\theta = R^3\int \sin\theta\,d\theta = -R^3\cos\theta\Big|_0^\pi = -R^3(\cos\pi - \cos 0)$

$\therefore \int y\,dA = 2R^3 \quad \rightarrow \quad \bar{y} = \frac{\int y\,dA}{\int dA} = \frac{2R^3}{\pi R^2} = \frac{2R}{\pi}$

3.26



$L = \frac{2\pi r}{4} = \frac{2\pi 8}{4}$ 2

$C = \frac{2r}{\pi}$ (circunferencia)

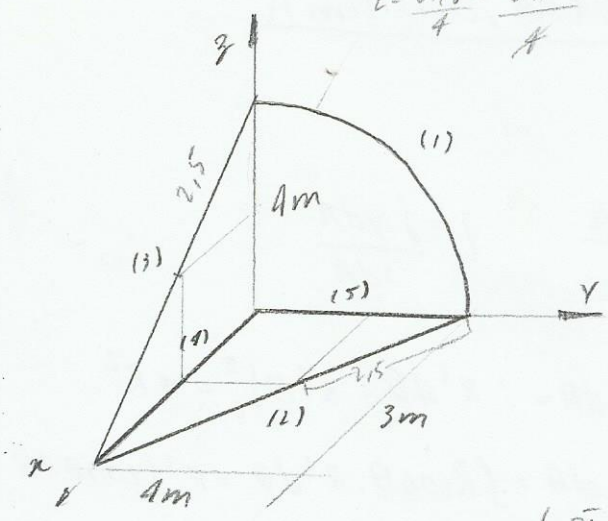
$\bar{x} = \frac{\Sigma \bar{r} L_i}{L_i}$

	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	L
1-)	3	0	4	10
2-)	3	4	0	10
3-)	0	$\frac{16}{\pi}$	$\frac{16}{\pi}$	4π
4-)	0	0	4	8

$\bar{x} = \frac{60}{28+4\pi} = 1,98$ $\quad \bar{y} = \frac{104}{28+4\pi} = 2,56$ $\quad \bar{z} = \frac{136}{28+4\pi} = 3,35$

3.27 -

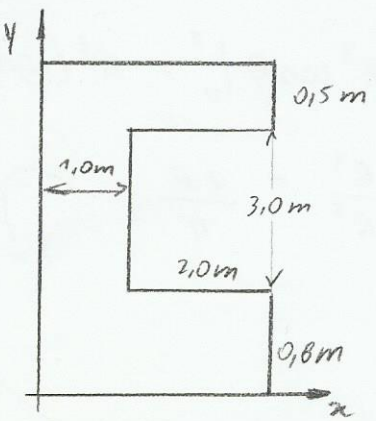
$L = \frac{2\pi r}{4} = \frac{2\pi 4}{4}$



	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	L
(1)	0	$8/\pi$	$8/\pi$	2π
(2)	1,5	2	0	5
(3)	1,5	0	2	5
(4)	1,5	0	0	3
(5)	0	2	0	4

$(\bar{x}, \bar{y}, \bar{z}) = (0,836; 1,46; 1,12)(m)$

3.31

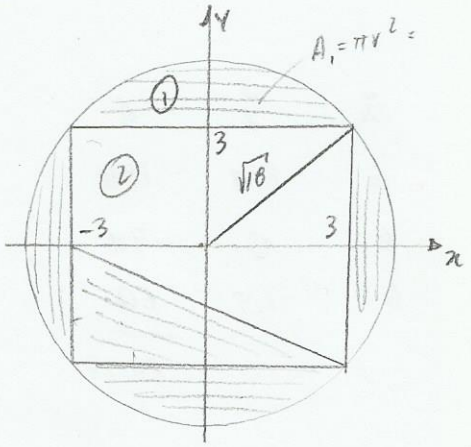


	$\tilde{x}$	$\tilde{y}$	A
(1)	1,5	2,15	12,9
(2)	2	2,3	-6

$\bar{x} = \frac{\Sigma x \cdot A}{A} = \frac{1,5 \cdot 12,9 - 2 \cdot 6}{6,9} = 1,07$

$\bar{y} = \frac{2,15 \cdot 12,9 - 6 \cdot 2,3}{6,9} = 2,02$

3.38

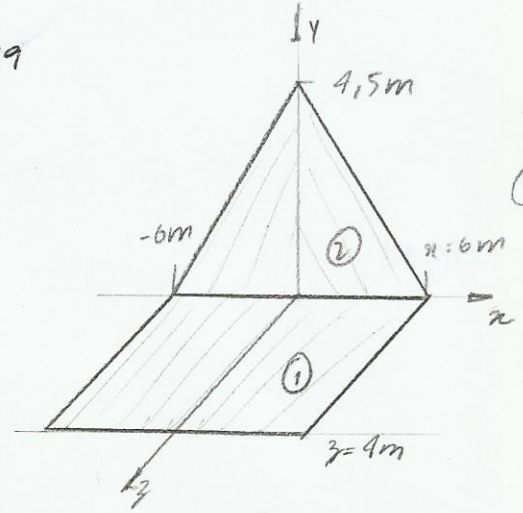


	$\tilde{x}$	$\tilde{y}$	A
①	0	0	18π
②	0	0	-36
③	-1	-2	9

$$\bar{x} = \frac{-9}{29,55} = \boxed{-0,305\ m}$$

$$\bar{y} = \frac{-18}{29,55} = \boxed{-0,609\ m} \quad \bar{z} = \boxed{-0,25\ m}$$

3.49



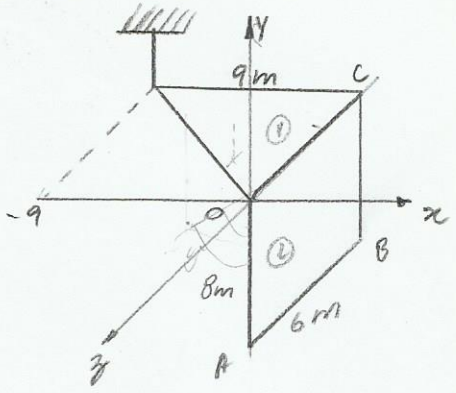
	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	A
①	0	0	2	48
②	0	1,5	0	27

$$\bar{x} = 0$$

$$\bar{y} = \frac{1,5 \cdot 27}{48+27} = 0,590\ m$$

$$\bar{z} = \frac{2 \cdot 48}{48+27} = 1,28\ m$$

1.º Nov 2009 (P1) Noturno



$C_1 = (-3, 0, -4)$ $A_1 = 27$

$C_2 = (0, -4, -3)$ $A_2 = 48$

$A_T = 75$

10) $C = (\bar{x}; \bar{y}; \bar{z}) = \boxed{(-1,08; -2,56; -3,36)}$

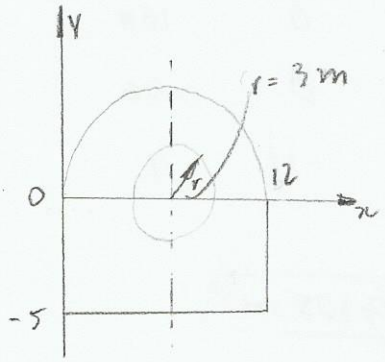
$$x = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$x = \frac{-3 \cdot 27}{75} = -1,08$$

$$y = \frac{-4 \cdot 48}{75} = -2,56$$

$$z = \frac{-4 \cdot 27 - 3 \cdot 48}{75} = -3,36$$

Centro de Massa: P1 sem 2011



	$\bar{x}$	$\bar{y}$	A
semicírculo	6	$8/\pi$	18π
círculo	6	0	-9π
retângulo	6	-2,5	-60
			88,27

$$\bar{x} = \frac{6.18\pi - 6.9\pi + 6.60}{88,27} = 6\ m$$

$$\bar{y} = \frac{8/\pi \cdot 18\pi - 2,5.60}{88,27} = -0,068\ m$$

3.52

(1) → (2; 0,75; 1,5) $V_1 = 18$ $V_T = 20,88$

(2) → (0,4; 2,3; 1) $V_2 = 2,88$

$$\bar{x} = \frac{2\,18 + 0,4.2,88}{20,88} = 1,76$$

$$\bar{y} = \frac{0,75.18 + 2,3.2,88}{20,88} = 0,96$$

$$\bar{z} = \frac{1,5.18 + 1.2,88}{20,88} = 1,43$$

3.54

$\rho_1 = 5,32\ g/cm^3$ $\rho_2 = 2,33\ g/cm^3$

$\rho = \frac{m}{V} \Rightarrow m = \rho.V$

(1) $V_1 = 15.30.6 = 2700 \Rightarrow m = 14364\ g$ $C_1 = (3; 15; 7,5)$

(2) $V_2 = 15.30.6 = 2700 \Rightarrow m = 6291\ g$ $C_2 = (3; 45; 7,5)$

$$\bar{x} = \frac{3(14364 + 6291)}{14364 + 6291} = 3 \qquad \bar{y} = \frac{15.14364 + 45.6291}{14364 + 6291} = 24,14$$

3.60

	$\bar{x}_i$	$\bar{y}_i$	$\bar{z}_i$	V_i	m_i
Silicio	9	4	0	864	2013,12
Germânico	9	-4	0	864	4596,48
Cobre1	21,5	4	0	28	250,88
Cobre2	21,5	-4	0	28	250,88

$$\bar{x} = \frac{9(2013,12 + 4596,48) + 21,5(250,88.2)}{7111,36} = 9,88\,mm$$

$$\bar{y} = \frac{4.2013,12 - 4.4596,48 + 4.250,88 - 4.250,88}{7111,36} = -1,45\,mm$$

e $\bar{z} = 0$

Teorema de Pappus

$A = \theta \bar{r} L$

$V = \theta \bar{r} A$

3.67 $A = \theta \bar{r} L = 2\pi \cdot \frac{r}{2} \cdot r = \pi r^2$

3.68 $A = \theta \bar{r} L = 2\pi \cdot r\, h$

3.69 $A = \theta \bar{r} L = 2\pi \cdot \frac{r}{2} \cdot g = \pi r g$

3.70 $V = \theta \bar{r} A = 2\pi \frac{r}{2} \cdot r h = \pi r^2 h$

3.71 $V = \theta \bar{r} A = 2\pi \cdot \frac{r}{3} \cdot \frac{hr}{2} = \frac{\pi r^2 h}{3}$

3.74 $V = \theta \bar{r} A = 2\pi \cdot (r+R) \cdot \pi r^2 = 2\pi^2 r^2 (R+r)$

3.76

	x	y	A
Retangulo	3	4,75	47,5
triangulo	4,5	9	-2,25
Cirmulo	3,6	3,6	$-0,36\pi$
			44,12

$$\bar{x} = \frac{3.47,5 - 4,5.2,25 - 3,6.0,36\pi}{44,12}$$

$\therefore \bar{x} = 2,908$

$$\bar{y} = \frac{4,75.47,5 - 9.2,25 - 0,36\pi\ 3,6}{44,12}$$

$\therefore \bar{y} = 4,56$

b) $V = \theta \bar{r} A$

$V = 2\pi \cdot 2{,}91 \cdot 44{,}12 \therefore \boxed{V = 806{,}7\ m^3}$

Torque e sistemas equivalentes

4.7 (4.7)

