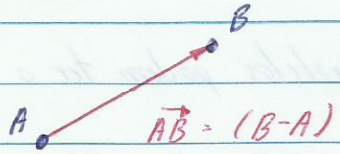
Prof: Novazzi   G.A. → Geometria Analítica

Vetores $\mathbb{R}$ = Real

$\mathbb{R}^2$ = Plano

I - Segmentos Orientados $\mathbb{R}^3$ = Espaço

Dados os pontos A e B do espaço $\mathbb{R}^3$ chama-se segmento orientado de origem A e extremidade B o par ordenado (A, B) que será indicado por $\vec{AB}$ ou $(B-A)$.



Ainda:

(1) Se $A = B$ então o segmento orientado $\vec{AA} = (A-A)$ denomina-se segmento orientado nulo

(2) O segmento orientado $\vec{BA}$ ou $(A-B)$ denomina-se o oposto do segmento orientado $\vec{AB}$ ou $(B-A)$ e é indicado por: $\vec{BA} = -\vec{AB}$ ou $\boxed{(A-B) = -(B-A)}$

(3) Fixada uma unidade de medida, a distância entre os pontos A e B denomina-se comprimento ou módulo do segmento orientado $\vec{AB} = (B-A)$ e/ou $\vec{BA} = (A-B)$ ou seja:

$$|\vec{AB}| = |(B-A)| = |\vec{BA}| = |(A-B)| = d(A, B)$$

Assim, é óbvio que:

$|\vec{AB}| = |(B-A)| \geq 0$ e $|\vec{AB}| = |(B-A)| = 0 \Leftrightarrow A = B$ (neste caso o segmento é nulo)

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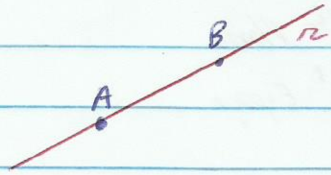
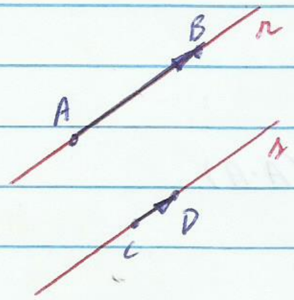
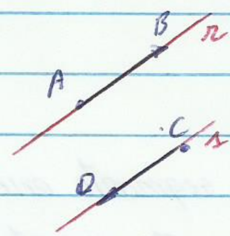
(4) Se $A \neq B$, então a reta que passa por A e por B denomina-se reta suporte ou direção do segmento orientado $\vec{AB} = (B-A)$ e/ou $\vec{BA} = (A-B)$



(5) Dois segmentos orientados e não nulos são paralelos ou têm a mesma direção se e só se as respativas retas suportes são paraleles

(6) Dois segmentos orientados e não nulos e paralelos podem ter o mesmo sentido ou sentido oposto



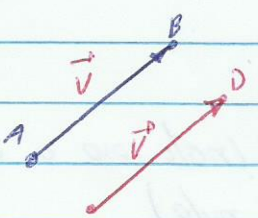


sentidos opostos

mesmo sentido

II- Vetores

Dado o segmento orientado $\vec{AB} = (B-A)$, denomina-se vetor determinado por este segmento e se indica por $\vec{v} = \vec{AB} = (B-A)$ o dado segmento orientado $\vec{CD}$ que tem o mesmo módulo, direção e sentido de $\vec{AB}$



O vetor não possui possição, já o segmento possui. Essa é a diferença entre ambos

Ainda:

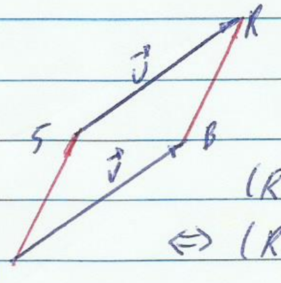
(a) Se $A = B$ então $\vec{v} = \vec{AA} = (A-A)$ denomina-se <u>vetor nulo</u> e é indicado por $\vec{v} = \vec{0}$

(b) O módulo de $\vec{v}$ é o módulo de $\vec{AB}$

A direção de $\vec{v}$ é a direção de $\vec{AB}$

(c) Se $\vec{u} = \vec{BA} = (A-B)$ então $\vec{u}$ denomina-se o <u>oposto</u> de $\vec{v}$ e é indicado por $\vec{u} = -\vec{v}$ ou $(A-B) = -(B-A)$

*(d) Se $\vec{u} = (R-S)$ e $\vec{v} = (B-A)$ então $\vec{u} = \vec{v} \iff (R-S) = (B-A) \iff (R-B) = (S-A)$



$(R-S) = (B-A)$

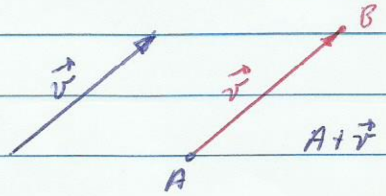
$\iff (R-B) = (S-A)$

Paralelograma (lados iguais são iguais)

III - Operação com vetores

(a) <u>soma de ponto com vetor</u>

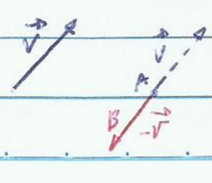
Dado o ponto A e o vetor $\vec{v}$, chama-se <u>soma</u> de A com $\vec{v}$ e se indica por $A + \vec{v}$ ao <u>ponto B</u> tal que $\vec{v} = (B-A)$



$A + \vec{v} = B \iff \vec{v} = (B-A)$

Obs.: Diferença

$A - \vec{v} = A + (-\vec{v})$, onde $(-\vec{v})$ é o oposto de $\vec{v}$



Propriedades:

S1) $A + \vec{0} = A$

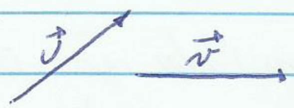
S2) $A + \vec{u} = B + \vec{u} \Rightarrow A = B$

S3) $A + \vec{u} = A + \vec{v} \Rightarrow \vec{u} = \vec{v}$

* S4) $A + (B - A) = B$

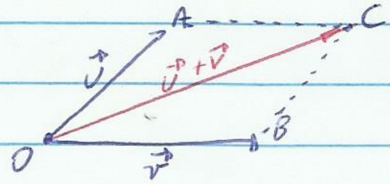
(b) Soma de vetores

b1) Regra do Paralelogramo



Seja O um ponto qualquer e sejam

$A = O + \vec{u}$ e $B = O + \vec{v}$

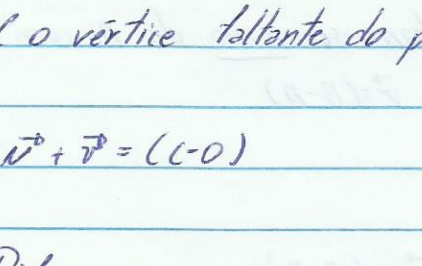


Seja C o vértice faltante do paralelogramo é OABC. Assim

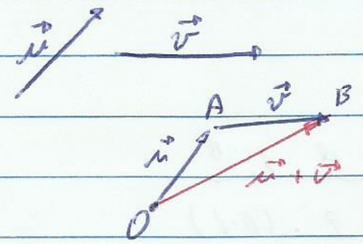
$\vec{u} + \vec{v} = (C - O)$

Obs.: Diferença

$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$



b2) Regra da Poligonal



Propriedade

(a) $\vec{u}+\vec{v}=\vec{0} \iff \vec{u}=-\vec{v}$ ou $\vec{v}=-\vec{u}$

(b) $\vec{u}+\vec{v}=\vec{v}+\vec{u}$

(c) $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$

* (d) $(A-B)=(A-T)+(T-B)$

(c) Produto nº real por vetor

Seja $\alpha \in \mathbb{R}$ e $\vec{v}$ um vetor. O produto $\alpha.\vec{v}$ é o vetor $\vec{w}$ tal que:

(i) Se $\alpha=0$ ou $\vec{v}=\vec{0}$ então $\vec{w}=\vec{0}$

(ii) Se $\alpha\neq 0$ e $\vec{v}\neq\vec{0}$ então $\vec{w}=\alpha.\vec{v}$ tem as seguintes característica:

(a) $|\vec{w}|=|\alpha.\vec{v}|=|\alpha|.|\vec{v}|$

(b) $\vec{w}$ e $\vec{v}$ são paralelos e terão o mesmo sentido se $\alpha>0$ e sentidos oposto se $\alpha<0$

$\mathbb{R}.\vec{v}$: (vetor paralelo ao vetor dado)

Obs.: Se $\vec{v}\neq\vec{0}$ e se $\alpha=\dfrac{1}{|\vec{v}|}$ então

$\vec{w}=\alpha.\vec{v}=\dfrac{1}{|\vec{v}|}.\vec{v}$ denomina-se <u>versor</u> de $\vec{v}$ pois $|\vec{w}|=1$

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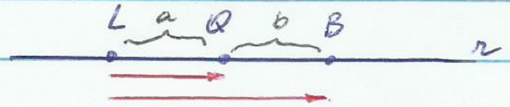
* Obs.: Posição de pontos numa reta



$$Q = L + \frac{a}{a+b} \cdot (B-L)$$

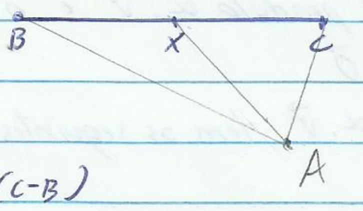
$$Q = L + \frac{7}{10} \cdot (B-L)$$

$$Q = B + \frac{b}{a+b} \cdot (L-B)$$

Exercícios

(1) Determinar o vetor $(X-A)$ em função de $(B-A)$ e $(C-A)$ nos casos

(a) X é o ponto médio de BC



$$X = B + \frac{1}{2}(C-B)$$

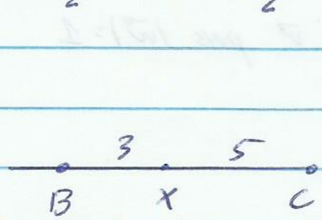
$$(X-A) = (B-A) + \frac{1}{2}(C-B)$$

$$(X-A) = (B-A) + \frac{1}{2}[(C-A)+(A-B)]$$

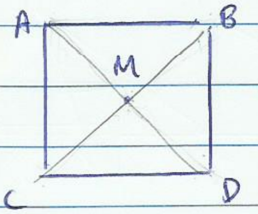
$$(X-A) = (B-A) + \frac{1}{2}(C-A) - \frac{1}{2}(B-A)$$

$$(X-A) = \frac{1}{2}(B-A) + \frac{1}{2}(C-A)$$

b)



Extra 2: Se ABCD são vértices de um quadrado de centro M. Prove que $\vec{AB} + \vec{AC} + \vec{AD} = 4\vec{AM}$



$\vec{AM} = \vec{AB} + \vec{BM}$

$\vec{AM} = \vec{AC} + \vec{CM}$

$\vec{AM} = \vec{AC} + \vec{CD} + \vec{DM}$

$\vec{AB} + \vec{AC} + \vec{AD} = \vec{AD} + \vec{AD} =$

$\vec{AB} + \vec{AC} + \vec{AD} = 2\vec{AD} = 2(2\vec{AM})$

$\vec{AB} + \vec{AC} + \vec{AD} = 4\vec{AM}$

9) A figura representa um hexágono regular ABCDEF, de centro O. Escreva $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$ em função de $\vec{AO}$

$\vec{AO}$ -

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Ex = 8 Livro

temos: $\begin{cases} \vec{AN} = \frac{1}{2}(\vec{AB} + \vec{AC}) \\ \vec{BP} = \frac{1}{2}(\vec{BA} + \vec{BC}) \\ \vec{CM} = \frac{1}{2}(\vec{CA} + \vec{CB}) \end{cases}$

$\vec{AN} + \vec{BP} + \vec{CM} = \frac{1}{2}(\vec{AB} + \vec{AC} + \vec{BA} + \vec{BC} + \vec{CA} + \vec{CB})$

$\vec{AN} + \vec{BP} + \vec{CM} = \frac{1}{2}(\vec{0}) = \vec{0}$

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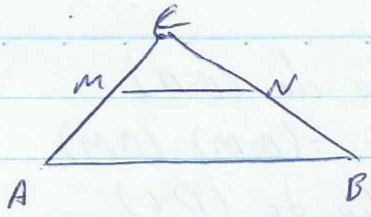
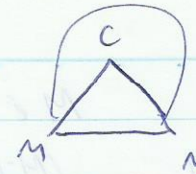
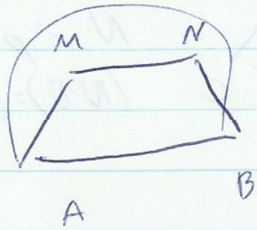
(5)





$\vec{MN} = \vec{MC} + \vec{CN}$



$\vec{MN} = \vec{MA} + \vec{AB} + \vec{BN}$

tese: $\begin{cases} \vec{MN} = \dfrac{1}{2}\vec{AB} \\ \vec{MN} \parallel \vec{AB} \end{cases}$

$2\vec{MN} = \vec{AB}$

$\vec{MN} = \dfrac{1}{2}\vec{AB}$

M é ponto médio de AC $\therefore$ $\vec{AM} = \vec{MC}$

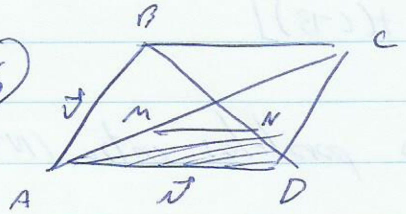
$-\vec{MA} = \vec{MC}$

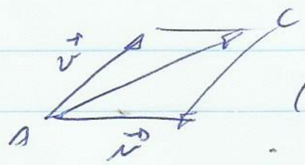
N é o ponto médio de BC $\therefore$ $\vec{BN} = \vec{BN}$

$\vec{BN} = \vec{CN}$

P/ def. de produto de um nº real p/ 1 vetor $\vec{MN} \parallel \vec{AB}$

(6)





$(C-A) = \vec{u} + \vec{v}$

$(M-A) = \dfrac{1}{2}(C-A) \therefore (M-A) = \dfrac{\vec{u}+\vec{v}}{2}$

$(N-A) = \vec{u} + (N-D)$

$(N-D) = \dfrac{1}{2}(B-D)$

$(B-D = \vec{v} - u$

$(N-D) = \dfrac{1}{2}(v-u)$

$* \ (N-A) = \vec{u} + \dfrac{\vec{v}-\vec{u}}{2}$

$(N-A) = \dfrac{\vec{u}+\vec{v}}{2}$

$(M-A) = (N-A)$

$M = N$

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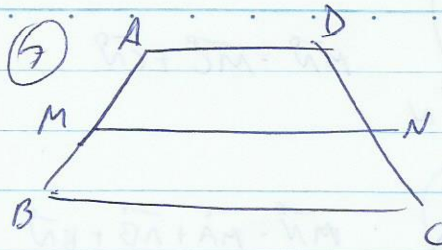
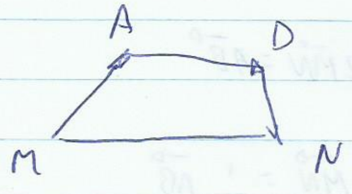


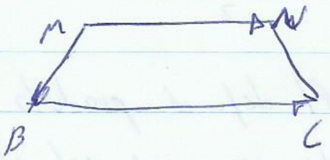
M é pto médio de $(B-A)$

$(M-B)=(A-M) \therefore -(B-M)=(AM)$

N é pto médio de $(D-C)$

$(N-D)=(C-N) \therefore (N-D) = -(N-C)$





$$(N-M) = (A-\cancel{M}) + (D-A) + (N-\cancel{D})$$

$$(N-M) = (B-\cancel{M}) + (C-B) + (N-\cancel{C})$$

$$2(N-M) = (D-A) + (C-B)$$

$$(N-M) = \frac{1}{2}\left[(D-A) + (C-B)\right]$$

Como $(N-M)$ é soma de 2 vetores paralelos então $(N-M)$ é $\parallel$ a ambos

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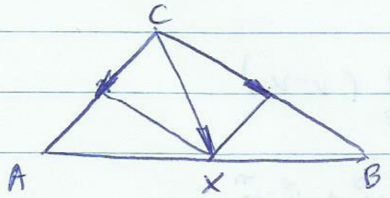
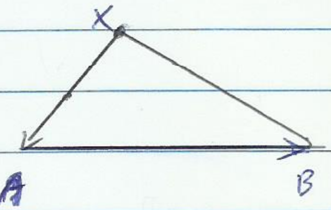
Geometricamente

Ex 3

$$\vec{CX} = \frac{1}{2}\vec{CB} + \frac{1}{2}\vec{CA}$$



1. Dado um ponto X qualquer do espaço e um vetor qualquer $\vec{v} = \vec{AB} = (B-A)$.
Prove que $\vec{AB} = \vec{XB} - \vec{XA}$



$\vec{XB} = \vec{XA} + \vec{XB} \Rightarrow$

$\vec{AB} = \vec{XB} - \vec{XA}$

2. Dados 4 pontos A, B, C e D qualquer. Mostre que: $\vec{AB} - \vec{CD} = \vec{DB} - \vec{CA}$

Sabemos que: $\vec{AB} = \vec{DB} - \vec{DA}$

$\vec{CD} = \vec{AD} - \vec{AC}$

Logo: $\vec{AB} - \vec{CD} = (\vec{DB} - \vec{DA}) - (\vec{AD} - \vec{AC})$

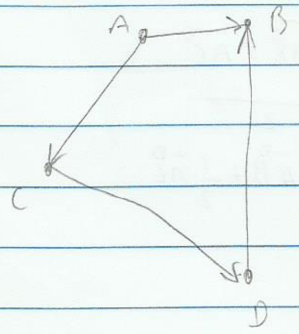
$\vec{AB} - \vec{CD} = \vec{DB} - \vec{DA} - \vec{AD} + \vec{AC}$

$\vec{AB} - \vec{CD} = \vec{DB} + \vec{AD} - \vec{AD} + \vec{AC}$

$\therefore \vec{AB} - \vec{CD} = \vec{DB} + \vec{0} + \vec{AC}$

$\vec{AB} - \vec{CD} = \vec{DB} + \vec{AC} \Rightarrow \vec{AB} - \vec{CD} = \vec{DB} - \vec{CA}$

Provamos que: $\vec{AB} = \vec{AC} + \vec{CD} + \vec{DB}$



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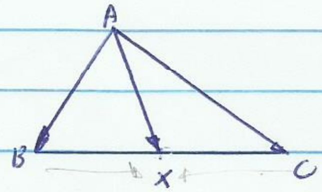
3) Dados os pontos ABC, escreva o vetor $\vec{AX}$ em função dos vetores $\vec{AB}$ e $\vec{AC}$, sabendo-se que x é o ponto médio do seg. BC



Como: $\vec{AX} = \vec{AB} + \vec{BX}$

$\vec{AX} = \vec{AC} + \vec{CX}$

temos que:

$\vec{AX} + \vec{AX} = 2\vec{AX}$

$(\vec{AB} + \vec{BX}) + (\vec{AC} + \vec{CX})$

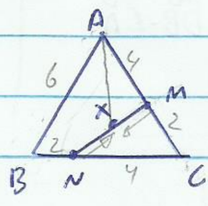
$(\vec{AB} + \vec{BX} + \vec{AC} + \vec{CX}$

$\vec{AB} + \vec{AC} + (\vec{BX} + \vec{CX}) = \vec{AB} + \vec{AC}$

$2\vec{AX} = \vec{AB} + \vec{AC}$

$\vec{AX} = \dfrac{\vec{AB} + \vec{AC}}{2}$ ou $\vec{AX} = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AC}$

Extra 1: O ΔABC é equilátero o seu lado mede 6cm. O seg. AM mede 4cm e o seg BN mede 2cm. Se x é o ponto médio do seg MN, escrever o vetor $\vec{AX}$ em função dos vetores $\vec{AB}$ e $\vec{AC}$



$\begin{cases} \vec{AX} = \vec{AB} + \vec{BN} + \vec{NX} \\ \vec{AX} = \vec{AM} + \vec{MX} \end{cases}$

$2\vec{AX} = \vec{AB} + \vec{BN} + \vec{NX} + \vec{AM} + \vec{MX}$

$2\vec{AX} = \vec{AB} + \vec{BN} + \vec{AM} + \vec{0}$

$2\vec{AX} = \vec{AB} + \frac{1}{3}\vec{BC} + \frac{2}{3}\vec{AC}$

$2\vec{AX} = \vec{AB} + \frac{1}{3}(\vec{AB} - \vec{AC}) + \frac{2}{3}\vec{AC}$

$2\vec{AX} = \vec{AB} + \frac{1}{3}\vec{AB} + \frac{1}{3}\vec{AC} + \frac{2}{3}\vec{AC}$

$2\vec{AX} = \frac{2}{3}\vec{AB} + \vec{AC}$

$\vec{AX} = \dfrac{\frac{2}{3}\vec{AB} + \vec{AC}}{2}$

$\vec{AX} = \frac{1}{3}\vec{AB} + \frac{1}{2}\vec{AC}$

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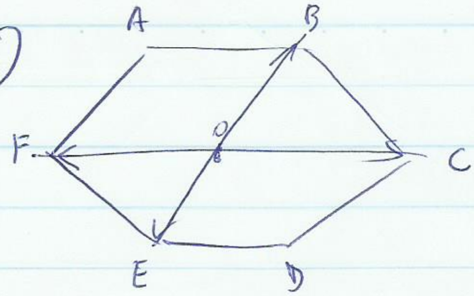


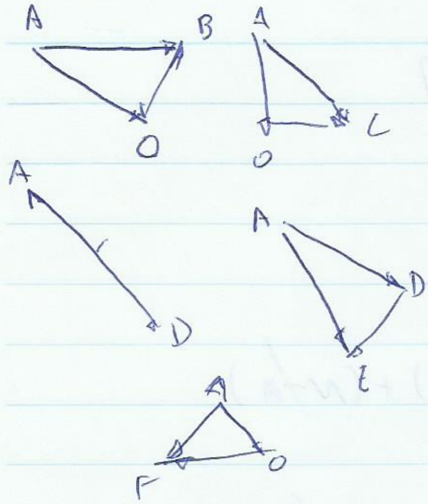
$(B-A) = (O-A) + (B-O)$

$(C-A) = (O-A) + (C-O)$

$(D-A) = 2(O-A)$

$(E-A) = (O-A) + (E-O)$

$(F-A) = (O-A) + (F-O)$



$(B-A) + (C-A) + (D-A) + (E-A) + (F-A) = 6(O-A)$

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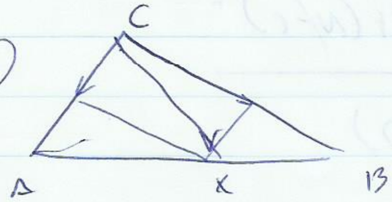
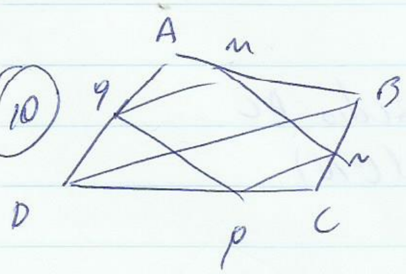
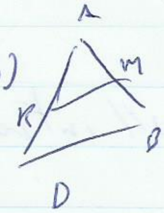


$\vec{CX} = \frac{\vec{CA}}{2} + \frac{\vec{CB}}{2}$

10

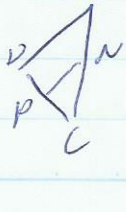


Δ(ABD)



$\begin{cases} (Q-M) = \frac{1}{2}(D-B) \\ (Q-M) \parallel (D-B) \end{cases}$

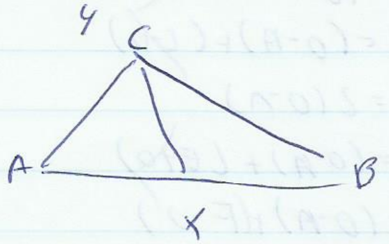
Δ(CBD)



$\begin{cases} (P-N) = \frac{1}{2}(D-B) \\ (P-N) \parallel (D-B) \end{cases}$

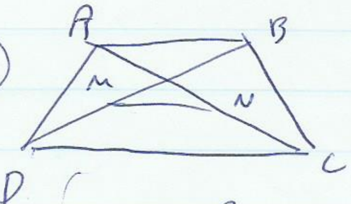
$\left.\begin{matrix} (Q-M) = (P-N) \\ (Q-M) \parallel (P-N) \end{matrix}\right\}$ (MNPQ é 1 paralelogramo

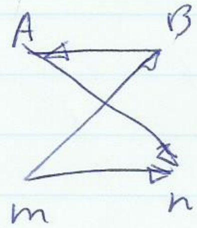
(11) $(X-A)=\frac{3}{4}(B-X)$ ou $4(X-A)=3(B-X)$



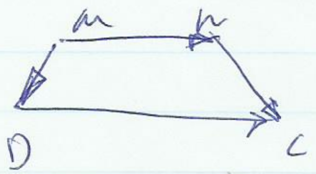
$$4[(C-A)+(X-C)]=3[(C-X)+(B-C)]$$

(12)





$$(N-M)=(B-\cancel{M})+(A-B)+(\cancel{N}-A)$$



$$(N-M)=(D-\cancel{M})+(C-D)+(\cancel{N}-C)\ +$$

$$2(N-M)=(A-B)+(C-D)$$

M é pto médio de $\overline{DB}$ | N é pto médio AC
$(M-D)=(B-M)$ | $(N-A)=(C-N)$

$$(N-M)=\frac{1}{2}\{(A-B)+(C-D)\}$$

Como $(N-M)$ é semi soma de 2 vetores // entre si

$(M-N)$ é // a ambos (base)

Dependências linear: Considere o conjunto $\{\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n\}$ e os escalares $a_1, a_2, \dots, a_n$ ($a_i \in \mathbb{R}$, $i = 1, \dots, n$)

Dizemos que:

(L.D.)

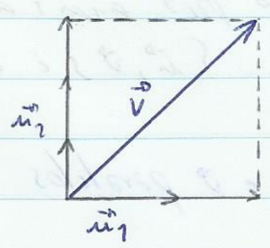
i) $\{\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n\}$ é linearmente dependente quando a equação vetorial $a_1\vec{u}_1 + a_2\vec{u}_2 + \dots + a_n\vec{u}_n = 0$ admitir uma solução não trivial, ou seja, com pelo menos um dos escalares não-nulos.

ii) $\{\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n\}$ é linearmente independente (L.I) quando a eq. vetorial $a_1\vec{u}_1 + a_2\vec{u}_2 + \dots + a_n\vec{u}_n = \vec{0}$ for satisfeita apenas com solução trivial ($a_1 = a_2 = \dots = a_n = 0$).

iii) o vetor $\vec{v}$ é uma combinação linear de $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n$ se existirem os escalares $a_1, a_2, \dots, a_n$ tais que:

$$\vec{v} = a_1\vec{u}_1 + a_2\vec{u}_2 + \dots + a_n\vec{u}_n$$

Exemplo: $\vec{v}$ escrito como combinação linear de $\vec{u}_1$ e $\vec{u}_2$



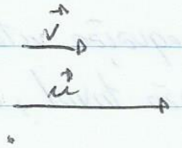
$$\vec{v} = 2\vec{u}_1 + 3\vec{u}_2$$

Vamos estudar a dependência linear dos conjuntos nos seguintes casos:

1) $\{\vec{u}\}$ com $\vec{u} \neq \vec{0}$. Observe que nesse caso, a única solução da equação vetorial $a_1\vec{u} = \vec{0}$ é $a_1 = 0$. Logo $\{\vec{u}\}$ é L.I.

2) $\{\vec{u}\}$ com $\vec{u} = \vec{0}$. Tomando $a_1 \neq 0$, temos que $a_1 \cdot \vec{0} = \vec{0}$. Portanto, $\{\vec{u}\}$ é L.D.

3) $\{\vec{u}, \vec{v}\}$ com $\vec{u}$ e $\vec{v}$ paralelos. Por exemplo, $\vec{u} = 3\vec{v}$



Assim, $\vec{u} - 3\vec{v} = \vec{0}$

Ou seja, a equação vetorial $a_1\vec{u} + a_2\vec{v} = \vec{0}$ é satisfeita com $a_1 = 1 \neq 0$ e $a_2 = -3 \neq 0$

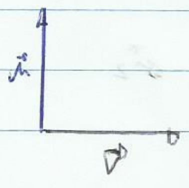
Segue que $\{\vec{u}, \vec{v}\}$ é L.D.

4) $\{\vec{u}, \vec{v}\}$ com $\vec{u} \neq \vec{0}$ e $\vec{v} = \vec{0}$

Nesse caso, $a_1\vec{u} + a_2\vec{0}$ é satisfeita tomando $a_1 = 0$ e $a_2 \neq 0$. Portanto, $\{\vec{v}, \vec{0}\}$ é L.D.

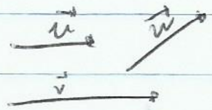
5) $\{\vec{u}, \vec{v}\}$ com $\vec{u}$ e $\vec{v}$ não paralelos e não-nulos.

$a_1\vec{u} + a_2\vec{v} = \vec{0}$



Como a única solução para $a_1\vec{u} + a_2\vec{v} = \vec{0}$ é $a_1 = a_2 = 0$, temos que: $\{\vec{u}, \vec{v}\}$ é L.I.

6) $\{\vec{u}, \vec{v}, \vec{w}\}$ ~~com $\vec{u}$~~ todos não-nulos, mas $\vec{u}$ e $\vec{v}$ paralelos



Por exemplo, $\cancel{\vec{u}} = 2\vec{v}$ e $\vec{w}$ um vetor qualquer

Como $\vec{v} = 2\vec{u}$, então

$\vec{v} - 2\vec{v} = \vec{0}$

$\vec{v} - 2\vec{v} + 0\vec{w} = \vec{0}$

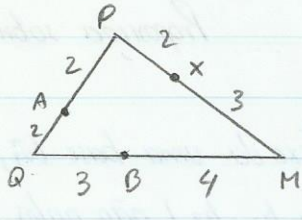
$\vec{v} - 2\vec{u} = \vec{0} \Rightarrow$

$\vec{v} - 2\vec{u} + \vec{0} = \vec{0}$

$\vec{v} - 2\vec{u} + 0 \cdot \vec{w} = \vec{0}$ Logo, $a_1 = 1$, $a_2 = -2$, $a_3 = 0$ é solução de $a_1\vec{u} + a_2\vec{u} + a_3\vec{w} = \vec{0}$

Portanto, $\{\vec{u}, \vec{v}, \vec{w}\}$ é L.D.

1) $\vec{u}+\vec{v} = (a_1+b_1 \ , a_2+b_2, a_3+b_3)$
2) $\beta\vec{u} = (\beta a_1, \beta a_2, \beta a_3) \ , \ \forall \beta \in \mathbb{R}$



$\vec{QX} \rightarrow \vec{QA}, \vec{QB}$

1) $\vec{u}+\vec{v} = (a_1\vec{i}, a_2\vec{j} + a_3\vec{k}) + (b_1\vec{i} + b_2\vec{j} + b_3\vec{k})$
$= (a_1+b_1)\vec{i} + (a_2+b_2)\vec{j} + (a_3+b_3)\vec{k}$
$= (a_1+b_1, a_2+b_2, a_3+b_3)$

2) $\beta\vec{v} = \beta(a_1, a_2, a_3)$
$= \beta a_1, \beta a_2, \beta a_3$

43) Escreva o vetor $\vec{r}=(3,12,11)$ como combinação linear dos vetores $\vec{u}, \vec{v}$, e $\vec{w}$.
Onde: $\vec{u} = (1,2,1)$, $\vec{v} = (2,1,0)$ e $\vec{w} = (1,3,3)$.

$$\vec{r} = a\vec{u} + b\vec{v} + c\vec{w}$$

$$(3,12,11) = a(1,2,1) + b(2,1,0) + c(1,3,3)$$

$$\begin{cases} a+2b+c=3 \ (\times -2) \\ 2a+b+3c=12 \\ a+0b+3c=11 \end{cases} \Rightarrow \begin{cases} a+2b+c=3 \ (\times -1) \\ -3b+c=6 \\ a+0.b+3c=11 \end{cases} \Rightarrow \begin{cases} a+2b+c=3 \\ -3b+c=6 \quad (\times -2) \\ -2b+2c=8 \quad (\times -3) \end{cases}$$

$$\begin{cases} a+2b+c=3 \\ -3b+c=6 \\ 4c=12 \end{cases} \Rightarrow \begin{cases} c=12/4=3, \\ -3b+3=6 \Rightarrow b=3/-3=-1, \\ a+2.(-1)+3=2; \end{cases}$$

Portanto,
$$\vec{r} = 2\vec{u} - \vec{v} + 3\vec{w}$$

Estudar os teoremas e as preposições
28 40

Proposições sobre vetores LD:

P_1: Fixada uma base $(\vec{v_1}, \vec{v_2}, \vec{v_3})$ do espaço, e dados os vetores $\vec{u} = (a_1, a_2, a_3)$ e $\vec{v} = (b_1, b_2, b_3)$ não nulos em relação a base, o conjunto de vetores $\{\vec{u}, \vec{v}\}$ é linearmente dependente (LD), se e somente se: $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$, com $b_1, b_2, b_3 \neq 0$.

$\vec{u}$ e $\vec{v}$: LD $\therefore$

$\alpha \vec{u} + \beta \vec{v} = \vec{0}$

$\alpha(a_1, a_2, a_3) + \beta(b_1, b_2, b_3) = (0,0,0)$

$(\alpha a_1, \alpha a_2, \alpha a_3) + (\beta b_1, \beta b_2, \beta b_3) = (0,0,0)$

$(\alpha a_1 + \beta b_1, \alpha a_2 + \beta b_2, \alpha a_3 + \beta b_3) = (0,0,0)$

$\alpha a_1 + \beta b_1 = 0$

$\alpha a_2 + \beta b_2 = 0$

$\alpha a_3 + \beta b_3 = 0$

A condição necessária e suficiente p/q o sistema linear homogêneo tenha solução diferente da trivial $(\alpha = 0, \beta = 0)$ é ___

$$M = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = 0 \quad a_1 b_2 - a_2 b_1 = 0 \quad \frac{a_1}{b_1} = \frac{a_2}{b_2}$

$\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} = 0 \quad a_1 b_3 - a_3 b_1 = 0 \therefore \frac{a_1}{b_1} = \frac{a_3}{b_3}$

$\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} = 0 \quad a_2 b_3 - a_3 b_2 = 0 \therefore \frac{a_2}{b_2} = \frac{a_3}{b_3}$

$\therefore \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

ou todos os determinantes de 2ª ordem são nulos

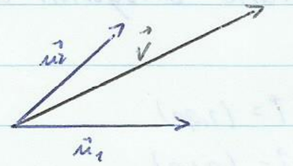
Bases

1) **Base na Reta:** $(\vec{v})$ é uma base na reta se o conjunto $\{\vec{v}\}$ é L.I.

Obs.: Qualquer vetor $\vec{v}$ paralelo ao vetor $\vec{u}$, poderá ser escrito da seguinte forma: $\vec{v} = \beta\vec{u}$, $\beta \in \mathbb{R}$

2) **Base no Plano:** $(\vec{u}_1, \vec{u}_2)$ é uma base no plano quando o conjunto $\{\vec{v}_1, \vec{u}_2\}$ é L.I.

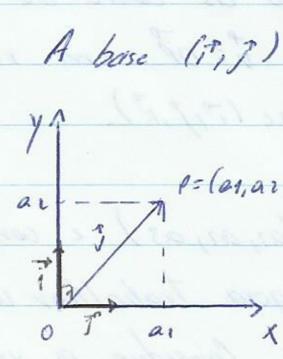
Obs.: Qualquer vetor $\vec{v}$ coplanar com vetores $\vec{v}_1$ e $\vec{v}_2$, poderá ser escrito (de maneira única) como comb. linear de $\vec{v}_1$ e $\vec{v}_2$. Existem os escalares a_1 e a_2 tais que:
$\vec{v} = a_1\vec{v}_1 + a_2\vec{u}_2$



$\vec{v} = a_1\vec{v}_1 + a_2\vec{v}_2 = (a_1, a_2)$ → a_1 e a_2: são as coordenadas do vetor $\vec{v}$ em relação à base $(\vec{v}_1, \vec{u}_2)$

• Quando $(\vec{v}_1, \vec{v}_2)$ é tal que os vetores são unitários ($|\vec{v}_1| = |\vec{v}_2| = 1$) e ortogonais ($\vec{u}_1 \perp \vec{u}_2$), a base será chamada de base ~~camônica~~ canônica. Denominamos esses vetores por $\vec{i}$ e $\vec{j}$.

A base $(\vec{i}, \vec{j})$ estabelece o plano cartesiano xOy.



Observe que $\begin{cases} \vec{i} = 1.\vec{i} + 0.\vec{j} = (1,0) \\ \vec{j} = 0.\vec{i} + 1.\vec{j} = (0,1) \end{cases}$

$\vec{v} = \vec{OP} = a_1\vec{i} + a_2\vec{j} = (a_1, a_2)$
⇓
São as coordenadas do vetor $\vec{v}$ em relação à base $(\vec{i}, \vec{j})$

3) <u>Base no Espaço</u>: $(\vec{v_1}, \vec{v_2}, \vec{v_3})$ é uma base no espaço quando o conjunto $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ é L.I.

Obs.: Quando $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ é L.I, qualquer vetor $\vec{v}$ poderá ser escrito combinação linear dos vetores $\vec{u_1}, \vec{u_2}$, e $\vec{u_3}$.

