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1º semestre de 2011 2º ciclo Engenharia Básica

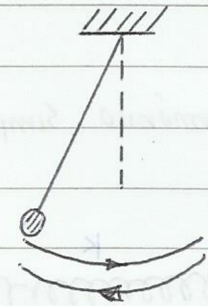
Física II (Parte I)

Física II

Movimentos Periódicos

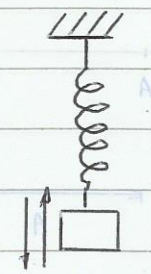
Exemplos:

a) movimento do pêndulo simples



Cada IDA + VOLTA é uma oscilação ou ciclo

b) movimento do pêndulo de mola



Para os movimentos periódicos definem-se:

Período (T): Tempo decorrido para a realização de cada ciclo

Frequência (f): número de ciclos realizados na unidade de tempo

Relação entre T e f

1 ciclo — T

f ciclos — 1 (un. de tempo)

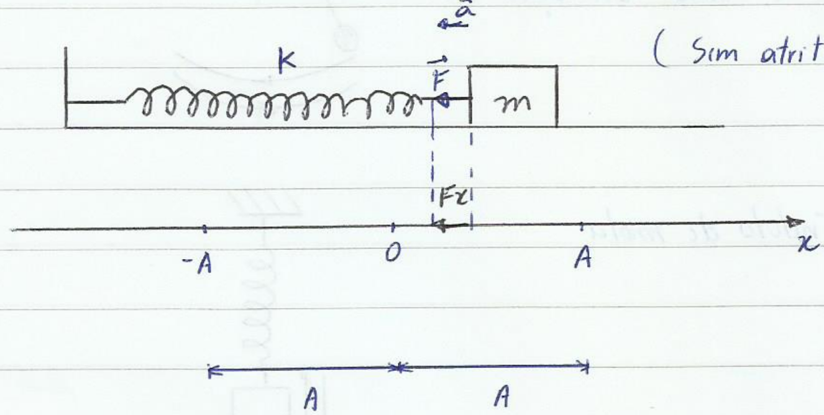
$f \times T = 1 \times 1$ $\boxed{f = \frac{1}{T}}$ $\boxed{T = \frac{1}{f}}$

Unidades (SI)

T em segundos (s)

f em $\frac{1}{s} = s^{-1}$ ou $\frac{ciclo}{s} \equiv Hz$ (hertz)

Movimento Harmônico Simples (MHS)



(A > 0 , sempre)

$\vec{F} = m\vec{a} \Rightarrow F_x = m a_x$

Porém: $F_x = -Kx$ (lei de Hooke)

$m a_x = -Kx$

$m a_x + Kx = 0$

$a_x + \frac{Kx}{m} = 0$

$v_x = \frac{dx}{dt}$

$a_x = \frac{dv_x}{dt}$

$a_x = \frac{d^2x}{dt^2}$

1 → $\boxed{\frac{d^2x}{dt^2} + \frac{K}{m}x = 0}$ Equação diferencial do movim

Uma possível solução da eq. 1 é a função:

$x = A . \cos(wt + \phi)$ → "Equação do 'espaço' ou da elongação"

desde que seja satisfeita a condição:

$$w = \sqrt{\frac{K}{m}}$$

"Obs: O 'A' e o 'w' sempre é positivo"

x: "posição" no instante t

A: amplitude do MHS

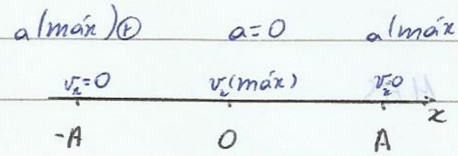
w: frequência angular

$wt + \phi$: fase no instante t

ϕ : fase no instante $t = 0$

ϕ depende da posição (x) e da velocidade (v_x) do móvel no instante $t = 0$

MHS



$$w = \sqrt{\frac{K}{m}}$$

Demonstra-se que $\frac{2\pi}{T} = \sqrt{\frac{K}{m}}$

$$w = 2\pi f = \frac{2\pi}{T}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{K}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

Portanto, T só depende das características físicas (m e K) do sistema oscilante.

"Espaço" ou elongação

$$\boxed{x = \underbrace{A}_{\text{Amplitude "sempre positiva"}} \cdot \cos(wt + \phi)} \quad 1$$

Velocidade: $v_x = \dfrac{dx}{dt}$

$v_x = A \cdot (-1) \operatorname{sen}(wt + \phi) \cdot (w + 0)$

$$\boxed{v_x = -wA \cdot \operatorname{sen}(wt + \phi)} \quad 2$$

Aceleração $a_x = \dfrac{dv_x}{dt}$

$a_x = -wA \cdot \cos(wt + \phi) \cdot (w + 0)$

$$\boxed{a_x = -(w)^2 \underbrace{A \cdot \cos(wt + \phi)}_{x}} \quad 3$$

Comparando 3 com 1:

$\boxed{a_x = -w^2 x}$ Eq. fundam. do MHS

Valores Máximos de v_x e a_x:

$v_x = -wA \operatorname{sen}(\cdots)$

v_x é máx quando $\operatorname{sen}(\ldots) = \pm 1$

$$\boxed{v_x(\text{MÁX}) = \pm wA}$$

$\xrightarrow{(+)}$ $\xleftarrow{(-)}$

Analogamente:

$$\boxed{a_x(MAX) = \pm w^2 A}$$

$\xrightarrow{(+)}$ $\xleftarrow{(-)}$

Exercícios

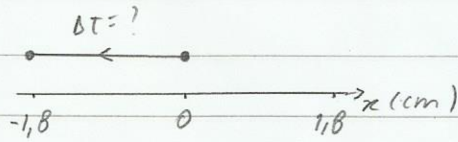
13.5 (p.63)

$f = 5\ Hz$

$A = 1{,}8\ cm$

$x = 0 \xrightarrow{\text{até}} x = -1{,}8\ cm$

$\Rightarrow \Delta t = ?$



$$T = \frac{1}{f} \Rightarrow T = \frac{1}{5} = 0{,}2\ s \qquad \Delta t = \frac{0{,}2}{4} = \underline{0{,}05\ s}$$

13.11 (p64)

$m = 2\ kg$

$K = 300\ N/m$

$t = 0 \begin{cases} x = 0 \\ v = -12\ m/s \end{cases}$

a) Amplitude = ? $A = ?$

b) angulo de fase = ? $\phi = ?$

c) escreva uma equação de x em função de T

$x = x(t) \Rightarrow x = A \cos(wT + \phi)$

$$w = \sqrt{\frac{K}{m}} = \sqrt{\frac{300}{2}} = \sqrt{150} = 12{,}25\ \frac{rad}{s}$$

$vx = wA$

$$A = \frac{vx}{w} = \frac{-12}{12{,}25} = \underline{-0{,}980}$$

Energia no MHS

En. potencial (elástica): $\boxed{E_p = \frac{1}{2} K x^2}$ → U

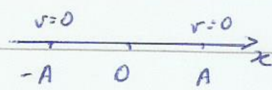
En. cinética: $\boxed{E_c = \frac{1}{2} m v^2}$

→ K (maiúscula)

Energia Mecânica (tota):

$\boxed{E = E_p + E_c}$ = constante

Cálculo de E



$$E = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

Quando $x = \pm A \Rightarrow v = 0$

$$E = \frac{1}{2} K (\pm A)^2 + 0$$

$$\boxed{E = \frac{1}{2} K A^2 = \text{constante}}$$

Energia cinética em função de x:

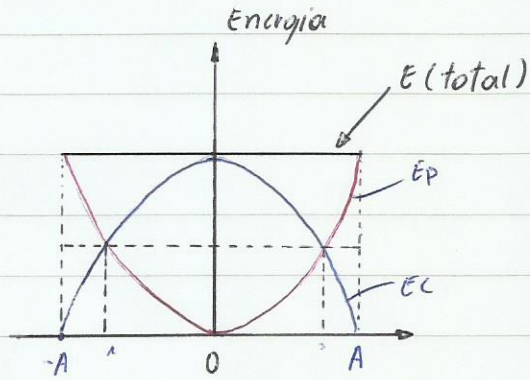
$E = E_p + E_c$

$E_c = E - E_p$

$$\boxed{E_c = \frac{1}{2} K A^2 - \frac{1}{2} K x^2}$$

parábola ∩

$\boxed{E_p = \frac{1}{2} K x^2}$ parábola ∪



Quais os valores de x para os quais $E_c = E_p$?