Para escrever uma força vetorialmente. Temos que

$$\vec{F} = F \cdot \frac{P-A}{|P-A|}$$

$\vec{r} = C-O = 0 + 20\vec{j} + 28\vec{k}$

$\vec{T} = 29 \cdot \dfrac{(7\vec{i} - 11\vec{j} - 28\vec{k})}{\sqrt{7^2+11^2+28^2}} = 6{,}57\vec{i} - 10{,}328\vec{j} - 26{,}289\vec{k}$

$$\vec{\sigma} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 20 & 28 \\ 6{,}57 & 10{,}328 & 26{,}289 \end{vmatrix} = 236{,}596\vec{i} - 183{,}96\vec{j} - 131{,}4\vec{k} = 327{,}24$$

2.12

$F = 240\,N$

$A - P = 1\vec{i} + 2\vec{j} - 7\vec{k} \rightarrow \vec{F} = F\lambda = F \cdot \dfrac{(A-P)}{|A-P|}$

$\vec{F} = 240 \dfrac{(-1\vec{i} + 2\vec{j} - 7\vec{k})}{\sqrt{1^2+2^2+7^2}} = -32{,}66\vec{i} + 65{,}32\vec{j} - 228{,}62\vec{k}$

$P - O = 4\vec{i} + 9\vec{j} + 7\vec{k}$

$$\vec{\sigma}_O^F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 9 & 7 \\ -32{,}66 & 65{,}32 & -228{,}62 \end{vmatrix} = -2514{,}82\vec{i} + 685{,}86\vec{j} + 555{,}22\vec{k}$$

4.8

$$\vec{T} = T.\lambda = T.\frac{(C-A)}{|C-A|} = 29.\frac{(11\vec{i}-7\vec{j}-28\vec{k})}{\sqrt{11^2+7^2+28^2}}$$

$$\vec{T} = 10{,}328\vec{i} - 6{,}572\vec{j} - 26{,}289\vec{k}$$

$$(A-O) = 0 + 20\vec{j} + 28\vec{k}$$

$$\vec{\sigma}_O^F = \begin{vmatrix} -i & j & k \\ 0 & 20 & 28 \\ 10{,}328 & -6{,}572 & -26{,}289 \end{vmatrix} = -341{,}769\vec{i} + 289{,}189\vec{j} - 206{,}56\vec{k}$$

$$\therefore |\vec{\sigma}_O^F| = 493 \text{ kN.m}$$

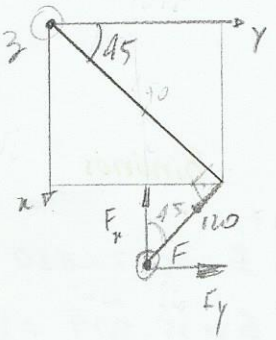
4.9

$F = 120N$

$$(A-O) = 20\cos 45\vec{i} + 20\sin 45\vec{j} + 5\vec{k}$$
$$= 10\sqrt{2}\,\vec{i} + 10\sqrt{2}\,\vec{j} + 5\vec{k}$$

$$\vec{\sigma}_r = F \times r = |F||r|\sin\theta$$
$$= 120.\sqrt{(10\sqrt{2})^2+(10\sqrt{2})^2+5^2}.\sin 90$$

$$\therefore \sigma_r = 2473{,}86 \text{ Ncm} = 24{,}74 \text{ Nm}$$



4.10

$$(A-D) = 0 + 6\vec{j} + 3\vec{k}$$

(a) $$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -10 & 6 & 3 \\ 0 & 6 & 3 \end{vmatrix} = 0\vec{i} - 30\vec{j} + 60\vec{k} \text{ N.m}$$

(b) $$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 5 \\ 8 & 0 & 12 \end{vmatrix} = -64\vec{j} \text{ N.m}$$

(c) $$F_r = (-10-2)\vec{i} + 6\vec{j} + (5+3)\vec{k} \quad \therefore \quad \vec{F}_r = -12\vec{i} + 6\vec{j} + 8\vec{k}$$

$$\vec{\sigma}_r = \sum_i \vec{\sigma}_i = (60)\vec{i} + (-30-64-90)\vec{j} + (60+120)\vec{k}$$

$$\therefore \vec{\sigma}_r = 60\vec{i} - 184\vec{j} + 180\vec{k} \text{ (N.m)}$$

4.12 $\vec{F} = 6\vec{i} - 8\vec{j} + 5\vec{k}$ (N)

$\vec{\tau}_3 = -6.0{,}8 - 8.1{,}5 = -16{,}8$ Nm

4.13

$\vec{\tau}_{AB} = \frac{1}{\sqrt{5}} \cdot 200 \cdot 8 = 715{,}54$ (Nm)

4.14 $\Sigma F = \vec{F}_r = -4\vec{i} + 7\vec{j} + (12-2)\vec{k} \therefore \vec{F}_r = -4\vec{i} + 7\vec{j} + 10\vec{k}$

$\tau_1 = 7.4 = 28$ Nm $\quad \tau_2 = 12.5 = 60$ Nm $\quad \tau_3 = 4.3 = 12$ Nm

$\vec{\tau}_1 = 28\vec{k}$ (Nm) $\quad \vec{\tau}_2 = 60\vec{i}$ (Nm) $\quad \vec{\tau}_3 = -12\vec{j}$ (Nm) $\quad \vec{\tau}_i = -30\vec{j}$

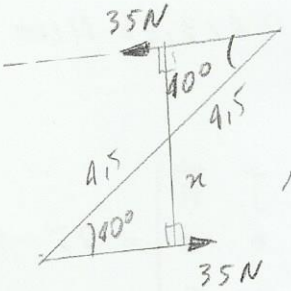
$$\tau_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & 3 \\ 0 & 0 & -2 \end{vmatrix} = -6\vec{i} + 8\vec{j} \qquad \vec{\tau}_r = 54\vec{i} - 34\vec{j} + 28\vec{k}$$ (Nm)

Binários

4.17 $\tau_1 = 60.7 = 420 \rightarrow \vec{\tau}_1 = -420\vec{k}$ (Nm)

$\tau_2 = 35.5{,}79 = 202{,}5 \rightarrow \vec{\tau}_2 = 202{,}5\vec{k}$ (Nm)

$\vec{\tau}_r = -217{,}5\vec{k}$ (N.m)



$\operatorname{tg} 40 = \frac{x}{4{,}5}$

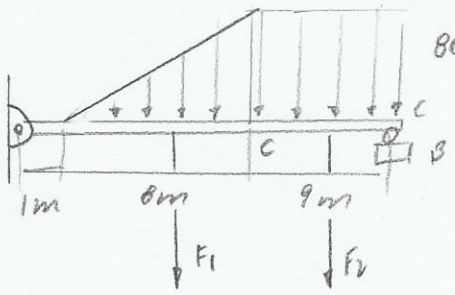
4.18

$\vec{\tau}_1 = -24\vec{i} - 24\vec{k}$ (Nm) $\quad \vec{\tau}_2 = 60\vec{i} - 42\vec{j}$ (Nm)

$\vec{\tau}_3 = 36\vec{k}$ (Nm) $\quad \vec{\tau}_r = 42\vec{i} - 42\vec{j} + 12\vec{k}$ (Nm)

Sistemas de Forças contínuas e paralela

4.19



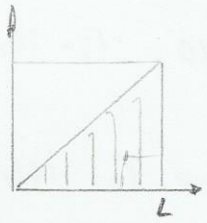
600 N — 1 m

F_2 — 9 m

$\therefore F_2 = 7200 \rightarrow \vec{F_2} = -7200\ N$

$x_2 = 13,5\ m$

F_1:

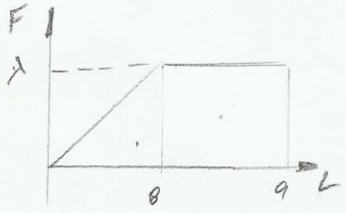


$A = \frac{\lambda L}{2} \overset{N}{=} F_1 \quad \therefore F_1 = \frac{800.8}{2} = 3200 \rightarrow \vec{F_1} = -3200\ N$

$x = \frac{2}{3}.8 + 1 = 6,33$

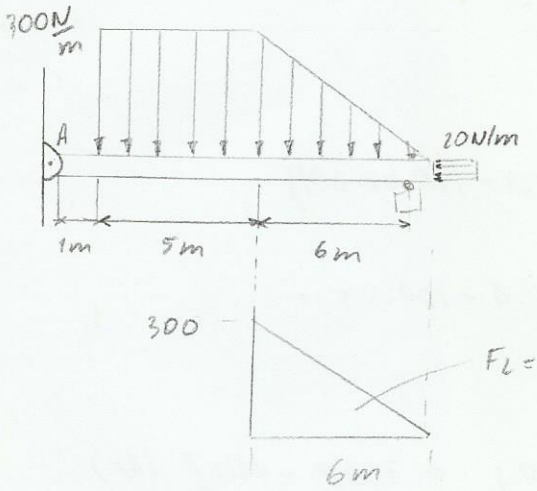
$\vec{F_T} = -10400\ N \quad x =$

Como $\bar{x} = \frac{\int x\,dF}{\int dF} = \frac{\int x\,dA}{\int dA} = \frac{\int x\lambda\,dL}{\int \lambda\,dL} = \frac{6,33.3200 + 13,5.7200}{10400}$



$\int \lambda\,dL = 10400 \quad \therefore x = 11,29\ m$

4.20



300 N — 1 m

F_1 — 5

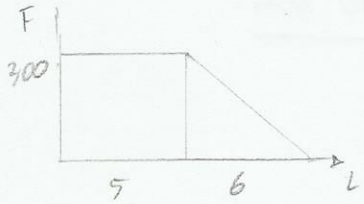
$\therefore \vec{F} = -1500\vec{j}\ (N)$

$x_1 = 3,5\ m$ de A

$F_2 = \frac{300.6}{2} = 900 \rightarrow \vec{F_2} = -900\vec{j}\ (N)$

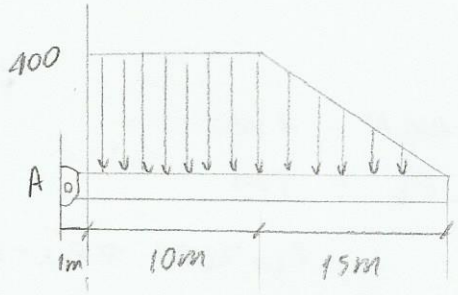
$x_2 = 8m$ de A

$F_3 = 20.0,5 = 10 \rightarrow \vec{F_3} = -10\vec{i}\ (N)$



$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{3,5.5.300 + 8.300.6/2}{(11+5).300} = 5,19\ m$