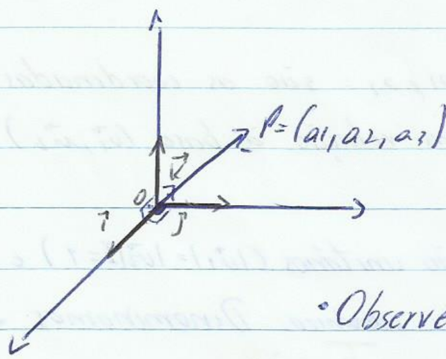
Ou seja, existem as escalares a_1, a_2 e a_3 (únicos) tais que: ~~$\vec{v} = a_1$~~

$\vec{v} = a_1\vec{u_1} + a_2\vec{u_2} + a_3\vec{u_3} = (a_1, a_2, a_3)$

São as coordenadas do vetor $\vec{v}$ em relação à base $(\vec{v_1}, \vec{v_2}, \vec{v_3})$

• Também podemos formar uma base canônica $(\vec{v_1}, \vec{v_2}, \vec{v_3})$ no espaço, desde que $(|\vec{v_1}| = |\vec{v_2}| = |\vec{v_3}| = 1)$ e $(\vec{v_1} \perp \vec{v_2} \perp \vec{v_3} \perp \vec{v_1})$.

Tais vetores no espaço com essas características serão chamados de $\vec{i}, \vec{j}$ e $\vec{k}$. A base $(\vec{i}, \vec{j}, \vec{k})$ determina o sistema cartesiano ortogonal (O, x, y, z).



Nesse caso, temos $\begin{cases} \vec{i} = (1,0,0) \\ \vec{j} = (0,1,0) \\ \vec{k} = (0,0,1) \end{cases}$

• Observe que $\vec{v} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} = (a_1, a_2, a_3)$

São as coord. do vetor $\vec{v} = \vec{OP}$ em relação base $(\vec{i}, \vec{j}, \vec{k})$.

Uma outra maneira de representar o vetor $\vec{u} = (a_1, a_2, a_3)$ é com notação de matriz, $\vec{u} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$. Além do mais, podemos agora trabalhar com as operações de vetores já estudos de maneira algébrica. Considere os vetores $\vec{u} = (a_1, a_2, a_3)$ e $\vec{v} = (b_1, b_2, b_3)$. Temos que:

P_2: Os vetores $\vec{u}$ e $\vec{v}$ são LI se e somente se não valer a proporcionalidade entre suas coordenadas ou pelo menos 1 dos determinantes de 2° ordem é $\neq 0$

$\vec{u} = (2,3,6)$

$\vec{v} = (4,6,12)$

$\frac{2}{4} = \frac{3}{6} = \frac{6}{12}$

$\vec{u} = (2,0,3)$

$\vec{v} = (4,2,0)$

$$M = \begin{vmatrix} 2 & 0 & 3 \\ 4 & 2 & 0 \end{vmatrix}$$

$\begin{vmatrix} 2 & 0 \\ 4 & 2 \end{vmatrix} = 4$ $\begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = -12$ $\begin{vmatrix} 0 & 3 \\ 2 & 0 \end{vmatrix} = -6$

$\therefore$ LI

$\vec{u} = (1,3,5)$

$\vec{v} = (2,6,13)$

$\frac{1}{2} = \frac{3}{6} \neq \frac{5}{13}$ $\therefore$ LI

P_3: Tres vetores $\vec{u} = (a_1, a_2, a_3)$, $\vec{v} = (b_1, b_2, b_3)$ e $\vec{w} = (c_1, c_2, c_3)$ $\vec{s}$ LD $\therefore$

$\alpha\vec{u} + \beta\vec{v} + \gamma\vec{w} = \vec{0}$

$(\alpha a_1, \alpha a_2, \alpha a_3) + (\beta b_1, \beta b_2, \beta b_3) + (\gamma c_1, \gamma c_2, \gamma c_3) = (0,0,0)$

$$\begin{cases} \alpha a_1 + \beta b_1 + \gamma c_1 = 0 \\ \alpha a_2 + \beta b_2 + \gamma c_2 = 0 \\ \alpha a_3 + \beta b_3 + \gamma c_3 = 0 \end{cases}$$

Pq o sistema lin. hom. tinha sol $\neq$ da trivial

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

2

P4. Tres vetores $\vec{u}, \vec{v}$ e $\vec{w}$ são LI se e somente se $\Delta \neq 0$

$\{(3,-1,0), (1,1,2), (4,0,2)\}$

$$\Delta = \begin{vmatrix} 3 & -1 & 0 \\ 1 & 1 & 2 \\ 4 & 0 & 2 \end{vmatrix} = 6 - 8 + 0 - 0 - 0 + 2 = 0$$

LD

$\{(1,2,2), (0,2,2), (0,0,1)\}$

$$\Delta = \begin{vmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 2 + 0 + 0 - 0 - 0 - 0 = 2 \neq 0$$

LI

40) $\{(1,2,1), (2,4,2)\}$

LD $\therefore$ o 2º é o dobro do 1º

$\frac{1}{2} = \frac{2}{4} = \frac{1}{2}$ V

41) $\{(0,1,0); (2,0,3)\}$ LI

$$M = \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} \quad \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -2 \neq 0$$

$$\begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = -3 \neq 0$$

44-) $\{(1,2,1),(2,1,0),(1,3,3)\}$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 3 & 3 \end{vmatrix} = \begin{matrix} 3+0+6 \\ -1-0-12 \end{matrix} = 0 \quad LI$$

45-

$$\Delta = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 5 \\ 4 & -1 & 11 \end{vmatrix} = \begin{matrix} 11-20-6 \\ -12+5+22 \end{matrix} = 0 \quad LD$$

46-) $\vec{u} = (3, a^2-4, 6)$

$\vec{v} = (1, a, b)$

$\frac{3}{1} = \frac{6}{b} = 3b = 6 \quad b=2$

$\frac{3}{1} = \frac{a^2-4}{a} = \frac{6}{b}$

$\frac{3}{1} = \frac{a^2-4}{a} = a^2-4 = 3a$

$a^2-3a-4=0$

$a = \frac{3 \sqrt{9+16}}{2}$ $\begin{cases} a=4 \text{ ou} \\ a=-1 \end{cases}$

$a=4$ e $b=2$ ou

$a=-1$ e $b=2$

48 $\vec{u} = CL(\vec{v},\vec{w})$

$\vec{u} = (1,1,2)$
$\vec{v} = (2,1,3)$
$\vec{w} = (5,4,7)$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 5 & 4 & 7 \end{vmatrix} = \begin{matrix} 7+15+16 \\ -5-12-14 \end{matrix} \neq 0$$

$\vec{u}$ n é CL de $\vec{v}$ e $\vec{w}$

(49) $\vec{u} = (1,0,3)$
$\vec{v} = (2,1,4)$
$\vec{w} = (7,2,17)$

$$\Delta = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 7 & 2 & 17 \end{vmatrix} = \begin{matrix} 17+0+12 = 0 \\ -21-8-0 \end{matrix}$$

$\therefore$ LD

$\vec{u} = a\vec{v} + b\vec{w}$

$(1,0,3) = (2a, a, 4a) + (7b, 2b, 17b)$

$2a + 7b = 1$

$a + 2b = 0$

$3a + 4a + 17b = 3$ $\quad a = \frac{2}{3}$ $\quad b = 1/3$

Verificando a terceira equação

$4\left(\frac{-2}{9}\right) + 17\left(\frac{1}{3}\right) = 3$ $\quad \frac{9}{3} = 3$ (v)

$$\boxed{\vec{u} = \frac{2}{3}\vec{v} + \frac{1}{3}\vec{w}}$$

LI - forma base

Mudança de Base: Considere as bases $F = (\vec{f_1}, \vec{f_2}, \vec{f_3})$, $E = (\vec{e_1}, \vec{e_2}, \vec{e_3})$ onde: $\vec{f_1} = (a_{11}, a_{21}, a_{31})$, $\vec{f_2} = (a_{12}, a_{22}, a_{32})$ e $\vec{f_3} = (a_{13}, a_{23}, a_{33})$ e ainda ~~$\vec{v} = (x_E, y_E, z_E)$~~ $\vec{v} = (x_E, y_E, z_E)_E$. Conhecendo-se as coord. do vetor $\vec{v}$ na base E, como determinar as coord. desse vetor na base F?

Observe que precisamos encontrar os escalares x_F, y_F, e z_F tais que: $\vec{v} = x_F\vec{f_1} + y_F\vec{f_2} + z_F\vec{f_3}$, sabendo que $\vec{v} = x_E\vec{e_1} + y_E\vec{e_2} + z_E\vec{e_3}$. Segue que:

$$\vec{v} = x_F \underbrace{(a_{11}\vec{e_1} + a_{21}\vec{e_2} + a_{31}\vec{e_3})}_{F_1} + y_F\underbrace{(a_{12}\vec{e_1} + a_{22}\vec{e_2} + a_{32}\vec{e_3})}_{F_2} + z_F\underbrace{(a_{13}\vec{e_1} + a_{23}\vec{e_2} + a_{33}\vec{e_3})}_{F_3} \quad *$$

$$\vec{v} = \underbrace{(x_F a_{11} + y_F a_{12} + z_F a_{13})}_{x_E}\vec{e_1} + \underbrace{(x_F a_{21} + y_F a_{22} + z_F a_{23})}_{y_E}\vec{e_2} + \underbrace{(x_F a_{31} + y_F a_{32} + z_F a_{33})}_{z_E}\vec{e_3} \Rightarrow$$

$$\begin{cases} x_E = x_F a_{11} + y_F a_{12} + z_F a_{13} \\ y_E = x_F a_{21} + y_F a_{22} + z_F a_{23} \\ z_E = x_F a_{31} + x_F a_{32} + x_F a_{33} \end{cases} \Rightarrow \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{23} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} x_F \\ y_F \\ z_F \end{bmatrix} \Rightarrow \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} = M_{E \to F} \cdot \begin{bmatrix} x_F \\ y_F \\ z_F \end{bmatrix},$$

onde $M_{E \to F}$ é chamada matriz mudança de base de $\underline{E}$ para $\underline{F}$, onde $M_{F \to E} = M^{-1}_{E \to F}$

Exemplo: Sejam $E = (\vec{i}, \vec{j}, \vec{k})$, $F = (\vec{f_1}, \vec{f_2}, \vec{f_3})$ duas bases e $\vec{u} = (5, 3, 2)$. Onde:

$$\begin{cases} \vec{f_1} = 2\vec{i} + 1\vec{i} + 0\vec{k} = (2,1,0)_E \\ \vec{f_2} = 1\vec{i} + 3\vec{j} + 1\vec{k} = (1,3,1)_E \\ \vec{f_3} = 3\vec{i} + 0\vec{j} - 1\vec{k} = (3,0,-1)_E \end{cases}$$

Vamos determinar $\vec{v} = (x_F, y_F, z_F)_F$

$$\boxed{\text{Método (livro!)}: \; M^{-1} = \frac{1}{\Delta} \cdot (\text{cof. } M)^T}$$

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$$M_{E \to F} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

1- $\begin{vmatrix} 2 & 1 & 3 \\ 1 & 3 & 0 \\ 0 & 1 & -1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 1 & 3 \\ 0 & 1 \end{matrix} = -6 + 3 + 1 = -2$

2- $\text{Cof. } M_{E \to F} = \begin{bmatrix} -3 & 1 & 1 \\ 4 & -2 & -2 \\ -9 & 3 & 5 \end{bmatrix}$

3- $[\text{Cof. } M_{E \to F}]^T = \begin{bmatrix} -3 & 4 & -9 \\ 1 & -2 & 3 \\ 1 & -2 & 5 \end{bmatrix}$

Logo, $M_{F \to E} = \begin{bmatrix} 3/2 & -2 & 9/2 \\ -1/2 & 1 & -3/2 \\ -1/2 & 1 & -5/2 \end{bmatrix}$

Portanto,

$$\begin{bmatrix} x_F \\ y_F \\ z_F \end{bmatrix} = \begin{bmatrix} 3/2 & -2 & 9/2 \\ -1/2 & 1 & -3/2 \\ -1/2 & 1 & -5/2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 21/2 \\ -5/2 \\ -9/2 \end{bmatrix}$$

$\frac{15}{2} - \frac{12}{2} + \frac{18}{2} =$

$\frac{-5}{2} + \frac{6}{2} - \frac{6}{2}$

$$\vec{u} = \left(\frac{21}{2}, \frac{-5}{2}, \frac{-9}{2} \right)_F$$

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77) $\begin{cases} \vec{f_1} = -\vec{e_1} + 2\vec{e_2} = (-1,2,0) \\ \vec{f_2} = \vec{e_1} + \vec{e_2} = (1,1,2) \\ \vec{f_3} = 4\vec{e_1} + \vec{e_3} = (4,0,1) \end{cases}$

$E \to F$

M_{EF}

$M_{FE} = M_{EF}^{-1}$

$DET(M_{EF}) = 1 - 2 = -3 \neq 0$

$$M_{EF} = \begin{bmatrix} -1 & 1 & 4 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -3 \neq 0$$

Base

$$cof(M) = \begin{bmatrix} 1 & -2 & 0 \\ -1 & -1 & 0 \\ -4 & 8 & -3 \end{bmatrix}$$

$$Adj(M) = \begin{bmatrix} 1 & -1 & -4 \\ -2 & -1 & 8 \\ 0 & 0 & 3 \end{bmatrix} \qquad M^{-1} = \frac{-1}{3} \begin{bmatrix} 1 & -1 & -4 \\ -2 & -1 & -8 \\ 0 & 0 & -3 \end{bmatrix}$$

c) $E \to F$

$$\begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix} = M \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = M^{-1} \cdot \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & -1 & -4 \\ -2 & -1 & 8 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8/3 \\ -31/3 \\ -4 \end{bmatrix}$$

$$\vec{V}_F = \left(\frac{8}{3}, -\frac{31}{3}, -4 \right) = \frac{8}{3}\vec{f_1} - \frac{31}{3}\vec{f_2} - 4\vec{f_3}$$

$\overleftarrow{F \to E}$

$\overleftarrow{E \to G}$

$$\begin{tikzcd} F \arrow[r] \arrow[rr, bend left, "M_{FG}"] & E \arrow[r] & G \end{tikzcd}$$

M_{FE} $\quad$ M_{Eg}

$$M_{FE} = M_{FE} \cdot M_{EG}$$

Produto escalar entre vetores: Considere os vetores $\vec{u}=(x_1, y_1, z_1)$, $\vec{v}=(x_2, y_2, z_2)$ e $\vec{w}=(x_3, y_3, z_3)$. Definimos o produto escalar de $\vec{u}$ por $\vec{v}$ como o escalar:

$$\vec{u}\cdot\vec{v} = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2$$

$\otimes$ = Todos em relação à base $(\vec{i}, \vec{j}, \vec{k})$

Exemplo: $\vec{u}=(1,2,3)$ e $\vec{v}=(2,-2,4)$. Assim:

$$\vec{u}\cdot\vec{v} = 1.2 + 2.(-2) + 3.4 = 10$$

Obs.:

1) $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$

2) $\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$

3) $(\beta\vec{u})\cdot\vec{v} = \vec{u}\cdot(\beta\vec{v}) = \beta.(\vec{u}\cdot\vec{v})$, $\forall \beta \in \mathbb{R}$

4) $|\vec{u}| = \sqrt{x_1^2+y_1^2+z_1^2}$ ou $|\vec{u}|^2 = \vec{u}\cdot\vec{u}$

5) $\vec{u}\cdot\vec{v} = |\vec{u}|.|\vec{v}|.\cos\theta$, onde θ é o ângulo formado entre os vetores $\vec{u}$ e $\vec{v}$.

Demonstração:

Sabemos que:

~~$|\vec{v}-\vec{v}| = (\vec{u}-\vec{v})(\vec{u}-\vec{v})$~~

$$|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|.\cos\theta$$

Mas, $|\vec{u}-\vec{v}|^2 = (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$$= |\vec{u}|^2 + |\vec{v}|^2 - 2\vec{u}\cdot\vec{v}$$

Logo, $-2\vec{u}\cdot\vec{v} = -2|\vec{u}||\vec{v}|.\cos\theta$

$$\vec{u}\cdot\vec{v} = |\vec{u}|.|\vec{v}|.\cos\theta$$

6) Dois vetores $\vec{u}$ e $\vec{v}$ são ortogonais quando $\vec{u}\cdot\vec{v} = 0$

Exercícios: 1) Dados os vetores $\vec{u} = (1,2,1)$; $\vec{v} = (2,1,0)$ e $\vec{w} = (1,1,m)$; $m \in \mathbb{R}$, em relação a uma base ortonormal $(\vec{i}, \vec{j}, \vec{k})$. Pede-se calcular:

a) $\vec{u} \cdot \vec{v}$

b) $|\vec{u}|, |\vec{v}|$

c) $\cos\theta$ (θ é o ângulo formado entre $\vec{u} \cdot \vec{v}$

d) o valor de m para que o vetor $\vec{u}$ seja ortogonal ao vetor $\vec{w}$ — Quando o produto é $\emptyset$

e) um vetor $\vec{p}$, de módulo $\sqrt{14}$ que seja simultaneamente ortogonal aos vetores $\vec{u}$ e $\vec{v}$ e que tenha cota positiva.

a) $\vec{u} = (1,2,1)$ $\vec{v} = (2,1,0)$ $\vec{u} \cdot \vec{v} = 2 + 2 + 0 = 4$

b) $|\vec{u}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$|\vec{v}| = \sqrt{2^2 + 1^2 + 0^2} = \sqrt{5}$

c) $\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cos\theta$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} = \frac{4}{\sqrt{6} \cdot \sqrt{5}} = \frac{4}{\sqrt{30}} \cdot \frac{\sqrt{30}}{\sqrt{30}} = \frac{4\sqrt{30}}{30} = \frac{2\sqrt{30}}{15}$$

d) $\vec{u} = (1,2,1)$ $\quad 1 + 2 + m = 0$

$\vec{w} = (1,1,m)$ $\quad m = -3$

e) Seja $\vec{p} = (a, b, c)$

$$\begin{cases} |\vec{p}| = \sqrt{14} \\ \vec{p} \cdot \vec{u} = 0 \\ \vec{p} \cdot \vec{v} = 0 \end{cases} \Rightarrow \begin{cases} a^2 + b^2 + c^2 = 14 \\ a + 2b + c = 0 \\ 2a + b = 0 \end{cases} \Rightarrow \begin{cases} b = -2a \\ a - 4a + c = 0 \\ c = 3a \end{cases}$$

Logo, $a^2 + 4a^2 + 9a^2 = 14 \Rightarrow a^2 = 1$

Portanto, $\vec{p} = (1, -2, 3)$

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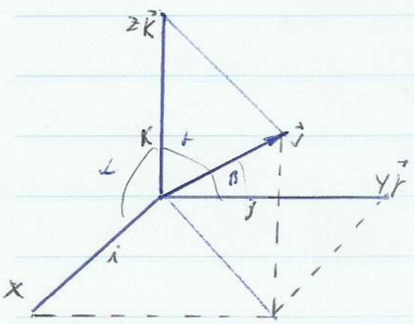
Aplicações do produto escalar

Cossenos Diretores de um vetor

Def: São os cossenos dos ângulos formados pelo vetor com cada um dos vetores da base $(\vec{i}, \vec{j}, \vec{k})$



$\vec{v} = (x, y, z)$

$\vec{i} = (1, 0, 0)$

$\vec{j} = (0, 1, 0)$

$\vec{k} = (0, 0, 1)$

$$\cos\alpha = \cos(\vec{v}, \vec{i}) = \frac{\vec{v} \cdot \vec{i}}{|\vec{v}||\vec{i}|} = \frac{x}{|\vec{v}|} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\beta = \cos(\vec{v}, \vec{j}) = \frac{\vec{v} \cdot \vec{j}}{|\vec{v}||\vec{j}|} = \frac{y}{|\vec{v}|} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\gamma = \cos(\vec{v}, \vec{k}) = \frac{\vec{v} \cdot \vec{k}}{|\vec{v}||\vec{k}|} = \frac{z}{|\vec{v}|} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$\vec{u} = (1, -3, 5)$

$|\vec{u}| = \sqrt{1+9+25} = \sqrt{35}$

$\cos\alpha = \frac{1}{\sqrt{35}}$

$\cos\beta = \frac{-3}{\sqrt{35}}$

$\cos\gamma = \frac{5}{\sqrt{35}}$

Versor de $\vec{v}$

$$\frac{\vec{v}}{|\vec{v}|} = \left(\frac{x}{|\vec{v}|}, \frac{y}{|\vec{v}|}, \frac{z}{|\vec{v}|} \right)$$

$$\frac{\vec{V}}{|\vec{v}|} = (\cos\alpha, \cos\beta, \cos\gamma)$$

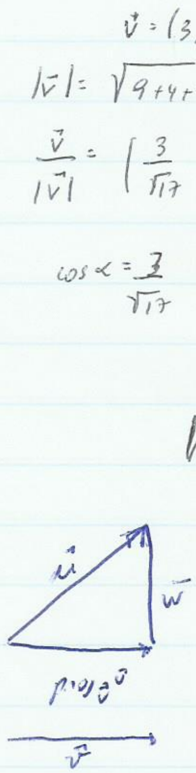
$$\vec{v} = (3, -2, 2)$$

$$|\vec{v}| = \sqrt{9+4+4} = \sqrt{17}$$

$$\frac{\vec{v}}{|\vec{v}|} = \left(\frac{3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{2}{\sqrt{17}} \right)$$

$$\cos\alpha = \frac{3}{\sqrt{17}} \qquad \cos\beta = \frac{-2}{\sqrt{17}} \qquad \cos\gamma = \frac{2}{\sqrt{17}}$$

Vetor projeção ortogonal de um vetor



$$Proj_{\vec{v}} \vec{u} = \alpha \vec{v}$$

$$\vec{u} \times \vec{v} = \alpha\, \vec{v} \times \vec{v} + \vec{w} \times \vec{v}$$

Como $\vec{w} \perp \vec{v} \rightarrow \vec{w} \times \vec{v} = 0$

$$\vec{u} \times \vec{v} = \alpha\, \vec{v} \times \vec{v}$$

$$\alpha = \frac{\vec{u} \times \vec{v}}{\vec{v} \times \vec{v}}$$

$$\vec{u} = proj_{\vec{v}} \vec{u} + \vec{w}$$

$$\vec{u} = \alpha \vec{v} + w \times \vec{v}$$

$$Proj_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \times \vec{v}}{\vec{v} \times \vec{v}} \right) \vec{v}$$

$$Proj_{\vec{u}} \vec{v} = \left(\frac{\vec{u} \times \vec{v}}{\vec{u} \cdot \vec{u}} \right) \vec{u}$$

Ex: $\vec{u} = (2,3,-4)$

$\vec{v} = (-1,2,-3)$

$\vec{u} \times \vec{v} = -2+6+12 = 16$

$\vec{u} \times \vec{u} = 4+9+16 = 29$

$\vec{v} \times \vec{v} = 1+4+9 = 14$

$Proj_{\vec{v}} \vec{u} = \frac{16}{14} \vec{v} = \frac{8}{7}(-1,2,-3)$

$Proj_{\vec{v}} \vec{u} = \left(\frac{-8}{7}, \frac{16}{7}, \frac{-24}{7}\right)$

$Proj_{\vec{u}} \vec{v} = \frac{16}{29} \vec{u} = \frac{16}{29}(2,3,-4)$

$Proj_{\vec{u}} \vec{v} = \left(\frac{32}{29}, \frac{48}{29}, \frac{-64}{29}\right)$

(25) $\vec{v} = (3,-4,2)$

$\vec{v} = \vec{p} + \vec{w}$

$\vec{p} \parallel \vec{u} = (1,1,0)$

$\vec{w} \perp \vec{u}$

$\vec{p} = proj_{\vec{u}} \vec{v} =$

$\vec{u} \times \vec{v} = 3-4+0 = -1$

$\vec{u} \times \vec{u} = 1+1+0 = 2$

$\vec{p} = \frac{1}{2} \vec{u} = \frac{-1}{2}(1,1,0)$

$\vec{p} = \left(-\frac{1}{2}, -\frac{1}{2}, 0\right)$

$\vec{w} = \vec{v} - \vec{p} = (3,-4,2) - \left(-\frac{1}{2}, -\frac{1}{2}, 0\right)$

$\vec{w} = \left(\frac{7}{2}, -\frac{7}{2}, 2\right)$

(42) (79) $\vec{u} = (1,0,-2)$

$\vec{v} = (2,1,1)$

$\vec{w} = (a,b,c)$

$|\vec{w}| = 1$

$\vec{w} \perp \vec{u}$

$\vec{w} \perp \vec{v}$

$\sqrt{a^2+b^2+c^2} = 1$

$\vec{w} \times \vec{u} = 0$

$\vec{w} \times \vec{v} = 0$

$$\begin{cases} a^2+b^2+c^2 = 1 \\ a - 2c = 0 \\ 2a+b+c = 0 \end{cases}$$

$a - 2c = 0 \rightarrow \boxed{a = 2c}$

$2a+b+c = 0 \rightarrow 4c+b+c = 0$

$5c + b = 0$

$b = -5c$

$(2c)^2 + (-5c)^2 + c^2 = 1$

$4c^2 + 25c^2 + c^2 = 1$

$30c^2 = 1$

$c^2 = \frac{1}{30}$ $\quad c \pm \frac{1}{\sqrt{30}}$

$a = \pm \frac{2}{\sqrt{30}}$ $\quad c = \pm \frac{\sqrt{1}}{\sqrt{30}}$

$b = \pm \frac{5}{\sqrt{30}}$

$\vec{w} \pm \left(\frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}}, \frac{1}{\sqrt{30}}\right)$

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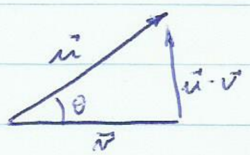
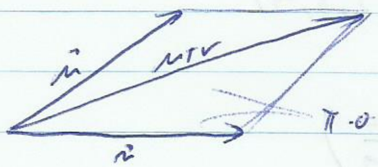
(86) $|\vec{u}| = 3 \qquad |\vec{v}| = 2 \qquad \theta = \frac{\pi}{3}$

$$\cos[(u+v),(u-v)] = \frac{(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v})}{|\vec{u}+\vec{v}|\,|\vec{u}-\vec{v}|} = \frac{|\vec{u}|^2 - |\vec{v}|^2}{|\vec{u}+\vec{v}|\,|\vec{u}-\vec{v}|}$$



$$|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos\theta$$



$$|\vec{u}+\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + 2|\vec{u}||\vec{v}|\cos\theta$$

$\pi - \theta$ $\qquad \cos(\pi-\theta) = -\cos\theta$

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Produto vetorial:

Considere os vetores $\vec{u}=(x_1,y_1,z_1)$ e $\vec{v}=(x_2,y_2,z_2)$ em relação à base positiva $(\vec{i},\vec{j},\vec{k})$. Definimos o produto vetorial de $\vec{u}$ por $\vec{v}$ como sendo o vetor:

$$\vec{u}\wedge\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

Exemplo: Dados $\vec{u}=(1,0,2)$ e $\vec{v}=(1,1,-2)$. Logo

$$\vec{u}\wedge\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 1 & 1 & -2 \end{vmatrix} = 2\vec{j}+\vec{k}-2i+2j = (-2,4,1)$$

Propriedades:

1) $\vec{u}\wedge\vec{v} = -\vec{v}\wedge\vec{u}$

2) $(\beta\vec{u})\wedge\vec{v} = \vec{u}\wedge(\beta\vec{v}) = \beta(\vec{u}\wedge\vec{v}),\ \forall\beta\in\mathbb{R}$

3) $\vec{u}\wedge(\vec{v}+\vec{w}) = \vec{u}\wedge\vec{v}+\vec{u}\wedge\vec{w}$

4) $|\vec{u}\wedge\vec{v}| = |\vec{u}|.|\vec{v}|.\text{sen}\,\theta$, onde θ é o ângulo entre $\vec{u}$ e $\vec{v}$.

Demonstração:

Pela Id. de Lagrange, temos:

$|\vec{u}\wedge\vec{v}|^2+(\vec{u}.\vec{v})^2 = |\vec{u}|^2.|\vec{v}|^2 \Rightarrow$

$|\vec{u}\wedge\vec{v}|^2 = |\vec{u}|^2.|\vec{v}|^2-(|\vec{u}|.|\vec{v}|\cos\theta)^2 \Rightarrow$

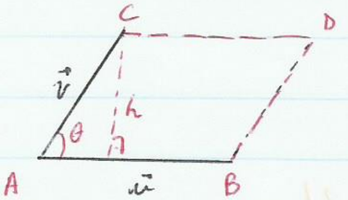
$|\vec{u}\wedge\vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2.(1-\cos^2\theta) \Rightarrow$

$|\vec{u}\wedge\vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2.\text{sen}^2\theta$

5) $\vec{u} \wedge \vec{v}$ é ortogonal aos vetores $\vec{u}$ e $\vec{v}$ e o seu sentido é tal que $(\vec{u}, \vec{v}, \vec{u} \wedge \vec{v})$ é positiva

Interpretação Geométrica:

Dados: $\begin{cases} \vec{u} = \vec{AB} \\ \vec{v} = \vec{AC} \\ \vec{u} + \vec{v} = \vec{AD} \end{cases}$, onde:



θ é o ângulo entre $\vec{u}$ e $\vec{v}$

$A_{\square ABCD}$ = "Área do paralelogramo ABCD"

$A_{\Delta ABC}$ = "Área do triângulo ABC"

Temos que: $A_{\square ABCD} = |\vec{u}| . h$. Mas $\operatorname{sen} \theta = \dfrac{h}{|\vec{v}|}$. Logo

$$A_{\square ABCD} = |\vec{u}| . \underbrace{|\vec{v}| . \operatorname{sen} \theta}_{h} = |\vec{u} \wedge \vec{v}|$$

Portanto,

$$A_{ABC} = \frac{1}{2} . |\vec{u} \wedge \vec{v}|$$

Exercícios: Determine um vetor ortogonal aos vetores $\vec{u} = 2\vec{j} - 2\vec{k}$ e $\vec{v} = 2\vec{i} + \vec{j} + \vec{k}$, que tenha módulo 2 e abcissa negativa

$$\vec{u} \wedge \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -2 \\ 2 & 1 & 1 \end{vmatrix} = 2\vec{i} - 4\vec{j} - 4\vec{k} + 2\vec{i} = (4, -4, -4)$$

O vetor $\vec{p}$ possui a mesma direção de $(1, -1, -1)$. Portanto $\vec{p} = \frac{-2}{\sqrt{3}} . (1, -1, -1)$

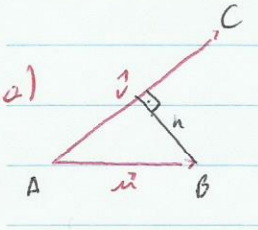
2) Dados $\vec{u} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{v} = -\vec{i} + 2\vec{j} + 2\vec{k}$, onde $\begin{cases} \vec{u} = B - A \\ \vec{v} = C - A \end{cases}$

a) Calcule a área do triângulo ABC

b) Determine a medida da altura do ΔABC relativa ao lado AC

c) " " " " " " " " " " AB.

a)



$$\vec{u} \wedge \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 1 \\ -1 & 2 & 2 \end{vmatrix} = -6\vec{i} + 4\vec{k} - \vec{j} - 3\vec{k} - 2\vec{i} - 4\vec{j} = (-8, -5, 1)$$

$$|\vec{u} \wedge \vec{v}| = \sqrt{64 + 25 + 1} = \sqrt{90} = 3\sqrt{10}$$

Portanto, $A_{\Delta ABC} = \dfrac{3\sqrt{10}}{2}$

b) $= |\vec{v}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$$\frac{|\vec{v}| \cdot h}{2} = \frac{3\sqrt{10}}{2} \Rightarrow h = \sqrt{10}$$

Produto Misto de vetores

Base: $B = (\vec{i}, \vec{j}, \vec{k})$

$\vec{u} = (x_1, y_1, z_1)$

$\vec{v} = (x_2, y_2, z_2)$

$\vec{w} = (x_3, y_3, z_3)$

Def: Produto misto dos vetores $\vec{u}, \vec{v}$ e $\vec{w}$ é o nº real dado por:

$$(\vec{u} \wedge \vec{v}) \times \vec{w} = [\vec{u}, \vec{v}, \vec{w}] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$\vec{u} = (2, -1, 0)$ $\vec{v} = (1, 2, -1)$ $\vec{w} = (3, 3, 1)$

$$[\vec{u}, \vec{v}, \vec{w}] = \begin{vmatrix} 2 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 3 & 1 \end{vmatrix} = 4 + 3 + 0 - 0 + 6 + 1 = \boxed{14}$$

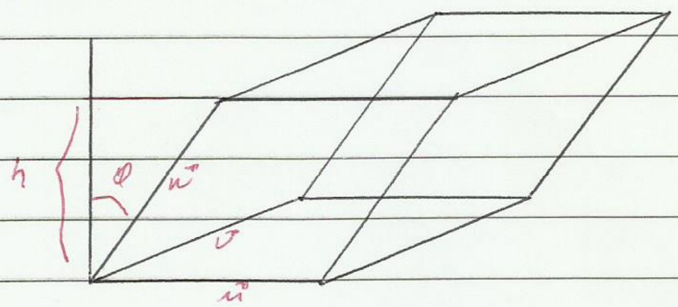
Propriedades

1) $[\vec{u}, \vec{v}, \vec{w}] = 0$ se $\vec{u}, \vec{v}, \vec{w}$: LD
2) $[\alpha \vec{u}, \vec{v}, \vec{w}] = \alpha [\vec{u}, \vec{v}, \vec{w}]$
3) $[\vec{u} + \vec{v}, \vec{w}, \vec{t}] = [\vec{u}, \vec{w}, \vec{t}] + [\vec{v}, \vec{w}, \vec{t}]$
4) $[\vec{u}, \vec{v}, \vec{w}] = [\vec{v}, \vec{w}, \vec{u}] = [\vec{w}, \vec{u}, \vec{v}]$

$$[\vec{u}, \vec{v}, \vec{w}] = -[\vec{v}, \vec{u}, \vec{w}]$$

DATA / /

1) Interpretação Geométrica



Vol. Paralelepípedo = A base . h

$V_{par} = |\vec{u} \wedge \vec{v}| \cdot h$

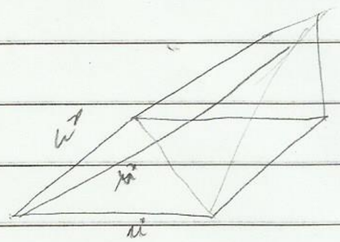
$\cos\theta = \frac{h}{|\vec{w}|}$

$h = |\vec{w}| \cdot \cos\theta$

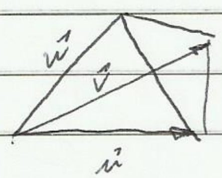
$V_{par} = |\vec{u} \wedge \vec{v}| |\vec{w}| \cos\theta$

$V_{par} = |\vec{u} \wedge \vec{v} \times \vec{w}|$

$V_{par} = |[\vec{u}, \vec{v}, \vec{w}]|$



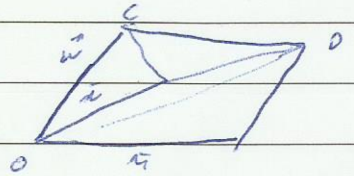
$V_{pirâmide} = \frac{1}{3} |[\vec{u}, \vec{v}, \vec{w}]|$



$V_{tetraedro} = \frac{1}{6} |[\vec{u}, \vec{v}, \vec{w}]|$

DATA / /

(106) $\vec{u}=(4,2,0)$ $\vec{v}=(-2,4,1)$ $\vec{w}=(1,1,3)$



$V_{par}(OABDC) = 58$ u.v

$$[\vec{u},\vec{v},\vec{w}] = \begin{vmatrix} 4 & 2 & 0 \\ -2 & 4 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 48+2+0-0-4+12 = 58$$

b) $V_{PIR}(O,A,B,DC) = \frac{1}{3}\cdot 58 = \frac{58}{3}$ u.v

c) $V_{tet}(OABC) = \frac{1}{6}\cdot 58 = \frac{58}{6}$ u.v

d) $\det \neq 0$ $\nexists$ são coplanares

e) Como $\det = 58 > 0$ base positiva

DATA / /

16) $\begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 3 & 2 \end{vmatrix} \begin{matrix} 2 & 1 \\ 1 & 1 \\ -1 & 3 \end{matrix} = 4-1-6-2 = -5$

a) $\frac{1}{6} \cdot (-5) = \frac{-5}{6}$

b) $V_{paralelepipedo} = A_{paralelo} \cdot h$

$h = \frac{|[\vec{u}, \vec{v}, \vec{w}]|}{|\vec{u} \wedge \vec{v}|} = \frac{5}{\sqrt{6}} = \frac{5\sqrt{6}}{\sqrt{6}}$

$\vec{u} \wedge \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (1, -2, 1)$

$|\vec{u} \wedge \vec{v}| = \sqrt{1+4+1} = \sqrt{6}$

Duplo produto vetorial

Det: vetor $(\vec{u} \wedge \vec{v}) \wedge \vec{w}$

$\vec{u} \wedge \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (1 -2, 1)$

$(\vec{u} \wedge \vec{v}) \wedge \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ -1 & 3 & 2 \end{vmatrix} = (-7, -3, 1)$

DATA / /

Exprimir o duplo produto vetorial como combinação linear de $\vec{u}$, $\vec{v}$

Demonstrar-se que

$(\vec{u} \wedge \vec{v}) \wedge \vec{w} = (\vec{u} \times \vec{w})\vec{v} - (\vec{v} \times \vec{w})\vec{u}$

$\vec{u} \times \vec{w} = 2 + 3 + 0 = 1$

$v \times \vec{w} = -1 + 3 + 2 = 4$

$(\vec{u} \wedge \vec{v}) \wedge \vec{w} = 1.\vec{v} - 4\vec{u} = (1,1,1) - (8,4,0) = (-7,3,1)$

ou

$(\vec{u} \wedge \vec{v}) \wedge \vec{w} = a\vec{v} + b\vec{u}$ $\quad$ $(\vec{u} \wedge \vec{v}) \wedge \vec{w} = 4\vec{u} + 1\vec{v}$

$(-7,-3,1) = (2a, a, 0) + (b,b,b)$

$2a + b = -7$

$a + b = -3$ $\quad$ $\underline{a = -4}$

$b = \underline{1}$

Vale p/a 1ª Eq:

$2(-4) + 1 = -7$

$-7 = -7$ V

B $\quad BX = \frac{1}{3} BC \quad \frac{2}{3} \quad CX = \frac{2}{3} CB \quad \vec{CB} = \vec{CA} + \vec{AB} \quad \vec{AB} - \vec{CA}$

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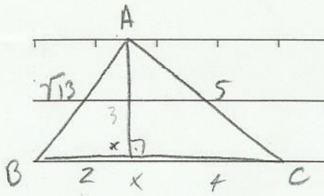


$\vec{AX} = \vec{AB} + \vec{BX}$

$\vec{AX} = \vec{AC} + \vec{CX}$

$2\vec{AX} = \vec{AB} + \vec{AC} + \vec{BX} + \vec{CX}$

$2\vec{AX} = \vec{AB} + \vec{AC} + \frac{1}{3}(\vec{BA} + \vec{AC}) + \frac{2}{3}(CA + AB)$

$2\vec{AX} = \vec{AB} + \vec{AC} - \frac{1}{3}\vec{AB} + \frac{1}{3}\vec{AC} - \frac{2}{3}AC + \frac{2}{3}AB$

$2\vec{AX} = \frac{4}{3}AB + \frac{2}{3}AC$

$(\sqrt{13})^2 = 2^2 + x^2$

$13 = 4 + x^2$

$x^2 = 9$

$x = \sqrt{9} = 3$

$5^2 = 3^2 + x^2$

$25 = 9 + x^2$

$x = \sqrt{16}$

$x = 4$

a) $\vec{AX} = \frac{2}{3}AB + \frac{1}{6}AC$

2-) $\begin{vmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 1 & -1 \\ 2 & 0 \end{matrix} = -2 + 2 - 1 = -1$

$M = \begin{bmatrix} 2 & 1 & 2 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Cof } M \begin{bmatrix} -1 & -1 & 0 \\ -1 & 2 & -0 \\ 2 & +2 & -3 \end{bmatrix} \quad \text{Adj } M \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

$M_{E \to F}$

$M_{F \to E}$

$M_{F \to E} \begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & -2 \\ 0 & 0 & 3 \end{bmatrix} \quad M_{E \to F} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

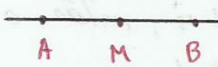
B) $|\vec{AB}| \cdot |\vec{AX}| \cos\theta$

$\sqrt{13} \cdot 3 \cdot \frac{3}{\sqrt{13}} = 9$

3b) $\begin{bmatrix} i & j & K \\ 2 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix} = 0 + 2j - 2K - K + 2i = 2i + 2j - 2K$

$\begin{bmatrix} -2 & -1 & -2 \\ -1 & +1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} = \begin{matrix} -4 - 2 + 4 \\ \\ 2 \end{matrix} \begin{bmatrix} 2 \\ \\ -2 \end{bmatrix}_E$

Coordenadas do Ponto Médio: Considere $A=(x_a, y_a, z_a)$, $B=(x_b, y_b, z_b)$ e $M=(x_m, y_m, z_m)$ onde M é o ponto Médio do segmento AB.