$E = E_p + E_c \rightarrow E = E_p + E_p \rightarrow E = 2E_p$

$\frac{1}{2} K A^2 = 2 \cdot \frac{1}{2} K x^2$

$A^2 = 2x^2$

$x^2 = \frac{A^2}{2} \Rightarrow x = \pm \frac{A}{\sqrt{2}}$ ou $\pm \frac{A\sqrt{2}}{2}$

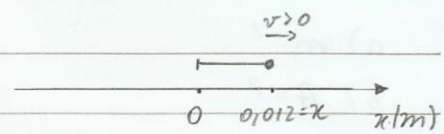
13.26 (p.64)

$m = 0{,}15\ kg$ a) $E = ?$

$k = 300\ N/m$ b) $A = ?$

$x = 0{,}012\ m \Rightarrow v = 0{,}3\ m/s$ c) $v_{máx} = ?$

a) $E = Ep + Ec = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$



$E = \frac{1}{2}(300 \times 0{,}012^2 + 0{,}15 . 0{,}3^2)$

$E = 0{,}02835\ J$

b) $E = \frac{1}{2}KA^2$ $\underline{0{,}028} = \frac{1}{2} \times 300\ A^2$

$A = +\sqrt{\frac{0{,}028}{150}} = 0{,}0137\ m$

c) $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

v é máx quando $x = 0$

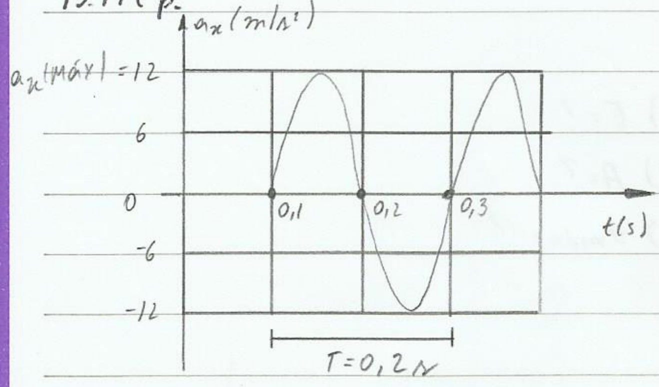
$E = 0 + \frac{1}{2}mv^2_{max}$

$\downarrow$

$0{,}028 = \frac{1}{2} \times 0{,}15\ v^2_{max}$

$v_{max} = \pm\ 0{,}611\ m/s$

13.17 (p.



$K = 2{,}5 \frac{N}{cm} = 2{,}5 \frac{N}{\frac{1}{100} m}$

$K = 250 \ N/m$

a) $m = ?$

b) $A = ?$

c) $F_x(MÁX.) = ?$

a) $T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow T^2 = 4\pi^2 \frac{m}{K} \Rightarrow m = \frac{T^2 k}{4\pi^2}$

$m = \frac{0{,}2^2 . 250}{4\pi^2} = 0{,}25$ kilograma (kg)

b) $a_x = -w^2 x$

a_x é MÁX quando $x = \pm A$

$\begin{cases} a_x(MÁX) = +12 \ m/s^2 \Rightarrow x < 0 \Rightarrow x = -A \\ \text{se } a_x(Máx) = -12 m/s^2 \Rightarrow x > 0 \end{cases}$

$a_x(MÁX) = -w^2 . (-A)$ $w = \frac{2\pi}{T} = \frac{2\pi}{0{,}2} = 31{,}42$

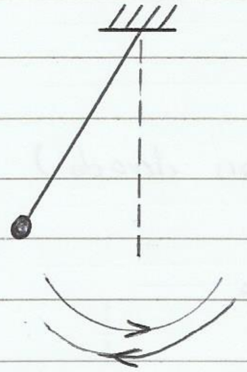
$a_x(MÁX) = w^2 A$

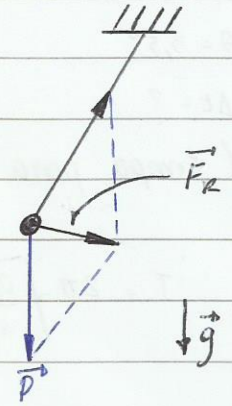
$A = \frac{a_x(MÁX)}{w^2} = \frac{12}{(31{,}4)^2} = 0{,}012 \ m$

c) $F_x(máx) = m . a(máx)$

$F_x(máx) = 0{,}25 . 12 = 3 \ N$

Pêndulos Simples





A partícula de massa m descreve um MHS desde que o (amplitude) seja "pequena": ~ 10°

Neste caso demonstra-se que

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

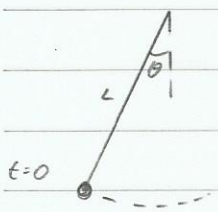
Para "grandes" amplitudes pode ser usada a expressão aproximada:

em radianos

$$\boxed{T \cong 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{4}\operatorname{sen}^2\frac{\theta}{2}\right)}$$

Note que esta última expressão também vale para "pequenas" amplitudes

13.41 (p.65)



$L = 0{,}24\,m$

$\theta = 3{,}5^\circ$

$\Delta t = ?$

(tempo para atingir a velocidade mais elevada)

$\Delta t = \frac{1}{4} T$ $\quad$ $T = 2\pi\sqrt{\frac{0{,}24}{9{,}8}} \approx 0{,}98$ $\quad$ $\Delta t = 0{,}25\,s$

13.44 (p. 66)

Um pêndulo em Marte

1) Terra $T_1 = 1{,}6\,s$ $g_1 = 9{,}81\ m/s^2$

2) Marte $T_2 = ?$ $g_2 = 3{,}71\ m/s^2$

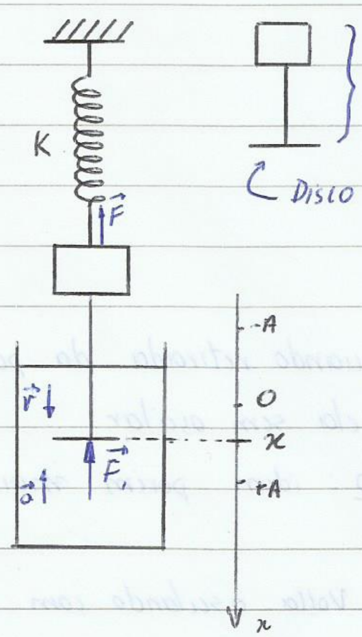
$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T_1 = 2\pi\sqrt{\frac{L}{g_1}} \qquad T_2 = 2\pi\sqrt{\frac{L}{g_2}}$$

$$\frac{T_2}{T_1} = \frac{\sqrt{\frac{L}{g_2}}}{\sqrt{\frac{L}{g_1}}} \qquad T_2 = T_1\sqrt{\frac{g_1}{g_2}}$$

$$T_2 = 1{,}6\sqrt{\frac{9{,}81}{3{,}71}} = T_2 = 2{,}6\,s$$

Oscilações Amortecidas



$\vec{F}$: força elástica

$$\boxed{F_x = -kx}$$

$\vec{F}'$ = força exercida pelo fluido

$$\boxed{F'_x = -b v_x}$$

b: constante de amortecimento

$F_x + F'_x = m a_x$

$-kx - b v_x = m a_x$

$m a_x + kx + b v_x = 0$

$a_x + \frac{b}{m} v_x + \frac{k}{m} x = 0$

$$\boxed{\frac{d^2x}{dt^2} + \frac{b}{m} \cdot \frac{dx}{dt} + \frac{k}{m} x = 0} \quad (1)$$

(Eq. dif. do movim.)

<u>Nota</u>:

$w_a = \frac{2\pi}{T_a}$

$f_a = \frac{1}{T_a}$

$\therefore w_a = 2\pi f_a$

As soluções da eq (1) dependem da constante

$$\boxed{w_a = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

w_a: frequência angular das oscilações amortecidas

Para um valor de b, suficientemente elevado tem-se $w_a = 0$. Nesse caso:

$\frac{k}{\cancel{m}} - \frac{b^2}{4m^{\cancel{2}}} = 0$

$b^2 = 4mk \Rightarrow \boxed{b = 2\sqrt{mk}}$

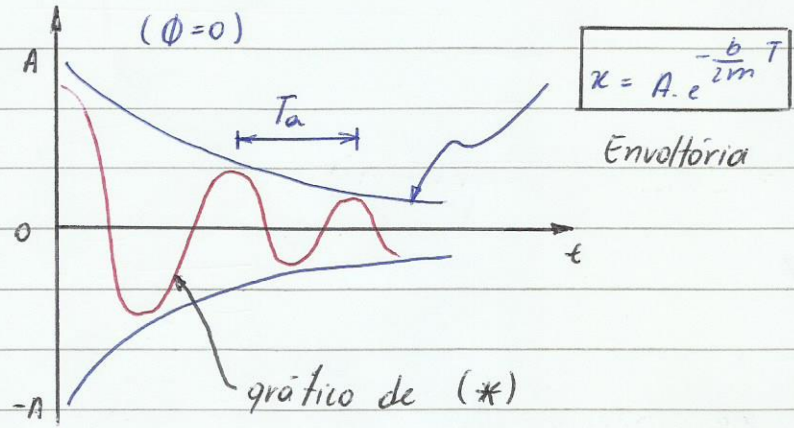
Tipos de Amortecimento

- Se $b = 2\sqrt{mk}$ → amortecimento <u>CRÍTICO</u>: quando retirada da posição de equilíbrio e liberada a massa volta a ela sem oscilar
- Se $b > 2\sqrt{mk}$ → amortecimento <u>SUPER CRÍTICO</u>: idem porém mais lentamente que no caso crítico
- Se $b < 2\sqrt{mk}$ → amortecimento <u>SUB CRÍTICO</u>: ... Volta oscilando com amplitude decrescente.