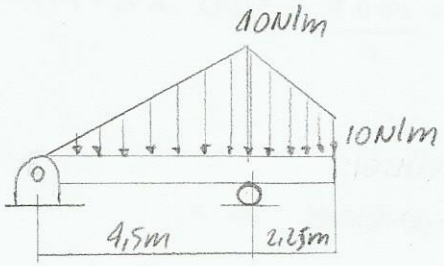
4.21

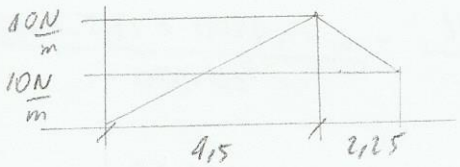


$F_1 = \frac{400.10}{2} = 4000$ $x_1 = 6\,m$ de A

$F_2 = \frac{400.15}{2} = 3000$ $x_2 = 16\,m$ de A

4.26





$F_1 = \frac{40.4,5}{2} = 90$ $F_2 = 56,25$

$F_V = 146,25 \rightarrow \vec{F_r} = -146,25\,N$

$\bar{x} = \frac{3.90 + 5,4.56,25}{146,25} = 3,92$

$x_2 = \frac{0,75.33,75 + 1,125.22,5}{56,25} = 0,9$

Sistemas equivalentes de Forças e torques

4.29

$F_V = \left(\frac{4}{5}.450 + 120\cos 60\right)\vec{i} + \left(\frac{3}{5}450 - 120\,sen\,60\right)\vec{j}$

$F_V = -420\,\vec{i} + 166\,\vec{j}\ (N)$ | $\mathcal{C} = 360,4 - 104.2,5 =$

4.30 $F_V = \left(\frac{4}{5}.700 + 600 - 400\right)_i - \frac{3}{5}.700\,j = 760\vec{x} - 420\vec{j}\ (N)$

$\vec{\mathcal{C}}_R = 360.4 - 103,92.2,5 \therefore \vec{\mathcal{C}}_r = 1180 - 340 = 840\ Nm$

$\mathcal{C}_r = 840 = 420d \therefore d = 2$

4.30

$$F_r = \left(\frac{4}{5}700 + 600 - 400\right)\vec{i} - \frac{3}{5}700\,\vec{j} = 760\vec{i} - 420\vec{j}\ (N)$$

$$\vec{G}_r = \frac{3}{5}700.9 + 600.6 = 7360 \rightarrow \vec{G}_r = 7380\vec{k}\ (N.m)$$

$$7380 = 760d \quad \therefore \quad d = 9{,}71\ m$$

4.31

$$F_r = (25 - 15\cos 30)\vec{j} + (12 - 15\sin 30)\vec{k}$$

$$\therefore \quad F_r = 12\vec{j} + 4{,}5\vec{k}\ (N)$$

$$G_r = -25.1{,}5 + 12.3 + 15\cos 30 \cdot 1{,}5 - 15\sin 30.3$$

$$\therefore \quad G_r = -9{,}51\vec{i}$$

$$-4{,}51 = 12d \quad \therefore \quad d = 0{,}38\ m$$

4.35 (Fayr dipoir)

4.34

$$F_r = (-8 + 10\sin 35)\vec{i} + (-12 - 10\cos 35)\vec{j}$$

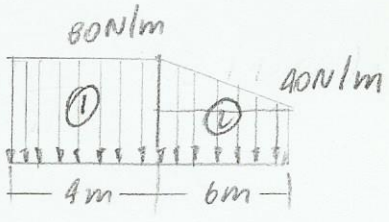
$$\therefore \quad \vec{F}_r = -2{,}26\vec{i} - 20{,}19\vec{j}\ (N)$$

$$G_r = -12.10 + 8.8 - 10\cos 35.23 - 10\sin 35.15 + 300{,}4 = -30{,}04$$

$$\therefore \quad \vec{G}_r = -30{,}04\vec{k}\ (N.m)$$

$$-30{,}04 = -20{,}19\,d \quad \therefore \quad d = 1{,}49\ m$$

4.36



$F_1 = 80.4 = 320\ N \quad x_1 = 2\ m \quad F_e = -660\ N$

$$F_2 = \frac{(80+40).6}{2} = 360\ N$$

$$x_2 = \frac{2.120 + 3.240}{360} = 8{,}13\ m$$

$$F_r = \left(\frac{2}{\sqrt{13}}.540 + 4500\right)\vec{i} + \left(\frac{3}{\sqrt{13}}.540 - 320 - 360\right)\vec{j}$$

$$F_r = 7500\vec{i} + 3620\vec{j}\ (N)$$

$$\frac{2.320 + \frac{20}{3}.360}{660} \Rightarrow$$

$$\therefore \quad x = 4{,}47\ m$$

Equilíbrio

5.5

$\Sigma F_x = 0$: $A_x - 5 \operatorname{sen} 30 = 0$ $\therefore$ $A_x = 2,5\ N$

$\Sigma F_y = 0$: $B_y + A_y - 5 \cos 30 - 50 = 0$

$\Sigma \tau_v = 0$: $-10 B_y + 5 \cos 30 \cdot 8 + 50 \cdot 5 = 0$ $\therefore$ $B_y = 28,46\ N$

$A_y = 50 + 5 \cos 30 - 28,46$ $\therefore$ $A_y = 82,79\ N$

$\therefore$ $\vec{A} = 2,5\,\vec{i} + 82,79\,\vec{j}\ (N)$ e $\vec{B} = 28,46\,\vec{j}\ (N)$

5.6

$m = 2\,kg$

$F = 300\,N$

$\Sigma F_x = 0$: $A_x + B_x = 0$

$\Sigma F_y = 0$: $A_y - 300 - 2 \cdot 9,8 = 0$ $\therefore$ $A_y = 319,6\ N$

$\Sigma \tau_v = 0$: $20 B_x - 20 \cdot 2 \cdot 9,8 - 300 \cdot 30 = 0$

$G = (\bar{x}, \bar{y}) = (20, \frac{10}{3})$

$B_x = \dfrac{300 \cdot 30 + 20 \cdot 29,8}{20} = 470\ N.$

$\therefore$ $A_x = -B_x = -470\,N$

$\therefore$ $\vec{A} = -470\,\vec{i} + 319,6\,\vec{j}\ (N)$ $B_x = 470\,\vec{i}\ (N)$

5.9

I $A_x - T \operatorname{sen} 40 = 0$

II $A_y + T \cos 40 - 20 \cdot 9,8 = 0$

III $T \cos 40 \cdot 8 \cos 20 - 20 \cdot 9,8 \cdot 4 \cos 20 - T \operatorname{sen} 40 \cdot 8 \operatorname{sen} 20 = 0$

III : $T(\cos 40 \cdot 8 \cdot \cos 20 - \operatorname{sen} 40 \cdot 8 \operatorname{sen} 20) = 20 \cdot 9,8 \cdot 4 \cos 20$

$\vec{T} = \dfrac{20 \cdot 9,8\, 4 \cos 20}{\cos 40 \cdot 8 \cdot \cos 20 - \operatorname{sen} 40 \cdot 8 \operatorname{sen} 20} = 184,18$

$\boxed{T = -118,4\,\vec{i} + 141,1\,\vec{j}\ (N)}$

$A_x = -118,4$ $A_y = 20 \cdot 9,8 - 141,1 = 54,91$

$\boxed{\vec{A} = -118,4\,\vec{i} + 54,91\,\vec{j}\ (N)}$

5.13

$A_x + 150 \operatorname{sen} 30 = 0 \therefore A_x = -75$

$A_y + B - 200 - 245 - 150 \cos 30 = 0$

$1,5B - 200.4 - 245.4,36 - 150.6.\cos 30 + 150.3.\operatorname{sen} 30 = 0$

↳ D $\therefore B = 1615,08\ N \therefore \boxed{\vec{B} = 1615,08\,\vec{j}\ (N)}$

$A_y = 200 + 245 + 150\cos 30 - 1615,08 \therefore A_y = -1040,2$

$\boxed{\vec{A} = -75\,\vec{i} - 1040,2\,\vec{j}\ (N)}$

5.14

$m = 2200\ kg \rightarrow P = 21560N = 21,56\ kN$

- Com o motor desligado

$N_B + N_A - 21,56 = 0 \therefore \boxed{N_B = 2,65\ kN}$ e $\boxed{fat_A = 0}$

$-21,56.5 + 5,7\,N_A = 0 \therefore \boxed{N_A = 18,9\ kN}$

- Com o motor ligado

$4\cos 15 - fat_A = 0$

$4\operatorname{sen} 15 + N_A + N_B - 21,56 = 0$

$-21,56.5 + 5,7\,N_A + 4\operatorname{sen} 15.5 - 4\cos 15.1,6 = 0$

$\therefore N_A = 19,1\ kN \quad N_B = 1,49\ kN \quad fat_A = 3,86\ kN$

5.15

$A_x - 700 - \frac{4}{5}500 = 0 \quad \therefore \quad A_x = 1100$

$A_y - 600 - 400 - \frac{3}{5}.500 + B = 0 \rightarrow A_y + B = 1300$

$-2000 - 600.2 - 400.4 - 700.1,5 - \frac{3}{5}500.6 + 6B = 0 \quad \therefore \boxed{B = 1275\vec{j}\ N}$

$A_y = 1300 - 1275 \quad \therefore \quad \vec{A_y} = 25\vec{j} \quad \therefore \boxed{\vec{A} = 1100\vec{i} + 25\vec{j}}$

5.35

$20.9,8 - T = 0 \quad \therefore \boxed{T = 196N}$

$N - 30.9,8 - 9,8m - 98 = 0$

$0,2.9,8m - 0,5.98 - 20.9,8.0,5 = 0 \quad \therefore \boxed{m_A = 75\ kg}$

$N = 98 + 9,8.75 + 30.9,8 \quad \therefore \boxed{N = 1127N}$

5.41

$B_x + A_x - 130 - 90 \operatorname{sen} 30 = 0 \rightarrow B_x + A_x = 175$

$B_z + A_z - F - 90 \cos 30 = 0 \rightarrow B_y + A_y - F = 77,94$

$0,3F - 0,3.90 + 0,2(50 - 80) = 0 \quad \therefore F = 110 \quad \therefore \boxed{\vec{F} = -110\vec{j}\ (N)}$

$(80+50).1,5 + 90 \operatorname{sen} 30.2,3 - A_x.3,5 = 0 \quad \therefore A_x = 85,29 \text{ e } B_y = 89,7$