Temos que $\vec{AM} = \vec{MB}$, logo

$$\begin{cases} x_m - x_a = x_b - x_m \\ y_m - y_a = y_b - y_m \\ z_m - z_a = z_b - z_m \end{cases} \Longrightarrow M = \left(\frac{x_a + x_b}{2}, \frac{y_a + y_b}{2}, \frac{z_a + z_b}{2} \right).$$

A Reta

Dados dois pontos distintos, existe uma única reta que os contém. Essa reta é determinada por esses dois pontos.

Observe que se $X \in r$, então os vetores $(X-A)$ e $\vec{u}$ são paralelos. Logo:

$\exists \lambda \in \mathbb{R} : (X-A) = \lambda \vec{u}$, ou seja,

$\boxed{r: X = A + \lambda \vec{u}, \lambda \in \mathbb{R}}$ é a equação vetorial da reta r, que contém o ponto A e é paralela ao vetor $\vec{u}$.

Sejam $A=(x_0, y_0, z_0)$, $X=(x, y, z)$ e $\vec{u}=(l, m, n)$ Segue que:

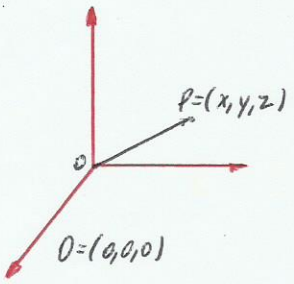
$(x, y, z) = (x_0, y_0, z_0) + \lambda (l, m, n)$, $\lambda \in \mathbb{R}$ ou seja,

$$\boxed{r: \begin{cases} x = x_0 + \lambda l \\ y = y_0 + \lambda m \\ z = z_0 + \lambda n \end{cases}, \lambda \in \mathbb{R}}$$

São as equações paramétricas da reta r

Sistemas de coordenadas cartesianas:

É um conjunto formado por um ponto O (origem) e pelos vetores $\vec{e_1}, \vec{e_2}, \vec{e_3}$ que formam uma base no espaço. Not.: $S = (O, \vec{e_1}, \vec{e_2}, \vec{e_3})$.



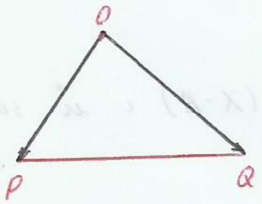
Já vimos que se $\vec{OP} = x\vec{e_1} + y\vec{e_2} + z\vec{e_3}$, então o vetor $\vec{OP}$ também pode ser representado da seguinte forma: $\vec{OP} = (x, y, z)$.

- Os escalares x, y e z são as coordenadas do ponto P e também do vetor $\vec{OP}$.

Propriedades: Sejam $P = (x_1, y_1, z_1)$, $Q = (x_2, y_2, z_2)$ e $\vec{u} = (a, b, c)$ em relação ao sistema $S = (O, \vec{e_1}, \vec{e_2}, \vec{e_3})$, temos que:

1) $(Q - P) = \vec{PQ} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

Demonstração:



Temos que:

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$
$$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$$
$$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

2) $P + \vec{u} = (x_1 + a, y_1 + b, z_1 + c)$

Estudar demonstração no livro!

Exemplo: Dados $A = (1, 2, 1)$ e $B = (3, 0, 5)$, temos

$$(B - A) = \vec{AB} = (2, -2, +4)$$

Isolando λ nas equações acima, temos:

$$\boxed{r: \frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}, \quad l, m, n \neq 0}$$ → são as equações simétricas da reta r

Exercícios: 1) Seja a reta dada pelos pontos $A=(0,1,1)$ e $B=(-3,1,0)$

a) Verifique se o ponto $C=(3,1,2)$ pertence à reta r.

$r: X = A + \lambda \overrightarrow{BA}, \ \lambda \in \mathbb{R}$

onde $\overrightarrow{BA} = A - B = (3,0,1)$

Se $C \in r$, então a eq. $C = A + \lambda \overrightarrow{BA}$ deve ser satisfeita para algum λ. Ou seja,

$$(3,1,2) = (0,1,1) + \lambda(3,0,1) \Rightarrow \begin{cases} 3 = 0 + 3\lambda \\ 1 = 1 + 0.\lambda \\ 2 = 1 + 1.\lambda \end{cases} \Rightarrow \lambda = 1$$

b) Verifique se o ponto $D = (2,-1,5)$ pertence à reta r.

$$(2,-1,5) = (0,1,1) + \lambda(3,0,1) \Rightarrow$$

$$\begin{cases} 2 = 0 + 3\lambda \\ -1 = 1 + 0\lambda \text{ (ABSURDO!)} \\ 5 = 1 + \lambda \end{cases}$$

$\therefore \ D \notin r$

3) Escreva a eq. vetorial da reta $r: \frac{x+1}{5} = \frac{3y-6}{-9} = \frac{7-z}{1}$

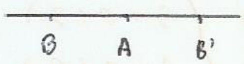
Temos $\frac{x-(-1)}{5} = \frac{y-2}{-3} = \frac{z-7}{-1}$

Logo, $r: X = (-1,2,7) + \lambda(5,-3,-1), \lambda \in \mathbb{R}$

4) Verifique se os pontos $A=(3,-2,1)$ e $B=(-1,2,0)$ pertencem à reta determinada pelos pontos $C=(2,1,-1)$ e $D=(4,-5,3)$ $A \in r$ $B \notin r$

5) Dados $A=(2,-3,4)$, $B=(4,7,6)$. Determine o ponto B', simétrico ao ponto B em relação à A.

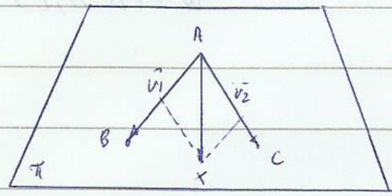
$B' = (0,-13,2)$



4) $C=$

Estudo do plano

Equação vetorial



$(X-A), \vec{v_1}$ e $\vec{v_2}$ são LD

$(X-A) = \lambda \vec{v_1} + \rho \vec{v_2}$

$\pi: \ X = A + \lambda \vec{v_1} + \rho \vec{v_2}$

Exemplo: $A = (1, 2, -1)$

$\vec{v_1} = (2, -1, 4)$

$\vec{v_2} = (1, 3, 3)$

Equação vetorial

$X = (1, 2, -1) + \lambda (2, -1, 4) + \rho (1, 3, 3)$

Equação paramétrica

$(x, y, z) = (x_0, y_0, z_0) + \lambda (l_1, m_1, n_1) + \rho (l_2, m_2, n_2)$

$$\pi: \begin{cases} x = 1 + 2\lambda + \rho \\ y = 2 - \lambda + 3\rho \\ z = -1 + 4\lambda + 3\rho \end{cases}$$

$ax + by + cz + d = 0$

Equação Geral

$$\begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & -1 & 4 \\ 1 & 3 & 3 \end{vmatrix} = 0$$

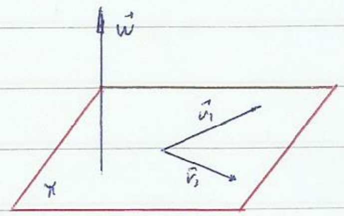
$-15(x-1) - 2(y-2) + 7(z+1) = 0$

$-15x - 2y + 7z + 26 = 0$

ou

$15x + 2y - 7z - 26 = 0$

Vetor normal a um plano



$\vec{w} \perp \pi$

$\vec{w} = \vec{v_1} \wedge \vec{v_2}$

Demostra-se que

$\vec{w} = (a, b, c)$

$$\vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 4 \\ 1 & 3 & 3 \end{vmatrix} = (-15, -2, 7)$$

(165-) $A = (1,1,3)$

$B = (2,-1,0)$

$C = (1,1,1)$

Equação vetorial:

$X = A + \lambda(B-A) + \rho(C-A)$

$X = (1,1,3) + \lambda(1,-2,-3) + \rho(0,0,-2) \; ; \; \lambda, \rho \in \mathbb{R}$

Equação paramétrica

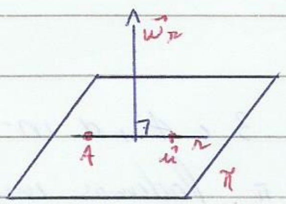
$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & -2 & -3 \\ 0 & 0 & -2 \end{vmatrix} = 0$$

$4(x-1) + 2(y-1) = 0$

$\frac{4x + 2y - 6 = 0}{2} = 2x + y - 3 = 0$

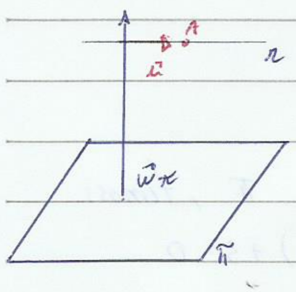
Posições Relativas entre Reta e Plano: Considere a reta $r: X = A + \lambda \vec{u}, \lambda \in \mathbb{R}$ e o plano π de equação geral $\pi: ax + by + cz + d = 0$. Podem ocorrer as seguintes posições entre r e π:

1) Reta contida no Plano ($r \subset \pi$): Nesse caso, os vetores $\vec{w_\pi}$ e $\vec{u}$ são ortogonais e assim:



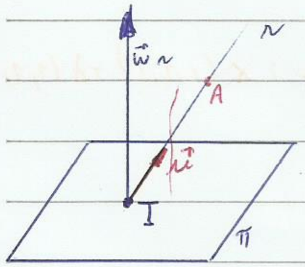
$\vec{w_\pi} \cdot \vec{u} = 0$ e ainda $A \in \pi$

2) Reta paralela ao plano: Também temos $\vec{u}$ e $\vec{w_\pi}$ ortogonais, ou seja,



$\vec{w_\pi} \cdot \vec{u} = 0$. Porém $A \notin \pi$

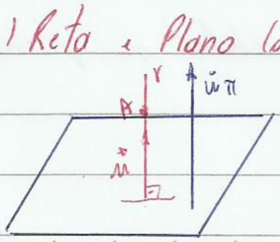
3) Reta e Plano Concorrentes Não Perpendiculares



$\vec{w_\pi}$ e $\vec{u}$ não são ortogonais e não são paralelos, logo:

$\vec{w_\pi} \cdot \vec{u} \neq 0$ e $\{\vec{w_\pi}, \vec{u}\}$ é L.I

4) Reta e Plano Concorrentes Perpendiculares:



$\vec{u}$ e $\vec{w_\pi}$ são paralelos, assim:

$\vec{w_\pi} \cdot \vec{u} \neq 0$ e $\{\vec{w_\pi}, \vec{u}\}$ é LD

/ /

Exemplo: Verifique a posição relativa entre r e π nos seguintes casos:

1) $r: X=\overset{A}{(1,2,1)} + \lambda \overset{\vec{u}}{(1,1,1)}, \lambda \in \mathbb{R}$ e $\pi: 2x+y-2z+3=0$

$\vec{w}_\pi = (2,1,-2)$

Como $\vec{w}_\pi \cdot \vec{u} = 2+1-2 = 1 \neq 0$ e $\{\vec{u}, \vec{w}_\pi\}$ é LI, temos que r e π são concorrentes não perpendiculares

Intersecção entre Reta e Plano: Observem que nos casos 3 e 4, a intersecção entre r e π determinam um ponto $I = r \cap \pi$. Podemos encontrar I substituindo as eq. paramétricas de r na eq. geral de π e resolver o sistema formado.

Ex. 2: Obtenha $I = r \cap \pi$ do último exemplo.

$$r: \begin{cases} x = 1+\lambda \\ y = 2+\lambda \\ z = 1+\lambda \end{cases}, \lambda \in \mathbb{R}$$

Subst. na eq. geral de π, temos:

$$(2+2\lambda) + (2+\lambda) + (-2-2\lambda) + 3 = 0$$

$$\lambda = -5$$

Portanto, $I = r \cap \pi = (-4,-3,-4)$

Ex. 3: Verifique a posição relativa entre a reta $r: X=(1,2,1)+\lambda(1,1,1), \lambda \in \mathbb{R}$ e o plano $\beta: x+y-2z+3=0$

$\vec{w}_\beta = (1,1,-2)$

- $\vec{u} \cdot \vec{w}_\beta = 1+1-2 = 0$ (*)
- $(1)+(2)-2(1)+3 = +4 \neq 0$, ou seja, $A \notin \beta$ (**)

De (*) e (**), temos que a reta r é paralela ao plano β ($r \parallel \beta$)

Prova

Ex. 4) Dado o plano $\pi: 2x-y+z-4=0$ e a reta $r: 1-x = \frac{y-1}{2} = \frac{z-3}{4}$

a) Mostre que o plano π contém a reta r.

b) Determine a equação geral do plano α que é perpendicular ao plano π e cuja interseção com π é a reta r

c) Encontre a eq. vetorial do plano π.

(5) Dados os pontos $A=(1,1,5)$, $B=(1,2,3)$ e $C=(4,2,-1)$. Seja X o ponto sobre o segmento $\overline{BC}$ de modo que $\overline{BX} = 3\,cm$. Escreva as eq. paramétricas da reta r que contém os pontos A e X.

(4) $\pi: 2x-y+z-4=0$ $\quad w_r = (2,-1,1)$

a) $r: 1-x = \frac{y-1}{2} = \frac{z-3}{4}$

$$r: \begin{cases} x = 1-\lambda \\ y = 1+2\lambda \\ z = 3+4\lambda \end{cases}$$

$$(2-2\lambda)+(-1-2\lambda)+(3+4\lambda)-4=0$$

$$(2-1+3-4)(-2\lambda-2\lambda+4\lambda)=0$$

$$0=0$$

ou

$$(2,-1,1)(-1,2,4)=0$$

$$-2+(-2)+4=0$$

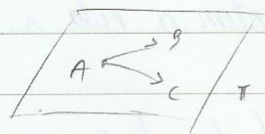
$$0=0$$

b) $\alpha: x = ($

c) $A = (1,1,3) \in \pi$ $\quad \pi = 2x - y + z - 4 = 0$

$B = (0,0,4) \in \pi$

$C = (2,0,0) \in \pi$

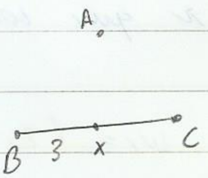


$$\begin{cases} \vec{AB} = (B - A) = (-1,-1,1) \\ \vec{AC} = (C - A) = (1,-1,-3) \end{cases} \quad LI$$

$$x = A + \lambda \vec{AB} + \rho \vec{AC}$$

$$x = (1,1,3) + \lambda(-1,-1,1) + \rho(1,-1,-3)$$

(5) $A = (1,1,5)$ $\quad B = (1,2,3)$ $\quad C = (4,2,-1)$



$$\vec{AX} = \vec{AB} + \vec{BX}$$

$$\vec{AX} = (0,1,-2) + \left(\frac{9}{5}, 0, \frac{-12}{5}\right)$$

$$\vec{AX} = \left(\frac{9}{5}, 1, \frac{-22}{5}\right)$$

$\vec{AB} = (0,1,-2)$

$\vec{BC} = (3,0,-4)$. $\quad |BC| = 5$

$$\vec{BX} = \frac{3}{5}\vec{BC}$$

$$\vec{BX} = \frac{3}{5}(3,0,-4)$$

$$\vec{BX} = \left(\frac{9}{5}, 0, \frac{-12}{5}\right)$$

$$x = (1,1,5) + \lambda\left(\frac{9}{5}, 1, \frac{-22}{5}\right)$$

$$r = \begin{cases} x = 1 + \frac{9}{5}\lambda \\ y = 1 + \lambda \\ y = 5 + \left(-\frac{22}{5}\right)\lambda \end{cases} \quad \text{ou} \quad r: \begin{cases} x = 1 + 9\lambda \\ y = 1 + 5\lambda \\ z = 5 - 22\lambda \end{cases}$$

Exercícios

1) Dadas as retas $r: X = (1,-3,0) + \lambda(2,2,-4)$ e $s: x = y-1 = \frac{z-3}{-2}$. Determine (se existir) a eq. geral do plano π que contém r e s.

2) Seja $\alpha: x+2y-z+7=0$, determine o ponto A', simétrico do ponto $A=(2,5,-1)$ em relação ao plano α.

3) Dada a reta r que contém os pontos $A=(0,-1,2)$ e $B=(1,0,3)$ e s a reta que contém o ponto $C=(-3,-4,-1)$ e é simultaneamente ortogonal às retas $r_1: X=(0,0,0)+\lambda(1,1,1)$, $\lambda \in \mathbb{R}$ e $r_2: x=1+2t$; $y=2$, $z=1+t$; $t \in \mathbb{R}$.
Pede-se:

(a) Determinar as eq. paramétricas das retas r e s
(b) Verificar se r e s são coplanares ou reversas
(c) Encontre $I = r \cap s$

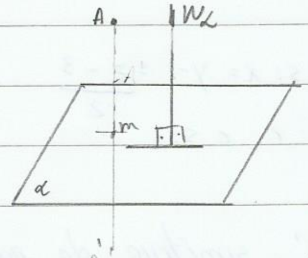
4) Determine a eq. geral do plano π, que contém o ponto $P=(1,1,1)$ e é paralelo ao plano $\beta: 3x-y+z+1=0$

5) Para que valores de m a reta $r: X=(2,0,-3)+\lambda(-5,1,4)$, $\lambda \in \mathbb{R}$ é paralela ao plano $\alpha: mx+2y+2z+4=0$

1) $r: X=(1,-3,0)+\lambda(2,2,-4)$
$s: X=(0,1,3)+\lambda'(1,1,-2)$

$$\begin{vmatrix} x-1 & y+3 & z \\ 2 & 2 & -4 \\ -1 & 4 & 3 \end{vmatrix} = 0$$

$22(x-1)-2(y+3)+10z=0$
$11(x-1)-(y+3)+5z=0$
$11x-y+5z-14=0$

(2) 

$\alpha: x+2y-z+7=0 \quad \vec{w_\alpha}=(1,2,-1)$

$A=(2,5,-1)$ $\quad B=(0,0,7)$

$C=(-7,0,0)$

$X=(2,5,-1)+\lambda(1,2,-1)$

$x=2+\lambda;\ y=5+2\lambda;\ z=-1-\lambda$

$\alpha: x+2y-z+7=0$

$2+\lambda+10+4\lambda+1+\lambda+7=0$

$6\lambda=-20$

$\lambda=\frac{-10}{3}$

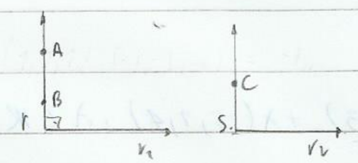
$X=(2,5,-1)+\lambda(1,2,-1)$

$X=(2,5,-1)-\frac{10}{3}(1,2,-1)$

$X=\left(\frac{-4}{3},\frac{-5}{3},\frac{7}{3}\right)$

$\left(\frac{-4}{3},\frac{-5}{3},\frac{7}{3}\right)=\left(\frac{2+x}{2},\frac{5+y}{2},\frac{-1+z}{2}\right)$

$A'=\left(\frac{-14}{3},\frac{-25}{3},\frac{17}{3}\right)$

(3) 

$A=(0,-1,2) \quad B=(1,0,3) \quad C=(-3,-4,1)$

$r_1: X=(0,0,0)+\lambda(1,1,1)$

$r_2: X=(1,2,1)+t(2,0,1)$

(a) r? e s?

$r: X=(0,-1,2)+a(1,1,1)$

$s: X=(-3,-4,1)+b(1,1,-2)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \vec{i}+\vec{j}-2\vec{k} \quad (1,1,-2)$$

(b) $$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \\ -3 & -3 & -3 \end{vmatrix} = -9+9=0$$ São coplanares e concorrentes

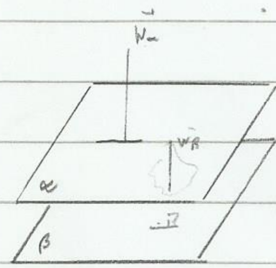
c) $a = -3 + b$ $\quad 2 + a = -1 - 2b$ $\quad a = -3 + b$

$-1 + a = -4 + b$ $\quad 2 - 3 + b = -1 - 2b$ $\quad a = -3$

$2 + a = -1 - 2b$ $\quad 3b = 0$

$b = \frac{0}{3} = 0$

$I = (-3, -4, -1)$



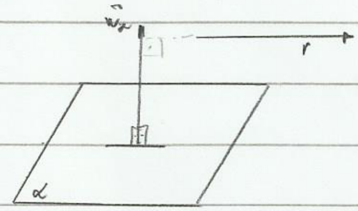
4) $P = (1,1,1)$ // $\beta: 3x - y + z + 1 = 0$

$\alpha: 3x - y + z + d = 0$

$3 \cdot 1 - 1 + 1 + d = 0$

$d = -3$

$\alpha: 3x - y + z - 3 = 0$



5) $r: X = (2,0,3) + \lambda(-5,1,4)$

$\alpha: mx + 2y + 2z + 4 = 0$

$\vec{n}_\alpha \; (m, 2, 2)$

$(m,2,2) \cdot (-5,1,4) = 0$

$-5m + 2 + 8 = 0$

$-5m = -10$

$m = 2$

Posições relativas

Ex 4) Obtenha o centro e o raio da superfície esférica
$S: x^2+y^2+z^2+4x-10z-71=0$

Ex 5) Determine o centro e o raio da superfície esférica
$S: x^2+y^2+z^2-2x-2y+6z-38=0$

Ex 6) Determine o centro e o raio da superfície esférica $S: x^2+y^2+z^2-2x-y-z-\frac{5}{2}$ e a eq. geral dos planos tangentes à superfície que são perpendiculares à reta $r: X=(0,-2,1)+\lambda(1,2,-1);\ \lambda\in\mathbb{R}$

Respostas

Ex 4) $\begin{cases} -2a = 4 \\ -2b = 0 \\ -2c = -10 \end{cases} \Rightarrow C=(-2,0,5)$

$4+25-R^2=-71$

$R^2=100 \quad R=10$

Ex 5) $\begin{cases} -2a = -2 \\ -2b = -2 \\ -2c = 6 \end{cases} \Rightarrow C=(1,1,-3)$

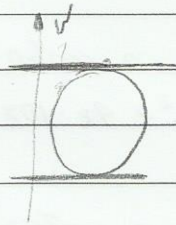
$1+1+9-R^2=-38$

$R^2=49$

$R=7$

6-) $s: x^2+y^2+z^2-2x-y-z-\frac{5}{2}=0$

$r: X=(0,-2,1)+\lambda(1,2,-1),\ \lambda \in \mathbb{R}$



$$\begin{cases} -2a=-2 \\ -2b=-1 \\ -2c=-1 \end{cases} \Rightarrow C=\left(1,\frac{1}{2},\frac{1}{2}\right)$$

$1+\frac{1}{4}+\frac{1}{4}-R^2=-\frac{5}{2} \qquad R^2=4 \Rightarrow R=2$

$\vec{W_\pi}=(1,2,-1)$

$\pi: x+2y-z+d=0$

$d(C,\pi)=\dfrac{|1+1-\frac{1}{2}+d|}{\sqrt{6}}=2 \qquad |\frac{3}{2}+d|=2\sqrt{6}$

$$\begin{cases} \frac{3}{2}+d=2\sqrt{6} \\ -\frac{3}{2}-d=2\sqrt{6} \end{cases} \Rightarrow \begin{matrix} d=2\sqrt{6}-\frac{3}{2} \\ d=-2\sqrt{6}+\frac{3}{2} \end{matrix}$$

$\pi_1: x+2y-z+2\sqrt{6}-\frac{3}{2}=0$

$\pi_2: x+2y-z-2\sqrt{6}+\frac{3}{2}=0$

STQQSSD Assunto: Data: / /

Ex7) Dada a sup. esf. $S: x^2+y^2+z^2-6x+2y-4z-24=0$, determine:

a) a eq. geral do plano tangente à superfície no ponto $P=(2,5,1)$
b) a eq. geral do plano que é paralelo ao plano tangente em P e que divide S em partes iguais.

$$\begin{cases} -2a = -6 \\ -2b = 2 \\ -2c = -4 \end{cases} \Rightarrow C=(3,-1,2)$$

$9+1+4-R^2=-24$

$R^2=38 \quad R=\sqrt{38}$

$\vec{u}=(1,-6,1)$

$\pi: x-6y+z+d=0$

$2-30+1+d=0$

$d=27$

a) $\pi_1: x-6y+z+27=0$

$\pi_2: 3+6+2+d=0$

$d=-11$

$\pi_2: x-6y+z-11=0$

Geometria Analítica - Exercícios P2

Treino com operações:

$A=(1,2,1)$ $B=(2,-1,0)$ $C=(1,3,3)$

$\vec{AB}=(B-A)=(2-1,-1-2,0-1)=(1,-3,-1)$

$\vec{BC}=(C-B)=(1-2,3-(-1),3-0)=(-1,4,3)$

$P=A+3(C-B)=(1,2,1)+3(-1,4,3)=(-2,14,10)$

— // —

(112-) $A=(1,2,1)$ $B=(2,1,0)$ $C=(1,3,3)$ $S=(0,\vec{i},\vec{j},\vec{k})$

a) A M B $\quad AB=(\ ,\ ,\ )$?

$\vec{AM}=\vec{MB}$

$X_m - X_a = X_b - X_m$	$Y_m - Y_a = Y_b - Y_m$	$Z_m - Z_a = Z_b - Z_m$
$2X_m = X_b + X_a$	$2Y_m = Y_b + Y_a$	$2Z_m = Z_b + Z_a$
$X_m = \frac{X_b + X_a}{2}$	$Y_m = \frac{Y_b + Y_a}{2}$	$Z_m = \frac{Z_b + Z_a}{2}$
$X_m = \frac{2+1}{2} = \frac{3}{2}$	$Y_m = \frac{1+2}{2} = \frac{3}{2}$	$Z_m = \frac{0+1}{2} = \frac{1}{2}$

$M=\left(\frac{3}{2}, \frac{3}{2}, \frac{1}{2}\right)$

b) (Dúvida: "É assunto de prova?")

(113-) $A=(2,-3,4)$ $B=(4,7,6)$ $S=(0,\vec{i},\vec{j},\vec{k})$

B A B'

$\vec{BA} = \vec{AB'}$

$Xa - Xb = Xb' - Xa$ $Ya - Yb = Yb' - Ya$ $Za - Zb = Zb' - Za$

$Xb' = 2Xa - Xb$ $Yb' = 2Ya - Yb$ $Zb' = 2Za - Zb$

$Xb' = 2.2 - 4$ $Yb' = 2.(-3) - 7$ $Zb' = 2.4 - 6$

$= 0$ $Yb' = -13$ $Zb' = 2$

$B' = (0, -13, 2)$

— // —

(114-) $A=(2,-3,4)$ $B=(4,7,6)$ $C=(3,1,-2)$

"Observação: Pontos colineares são pontos que pertecem a mesma reta" (e são paralelos.

$\vec{AB} \parallel \vec{AC}$

$\vec{AB} = m\,\vec{AC}$

$(4-2, 7-(-3), 6-4) = m(3-2, 1-(-3), -2-4)$

$(2, 10, 2) = m(1, 4, 6)$

$m = \frac{2}{1} \neq \frac{10}{4} \neq \frac{2}{6}$

R: Eles não são colineares, pois o sistema linear da equação não existe solução

(115-) $A=(1,2,1)$ $B=(1,0,0)$ $C=(1,y,z)$

$\vec{BA} = m\vec{BC}$

$(0,2,1) = m(0,y,z)$

$my = 2$

$mz = 1$ $\qquad C=(1,2z,z)$

—//—

(116-) $A=(2,6,3)$ $B=(3,2,2)$ $C=(0,5,4)$ $D=(1,1,3)$

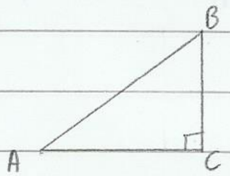
$\vec{AB} = (1,-4,-1)$ $\qquad \vec{AC} = (-2,-1,1)$

$\vec{CD} = (1,-4,-1)$ $\qquad \vec{BD} = (-2,-1,1)$

$\vec{AB} \parallel \vec{CD}$ e $\vec{AC} \parallel \vec{BD}$

—//—

(117-) $A=(1,-1,0)$ $B=(4,3,5)$ $C=(3,0,-2)$



$\vec{AC} = (2,1,-2)$

$\vec{BC} = (-1,-3,-7)$

$\vec{AC} \cdot \vec{BC} = -2 - 3 + 14 = 8$ (Não é retângulo em C)

$\vec{AC} = (2,1,-2)$

$\vec{AB} = (3,4,5)$

$\vec{AC} \cdot \vec{AB} = 6 + 4 - 10 = 0$ (É retângulo no vértice A)

—//—

(118-) a-) $\vec{AB} = m\vec{AC}$

$(2,10,2) = m(3,15,3)$

$m = \frac{2}{3} = \frac{10}{15} = \frac{2}{3}$ (Verdadeira, Portanto são colineares)

b) $(-1,2,-1) = m(1,-2,1)$

$m = \frac{-1}{1} = \frac{2}{-2} = \frac{-1}{1}$ (Verdadeira, Portanto são colineares)

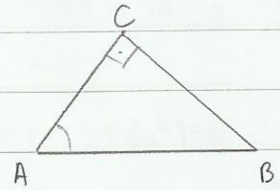
(119-) $A=(3,-1,-2)$ $B=(2,1,0)$ $C=(1,0,-1)$ $D=(0,y,2)$

a) $G = O + \dfrac{m_A(O-A) + m_B(O-B) + m_C(O-C)}{m_A + m_B + m_C}$

$G = (0,0,0) + \dfrac{1(3,-1,-2) + 2(2,1,0) + 3(1,0,-1)}{6}$

$G = \dfrac{(3,-1,-2) + (4,2,0) + (3,0,-3)}{6}$

$G = \left(\dfrac{10}{6}, \dfrac{1}{6}, -\dfrac{5}{6}\right)$



b) $\vec{AB} = (-1,2,2)$

$\vec{AC} = (-2,1,1)$

$\vec{AB} \cdot \vec{AC} = 2+2+2 = 6$

$\vec{CA} = (2,-1,-1)$

$\vec{CB} = (1,1,1)$

$\vec{CA} \cdot \vec{CB} = 2-1-1 = 0$ (Portanto, é retangulo em C)

c) $\vec{BC} = m\,\vec{BD}$

$(-1,-1,-1) = m(-2, y-1, z)$

$m = \dfrac{-1}{-2} = \dfrac{-1}{y-1} = \dfrac{-1}{z}$

$y-1 = -2$ $\underline{z = -2}$

$\underline{y = -1}$

(120-) $A = \left(\frac{1}{2}, 3, -1\right)$ $B = \left(\frac{3}{2}, 4, 2\right)$ $C = (5, 0, 8)$ $D(4, -1, 5)$

$\vec{AB} = (1, 1, 3)$ $\vec{AB} \parallel \vec{DC}$ É retângulo

$\vec{CD} = (-1, -1, -3)$ $\vec{AD} \parallel \vec{BC}$

$\vec{AD} = \left(\frac{7}{2}, -4, 6\right)$

$\vec{BC} = \left(\frac{7}{2}, -4, 6\right)$

— // —

(121-) $A = (3, -5, 1)$ $B = (4, -5, 3)$

A B A'

$\vec{AB} = \vec{BA'}$

$X_B - X_A = X_{A'} - X_B$ $Y_B - Y_A = Y_{A'} - Y_B$ $Z_B - Z_A = Z_{A'} - Z_B$

$X_{A'} = 2X_B - X_A$ $Y_{A'} = 2Y_B - Y_A$ $Z_{A'} = 2Z_B - Z_A$

$X_{A'} = 2.4 - 3$ $= 2.(-5) - (-5)$ $= 2.3 - 1$

$= 5$ $= -5$ $= 5$

$A' = (5, -5, 5)$

— // —

(122-) $A = (2, -1, 3)$ $B = (3, 1, 4)$

$$G = (0,0,0) + \frac{1(2,-1,3) + 3(3,1,4)}{1+3}$$

$$G = \frac{(2,-1,3) + (9,3,12)}{4}$$

$$G = \left(\frac{11}{4}, \frac{2}{4}, \frac{15}{4}\right)$$

/ /

(123-) A M B $A=(2,-1,3)$ $B=(3,1,4)$

$$\vec{AM}=\vec{MB}$$

$$X_m - X_A = X_B - X_m \qquad Y_m - Y_a = Y_B - Y_m \qquad Z_m - Z_a = Z_b - Z_m$$

$$X_m = \frac{X_B + X_A}{2} \qquad Y_m = \frac{Y_B + Y_a}{2} \qquad Z_m = \frac{Z_b + Z_a}{2}$$

$$X_m = \frac{3+2}{2} = \frac{5}{2} \qquad Y_m = \frac{1+(-1)}{2} = 0 \qquad Z_m = \frac{4+3}{2} = \frac{7}{2}$$

$$G = \left(\frac{5}{2}, 0, \frac{7}{2}\right)$$

— // —

(124-) "Qual é a condição dos vértices do tetraedro?"