Nesse caso, uma solução da eq. 1 é:

$$\boxed{x = Ae^{\frac{-b}{2m}.t} . \cos(w_a t + \phi)} \quad (*)$$

onde: $A(t) = Ae^{\frac{-b}{2m}.t}$



Exercícios

13.56 (pág. 67)

$m = 2,2\,kg$

$k = 250\,N/m$

$T_a = 0,615\,s$

a-) Se não fosse amortecido:

Calc. sem amortecimento

$\cancel{w = \sqrt{\frac{k}{m}}}$ ou $T_0 = 2\pi\sqrt{\frac{m}{k}}$ $\therefore T_0 = 2\pi\sqrt{\frac{2,2}{250}} = 0,589\,s$

$\therefore$ Como $T > T_0 \Rightarrow$ movimento amortecido

$w_a = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$; $w_a = \frac{2\pi}{T}$

$\frac{2\pi}{T} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \Rightarrow \frac{4\pi^2}{T^2} = \frac{k}{m} - \frac{b^2}{4m^2}$

$\Rightarrow \frac{b^2}{4m^2} = \frac{k}{m} - \frac{4\pi^2}{T^2}$

$b^2 = \left(\frac{k}{m} - \frac{4\pi^2}{T^2}\right) . 4m^2$

$\therefore b^2 = 179,24 \quad \therefore b = 13,39\,Kg/s$

/ /

Se crítico $b = 2\sqrt{mk} \Rightarrow 13,39 \neq 46,90$ X

Se super crítico $b > 2\sqrt{mk}$ X

Se sub crítico $13,39 < 46,90$ ✓

∴ O movimento é subcrítico

13.57 (p67)

$m = 0,3\ kg$

$k = 2,5\ \frac{N}{m}$

a) $b = 0,9\ \frac{kg}{s} \Rightarrow f_a = ?$

b) Amort crítico
$\Rightarrow b = ?$

a) $w_a = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \Rightarrow f_a = \frac{1}{2\pi}\sqrt{\frac{2,5}{0,3} - \left(\frac{0,9}{2 \times 0,3}\right)^2}$

$f_a = 0,393\ Hz$

b) Amort. crítico

$b = 2\sqrt{mk} = 2\sqrt{0,3 \times 2,5}$

$b = 1,73\ kg/s$

Note que $0,9 < 1,73$

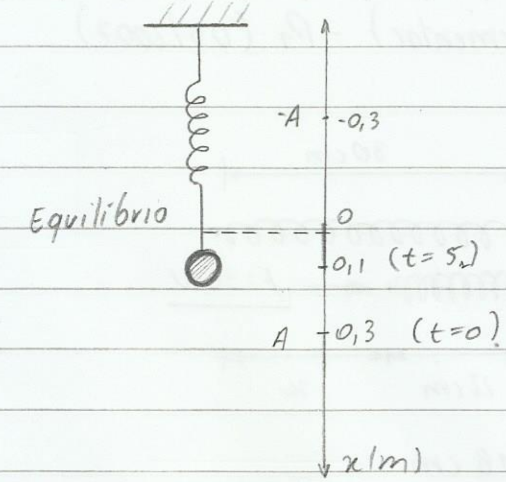
sub-crit crit.

13.58

$m = 50g = 0{,}05$

$k = 25\ N/m$

$x = 0{,}3\ m$



$A(t) = A \cdot e^{\frac{-b}{2m} \cdot t}$

Quando $t = 5s$ $A = 0{,}1$

$0{,}1 = 0{,}3 \cdot e^{\frac{-b}{2 \cdot 0{,}05} \cdot 5}$

$0{,}33 = e^{\frac{-b}{0{,}02}}$ $\quad \ln 0{,}33 = \ln e^{\frac{-b}{0{,}02}}$

$-1{,}0986 = \frac{-b}{0{,}02}$ $\quad \therefore\ \underline{b = 0{,}022\ kg/s}$

Se fosse crítico

$b = 2\sqrt{mk} = 2\sqrt{0{,}05 \cdot 25} = 2{,}23\ kg/s$

Para ser super crítico:

$b_{sc} > b_c$

→ Calcular o número de oscilações ocorridas de $t=0$ até $t=5s$

I $\quad w_a = \frac{2\pi}{T_a}$ $\qquad$ Relaciona I e II

II $\quad w_a = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$w_a = \sqrt{\frac{25}{0{,}05} - \left(\frac{0{,}022}{2 \cdot 0{,}05}\right)^2} = 22{,}36\ \frac{rad}{s}$ $\quad w_a = \frac{2\pi}{T_a}$ $\quad \therefore T_a = \frac{2\pi}{22{,}36} = 0{,}28\ s$

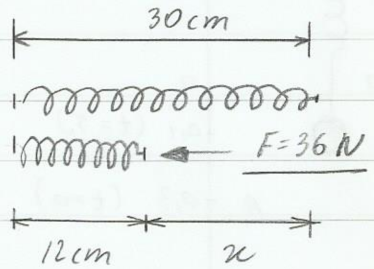
Regra de três: $\quad 1\ osc - T_a = 0{,}281$ $\quad \therefore n = \underline{17{,}79}$

$\qquad\qquad\qquad\quad n - 5$

spiral

6-) (Lista complementar) - P_1 (03/2007)

Mola:

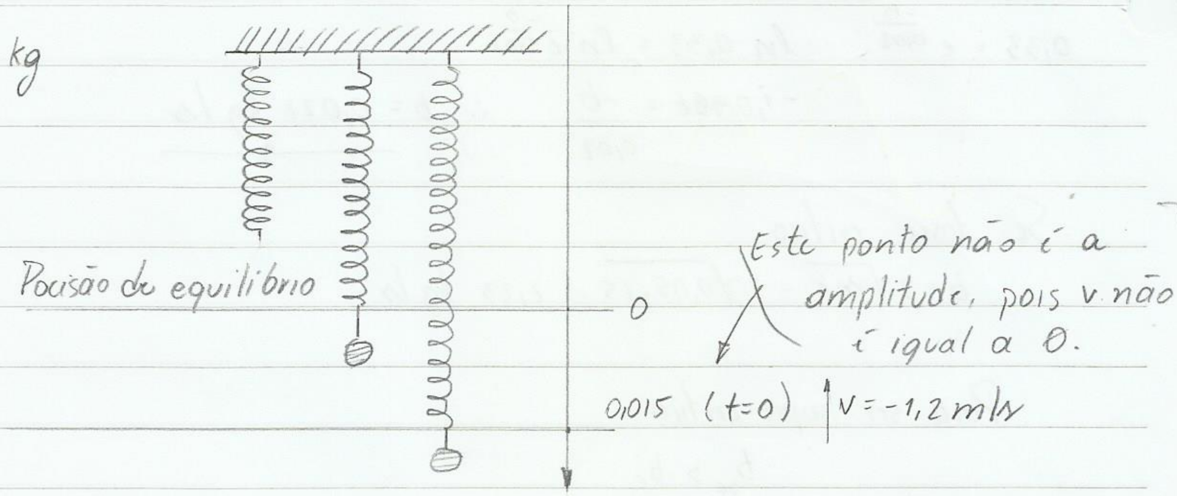


$F = kx$

$k = \frac{F}{x} = \frac{36}{0,18} = 200\ N/m$

$x = 30 - 12 = 18\ cm$

$m = 0,22\ kg$



$k = 200\ N/m$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0,22}} = 30,15\ rad/s$

$E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{0,22\,(-1,2)^2}{2} + \frac{200.(0,015)^2}{2} = 0,1809$

$E = \frac{1}{2}kA^2 \Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2.\,0,1809}{200}} = 0,042\ m$

$x(t) = 0,042 \cos(30,15t + \phi)$ $\quad \therefore\ x(t) = 0,042 \cos(30,15t + 1,21)$

$t = 0 \quad x = 0,015$

$0,015 = 0,042 \cos(30,15.0 + \phi)$

$\therefore \theta = 1,21$

/ /

Exercícios de MHS

1-) $m = 1kg$ $\quad t=0 \begin{cases} x = 0,3m \\ v = -2 m/s \end{cases}$

$k = 25 N/m$

a) ⇒ $F = kx$ ⇒ $mg = kx$

$x = \frac{mg}{k} = \frac{1.10}{25} = 0,4m$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{1}} = \underline{5 rad/s}$

$E = \frac{mv^2}{2} + \frac{kx^2}{2}$

$E = \frac{1.(-2)^2}{2} + \frac{25.0,3^2}{2} = 3,125$ $\quad E = \frac{1}{2}kA^2 \therefore A = \sqrt{\frac{2E}{k}}$

$A = \sqrt{\frac{2.3,125}{25}} = \underline{0,5 m}$ $\quad x(t) = A\cos(wt + \emptyset)$

Quando $t=0$ ⇒ $x = 0,3m$ e $V = -2m/s$

$\therefore 0,3 = 0,5\cos(5.0 + \emptyset)$

$0,3 = 0,5\cos\emptyset \quad \emptyset = \arccos\left(\frac{0,3}{0,5}\right) = \underline{0,93 rad}$