$-110.2,3 - 90 \cos 30.2,3 + A_y.3,5 = 0 \quad \therefore A_y = 123,50 \text{ e } B_x = 51,5$

$\boxed{\vec{A} = 85,29\vec{i} + 123,50\vec{j}}\ (N) \quad \boxed{\vec{B} = 89,7\vec{i} + 51,5\vec{j}}\ (N)$

Momento de Inércia

6.1

$$I_x = I_x^1 + I_x^2 + I_x^3$$

□: $I_x^1 = \frac{B h^3}{3} = \frac{a^4}{3} = \frac{6^4}{3} = 432$

◺: $I_x^2 = \frac{1}{16}\pi r^4 = \frac{1}{16}.\pi.6^4 = 81\pi$

○: $I_x^3 = \frac{1}{4}\pi r^4 + A.d^2 = \frac{1}{4}\pi.0{,}5^4 + 0{,}25\pi.5^2$

$$I_x^A = 432 - 81\pi - \frac{1}{4}\pi 0{,}5^4 - 0{,}25\pi.5^2 \quad \boxed{I_x^A = 157{,}84\ m^2}$$

$$I_m = \frac{m}{A}.I_x^A \;;\; A = a^2 - \frac{\pi a^2}{4} - \pi r^2 = 6^2 - \frac{\pi.6^2}{4} - \pi.0{,}5^2 = 6{,}99\ m^2$$

$$I_m = \frac{3}{6{,}99}.157{,}84$$

Calculo do Momento de Inércia de Figuras simples

6.6

$$I = \int r^2 dm$$

$$\rho = \frac{m}{L} = \frac{dm}{dL} \;;\; dm = \rho dL \qquad I = \int r^2 \rho dL = \frac{m}{L}.I_c^L$$

$$I_c^L = \int r^2 dL = \int \left(\frac{L}{2}\right)^2 dL = \frac{1}{4}\int L^2 dL = \frac{L^3}{12}$$

$$I = \frac{L^3}{12}.\frac{m}{L} = \frac{L^2.m}{12} = \frac{8^2.3}{12} = 16\ Kgm^2$$

$$I_{11} = \frac{L^3}{12} + L.\frac{L^2}{4}$$

$$= \frac{L^3 + 3L^3}{12} = \frac{4L^3}{12} = \frac{L^3}{3}$$

6.7 $\rho = \frac{m}{L} = \frac{dm}{dt} \;;\; dm = \rho dL \qquad I = \int r^2 dm = \int r^2 \rho dL = \rho \underbrace{\int r^2 dL}_{I_c^L}$

$$\therefore\ I_c^m = \frac{m}{L} I_c^L \;;\; I_c^L = \int r^2 dL$$

$$I_c^L = \int R^2.dL = R^2.L\Big|_0^{2\pi R} = \boxed{2\pi R^3}$$

$$I_c^m = \frac{m}{L} I_c^L = \frac{m}{2\pi R}.2\pi R^3 \;\therefore\; \boxed{I_c^m = mR^2}$$

6.8

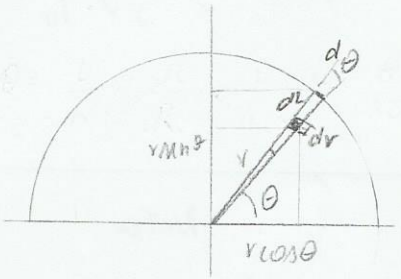
$$\rho = \frac{m}{L} = \frac{dm}{dL} \rightarrow dm = \rho\, dL \qquad I = \int r^2\, dm = \rho \int r^2 dL$$

$$\therefore \quad I_\theta^m = \frac{m}{L} I_c^L \; ; \; I_c^L = \int r^2 dL$$

$$I_x^L = \int R^2 dL \; ; \; dL = R\,d\theta$$

$$= \int_0^\pi R^3 d\theta = R^3\theta\Big|_0^\pi = \pi R^3$$

6.18



$$dA = dL\,dr \qquad d\theta = \frac{dL}{r} \rightarrow dL = r\,d\theta$$

$$dA = r\,dr\,d\theta$$

$$I_x = \int y^2 dA = \int (r\sin\theta)^2 r\,dr\,d\theta = \int r^3 dr \int \sin^2\theta\, d\theta$$

$$= \frac{r^4}{4}\Big|_0^R \cdot \frac{1}{2}\left(\theta - \sin\theta\cos\theta\right)\Big|_0^\pi$$

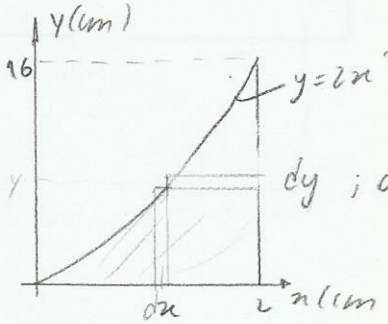
$$= \frac{R^4}{4} \cdot \frac{1}{2}\left(\theta - \sin 2\theta\Big|_0^\pi\right) = \frac{R^4}{8}\left(\pi + \sin 2\pi^{\,0}\right) = \frac{R^4\pi}{8}$$

$$\therefore \quad I_x = \frac{\pi R^4}{8} = I_y$$

$$I_{x'} = \frac{\pi R^4}{8} - \frac{\pi R^2}{2} \cdot \left(\frac{4R}{3\pi}\right)^2 = \frac{\pi R^4}{8} - \frac{\pi R^2}{2} \cdot \frac{16R^2}{9\pi^2}$$

$$= \frac{\pi R^4}{8} - \frac{8R^2}{9\pi}$$

6.13



$y = 2x^3$

$dy = 6x^2 dx$

$dy \; ; \; dA = (2-x)\,dy$

$$I_x = \int r^2 dA \rightarrow \int y^2 (2-x)\,dy$$

$$= \int 4x^6 (2-x) \cdot 6x^2 dx$$

$$= \int 48x^8 - 24x^9\, dx$$

$$= \frac{48x^9}{9} - \frac{24x^{10}}{10}\Big|_0^2$$

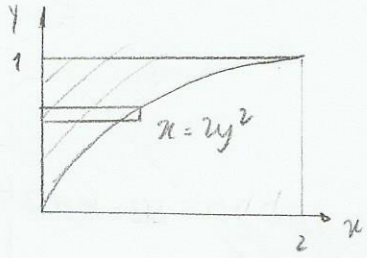
$$\therefore \quad \boxed{I_x = 273\ cm^4}$$

I_y:

$dA = y\,dx \qquad I_y = \int x^2 dA = \int x^2 y\,dx = \int x^2 \cdot 2x^3\,dx = 2\int x^5\,dx$

$= \frac{2x^6}{6}\Big|_0^2 = \underline{21{,}33\ cm^4}$

6.16



$dA = x\,dy \qquad I_x = \int y^2 dA$

$= \int y^2 x\,dy$

$= \int 2y^4\,dy = \frac{2}{5}y^5\Big|_0^1 = 0{,}4\ m^4$

6.33

$A_1 = 6 \qquad I_y^1 = \frac{3.2^3}{12} + 6.4^2 = 96$

$A_2 = 4{,}5 \qquad I_y^2 = \frac{3.3^3}{36} + 4{,}5.6^2 = 164{,}25$

$\therefore \boxed{I = 262{,}25\ m^4}$

6.34

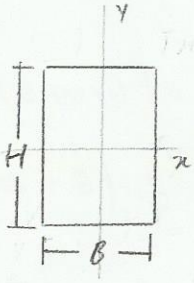
$A_1 = 3.4 = 12 \qquad I_y^1 = \frac{3.4^3}{12} + 12.5^2 = 316\ m^4$

$A_2 = \frac{\pi.1{,}5^2}{2} = 3{,}53 \qquad I_y^2 = \frac{\pi.1{,}5^4}{4} + 3{,}53.\left(7 - \frac{4.1{,}5}{3\pi}\right)^2$

$\therefore I_y^2 = 147{,}1$

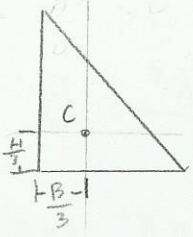
$I_y = 169\ m^4$

Nota



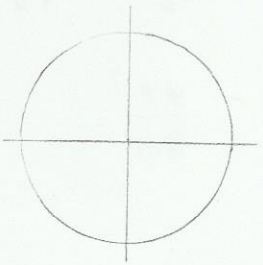
$I_x = \frac{BH^3}{12}$

$I_y = \frac{H.B^3}{12}$



$I_x = \frac{BH^3}{36}$

$I_y = \frac{HB^3}{36}$



$I_x = I_y = \frac{\pi R^4}{4}$

6.35

① $I_y^1 = \frac{1}{8}\pi R^4 + \frac{\pi R^2}{2}.R^2 = \frac{\pi R^4 + 4\pi R^4}{8} = \frac{5\pi R^4}{8}$

② $I_y^2 = I_y^1 = \frac{5\pi R^4}{8}$

$\therefore I_y = \frac{2.5\pi R^4}{8} = \frac{5\pi R^4}{4}$

6.38

1: $I_y^{A1} = \frac{18.12^3}{12} + 12.18.6^2 = 10368\,m^4$

2: $I_y^{A2} = \frac{9.6^3}{36} + 9.3.8^2 = 1782\,m^4$ $\quad \therefore I_y^A = 7769\,m^4$

3: $I_y^{A3} = \frac{\pi.2^4}{4} + \pi.2^2.8^2 = 816\,m^4$

6.42

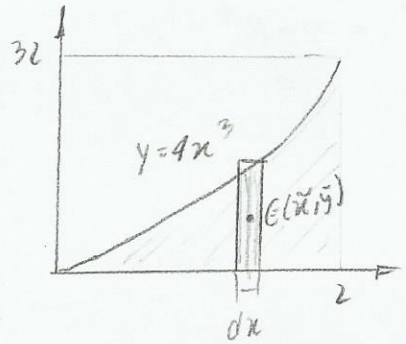
$I_x^{A1} = \frac{9.6^3}{12} = 162 \quad I_x^{A2} = \frac{\pi.2^4}{4} = 4\pi$

$I_x^{A3} = \frac{2,9.7,5^3}{36} + \frac{(1,5.2,9)}{2}.\left(\frac{1,5}{3}\right)^2 = 0,675 = I_x^{A4}$

$\therefore I_x^A = 148,08\,m^4$

Exercícios diversos (Lista Base)