(125-) $\vec{AB} = m\vec{AC}$

$(3,6,3) = m(-1,-2,-1)$

$m = \frac{3}{-1} = \frac{6}{-2} = \frac{3}{-1}$ (São colineares)

$\vec{AB} = m\vec{AC}$

$(3,-3,6) = m(5,-5,10)$

$m = \frac{3}{5} = \frac{-3}{-5} = \frac{6}{10}$ (São colineares)

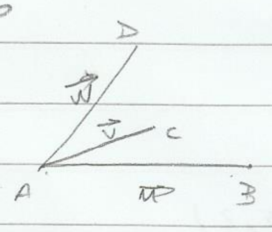
$\vec{AB} = m\vec{AC}$

$(1,-2,1) = m(-1,-1,5)$

$m = \frac{1}{-1} \neq \frac{-2}{-1} \neq \frac{1}{5}$ (Não são colineares)

(126-) $A=(0,1,1)$ $B=(1,3,2)$ $C=(2,2,1)$ $D=(1,4,m)$ $V_t = \frac{2}{3}$

$V_t = \frac{[\vec{u}, \vec{v}, \vec{w}]}{6}$ $\vec{AB} = . (B-A)$



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(127-) $A=(-5,1,1)$ $B=(1,3,2)$

$B = A + \frac{1}{3}\vec{v}$ $(1,3,2) = (-5,1,1) + \frac{1}{3}\vec{v}$

$\frac{1}{3}\vec{v} = (6,2,1)$ $\vec{v} = (18,6,3)$ Resposta: Alternativa A

(128-) $A=(0,1,-1)$ $B=(1,2,-1)$ $\vec{u}=(-2,-1,1)$ $\vec{v}=(3,0,-1)$ $\vec{w}=(-2,2,2)$

$\vec{w} = a\,\vec{AB} + b\,\vec{u} + c\,\vec{v}$

$(-2,2,2) = a(1,1,0) + b(-2,-1,1) + c(3,0,-1)$

$$\begin{cases} a - 2b + 3c = -2 \\ a - b = 2 \\ b - c = 2 \end{cases}$$

$-a + 2b - 3c = 2$

$a - b = 2$

$-b + 3c = 4$

$b - c = 2$

$2c = -2$

$a - 2b + 3c = -2$

$b - 3c = 4$

$2c = -2$

$c = -1$

$b = 1$

$a = 3$

Resposta A

(129-) $A=(3,-4,-2)$ $B=(-2,1,0)$

$\vec{AN} = \frac{2}{5}\vec{AB}$ $(x-3, y+4, z+2) = \frac{2}{5}(-5,5,2)$

$(x-3, y+4, z+2) = (-2, 2, \frac{4}{5})$

$x - 3 = -2 \therefore x = 1$

$y + 4 = 2 \therefore y = -2$

$z + 2 = \frac{4}{5} \therefore z = -\frac{6}{5}$

Resposta B

spiral

Exemplo livro

$A=(1,2,1)$ // $\vec{v}=(2,1,3)$

$X=(1,2,1)+\lambda(2,1,3)$

(130-) A P B $A=(2,-3,4)$ $B=(4,7,6)$

$\vec{AP}=m\,\vec{AB}$

$(x_P, y_P, z_P)-(2,-3,4)=m(2,10,2)$

$(x-2, y+3, z-4)=m(2,10,2)$

$P=(2+2m, -3+10m, 4+2m)\ ;\ m\in\mathbb{R}$

(131-) $r: \dfrac{2x-1}{3}=\dfrac{1+y}{2}=-z$

Eq. vetorial $X=A+\lambda\vec{v}$

Eq. paramétrica $(x,y,z)=(x_0,y_0,z_0)+\lambda(l,m,n)$

Eq. simétrica $\dfrac{x-x_0}{l}=\dfrac{y-y_0}{m}=\dfrac{z-z_0}{n}$

$\dfrac{x-\frac{1}{2}}{\frac{3}{2}}=\dfrac{y-(-1)}{2}=\dfrac{-z}{1}$

Equação vetorial:

$r: x=(1/2,-1,0)+\lambda(3/2,2,-1)$

Equação paramétrica

$x=\dfrac{1}{2}+\dfrac{3\lambda}{2}\ ;\ y=-1+2\lambda\ ;\ z=-\lambda\ ;\ \lambda\in\mathbb{R}$

/ /

132-) $A=(1,2,1)$ $B=(3,5,1)$

Equação vetorial: $X = A + \lambda\vec{v}$

$X = (1,2,1) + \lambda(2,3,0)$; $\lambda \in \mathbb{R}$

Equação paramétricas: $x = 1 + 2\lambda$; $y = 2 + 3\lambda$; $z = 1$; $\lambda \in \mathbb{R}$.

Equação simétrica: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-1}{1}$

(133-) $P=(2,1,-4)$ e $Q=(2,0,5)$ $\in$ $X=(1,1,3)+\lambda(1,-1,2)$, $\lambda \in \mathbb{R}$

$(2,1,-4) = (1,1,3) + \lambda(1,-1,2)$

$\lambda = \frac{2-1}{1} \neq \frac{1-1}{-1} \neq \frac{-4-3}{2}$ "O ponto P não pertence a esta reta, pois não é paralela".

$(2,0,5) = (1,1,3) + \lambda(1,-1,2)$

$\lambda = \frac{2-1}{1} = \frac{0-1}{-1} = \frac{5-3}{2}$ "O ponto Q pertence a esta reta, pois ele é paralelo a reta"!

(134-) a) $A=(1,-1,2)$; $\vec{u}=(-3,0,2)$; $\vec{v}=(1,2,-1)$

$$\vec{u} \wedge \vec{v} = \left[\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ -3 & 0 & 2 & -3 & 0 \\ 1 & 2 & -1 & 1 & 2 \end{array}\right] = 2\vec{j} - 6\vec{k} - 4\vec{i} - 3\vec{j} = (-4,-1,-6)$$

Equação vetorial: $X = (1,-1,2) + \lambda(-4,-1,-6)$; $\lambda \in \mathbb{R}$

Equação paramétrica: $x = 1 - 4\lambda$; $y = -1 - \lambda$; $z = 2 - 6\lambda$; $\lambda \in \mathbb{R}$

Equação simétrica: $\frac{x-1}{4} = \frac{y+1}{-1} = \frac{z-2}{+6}$

b) $A=(-2,0,1)$; $\vec{u}=(2,1,-1)$; $\vec{v}=(-1,0,2)$

$$\vec{u}\wedge\vec{v}=\left[\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 2 & 1 & -1 & 2 & 1 \\ -1 & 0 & 2 & -1 & 0 \end{array}\right] = 2\vec{i}+\vec{j}+\vec{k}-4\vec{j} = (2,-3,1)$$

Equação vetorial: $X=(-2,0,1)+\lambda(2,-3,1)$

Equação paramétrica: $x=-2+2\lambda$; $y=-3\lambda$; $z=1+\lambda$; $\lambda\in\mathbb{R}$

Equação simétrica: $\dfrac{x+2}{2}=\dfrac{-y}{3}=\dfrac{z-1}{1}$

c) $A=(0,0,0)$; $\vec{u}=(2,-1,0)$; $\vec{v}=(-3,1,0)$

$$\vec{u}\wedge\vec{v}=\left[\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 2 & -1 & 0 & 2 & -1 \\ -3 & 1 & 0 & -3 & 1 \end{array}\right] = 2\vec{k}-3\vec{k} = (0,0,-1)$$

Equação vetorial: $X=\lambda(0,0,-1)$; $\lambda\in\mathbb{R}$

Equação paramétrica: $x=0$; $y=0$; $z=-\lambda$; $\lambda\in\mathbb{R}$

Equação simétrica: $x=y=0$; $z=0$

d) $A=(0,0,0)$; $\vec{u}=(1,0,0)$; $\vec{v}=(0,1,0)$

$$\vec{u}\wedge\vec{v}=\left[\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right] = \vec{k} = (0,0,1)$$

Equação vetorial: $X=\lambda(0,0,1)$

Equação paramétrica: $x=0$; $y=0$, $z=\lambda$; $\lambda\in\mathbb{R}$

Equação simétrica: $x=y=0$

/ /

Exemplos: "Posição Relativas entre duas retas"

A) $r: X = (1,3,5) + \lambda(1,3,3)$ $\{\vec{u}, \vec{v}, \vec{AB}\}$

$s: X = (2,5,6) + \lambda'(2,1,0)$

$$\left|\begin{array}{ccc|cc} 1 & 3 & 3 & 1 & 3 \\ 2 & 1 & 0 & 2 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{array}\right| = 1 + 12 - 3 - 6 = 5, \text{ que } \neq 0$$

"As retas r e s são reversos"

B) $r: X = (1,1,-1) + \lambda(1,1,2)$

$s: X = (5,8,4) + \lambda'(2,5,1)$

$$\left|\begin{array}{ccc|cc} 1 & 1 & 2 & 1 & 1 \\ 2 & 5 & 1 & 2 & 5 \\ 4 & 7 & 5 & 4 & 7 \end{array}\right| = 25 + 4 + 28 - 40 - 7 - 10 = 57 - 57 = 0$$

$\vec{u} = m\vec{v} \therefore (1,1,2) = m(2,5,1) / m = \frac{1}{2} \neq \frac{1}{5} \neq \frac{1}{2}$ "As retas r e s são coplanares (concorrentes)"

c) $r: X = (1,1,-1) + \lambda(1,1,2)$

$s: X = (5,8,4) + \lambda(2,2,4)$

$$\left|\begin{array}{ccc|cc} 1 & 1 & 2 & 1 & 1 \\ 2 & 2 & 4 & 2 & 2 \\ 4 & 7 & 5 & 4 & 7 \end{array}\right| = 10 + 16 + 28 - 16 - 28 - 10 = 54 - 54 = 0$$

$\vec{u} = m\vec{v}$

$(1,1,2) = m(2,2,4)$ "As retas r e s são coplanares (paralelas)"

$m = \frac{1}{2} = \frac{1}{2} = \frac{2}{4}$

Exemplo: "Intersecção entre duas retas"

$r: X = (1,2,1) + \lambda(2,1,0)$

$s: X = (1,3,4) + \lambda'(0,1,3)$

$$r: \begin{cases} x = 1 + 2\lambda \\ y = 2 + \lambda \\ z = 1 \end{cases} ; \lambda \in \mathbb{R} \qquad s: \begin{cases} x = 1 \\ y = 3 + \lambda' \\ z = 4 + 3\lambda' \end{cases} ; \lambda \in \mathbb{R}$$

$$\begin{cases} 1 + 2\lambda = 1 & \lambda = 0 \\ 2 + \lambda = 3 + \lambda' & \\ 1 = 4 + 3\lambda' & \lambda' = -1 \end{cases}$$

Substituindo λ e λ' nas equações paramétricas obtemos: o ponto de interseção: é:

$T = (1,2,1)$

Exemplo: "Perpendicularismo entre duas retas"

a) $r: X = (1,2,1) + \lambda(1,1,1)$

$s: X = (2,1,0) + \lambda'(1,5,-6)$

$(1,1,1) \cdot (1,5,-6) = 1 + 5 - 6 = 0$, Portanto ele formo um ânguloreto.

$$\left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 5 & 6 & 1 & 5 \\ 1 & -1 & -1 & 1 & -1 \end{array}\right| = -5 + 6 - 1 - 5 + 6 + 1 = 2 \neq 0$$

, Portanto as retas são ortogonais reversas.

b) $r: X = (1,2,1) + \lambda(1,1,1)$

$s: X = (2,7,-5) + \lambda'(1,5,-6)$

$(1,1,1).(1,5,-6) = 1+5-6 = 0$ "Angulo reto"

$$\det[\vec{u}, \vec{v}, \vec{AB}] = \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 5 & -6 & 1 & 5 \\ 1 & 5 & -6 & 1 & 5 \end{array}\right| = -30-6+5-5+30+6 = 0.$$

Portanto as retas são verdadeiramente perpendicular.

$$\begin{cases} x = 1+\lambda \\ y = 2+\lambda \\ z = 1+\lambda \end{cases} \quad \begin{cases} x = 2+\lambda' \\ y = 7+5\lambda' \\ z = -5-6\lambda' \end{cases} \quad \begin{cases} 1+\lambda = 2+\lambda' \\ 2+\lambda = 7+5\lambda' \\ 1+\lambda = -5-6\lambda' \end{cases}$$

$4+3\lambda = 4$

$\lambda = 0$

$T = (1,2,1)$

Exercícios "Apostila"

(135-) 1. $r: X = (1,-1,2) + t(3,0,-2)$

$s: X = (2,1,0) + t'(2,-2,0)$

- São coplanares ✓
- São concorrentes ✓

$$\left|\begin{array}{ccc|cc} 3 & 0 & -2 & 3 & 0 \\ 2 & -2 & 0 & 2 & -2 \\ 1 & 2 & -2 & 1 & 2 \end{array}\right| = 12 - 8 - 4 = 0$$

$\vec{u} = m\vec{v}$

$(3,0,-2) = m(2,-2,0)$

$m = \frac{3}{2} \neq \frac{0}{-2} \neq \frac{-2}{0}$

$\begin{cases} x = 1 + 3t \\ y = -1 \\ z = 2 - 2t \end{cases}$ $\begin{cases} x = 2 + 2t' \\ y = 1 - 2t' \\ z = 0 \end{cases}$ $\begin{cases} 1 + 3t = 2 + 2t' \\ -1 = 1 - 2t' \\ 2 - 2t = 0 \end{cases}$

$t = 1$

$t' = 1$

$T = (4,-1,0)$

II. $r: X = (2,-1,3) + t(-3,1,-2)$

$s: X = (-1,0,2) + t'(6,-2,4)$

- São coplanares ✓
- São paralelos ✓

$$\left|\begin{array}{ccc|cc} -3 & 1 & -2 & -3 & 1 \\ 6 & -2 & 4 & 6 & -2 \\ -3 & 1 & -1 & -3 & 1 \end{array}\right| = -6 - 12 - 12 + 12 + 12 + 6 = 0$$

$\vec{u} = m\vec{v}$

$(-3,1,-2) = m(6,-2,4)$

$m = \frac{-3}{6} = \frac{1}{-2} = \frac{-2}{4}$

/ /

$\overset{A}{}$ $\overset{\vec{u}}{}$

III. $r: X = (-2,1,2) + t(-1,2,3)$

$s: X = (1,0,-1) + t'(0,1,4)$ — São reversos ✓

$$\begin{vmatrix} -1 & 2 & 3 \\ 0 & 1 & 4 \\ 3 & -1 & -3 \end{vmatrix}\begin{matrix} -1 & 2 \\ 0 & 1 \\ 3 & -1 \end{matrix} = 3+24-9-4+6 = 20 \neq 0$$

Reta r:

$\alpha = xOy = z = 0$ $\quad z = 2 + 3t \Rightarrow 0 = 2 + 3t \therefore t = -2/3$ $\quad r \cap \alpha = \left(\frac{-4}{3}, \frac{-1}{3}, 0\right)$

$x = -2 + (-1)(-2/3) \therefore x = -4/3$

$y = 1 + 2 \cdot (-2/3) \therefore y = -1/3$

$\beta = xOz = y = 0$ $\quad y = 1 + 2t \Rightarrow 0 = 1 + 2t \therefore t = -1/2$ $\quad r \cap \beta = (-3/2, 0, 1/2)$

$x = -2 - (-1/2) \therefore x = -3/2$

$z = 2 + 3(-1/2) \therefore z = 1/2$

$\gamma = yOz = x = 0$ $\quad x = -2 - t \Rightarrow 0 = -2 - t \therefore t = -2$ $\quad r \cap \gamma = (0, -3, -4)$

$y = 1 + 2(-2) \therefore y = -3$

$z = 2 + 3(-2) \therefore z = -4$

IV. $r: X = (2,-3,1) + t(1,1,-2)$; $t \in \mathbb{R}$

$s: X = (4,-1,-3) + t'(-2,-2,4)$; $t' \in \mathbb{R}$ · São coplanares ✓ · São paralelas ✓

$$\begin{vmatrix} 1 & 1 & -2 \\ -2 & -2 & 4 \\ 2 & 2 & -4 \end{vmatrix}\begin{matrix} 1 & 1 \\ -2 & -2 \\ 2 & 2 \end{matrix} = \cancel{8} + \cancel{8} + \cancel{8} - \cancel{8} - \cancel{8} - \cancel{8} = 0$$

$\vec{u} = m\vec{v}$

$(1,1,-2) = m(-2,-2,4)$

$m = \frac{1}{-2} = \frac{1}{-2} = \frac{-2}{4}$

(137-) $C=(2,1,-1)$ $D=(4,-5,3)$ $A=(3,-2,1)$ $B=(-1,2,0)$

$r: X=(2,1,-1)+m(2,-6,4)$

A-

$(3,-2,1)=(2,1,-1)+m(2,-6,4)$

$m(2,-6,4)=(1,-3,2)$

$m=\frac{1}{2}=\frac{-3}{-6}=\frac{2}{4}$; Portanto, o ponto A pertence a esta reta

B-

$(-1,2,0)=(2,1,-1)+m(2,-6,4)$

$m(2,-6,4)=(-3,1,1)$

$m=\frac{-3}{2}\neq\frac{1}{-6}\neq\frac{1}{4}$; Portanto, o ponto B não pertence a esta reta

(138-) $P=(-1,1,3)$ $Q=(4,-2,1)$

$r: X=(-1,1,3)+m(5,-3,-2)$

$$\begin{cases} x=-1+5m \\ y=1-3m \\ z=3-2m \end{cases}$$

$xOy \therefore z=0$

$0=3-2m=3/2$

$x=-1+5(3/2)=13/2$

$y=1-3(3/2)=-7/2$

$r\cap xOy\equiv\left(\frac{13}{2},\frac{-7}{2},0\right)$

$xOz \therefore y=0$

$0=1-3m=1/3$

$x=-1+5(1/3)=2/3$

$y=1-3(1/3)=0$

$z=3-2(1/3)=7/3$

$r\cap xOz\equiv(2/3,0,7/3)$

$yOz \therefore x=0$

$0=-1+5m=1/5$

$y=1-3(1/5)=2/5$

$z=3-2(1/5)=13/5$

$r\cap xOz=(0,2/5,13/5)$

(/ /)

(139.) $A=(2,-1,4)$ $B=r_1 \cap r_2$

$r_1: \dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-1}{-2}$ $r_2:$ $x=2t$

$y=1+2t$

$z=2+t$

$r_1 = X=(1,3,1)+\lambda(2,4,-2)$

$x=1+2t'$ $1+2t'=2t$ $t=\dfrac{1+2t'}{2}$

$y=3+4t'$ $3+4t'=1+2t$

$z=1-2t'$ $1-2t'=2+t$ $3+4t'=1+2\left(\dfrac{1+2t'}{2}\right)$

$3+4t'=1+\dfrac{2+4t'}{2}$

$B=(0,1,2)$ $1+2t'=2t$ $3+4t'=\dfrac{2+2+4t'}{2}$ $1+2t'$

$1+2.(-1/2)=2t$ $3+4t'=2+2t'$ $2\,2t^2$

$s: X=(2,-1,4)+m(-2,2,-2); m\in\mathbb{R}$ $0=2t$ $2t'=-1$

$t=0$ $t'=-1/2$

(140.) $A=(2,-1,3)$ // $r=$ $P=(3,1,4)$ $Q=(5,3,4)$

Equação vetorial:

$r: X=(2,-1,3)+\lambda(2,2,0); \lambda\in\mathbb{R}$

Equação paramétrica

$x=2+2\lambda; y=-1+2\lambda; z=3$; $\lambda\in\mathbb{R}$

Equação simétrica

$\dfrac{x-2}{2} = \dfrac{y+1}{2}$; e $z=3$

(141-) $A=(4,-3,2)$ $B=(5,-4,4)$

$$\left(p,q,\frac{7}{2}\right)=(4,-3,2)+\lambda(1,-1,2) \ ; \ \lambda \in R$$

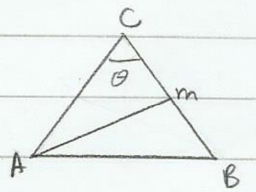
$p = 4+\lambda$ $\quad p = 19/4$

$q = -3-\lambda$ $\quad q = -15/4$

$\frac{7}{2} = 2+2\lambda$ $\quad \lambda = 3/4$

$$p = \frac{19}{4} \quad q = \frac{-15}{4}$$

(142-) $A=(4,3,-2)$ $B=(5,5,-1)$ $C=(6,4,-3)$



m: $(4,3,-2)+t(3/2, 3/2, 0)$

$x = 4+\frac{3}{2}t$ $\quad y = 3+\frac{3}{2}t$ $\quad z=-2$

$CM = MB$

$M_x - M_C = M_b - M_x$

$M_x = \frac{M_b + M_c}{2}$

$m = (11/2, 9/2, -2)$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \frac{(-2,-1,1).(-1,1,2)}{\sqrt{6}\,.\,\sqrt{6}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2} \text{ ou } 60^\circ$$

(/ /)

(143.)

r: $x = 3 + t$; s: $x = 5 + \frac{1}{2}t'$;

$y = -4 - 2t$; $y = -8 - t'$;

$z = 5 - t$; $z = 3 - \frac{1}{2}t'$;

r: $X = (3, -4, 5) + t(1, -2, -1)$

s: $X = (5, -8, 3) + t'(\frac{1}{2}, -1, -\frac{1}{2})$

$$\left|\begin{array}{ccc|cc} 1 & -2 & -1 & 1 & -2 \\ \frac{1}{2} & -1 & -\frac{1}{2} & \frac{1}{2} & -1 \\ 2 & -4 & -2 & 2 & -4 \end{array}\right| = 2 + 2 + 2 - 2 - 2 - 2 = 0$$

$\vec{u} = m\vec{v}$

$(1, -2, -1) = m(\frac{1}{2}, -1, -\frac{1}{2})$

$m = 2 = 2 = 2$

Portanto, são:

- coplanares
- paralelas

$r \cap s =$

$$\begin{cases} x = 3 + t \\ y = -4 - 2t \\ z = 5 - t \end{cases} \quad \begin{cases} x = 5 + \frac{1}{2}t' \\ y = -8 - t' \\ z = 3 - t' \end{cases}$$

$3 + t = 5 + \frac{1}{2}t'$

$-4 - 2t = -8 - t'$

$5 - t = 3 - t'$

$t = 2 + t'$

$-4 - 2t = -8 - t'$

$-4 - 2(2 + t') = -8 - t'$

$-4 - 4 - 2t' = -8 - t'$

$-2t' = -t'$

$-t' = 0$

$5 - t = 3 - t'$

$5 - t = 3 - 0$

$-t = -2$

$t = 2$

Obs.: As retas não se intercepta

/ /

(144.) $r: x = (K, -1, 2) + t(2, -1, 1)$

equações paramétricas

$x = K + 2t$

$y = -1 - t$

$z = 2 + t$

$s: \frac{x-6}{1} = \frac{y}{2} = \frac{z}{-3} = t$

$x = 6 + t'$

$y = 2t'$

$z = -3t'$

$s: x = (6,0,0) + t(1,2,-3)$

$K + 2t = 6 + t'$

$-1 - t = 2t'$

$2 + t = -3t'$

$t = -2t' - 1$

$2 - 2t' - 1 = -3t'$

$-2t + 1 = -3t'$

$1 = -t'$

$t' = -1$

$2 + t = -3t'$

$2 + t = -3 \cdot (-1)$

$2 + t = 3$

$t = 1$

$K + 2t = 6 + t'$

$K + 2 = 5$

$\underline{K = 3}$ $\qquad \underline{P = (5, -2, 3)}$

(145.) $A = (4, -3, 2)$ // $s: x = 1 + 3\lambda;$ $\quad P = (m, n, -5)$

$y = 2 - 4\lambda;$

$z = 3 - \lambda$

$r: x = (4, -3, 2) + \lambda'(m-4, n+3, -7)$

$s: x = (1, 2, 3) + \lambda(3, -4, -1)$

$\vec{u} = \alpha \vec{v}$

$(m-4, n+3, -7) = \alpha(3, -4, -1)$

$\alpha = \frac{m-4}{3} = \frac{n+3}{-4} = \frac{-7}{-1} = 7$

$\frac{m-4}{3} = 7 \Rightarrow m - 4 = 21$ $\qquad \frac{n+3}{-4} = 7 \Rightarrow n + 3 = -28$

$\underline{m = 25}$ $\qquad \underline{n = -31}$

spiral®

(146.) $A=(2,4,0)$ $B=(0,0,4)$ $C=(0,3,0)$ $D=(2,3,0)$ $E=(2,0,0)$

a) A e B

$r: X=(2,4,0)+\lambda(-2,-4,4)$ (Diferente do livro)

$x=2-2\lambda;\ y=4-4\lambda;\ z=0\ ;\ \lambda\in\mathbb{R}$

b) C e D.

$r: X=(0,3,0)+\lambda(2,0,0)$

$x=2\lambda;\ y=3;\ z=0\ ;\ \lambda\in\mathbb{R}$

c) A e D

$A=(2,4,0)$ $D=(2,3,0)$ (Diferente do livro)

$(x,y,z)=(2,4,0)+t(0,-1,0)$

$x=2;\ y=4-t;\ z=0;\ t\in\mathbb{R}$

ou

$(x,y,z)=(2,3,0)+t(0,1,0);$

$x=2;\ y=3+t;\ z=0$

d) $B=(0,0,4)$ $C=(0,3,0)$

$(x,y,z)=(0,0,4)+t(0,3,-4)$

$x=0;\ y=3t;\ z=4-4t\ ;\ t\in\mathbb{R}$

e) $D=(2,3,0)$ $E=(2,0,0)$

$(x,y,z)=(2,3,0)+t(0,-3,0)$

$x=2\ ;\ y=3-3t;\ z=0\ ;\ t\in\mathbb{R}$

f) $B=(0,0,4)$, $D=(2,3,0)$

$(x,y,z)=(0,0,4)+t(2,3,-4)$

$x=2t;\ y=3t;\ z=4-4t;\ t\in\mathbb{R}$

(147-) $P=(m,1,n)$ $A=(3,-1,4)$ $B=(4,-3,-1)$

$r: (x,y,z) = (3,-1,4) + t(1,-2,-5)$

$(m,1,n) = (3,-1,4) + t(1,-2,-5)$

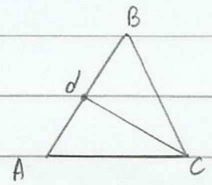
$1 = -1-2t$ $\quad m = 3+t$ $\quad n = 4-5t$

$-2t = 2$ $\quad m = 3-1$ $\quad m = 4+5$

$t = -1$ $\quad m = 2$ $\quad m = 9$

$P = (\underline{2,1,9})$

(148-) $A=(-1,4,-2)$ $B=(3,-3,6)$ $C=(2,-1,4)$



$\vec{Ad} = dB$ $\quad X_d = 1$ $\quad d = (1, 1/2, 2)$

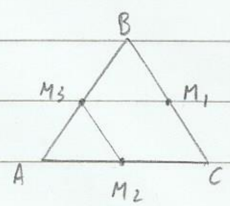
$X_d - X_a = X_b - X_d$ $\quad Y_d = 1/2$

$X_d \cdot \frac{X_b + X_a}{2}$ $\quad Z_d = 2$

$r: (x,y,z) = (2,-1,4) + t(1, \frac{-3}{2}, 2)$

$x = 2+t;\ y = -1 - \frac{3}{2}t;\ z = 4+2t;\ t \in \mathbb{R}$

(149-) $M_1 = (2,-1,3)$ $M_2 = (1,-3,0)$ $M_3 = (2,1,-5)$ $\quad (1,4,-5)$



$m_1: (x,y,z) = (2,-1,3) + t(1,4,-5)$

$x = 2+t;\ y = -1+4t;\ z = 3-5t^3;\ t \in K$

? (Mas Por que?)

(150-)

A) $r_1: \begin{cases} x = -3 + 2at \\ y = 1 + 3t \\ z = -1 + t \end{cases}$ $r_2: \begin{cases} x = -1 + 2t' \\ y = t' \\ z = 4 - t' \end{cases}$

$r_1: (x, y, z) = (-3, 1, -1) + t(2a, 3, 1)$

$r_2: (x, y, z) = (-1, 0, 4) + t'(2, 1, -1)$

$\vec{u} \cdot \vec{v} = 0$

$(2a, 3, 1) \cdot (2, 1, -1) = 0$

$4a + 3 - 1 = 0$

$4a = -2$

$a = -1/2$

B) $r_1: \begin{cases} x = t \\ y = 3 + at \\ z = -1 + t \end{cases}$ $r_2: A = (1, 0, a) \quad B = (-2, 2a, 2a)$

$r_1: (x, y, z) = (0, 3, -1) + t(1, a, 1)$

$r_2: (x, y, z) = (1, 0, a) + t'(-3, 2a, a)$

$\vec{u} \cdot \vec{v} = 0$

$(1, a, 1) \cdot (-3, 2a, a) = 0$

$-3 + 2a^2 + a = 0$

$2a^2 + a - 3 = 0$

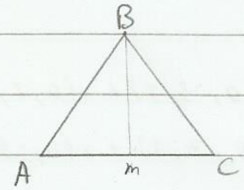
$a = \dfrac{-1 \pm \sqrt{1 + 24}}{4}$

$a_1 = 1$

$a_2 = -3/2$

$a = 1$ ou $-\dfrac{3}{2}$

(151.) $A=(-1,1,3)$ $B=(2,1,4)$ $C=(3,-1,-1)$



$\underline{AB}$: $(x,y,z)=(-1,1,3)+t(3,0,1)$

$x=-1+3t;\ y=1;\ z=3+t;\ t\in\mathbb{R}$

$\underline{AC}$: $(x,y,z)=(-1,1,3)+t(4,-2,-4)$

$x=-1+4t;\ y=1-2t;\ z=3-4t;\ t\in\mathbb{R}$

$m=(1,0,1)$

$\underline{BC}$: $(x,y,z)=(2,1,4)+t(1,-2,-5)$

$x=2+t;\ y=1-2t;\ z=4-5t;\ t\in\mathbb{R}$

Ponto ou Vetor

$r: (x,y,z)=(2,1,4)+t(1,0,1)$

$x=2+t;\ y=1;\ z=4+t;\ t\in\mathbb{R}$

(152.) (A) $r_1: (3,-1,0)+m(0,0,1)$ $t=3$

$$\begin{cases} x=t \\ y=t-3 \\ z=-2t+3 \end{cases}$$

$r_2: (0,-3,3)+t(1,1,-2)$ $+t$

vetorial

B) $r_1: (x,y,z)=(0,0,3)+t(2,1,2)$ $(2,6,-5)$

$r_2: (x,y,z)=(0,1,2)+t'(3,-1,0)$

$2a+b+2c=0$ $\Rightarrow$ $5a+2c=0$ $3a-b=0$ $b=\dfrac{-6c}{5}$

$3a+(-b)=0$ $a=\dfrac{-2c}{5}$ $\dfrac{-6c}{5}-b=0$

$a=1$	2
$b=3$	6
$c=-5/2$	-5

$(x,y,z)=(0,0,0)+t(2,6,-5)$

$x=2t;\ y=6t;\ z=-5t;\ t\in\mathbb{R}$

(C) $r_1: X=(2,-1,0)+t(2,1,2)$ $2+2t=1-m$ $t=-1+m$

$r_2: X=(1,1,-2)+m(-1,-1,2)$ $-1+t=1-m$ $-1-1+m=1-m$

$2t=-2+2m$ $-2-1=-2m$ $m=3/2$

$x=(1/2,-1/2,-3)+$

spiral

(153-) A)

r1: $(x,y,z) = (0,-3,5) + x(0,2,-1)$

r2: $(x,y,z) = (0,7,1) + x(0,-3,1)$

$$\begin{vmatrix} 0 & 2 & -1 \\ 0 & -3 & 1 \\ 0 & 10 & -4 \end{vmatrix} = 0$$

- São coplanares
- São concorrentes

$-3+2x = 7-3x$

$5-x = 1+x$

$5-2x = 1$

$-2x = -4$

$x = 2$

R= $(2,1,3)$

B) r1: $(x,y,z) = (3,-1,2) + t(2,-3,4)$

r2: $(x,y,z) = (-1,4,-8) + t'(1,-1,3)$

$3+2t = -1+t'$

$-1-3t = 4-t'$

$2+4t = -8+3t'$

$t' = 2t+4$

$-1-3t = 4-2t-4$

$t = -1$

$-1-3t = 4-t'$

$-1-3\cdot -1 = 4-t'$

$2 = 4-t'$

$t' = 2$

R= $(1,2,-2)$

c) r1: $(x,y,z) = (0,-3,-10) + x(0,2,-1)$

r2: $(x,y,z) = (4,4,-1) + t(3,3,-2)$

$$\begin{array}{ccc|cc} 0 & 2 & -1 & 0 & 2 \\ 3 & 3 & -2 & 3 & 3 \\ 4 & 7 & 9 & 4 & 7 \end{array} = -16-21+12-54 = -79 \neq 0$$

Portanto são reversos, e não são concorrentes.

D) $r1: X=(2,3,6)+t(-1,-5,-6)$ — São coplanares

$r2: X=-3+6h;\ y=1+7h;\ z=-1+13h$ — São concorrentes

$r2: (x,y,z)=(-3,1,-1)+h(6,7,13)$

$$\begin{vmatrix} -1 & -5 & -6 \\ 6 & 7 & 13 \\ -5 & -2 & -7 \end{vmatrix}\begin{matrix} -1 & -5 \\ 6 & 7 \\ -5 & -2 \end{matrix} = 49+325+72-210-26-210=0$$

$\vec{u}=m\vec{v}$

$(-1,-5,-6)=m(6,7,13)$

$m=\frac{-1}{6}\neq\frac{-5}{7}\neq\frac{-6}{13}$

$2-t=-3+6h$

$3-5t=1+7h$

$6-6t=-1+13h$

$t=5-6h$

$3-5(5-6h)=1+7h$

$3-25+30h=1+7h$

$-22+30h=1+7h$

$23h=23$

$h=1$

$t=5-6h$

$t=5-6.1$

$t=-1$

$6-6t=-1+13h$

$6-6(-1)=-1+13.1$

$12=12$

V

$T=(3,8,12)$

E) $r1: (x,y,z)=(2,4,1)+t(1,-2,3)$

$r_2: (x,y,z)=(-1,2,5)+h(4,3,-2)$

$$\begin{vmatrix} 1 & -2 & 3 \\ 4 & 3 & -2 \\ -3 & -2 & 4 \end{vmatrix}\begin{matrix} 1 & -2 \\ 4 & 3 \\ -3 & -2 \end{matrix} = 12-12-24+27-4+32=31\neq0$$

Portanto são reversos

(154-) $r1: y = 2x - 5 \; ; \; z = -x + 2$

$r2: x - 5 = \frac{y}{m} = z + 1$

$r1: (x, y, z) = ("0", -5, 2) + x("0", 2, -1)$

$r2: (x, y, z) = (5, 0, -1) + t(1, m, 1)$

$$\left|\begin{matrix} 0 & 2 & -1 \\ 1 & m & 1 \\ 5 & 5 & -3 \end{matrix}\right| \begin{matrix} 0 & 2 \\ 1 & m \\ 5 & 5 \end{matrix} = 10 - 5 + 5m + 6 = 5m + 11 = 0$$

$5m = -11$

$m = \frac{-11}{5}$

B) $r1: x = m - t \; ; \; y = 1 + t \; ; \; z = 2t$ $\quad r1: (x, y, z) = (m, 1, 0) + t(-1, 1, 2)$

$r2: \frac{x-1}{2} = \frac{y+2}{1} = \frac{z}{-2}$ $\quad (x, y, z) = (1, -1, 0) + t'(2, 1, -2)$

$$\left|\begin{matrix} -1 & 1 & 2 \\ 2 & 1 & -2 \\ 1-m & -2 & 0 \end{matrix}\right| \begin{matrix} -1 & 1 \\ 2 & 1 \\ 1-m & -2 \end{matrix} = -2 + 2m - 8 - 2 + 2m + 4 = 4m - 8 = 0$$

$4m = 8$

$m = 2$

$-2(1 - m) - 8 \quad -2(1 - m) - 4$

$-2 + 2m - 8 - 2 + 2m - 4$

(155-)

Equação vetorial:

$r: (x,y,z) = (3,2,-4) + t(8,0,5)$

$r: X = (3,-1,2) + t(8,0,-5)$

Equação simétrica

$\frac{x-3}{8} = \frac{z-2}{-5}\ ;\ y = -1$

Equação paramétrica

$x = 3+8t\ ;\ y = -1\ ;\ z = 2-5t\ ;\ t \in \mathbb{R}$

(156-) $r: X = (3,-1,2) + t(2,-1,1)$

$s: x-6\ ;\ \frac{y}{2} = \frac{z}{-3}$

$r: (x,y,z) = (3,-1,2) + t(2,-1,1)$ - São coplanar

$s: (x,y,z) = (6,0,0) + t'(1,2,-3)$ - São concorrentes

$$\left|\begin{matrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & 1 & -2 \end{matrix}\right| \begin{matrix} 2 & -1 \\ 1 & 2 \\ 3 & 1 \end{matrix} = -8+9+1-6+6-2 = 0$$

$m = \frac{2}{1} \neq \frac{-1}{2} \neq \frac{1}{-3}$

$3+2t = 6+t'$

$-1-t = 2t'$

$2+t = -3t'$

$t' = 2t-3$

$-1-t = 2(2t-3)$

$-1-t = 4t-6$

$-1-5t = -6$

$-5t = -5$

$t = 1$

$-1-t = 2t'$

$-1-1 = 2t'$

$-2 = 2t'$

$t' = -1$

$T = (5,-2,3)$

/ /

(157-) r: $X = (1,-2,1) + t(1,3,3)$; $t \in \mathbb{R}$

s: $\frac{2-x}{2} = \frac{1-y}{6} = \frac{-z}{6}$

r: $(x,y,z) = (1,-2,1) + t(1,3,3)$ · São coplanares

s: $(x,y,z) = (2,1,0) + t'(-2,-6,-6)$ · São Paralelas

$$\begin{vmatrix} 1 & 3 & 3 \\ -2 & -6 & -6 \\ 1 & 3 & -1 \end{vmatrix} \begin{matrix} 1 & 3 \\ -2 & -6 \\ 1 & 3 \end{matrix} = \cancel{6} - \cancel{18} - \cancel{18} + \cancel{18} + \cancel{18} - \cancel{6} = 0$$

$$m = \frac{1}{-2} = \frac{3}{-6} = \frac{3}{-6}$$

(158-) r: $x-3 = y-2 = z+1$

r: $(x,y,z) = (3,2,-1)$

Obs.: "Esta reta é apenas um ponto no espaço, portanto, qualquer seja o outro vetor ela será ainda uma reta concorrente, exceto se for proporcional as cordenadas".

$$(x,y,z) = (3,2,-1) + t(9,8,7)$$

(159-) $a + 5b + c = 0$ (-2) $\quad -2a - 10b - 2c = 0$ $\quad -13b + c = 0$

$2a - 3b + 3c = 0$ $\quad 2a - 3b + 3c$ $\quad c = 13b$ $\quad b = \frac{c}{13}$

$a = -13b - 5b$

$a = -18b$

$a = -18$ $\quad (x,y,z) = (3,-4,1) + t(-18,1,13)$

$b = 1$ $\quad \frac{3-x}{18} = y+4 = \frac{z-1}{13}$

$c = 13$ } São iguais

$\frac{-3+x}{18} = -y-4 = \frac{-z+1}{13}$ (Livro)