$x(t) = 0,5\cos(5t + 0,93)$ $\quad \nearrow \quad -2,5\,sen(5t + 0,93) = 0$

$v(t) = -2,5\,sen(5t + 0,93)$

$a(t) = -1,25\cos(5t + 0,93)$

$V_{máx} = \pm wA \therefore v_{máx} = 5.0,5 = 2,5 m/s$

$a_{máx} = \pm wA^2 \therefore a_{máx} = 5.0,5^2 = 1,25 m/s^2$

a-) (O que é x estático)

b-) 0,5m

c-) $-2,5\,sen(5t + 0,93)$

d-) 5 rad/s (Velocidade angular é = frequencia angular?)

e-) $v_{máx} = 2,5 m/s$ $\quad a_{máx} = 1,25 m/s^2$

(/ /)

2-) $m = 4kg$ $\quad t=0 \begin{cases} x = 1m \\ v = -2 m/s \end{cases}$

$k = 16 N/m$

a-) $w = ?$; $T = ?$; $f = ?$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{16}{4}} = \underline{2 \text{ rad/s}}$

$w = \frac{2\pi}{T} \sim T = 2\pi : \sqrt{\frac{k}{m}} = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{4}{16}} = \underline{3{,}14 s}$

$f = \frac{1}{T} \therefore f = \frac{1}{3{,}14} = \underline{0{,}32 \text{ hertz}}$

b-) equação horária $\quad E = \frac{mv^2}{2} + \frac{kx^2}{2}$

$x(t) = A\cos(wt + \emptyset)$ $\quad E = \frac{4.(-2)^2}{2} + \frac{16.1^2}{2} = 16 J$

$x(t) = 1{,}41 \cos(2t + \emptyset)$

Quando $t=0 \rightarrow x = 1$ $\quad E = \frac{kA^2}{2} \therefore A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2.16}{16}} = 1{,}41$

$\therefore 1 = 1{,}41 \cos(2.0 + \emptyset)$

$\emptyset = \arccos\left(\frac{1}{1{,}41}\right) = 0{,}785$ $\quad \underline{x(t) = 1{,}41 \cos(2t + 0{,}785)}$

c) posição de x, quando $t = 5s$

$x(5) = 1{,}41 . \cos(2.5 + 0{,}785)$

$x(5) = -0{,}29 m$

d-) $v_{máx} = ?$

$v_{máx} = \pm wA = 2.1{,}41 = 2{,}82 m/s$

e-) $a_{máx} = ?$

$a_{máx} = \pm w^2 A = 2^2 . 1{,}41 = 5{,}64 m/s^2$

3-)

$m = 3\,kg$

comp. da mola = 15cm = 0,15 m

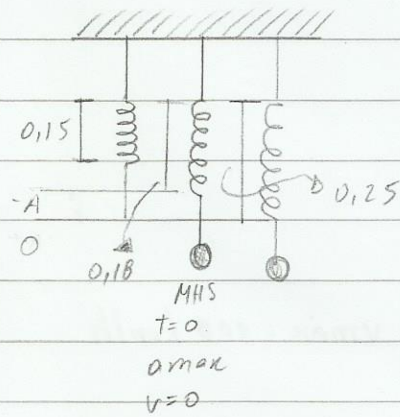
quando em equilibrio a mola atinge 25cm = 0,25 m → $x = 0$

$t=0 \begin{cases} \text{mola atinge} = 18cm \rightarrow x = -0{,}07cm \\ a\,max \sim v = 0 \end{cases}$

$F + p = 0$

$F = mg \quad \therefore \quad F = 3 \cdot 10 = 30\,N$

$X = 0{,}25 - 0{,}15 = 0{,}10$



$F = kx$

$K = \frac{F}{x} = \frac{30}{0{,}1} \quad \therefore \quad \underline{k = 300\,N/m}$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{3}} \quad \therefore \quad w = 10$

$E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{3 \cdot 0^2}{2} + \frac{300 \cdot 0{,}07^2}{2} \quad \therefore \quad \underline{E = 0{,}735\,J}$

$E = \frac{kA^2}{2} \sim A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2 \cdot 0{,}735}{300}} \quad \therefore \quad \underline{A = 0{,}07\,m}$

$x(t) = A\cos(wt + \emptyset)$

$x(t) = 0{,}07\cos(10t + \emptyset)$ Quando $t=0 \begin{cases} a\,máx \rightarrow v = 0 \\ x = 0{,}07 \end{cases}$

$\therefore -0{,}07 = 0{,}07\cos(10 \cdot 0 + \emptyset)$

$\emptyset = \arccos(-1) \quad \therefore \emptyset = \pi$ ou $3{,}14$

$\therefore \underline{x(t) = 0{,}07\cos(10t + \pi)}$

4) $m = 2000\ kg$ $\qquad F = -kx$

$\Delta x = 0,04\ m$ $\qquad k = \frac{F}{x} = \frac{20000}{0,16} = 1,25 \cdot 10^5\ N/m$

$x' = 4 \cdot 0,04 = 0,16$

$F = -bv$

?

5) $t = 0 \rightarrow v_{máx} = 108\ km/h$

$m = 60\ kg$

tempo de $2A = 9,8\ s$, ou seja, $\underline{T = 19,6\ s}$ $\qquad w = \frac{2\pi}{T}$

$w = \sqrt{\frac{k}{m}} \therefore k = w^2 \cdot m$ $\qquad w = \frac{2\pi}{19,6} = 0,32$

$k = 0,32^2 \cdot 60 = \underline{6,14\ N/m}$

$E = 0,5\ kA^2 = 0,5 \cdot 6,14 \cdot 2(2)$

$\therefore E =$?

$v_{max} = \pm wA \quad \therefore A = \frac{v_{max}}{w} = \frac{30}{0,32} = 93,75$

$x(t) = 93,75 \cos(0,32t + \emptyset)$

$v(t) = -30 \operatorname{sen}(0,32t + \emptyset) \Rightarrow t = 0 \rightarrow v = 30\ m/s$

$\therefore 30 = -30 \operatorname{sen}(0,32 \cdot 0 + \emptyset)$

$-1 = \operatorname{sen} \emptyset \quad \therefore \emptyset = 1,57 = \frac{\pi}{2}$

$\therefore \underline{x(t) = 93,75 \cos(0,32t + \pi/2)}$

$b = 2\sqrt{mk} = 2\sqrt{60 \cdot 6,14} = 38,39\ kg/s$

6-) $k = 200\ N/m$

$t=0 \begin{cases} v = 1{,}2\ m/s \\ E_c = 0{,}36\ J \end{cases}$

$E_c = \frac{mv^2}{2} \therefore m = \frac{2E_c}{v^2}$

$m = \frac{2.0{,}36}{1{,}2^2} = 0{,}5\ kg$

a) $\underline{m = 0{,}5\ kg}$

b) $w = ?$ $\quad w = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0{,}5}} \therefore w = \underline{20\ rad/s}$

$A = ?$ $\quad E_c = \frac{kA^2}{2} \rightarrow A = \sqrt{\frac{2E_c}{k}} = \sqrt{\frac{2.0{,}36}{200}} \therefore \underline{A = 0{,}06}$

c) $x(t) = 0{,}06 \cos(20t + \phi)$

$v(t) = -1{,}2\ sen(20t + \phi) \rightarrow t=0 \{ v = 1{,}2$

$1{,}2 = -1{,}2\ sen(20.0 + \phi)$

$\phi = arcsen(-1) = -1{,}57 = -\pi/2$

$\therefore (x(t) = 0{,}06 \cos(20t - \pi/2))\ m$

d) $F = kx$

$x = \frac{F}{K} = \frac{5}{200} = \underline{0{,}025\ m}$

7-) $m = 0{,}01\ kg$ $\quad t=0 \{ x = 0$

$k = 0{,}16\ N/m$

$Em = 32 \times 10^{-4}\ J$ $\quad Ep = 0$ { Ponto de equilíbrio

$E = \frac{kA^2}{2}$ $\quad A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2.32.10^{-2}}{0{,}16}} \therefore A = 2\ m$

$f = \frac{w}{2\pi} \rightarrow w = \sqrt{\frac{k}{m}} \therefore f = \sqrt{\frac{k}{m}} \div 2\pi = \sqrt{\frac{0{,}16}{0{,}01}} : 2\pi = 0{,}64$ hertz

spiral

b) $x(t) = 2\cos(4t + \theta)$

Quando $t = 0 \rightarrow x = 0$

$0 = 2\cos(4.0 + \theta)$

$\cos\theta = 0 \quad \therefore \theta = \pi/2$

$\therefore \boxed{x(t) = 2\cos(4t + \pi/2)\ m}$

c) $b = 2\sqrt{mK}$

$= 2\sqrt{0{,}01 . 0{,}16}$

$\therefore b = 0{,}08\ Kg/s$

8) $m = 2{,}5\ kg$

$k = 1000\ N/m$

$t = 0 \ \{ \ x = 28\ mm = 0{,}028\ m$

$w = \sqrt{\frac{K}{m}} = \sqrt{\frac{1000}{2{,}5}} = 20\ rad/s$

$w = 2\pi f \rightarrow f = \frac{w}{2\pi} = \frac{20}{2\pi} = 3{,}18$ hertz

a) frequencia = ? $\quad$ $f = 3{,}18$ hertz

$E = \frac{mv^2}{2} + \frac{Kx^2}{2} = \frac{2{,}5 . 0^2}{2} + \frac{1000 . (0{,}028)^2}{2} = 0{,}392$