①



$dA = y\,dx \qquad \bar{x} = \dfrac{\int x\,dA}{\int dA}$

$$\int dA = \int y\,dx = \int_0^2 4x^3\,dx = x^4\Big|_0^2 = 16$$

$$\int x\,dA = \int xy\,dx = \int 4x^4\,dx = \frac{4}{5}x^5\Big|_0^2 = 25{,}6$$

$$\bar{x} = \frac{25{,}6}{16} = 1{,}6\ m$$

②

$V = \Theta \vec{r} A$

$= 2\pi \cdot \frac{r}{2} \cdot rh \quad \therefore \boxed{V = \pi r^2 h}$

5.26

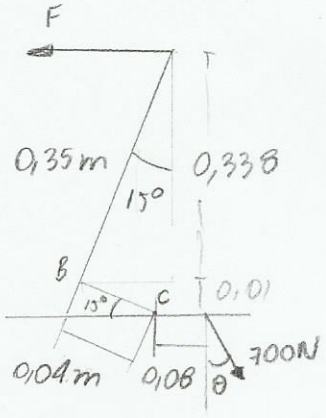
$A_y = 1{,}5 \cdot 9{,}8 + 0{,}2 \cdot 200 + 0{,}3 \cdot 200 = 115\,\vec{j}$

$A_x + B = 0$

$0{,}09B - 0{,}13 \cdot 0{,}2 \cdot 200 - 0{,}3 \cdot 200 \cdot 0{,}98 - 0{,}217 \cdot 1{,}5 \cdot 98 = 0 \quad \therefore B = 413{,}22\,\vec{i}\ (N)$

$$\vec{A} = -413\,\vec{i} + 115\,\vec{j}\ (N)$$

4.35



$0{,}348F - 606{,}21 \cdot 0{,}08 = 0$

$\therefore \vec{F} = -139{,}1\,N$

$\sin 15 = \frac{x}{0{,}04}$

4.46

$\vec{F}_r = (10+10)\vec{i} - 8\vec{j} + (14-6)\vec{k} \quad \therefore \quad \vec{F}_r = 20\vec{i} - 8\vec{j} + 8\vec{k} \ (kN)$

$\vec{G}_g^{Fa} = (6.3)\vec{i} - (6.0{,}8)\vec{j} \quad \therefore \quad \vec{G}_g^{Fa} = 18\vec{i} - 4{,}8\vec{j} \ (kNm)$

$\vec{G}_g^{Fb} = (14.9)\vec{j} + (14.1{,}2)\vec{i} \quad \therefore \quad \vec{G}_g^{Fb} = 16{,}8\vec{i} + 126\vec{j} \ (kNm)$

$\vec{G}_g^{Fc} = (8.9)\vec{k} + (8.0{,}7)\vec{i} \quad \therefore \quad \vec{G}_g^{Fc} = 5{,}6\vec{i} + 72\vec{k} \ (kNm)$

$\vec{G}_g^{T1} = -2.10\,\vec{k} \quad \therefore \quad \vec{G}_g^{T1} = -20\vec{k}\,kNm \ e \ \vec{G}_g^{T2} = 20\,kNm.$

$\vec{G}_r = 40{,}4\vec{i} + 121{,}2\vec{j} + 72\vec{k} \ (kNm)$

5.35

$-T + 20.9{,}8 = 0 \quad \therefore \boxed{T = 196\,N}$

$N - 30.9{,}8 - 9{,}8m - 98 = 0$

$9{,}8.0{,}2\,m - 98.0{,}5 - 20.9{,}8.0{,}5 = 0 \quad \therefore \boxed{m = 75\,kg} \quad \therefore \boxed{N = 1127\,N}$

6.5

$I_{cm} = \frac{m}{2}(b^2+h^2) \qquad I_{cm} = \frac{2}{5}mr^2$

$I_p^c = 160{,}9 \rightarrow I_p^A = 160{,}9 + 2^2.20 = 240{,}9$

$I_e^c = \frac{2}{5}.30.0{,}6^2 = 4{,}32 + (4+0{,}6)^2.30 = 639{,}12$

$\therefore I_m^A = 880{,}02\,kgm$

7.14

$w_{A0} = 3\,rad/s \rightarrow \vec{w}_{A0} = -3\vec{k}\ (rad/s) \qquad \vec{\alpha} = -2\vec{k}\ (rad/s^2)$

2voltas = ? $\qquad$ 1volta $= 2\pi$

2volta $= 4\pi$

Determine: $w_A\,(\theta = 4\pi) = ?$

$w_A^2 = w_{A0}^2 + 2.\alpha\,\Delta\theta$

$w_A^2 = 3^2 + 2.2.4\pi \quad \therefore \quad w = 7{,}7\ rad/s \quad \therefore \boxed{\vec{w}_A = -7{,}7\ \frac{rad}{s}}$

$w_a r_a = w_b r_b$

$w_b = \frac{7{,}70.3}{6}$ $\therefore$ $\vec{w}_b = 3{,}85\,\vec{k}$ (rad/s) $= \vec{w}_c$

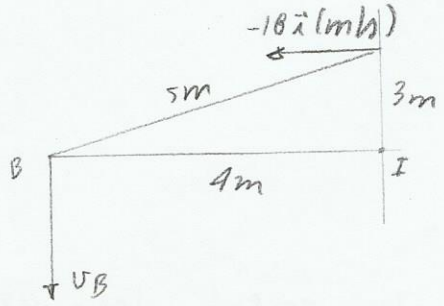
$3{,}85.4 = w_d . 6$ $\therefore$ $\vec{w}_d = -2{,}57\,\vec{k}$ (rad/s)

7.29

- (a) $v_a = ?$

$w_a = \frac{v_a}{r}$ $\therefore$ $v_a = 6.3 = 18$ m/s $\rightarrow$ $\boxed{\vec{v}_a = -18\,\vec{i}\ (m/s)}$

(b)



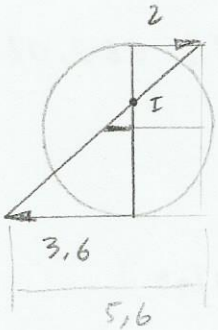
$w_{ab} = \frac{18}{3} = 6 \rightarrow \vec{w}_{ab} = 6\,\vec{k}\ \frac{rad}{s}$

7.30

(a) $w_s = \frac{v_a}{R}$ $\therefore$ $v_s = v_a = 4\ 0{,}5$ $\therefore$ $\boxed{\vec{v}_a = 2\,\vec{i}\ (m/s)}$

(b) $w_c = \frac{v_c}{R_c}$ $\therefore$ $v_c = 4 . 0{,}9$ $\therefore$ $\boxed{\vec{v}_c = -3{,}6\,\vec{i}\ (m/s)}$

(c)



$\frac{5{,}6}{0A} = \frac{3{,}6}{1B}$ $\therefore$ $\boxed{IB = 0{,}257\ m}$

7.52 $\vec{v}_b = -3\,\vec{j}\ (m/s)$ $\vec{a}_b = 5\,\vec{j}\ (m/s^2)$

$\vec{i}\ \vec{j}\ \vec{k}\ \vec{i}\ \vec{j}$ (+ / −)

$v_a = v_b + w_{AB} \wedge (A - B)$

$v_{a_i} = -3\,\vec{j} + w_k \wedge (-4{,}33\,\vec{i} + 2{,}5\,\vec{j})$

$v_{ai} = -3\,\vec{j} - 4{,}33\,w\,\vec{j} - 2{,}5\,w\,\vec{i}$

$v_a = -2{,}5\,w$

$0 = -3 - 4{,}33\,w$ $\therefore$ $w = -0{,}693$ $\rightarrow$ $\boxed{\vec{w}_{AB} = -0{,}693\,\vec{k}\ \left(\frac{rad}{s}\right)}$

$\therefore \vec{v}_a = 1{,}73\,\vec{i}\ (m/s)$

7.54

$\vec{v}_a = \vec{v}_b + \vec{w}_{ab} \times (A-B)$

$v_{ai}\,\vec{i} = 4\vec{i} - 6\vec{k} \times (-0{,}7\vec{j})$

$v_{ai}\,\vec{i} = 4\vec{i} - 4{,}2\vec{i} \quad \therefore \boxed{\vec{v}_a = -0{,}2\,\vec{i}\ (m/s)}$

$\vec{a}_a = \vec{a}_b + \vec{\alpha}_{OB} \times (A-B) - \omega^2 (A-B)$

$\vec{a}_A = -2\vec{i} + 3\vec{k} \times (-0{,}7\vec{j}) - 36\,(-0{,}7\vec{j})$

$\vec{a}_A = -2\vec{i} + 2{,}1\,\vec{i} + 25{,}2\,\vec{j} \quad \therefore \boxed{\vec{a}_A = -0{,}1\,\vec{i} + 25{,}2\,\vec{j}\ (m/s^2)}$

7.56

(a) $\vec{v}_a = \vec{v}_b + \vec{w}_{ab} \times (A-B)$

$-2\vec{j} = -3\vec{j} + w\vec{k} \times (-1{,}6\,\vec{i})$

$-2\vec{j} = -3\vec{j} - 1{,}6w\vec{j} \quad \therefore \boxed{\vec{w} = -0{,}625\,\vec{k}\ (rad/s)}$

(b) $\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \times (A-B) + w_{AB}^2 (A-B)$

$-0{,}15\vec{j} = 0{,}8\vec{j} + \alpha\vec{k} \times (-1{,}6\,\vec{i}) + 0{,}625^2 (-1{,}6\vec{i})$

$-1{,}3\vec{j} = -1{,}6\alpha\vec{j} - 0{,}625\,\vec{i} \quad \therefore \boxed{\vec{\alpha} = 0{,}813\,\vec{k}\ (rad/s^2)}$

Dinâmica de Impulso e Momento

8.5

$m = 15\,000\ kg \qquad F = (50t + 20)\ kN \qquad p = m \cdot g = 15000 \cdot 9{,}8 = 147000\ N$

Determine v após 7s

$m\vec{v}_0^{\,0} + I = mv'$

$\int F\,dt = mv'$

$\int_0^7 50\,000t + 20\,000 - 147\,000\,dt = 15000\,v'$

$25\,000t^2 - 127\,000t\Big|_0^2 = 15000\,v' \quad \therefore \boxed{v' = 22{,}4\ m/s}$

8.8

$m = 1500\ kg$ $\quad v_0 = 20\ m/s$

$T = 600t + 4000\ (N)$ $\quad v(t) = ?$ $\ no\ t = 10\ s$

$$I = \Delta p \rightarrow mv_0 + \int F\,dt = mv'$$

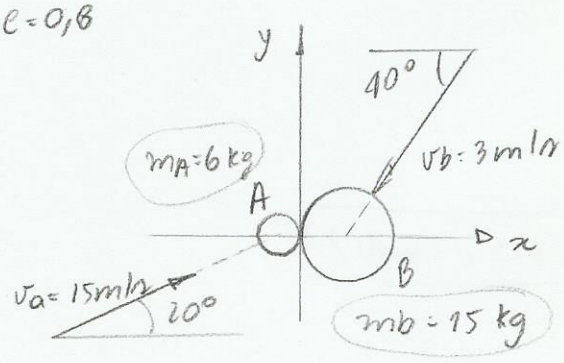
$$1500 \cdot 20 + \int_0^{10} 600t + 4000\ dt = 1500\,v'$$

$$30000 + 300t^2 + 4000t \Big|_0^{10} = 1500\,v'$$

$$\therefore \boxed{v' = 66{,}67\ m/s}$$

8.9

$e = 0{,}8$



Conservação do momentum

$$mv_0 = mv'$$

Velocidades iniciais:

$\vec{v}_{ax} = 14{,}095\,\vec{i}\ (m/s)$ $\quad \vec{v}_{bx} = -2{,}298\,\vec{i}\ (m/s)$

$\vec{v}_{ay} = 5{,}130\,\vec{j}\ (m/s)$ $\quad \vec{v}_{by} = -1{,}928\,\vec{j}\ (m/s)$

Obs.: As velocidades no eixo y são constantes

No eixo x:

$$14{,}095 \cdot 6 - 2{,}298 \cdot 15 = 6v'_{ax} + 15v'_{bx}$$

$$\therefore\ 6v'_{ax} + 15v'_{bx} = 50{,}1$$

$$e = \frac{v'_{ax} - v'_{bx}}{v_{bx} - v_{ax}} \rightarrow v'_{ax} - v'_{bx} = -13{,}1194$$

$$\begin{cases} 6v'_{ax} + 15v'_{bx} = 50{,}1 \\ v'_{ax} - v'_{bx} = -13{,}1194 \end{cases} \sim \begin{cases} 6v'_{ax} + 15v'_{bx} = 50{,}1 \\ 21v'_{bx} = 128{,}786 \end{cases}$$

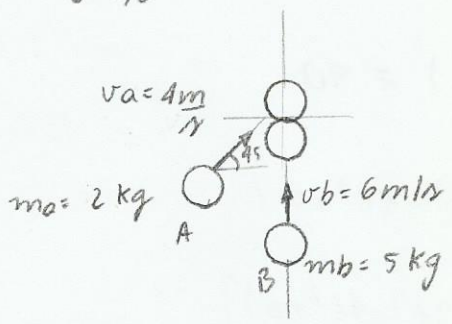
$$\therefore v'_{bx} = 6{,}13 \quad v'_{ax} = -6{,}98$$

R: $\vec{v}'_a = -6{,}98\,\vec{i} + 5{,}130\,\vec{j}$

$\vec{v}'_b = 6{,}13\,\vec{i} - 1{,}928\,\vec{j}$ $\left(\frac{m}{s}\right)$

8.17

$e = 0,8$



$\vec{v_a} = 2,828\,\vec{i} + 2,828\,\vec{j}$ (m/s)

$\vec{v_b} = 6\,\vec{j}$ (m/s)

Obs: As velocidades em x, mantem constante, pois a linha perpendicular entre a linha formada entre os centros de massa são sempre constantes.

Analisando os momentos do eixo y:

$$2,828.2 + 6.5 = 2v_{a'y} + 5v_{b'y} \rightarrow 2v_{a'y} + 5v_{b'y} = 35,657$$

$$e = \frac{v_{a'y} - v_{b'y}}{v_{by} - v_{ay}} \rightarrow 0,8 = \frac{v_{a'y} - v_{b'y}}{6 - 2,828} \rightarrow v_{a'y} - v_{b'y} = 2,54$$

$$\begin{cases} v_{a'y} - v_{b'y} = 2,54 \\ 2v_{a'y} + 5v_{b'y} = 35,657 \end{cases} \rightarrow \begin{cases} 2v_{a'y} + 5v_{b'y} = 35,657 \\ 7v_{b'y} = 30,58 \end{cases}$$

$\therefore v_{b'y} = 4,37 \qquad v_{a'y} = 6,906$

$\vec{v_{a'}} = 2,828\,\vec{i} + 6,906\,\vec{j}$ (m/s)

$\vec{v_{b'}} = 4,37\,\vec{j}$ (m/s)

8.20

$\vec{L_o^A} = 48\,\vec{k}$ kg m²/s $\qquad \vec{L_o^B} = 60\,\vec{i} - 75\,\vec{j}$ kg m²/s

$\vec{L_o^C} = -96\,\vec{i} + 80\,\vec{k}$ Kg m²/s

$\vec{L_o^R} = -36\,\vec{i} - 75\,\vec{j} + 128\,\vec{k}$ (kg.m²/s)

8.21

$\vec{L} + \int \vec{\tau} dt = \vec{L}'$

$L' = Iw^2 = \frac{5.3^2}{3} \cdot (2.2^3 - 4.2^2 + 6) = 90$

$\int \tau dt = L' \rightarrow \int \tau dt = \frac{ml^2}{3} \cdot (2t^3 - 4t^2 + 6)$

$\tau = \frac{d}{dt}\left[\frac{ml^2}{3}(2t^3 - 4t^2 + 6)\right]$

$\tau = \frac{ml^2}{3} \cdot (6t^2 - 8t) = \frac{5.3^2}{3} \cdot (6.2^2 - 8.2) \therefore \vec{\tau} = 120\vec{k}\ (kg\,m^2/s)$

8.22

$m = 2t = 2000\ kg$

$F = 3000t^2 + 50t + 80\ (N)$, $\Delta t = 12s$ $g = 3{,}2\ m/s^2$ $v_0 = 250 m/s$

$v = ?$

$2000 \cdot 250 + \int 3000t^2 + 50t + 80 - 3{,}2 \cdot 2000\ dt = v \cdot 2000$

$2000 \cdot 250 + 1000t^3 + 25t^2 - 6320t \Big|_0^{12} = v \cdot 2000$

$\therefore \vec{v} = 1078\vec{j}\ (m/s)$

8.23

$m = 2t = 2000\ kg$

$T = 32000 - 2000e^{-t/5}\ (N)$

$v_0 = 180\ m/s$ $\Delta t = 10s$

$\int e^{-t/5} dt = \int e^{-t/5} \cdot \frac{(-5)}{5} dt$

$= +5 \int e^{-t/5} d\left(\frac{t}{5}\right)$

$= 5e^{-t/5}$

$2000 \cdot 180 + \int 32000 - 2000e^{-t/5} dt = 2000v$

$180 + \int 16 - e^{-t/5} dt = v$

$180 + 16t - 5e^{-t/5} \Big|_0^{10} = v$

$\therefore \vec{v} = 336\vec{i}\ m/s$

8.24

$$L + \int \tau dt = L'$$

$$5 \cdot 4^2 \cdot 8 + \int 8t + 10 \, dt = L'$$

$$\boxed{640 + 4t^2 + 10t = L'}$$

Para $t = 4 \rightarrow L' = 744 \vec{k}$ $(kg\,m^2/s)$

$$w = 0{,}05\,t^2 + 0{,}125t + 8$$

8.25

(a) $\vec{L} + \int \tau dt = \vec{L}'$

$$120 \cdot \frac{150 \cdot 1{,}2^2}{2} + \int 30 \, dt = \frac{150 \cdot 1{,}2^2}{2} w$$

$$\therefore \quad \vec{w} = (120 - 0{,}278t)\,\vec{k} \quad (rad/s)$$

(b) $0 = 120 - 0{,}278t \quad \therefore \quad t = 432\,s$

(c) $w = w_0 + \alpha t$

$$\alpha = \frac{w - w_0}{t} = \frac{0 - 120}{432} = -0{,}278$$

$$w^2 = w_0^2 + 2\alpha \Delta\theta$$

$$\Delta\theta = \frac{w^2 - w_0^2}{2\alpha} = \frac{0 - 120^2}{2 \cdot (-0{,}278)} \quad \therefore \quad \Delta\theta = 25920\ rad$$

2π = 1 volta

25920 = x $\quad \therefore \quad \boxed{x = 4125{,}3\ voltas}$

8.26

$I = \frac{2}{5} m r^2 \quad ; m = 3\,kg \quad ; r = 0{,}5\,m$

$I_D^A = \frac{2}{5}\ 3 . 0{,}5^2 + 3 . 1{,}5^2 = 7{,}05$

$7{,}05 . \frac{5}{0{,}5} + \int 9t^2 - 6t + 3\, dt = 7{,}05\, w$

$7{,}05 . \frac{5}{0{,}5} + 3t^3 - 3t^2 + 3t \Big|_0^4 = 7{,}05\, w \quad \therefore w = 27{,}18$

8.27

$\vec{P}^0 + I = \vec{P}^1$

$20\vec{i} - 30\cos 25\,\vec{i} - 30 \operatorname{sen} 25\,\vec{j} = 50\vec{v}$

$\therefore \vec{v} = -0{,}144\,\vec{i} - 0{,}254\,\vec{j} \quad (m/s)$

$\sum I^O = -20 . 1 - 30\cos 25 . 1{,}5 - 30 \operatorname{sen} 25 . 1{,}2$

$\therefore \sum I\theta = -76\,\vec{k} \quad N s m$

$-76 = \frac{50 . (4^2 + 3^2)}{12} . w \quad \therefore \vec{w} = -0{,}73\,\vec{k} \quad (rad/s)$

8.29 $m = 3000\,kg \quad K_g = 2{,}5\,m \quad T_A = T_B = 40000\,N \xrightarrow{v = 500\,m/s} T_B' = 2000$

Após $t = 4\,s$

(a) $3000 . 500 + \int_0^4 40000 + 20000 - 3000 . 9{,}8\, dt = 3000 . v$

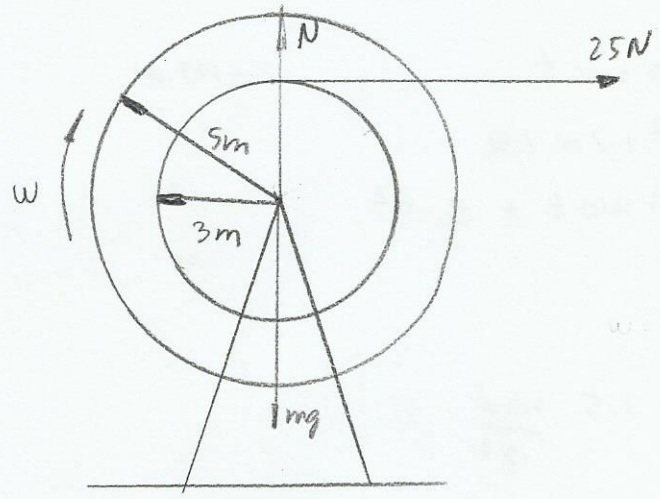
$1500000 + 30600\, t \Big|_0^4 = 3000\, v \quad \therefore v = 540{,}8\,m/s$