/ /

(160-) r1: $X=(1,3,1)+t(2,4,-2);\ t\in\mathbb{R}$

r2: $X=(0,1,2)+t'(2,2,1);\ t'\in\mathbb{R}$

$1+2t=2t'$

$3+4t=1+2t'$

$1-2t=2+t'$

$t=\frac{2t'-1}{2}$

$3+\frac{8t'-4}{2}=1+2t'$

$3+4t'-2=1+2t'$

$4t'+1=1+2t'$

$2t'=0$

$t'=0$

$3+4t=1+2t'$

$3+4t=1+2\cdot 0$

$4t=-2$

$t=-1/2$

$T=(0,1,2)$

$X=(2,-1,4)+t(-2,2,-2)$

(161-) r1: $X=(1,3,1)+t(2,4,-2)$

A) r2: $X=(0,1,2)+t'(2,2,1)$

$1+2t=2t'$

$3+4t=1+2t'$

$1-2t=2+t'$

$t'=-2t-1$

$3+4t=1+2(-2t-1)$

$3+4t=1-4t-2$

$3+4t=-4t-1$

$8t=-4$

$t=-1/2$

$3+4t=1+2t'$

$3+4(-1/2)=1+2t'$

$3+(-2)=1+2t'$

$2t'=0$

$t'=0$

$T=(0,1,2)\neq(3,-7,-1)$

O ponto P não intercepta

/ /

B) $r1: X=(0,15,0)+t(1,4,3)$

$r2: X=(1,-8,-12)+h(2,-1,1)$

$t = 1+2h$ $\quad$ $3(1+2h) = -12+h$ $\quad$ $3t = -12+h$

$15+4t = -8-h$ $\quad$ $3+6h = -12+h$ $\quad$ $3t = -12+(-3)$

$3t = -12+h$ $\quad$ $5h = -15$ $\quad$ $3t = -15$

$h = -3$ $\quad$ $t = -5$

$T = (-5,-5,-15) = P = (-5,-5,-15)$

Portanto o ponto $P = r1 \cap r2$

C) $r1: X=(1,1,7)+t(1,0,2)$

$r2: X=(6,-1,1)+h(4,-1,0)$

$1+t = 6+4h$ $\quad$ $h = -2$ $\quad$ $1+t = 6+4h$

$1 = -1-h$ $\quad$ $1+t = 6+4(-2)$ $\quad$ $1-3 = 6+4h$

$7+2t = 1$ $\quad$ $1+t = 6-8$ $\quad$ $h = \frac{-8}{4}$

$t = -3$ $\quad$ $h = -2$

$T = (-2,1,1) = P = (-2,1,1)$

162

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(163-) r: $x + y - z + 1 = 0$ (I)

$2x - y + 3z + 2 = 0$ (II)

$\alpha + m\beta = 0$

$x + y - z + 1 + m(2x - y + 3z + 2) = 0$

$x + y - z + 1 + 2xm - ym + 3zm + 2m = 0$

$x(1+2m) + y(1-m) + z(-1+3m) + 1 + 2m = 0$

(A) // Ox: $1 + 2m = 0$ $\quad \frac{3}{2}y - \frac{5}{2}z = 0$

$m = -\frac{1}{2}$

$3y - 5z = 0$

(B) // Oy: $1 - m = 0$ $\quad 3x + 2z + 3 = 0$

$m = 1$

(C) // Oz: $-1 + 3m = 0$ $\quad \frac{5}{3}x + \frac{2}{3}y + \frac{5}{3} = 0$

$m = \frac{1}{3}$

$5x + 2y + 5 = 0$

(164-) $A = (1, 1, 2)$

r: $x - z - 1 = 0$

$y - 2z + 1 = 0$

$x - z - 1 + m(y - 2z + 1) = 0$

$x - z - 1 + my - 2zm + m = 0$

$x + my + z(-1 - 2m) - 1 + m = 0$

$\rightarrow 1 + m - 2 - 4m - 1 + m = 0$

$-2m = +2$

$m = -\underline{1}$

$\alpha = x - y + z - 2 = 0$

spiral®

/ /

(165-) $A=(1,1,3)$

$B=(2,-1,0)$

$C=(1,1,1)$

Equação vetorial

$X=(1,1,3)+m(1,-2,-3)+n(0,0,-2)$

Equação paramétrica

$x=1+m$

$y=1-2m$

$z=3-3m-2n$

Equação geral

$$\begin{vmatrix} x-1 & y-1 & z-3 \\ 1 & -2 & -3 \\ 0 & 0 & -2 \end{vmatrix} = 0$$

$4(x-1)+2(y-1)=0$

$2(x-1)+(y-1)=0$

$2x+y-2=0$

(166-) $A=(1,3,-4)$

$r: X=(1,-5,6)+\lambda(0,1,-2)$

Equação vetorial:

$X=(1,3,-4)+\lambda(0,-8,10)+\rho(0,1,-2)$; $\lambda,\rho \in \mathbb{R}$

Equação paramétrica

$x=1$; $y=3-8\lambda+\rho$; $z=-4+10\lambda-2\rho$; $\lambda,\rho \in \mathbb{R}$

Equação Geral

$$\begin{vmatrix} x-1 & y-3 & z+4 \\ 0 & -8 & 10 \\ 0 & 1 & -2 \end{vmatrix} = 0$$

$6(x-1)=0$

$6x-6=0$

$x=\frac{6}{6}=1$

$\underline{x=1}$

spiral

(167-) $A = (1,-2,1)$

$\vec{w}_\pi = (2,-1,0)$

$2x - y + d = 0$

$x = 1 \quad y = -2$

$2.1 - (-2) + d = 0$

$2 + 2 + d = 0$

$4 + d = 0$

$d = -4$

$\pi: 2x - y - 4 = 0$

(168-) $r: X = (1,-1,2) + \lambda(-2,2,4)$

$s: X = (3,1,0) + \lambda'(1,-1,-2)$

Equação Vetorial

$X = (1,-1,2) + \lambda(2,2,-2) + \lambda'(1,-1,-2)$

Equações Paramétricas

$x = 1 + 2\lambda + \lambda'$

$y = -1 + 2\lambda - \lambda'$

$z = 2 + 2\lambda - 2\lambda'$

Equação Geral

$$\begin{vmatrix} x-1 & y+1 & z-2 \\ 2 & 2 & -2 \\ 1 & -1 & -2 \end{vmatrix} = 0$$

$-6(x-1) + 2(y+1) - 4(z-2) = 0$

$-3(x-1) + (y+1) - 2(z-2) = 0$

$-3x + 3 + y + 1 - 2z + 4 = 0$

$\pi: -3x + y - 2z + 8 = 0$

ou

$3x - y + 2z - 8 = 0$

/ /

(169-) $\pi: x+y-z-1=0$ $\quad \vec{w}_\pi = (1,1,-1)$

$A=(1,0,0)$

$B=(0,1,0)$

$C=(0,0,-1)$

Equação vetorial

$$X=(1,0,0)+\lambda(-1,1,0)+\rho(-1,0,-1)\ ;\ \lambda,\rho\in\mathbb{R}$$

Equação paramétrica

$$X:\begin{cases} x=1-\lambda-\rho \\ y=\lambda \\ z=-\rho \end{cases}$$

(170-) $r: X=(1,-2,1)+\lambda(1,3,3)$

$s: X=(2,1,0)+\lambda'(1,3,-1)$

Equação vetorial

$$X=(1,-2,1)+\lambda(1,3,3)+\lambda'(1,3,-1)\ ;\ \lambda,\lambda'\in\mathbb{R}$$

Equação paramétrica

$x=1+1\lambda+\lambda'$

$y=-2+3\lambda+3\lambda'$

$z=1+3\lambda-\lambda'$

Equação Geral

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & 3 & 3 \\ 1 & 3 & -1 \end{vmatrix}=0$$

$-12(x-1)+4(y+2)=0 \quad (\div 4)$

$-3(x-1)+(y+2)=0$

$-3x+y+5=0$ ou $3x-y-5=0$

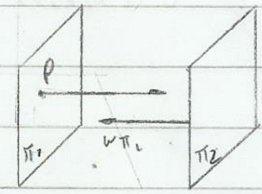
(171-) π: $P=(1,1,2)$ $\quad \pi // \pi_1$ $\quad A=(0,1,0)$

$\pi_1: x-y+2z+1=0 \quad w_{\pi_1}=(1,-1,2)$ $\quad B=(-1,0,0)$

$X=(1,1,2)+\lambda(-1,0,-2)+\rho(-2,-1,-2)$

$$\begin{vmatrix} x-1 & y-1 & z-2 \\ -1 & 0 & -2 \\ -2 & -1 & -2 \end{vmatrix} \quad -2(x-1)+2(y-1)+(z-2)=0$$

$-2x+2y+z-2=0$



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(172-) $A=(1,3,-4)$ $\quad \pi \perp \alpha$

$\alpha: 2x-y+z+1=0 \quad \vec{W}_\alpha=(2,-1,1)$

$X: (1,3,-4)+t(2,-1,1)$

$\frac{x-1}{2}=3-y=z+4$

(173-) π: $X=(1,0,0)+\lambda_1(-1,1,1)+\lambda_2(-1,1,0)$

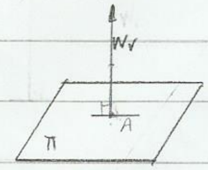
$A=(0,0,0)$

$$\begin{vmatrix} x-1 & y & z \\ -1 & +1 & 1 \\ -1 & 1 & 0 \end{vmatrix} = \quad -1(x-1)-1(y)+2z=0$$

$-x-y+2z+1=0$

$(-(1,1,-2)$

$X=(0,0,0)+t(1,1,-2)$



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(174-) π: $A=(1,1,1)$ $\quad X: (1,1,1)+\lambda(0,-1,0)+\rho(0,1,2)$

(A) $B=(1,0,1)$

$C=(1,2,3)$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 0$$

$-2(x-1)=0$

$-2x+2=0$

$-x+1=0$ ou $x-1=0$

$\vec{w}_\pi (1,0,0)$.

(B) π: $X=(1,2,0)+\alpha(1,-1,1)+\beta(0,1,-2)$

$$\begin{vmatrix} x-1 & y-2 & z \\ 1 & -1 & 1 \\ 0 & 1 & -2 \end{vmatrix} = 0$$

$(x-1)+2(y-2)+z=0$

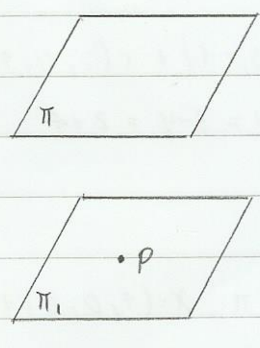
$x+2y+z-5=0$

$\vec{w}_\pi(1,2,1)$

(C) $\pi // \pi_1$

π_1: $x-2y+4z+1=0$ e $\supset P=(2,-1,1)$

$\vec{W}_\pi=(1,-2,4)$



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(175-) $\pi: x + 3y - 4z + 8 = 0$

$A = (-1, 7, z)$

$B = (11, -5, 1)$

$C = (1/2, -4, 7)$

$D = (3, -3, 4)$

$E = (3/2, -1/2, 2)$

(A-) $z = ?$ $A \in \pi$

$x + 3y - 4z + 8 = 0$

$-1 + (3.7) - 4z + 8 = 0$

$-1 + 21 - 4z + 8 = 0$

$-4z = -28$

$z = 7$

(B-) $x + 3y - 4z + 8 = 0$

$11 + (-5.3) - (4.1) + 8 = 0$

$11 - 15 - 4 + 8 = 0$

$x + 3y - 4z + 8 = 0$

$1/2 - 12 - 28 + 8 = 31,5 \neq 0$

$x + 3y - 4z + 8 = 0$

$3 - 9 - 16 + 8 = -14 = 0$

$x + 3y - 4z + 8 = 0$

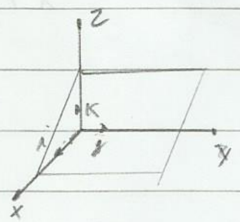
$3/2 - 3/2 - 8 + 8 = 0$

Pontos pertencentes: B, E

Pontos não pertence: C, D

(176-) $X = (3, 0, -2) + t(4, -1, 1)$

$A = (x, 0, 1)$



(177-) α: $x = 2 + t_1 + 3t_2$
$y = -1 - 2t_1$
$z = 1 + t_1 + 5t_2$

$\alpha: X = (2,-1,1) + t_1(1,-2,1) + t_2(3,0,5)$

β: $x = 4 + 2t'_1 + t'_2$
$y = 1 - t'_1 + t'_2$
$z = 5 + 3t'_1 + 2t'_2$

$\beta: X = (4,1,5) + t'_1(2,-1,3) + t'_2(1,1,2)$

$$\begin{vmatrix} x-2 & y+1 & z-1 \\ 1 & -2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$-5(x-2) - 1(y+1) + 3(z-1) = 0$

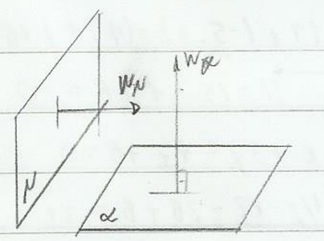
$-5x - y + 3z + 6 = 0$

00

$5x + y - 3z - 6 = 0$

(178-) $[A=(1,1,-3) \quad B=(1,-3,1)] \perp$ $\alpha: 2x - y - z - 6 = 0$

$r: X = (1,1,-3) + t(0,-4,4)$ $\quad \vec{w}_\alpha = (2,-1,-1)$



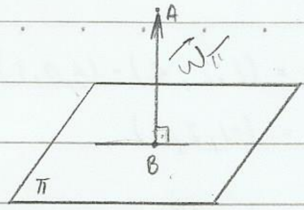
$$\begin{vmatrix} x-1 & y-1 & z+3 \\ 0 & -4 & 4 \\ 2 & -1 & -1 \end{vmatrix} = 0$$

$8(x-1) + 8(y-1) + 8(z+3) = 0$

$(x-1) + (y-1) + (z+3) = 0$

$x + y + z + 1 = 0$

(179-)



$\vec{BA} = (A-B) = (8,0,1) - (5,-2,3) = (3,2,-2)$

a, b, c

$3x + 2y - 2z + d = 0$

$3.5 + 2.-2 + -2.3 + d = 0$

$15 - 4 - 6 + d = 0$

$d = -5$

$3x + 2y - 2z - 5 = 0$

(180-) $A=(2,0,0)$ $B=(0,-4,1)$ $C=(4,4,1)$

$X = (2,0,0) + \rho(-2,-4,1) + \lambda(2,4,1)$

(A)

$$\begin{vmatrix} x-2 & y & z \\ -2 & -4 & 1 \\ 2 & 4 & 1 \end{vmatrix} = 0$$

$-8(x-2) + 4y = 0$

$-2(x-2) + y = 0$

$-2x + y + 4 = 0$ ou $2x - y - 4 = 0$

(B) $A=(6,0,0)$ $X=(3,-1,2)+t(2,-1,1)$

$$\begin{vmatrix} x-6 & y & z \\ 2 & -1 & 1 \\ -3 & -1 & 2 \end{vmatrix} = 0$$

$-1(x-6) - 7y - 5z = 0$

$-x - 7y - 5z + 6 = 0$

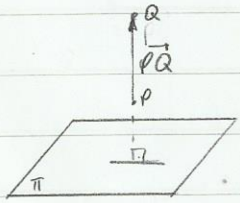
ou

$x + 7y + 5z - 6 = 0$

/ /

(181-) $P=(3,0,1)$ $Q=(2,2,4)$ $\qquad \vec{PQ} = Q-P = (2,2,4)-(3,0,1)$

$\vec{PQ}$ é ortogonal π e $Q \in \pi$ $\qquad = (-1,2,3)$



$$\begin{vmatrix} x-2 & y-2 & z-4 \\ -1 & 2 & 3 \\ & & \end{vmatrix} = 0$$

$$-(x-2)+2(y-2)+3(z-4)=0$$

$$-x+2y+3z-14=0 \quad \text{ou} \quad x-2y-3z+14=0$$

"Descobrir qual ponto utilizar"

(182-) $A=(2,0,0)$ $B=(0,-4,1)$ $C=(4,4,-5)$

$$x=(2,0,0)+\lambda(-2,-4,1)+\rho(2,4,-5)$$

$$\begin{vmatrix} x-2 & y & z \\ -2 & -4 & 1 \\ 2 & 4 & -5 \end{vmatrix} = 0$$

$$16(x-2)-8y=0$$

$$2(x-2)-y=0$$

$$2x-y-4=0$$

$$\vec{w}_\pi = (2,-1,0)$$

Qualquer vetor paralelo ao vetor normal $\vec{w}_\pi = (2,-1,0)$

spiral

(183-) $\pi: X = (3,-6,7) + a(1,-3,5) + b(2,-1,3)$

$$\begin{vmatrix} x-3 & y+6 & z-7 \\ 1 & -3 & 5 \\ 2 & -1 & 3 \end{vmatrix} = 0$$

$-4(x-3)+7(y+6)+5(z-7)=0$

$-4x+7y+5z-5=0 \quad \vec{W_\pi} = (-4,7,5)$

(184-) $X = a(1,0,1) + b(0,0,1)$

$$\begin{vmatrix} x & y & z \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$y = 0$

$\vec{W_\pi} = (0,1,0)$

(185-) $P=(2,-1,3) \perp \alpha: x+y+z+1=0$

(A) $X = (2,-1,3) + t(1,1,1)$

Obs.: Uma reta perpendicular ao plano, possui um ponto e o vetor normal do plano.

(186-) $P=(m,1,n) \quad A=(3,-1,4) \quad B=(4,-3,-1)$

$r: X=(3,-1,4)+t(1,-2,-5)$

$(m,1,n) = (3,-1,4) + t(1,-2,-5)$

$$\begin{cases} m = 3+t \\ 1 = -1-2t \\ n = 4-5t \end{cases}$$

$1 = -1-2t$

$t = \dfrac{2}{-2} = -1$

$m = 3-1 = 2$

$n = 4+5 = 9$

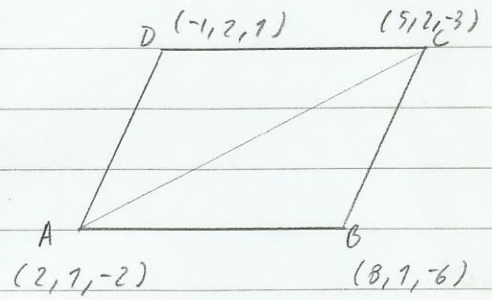
(D) $13m - 5n + 19 = 0$

(187-) $P=(2,-1,1)$

$r: X=(3,-1,0)+t(2,3,1)$

$p: X=(2,-1,1)+t(2,3,1);$

(188-)



$(2,1,-2)-(-1,2,1)=(3,-1,-3)$

$(8,1,-6)-(5,2,-3)=(3,-1,-3)$

$(2,1,-2)-(8,1,-6)=(-6,0,4)$

$(-1,2,1)-(5,2,-3)=(-6,0,4)$

$X=(2,1,-2)+t(3,1,-1)$

$\frac{x-2}{3}=y-1=-z-2$

(189-) $r: X=(m,1,0)+t(-1,1,2)$

$s: X=(1,-2,0)+t'(3,1,-2)$

$m-t=1+3t'$

$1+t=-2+t'$

$2t=-2t'$

$2t=-2t'$

$t=-t'$

$1-t'=-2+t'$

$2t'=3$

$t'=3/2$

$m+t'=1+3t'$

$m=1+2t'$

$m=1+3=4$ $\qquad \underline{m=4}$

/ /

(190-) r: $X=(-1,2,0)+t(a+1,6,b)$
s: $X=(2,2,3)+t'(-1,-2,-1)$

$\vec{u}=m\vec{v}$

$(a+1,6,b)=m(-1,-2,-1)$

$m=\frac{a+1}{-1}=\frac{6}{-2}=\frac{b}{-1}=-3$

$\frac{a+1}{-1}=-3$ $\frac{b}{-1}=-3$

$a=2$ $b=3$

$a+b=5$

(191-) r: $X=(4,3,7)+\lambda(9,6,4)$
s: $X=(1,-2,-6)+\rho(3,5,13)$

$4+9\lambda=1+3\rho$
$3+6\lambda=-2+5\rho$
$7+4\lambda=-6+13\rho$

$4+9\lambda=1+3\rho$
$9\lambda=-3+3\rho$
$\lambda=\frac{-3+3\rho}{9}$ $\lambda=\frac{-1+\rho}{3}$

$3+6\lambda=-2+5\rho$

$3+6\left(\frac{-1+\rho}{3}\right)=-2+5\rho$

$3+\left(\frac{-6+6\rho}{3}\right)=-2+5\rho$

$\frac{9-6+6\rho}{3} \Rightarrow 1+2\rho=-2+5\rho$
$3\rho=3$
$\rho=1$

$3+6\lambda=-2+5\rho$
$6\lambda=-2+5-3$
$\lambda=0$

$T=(4,3,7)$
abscissa $=4$

(192-) $r: X=(2,-1,0)+\lambda(m,2,-1)$

$s: X=(1,-1,-1)+\rho(1,2,2)$

$$\left|\begin{array}{ccc|cc} m & 2 & -1 & m & 2 \\ 1 & 2 & 2 & 1 & 2 \\ -1 & 0 & -1 & -1 & 0 \end{array}\right| = -2m-4-8+2 \neq 0 \qquad -2m \neq 4$$

$$m \neq -2.$$

Posições Relativas entre reta e plano

(193-) $[A=(1,1,-2)] \perp \pi: B=(1,2,3) \quad C=(4,-1,5) \quad D=(2,-1,0)$

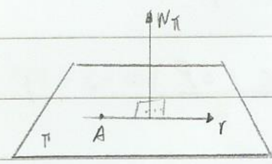
$\pi: X=(1,2,3)+m(3,-3,2)+n(1,-3,-3)$

$$\left|\begin{array}{ccc} x-1 & y-2 & z-3 \\ 3 & -3 & 2 \\ 1 & -3 & -3 \end{array}\right| = 0 \qquad \begin{array}{l} 15(x-1)+11(y-2)-6(z-3)=0 \\ 15x+11y-6z-19=0 \\ \vec{W}_\pi=(15,11,-6) \end{array}$$

$r: X=(1,1,-2)+t(15,11,-6)$

(194-) $\pi: x+by-2z-6=0$

$r: X=(a,2,-1)+t(1,-1,2)$



$\vec{W}_\pi=(1,b,-2)$ $\qquad\qquad \pi: x-3y-2z-6=0$

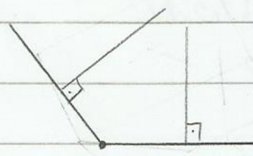
$(1,b,-2).(1,-1,2)=0$

$1-b-4=0 \qquad\qquad a-6+2-6=0$

$\underline{b=-3} \qquad\qquad \underline{a=10}$

(195-) $\to 2x - y + 3z + 3 = 0$

$\to 3x - y + 2z - 1 = 0$



$\vec{W}_\alpha = (2, -1, 3)$

$\vec{W}_\beta = (3, -1, 2)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ 2 & -1 & 3 \\ 3 & -1 & 2 \end{vmatrix}\begin{matrix} \vec{i} & \vec{j} \\ 2 & -1 \\ 3 & -1 \end{matrix} = -2\vec{i} + 9\vec{j} - 2\vec{K} + 3\vec{K} + 3\vec{i} - 4j$$

$$= \vec{i} + 5\vec{j} + \vec{K} \quad (1, 5, 1)$$

I- $2x - y + 3 = 0$ $\qquad x - 4 = 0$ $\qquad 2.4 - y + 3 = 0$

II- $3x - y - 1 = 0$ $\qquad x = 4$ $\qquad y = 11$

$I = (4, 11, 0)$

r: $X = (4, 11, 0) + t(1, 5, 1)$

(196-) $P = (1, -3, 5)$ $\qquad \alpha: 3x - 4y + z - 12 = 0$ $\qquad \vec{W}_\alpha = (3, -4, 1)$

$\beta: 4x - 7y + 3z + 4 = 0$ $\qquad \vec{W}_\beta = (4, -7, 3)$

$$\begin{vmatrix} i & j & K \\ 3 & -4 & 1 \\ 4 & -7 & 3 \end{vmatrix}\begin{matrix} i & j \\ 3 & -4 \\ 4 & -7 \end{matrix} = -12\vec{i} + 4j - 21\vec{K} + 16\vec{K} + 7i - 9j$$

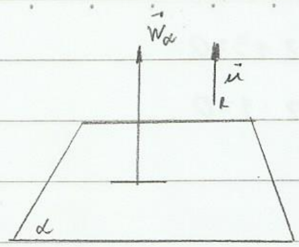
$$= -5\vec{i} - 5\vec{j} - 5\vec{K} \quad (-5, -5, -5)$$

r: $X = (1, -3, 5) + t(-5, -5, -5)$

(197-) $X=(1,-1,0)+t(1,3,1)$

$4x-3y-z+2=0$

$\vec{w}_\alpha=(4,-3,-1)$



$$\begin{vmatrix} x-1 & y+1 & z \\ 4 & -3 & -1 \\ 1 & 3 & 1 \end{vmatrix}=0$$

$-5(y+1)+15(z)=0$

$-(y+1)+3(z)=0$

$-y+3z-1=0$

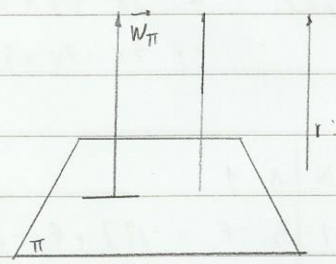
ou

$y-3z+1=0$

(198-) $A=(2,-4,1)$

$/\!/ \; X=(2,-5,3)+t(1,3,1) \quad \perp \quad 3x+y-z+1=0$

$\vec{w}_r=(3,1,-1)$



$X=(2,-4,1)+t(3,1,-1)$

$$\begin{vmatrix} x-2 & y+4 & z-1 \\ 3 & 1 & -1 \\ 1 & 3 & 1 \end{vmatrix}=0$$

$4(x-2)-4(y+4)+8(z-1)=0$

$(x-2)-(y+4)+2(z-1)=0$

$x-y+2z-8=0$

(/ /)

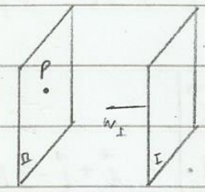
(199-) $x-2y+4z+1=0$

$\vec{W}_\alpha=(1,-2,4)$

$P=(2,-1,1)$

$2-2.(-1)+4.1+d=0$

$d=-8$



$R = x-2y+4z-8=0$

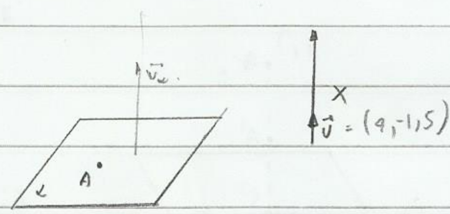
$\vec{W}_R\,(1,-2,4)$ $\quad A=(0,0,2)\quad B=(8,0,0)$

$X=(2,-1,1)+a(-2,1,1)+b(6,1,-1)$

$x=2-2a+6b\;;\; y=-1+a+b\;;\; z=1+a-b$

200-) $X=(1,1,1)+t(4,-1,5)$

$A=(-3,1,2)$



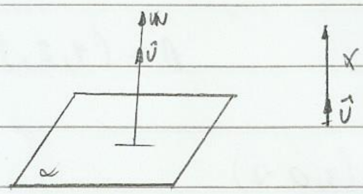
$4(x+3)-1(x-1)+5(x-2)=0$

$R: 4x-x+5x+3=0$

201) $A=(1,2,3)$

$\alpha: 3x-y+z+4=0$

$\vec{W}_\alpha=(3,-1,1)$



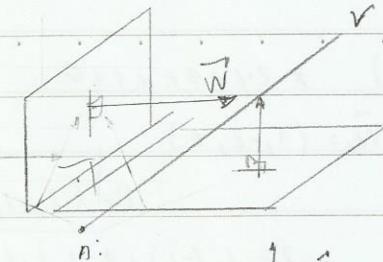
$X=(1,2,3)+t(3,-1,1)$

$\frac{x-1}{3}=2-y=z-3$

spiral

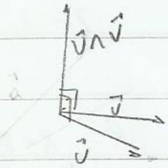
/ /

(202). $A=(3,-1,4)$



$\alpha: x+2y+z+8=0 \quad \bar{W}_\alpha=(1,2,1)$

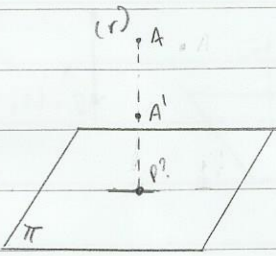
$\beta: 2x+y+z-4=0 \quad \bar{W}_\beta=(2,1,1)$



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \vec{i}+\vec{j}-3\vec{k} \quad (1,1,-3)$$

$r: X=(3,-1,4)+t(1,1,-3);\ t\in\mathbb{R}$

(203-)



$\pi: 4x-2y+4z-15=0 \qquad A=(2,3,1)$

$\vec{W}_\pi=(4,-2,4)$

$r: x=(2,3,1)+t(4,-2,4)$

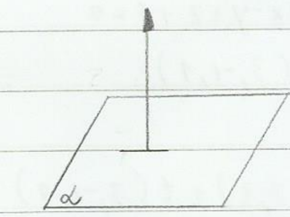
Admitindo $P=\frac{1}{2}$:

$(x,y,z)=(2,3,1)+\frac{1}{2}(4,-2,4)$

$A'=(4,2,3)$

(204-) $r: X=(1,-1,2)+t(3,0,-2)$

$\alpha: 2x+y-5z+1=0 \quad \vec{W}_\alpha=(2,1,-5)$



$(3,-2,0).(2,1,-5)=6-2=4\neq 0$, (Não é perpendicular)

$2(1+3t)+(-1)-5(2-2t)+1=0$

$(3,-2,0)=m(2,1,-5)$

$6t+10t=-2+1+10-1$

$x=1+3\cdot\frac{1}{2}=\frac{5}{2}$

$m=\frac{3}{2}\neq\frac{-2}{1}\neq\frac{0}{-5}$ (LI)

$16t=8$

$y=-1$

$t=\frac{8}{16}=\frac{1}{2}$

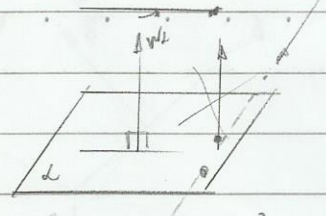
$z=2-2\cdot\frac{1}{2}=1$

$T=(\frac{5}{2},-1,1)$

spiral

1205-) $r: X=(1,2,-1)+t(1,-1,2);\ t\in\mathbb{R}$

$\alpha: 3x+y-z-8=0 \qquad \vec{w}_\alpha=(3,1,-1)$



Perpendicular: se o $\vec{u}=m\vec{w}$ e $\vec{v}.\vec{w}\neq 0$

$(1,-1,2)=m(3,1,-1)$

$m=\frac{1}{3}\neq\frac{-1}{1}\neq\frac{2}{-1}$ X

Paralelo: se $\vec{u}.\vec{v}=0$ e $P\notin\alpha$

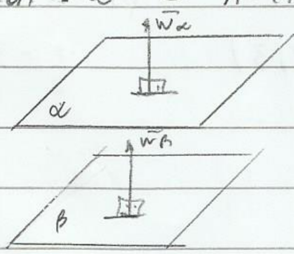
$(1,-1,2).(3,1,-1)=3-1-2=0$

$3.1+2+1-8=0$

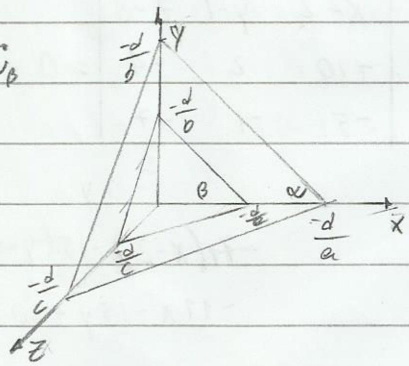
$-2\neq 0$

R: A reta é paralela ao plano e sua interseção é vazia

1206-) eq. geral α c $A=(1,2,1)$ // $\beta: 2x+y-z+1=0$



$\vec{w}_\alpha=m\vec{w}_\beta$



$\beta: 2x+y-z+1=0$

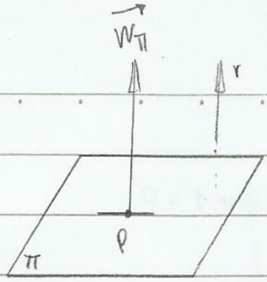
$\alpha: 2x+y-z+d=0$ Subs. $A=(1,2,1)$

$2.1+2-1+d=0$

$d=-3$

$\alpha=2x+y-z-3=0$

(208-)



$r: X = (1,-2,0) + t(3,1,-2)$

$P = (-1,3,-1)$

$\vec{w}_\pi = \vec{u}$

$\vec{w}_\pi = (3,1,-2)$

$3(x+1) + (y-3) - 2(z+1) = 0$

$3x + y - 2z - 2 = 0$

(209-) $A = (1,2,1)$

$X = (2,1,0) + t(1,3,3)$

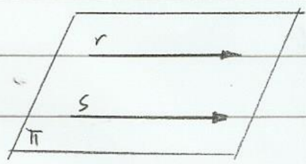
$$\begin{vmatrix} x-1 & y-2 & z-1 \\ 1 & 3 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

$4(y-2) - 4(z-1) = 0$

$(y-2) - (z-1) = 0$

$y - z - 1 = 0$

(210)



$r: X = (1,1,1) + a(2,1,0)$

$s: X = (2,2,2) + b(2,1,0)$

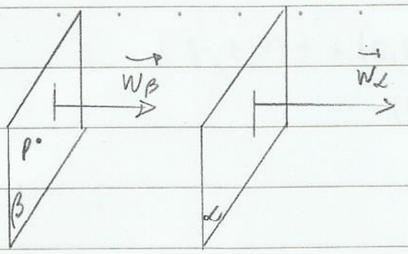
$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$(x-1) - 2(y-1) + (z-1) = 0$

$x - 2y + z = 0$

/ /

(211-)



$P = (1,1,1)$

$\alpha: 2x + y + z + 1 = 0$

$\vec{W}_\alpha = (2,1,1)$

$2x + y + z + d = 0$

subst. por P.

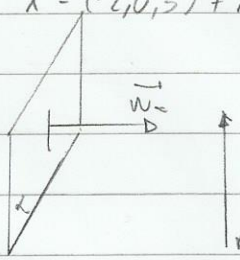
$2.+1+1+d = 0$

$d = -4$

$\beta: 2x + y + z - 4 = 0$

(212-) $r: X = (-2,0,3) + \lambda(-5,1,4);\ \lambda \in \mathbb{R}$ // $\alpha: x + y + mz + 2 = 0$

$\vec{w}_\alpha = (1,1,m)$



$\vec{u} \cdot \vec{w}_\alpha = 0$

$(-5,1,4).(1,1,m) = 0$

$-5 + 1 + 4m = 0$

$4m = 4$

$m = 1$

(213-) $\alpha: x + y + 2z - 4 = 0$

$\beta: 2x + 3y + 5z - 13 = 0$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 2 & 3 & 5 \end{vmatrix} = -\vec{i} - \vec{j} + \vec{k}$ $(-1,-1,1)$

Admitindo $z = 0$

$x + y - 4 = 0$ $\quad x = 4 - y$ $\quad x = 4 - y$

$2x + 3y - 13 = 0$ $\quad 2(4-y) + 3y = 13$ $\quad x = 4 - 5$

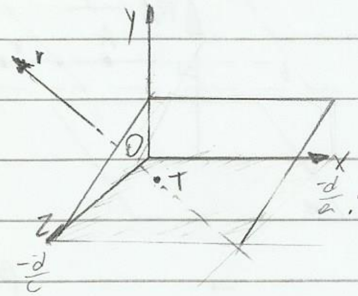
$8 - 2y + 3y = 13$ $\quad x = -1$

$y = 5$

$X = (-1,5,0) + \lambda(1,-1,1);\ \lambda \in \mathbb{R}$

(214-) r: $2x-1 = 2y-5 = z+1 \quad (\div 2)$

$x - \frac{1}{2} = y - \frac{5}{2} = \frac{z+1}{2}$



$\therefore\ y=0$

r: $X = (1/2, 5/2, -1) + a(1,1,2)$

$x = \frac{1}{2} + a \quad y = \frac{5}{2} + a \quad z = -1 + 2a$

$ax + cz + d = 0$

Admitindo $a = c = 1$ e $d = 0$

$x + z = 0$

$\frac{1}{2} + a + (-1) + 2a = 0$

$3a = \frac{1}{2}$

$a = \frac{1}{6}$

$x = \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3}$

(215-) r: $X = (1,3,1) + \lambda(1,1,3);\ \lambda \in \mathbb{R}$

α: $3x - y - z = 0$

$x = 1 + \lambda \qquad 3(1+\lambda) - (3+\lambda) - (1+3\lambda) = 0$

$y = 3 + \lambda \qquad -1 - 3\lambda = 0$

$z = 1 + 3\lambda \qquad -3\lambda = 1 \qquad y = 3 + \lambda \qquad 3 - \frac{1}{3} = \frac{9-3}{3} = \frac{6}{3} = 2$

$\lambda = \frac{1}{-3} \qquad y = 3 + \frac{1}{-3}$

$y = 2$

(216-) α: $x + y + 2z + 3 = 0 \quad \vec{w}_\alpha = (1,1,2)$

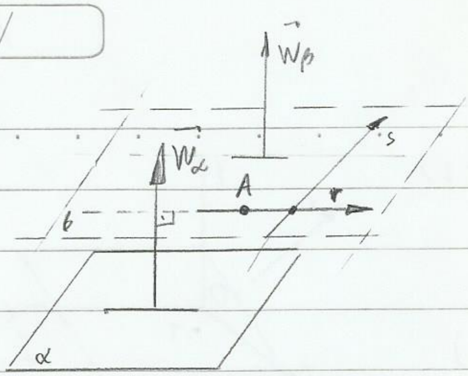
β: $2x - y + z - 1 = 0 \quad \vec{w}_\beta = (2,-1,1)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 3i + 3j - 3\vec{k}$$

$(3,3,-3)$

r: $X = (1,-2,3) + t(3,3,-3)$

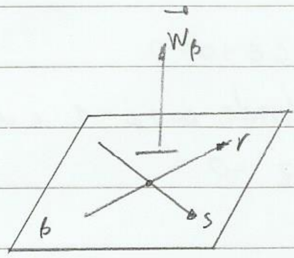
(217-)



$A = (1,-3,2)$

$\alpha: x+2y+z+5=0 \quad \vec{w}_\alpha = (1,2,1) \quad \vec{w}_\beta = (1,2,1)$

$s: (2,1,0) + \lambda(1,1,3) ; \lambda \in \mathbb{R}$



$\beta: x+2y+z+d=0$

$1 + 2.(-3) + 2 + d = 0$

$d = 3$

$\beta: x+2y+z+3=0$

$\vec{w}_\beta = (1,2,1)$

$2+\lambda + 2(1+\lambda) + 3\lambda + 3 = 0$

$6\lambda = -7$

$\lambda = \frac{-7}{6}$

$I = \left(\frac{5}{6}, \frac{-1}{6}, \frac{-7}{2}\right)$

$r: X = A + \lambda(B-A)$

$r: X = (1,-3,2) + \lambda'\left(\frac{-1}{6}, \frac{17}{6}, \frac{-11}{2}\right) ; \lambda' \in \mathbb{R}$

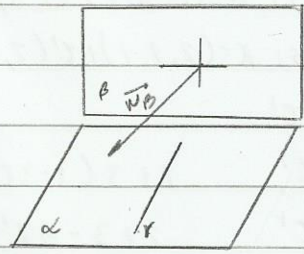
(218-) $A = (1,-2,3)$ $r: X = (1,-1,3) + \lambda(1,0,2)$ $s: X = (2,1,0) + \lambda'(0,1,4)$

(226-) $\alpha \subset r \quad \alpha \perp \beta$

$r: x=t;\ y=-t;\ z=t+2;\ t \in \mathbb{R}$

$\beta: x-2y+z-1=0$

equação vetorial: α: ?