$E = \frac{kA^2}{2} \rightarrow A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2 . 0{,}392}{1000}} = 0{,}028$

$\therefore x(t) = 0{,}028\cos(20t + \theta)$

$t = 0 \rightarrow x = 0{,}028$

$-0{,}028 = 0{,}028\cos(20.0 + \theta)$

$\cos\theta = -1 \quad \therefore \theta = \pi$

$\therefore x(t) = 0{,}028\cos(20t + \pi)$

b) $\quad t = 0{,}11\ s$

/ /

9) $m = 2{,}0\ kg$

$k = 300\ N/m$

$t=0 \begin{cases} v = 12 m/s \\ x = 0 \end{cases}$

$E = \frac{mv^2}{2} + \frac{kx^2}{2}$

$= \frac{2.12^2}{2} + \frac{300.0^2}{2} \therefore E = 144$

$w = \sqrt{\frac{K}{m}} = \sqrt{\frac{300}{2}} = 12{,}24$

$E = \frac{kA^2}{2} \therefore A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2.144}{300}} = 0{,}98$

a) $A = 0{,}98\ m$

b) $x(t) = 0{,}98 \cos(12{,}24t + \phi)$

$t=0 \to x=0$

$0 = 0{,}98 \cos(12{,}24.0 + \phi)$

$0 = \cos\phi \quad \phi = \pi/2$

Porque não pode ser $\frac{\pi}{2}$; pois na resposta esta $\frac{3}{2}\pi$?

10-) $F = -bv$

$m = 0{,}05\ kg$

$k = 25\ N/m$

$F \to \begin{cases} 1/3\ A \\ t = 5s \end{cases}$

Para ser criticamente amortecido

$b = 2\sqrt{mk} = 2\sqrt{0{,}05.25} \therefore b = 2{,}24\ kg/s$

$w_a = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$x = Ae^{\frac{-b}{2m}.t}$

$\to \ln x = \ln A - \frac{b}{2m}.t$

$\ln \frac{1}{3}A = \ln A - \frac{b}{0{,}1}.5 \quad \therefore b = \frac{0{,}1}{5}.\ \ln A - \ln \frac{1}{3}A \quad b = 0{,}02.\ \ln\left|A : \frac{1}{3}A\right|$

$b = 0{,}02\ \ln(3A)$

12-) $m = 0,5\ kg$

$k = 450\ N/m$

$t=0 \begin{cases} x = -1,4\ cm = \\ v_0 = 18\ cm/s = \end{cases}$

$E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{0,5 \cdot 18^2}{2} + \frac{450 \cdot (-1,4)^2}{2} = 522$

$E = \frac{kA^2}{2} \Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2 \cdot 522}{450}} = \underline{1,52\ cm}$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{450}{0,5}} = 30$

$x(t) = 1,52 \cos(30t + \phi)$

$t = 0 \rightarrow x = -1,4$

$-1,4 = 1,52 \cos(30 \cdot 0 + \phi)$

$-0,92 = \cos\theta \quad \therefore \quad \underline{\phi = 2,74\ rad}$

$x(t) = 1,52 \cos(30t + 2,74)\ (cm)$

13-) $m = 0,5\ kg$ $\quad E_m = 36\ J$ $\quad x(t) = 0,6 \cos(20t + \phi)$

$k = 200\ N/m$ $\quad t = 0 \rightarrow x = 0$

$t=0 \begin{cases} v = + \\ x = 0 \end{cases}$ $\quad 0 = 0,6 \cos(20 \cdot 0 + \phi)$

$\therefore \phi = -\frac{\pi}{2}$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0,5}} = 20$

"Como determinar o sinal do ϕ"

$E = \frac{kA^2}{2} \quad \therefore \quad A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2 \cdot 36}{200}} = 0,6$

a) $x(t) = 0,6 \cos(20t - \pi/2)\ (m)$

b) $x(t) = 0,6\cos(20t - \pi/2)$

$0,2 = 0,6\cos(20t - \pi/2)$

$1,23 = 20t - \pi/2$

$t = 0,14\,s$

$\omega = \frac{2\pi}{T} \therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{20} = 0,314$

$t = 4 \cdot 0,314 + 0,14 = 1,4\,s$ (Dúvida)

c) $a(t) = -240\cos(20t - \pi/2)$

$a(1,4) = -65\,m/s^2$

14-) $m = 3\,kg$

$k = 18\,N/m$

$A_0 = 0,2\,m$

Quando realizado 2 oscilações $\Rightarrow A = \frac{2}{5} \cdot 0,2 = 0,08$

$\omega_a = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$x = Ae^{\frac{-b}{2m}t}$

$\ln x = \ln A - \frac{b}{2m} \cdot t$

$\omega_a = \frac{2\pi}{T_a}$

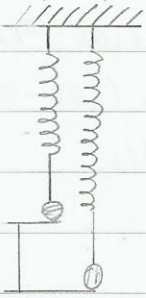
$b = (\ln 0,2 - \ln 0,08) \cdot \frac{6}{T_a} \Leftarrow b = (\ln A - \ln x) \cdot \frac{2m}{T}$

$\therefore b = \frac{5,5}{T}$ $\quad \omega_a = \frac{2\pi}{T}$ $\quad T = \frac{2\pi}{\omega_a} = 2,56$ $\quad 2T = 5,13$

$\therefore b = 5,5/5,13 = 1,07$ $\quad \therefore b = 1,07\,kg/s$

15-) $m = 2\ kg$

$t = 0 \begin{cases} v = 0 \\ x = + \end{cases}$



$F = mg$ e $F = -kx$

$\therefore mg = -kx \Rightarrow k = \frac{mg}{x} = \frac{2.10}{0,08} = \underline{250\ N/m}$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{2}} = 11,18\ rad/s$

$E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{2\,0^2}{2} + \frac{250.\,0,08^2}{2} = 0,8$

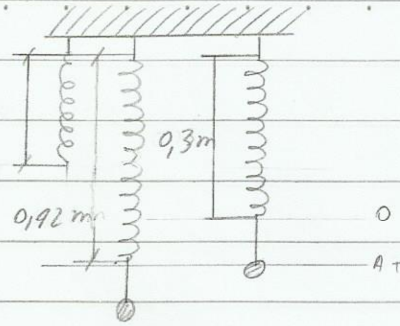
?

16-) $m = 2\,kg$

$l = 0{,}20\,m$

$t = 0 \begin{cases} x = 0{,}42\,m \\ a = 0 \quad v = max \end{cases}$

equilibrio estático:



$A = 0{,}42 - 0{,}3 = 0{,}12$

$F = m.g = kx$

$k = \dfrac{m.g}{x} = \dfrac{2.10}{0{,}1} = \underline{200\,N/m}$

$w = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{200}{2}} = 10\,rad/s$

$x(t) = 0{,}12\cos(10t + \theta)$

Quando $x = 0{,}12 \quad t = 0$

$0{,}12 = 0{,}12\cos(10.0 + \theta)$

$1 = \cos\theta \quad \therefore \quad \theta = 0$

$\therefore \; x(t) = 0{,}12\cos(10t)\ (m)$

$v(t) = -1{,}2\,sen(10t)\ (m/s)$

$a(t) = -12\cos(10t)\ (m/s^2)$

$E = \dfrac{kA^2}{2} = \dfrac{200.\,0{,}12^2}{2} = \underline{1{,}44\,J}$

Se $b = 0{,}3\,kg/s$

$A = \frac{1}{3}A_0$

$x = Ae^{\frac{-b}{2m}t}$

$\ln x = \ln A - \dfrac{b}{2m}t$

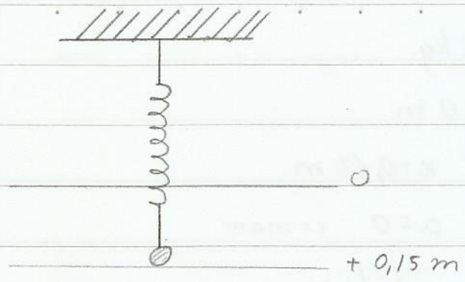
$t = (\ln A - \ln x).\dfrac{2m}{b}$

$t = (\ln 0{,}12 - \ln 0{,}04).\dfrac{2.2}{0{,}3} \Rightarrow \therefore t = \underline{14{,}65\,s}$

/ /

17-) $m = 2\,kg$

$f = 20\,Hz$



a) $k = ?$

$w = 2\pi f = 2\pi \cdot 20 = 125,66$

$w = \sqrt{\frac{k}{m}} \therefore k = w^2 \cdot m = (125,66^2) \cdot 2 = \underline{31582,73\ N/m}$

b) equação horária = ?