Dinâmica de Energia e trabalho

10.5

$m = 50\ kg$

$R = 5m \quad r = 3m \quad \Delta S = 16m \quad F = 25N \quad k_g = 1{,}5m$



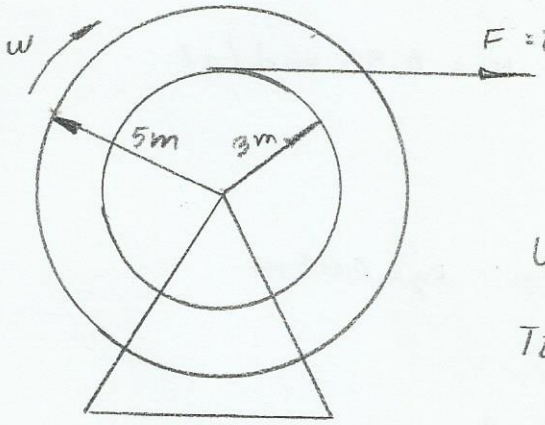
$T_1^{\to 0} + \Sigma U_{1\to 2} = T_2$

$U_{1\to 2} = U^F = \int_0^{16} F\,dr = F.r = 25.16 = 400$

$T_2 = \frac{1}{2} I w^2 = \frac{1}{2}.50.1{,}5^2.w^2$

$400 = \frac{1}{2}\,50.1{,}5^2\,w^2 \quad \therefore \boxed{w = 2{,}67\ \frac{rad}{s}}$

10.7



$w_0 = 0 \quad \Delta S = 5m \quad w = ?$

$m = 90\ kg \quad k_g = 3{,}2\ m$

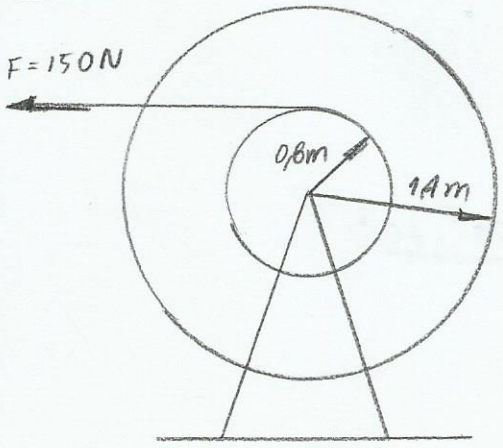
$T_1^{\to 0} + U_{1\to 2} = T_2$

$U_{1\to 2} = U^F = \int F\,dr = 200.5 = 1000$

$T_2 = \frac{1}{2} I w^2 = \frac{1}{2}.90.3{,}2^2.w^2$

$1000 = \frac{1}{2}.90.3{,}2^2.w^2 \quad \boxed{\therefore w = 1{,}47\ rad/s^2}$

10.8



$m = 100\ kg \quad k_g = 0{,}9m$

1 volta $= 2\pi.0{,}6$

4 voltas $= 8\pi.0{,}6$

(a) w para 4 voltas $= 8\pi$

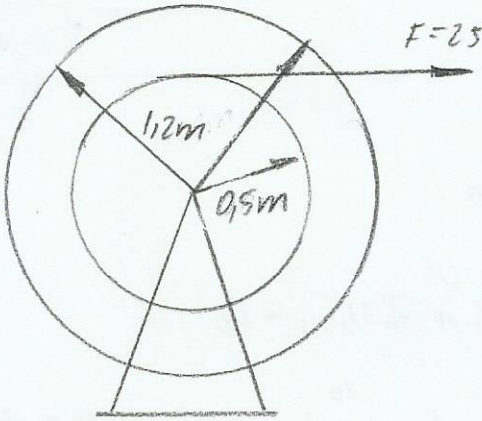
$T_1^{\to 0} + \Sigma U_{1\to 2} = T_2$

$U_{1\to 2} = U^F = 150.8\pi.0{,}6 = 960\pi$

$T_2 = \frac{1}{2}.100.0{,}9^2.w^2$

$960\pi = \frac{1}{2}.100.0{,}9^2.w^2 \quad \boxed{\therefore \vec{w} = 8{,}63\vec{k}\ \left(\frac{rad}{s}\right)}$

10.9



$m = 200\ kg \qquad k_g = 0{,}7\ m$

a) $\omega = ?$ para 3 voltas

1 volta $= 2\pi . 0{,}5$

3 volta $= 3\pi$

$\omega = \omega_0 + \alpha t \qquad F = m.a$

$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$

$\theta = \theta_0 + \omega_0 t + \frac{\alpha}{2} t^2$

$F = ma$

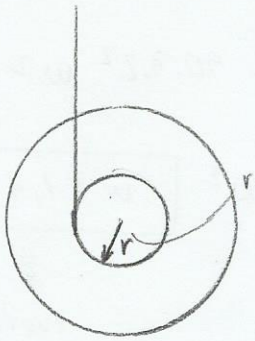
$a = \frac{250}{200} = 1{,}25 \qquad \alpha = \frac{1{,}25}{0{,}5} = 2{,}5\ \frac{rad}{s^2}$

$\omega^2 = \cancel{\omega_0^2}^{\to 0} + 2\alpha\Delta\theta$

$\omega^2 = 2 . 2{,}5 . 3\pi \quad \therefore \boxed{\omega = 6{,}86\ rad/s^2}$

$250 . 3\pi = \frac{1}{2} . 200 . 0{,}7^2 . \omega^2 \quad \therefore \omega = 6{,}93\ rad/s^2$

10.10



$\omega' = 60\ rad/s \qquad k_g = 0{,}06\ m$

$m = 0{,}3\ kg$

$T_1 + U_{1\to 2} = T_2$

$\cancel{m} g h = \frac{1}{2} \cancel{m} v^2 + \frac{1}{2} \cancel{m} k_g^2 . \omega^2$

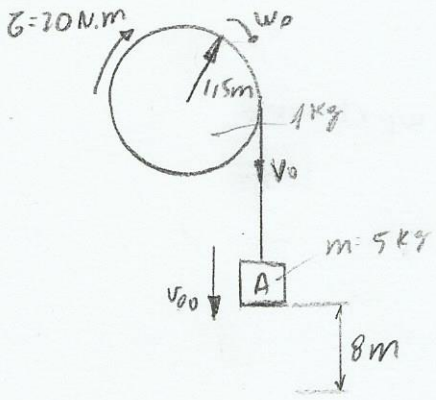
$h = \frac{v^2 + k_g^2 . \omega^2}{2g} = \frac{(60 . 0{,}02)^2 + 0{,}06^2 . 60^2}{2 . 9{,}8}$

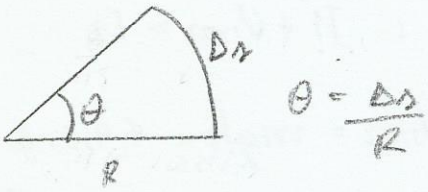
$\therefore \boxed{h = 0{,}735\ m}$

10.13

$m = 1\,kg \quad r = 1{,}5\,m \quad kg = 1{,}2\,m \quad G = 20\,N\,m$

$t = 0 \rightarrow w = 6\,rad/s \quad m = 5\,kg$





$T_1 + U_1 = T_2 + V_2$

$T_1 = \frac{1}{2} m v_0^2 + \frac{1}{2} I w_0^2 = \frac{1}{2} \cdot 5 \cdot (6 \cdot 1{,}5)^2 + \frac{1}{2} \cdot 4 \cdot 1{,}2^2 \cdot 6^2 = 306{,}18$

$U_1 = U^G + U^P = G \cdot \theta + mgh = 20 \cdot \frac{8}{1{,}5} + 5 \cdot 9{,}8 \cdot 6 = 498{,}67$

$T_2 = \frac{1}{2} m v^2 + \frac{1}{2} I w^2 = \frac{1}{2} 5 \cdot v^2 + \frac{1}{2} \cdot 4 \cdot 1{,}2^2 \cdot \frac{v^2}{1{,}5^2}$

$U_2 = 0$

$306{,}18 + 498{,}67 = v^2 \left(\frac{5}{2} + \frac{4}{2} \cdot \frac{1{,}2^2}{1{,}5^2} \right) \therefore \boxed{v = 14{,}59\ m/s}$

10.15

$m = 50\,kg \quad r = 0{,}2\,m \quad kg = 0{,}134\,m \quad m_b = 20\,kg \quad v_{g0} = 2\,m/s$

$w_1 = 20\,rad/s$

$T_1 + U_1 = T_2 + U_2$

$T_1 = \frac{1}{2} m v_0^2 + \frac{1}{2} I w^2 = \frac{1}{2} 20 \cdot 2^2 + \frac{1}{2} \cdot 50 \cdot 0{,}134^2 \cdot (2 \cdot 0{,}2)^2 = 84{,}89$

$U_1 = mgh = 20 \cdot 9{,}8 \cdot h = 196 \cdot h$

$T_2 = \frac{1}{2} m v^2 + \frac{1}{2} I w^2 = \frac{1}{2} \cdot 20 \cdot (20 \cdot 0{,}2)^2 + \frac{1}{2} \cdot 50 \cdot 0{,}134^2 \cdot 20^2 = 339{,}56$

$U_2 = 0$

$84{,}89 + 196 \cdot h = 339{,}56 \quad \therefore \boxed{h = 1{,}3\ m}$

10.24

Determinar v_b no $\Delta h = 6m$

$m_c = 20\,kg \quad r_c = 0{,}25\,m$

$m_p = 8\,kg \quad r_p = 0{,}125\,m$

$m_b = 5\,kg$

$\omega_c r_c = \omega_p r_b = v_b$

$$\cancel{T_1}^{\,0} + U_{1\to 2} = T_2$$

$$U_{1\to 2} = mgh = 5 \cdot 9{,}8 \cdot 6 = 294$$

$$T_2 = \frac{1}{2} m_B v_b^2 + \frac{1}{2} I_c \omega_c^2 + \frac{1}{2} I_p \omega_p^2$$

$$= \frac{1}{2} m_b v_b^2 + \frac{1}{2} \frac{m_c r_c^2 \omega_c^2}{2}^{v_b^2} + \frac{1}{2} \cdot \frac{m_p r_p^2 \omega_p^2}{2}^{v_b^2}$$

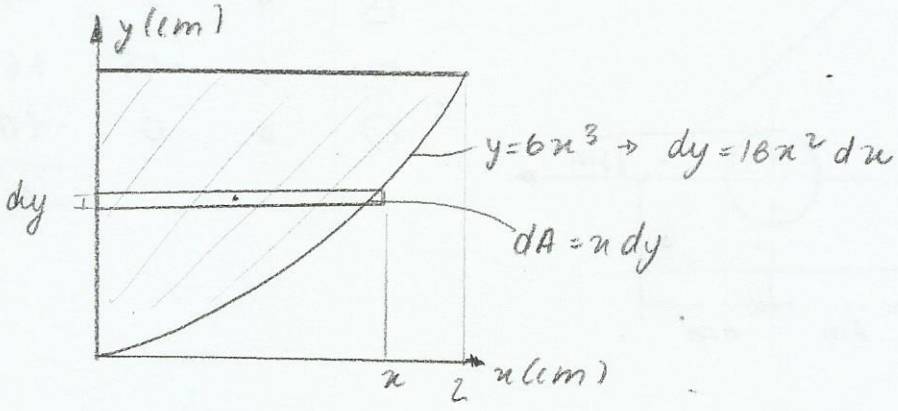
$$= \frac{v_b^2}{2}\left(m_b + \frac{m_c}{2} + \frac{m_p}{2}\right)$$

$$= \frac{v_b^2}{2}\left(5 + \frac{20}{2} + \frac{8}{2}\right) \quad \therefore T_2 = 9{,}5\, v_b^2$$