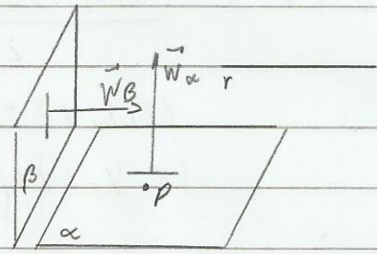
$r: X=(0,0,2)+t(1,-1,1);\ t \in \mathbb{R}$

$\vec{w}_\beta = (1,-2,1)$

$\alpha: X=(0,0,2)+a(1,-1,1)+b(1,-2,1);\ a,b \in \mathbb{R}$

(227-) equação vetorial α = ?

$\alpha \subset P \quad \alpha \parallel r \quad \alpha \perp \beta$



$P=(2,1,3)$

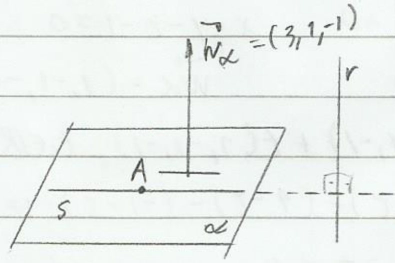
$r: X=(1,2,-3)+t(-2,1,2);\ t \in \mathbb{R}$

$\beta: x-y+2z-4=0 \quad \vec{v}_\beta=(1,-1,2)$

$\alpha: X=(2,1,3)+a(1,-1,2)+b(-2,1,2);\ a,b \in \mathbb{R}$

(228-) equações paramétricas r: ?

$s \subset A \quad s \perp r$



$A=(2,1,-1)$

$r: X=(2,0,0)+t(3,1,-1);\ t \in \mathbb{R}$

$\alpha: 3x+y-z+d=0 \quad (2,1,-1)$

$3.2+1-(-1)+d=0 \quad d=-8 \qquad \alpha: 3x+y-z-8=0$

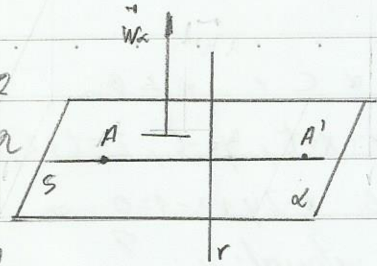
$\alpha \cap r = 3(2+3t)+t+t-8=0$

$11t=2 \quad t=\frac{2}{11} \qquad I=\left(\frac{28}{11}, \frac{2}{11}, -\frac{2}{11}\right)$

$s: X=(2,1,-1)+a\left(\frac{6}{11}, \frac{-9}{11}, \frac{9}{11}\right)$ ou $X=(2,1,-1)+a(2,-3,3);\ a \in \mathbb{R}$

(229-) $A=(2,1,-1)$ $r: X=(2,0,0)+t(3,1,-1);\ t\in\mathbb{R}$

$s: X=(2,1,-1)+t'(2,-3,3);\ t'\in\mathbb{R}$



$2+3t=2+2t'$

$t=1-3t'$ $\quad 2+3(1-3t')=2+2t'$

$-t=-1+3t'$ $\quad 2+3-9t'=2+2t'$ $\quad t=1-3t'$

$-11t'=-3$ $\quad =1-\frac{9}{11}$

$t'=\frac{3}{11}$

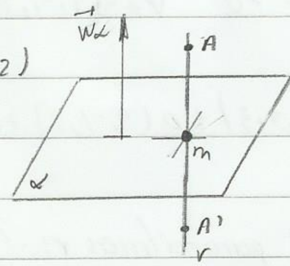
$I=\left(\frac{28}{11},\frac{2}{11},\frac{-2}{11}\right)$ $\quad t=\frac{2}{11}$

$\frac{A+A'}{2}=I \therefore \frac{(2,1,-1)+(x,y,z)}{2}=\left(\frac{28}{11},\frac{2}{11},\frac{-2}{11}\right)$

$(2,1,-1)+(x,y,z)=\left(\frac{56}{11},\frac{4}{11},\frac{-4}{11}\right)$

$(x,y,z)=\left(\frac{-34}{11},\frac{7}{11},\frac{-7}{11}\right)$ ou $\left(\frac{34}{11},\frac{-7}{11},\frac{7}{11}\right)$

(230-) $A=(3,4,-1)$ $\alpha: X=(1,0,0)+a(2,1,1)+b(1,-1,2)$



$\begin{vmatrix} x-1 & y & z \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}=0$ $\quad 3(x-1)-3(y)-3(z)=0$

$x-y-z-1=0$

$\vec{W}_\alpha=(1,-1,-1)$

$r: X=(3,4,-1)+t(1,-1,-1),\ t\in\mathbb{R}$

$(3+t)-(4-t)-(-1-t)-1=0$

$3t=1$

$t=\frac{1}{3}$ $\quad I=\left(\frac{10}{3},\frac{11}{3},\frac{-4}{3}\right)$

$\frac{(3,4,-1)+(x,y,z)}{2}=\left(\frac{10}{3},\frac{11}{3},\frac{-4}{3}\right)$

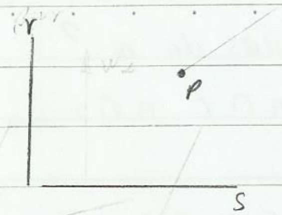
$(x,y,z)=\left(\frac{20}{3},\frac{22}{3},\frac{-8}{3}\right)-(3,4,-1)=\left(\frac{11}{3},\frac{10}{3},\frac{-5}{3}\right)$

/ /

(232-)

r: $X=(1,1,0)+t(1,2,-1)$

s: $X=(0,0,0)+t(2,1,-2)$

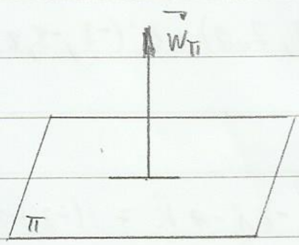


$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 2 & 1 & -2 \end{vmatrix} = -3\vec{i} - 3\vec{k} = \underset{x \quad y \quad z}{(-3,0,-3)}$$

Conclusão: $x=z$ e $y=0$

(233-) $\vec{w}_\pi=(-2,3,6)$

$\bar{O}=(-2,3,6)$



$-2x+3y+6z+d=0$

$-2(-2)+3(3)+6(6)+d=0$

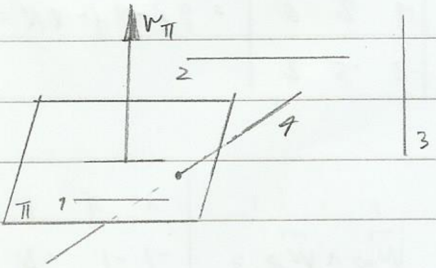
$+4+9+36+d=$

$d=-49$ R: $-2x+3y+6z-49=0$

(234-) r: $x=a+2t$; $y=3+bt$; $z=1+t$; $t\in\mathbb{R}$

→ r: $X=(a,3,1)+t(2,b,1)$

π: $2x-y+3z-10=0$ $\vec{w}_\pi=(2,-1,3)$



1-) $(2,-1,3).(2,b,1)=0$ (contida no plano) ✓

$4-b+3=0$ $b=7$

$2a-3+3-10=0$

$a=5$

2-) $(2,-1,3).(2,b,1)=0$ (paralela) ✓

$b=7$

$2a-3+3-10\neq0$

$a\neq5$

3-) $(2,-1,3)=m(2,b,1)$ (Perpendicular) ✗

$m=\frac{2}{2}=\frac{-1}{b}=\frac{3}{1}$

4) $(2,-1,3)(2,b,1)\neq0$

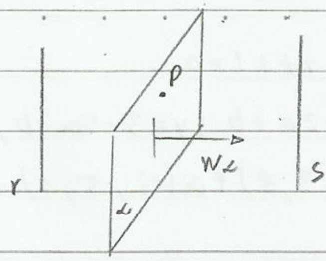
$b\neq7$

R: Se $a=5$, $b=7$ ele está contido no plano / se $b=7$ e $a\neq5$ ele é paralelo / não existe solução para que ele seja perpendicular / e ele interceptará com o plano somente se $b\neq7$

(235-) $P=(3,-1,2)$

$r: X=(-9,-1,-9)+a(4,2,3);\ a\in\mathbb{R}$

$s: X=(-3,-3,-3)+b(2,3,5);\ b\in\mathbb{R}$



$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 4 & 2 & 3 \\ 2 & 3 & 5 \end{vmatrix} = 0$$

$(x-3)-14(y+1)+8(z-2)=0$

$x-14y+8z-33=0$

(236-) $\alpha: 5x+3y+13z-1=0$ $\quad \vec{w}_\alpha=(5,3,13)$

$\beta: 3x+8y-3z+8=0$ $\quad \vec{w}_\beta=(3,8,-3)$

Paralelo $\Rightarrow$ $\vec{w}_\alpha = m\vec{w}_\beta$ ✗

$(5,3,13)=m(3,8,-3)$

$m=\frac{5}{3}\neq\frac{3}{8}\neq\frac{13}{-3}$

Perpendicular. $\vec{w}_\alpha\cdot\vec{w}_\beta=0$ ✓

$(5,3,13)\cdot(3,8,-3)=0$

$15+24-39=0$

Planos coincidentes

$\frac{5}{3}\neq\frac{3}{8}\neq\frac{13}{-3}\neq\frac{-1}{8}$ ✗

Planos secantes e não perpendiculares

$\vec{w}_\alpha\cdot\vec{w}_\beta\neq 0$

$(5,3,13)(3,8,-3)\neq 0$ ✗

$0=0$

(237-) $\alpha: 2x - y + z + 1 = 0$

$\beta: x = 1 - 2a + b \; ; y = 3 - 5a - b \; ; z = 4 + a - 2b \; ; a, b \in \mathbb{R}$

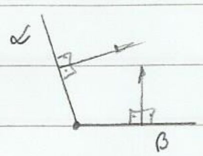
$X = (1,3,4) + a(-2,-5,1) + b(1,-1,-2) \; ; a, b \in \mathbb{R}$

$$\begin{vmatrix} x-1 & y-3 & z-4 \\ -2 & -5 & 1 \\ 1 & -1 & -2 \end{vmatrix} = 0 \qquad 9(x-1) - 3(y-3) + 7(z-4) = 0$$

$\beta: 9x - 3y + 7z - 28 = 0$

$\vec{w}_\alpha = (2,-1,1) \quad \vec{w}_\beta = (9,-3,7)$ É secante e não perpendicular

$(2,-1,1) \cdot (9,-3,7) = 18 + 3 + 7 = 28 \neq 0$



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 9 & -3 & 7 \end{vmatrix} = -4\vec{i} - 5\vec{j} + 3\vec{k} \qquad (-4,-5,3)$$

$\alpha: 2x - y + z + 1 = 0$

$\beta: 9x - 3y + 7z - 28 = 0$

Admitindo $z = 0$

$2x - y + 1 = 0$

$9x - 3y - 28 = 0$

$y = 2x + 1$

$9x - 3(2x+1) - 28 = 0$

$9x - 6x = 28 + 3$

$3x = 31$

$x = \frac{31}{3}$

$9 \cdot \frac{31}{3} - 3y - 28 = 0$

$-3y = -65$

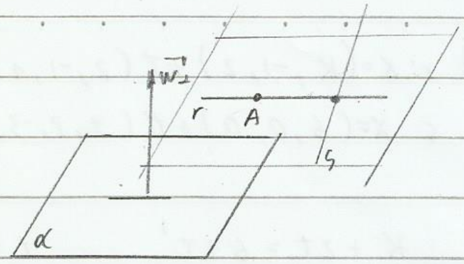
$y = \frac{\cancel{-}65}{3}$

$X = \left(\frac{31}{3}, \frac{65}{3}, 0\right) + t(-4,-5,3) \; ; t \in \mathbb{R}$

/ /

(238-) $\alpha: x+2y+z-1=0$

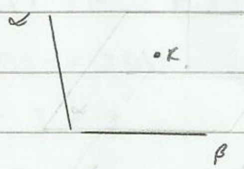
$s: x=2y=2z$



? como achar as equações da reta s:

(239-) $\alpha: x+y+z+1=0 \quad \vec{w}_\alpha = (1,1,1)$

$\beta: 2x-y+3z-1=0 \quad \vec{w}_\beta = (2,-1,3)$



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 4i - j - 3k = (4,-1,-3)$$

$X = (1,1,1) + t(4,-1,-3) \; ; t \in \mathbb{R}$

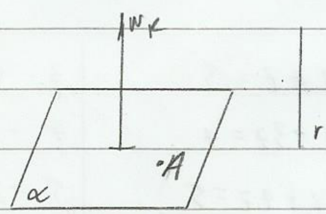
240-) $r: X = (1,0,1) + t(1,1,-1) \; ; t \in \mathbb{R}$

$x+y-z+d=0$

$1+2-1+d=0$

$d=-2$

eq. geral: $x+y-z-2=0$



spiral

/ /

(241-) $r: X=(K,-1,2)+t(2,-1,1);\ t\in\mathbb{R}$

$s: X=(6,0,0)+t'(1,2,-3);\ t'\in\mathbb{R}$

$K+2t=6+t'$ $\quad$ $t=-3t'-2$ $\quad$ $2+t=-3.t'$

$-1-t=2t'$ $\quad$ $-1-(-3t'-2)=2t'$ $\quad$ $2+t=-3.(-1)$

$2+t=-3t'$ $\quad$ $-1+3t'+2=2t'$ $\quad$ $t=1$

$t'=-1$ $\quad$ $K=6-1-2$

$K=3$

$I=(5,-2,3)$

(242-) $r: X=(2,-1,4)+t(1,3,-1);\ t\in\mathbb{R}$

$\pi: 2x-3y+3z+1=0$

$2(2+t)-3(-1+3t)+3(4-t)+1=0$

$4+2t+3-9t+12-3t+1=0$

$-10t=-20$

$t=2$

$P=(4,5,2)$

(243-) $\alpha: x+y+z-3=0$

$\beta: 2x-y+3z-4=0$

$\pi: 3x-2y+4z-5=0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & -2 & 4 \end{vmatrix} = 2+1-1=2$$

$$\begin{cases} x+y+z=3 \\ 2x-y+3z=4 \\ 3x-2y+4z=5 \end{cases}$$

$$\begin{vmatrix} 3 & 1 & 1 \\ 4 & -1 & 3 \\ 5 & -2 & 4 \end{vmatrix} = 6-1+3=2 \qquad x=2/2=1$$

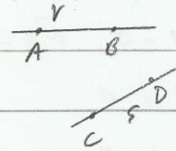
$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 3 \\ 3 & 5 & 4 \end{vmatrix} = 1+3-2=2 \qquad y=2/2=1$$

$$\begin{vmatrix} 1 & 1 & 3 \\ 2 & -1 & 4 \\ 3 & -2 & 5 \end{vmatrix} = 3+2-3=2 \qquad z=2/2=1$$

$I=(1,1,1)$

(244-) r: $X=(1,2,1)+t(1,-1,-1);\ t\in\mathbb{R}$

s: $X=(4,1,0)+t'(1,-1,-1);\ t\in\mathbb{R}$



$\vec{u}=m\vec{v}$

$(1,-1,-1)=m(1,-1,-1)$

$m=\frac{1}{1}=\frac{-1}{-1}=\frac{-1}{-1}$ Portanto é LD e não possui intersecção entre as retas

(245-) $\alpha: 2x+2y+2z+1=0$ $\vec{w}_\alpha=(2,2,2)$

$X=(2,-1,3)+a(2,2,2);\ a\in\mathbb{R}$

$\frac{x-2}{2}=\frac{y+1}{2}=\frac{z-3}{2}$

(246-) r: $X=(2,0,-3)+t(-5,1,4);\ t\in\mathbb{R}$

$\alpha: mx+2y+2z+4=0$ $\vec{w}_\alpha=(m,2,2)$

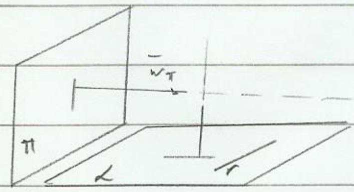
$(m,2,2)\cdot(-5,1,4)=0$

$-5m=-10$

$m=2$

(247-) $\pi: -x+2y+z-3=0$ $\vec{w}_\pi=(-1,2,1)$

r: $X=(1,-1,0)+t(3,-2,-1);\ t\in\mathbb{R}$



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 1 \\ 3 & -2 & -1 \end{vmatrix} = 2\vec{j}-4\vec{k} \quad (0,2,4)$$

$2y+4z+d=0$

$2-4+d=0$

$d=2$

$2y+4z+2=0$

/ /

(248-) $\beta: 3x-y+z+1=0 \quad \vec{w}_\beta=(3,-1,1)$

$\alpha: 3x-y+z+d=0$

$3-1+1+d=0$

$d=-3 \qquad \alpha: 3x-y+z-3=0$

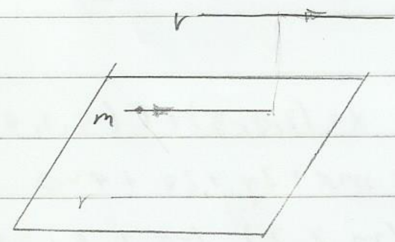
(249-) $\alpha: 2x-y+z+7=0 \quad \vec{v}_\alpha=(2,-1,1)$

$r: X=(1,1,1)+t(2,-1,1) \ ; t\in\mathbb{R}$

$\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z-1}{1}$

(250-) $m: x=y=z \qquad r: x=(0,-2,2)+t'(1,1,2) \ ; t'\in\mathbb{R}$

$m: x=(0,0,0)+t(1,1,1)$



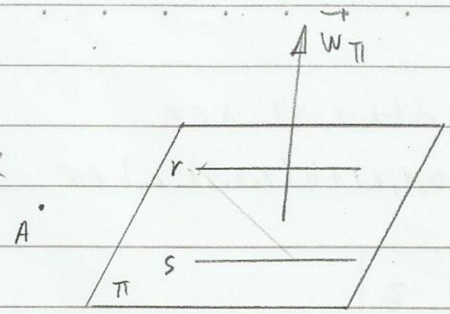
x	y	z
1	1	1
1	1	2

$x-y=0$

(251-) $A = (1,8,1)$

$r: X = (1,2,1) + t(1,-1,3);\ t \in \mathbb{R}$

$s: X = (3,4,-1) + t'(1,-1,3);\ t \in \mathbb{R}$



$$\begin{vmatrix} x-1 & y-2 & z-1 \\ 1 & -1 & 3 \\ 2 & 2 & -2 \end{vmatrix} = 0$$

$-4(x-1) + 8(y-2) + 4(z-1) = 0$

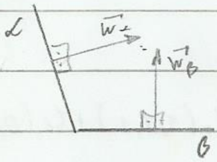
$-(x-1) + 2(y-2) + (z-1) = 0$

$-x + 2y + z - 4 = 0$ ou $x - 2y - z + 4 = 0$

(252-)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -2 \\ 1 & 0 & -1 \end{vmatrix} = -2i - j - 2k = (-2,-1,-2)$$

ou $(2,1,2)$



$(x, y, z) = t(2,1,2);\ t \in \mathbb{R}$

(253-)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -3 & 2 \\ 1 & -3 & -3 \end{vmatrix} = 15i + 11j - 6k = (15,11,-6)$$

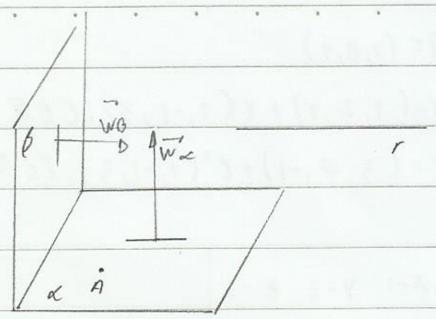
$X = (1,1,-2) + t(15,11,-6);\ t \in \mathbb{R}$

/ /

(254-) $A=(1,1,1)$

$r: X=(2,1,3)+\lambda(-1,-2,-3);\ \lambda\in\mathbb{R}$

$\beta=(1,0,0)+a(0,1,2)+b(1,2,3);\ a,b\in\mathbb{R}$



$$\begin{vmatrix} x-1 & y & z \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}=0 \quad -(x-1)+2y-z=0$$

$-x+2y-z+1=0$ ou $x-2y+z-1=0$

$\vec{w}_\beta=(1,-2,1)$

$\alpha: X=(1,1,1)+a(1,-2,1)+b(-1,-2,-3);\ a,b\in\mathbb{R}$

R: $x=1+a-b;\ y=1-2a-2b;\ z=1+a-3b;\ a,b\in\mathbb{R}$

(255-) $r: X=(2,-1,3)+t(-6,2,-4);\ t\in\mathbb{R}$

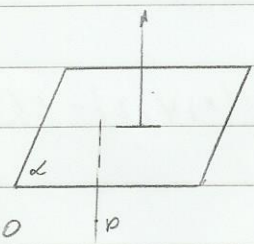
$-6x+2y-4z+d=0$

$-6+2-4+d=0$ $\quad -6x+2y-4z+8=0$

$d=8$ $\quad -3x+y-2z+4=0$ ou $3x-y+2z-4=0$

(256-) $X=(1,0,0)+t_1(1,1,1)+t_2(0,0,3);\ t_1,t_2\in\mathbb{R}$

$P=(2,-1,0)$



$$\begin{vmatrix} x-1 & y & z \\ 1 & 1 & 1 \\ 0 & 0 & 3 \end{vmatrix}=3(x-1)-3y=0 \Rightarrow 3x-3y-3=0$$

$r: X=(2,-1,0)+a(3,-3,0);\ a\in\mathbb{R}$

$3(2+3a)-3(-1-3a)-3=0$ $\quad I=(1,0,0)$

$6+9a+\cancel{3}+9a-\cancel{3}=0$

$18a=-6$

$a=\frac{-6}{18}=\frac{-1}{3}$

(257-) r_1: $X=(1,8,-1)+t_1(5,-1,4);\ t_1 \in \mathbb{R}$

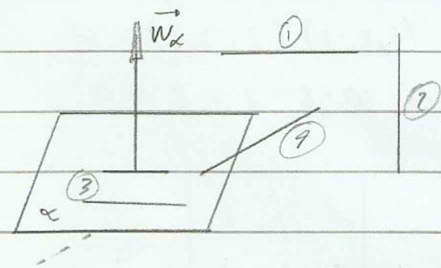
r_2: $X=(0,5,2)+t_2(2,6,-1);\ t_2 \in \mathbb{R}$

$$\begin{vmatrix} 5 & -1 & 4 \\ 2 & 6 & -1 \\ -1 & -3 & 3 \end{vmatrix} \begin{matrix} 5 & -1 \\ 2 & 6 \\ -1 & -3 \end{matrix} = 90-1-24+24-15+6 = 85 \neq 0$$

São reversas (não coplanares)

(258-) r: $X=(2,-1,3)+t(-6,2,-4);\ t \in \mathbb{R}$

α: $3x-y+2z-8=0$ $\quad \vec{w}_\alpha=(3,-1,2)$



Paralelo $(-6,2,-4).(3,-1,2)=0$

$-18-2-8 \neq 0$ X

Perpendicular $(-6,2,-4)=m(3,-1,2)$ ✓

$m = \frac{-6}{3} = \frac{2}{-1} = \frac{-4}{2}$

$3x-y+2z-8=0$

$3(2-6t)-(-1+2t)+2(3-4t)-8=0$

$6-18t+1-2t+6-8t-8=0$

$-28t=-5$

$t=\frac{5}{28}$

$I=\left(\frac{13}{14}, \frac{-9}{14}, \frac{16}{7}\right)$

/ /

(1259-) $\alpha: 5x + 2y - z + 1 = 0$

$\beta: 2x - 3y + 2z + 2 = 0$

$\vec{w_\alpha} = (5,2,-1)$ $\vec{w_\beta} = (2,-3,2)$

I - Paralelos ×

II - Perpendicular ×

III - Secantes e não perpendiculares

IV - Coincidentes ×

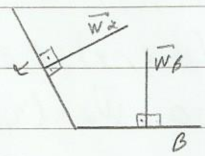
I $(5,2,-1) = m(2,-3,2)$

$m = \frac{5}{2} \neq \frac{2}{-3} \neq \frac{-1}{2}$

IV $\frac{5}{2} \neq \frac{2}{-3} \neq \frac{-1}{2} \neq \frac{1}{2}$

II $(5,2,-1)(2,-3,2) = 0$

$10 - 6 - 2 = 2 \neq 0$



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 2 & -1 \\ 2 & -3 & 2 \end{vmatrix} = \vec{i} - 12\vec{j} - 19\vec{k} \quad (1,-12,-19)$$

Admitindo $x = 0$

$2y - z = -1 \quad (\times 2)$

$-3y + 2z = -2$

$\begin{cases} 4y - 2y = -2 \\ -3y + 2y = -2 \end{cases}$

$y = -4$

$2y - z = -1$

$2(-4) - z = -1$

$z = -7$

$x = (0,-4,-7) + t(1,-12,-19);\ t \in \mathbb{R}$

$$\begin{cases} x = t \\ y = -4 - 12t \\ z = -7 - 19t \end{cases} ;\ t \in \mathbb{R}$$

(260-) (261)

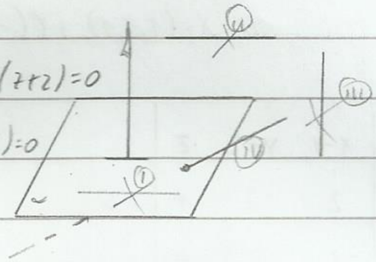
$$\begin{vmatrix} x-4 & y-3 & z+2 \\ 1 & -3 & 1 \\ -4 & -8 & 4 \end{vmatrix} = 0$$

$-4(x-4)-8(y-3)-20(z+2)=0$

$-(x-4)-2(y-3)-5(z+2)=0$

$-x-2y-5z=0$

ou



$\alpha: x+2y+5z=0 \quad \vec{n}_\alpha=(1,2,5)$

$r: X=(1,-2,0)+t(3,1,-2);\ t\in\mathbb{R}$

(I) $1-4+5=2\neq 0$ ×

(II) $=(3,1,2)(1,2,5)=3+2+10=15\neq 0$ ×

(III) $(3,1,2)=m(1,2,5) \Rightarrow m=\frac{3}{1}\neq\frac{1}{2}\neq\frac{2}{5}$ ×

São secantes e não perpendiculares

$x+2y+5z=0$

$(1+3t)+2(-2+t)+5(-2t)=0$

$3t+2t-10t=-1+4$

$-5t=3$

$t=\frac{-3}{5}$

Ponto de Intersecção é

$I=\left(\frac{-4}{5}, \frac{-13}{5}, \frac{6}{5}\right)$

/ /

(261-) (260) $r: X=(1,1,1)+t(-1,5,2);\ t\in\mathbb{R}$

$u: 8-2x = 2y-2 = z$

$$\begin{vmatrix} x-1 & y+2 & z \\ 2 & -2 & 4 \\ -2 & 2 & 4 \end{vmatrix} = 0$$

$$\frac{4-x}{2} = \frac{y-1}{2} = \frac{z}{4}$$

$v: X=(4,1,0)+t(-2,2,4)$

$-16(x-1)-16(y+2)=0$

$-x-y-1=0$ ou $x+y+1=0$

$1-t+1+5t+1=0$

$4t=-3$

$t=\frac{-3}{4}$

$I=\left(\frac{7}{4}, \frac{-11}{4}, \frac{-2}{4}\right)$

(262-)

$$\begin{vmatrix} x-4 & y-3 & z+2 \\ 1 & -3 & 1 \\ -4 & -8 & 4 \end{vmatrix} = 0$$

$-4(x-4)-8(y-3)+4(z+2)=0$

$-(x-4)-2(y-3)+(z+2)=0$

$-x-2y+z+8=0$ ou $x+2y-z-8=0$

$w=(1,2,-1)$

$(1,2,-1)(1,1,1) = 1+2-1 = 2 \neq 0$ x

$(1,2,-1)=m(1,1,1) \Rightarrow m = 1 \neq 2 \neq -1$ x

$r: X=(1,1,1)+t(-1,-1,-1)$

$x+2y-z-8=0$

$(1-t)+2(1-t)-(1-t)-8=0$

$\cancel{-t}-2t+\cancel{t}+\cancel{1}+2-\cancel{1}-8=0$

$-2t-6=0$

$t=\frac{6}{-2}=-3$

$I=(4,4,4)$

(263-) $r1: X=(4,2,1)+t(1,-3,4);\ t\in\mathbb{R}$

$r2: X=(3,3,11)+t'(2,-5,1);\ t'\in\mathbb{R}$

$$\begin{vmatrix} 1 & -3 & 4 \\ 2 & -5 & 1 \\ -1 & 1 & 10 \end{vmatrix}\begin{matrix} 1 & -3 \\ 2 & -5 \\ -1 & 1 \end{matrix} = \cancel{-50}+3+\cancel{8}-\cancel{20}-\cancel{1}+\cancel{60} = 0 \quad LD.$$

$(1,-3,4)=m(2,-5,1)$

$m=\frac{1}{2}\neq\frac{-3}{-5}\neq\frac{4}{1}$

São concorrente

$4+t=3+2t'$ $\qquad$ $t=-1+2t'$ $\qquad$ $t=-1+2.2$

$2-3t=3-5t'$ $\qquad$ $2-3(-1+2t')=3-5t'$ $\qquad$ $t=3$

$1+4t=11+t'$ $\qquad$ $2+3-6t'=3-5t'$ $\qquad$ $\Rightarrow 1+12=11+2$

$t'=5-3=2$

$I=(7,-7,13)$

(264-) $r1: X=(3,2,-1)+t(1,5,5);\ t\in\mathbb{R}$

$r2: X=(4,2,-1)+t'(1,5,5);\ t\in\mathbb{R}$

Retas Paralelas / Intersecção vazia.

(265-) $r: X=(5,2,4)+t(4,1,3);\ t\in\mathbb{R}$

$s: X=(0,5,2)+t'(1,-5,1);\ t'\in\mathbb{R}$

$$\begin{vmatrix} 4 & 1 & 3 \\ 1 & -5 & 1 \\ -5 & 3 & -2 \end{vmatrix}\begin{matrix} 4 & 1 \\ 1 & -5 \\ -5 & 3 \end{matrix} = 40-\cancel{5}+9-75-12+\cancel{2} = -41 \neq 0$$

LI

Portanto. São reversas

Intersecção Vazia

/ /

(266-)

$$\begin{vmatrix} 4 & -5 & 1 \\ 3 & -1 & 2 \\ 2 & 7 & 2 \end{vmatrix} \begin{matrix} 4 & -5 \\ 3 & -1 \\ 2 & 7 \end{matrix} = -8 - 20 + 21 + 2 - 56 + 30 = 21 \neq 0$$

As retas são reversas e não existe equação geral do plano, pois para que exista o conjunto $\{(X-A), \vec{v}_1, \vec{v}_2\}$ tem que ser LD e neste caso é LI

(267-) $r1: X=(7,-4,1)+t_1(4,-5,1); t_1 \in \mathbb{R}$

$r2: X=(2,5,1)+t_2(3,-1,2); t_2 \in \mathbb{R}$

$$\begin{vmatrix} 4 & -5 & 1 \\ 3 & -1 & 2 \\ -5 & 9 & 0 \end{vmatrix} \begin{matrix} 4 & -5 \\ 3 & -1 \\ -5 & 9 \end{matrix} = 50 + 27 - 5 - 72 = 0$$

As retas são coplanares, pois o conjunto $\{(X-A); \vec{v}_1; \vec{v}_2\}$ é linearmente dependente e também é concorrente pois não existe um escalar m para que $\vec{v}_1 = m\vec{v}_2$.

$(4,-5,1)=m(3,-1,2)$

$m = \frac{4}{3} \neq \frac{-5}{-1} \neq \frac{1}{2}$

$$\begin{vmatrix} x-2 & y-5 & z-1 \\ 3 & -1 & 2 \\ 4 & -5 & 1 \end{vmatrix} = 0$$

$9(x-2)+5(y-5)-11(z-1)=0$

$9x+5y-11z-32=0$

spiral

Distancia

Exemplo 4) Calcule a distância entre a reta r e o plano π

$r: X = (1,0,2) + \lambda(1,0,1);\ \lambda \in \mathbb{R}$

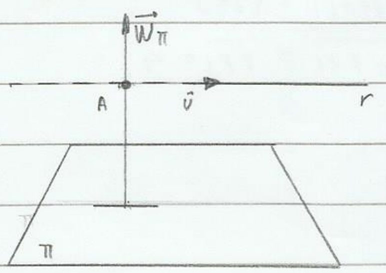
$\pi: 3x + 6y - 3z + 12 = 0$

Verificar a posição da reta e plano

$\vec{u} \cdot \vec{w}_\pi = (1,0,1)(3,6,-3) = 3 - 3 = 0$

$3.1 + 6.0 - 3.1 + 12 = 12 \neq 0$

Portanto, a reta r é paralela ao plano π



$$\delta(A,\pi) = \frac{|(1,0,2)(3,6,-3)+12|}{\sqrt{3^2 + 6^2 + (-3)^2}}$$

$$= \frac{|3 - 6 + 12|}{\sqrt{54}} = \frac{9}{\sqrt{54}}$$

Exemplo 5) Obtenha entre os planos $\alpha: x + 3y - 2z + 4 = 0$ e $\beta: 2x + 6y - 4z + 9 = 0$

Verificar posição dos planos $\frac{x}{2x} = \frac{3y}{6y} = \frac{-2z}{-4z}$, Portanto os planos são paralelos

$A \in \alpha = (0,0,2)$ $\qquad \delta = (A,\beta) = \frac{|-8+9|}{\sqrt{4+36+16}} = \frac{1}{\sqrt{56}}$

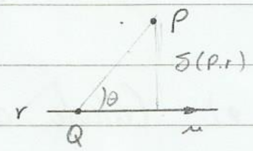
Exemplo 6) Determine a distância entre o ponto $P = (1,2,-1)$ e a reta

$r: \frac{x-1}{2} = \frac{y}{-1} = \frac{3+z}{3}$

Exemplo 7) Verifique se o triângulo ABC é isósceles, onde $A = (-1,-3,4)$, $B = (-2,1,-4)$ e $C = (3,-11,5)$.

6-) $r: X = \overset{Q}{(1,0,-2)} + t\overset{\vec{u}}{(2,-1,3)}\ ;\ t \in \mathbb{R}$

$P = (1,2,-1)$

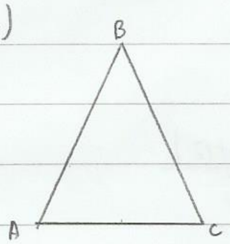


$$\delta(P,r) = \frac{|(P-Q)\wedge\vec{u}|}{|\vec{u}|} = \frac{|(0,2,1)\wedge(2,-1,3)|}{\sqrt{4+1+9}}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 7i + 2j - 4\vec{k}$$

$$= \frac{\sqrt{49+4+16}}{\sqrt{14}} = \frac{\sqrt{69}}{\sqrt{14}} \text{ u.m}$$

7-)



$\vec{AB} = (-1,4,-8) \quad |\vec{AB}| = 9$

$\vec{BC} = (5,-12,9) \quad |\vec{BC}| = \sqrt{25+144+81} = \sqrt{250}$

$\vec{CA} = (-4,8,-1) \quad |\vec{CA}| = \sqrt{16+64+1} = \sqrt{81} = 9$

$\vec{AB} = \vec{CA} = 9$

Distância entre duas retas paralelos

A distância entre duas retas paralelas, é a distancia de qualquer ponto de uma reta até a outra reta.