$x(t) = 0,15 \cos(125,66\,t + \theta)$ $\quad$ $\underline{x(t) = 0,15 \cos(125,66\,t)\ (m)}$

$t = 0 \to x = 0,15$

$0,15 = 0,15 \cos(125,66 \cdot 0 + \theta)$

$1 = \cos\theta \therefore \theta = 0$

c) $-0,04 = 0,15 \cos(125,66t)$

$-0,27 = \cos(125,66t)$

$105,46 = 125,66t \therefore t = 0,84$

$a(t) = -2368,56 \cos(125,66\,t)$

$a(0,84) = \underline{631,62\ m/s^2}$

18-) $m = 0,02\,kg$

$A = 0,5\,m$

$f = 0,2\,Hz$

$t = 0 \to x = -0,5\,m$

$w = 2\pi f = 2\pi \cdot 0,2 \therefore w = 0,4\pi$

$x(t) = 0,5 \cos(0,4\pi t + \theta)$

$t = 0 \to x = -0,5$

$-0,5 = 0,5 \cos(0,4\pi \cdot 0 + \theta)$

$-1 = \cos\theta \therefore \theta = \pi$

$\therefore \underline{x(t) = 0,5 \cos(0,4\pi\,t + \pi)\ (m)}$

$x(t) = 0{,}5\cos(0{,}4\pi t + \pi)$ $\qquad w = \sqrt{\frac{k}{m}} \therefore k = w^2 . m = (0{,}4\pi)^2 . 0{,}2$

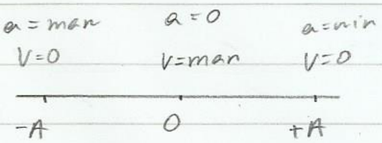
$x(0{,}7) = -0{,}318$ $\qquad k = 0{,}3158$

$F = -kx = -0{,}318 . 0{,}3158 = 0{,}006 = 0{,}01\,N$? Dúvida

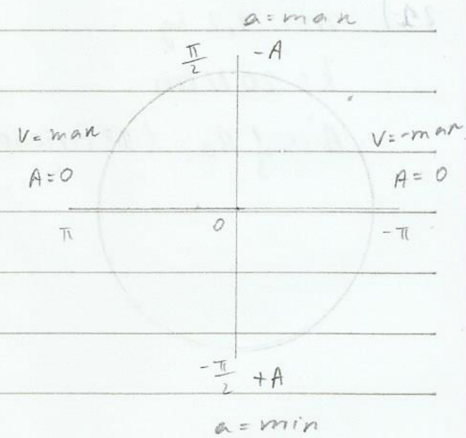
19-) $m = 0{,}2\,kg$

$k = 400\,N/m$

$t = 0 \begin{cases} x = \text{posição de equilíbrio} \\ v = 8\,m/s \end{cases}$



$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{0{,}2}} = 44{,}72\,rad/s$



$v_{max} = Aw \therefore A = \frac{v}{w} = \frac{8}{44{,}72} = 0{,}1788 \cong 0{,}18$

$x(t) = 0{,}18\cos(44{,}72t + \theta)$

$t = 0 \rightarrow x = 0$

$0 = 0{,}18\cos(44{,}72 . 0 + \theta)$

$0 = \cos\theta \therefore \theta = \pi/2$

$\therefore \underline{x(t) = 0{,}18\cos(44{,}72t + \pi/2)\,(m)}$

$E = E_p + E_c \therefore E_c = E - E_p \therefore E_c = \frac{kA^2}{2} - \frac{kx^2}{2}$

$x(0{,}5) = 0{,}06489$

$E_c = \frac{400}{2}(0{,}18^2 - 0{,}06489^2) = \underline{5{,}64\,J}$

20-) $F = -bv$ $\qquad x = A^{\frac{-b}{2m} t}$

$m = 1kg$ $\qquad \ln x = \ln A - \frac{b}{2m} . t$

$k = 25 N/m$

$t = 1 \Rightarrow A\frac{1}{3} = A'$ $\qquad b = (\ln A - \ln x) . \frac{2m}{t} \Rightarrow b = (3A)2 \Rightarrow b = 6A$

$w = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ $\qquad b^2 \left(\frac{k}{m} - w^2\right) . 4m = \left(\frac{25}{1} - 39,97\right.$.

21.) $m = 1,2 \ kg$

$k = 20 N/m$

$A' = \frac{1}{3} A_0$ (1.º Periodo)

/ /

23-) $m = 0{,}02\ kg$ $\quad w = \sqrt{\frac{K}{m}} = 2\pi f$ $\quad w = \sqrt{\frac{K}{m}} = \sqrt{\frac{0{,}5}{0{,}02}} = 5$

$k = 0{,}5\ N/m$

$t = 0 \begin{cases} x = 0 \\ E = 16 \times 10^{-4} \end{cases}$ $\quad \therefore f = \sqrt{\frac{K}{m}} : 2\pi = \sqrt{\frac{0{,}5}{0{,}02}} : 2\pi = 0{,}795 \cong \underline{0{,}8\ Hz}$

$E = \frac{KA^2}{2} \quad A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2.16.10^{-4}}{0{,}5}} = \underline{0{,}08\ m}$

$x(t) = 0{,}08 \cos(5t + \theta)$

$t = 0 \to x = 0$

$0 = 0{,}08 \cos(5.0 + \theta)$

$0 = \cos\theta \quad \therefore \theta = \pi/2$

$\therefore x(t) = 0{,}08 \cos(5t + \pi/2)\ (m)$

c) $\quad b = 2\sqrt{mk} = 2\sqrt{0{,}02 . 0{,}5} = 0{,}2\ kg/s$

24-) $m = 0{,}03\ kg$ $\quad w = \frac{2\pi}{T} = \frac{2\pi}{0{,}6} = 3{,}33\pi$

$A = 0{,}2\ m$

$T = 0{,}6\ s$ $\quad x(t) = 0{,}2 \cos(3{,}33\pi t + \theta)$

$t = 0 \to x = -0{,}2\ m$ $\quad t = 0 \to x = -0{,}2$

$-0{,}2 = 0{,}2 \cos(3{,}33\pi . 0 + \theta)$

$-1 = \cos\theta \quad \therefore \theta = \pi$

$\therefore x(t) = 0{,}2 \cos(3{,}33\pi t + \pi)\ (m)$

spiral

$x(t) = 0{,}2\cos(3{,}33\pi t + \pi)$

$0{,}12 = 0{,}2\cos(3{,}33\pi t + \pi)$

$0{,}6 = \cos(3{,}33\pi t + \pi)$

$0{,}92729 = 3{,}33\pi t + \pi \quad \therefore \quad \underline{t = 0{,}2117\ s}$

$w = \sqrt{\frac{k}{m}} \quad \therefore \quad k = w^2 \cdot m$

$k = (3{,}33\pi)^2 \cdot 0{,}03 = 3{,}2833$

$E = E_p + E_c \quad \therefore \quad E_c = E - E_p = \frac{kA^2}{2} - \frac{kx^2}{2} = \frac{3{,}2833}{2}(0{,}04 - 0{,}17^2)$

$\therefore E_c = \underline{0{,}018\ J}$

25-) $m = 0{,}1\ kg$

$k = 40\ N/m$

$t = 0 \begin{cases} v_{max} = 15\ m/s \\ x = 0 \end{cases}$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{0{,}1}} = 20\ rad/s$

$v_{max} = Aw$

$A = \frac{v_{max}}{w} = \frac{15}{20} = 0{,}75\ m$

$x(t) = 0{,}75\cos(20t + \theta)$

$t = 0 \rightarrow x = 0$

$0 = 0{,}75\cos(20\cdot 0 + \theta)$

$0 = \cos\theta \quad \therefore \quad \theta = -\pi/2$

$\therefore \quad \underline{x(t) = 0{,}75\cos(20t - \pi/2)}$

$v(t) = -15\sin(20t - \pi/2)$

$v(0{,}5) = \underline{-12{,}59\ m/s}$

$x(t) = 0{,}75\cos(20t - \pi/2)$

$-0{,}3 = 0{,}75\cos(20t - \pi/2)$

$\therefore t = \underline{0{,}178\ s}$

26-) $m = 5\ g = 0{,}005\ kg$

$b = 0{,}8\ kg/s$

$b = 2\sqrt{mk}$

$\left(\frac{b}{2}\right)^2 = mk$

$\therefore\ k = \frac{b^2}{4m}$

$k = \frac{0{,}8^2}{4 \cdot 0{,}005} = \underline{32\ N/m}$

$x = A\, e^{\frac{-b}{2m}t}$

27-) $k = 12\ N/m$

$b = 0{,}4\ kg/s$

$A_0 = 0{,}35\ m$

$A = 0{,}25\ m$

$t = 5$

$x = A e^{\frac{-b}{2m}t}$

$\ln x = \ln A - \frac{b}{2m} t$

$(\ln x - \ln A) \cdot 2m = -bt$

$\therefore\ m = \frac{-bt}{2(\ln x - \ln A)} = \frac{-0{,}4 \cdot 5}{2(\ln 0{,}25 - \ln 0{,}35)}$

28-) $m = 0{,}3\ kg$ $\quad w = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0{,}3}} = 25{,}82\ rad/s$

$k = 200\ N/m$

$A = 0{,}12\ m$ $\quad x(t) = 0{,}12 \cos(25{,}82 t + \theta)$

$t = 0 \Rightarrow x = -0{,}12 m$ $\quad -0{,}12 = 0{,}12 \cos(25{,}82 . 0 + \theta)$

$-1 = \cos\theta \quad \therefore \quad \theta = \pi$

$\therefore$ $\underline{x(t) = 0{,}12 \cos(25{,}82 t + \pi)\ (m)}$

$a = -w^2 x \sim a = -666{,}67 . 0{,}06 = \underline{-40\ m/s^2}$

$Em = Ep + Ec \quad \therefore \quad Ec = E - Ep = \frac{1}{2} K (A^2 - x^2) = \frac{1}{2} . 200 (0{,}12^2 - 0)$