$294 = 9{,}5\, v_b^2 \quad \therefore \quad v_b = 5{,}56\ m/s \rightarrow \boxed{\vec{v}_b = -5{,}56\,\vec{j}\ (m/s)}$

Revisão para prova

3.19



$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} \qquad \bar{y} = \frac{\int \tilde{y}\,dA}{\int dA}$$

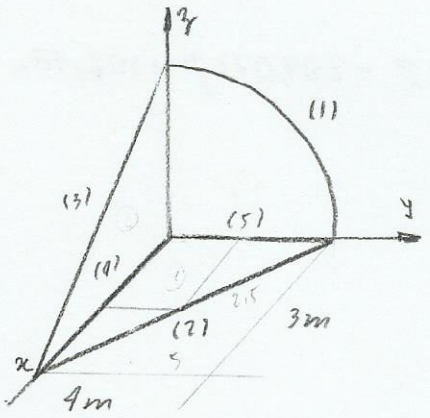
$$\int dA = \int x \cdot 18x^2\,dx = \frac{18x^4}{4}\Big|_0^2 = 72$$

$$\int \tilde{x}\,dA = \int x \cdot x \cdot 18x^2\,dx = \int 18x^4\,dx = \frac{18x^5}{5}\Big|_0^2 = 115{,}2$$

$$\int y\,dA = \int y \cdot x\,dy = \int 6x^3 \cdot x\,18x^2\,dx = \int 108x^6\,dx = \frac{108}{7}x^7\Big|_0^2 = 1974{,}86$$

$$(\bar{x}; \bar{y}) = (0{,}8\ ;\ 27{,}43)\ cm$$

3.27



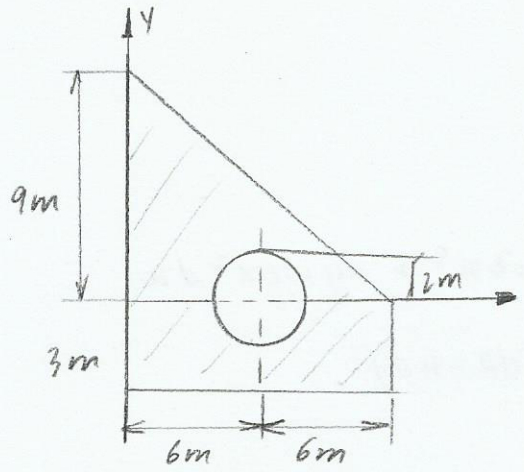
	x	y	z	L
1	0	$8/\pi$	$8/\pi$	2π
2	1,5	2	0	5
3	1,5	0	2	5
4	1,5	0	0	3
5	0	2	0	4

$L_T = 23{,}28$

$$\bar{x} = \frac{1{,}5 \cdot 5 \cdot 2 + 3 \cdot 1{,}5}{23{,}28} = 0{,}838\ m \qquad \bar{y} = \frac{8/\pi \cdot 2\pi + 2 \cdot 5 + 2 \cdot 4}{23{,}28} = 1{,}46\ m$$

$$\bar{z} = \frac{(8/\pi) \cdot 2\pi + 2 \cdot 5}{23{,}28} = 1{,}12\ m$$

3.35



	x	y	A
△	4	3	54
□	6	-1,5	36
○	6	0	-4π

$A_T = 77,43$

$$\bar{x} = \frac{4.54 + 6.36 - 6.4\pi}{77,43} \therefore \bar{x} = 4,61\ m$$

$$\bar{y} = \frac{3.54 - 1,5.36}{77,43} \therefore \bar{y} = 1,39\ m$$

4.8

$\tau = F \times r$

$$\hat{u} = \frac{11\vec{i} - 7\vec{j} - 26\vec{k}}{\sqrt{11^2 + 7^2 + 26^2}} \therefore \hat{u} = 0,356\,\vec{i} - 0,227\,\vec{j} - 0,907\,\vec{k}$$

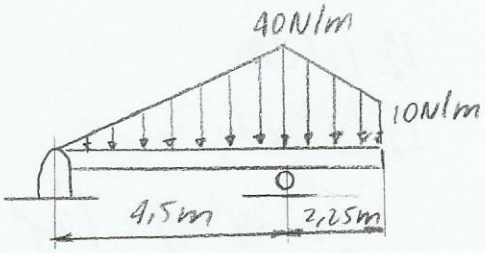
$T = 10,324\,\vec{i} - 6,583\,\vec{j} - 26,303\,\vec{k}$

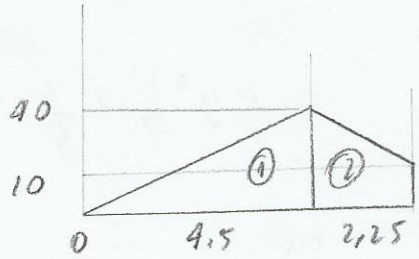
$r = 20\vec{j} + 26\vec{k}$

$$\tau = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10,324 & -6,583 & -26,303 \\ 0 & 20 & 26 \end{vmatrix} = 341,736\,\vec{i} - 289,072\,\vec{j} + 206,48\,\vec{k}$$

$\tau = 492,93\ Nm$

4.26





① $F_1 = 90N \quad x = 3$

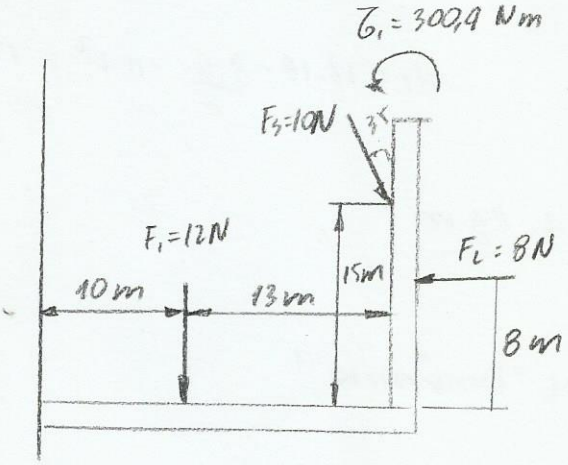
② $F_2 = 56,25N \quad x = 5,4$

$$\bar{x}' = \frac{1,125 \cdot 22,5 + 0,75 \cdot 33,75}{22,5 + 33,75} = 0,9$$

$$\bar{F}_r = -146,75\,\vec{j}\ (N)$$

$$\bar{x} = \frac{3 \cdot 90 + 5,4 \cdot 56,25}{146,25} = 3,92\ m$$

4.34



$F_r = F_1 + F_2 + F_3$

$= -12\,\vec{j} - 8\,\vec{i} + 10 \sin 35\,\vec{i} - 10 \cos 35\,\vec{j}$ $\therefore$ $F_r = -2,26\,\vec{i} - 20,19\,\vec{j}\ (N)$

$C_r = -12.10 + 8.8 - 10 \sin 35 . 15 - 10 \cos 35 . 23 + 300,4$ $\therefore$ $C_r = -30,04\,k\ (Nm)$

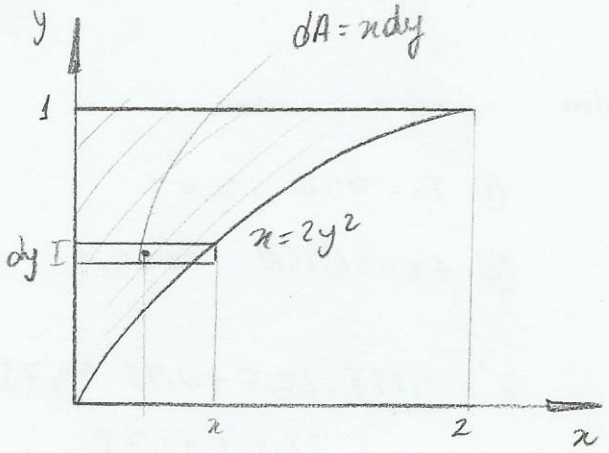
5.36

$$A_x - 500.\frac{4}{5} - 700 = 0 \quad \therefore A_x = 1100\,N$$

$$A_y + B - 600 - 400 - \frac{3}{5}.500 = 0 \quad A_y = 75\,N$$

$$6B - 2.600 - 4.400 - 700.1,5 - 500.\frac{3}{5}.6 - 2000 = 0 \quad \therefore B = 1275\,N$$

6.19



$$I_x^A = \int r^2 dA$$

$$I_x^A = \int y^2 \cdot x\, dy$$

$$= \int_0^1 y^2 \cdot 2y^2\, dy = \int_0^1 2y^4\, dy$$

$$= \frac{2}{5} y^5 \Big|_0^1 = \frac{2}{5} = 0{,}4\ m^4$$

6.38

$$I_y^{A1} = \frac{18 \cdot 12^3}{12} + 12 \cdot 18 \cdot 6^2 = 10368\ m^4$$

$$I_y^{A2} = \frac{9 \cdot 6^3}{36} + 8^2 \cdot \frac{6\,9}{2} = 1782\ m^4$$

$$I_y^{A3} = \frac{\pi \cdot 2^4}{4} + \pi 2^2 \cdot 8^2 = 816$$

$$I_y^A = 10368 - 1782 - 816$$

$$\therefore I_y^A = 7769\ m^4$$

$$A_T = 12 \cdot 18 - \frac{9 \cdot 6}{2} - \pi 2^2 = 176{,}43$$

$$I_y^m = \frac{5}{176{,}43} \cdot 7769 = 220{,}2\ kg\,m^2$$

Revisão de fórmulas, "Dinâmica"

$$\omega = \frac{v_a}{IA} = \frac{v_b}{IB} = \dots$$

Polia e engrenagem

$$\omega_a r_a = \omega_b r_b$$

$$\alpha_a r_a = \alpha_b r_b$$

$$\theta_a r_a = \theta_b r_b$$

$$v_a = v_b + \omega_{ab} \wedge (A-B)$$

$$a_a = a_b + \alpha_{AB} \wedge (A-B) - \omega_{AB}^2 (A-B)$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = \theta_0 + \omega_0 t$$

$$\rightarrow m v_0 + \int F dt = m v'$$

$$\rightarrow I\omega_0 + \int \tau dt = I\omega'$$

$$\rightarrow e = \frac{v_{b'y} - v_{a'y}}{v_{ay} - v_{by}}$$