Ex: 287-

(272-) A) $\sqrt{4+4+1} = \sqrt{9} = 3$ u.m

B) $(2,-6,3) = \sqrt{4+36+9} = \sqrt{49} = 7$ u.m

c) $(-1,0,-1) = \sqrt{1+1} = \sqrt{2}$ u.m

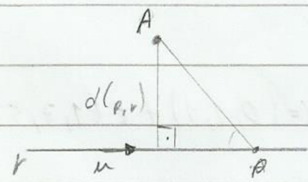
(273-) $\vec{AB} = (-1,3,-2)$ $|\vec{AB}| = \sqrt{1+9+4} = \sqrt{14}$

$\vec{BC} = (3,-2,-1)$ $|\vec{BC}| = \sqrt{9+4+1} = \sqrt{14}$ Triângulo equilátero

$\vec{CA} = (2,1,-3)$ $|\vec{CA}| = \sqrt{4+1+9} = \sqrt{14}$

(274-) $A = (1,2,-1)$

$r: X = (1,0,-2) + t(2,-1,3)\ ;\ t \in \mathbb{R}$



$$d(A,r) = \frac{|(2,-1,3) \wedge (0,-2,-1)|}{\sqrt{4+1+9}}$$

$$\left|\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 3 \\ 0 & -2 & -1 \end{matrix}\right| = 7i + 2j - 4k \Rightarrow \sqrt{49+4+16} = \sqrt{69} \qquad R: \frac{\sqrt{69}}{\sqrt{14}}$$

b) $P = (2,1,-3)$

$r: X = (1,1,1) + \lambda(2,1,-3)\ ;\ \lambda \in \mathbb{R}$

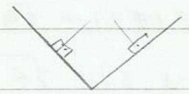
$$d(P,r) = \frac{|(2,1,-3) \wedge (-1,0,-4)|}{\sqrt{4+1+9}} \qquad R: \frac{\sqrt{42}}{\sqrt{14}}$$

$$\left|\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -3 \\ 1 & 0 & -4 \end{matrix}\right| = -4i - 5j - k \Rightarrow \sqrt{16+25+1} = \sqrt{42} \qquad R: \frac{\sqrt{42}}{\sqrt{14}}$$

$4i$

c) $P=(1,-1,0)$

$r: 2x-y+z=0 \; ; \; 3x+y-2z+1=0$



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 3 & 1 & -2 \end{vmatrix} = \vec{i}+7\vec{j}+5\vec{k} \qquad \vec{u}=(1,7,5)$$

$2x-y+z=0$ Admitindo $x=0$

$3x+y-2z=-1$ $\quad -y+z=0 \quad -z=-1 \quad y=1$

$y-2z=-1 \quad z=1$

$r: X=(0,1,1)+t(1,7,5)$

$$d(P,r)=\frac{|(1,-2,-1)\wedge(1,7,5)|}{\sqrt{1+49+25}}=\frac{\sqrt{126}}{\sqrt{75}}=$$

$$\begin{vmatrix} \hat{i} & \vec{j} & \vec{k} \\ 1 & -2 & -1 \\ 1 & 7 & 5 \end{vmatrix} = -3i-6j+9k = \sqrt{9+36+81}=\sqrt{126}$$

d) $P=(0,0,2)$

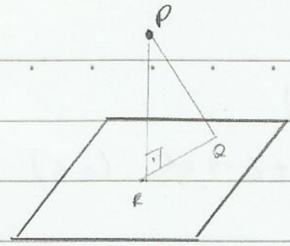
$r: X=(1,-2,0)+t(1,2,1)$

$$d(P,r)=\frac{|(1,2,1)\wedge(1,-2,-2)|}{\sqrt{1+4+1}}=\frac{\sqrt{29}}{\sqrt{6}}$$

$$\begin{vmatrix} \hat{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 1 & -2 & -2 \end{vmatrix} = -2i+3j-4k \qquad \sqrt{4+9+16}=\sqrt{29}$$

(275-) A) $P=(1,1,1)$

$\pi: 2x-y+z-1=0$



$$d(P,\pi)=\frac{|2.1-1.1+1.1-1|}{\sqrt{4+1+1}}=\frac{1}{\sqrt6}\cdot\frac{\sqrt6}{\sqrt6}=\frac{\sqrt6}{6}$$

B) $P=(2,1,-3)$

$\pi: X=(1,2,-1)+a(3,2,-1)+b(1,0,0)$

$$\begin{vmatrix} x-1 & y-2 & z+1 \\ 3 & 2 & -1 \\ 1 & 0 & 0 \end{vmatrix}=0$$

$-(y-2)-2(z+1)=0$

$-y-2z=0$

$y+2z=0$

$$d(P,\pi)=\frac{|1.1+2.(-3)|}{\sqrt{1+4}}=\frac{5}{\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}=\frac{5\sqrt5}{5}=\sqrt5$$

c) $P=(0,0,0)$

$\pi: x=2a+b$

$y=-a-b$

$z=1-2b$

$\pi: X=(0,0,1)+a(2,-1,0)+b(1,-1,-2), a,b\in\mathbb{R}$

$$\begin{vmatrix} x & y & z-1 \\ 2 & -1 & 0 \\ 1 & -1 & -2 \end{vmatrix}=0$$

$2x+4y-1(z-1)=0$

$2x+4y-z+1=0$

$$d(P,\pi)=\frac{|1|}{\sqrt{4+16+1}}=\frac{1}{\sqrt{21}}$$

(275.) A)

$P=(1,1,1)$

$\pi: 2x-y+z-1=0$

$$d(P,\pi)=\frac{|2-1+\cancel{1}-\cancel{1}|}{\sqrt{4+1+1}}=\frac{1}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}=\frac{\sqrt{6}}{6}$$

B) $P=(2,1,-3)$

$\pi: X=(1,2,-1)+a(3,2,-1)+b(1,0,0); a,b\in\mathbb{R}$

$$\begin{vmatrix} x-1 & y-2 & z+1 \\ 3 & 2 & -1 \\ 1 & 0 & 0 \end{vmatrix}=0 \qquad \begin{array}{l} -(y-2)-2(z+1)=0 \\ -y-2z=0 \quad \text{ou} \quad y+2z=0 \end{array}$$

$$d(P,\pi)=\frac{|1-6|}{\sqrt{1+4}}=\frac{5}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{5\sqrt{5}}{5}=\sqrt{5}$$

C) $P=(0,0,0)$

$\pi: X=(0,0,1)+a(2,-1,0)+b(1,-1,-2); a,b\in R$

$$\begin{vmatrix} x & y & z-1 \\ 2 & -1 & 0 \\ 1 & -1 & -2 \end{vmatrix}=0 \qquad \begin{array}{l} 2x-4y-(z-1)=0 \\ 2x-4y-z+1=0 \end{array}$$

$$d(P,\pi)=\frac{|1|}{\sqrt{4+16+1}}=\frac{1}{\sqrt{21}}$$

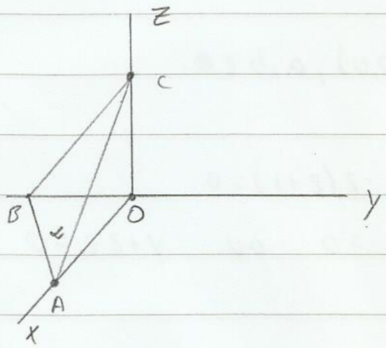
d) $P=(3,1,-1)$

$\pi: \frac{x}{2}+\frac{y}{3}+z=1$ $\quad \pi: \frac{x}{2}+\frac{y}{3}+z-1=0 \; (\times 6)$

$\pi: 3x+2y+6z-6=0$

$$d(P,\pi)=\frac{|9+2-6-6|}{\sqrt{9+4+36}}=\frac{1}{7}$$

(276-)



$\alpha: X=(3,0,0)+a(-3,-2,0)+b(-3,0,5)\;;a,b\in\mathbb{R}$

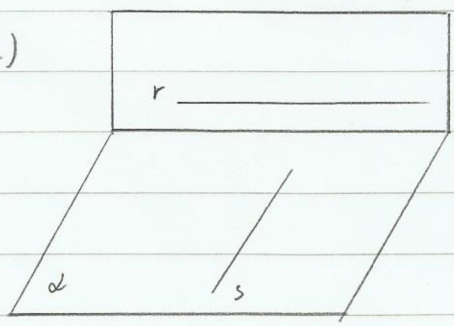
$$\begin{vmatrix} x-3 & y & z \\ -3 & -2 & 0 \\ -3 & 0 & 5 \end{vmatrix}=0$$

$-10(x-3)+15(y)-6(z)=0$

$-10x+15y-6z+30=0$

$$d(P,\pi)=\frac{|30|}{\sqrt{100+225+36}}=\frac{30}{19}$$

(277-)



$r: X=(1,0,3)+a(1,1,0)\;;a\in\mathbb{R}$

$s: X=(0,0,0)+b(2,-1,1),\; b\in\mathbb{R}$

$\alpha: X=(0,0,0)+b(2,-1,1)+a(1,1,0)\;;a,b\in\mathbb{R}$

$$\begin{vmatrix} x & y & z \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}=0$$

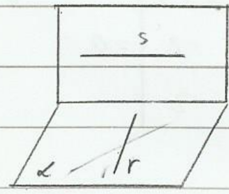
$-x+y+3z=0$

$$d(A,\alpha)=\frac{|8|}{\sqrt{1+1+9}}=\frac{8}{\sqrt{11}}\frac{\sqrt{11}}{\sqrt{11}}=\frac{8\sqrt{11}}{11}$$

(278-) r1: $X=(3,0,-2)+t(0,1,4);\ t\in\mathbb{R}$

r2: $X=(4,-1,1)+t'(2,1,0);\ t\in\mathbb{R}$

$$\begin{vmatrix} 0 & 1 & 4 \\ 2 & 1 & 0 \\ 1 & -1 & 3 \end{vmatrix}\begin{matrix} 0 & 1 \\ 2 & 1 \\ 1 & -1 \end{matrix} = -8-4-6 = -18 \neq 0$$ Retas são reversas



$\alpha: X=(3,0,-2)+a(0,1,4)+b(2,1,0);\ a,b\in\mathbb{R}$

$$\begin{vmatrix} x-3 & y & z+2 \\ 0 & 1 & 4 \\ 2 & 1 & 0 \end{vmatrix} = 0$$

$-4(x-3)+8(y)-2(z+2)=0$

$-2(x-3)+4(y)-(z+2)=0$

$-2x+4y-z+4=0$

$$d(P,\alpha)=\frac{|-8\cancel{-4}-1\cancel{+4}|}{\sqrt{4+16+1}} = \frac{9}{\sqrt{21}}\cdot\frac{\sqrt{21}}{\sqrt{21}} = \frac{9\sqrt{21}}{21} = \frac{3\sqrt{21}}{7}$$

(279-) r1: $X=(1,0,-2)+a(2,-1,3);\ a\in\mathbb{R}$

r2: $X=(1,2,-1)+b(4,-2,6);\ b\in\mathbb{R}$

$$\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ 0 & 2 & 1 \end{vmatrix}\begin{matrix} 2 & -1 \\ 4 & -2 \\ 0 & 2 \end{matrix} = \cancel{-4}+\cancel{24}-\cancel{24}+\cancel{4} = 0$$

$(2,-1,3)=m(4,-2,6)$

$m=\frac{2}{4}=\frac{-1}{-2}=\frac{3}{6}$

Reta Paralela ✓

$$d(P,r)=\frac{|(0,2,1)\wedge(2,-1,3)|}{\sqrt{4+1+9}} = \frac{\sqrt{69}}{\sqrt{14}}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 7\vec{i}-2\vec{j}-4\vec{k} \Rightarrow \sqrt{49+4+16} = \sqrt{69}$$

(280-) $A = (1,3,-4)$

$r: X = (1,5,-6) + t(0,1,-2); t \in \mathbb{R}$

a) $\alpha: X = (1,3,-4) + a(0,1,-2) + b(0,2,-2)$; $a, b \in \mathbb{R}$

$$\begin{vmatrix} x-1 & y-3 & z+4 \\ 0 & 1 & -2 \\ 0 & 2 & -2 \end{vmatrix} = 0$$

$2(x-1) = 0$

$2x - 2 = 0$

$x - 1 = 0$

$\underline{x = 1}$

b) $d(P, \pi) = \dfrac{|3-1|}{\sqrt{1}} = \dfrac{2}{1} = 2$

(281-)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 2 & -1 \\ 4 & 3 & -1 \end{vmatrix} = i + j + 7k = (1, +1, 7)$$

$5x + 2y - z + 1 = 0$

$4x + 3y - z + 2 = 0$

Admitindo $x = 0$

$P = (-2, 0, 1)$

$\begin{cases} 2y - z = -1 \quad (x-1) \\ 3y - z = -2 \end{cases}$ $\begin{cases} -2y + z = 1 \\ 3y - z = -2 \end{cases}$

$y = -1$

$2 \cdot (-1) - z = -1$

$-z = -1 + 2$

$-z = 1 \quad z = -1$

$r: X = (0,-1,-1) + a(1,+1,7); a \in \mathbb{R}$

$d(P, r) = \dfrac{|(1,+1,7) \wedge (-2,-1,-2)|}{\sqrt{1+1+49}} = \dfrac{\sqrt{290}}{\sqrt{51}}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & +1 & 7 \\ -2 & -1 & -2 \end{vmatrix} = 5i + 16j - 3k = \sqrt{25 + 256 + 9} = 290$$

/ /

(282-) $\alpha: X=(1,2,2)+a(2,1,0)+b(-1,1,-1), a,b \in \mathbb{R}$

$\beta: x-2y-3z+8=0$ $A=(-8,0,0)$

$$\begin{vmatrix} x-1 & y-2 & z-2 \\ 2 & 1 & 0 \\ -1 & 1 & -1 \end{vmatrix} = 0 \qquad -(x-1)+2(y-2)+3(z-2)=0$$

$$-x+2y+3z-9=0$$

Os planos são paralelos

$$d(\alpha,\beta)=\frac{|8-9|}{\sqrt{1+4+9}} = \frac{1}{\sqrt{14}} \text{ ou } \frac{\sqrt{14}}{14}$$

(283-) $\alpha: x+y-2z+3=0$ $r: X=(1,1,1)+t(-1,5,2); t \in \mathbb{R}$

$\beta: 3x+y-z-1=0$ $s: X=(0,5,4)+t'(1,-5,-2); t' \in \mathbb{R}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 3 & 1 & -1 \end{vmatrix} = i-5j-2k$$

Admitindo $x=0$

$$\begin{cases} y-2z=-3 \\ -y+z=-1 \end{cases} \qquad -y+4=-1$$

$y=5$

$-z=-4$

$z=4$

$(-1,5,2)=m(1,-5,-2)$

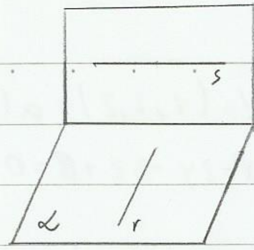
$m=\frac{-1}{1}=\frac{5}{-5}=\frac{2}{-2}$ "Como dois vetores são paralelos, as retas são linearmente dependente".

$$d(r,s)=\frac{|(1,-4,-3)\wedge(-1,5,2)|}{\sqrt{1+25+4}} = \frac{\sqrt{51}}{\sqrt{30}} \begin{matrix}\div 3\\ \div 3\end{matrix} = \frac{\sqrt{17}}{\sqrt{10}} = \sqrt{\frac{17}{10}}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -4 & -3 \\ -1 & 5 & 2 \end{vmatrix} = 7i+j+\vec{k} \Rightarrow \sqrt{49+1+1} = \sqrt{51}$$

(284-) r: $X=(1,-1,3)+a(1,0,2)$; $a \in \mathbb{R}$

s: $X=(2,1,0)+b(0,1,4)$; $b \in \mathbb{R}$



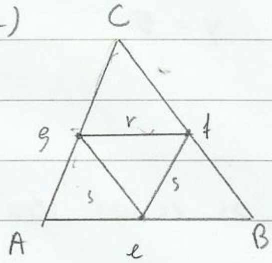
$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 1 & 2 & -3 \end{vmatrix}\begin{matrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{matrix} = -3-2-8=-13 \neq 0 \quad \therefore \text{Portanto São reversas}$$

α: $X=(1,-1,3)+a(1,0,2)+b(0,1,4)$; $a,b \in \mathbb{R}$

$$\begin{vmatrix} x-1 & y+1 & z-3 \\ 1 & 0 & 2 \\ 0 & 1 & 4 \end{vmatrix} = 0 \qquad \begin{aligned} &-2(x-1)-4(y+1)+1(z-3)=0 \\ &-2x-4y+z-5=0 \end{aligned}$$

$$d(P,\alpha)=\frac{|-2.2-4.1+1.0-5|}{\sqrt{4+16+1}} = \frac{13}{\sqrt{21}} = \frac{13\sqrt{21}}{21}$$

(285-)



$\frac{A+B}{2} = (-1,0,2)$ e / $\frac{B+C}{2} = (-2,0,0)$ f / $\frac{C+A}{2} = (1,1,1)$ g

$d(e,f)=\sqrt{1+0+4}=\sqrt{5}$

$d(f,g)=\sqrt{9+1+1}=\sqrt{11}$

$d(g,e)=\sqrt{4+1+1}=\sqrt{6}$

$P=\sqrt{5}+\sqrt{11}+\sqrt{6}$

(286-) r: $X=(1,1,1)+a(-3,-1,-1);\ a \in \mathbb{R}$

$$d(E,r)=\frac{|(-3,-1,-1)\wedge(-2,-1,1)|}{\sqrt{9+1+1}} = \frac{2\sqrt{30}}{\sqrt{11}}\cdot 2 = \frac{2\sqrt{30}}{11}$$

$$\left|\begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ -3 & -1 & -1 \\ -2 & -1 & 1 \end{array}\right| = -2i+5j+k \qquad |(-2,5,1)| = \sqrt{4+25+1} = \sqrt{30}$$

s: $X=(2,1,3)+a(2,0,4);\ a \in \mathbb{R}$

$$d(B,s)=\frac{|(2,0,4)\wedge(-6,-2,-2)|}{\sqrt{4+0+16}} = \frac{\sqrt{480}}{\sqrt{20}} = 2\sqrt{6}$$

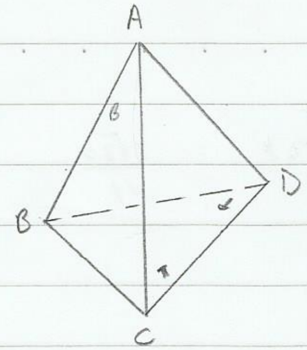
$$\left|\begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 4 \\ -6 & -2 & -2 \end{array}\right| = 8\vec{i}-20\vec{j}-4\vec{k} \Rightarrow |(8,-20,-4)| = \sqrt{480}$$

t: $X=(0,1,-1)+c(-4,-2,2);\ c \in \mathbb{R}$

$$d(A,t)=\frac{|(-4,-2,2)\wedge(2,0,4)|}{\sqrt{16+4+4}} = \frac{\sqrt{480}}{\sqrt{24}} = 2\sqrt{5}$$

$$\left|\begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ -4 & -2 & 2 \\ 2 & 0 & 4 \end{array}\right| = -8\vec{i}+20\vec{j}+4\vec{k} \Rightarrow |(-8,20,4)| = \sqrt{480}$$

(287-)



$\alpha: X=(3,1,0)+a(-5,0,0)+b(-4,1,-2); a,b \in \mathbb{R}$

$$\begin{vmatrix} x-3 & y-1 & z \\ -5 & 0 & 0 \\ -4 & 1 & -2 \end{vmatrix} = 0 \qquad \begin{array}{l} -10(y-1)-5(z)=0 \\ \alpha: -2y-z+2=0 \end{array}$$

$$d(A,\alpha)=\frac{|-2-4+2|}{\sqrt{4+1}} = \frac{4}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$$

$\beta: X=(1,1,4)+a(-3,0,-4)+b(2,0,-4); a,b \in \mathbb{R}$

$$\begin{vmatrix} x-1 & y-1 & z-4 \\ -3 & 0 & -4 \\ 2 & 0 & -4 \end{vmatrix} = 0 \qquad \begin{array}{l} -20(y-1)=0 \\ -20y+20=0 \\ -y+1=0 \end{array}$$

$$d(D,\beta)=\frac{|-2+1|}{\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \quad \frac{\sqrt{2}}{2}$$

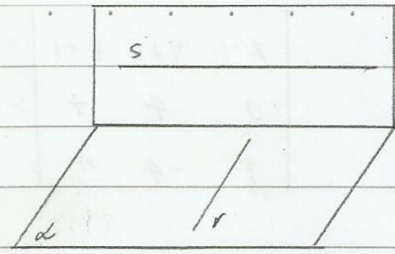
$\pi: X=(1,1,4)+a(2,0,-4)+b(-2,1,-6); a,b \in \mathbb{R}$

$$\begin{vmatrix} x-1 & y-1 & z-4 \\ 2 & 0 & -4 \\ -2 & 1 & -6 \end{vmatrix} = 0 \qquad \begin{array}{l} 4(x-1)+20(y-1)+2(z-4)=0 \\ 2(x-1)+10(y-1)+(z-4)=0 \\ 2x+10y+z-16=0 \end{array}$$

$$d(B,\pi)=\frac{|-4+10-16|}{\sqrt{4+100+1}} = \frac{10}{\sqrt{105}}\cdot\frac{\sqrt{105}}{\sqrt{105}} = \frac{10\sqrt{105}}{105} \Rightarrow \frac{2\sqrt{105}}{21}$$

| / / |

(288-) $r: X=(-4,0,-5)+a(3,4,-2); a \in \mathbb{R}$
$s: X=(m,7,5)+b(6,-4,-1); b \in \mathbb{R}$



$$\alpha: \begin{vmatrix} x+4 & y & z+5 \\ 3 & 4 & -2 \\ 6 & -4 & -1 \end{vmatrix} = 0 \qquad \begin{aligned} &-12(x+4)-9(y)-36(z+5)=0 \\ &-4(x+4)-3(y)-12(z+5)=0 \\ &-4x-3y-12z-76=0 \end{aligned}$$

$$d(s,r) = \frac{|-4m-21-60-76|}{\sqrt{16+9+144}} = \frac{|-4m-157|}{13} = 13$$

$$|-4m-157| = 169 \qquad \begin{cases} -4m-157=169 & m=-163/2 \\ 4m+157=169 & m=3 \end{cases}$$

(289-)

$$\begin{aligned} 1+t &= 1+2t' \\ 1-t &= 4+t' \\ 1+t &= t' \end{aligned} \qquad \begin{aligned} 1-t &= 4+1+t \\ -4 &= 2t \\ t &= -2 \quad t'=-1 \end{aligned} \qquad P=(-1,3,-1)$$

$$d(P,\alpha) = \frac{|-2-3-1+1|}{\sqrt{4+1+1}} = \frac{5}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$$

(290-)

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 0 \qquad \begin{aligned} &-2(x-2)+3(y-1)-z=0 \\ &-2x+3y-z+1=0 \end{aligned}$$

$$d(A,\alpha) = \frac{|-2+6+1|}{\sqrt{4+9+1}} = \frac{5}{\sqrt{14}} \text{ ou } \frac{5\sqrt{14}}{14}$$

/ /

(291-) $\begin{vmatrix} x-1 & y+1 & z-1 \\ 0 & 4 & 4 \\ 1 & -4 & 1 \end{vmatrix} = 0$ $\quad 20(x-1)+4(y+1)-4(z-1)=0$

$5(x-1)+(y+1)-(z-1)=0$

$5x+y-z-3=0$

$d(P,\pi) = \dfrac{|-10+10-3|}{\sqrt{25+1+1}} = \dfrac{3}{\sqrt{27}} = \dfrac{3\sqrt{27}}{27} \quad \dfrac{\sqrt{27}}{9}$

(292-) $d(A,r) = \dfrac{|(1,3,3)\wedge(1,-1,-1)|}{\sqrt{1+9+9}} = \dfrac{\sqrt{32}}{\sqrt{19}}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 3 \\ 1 & -1 & -1 \end{vmatrix} = +4\vec{j} - 4\vec{k} \quad |(0,4,4)| = \sqrt{32}$

(293-) $d(A,\alpha) = \dfrac{|2|}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

(294-) $\vec{AB} = (3,4,5) \quad |\vec{AB}| = \sqrt{50}$

$\vec{BC} = (-1,-3,-7) \quad |\vec{BC}| = \sqrt{59}$

$\vec{CA} = (2,1,-2) \quad |\vec{CA}| = 3$

$|AB|^2 = |BC|^2 - |CA|^2$

$50 = 59 - 9$

$50 = 50$

É retangulo

(295.)

(296.) $d(P,\pi) = \dfrac{|0-1+3-2|}{\sqrt{6}} = \dfrac{0}{\sqrt{6}} = \underline{\underline{0}}$

(297.) $d(P,r) = \dfrac{|(0,-1,2) \wedge (1,1,0)|}{\sqrt{5}} = \dfrac{3}{\sqrt{5}} = \dfrac{3\sqrt{5}}{5}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & 2 \\ 1 & 1 & 0 \end{vmatrix} = -2\vec{i} + 2\vec{j} + \vec{k} \quad |(-2,2,1)| = 3$$

(298.) $\begin{vmatrix} x & y & z-1 \\ 2 & -1 & 0 \\ 1 & -1 & -2 \end{vmatrix} = 0$ $\quad 2x + 4y - (z-1) = 0$

$2x + 4y - z + 1 = 0$

$d(P,\pi) = \dfrac{|1+1|}{\sqrt{21}} = \dfrac{2}{\sqrt{21}}$

(299.) $(0, a, 0)$

$$\sqrt{1+(1-a)^2+1} = \sqrt{4+(-2-a)^2+9}$$
$$1-2a+a^2+2 = 4+4+4a+a^2+9$$
$$-2a+a^2-4a-a^2 = 17-3$$
$$-6a = 14$$
$$a = \frac{14}{-6} = -\frac{7}{3}$$

(300.) $d(r,s) = \dfrac{|(1,-1,3)\wedge(2,2,2)|}{\sqrt{1+1+9}} = \dfrac{\sqrt{96}}{\sqrt{11}}$.

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & 3 \\ 2 & 2 & 2 \end{vmatrix} = -8i+4j+4k \qquad |(-8,4,4)| = \sqrt{96}$$

(301.) $\begin{vmatrix} x & y & z-7 \\ 2 & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$ $\quad -2x-2y+2(z-7)=0$

$-x-y+z-7=0$

$$d(P,\pi) = \frac{|-6|}{\sqrt{3}} = \frac{6}{\sqrt{3}}. \quad 2\sqrt{3}$$

(302.) $\begin{vmatrix} x-1 & y-1 & z-7 \\ 2 & 1 & 3 \\ -1 & 1 & 0 \end{vmatrix} = 0$ $\quad -3x-3y+3(z-7)=0$

$-x-y+z-7=0$

$$d(P,\alpha) = \frac{|-1-1+1-7|}{\sqrt{3}} = \frac{8}{\sqrt{3}} \text{ ou } \frac{8\sqrt{3}}{3}$$

(303-) $\begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & 1 & 3 \\ -1 & 1 & 0 \end{vmatrix} = 0$ $\quad -3(x-1)-3(y-1)+3(z-1)=0$

$-x-y+z+1=0$

$$d(P,\pi) = \frac{|1|}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

(304-) $r: X=(-2,1,-4)+a(5,-12,9); a \in \mathbb{R}$

$$d(P,r) = \frac{|(5,-12,9) \wedge (-1,4,-8)|}{\sqrt{25+144+81}} = \frac{\sqrt{4625}}{\sqrt{250}} = \frac{\sqrt{925}}{\sqrt{50}} = \frac{\sqrt{185}}{\sqrt{10}} = \frac{\sqrt{37}}{\sqrt{2}}.$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & -12 & 9 \\ -1 & 4 & -8 \end{vmatrix} = 60i + 31j + 8k = |(60,31,8)| = \sqrt{4625}$$

(305-) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -\vec{i}+\vec{j}+\vec{k} \Rightarrow (-1,1,1)$

$x+0y+z+1=0$ Admitindo $x=0 \Rightarrow y=0 \quad z=-1$

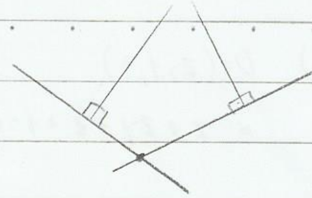
$0x+y-z-1=0$ $\quad r: X=(0,0,-1)+a(-1,1,1); a \in \mathbb{R}$

$$d(p,r) = \frac{|(-1,1,1) \wedge (-1,-1,-2)|}{\sqrt{3}} = \frac{\sqrt{14}}{\sqrt{3}}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ -1 & -1 & -2 \end{vmatrix} = -i-3j+2k \quad |(-1,-3,2)| = \sqrt{14}$$

(305-) $P=(1,1,1)$ $\quad \alpha: x+z+1=0 \quad W_\alpha (1,0,1)$

$\beta: y-z-1=0 \quad W_\beta (0,1,-1)$

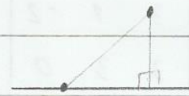


$W_\alpha \wedge W_\beta:$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -\vec{i}+\vec{j}+\vec{k} \Rightarrow (-1,1,1)$

Admitindo $x=0$

$z+1=0 \Rightarrow z=-1$

$y-z=1 \Rightarrow y-(-1)=1 \Rightarrow y=0$



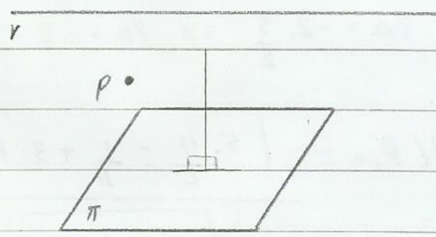
$r: X=(0,0,-1)+t(-1,1,1);\ t\in\mathbb{R}$

$d(P,r) = \dfrac{|(1,1,2)\wedge(-1,1,1)|}{|(1,1,1)|} \Rightarrow \dfrac{\sqrt{14}}{\sqrt{3}}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ -1 & 1 & 1 \end{vmatrix} = -\vec{i}-3\vec{j}+2\vec{k} \Rightarrow (-1,-3,2) \quad |(-1,-3,2)| \Rightarrow \sqrt{14}$

(306-) $\pi: 2x-3y+6z+1=0$

$X=(1,3,0)+t(3,4,0);\ t\in\mathbb{R}$



$W_\pi=(2,-3,6)$

$d(r,\pi) = \dfrac{|2.4-3.3+6.0+1|}{\sqrt{2^2+(-3)^2+6^2}} =$

(307-) $D=(2,1,3)$

$\alpha: x+2y+z+1=0$

$$d(P,\alpha) = \frac{|1.2+2.1.+1.3+1|}{\sqrt{1^2+2^2+1^2}} = \frac{8}{\sqrt{6}}.\frac{\sqrt{6}}{\sqrt{6}} = \frac{8\sqrt{6}}{6} = \frac{4\sqrt{6}}{3}$$

(308-) $r1: \ 4-x = y-1 = \frac{z}{2} \Rightarrow X=(4,1,0)+a(-1,1,2);\ a\in\mathbb{R}$

$r2: \ \frac{x-1}{3} = \frac{y+2}{1} = \frac{z}{-2} \Rightarrow X=(1,-2,0)+b(3,1,-2);\ b\in\mathbb{R}$

$\alpha: 5x+y-z+1=0$

$$\left|\begin{array}{ccc|cc} -1 & 1 & 2 & -1 & 1 \\ 3 & 1 & -2 & 3 & 1 \\ 3 & 3 & 0 & 3 & 3 \end{array}\right| = -6+18-6-6=0 \ \therefore$$ Portanto são coplanares

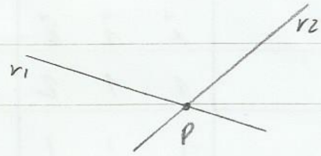
$(-1,1,2)=m(3,1,-2) \Rightarrow m = \frac{-1}{3} \neq \frac{1}{1} \neq \frac{2}{-2}$ ∴ Portanto são concorrentes

$4-a=1+3b \qquad a=\frac{-2b}{2} \Rightarrow a=-b$

$1+a=-2+b$

$2a=-2b \qquad 4-a=1+3b$

$4+b=1+3b \Rightarrow 2b=3 \Rightarrow b=3/2$



$2a=-2.\frac{3}{2} \Rightarrow 2a=-3 \quad a=\frac{-3}{2}$

$P=\left(\frac{11}{2}, \frac{-1}{2}, -3\right)$

$$d(P,\alpha) = \frac{\left|5.\frac{11}{2} - \frac{1}{2} + 3 + 1\right|}{\sqrt{5^2+1^2+(-1)^2}} = \frac{31}{\sqrt{27}}.\frac{\sqrt{27}}{\sqrt{27}} = \frac{31\sqrt{27}}{27} = \frac{31.3\sqrt{3}}{27}$$

$$d(P,\alpha) = \frac{31\sqrt{3}}{9}$$

(309.) $P = (4,-1,3)$

$X = (2,-1,1) + t(0,3,-2); t \in \mathbb{R}$ $\quad A: (1,-5,6)$

$$\begin{vmatrix} x-2 & y+1 & z-1 \\ 0 & 3 & -2 \\ 1 & 4 & -5 \end{vmatrix} = -7(x-2) - 2(y+1) - 3(z-1) = 0$$

$-7x - 2y - 3z + 15 = 0$ ou $7x + 2y + 3z - 15 = 0$

$$d(P,\alpha) = \frac{|7.4 + 2.(-1) + 3.3 - 15|}{\sqrt{7^2+2^2+3^2}} = \frac{20}{\sqrt{62}}$$

?