$\therefore Ec = \underline{1{,}44\ J}$

$Ec = \frac{1}{2} Ep \qquad E = Ep + \frac{1}{2} Ep = \frac{3}{2} Ep = \frac{3}{2} \left(\frac{1}{2} K x^2\right)$

$\therefore E = \frac{3}{4} K x^2 \quad ; \quad E = \frac{1}{2} K A^2$

Igualando $\quad \frac{3}{4} K x^2 = \frac{1}{2} K A^2$

$x^2 = \frac{1}{2} . \frac{4}{3} \frac{K A^2}{K} \quad \therefore \quad x^2 = \frac{2}{3} A^2 \quad \therefore \quad x = A \sqrt{\frac{2}{3}}$

$A = 0{,}12 \quad \therefore \quad x = 0{,}12 \sqrt{\frac{2}{3}} = \underline{\pm 0{,}098\ m}$

29-) $m = 0{,}2\ kg$ $\quad w = \frac{2\pi}{T} = \frac{2\pi}{2{,}4} = 2{,}618\ rad/s$

$T = 2{,}4 s$

$t = 0 \rightarrow x = 0{,}2$ $\quad w = \sqrt{\frac{K}{m}} \quad \therefore \quad K = w^2 . m = (2{,}618)^2 . 0{,}2$

$K = 1{,}37\ N/m$

$x(t) = 0{,}2\cos(2{,}62\,t + \theta)$

$t = 0 \rightarrow x = -0{,}2$

$-0{,}2 = 0{,}2\cos(2{,}62.0 + \theta)$

$-1 = \cos\theta \quad \therefore \theta = \pi$

$\therefore \underline{x(t) = 0{,}2\cos(2{,}62t + \pi)\ (m)}$

$a = -w^2 x = -(2{,}618)^2\, 0{,}05 = \underline{-0{,}343\ m/s^2}$

$x(t) = 0{,}2\cos(2{,}62\,t + \pi)$

$0{,}15 = 0{,}2\cos(2{,}62t + \pi)$

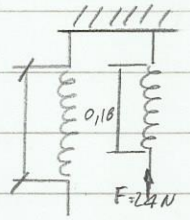
$0{,}75 = \cos(2{,}62t + \pi)$

$0{,}7227 = 2{,}62t + \pi) \quad \therefore t = \underline{0{,}923\ s}$

30-) $l = 0{,}3\ m$

$m = 0{,}22\ kg$

$t = 0 \begin{cases} x = 0{,}015\ m \\ v = -1{,}2\ m/s \end{cases}$



$F = kx \quad \therefore k = \frac{F}{x} = \frac{24}{0{,}12} = \underline{200\ N/m}$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0{,}22}} = \underline{30{,}15\ rad/s}$

$E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{0{,}22.(-1{,}2)^2}{2} + \frac{200\ (0{,}015)^2}{2} = 0{,}1805$

$E = \frac{kA^2}{2} = 0{,}1805 \quad \therefore A = \sqrt{\frac{2.0{,}1805}{200}} = 0{,}042$

$x(t) = 0{,}042\cos(30{,}15\,t + \theta)$

$t = 0 \rightarrow x = 0{,}015$

$0,015 = 0,042 \cos(30,15 . 0 + \theta)$

$0,353 = \cos(\theta) \quad \therefore \theta = 1,21$

$\therefore \underline{x(t) = 0,042 \cos(30,15t + 1,21)\ (m)}$

$\underline{v(t) = -1,266 \sin(30,15t + 1,21)\ (m/s)}$

$x = 0,03 \quad a(t) = -w^2 x(t)$

$a = -909,0225 . 0,03 \quad \therefore \underline{a = -27,27\ m/s^2}$

$E = \frac{1}{2} k A^2 = \frac{1}{2} . 200 . (0,042)^2 \quad \therefore \underline{E = 0,176\ J}$

31-) $T = 0,5$

$t = 0,1 \begin{cases} x = 0,049\ m \\ v = -1,8\ m/s \end{cases}$

$m = 0,2\ kg$

a) $w = \frac{2\pi}{T} = \frac{2\pi}{0,5} = 4\pi\ rad/s$

b) $k = ? \quad w = \sqrt{\frac{k}{m}} \quad \therefore k = w^2 . m = (4\pi)^2 . 0,2 = \underline{31,58\ N/m}$

c) $a = ?$ $E_p = ?$ Quando $x = 0,1$

$E_p = \frac{1}{2} k x^2 = \frac{1}{2} 31,58 . (0,1)^2 = \underline{0,158\ J}$

$a = -w^2 x(t)$

$a = -(4\pi)^2 . 0,1 = \underline{-15,79\ m/s^2}$

d-) A = ?

$$E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{0,2 \cdot (-1,8)^2}{2} + \frac{31,58 \cdot (0,049)^2}{2}$$

$\therefore E = 0,36$

$$E = \frac{1}{2} kA^2 \quad \therefore A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2 \cdot 0,36}{31,58}} = 0,15\,m \quad \therefore \underline{A = 0,15\,m}$$

$x(t) = 0,15 \cos(4\pi t + \theta)$

$t = 0,1 \rightarrow x = 0,049$

$0,049 = 0,15 \cos(4\pi \cdot 0,1 + \theta)$

$0,323 = \cos(4\pi\, 0,1 + \theta)$

$1,241 = 0,1 \cdot 4\pi + \theta \quad \therefore \quad \theta = -0,0163\ rad$

Verificar

32-) $a_{máx} = 2,2\,m/s^2$

$t = 0 \rightarrow x = 0,03\,m$

a) eq. $x(t)$

b) $x = ?$ quando $v = 0,61$ m/s

$v(t) = -1 . \sin(20t + 0,7)$

$0,61 = -1 \sin(20t + 0,7)$

$-0,656 = 20t + 0,7 \therefore t = 0,067$

$x(-0,067) = \underline{\pm 0,04 \text{ m}}$

c) $a = ?$ Quando $x = 0,02$m

$a = -w^2 . x$

$a = -(20)^2 . 0,02 = \underline{-8 \text{ m/s}^2}$

35-) $K = 0,8$ N/m

34-

/ /

33-) $m = 200g$

$k = 50 N/m$

Verificar com o Professor

34-) $m = 0,25 kg$ $\qquad w = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{0,25}} = 20$

(35-)

$t = 0 \begin{cases} x = 0,038 \, m \\ E_c = 0,051 \, J \end{cases}$

$k = 100 N/m$

$E = E_c + E_p \therefore E = E_c + \frac{kx^2}{2} = 0,051 + \frac{100.(0,038)^2}{2} = 0,1232$

$E = \frac{1}{2}kA^2 \Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2.0,1232}{100}} = 0,05 \quad \therefore \underline{A = 0,05 \, m}$

$x(t) = 0,05 \cos(20t + \theta)$

$t = 0 \quad x = 0,038$

$0,038 = 0,05 \cos(20.0 + \theta)$

$0,76 = \cos\theta \quad \therefore \underline{\theta = 0,707}$

38-) $m = 0,3\ kg$

$T = 1s$ $\qquad w = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi$

$v_{máx} = 5,03\ m/s$

$t = 0,1 \begin{cases} x = 0,34 \\ v = -4,54\ m/s \end{cases}$ $\qquad w = \sqrt{\frac{k}{m}} \therefore k = w^2 . m = 4\pi^2 . 0,3$

$\therefore \underline{k = 11,84\ N/m}$

a) $k = ?$ $\quad A = ?$

$\therefore \underline{k = 11,84}$

$E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{0,3 . (-4,54)^2}{2} + \frac{11,84 . 0,34^2}{2} = 3,735$

$E = \frac{1}{2} kA^2 \therefore A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2 . 3,735}{11,84}} = 0,79\ m$

$\therefore A = \underline{0,79\ m} \cong \underline{0,8\ m}$

b) $x(t) = 0,8 \cos(2\pi t + \theta)$

$t = 0,1 \to x = 0,34$

$0,34 = 0,8 \cos(2\pi . 0,1 + \theta)$

$0,425 = \cos(2\pi . 0,1 + \theta)$

$1,131 = 0,2\pi + \theta \qquad \therefore \theta = \underline{0,503\ rad}$

$x = 0,2\ m$

$0,2 = 0,8 \cos(2\pi t + 0,503)$

$0,25 = \cos(2\pi t + 0,503)$

$1,318 = 2\pi t + 0,503 \qquad \therefore t = \underline{0,13\ s}$

39-) $m = 200g$ $\qquad w = \frac{2\pi}{T} = \frac{2\pi}{0,8} = 2,5\pi$

$T = 0,8s$

$t = 0,12 \begin{cases} x = 0,032 \\ v = -12 m/s \end{cases}$ $\qquad k = w^2 m = (2,5\pi)^2 . 0,2 = \underline{12,34\ N/m}$

$$E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{0,2.(-12)^2}{2} + \frac{12,34.(0,032)^2}{2} = 14,406$$

$$E = \frac{1}{2} k A^2 \quad \therefore \quad A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2.14,406}{12,34}} = 1,528 \quad \therefore \underline{A = 1,528\ m}$$

$x(t) = 1,528 \cos(2,5\pi t + \theta)$

$t = 0,12 \rightarrow x = 0,032$

$0,032 = 1,528 \cos(2,5\pi . 0,12 + \theta)$

$0,021 = \cos(0,3\pi + \theta)$

$1,5496 = 0,3\pi + \theta \quad \therefore \underline{\theta = 0,607\ rad}$

$E = \frac{mv^2}{2} \rightarrow t = 0,18s$

$v(t) = -12 \sin(2,5\pi t + 0,607)$

$v(0,18) = -10,8\ m/s$

$$E = \frac{mv^2}{2} = \frac{0,2.(-10,8)^2}{2} = \underline{11,68\ J}$$

40-) $m = (320 kg)$ $\qquad w = \frac{2\pi}{T} = \frac{2\pi}{1,1} = 5,71 \qquad F = kx \therefore k = \frac{F}{x} = 6,4.10^4$

$\bar{x} = 0,05\ m$

$T = 1,1$ $\qquad w = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \therefore T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{64}{6,4\ 10^4}} = 0,198\ s$

?