(310.) $r: X = (0,1,3) + t(1,1,-2); t \in \mathbb{R}$

$s: X = (1,-3,0) + t'(3,3,-6); t' \in \mathbb{R}$

$$d(r,s) = \frac{|(1,1,-2) \wedge (1,-4,-3)|}{\sqrt{1^2+1^2+(-2)^2}} = \frac{\sqrt{(-11)^2+1^2+(-5)^2}}{\sqrt{6}} = \frac{\sqrt{147}}{\sqrt{6}}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 1 & -4 & -3 \end{vmatrix} = -11\vec{i} + \vec{j} - 5\vec{k} = (-11,1,-5)$$

$$R = \frac{\sqrt{147}}{\sqrt{6}} : 3 = \frac{\sqrt{49}}{\sqrt{2}} = \frac{7}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$$

(311-) $r_1: X=(1,4,0)+a(2,1,1);\ a\in\mathbb{R}$ $\qquad \alpha: 2x-y-z-6=0$

$r_2: X=(1,1,3)+b(1,-1,2);\ b\in\mathbb{R}$

$1+2a=1+b \qquad 1+2(3+2b)=1+b \qquad P=(-1,3,-1)$

$4+a=1-b \qquad 1+6+4b=1+b$

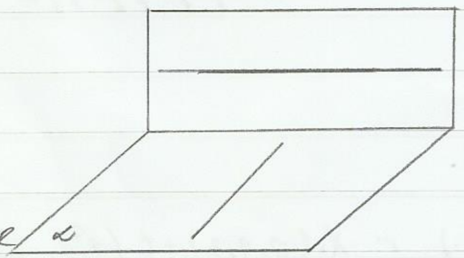
$a=3+2b \qquad 3b=-6$

$b=-2$

$$d(P,\alpha)=\frac{|2.(-1)-1.3-1.(-1)-6|}{\sqrt{2^2+1^2+1^2}}=\frac{12}{\sqrt{6}}.\frac{\sqrt{6}}{\sqrt{6}}=\frac{12\sqrt{6}}{6}=2\sqrt{6}$$

(312-) $r_1: X=(1,5,1)+t(1,5,-6);\ t\in\mathbb{R}$

$r_2: X=(2,3,-1)+t'(2,1,0);\ t'\in\mathbb{R}$



$\alpha: X=(1,5,1)+t(1,5,-6)+t'(2,1,0);\ t,t'\in\mathbb{R}$

$$\begin{vmatrix} x-1 & y-5 & z-1 \\ 1 & 5 & -6 \\ 2 & 1 & 0 \end{vmatrix}=0$$

$6(x-1)-12(y-5)-9(z-1)=0$

$2(x-1)-4(y-5)-3(z-1)=0$

$2x-4y-3z+21=0$

$$d=\frac{|2.2-4.3-3.(-1)+21|}{\sqrt{2^2+4^2+3^2}}=\frac{16}{\sqrt{29}}$$

(313) $O = (0,0,0)$

$$r: \begin{cases} x = 1+2t \\ y = 3t \\ z = 1-t \end{cases} \qquad r: X = (1,0,1) + t(2,3,-1);\ t \in \mathbb{R}$$

$\alpha: x - z + 3 = 0 \qquad \beta: x + y - 6 = 0$

$A = r \cap \alpha \qquad B = r \cap \beta$

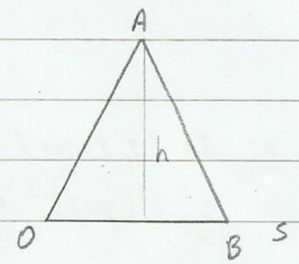
$x - z + 3 = 0$ $\qquad x = 1 + 2(-1) = -1$

$(1+2t) - (1-t) + 3 = 0$ $\qquad y = 3(-1) = -3$

$1 + 2t - 1 + t + 3 = 0$ $\qquad z = 1 - (-1) = 2$

$3t = -3$

$t = -1$ $\qquad A = (-1,-3,2)$



$x + y - 6 = 0$ $\qquad x = 1 + 2.1 = 3$

$(1+2t) + 3t - 6 = 0$ $\qquad y = 3.1 = 3$

$1 + 2t + 3t - 6 = 0$ $\qquad z = 1 - 1.1 = 0$

$5t = 5$

$t = 1$ $\qquad B = (3,3,0)$

$$BO = \sqrt{(3-0)^2 + (3-0)^2 + (0-0)^2} = \sqrt{18} = 3\sqrt{2}$$

$s: X = (0,0,0) + a(3,3,0);\ a \in \mathbb{R}$

$$d(A,s) = \frac{\|(3,3,0) \wedge (-1,-3,2)\|}{\sqrt{3^2+3^2+0^2}} = \frac{6\sqrt{3}}{3\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{3}.\sqrt{2}$$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 3 & 0 \\ -1 & -3 & 2 \end{vmatrix} = 6\bar{i} - 6j - 6K \qquad \|(6,-6,-6)\| = \sqrt{108} = 6\sqrt{3}$$

$$A = \frac{3\sqrt{2}.\sqrt{3}.\sqrt{2}}{2} = \frac{3.\cancel{2}\sqrt{3}}{\cancel{2}} = \underline{3\sqrt{3}}$$

(317-) $\alpha: x+y+z+m=0$

$S: x^2+(y-1)^2+z^2=4 \qquad C=(0,1,0) \quad R=2$

$d(C,\alpha)=\dfrac{|1+m|}{\sqrt{3}}$

Para que ele seja tangente $\dfrac{1+m}{\sqrt{3}}=2 \Rightarrow m=\pm 2\sqrt{3}-1$

Para que ele seja secante $\dfrac{1+m}{\sqrt{3}}<2 \Rightarrow -1-\sqrt{12}<m<-1+\sqrt{12}$

Para que ele seja externa $\dfrac{1+m}{\sqrt{3}}>2 \quad m>-1+\sqrt{12} \quad m<-1-\sqrt{12}$

(316-) $r: X=(1,2,m)+\lambda(1,1,1);\ \lambda\in\mathbb{R}$

$S: x^2+(y-1)^2+z^2=4 \qquad C=(0,1,0) \quad R=2$

(319-)

A) $r: X = (1,-3,5) + \lambda(0,1,4);\ \lambda \in \mathbb{R}$

$S: x^2+y^2+z^2-6x+4y-16=0$

$$\begin{cases} -2a = -6 & a = 3 \\ -2b = 4 & b = -2 \\ -2c = 0 & c = 0 \end{cases}$$

$C = (a,b,c)$

$C = (3,-2,0)$

$x^2+y^2+z^2-R^2 = -16$

$3^2+(-2)^2+0^2-R^2 = -16$

$9+4-R^2 = -16$

$R^2 = 29 \quad R = \sqrt{29}$

$$d(C,r) = \frac{|(0,1,4)\wedge(2,1,-5)|}{\sqrt{0^2+1^2+4^2}} = \frac{\sqrt{149}}{\sqrt{17}}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 4 \\ 2 & 1 & -5 \end{vmatrix} = -9\vec{i}+8\vec{j}-2\vec{k} \Rightarrow (-9,8,-2) \Rightarrow m = \sqrt{81+64+4} = \sqrt{149}$$

$x = 1 \quad y = -3+a \quad z = 5+4a$

$1^2+(-3+a)^2+(5+4a)^2-6+4(-3+a)-16=0$

$1+9-6a+a^2+25+40a+16a^2-6-12+4a-16=0$

$17a^2+38a+1=0$

$$x = \frac{-38 \pm \sqrt{1381}}{34}$$

NA1210 - Prova P2

Questão 1: $r: X=(1,2,1)+t(2,2,3),\ t\in\mathbb{R}$

$A=(0,1,3) \quad B=(4,5,9)$

a-)

$s: X=(0,1,3)+t'(4,4,6),\ t'\in\mathbb{R}$

$$\begin{vmatrix} 2 & 2 & 3 \\ 4 & 4 & 6 \\ 1 & 1 & -2 \end{vmatrix}\begin{matrix} 2 & 2 \\ 4 & 4 \\ 1 & 1 \end{matrix} = -\cancel{16}+12+12-12-12+\cancel{16}=0$$

∴ As retas são coplanares e paralelas, pois:

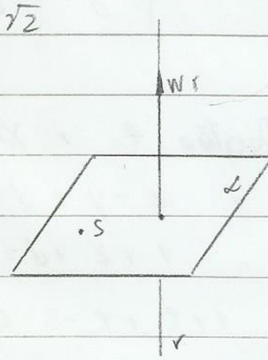
$(2,2,3)=m(4,4,6)$

$m=\frac{2}{4}=\frac{2}{4}=\frac{3}{6}$

b-) $d(r,s)=\dfrac{|(2,2,3)\wedge(1,1,-2)|}{\sqrt{2^2+2^2+3^2}} = \dfrac{7\sqrt{2}}{\sqrt{17}}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 3 \\ 1 & 1 & -2 \end{vmatrix} = -7\vec{i}+7\vec{j} \Rightarrow (-7,7,0)$$

$|(-7,7,0)| = \sqrt{98}$ ou $7\sqrt{2}$



Questão 2: $A=(2,1,1) \quad r: X=(1,-2,2)+t(-2,1,3);\ t\in\mathbb{R}$

$\alpha\ \ -2x+y+3z+d=0 \qquad \alpha: -2x+y+3z=0$

$-2.2+1+3.1+d=0 \Rightarrow d=0$

$-2(1-2a)+(-2+a)+3(2+3a)=0$

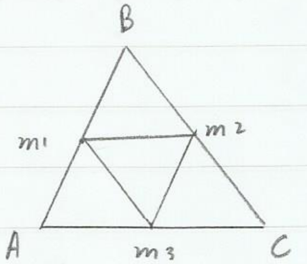
$-2+4a-2+a+6+9a=0 \quad 14a=-2 \Rightarrow a=-1/7$

$x=1+2/7=9/7 \qquad s: X=(2,1,1)+m(-5/7,-22/7,4/7),\ m\in\mathbb{R}$

$y=-2-1/7=-15/7 \qquad \dfrac{x-2}{-5}=\dfrac{y-1}{-22}=\dfrac{z-1}{4}$

$z=2-3/7=11/7$

Questão 3:



$M_1 = (2,-1,3)$ $M_2 = (1,-3,0)$ $M_3 = (2,1,-5)$

$\vec{v} = (2-1, 1+3, -5-0) \Rightarrow \vec{v} = (1,4,-5)$

$r: X = (2,-1,3) + t(1,4,-5); t \in \mathbb{R}$

$$r: \begin{cases} 2+t \\ -1+4t \\ 3-5t \end{cases} ; t \in \mathbb{R}$$

$d(M_1, M_2) = \sqrt{(2-1)^2 + (-1+3)^2 + (3-0)^2} = \sqrt{14}$

$d(M_2, M_3) = \sqrt{(1-2)^2 + (-3-1)^2 + (0+5)^2} = \sqrt{42}$

$d(M_3, M_1) = \sqrt{(2-2)^2 + (1+1)^2 + (-5-3)^2} = \sqrt{68}$

$P = 2 . (\sqrt{14} + \sqrt{42} + \sqrt{66})$

$\alpha: X = (2,-1,3) + a(1,2,3) + b(0,-2,8), a, b \in \mathbb{R}$

$$\begin{vmatrix} x-2 & y+1 & z-3 \\ 1 & -2 & 3 \\ 0 & -2 & 8 \end{vmatrix} = 0$$

$22(x-2) - 8(y+1) - 2(z-3) = 0$

$11(x-2) - 4(y+1) - (z-3) = 0$

$11x - 4y - z - 23 = 0$

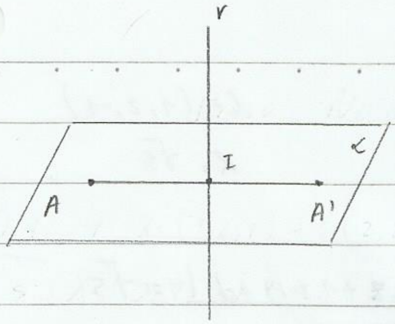
Questão 4: $r: X = (1,0,1) + t(1,1,0); t \in \mathbb{R}$ $A = (1,2,-1)$

$\alpha: x + y + d = 0$ $\alpha: x + y - 3 = 0$

$1 + 2 + d = 0$ $d = -3$

$1 + t + t - 3 = 0$ $t = 1$ $s: X = (1,2,-1) + m(1,-1,2); m \in \mathbb{R}$

$(x,y,z) = (2,1,1)$ $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$



$A = (1, 2, -1)$

$r: X = (1,0,1) + t(1,1,0);\ t \in \mathbb{R}$

$\vec{w}_\alpha = (1,1,0)$

$\alpha: x + y + d = 0 \qquad \alpha: x + y - 3 = 0$

$r \cap \alpha = (2,1,1) \qquad \dfrac{A+B}{2} = \dfrac{(1,2,-1)+(a,b,c)}{2} = (2,1,1)$

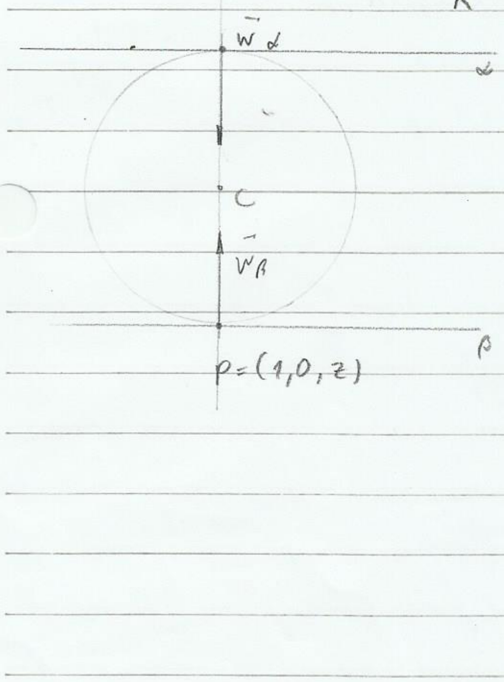
$(a,b,c) = (3,0,3)$

Questão 5: $\quad S: x^2 + y^2 + z^2 - 2x - 4y + 2z + 1 = 0$

$$\begin{cases} -2a = -2 \\ -2b = -4 \\ -2c = 2 \end{cases} \Rightarrow C = (a,b,c) = (1,2,-1)$$

$1^2 + 2^2 + (-1)^2 - R^2 = 1$

$R^2 = 5 \Rightarrow R = \sqrt{5}$



$d(C,P) = \sqrt{5}$

$d(C,P) = \sqrt{(1-1)^2 + (2-0)^2 (-1+z)^2} = \sqrt{5}$

$\cancel{4} + \cancel{1} - 2z + z^2 = \cancel{5}$

$z(z-2) = 0$

$z = 0 \quad z = 2$

$\alpha: (1-1)x + (2-0)y + (-1-0)z + d = 0$

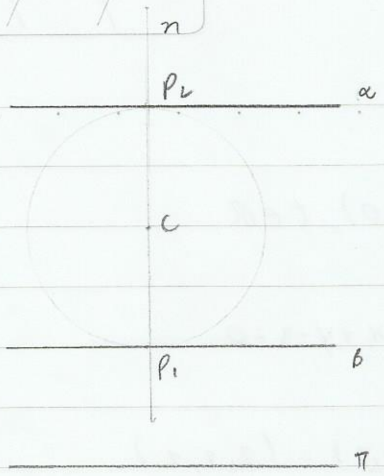
$2y - z + d = 0 \qquad \underline{\alpha: 2y - z = 0}$

$2.0 - 1.0 + d = 0$

$\beta: (1-1)x + (2-0)y + (-1-2)z + d = 0$

$2y - 3z + d = 0 \qquad \underline{\beta: 2y - 3z + 6 = 0}$

$2.0 - 3.2 + d = 0 \quad d = 6$



$\pi: x + 2y + z - 1 = 0$ $\quad C = (1,2,-1)$

$\vec{w}_\pi = (1,2,1)$ $\quad R = \sqrt{5}$

$$d(C,\rho) = \frac{|1.1 + 2.2 + 1.(-1) + d|}{\sqrt{1^2+2^2+(-1)^2}} = \sqrt{5}$$

$$d(C,\rho) = |4+d| = \sqrt{5}.\sqrt{6}$$

$$|4+d| = \sqrt{30}$$

$$d = -4 \pm \sqrt{30}$$

$\alpha: x + 2y + z - 4 + \sqrt{30} = 0$

$\beta: x + 2y + z - 4 - \sqrt{30} = 0$

MA1210 - Prova P2

1ª Questão: $r: X=(2,1,1)+t(1,-2,1);\ t\in\mathbb{R}$

$\alpha: X=(0,0,1)+a(0,1,-1)+b(1,0,-1);\ a,b,\in\mathbb{R}$

a) $\dfrac{x-2}{1}=\dfrac{y-1}{-2}=\dfrac{z-1}{1}$

b) $\begin{vmatrix} x & y & z-1 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{vmatrix}=0$ $\quad \alpha: -x-y-z+1=0$

ou

$\alpha: x+y+z-1=0$

$P=(1,0,0)$

$d(r,\alpha): \dfrac{|(1,-2,1)\wedge(1,1,1)|}{\sqrt{1^2+(-2)^2+1^2}} = \dfrac{3\sqrt{2}}{\sqrt{6}}$ ou $\sqrt{3}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -3\vec{i}+3\vec{k} \Rightarrow (-3,0,3)$

$|(-3,0,3)|=\sqrt{18}=3\sqrt{2}$

2ª Questão: $P=(1,1,1)$

$r: X=(1,-2,3)+t(0,-1,1);\ t\in\mathbb{R}$

$\alpha: x+y+z-1=0$

a) $\beta: x+y+z+d=0$, substituindo com o valor das coordenadas do ponto P

$1.1+1.1+1.1+d=0 \Rightarrow d=-3$

$\beta: x+y+z-3=0$

b) $\gamma: X=(1,-2,3)+a(0,-1,1)+b(1,1,1);\ a,b \in \mathbb{R}$

$$\begin{vmatrix} x-1 & y+2 & z-3 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$-2(x-1)+(y+2)+(z-3)=0$

$\gamma: -2x+y+z+1=0$ ou

$\gamma: 2x-y-z-1=0$

3ª Questão: $A=(1,2,-5)$ $B=(5,0,-7)$

$\alpha: 7x+9y+5z=0$

a) $7.1+9.2+5.(-5)=0$ $\therefore$ O ponto A pertence ao plano α

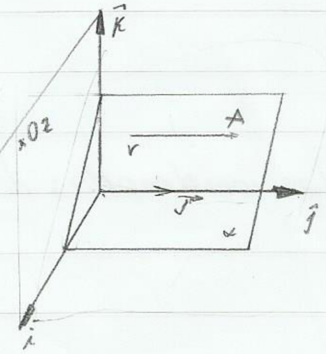
$7.5+9.0+5.(-7)=0$ $\therefore$ O ponto B pertence ao plano α

b) $P_m = \dfrac{A+B}{2} = \dfrac{(1+5,\ 2+0,\ -5-7)}{2} = (3,1,-6)$

$W\alpha = (7,9,5)$

$t: X=(3,1,-6)+a(7,9,5);\ a \in \mathbb{R}$

4ª Questão $A=(2,3,1)$



$X=(2,3,1)+a(0,1,0)$

$$\begin{cases} x=2+a \\ y=3+a \\ z=1+a \end{cases};\ a \in \mathbb{R}$$

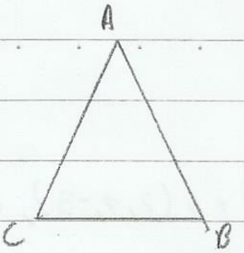
$xOz = y=0$

$y=3+a=0$ $a=-3$

$(2,0,1)$

$$\begin{vmatrix} x-2 & y-3 & z-1 \\ 0 & 1 & 0 \\ & & \end{vmatrix} = 0$$

c)



$AB = \sqrt{(-1)^2 + (-1)^2 + (-2)^2} = \sqrt{6}$

$BC = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$

$CA = \sqrt{(1-2)^2 + (1-3)^2 (-2-1)^2} = \sqrt{14}$

$P = 2\sqrt{14} + \sqrt{6}$

5-) $S: x^2 + y^2 + z^2 - 2x - 4y + 6z - 11 = 0$

$$\begin{cases} -2a = -2 \\ -2b = -4 \\ -2c = 6 \end{cases}$$

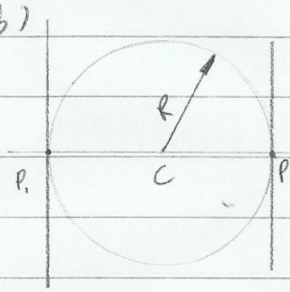
$C = (a, b, c)$

$\underline{C = (1, 2, -3)}$

$1^2 + 2^2 + (-3)^2 - R^2 = -11$

$14 - R^2 = -11 \Rightarrow R^2 = 25 \Rightarrow \underline{R = 5}$

b)



$$d(C,\pi) = \frac{|1 + 2.2 - 2.(-3) + d|}{\sqrt{1^2 + 2^2 + (-2)^2}} = \frac{|11 + d|}{\sqrt{9}} = 5$$

$|11 + d| = 15$

$11 + d = 15 \Rightarrow d = 4 \quad | \quad -11 - d = 15 \Rightarrow d = -26$

$\pi_1: x + 2y - 2z + 4 = 0 \qquad \pi_2: x + 2y - 2z - 26 = 0$

$P = (0, -2, 0)$

$$d(\pi_1, \pi_2) = \frac{|-2.2 - 26|}{\sqrt{9}} = \frac{|-30|}{\sqrt{9}} = 10$$

Questão 5: $\alpha: x + 2y + z + m = 0$

$S: x^2 + y^2 + z^2 - 2x + y - 1 = 0$

$$\begin{cases} -2a = -2 \\ -2b = 1 \\ -2c = -1 \end{cases}$$

$C = (a, b, c)$

$C = (1, \frac{-1}{2}, \frac{1}{2})$

$1 + \frac{1}{4} + \frac{1}{4} - R^2 = -1$

$R^2 = \frac{5}{2} \Rightarrow R = \sqrt{\frac{5}{2}}$

$$d(C, \pi) = \frac{|1 - 1 + \frac{1}{2} + m|}{\sqrt{6}} = \sqrt{\frac{5}{2}}$$

$\frac{1}{2} + m = \sqrt{15} \Rightarrow m = \frac{-1}{2} + \sqrt{15}$

$-\frac{1}{2} - m = \sqrt{15} \Rightarrow m = -\frac{1}{2} - \sqrt{15}$

MA1210 - Prova 02 - 1º semestre de 2009

$\alpha: X = (1,0,0) + a(0,1,2) + b(1,2,3); a, b \in \mathbb{R}$

$r: X = (2,1,3) + \lambda(-1,1,1); \lambda \in \mathbb{R}$ $\quad A = (2,1,4)$

A-1
$$\begin{vmatrix} x-1 & y & z \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

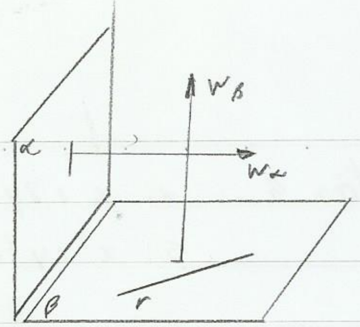
$-(x-1) + 2y - z = 0$

$\pi: -x + 2y - z + 1 = 0$ ou

$\pi: x - 2y + z - 1 = 0$

$1.2 - 2.1 + 4 + d = 0 \Rightarrow d = -4$

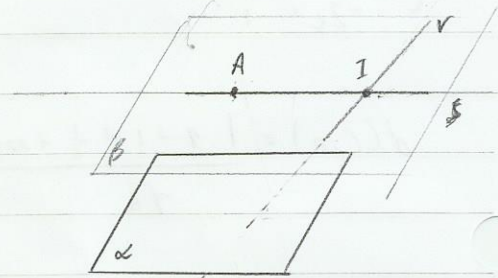
$\pi: x - 2y + z - 4 = 0$



B-) $\beta: X=(2,1,3)+a(-1,1,1)+b(1,-2,1), \; a,b \in \mathbb{R}$

$$\beta: \begin{cases} x = 2-a+b \\ y = 1+a-2b \\ z = 3+a+b \end{cases} ; \; a,b \in \mathbb{R}$$

$\beta: 3x+2y+z-11=0$



C-) $\beta: x-2y+z+d=0$

$2-2.1+4+d=0$

$d=-4$

$\beta: x-2y+z-4=0$

$$r: \begin{cases} x = 2-\lambda \\ y = 1+\lambda \\ z = 3+\lambda \end{cases} ; \; \lambda \in \mathbb{R}$$

$2-\lambda-2(1+\lambda)+3+\lambda-4=0$

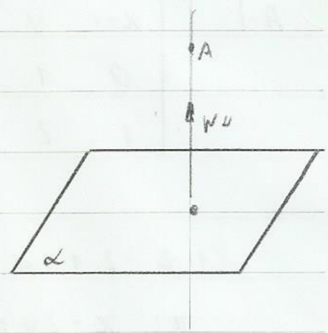
$\cancel{2}-\cancel{\lambda}-\cancel{2}-2\lambda+3+\cancel{\lambda}-4=0$

$2\lambda=-1 \Rightarrow \lambda = \frac{-1}{2}$

$(x,y,z)=\left(\frac{3}{2}, \frac{1}{2}, \frac{5}{2}\right)$

$s: X=(2,1,4)+a\left(\frac{-1}{2}, \frac{1}{2}, \frac{3}{2}\right)$

$$\frac{x-2}{-1} = \frac{y-1}{1} = \frac{z-4}{3}$$



D) $R: X=(2,1,4)+t(1,-2,1); \; t \in \mathbb{R}$

$\alpha: x-2y+z-1=0$

$\cancel{2}+t-\cancel{2}+4t+4+t-1=0$

$6t=-3$

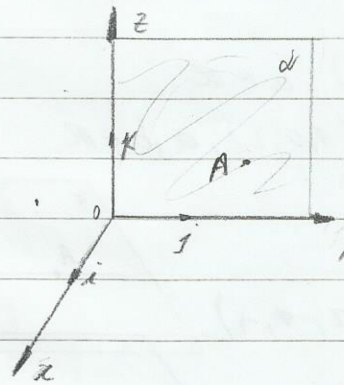
$t=\frac{-1}{2}$ $\quad B=\left(\frac{3}{2}, 2, \frac{7}{2}\right)$

E) $d(A,\alpha) = \dfrac{|1.2-2.1+4.1-1|}{\sqrt{1^2+(-2)^2+1^2}} = \dfrac{3}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} \quad \dfrac{\sqrt{6}}{2}$ u.d

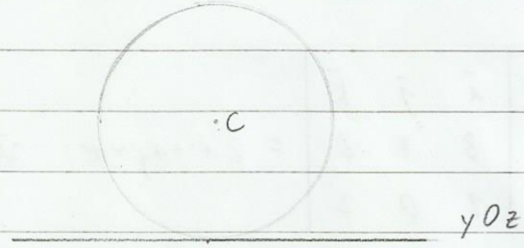
F) $d(A,r) = \dfrac{|(-1,1,1)\wedge(0,0,1)|}{\sqrt{3}} = \dfrac{\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{6}}{3}$ u.d

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i} + \vec{j} \Rightarrow (1,1,0) \quad |(1,1,0)| = \sqrt{2}$$

G) yOz



$C=(2,1,4)$ $\qquad \alpha: y+z=0$



$d(C,\pi) = \dfrac{|2|}{\sqrt{1}} = 2$

$S: (x-a)^2+(y-b)^2+(z-c)^2 = R^2$

$(x-2)^2+(y-1)^2+(z-4)^2 = 4$

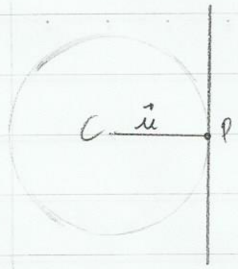
$P=(0,0,0)$

$\vec{v}=(0,0,1)$

$\vec{u}=(0,1,0)$

$$\begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0 \qquad \alpha: -x=0$$

H-)



$C=(2,1,4) \qquad P=(3,\ 2,\ 4-\sqrt{2})$

$S: (x-2)^2+(y-1)^2+(z-4)^2=4$

$\vec{u}=(1,1,-\sqrt{2})$

$\alpha: x+y-\sqrt{2}z+d=0$

$3+2-(4-\sqrt{2}).\sqrt{2}=0$

$5-4\sqrt{2}+2 = 7-4\sqrt{2}$

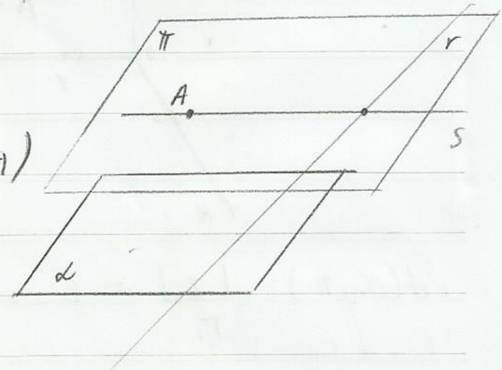
$\alpha: x+y-\sqrt{2}z+7-4\sqrt{2}=0$

Questão 4: $A=(2,1,0)$

$r: X=(1,-1,2)+a(2,1,-3);\ a\in\mathbb{R}$

$\alpha: X=(1,0,1)+a(3,1,2)+b(1,0,2);\ a,b\in\mathbb{R}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = 2i-4j-k \qquad \vec{w} \Rightarrow (2,-4,-1)$$



$\pi: 2x-4y-z+d=0$

$4-4+d=0 \qquad d=0$

$\pi: 2x-4y-z=0$

$2(1+2a)-4(-1+a)-(2-3a)=0$

$2+4a+4-4a-2+3a=0$

$3a=-4$

$a=-\frac{4}{3}$

$I=(-5/3,\ -7/3,\ 6)$

$s: X=(2,1,0)+t\left(-\frac{11}{3},\ \frac{-10}{3},\ 6\right);\ t\in\mathbb{R}$

$s: X=(2,1,0)+t(-11,-10,18),\ t\in\mathbb{R}$

Questão 5.)

$$\begin{cases} -2a = -2 \\ -2b = 1 \\ -2c = -1 \end{cases}$$

$C = (a,b,c)$

$C = (1, -\frac{1}{2}, \frac{1}{2})$

$1 + \frac{1}{4} + \frac{1}{4} - R^2 = -1$

$R^2 = \frac{5}{2}$ $R = \sqrt{\frac{5}{2}}$

$$d(C,\pi) = \frac{|\cancel{1} - \cancel{1} + \frac{1}{2} + d|}{\sqrt{6}} = \sqrt{\frac{5}{2}}$$

$1/2 + d = \sqrt{15}$ $d = \sqrt{15} - 1/2$ $\pi_1: x + 2y + z + \sqrt{15} - 1/2 = 0$

$-1/2 - d = \sqrt{15}$ $d = -\sqrt{15} - 1/2$ $\pi_2: x + 2y + z - \sqrt{15} - 1/2 = 0$

NA1210 - Prova P2 - 1º semestre de 2009

1.) $\alpha: X = (1,0,1) + a(3,1,2) + b(1,0,2);\ a,b \in \mathbb{R}$

$r: X = (1,-1,2) + m(2,1,-3);\ m \in \mathbb{R}$

$A = (2,1,0)$

A)

$$\alpha: \begin{vmatrix} x-1 & y & z-1 \\ 3 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = 0$$

$2(x-1) - 4(y) - (z-1) = 0$

$2x - 4y - z - 1 = 0$

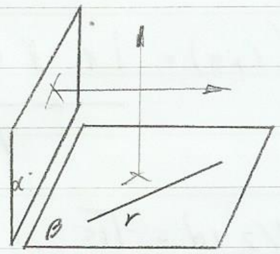
$$\alpha: \begin{cases} x = 1 + 3a; \\ y = a; \\ z = 1 + 2a \end{cases}$$

1 - 1 2

B) $\pi: X=(2,1,0)+a(2,1,-3)+b(1,2,-2); a,b \in \mathbb{R}$

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 1 & -3 \\ 1 & 2 & -2 \end{vmatrix} = 0 \qquad \begin{aligned} &4(x-2)+(y-1)+3(z)=0 \\ &\pi: 4x+y+3z-9=0 \end{aligned}$$

C) $\beta: X=(1,-1,2)+a(2,1,-3)+b(2,-4,-1), a,b \in \mathbb{R}$



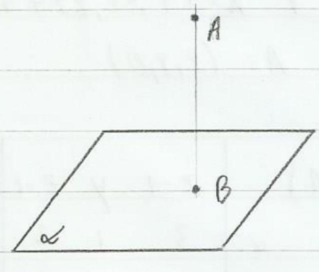
$$\begin{vmatrix} x-1 & y+1 & z-2 \\ 2 & 1 & -3 \\ 2 & -4 & -1 \end{vmatrix} = 0 \qquad \begin{aligned} &-13(x-1)-4(y+1)-10(z-2)=0 \\ &\beta: -13x-4y-10z+29=0 \end{aligned}$$

D) $(1+2m).2+(-1+m)-4+(2-3m)-1-1=0$

$\cancel{2}+\cancel{4m}+4-\cancel{4m}-\cancel{2}+3m-1=0$

$3m=-3$

$m=-1 \qquad I=(-1,-2,5)$



E) $\alpha: 2x-4y-z-1=0$

$N\alpha=(2,-4,-1)$

$t: X=(2,1,0)+a(2,-4,-1); a \in \mathbb{R}$

$2(2+2a)-4(1-4a)-(-a)-1=0$

$\cancel{4}+4a-\cancel{4}+16a+a-1=0$

$21a=1 \quad a=\frac{1}{21}$

$B=(44/21, 17/21, -1/21)$

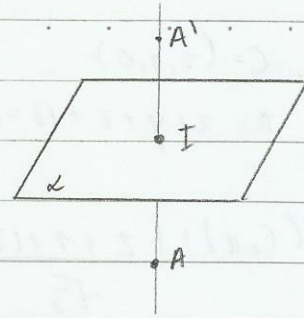
F-) $\alpha: 2x-4y-z-1=0$

$W\alpha = (2,-4,-1)$

$t: (2,1,0) + a(2,-4,-1); a \in \mathbb{R}$

$2(2+2a) - 4(1-4a) - (-a) - 1 = 0$

$I = (44/21, 17/21, -1/21)$



$$\frac{A' + A}{2} = I$$

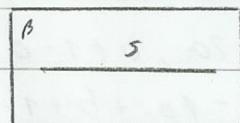
$$\frac{(a,b,c) + (2,1,0)}{2} = (44/21, 17/21, -1/21)$$

$(a,b,c) = (46/21, 13/21, -2/21)$

G-) $s: X = (0,0,1) + \lambda'(2,0,0); \lambda \in \mathbb{R}$

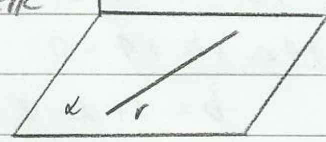
$r: X = (1,-1,2) + m(2,1,-3); m \in \mathbb{R}$

$$d\begin{vmatrix} 2 & 0 & 0 \\ 2 & 1 & -3 \\ 1 & -1 & 1 \end{vmatrix}\begin{matrix} 2 & 0 \\ 2 & 1 \\ 1 & -1 \end{matrix} = 2 - 6 = -4 \neq 0$$ ∴ São reversas



$\alpha: X = (1,-1,2) + m(2,1,-3) + \lambda'(2,0,0); m, \lambda' \in \mathbb{R}$

$$\begin{vmatrix} x-1 & y+1 & z-2 \\ 2 & 1 & -3 \\ 2 & 0 & 0 \end{vmatrix} = \begin{matrix} +6(y+1) + 2(z-2) = 0 \\ +3(y+1) + (z-2) = 0 \\ +3y + z - 1 = 0 \end{matrix}$$



$$d(P,\pi) = \frac{|-1-1|}{\sqrt{10}} = \frac{2}{\sqrt{10}} = \frac{\sqrt{10}}{5} \text{ u.d}$$

H) $C=(2,1,0)$

$\pi: x+y+z+12=0$

$S: (x-a)^2+(y-b)^2+(z-c)^2=R^2$

$S: (x-2)^2+(y-1)^2+z^2=R^2$

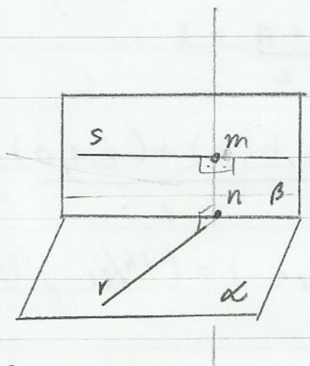
$d(C,\pi)=\dfrac{|2+1+12|}{\sqrt{3}} = \dfrac{15}{\sqrt{3}} = 5\sqrt{3}$

$S: (x-2)^2+(y-1)^2+z^2=75$

Questão 4-

$r: X=(1,-1,2)+a(2,1,-3); a\in\mathbb{R}$

$s: X=(1,0,1)+b(3,1,2), b\in\mathbb{R}$



$\begin{vmatrix} 2 & 1 & -3 \\ 3 & 1 & 2 \\ 0 & 1 & -1 \end{vmatrix}\begin{matrix} 2 & 1 \\ 3 & 1 \\ 0 & 1 \end{matrix} = -2-9-4+3 = -12 \neq 0$

$m=(1+3b, b, 1+2b)$

$n=(1+2a, -1+a, 2-3a)$

$m-n=\vec{v}=(3b-2a, b+1-a, -1+2b+3a)$

$(3b-2a, b+1-a, -1+2b+3a)\cdot(2,1,-3)=0$

$6b-4a+b+1-a+3-6b-9a$

$-14a+b+4=0$

$b=14a-4$

$(3b-2a, b+1-a, -1+2b+3a)\cdot(3,1,2)=0$

$9b-6a+b+1-a-2+4b+6a=0$

$-a+14b-1=0$

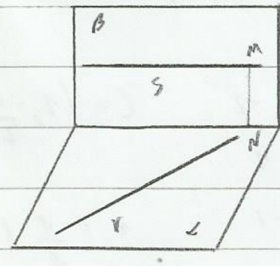
$-a+14(14a-4)-1=0$

$-a+196a-56-1=0$

$195a=57$

$a=\dfrac{57}{195}$

Exemplo:



$r: \ X=(1,2,1)+a(2,1,0) \ ; a \in \mathbb{R}$

$s: \ X=(1,3,3)+b(2,1,-1) \ ; b \in \mathbb{R}$

$M \in r: (1+2a, 2+a, 1)$

$N \in r: (1+2b, 3+b, 3-b)$

$(M-N)=\vec{v}=(2a-2b, -1+a-b, -2+b)$

$(2a-2b, -1+a-b, -2+b).(2,1,0)=0$

$4a-4b-1+a-b=0$

$5a-5b-1=0 \qquad a=\frac{5b+1}{5}$

$(2a-2b, -1+a-b, -2+b).(2,1,-1)=0$

$4a-4b-1+a-b+2-b=0$

$5a-6b+1=0$

$5.\left(\frac{5b+1}{5}\right)-6b+1=0 \Rightarrow 5b+1-6b+1=0$

$b=2 \qquad a=\frac{5.2+1}{5}=\frac{11}{5}$

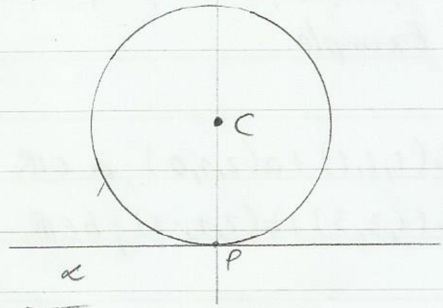
$\vec{v}=\left(\frac{22}{5}-4, -1+\frac{11}{5}-2, -2+2\right)$

$\vec{v}=(2/5, -4/5, 0) \qquad N=(5,5,1)$

$R: \ X=(5,5,1)+t(2/5, -4/5, 0) \ ; t \in \mathbb{R}$

Q. 5-) $\alpha: x + 2y + z + m = 0 \; ; m \in \mathbb{R}$

$S: x^2 + y^2 + z^2 - 2x + y - z - 1 = 0$



$$\begin{cases} -2a = -2 \\ -2b = 1 \\ -2c = -1 \end{cases} \qquad C = \left(1, -\frac{1}{2}, \frac{1}{2}\right)$$

$$1 + \frac{1}{4} + \frac{1}{4} - R^2 = -1 \qquad R = \sqrt{\frac{5}{2}}$$

$$d(\alpha, C) = \frac{\left|1 - 1 + \frac{1}{2} + m\right|}{\sqrt{6}} = \sqrt{\frac{5}{2}}$$

$1/2 + m = \sqrt{15} \qquad m = \sqrt{15} - 1/2$

$-1/2 - m = \sqrt{15} \qquad m = \sqrt{15} + 1/2$

MA1210 - Prova P2

1-) $\alpha: X = (2, -4, 1) + a(1, 3, 1) + b(3, 1, -1) \; , a, b \in \mathbb{R}$

$$\begin{vmatrix} x-2 & y+4 & z-1 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 0$$

$-4(x-2) + 4(y+4) - 8(z-1) = 0$

$-(x-2) + (y+4) - 2(z-1) = 0$

$\alpha: -x + y - 2z + 8 = 0$

2-) $\alpha: x - 4y + 2z - 3 = 0 \qquad W_\alpha = (1, -4, 2)$

$\beta: y + 2z + 4 = 0 \qquad W_\beta = (0, 1, 2)$

$(1, -4, 2) \cdot (0, 1, 2) = x$

$-4 + 4 = 0 = x$ ∴ Portanto é perpendicular