$*$ $m = 250\,g$ $\qquad t=0 \begin{cases} x = 0{,}12\,m \\ v = 0{,}6\,m/s \end{cases}$

$k = 50\,N/m$

$$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{0{,}25}} = 14{,}14\ rad/s$$

$$x = A\cos(wt+\theta) \rightarrow 0{,}12 = A\cos(14{,}14.0+\theta)$$

$$v_x = -wA\,sen(wt+\theta) \rightarrow 0{,}6 = -14{,}14A\,sen(14{,}14.0+\theta)$$

$$0{,}12 = A\cos\theta \qquad 0{,}6 = -14{,}14\,sen\theta\,A$$

$$A = \frac{0{,}12}{\cos\theta} \qquad A = \frac{0{,}6}{-14{,}14\,sen\theta}$$

$$\frac{0{,}12}{\cos\theta} = \frac{0{,}6}{-14{,}14\,sen\theta} \Rightarrow \frac{sen\theta}{\cos\theta} = \frac{0{,}6}{-14{,}14.0{,}12}$$

$$tg\,\theta = -0{,}35 \quad \therefore \quad \theta = -0{,}34\ rad$$

$$A = \frac{0{,}12}{\cos(-0{,}34)} = 0{,}127\,m$$

$$x = A\cos(wt+\theta)$$

a) $x = 0{,}127\cos(14{,}14t - 0{,}34)\ (m)$

b) $0{,}07 = 0{,}127\cos(14{,}14t - 0{,}34)$

$0{,}987 = 14{,}14t - 0{,}34$

$t = 0{,}0938$

$a(t) = -25{,}4\cos(14{,}14t - 0{,}34)$

$a(0{,}0938) = -14\ m/s^2$

ou

$mg = -Kx$

$$a = \frac{-kx}{m} = \frac{-50.0{,}07}{0{,}25} = \underline{-14\ m/s^2}$$

→ $m = 0{,}02\ kg$ $\qquad E = \frac{1}{2} k A^2$

$k = 0{,}5\ N/m$

$t = 0 \rightarrow x = \frac{mg}{k}$ $\qquad A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2.16.10^{-4}}{0{,}5}} = \underline{0{,}08\ m}$

$Em = 16.10^{-4}\ J$ $\qquad w = \sqrt{\frac{k}{m}} = \sqrt{\frac{0{,}5}{0{,}02}} = 5$

$w = 2\pi f \quad \therefore \quad f = \frac{w}{2\pi} = \frac{5}{2\pi} = \underline{0{,}8\ hertz}$

$x = A \cos(wt + \emptyset)$

$x = 0{,}08 \cos(5t + \emptyset)$ $\qquad x = 0{,}08 \cos(5t + \pi/2)$

$0 = 0{,}08 \cos(5.0 + \emptyset)$

$b = 2\sqrt{mk} = 2\sqrt{0{,}02.0{,}5} = \underline{0{,}2\ kg/s}$

→ $m = 0{,}03\ kg$ $\qquad x = A \cos(wt + \emptyset)$

$A = 0{,}2\ m$ $\qquad x = 0{,}2 \cos(3{,}33\pi t + \emptyset)$

$T = 0{,}6\ s$ $\qquad t = 0 \quad x = -0{,}2$

$t = 0 \rightarrow x = -0{,}2\ m$ $\qquad -0{,}2 = 0{,}2 \cos(3{,}33\pi.0 + \emptyset)$

$\therefore \emptyset = \pi$

$w = \frac{2\pi}{T} = \frac{2\pi}{0{,}6} = 3{,}33\pi$

$x = 0{,}2 \cos(3{,}33\pi t + \pi)$

$0{,}12 = 0{,}2 \cos(3{,}33\pi t + \pi)$

$\therefore t = \underline{0{,}211\ s}$ $\qquad 0{,}17 = 0{,}2 \cos(3{,}33\pi t + \pi)$

$\therefore t = 0{,}25\ s$

$Ec = \frac{mv^2}{2} = \frac{0{,}03(1{,}102)^2}{2}$ $\qquad v(t) = -2{,}09 \sin(3{,}33\pi t + \pi)$

$v(0{,}25) = 1{,}102$

$Ec = \underline{0{,}018\ J}$

/ /

$m = 0{,}1\ kg$

$k = 40\ N/m$

$v_{máx} = 15\ m/s \rightarrow t = 0$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{0{,}1}} = 20\ rad/s$

$Aw = v_{máx}$

$Aw^2 = a_{máx}$

$v_{máx} = wA$

$A = \frac{v_{máx}}{w} = \frac{15}{20} = 0{,}75$

$x(t) = 0{,}75 \cos(20t + \theta)$

$v(t) = -15 \sin(20t + \theta)$

$t = 0 \quad v = 15\ m/s$

$15 = -15 \sin(20.0 + \theta)$

$\therefore \theta = \frac{-\pi}{2}$

a) $x(t) = 0{,}75 \cos(20t - \frac{\pi}{2})$

$v(t) = -15 \sin(20t - \frac{\pi}{2})$

$v(0{,}5) = \underline{-12{,}59\ m/s}$

$-0{,}3 = 0{,}75 \cos(20t - \pi/2)$

$\therefore \underline{t = 0{,}18\ s}$

$m = 5g = 0{,}005\ kg$

$b = 0{,}8\ kg/s$

a) $k = ?$ $\quad \underline{k = 32\ N/m}$

$b = 2\sqrt{mk}$

$k = \left(\frac{b}{2}\right)^2 : m = \frac{b^2}{4m} = \frac{(0{,}8)^2}{4 . 0{,}005}$

$k = 32\ N/m$

b) $A_F = \frac{1}{4} A_I \rightarrow \frac{1}{4} \cancel{A_I} = \cancel{A_I}\, e^{\frac{-b}{2m} t}$

$1/4 = e^{-80t}$

$\ln(1/4) = \ln e^{-80t}$

$-1{,}386 = -80T$

$T = 0{,}017\ s$

$w_a = \frac{2\pi}{T_a} \quad w_a = \frac{2\pi}{0{,}017} = 362{,}59$

$w_a = \sqrt{\frac{K}{m}} \quad \therefore \quad K = w_a^2\, m$

$K = (362{,}59)^2\ 0{,}005$

$\underline{K = 657{,}4\ N/m}$

/ /

$k = 12\ N/m$ $\quad A_f = A_i\, e^{\frac{-b}{2m}\cdot t}$

$b = 0{,}4\ kg/s$ $\quad 0{,}035 = 0{,}025\, e^{\frac{-0{,}4}{2m}\cdot 4}$

$A_i = 0{,}035\ m$ $\quad 1{,}4 = e^{\frac{-0{,}8}{m}}$

$A_f = 0{,}025\ m$ } $t = 4\ s$ $\quad \ln 1{,}4 = \ln e^{\frac{-0{,}8}{m}}$

$0{,}336 = \frac{-0{,}8}{m}$ $\quad \therefore m = 2{,}38\ kg$

$m = 0{,}3\ kg$ $\quad w = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0{,}3}} = 25{,}82\ rad/s$

$k = 200\ N/m$

$A = 0{,}12\ m$ $\quad x(t) = 0{,}12 \cos(25{,}82t + \phi)$

$t = 0 \quad x = -0{,}12\ m$ $\quad t = 0 \rightarrow x = -0{,}12\ m$

$-0{,}12 = 0{,}12 \cos(25{,}82 \cdot 0 + \phi)$

$\therefore \phi = \pi$

$x(t) = 0{,}12 \cos(25{,}82t + \pi)$

$A = -w^2 x$

$A = -666{,}67 \cdot 0{,}06 = \underline{-40\ m/s^2}$

$F = -kx$ $\quad x = \frac{mg}{-k} = \frac{0{,}3 \cdot 10}{200} = -0{,}015$

$mg = -kx$

$-0{,}015 = 0{,}12 \cos(25{,}82t + \pi)$

$E_c = \frac{mv^2}{2}$ $\quad \therefore t = 0{,}06\ s$

$v(t) = -3{,}0984 \sin(25{,}82t + \pi)$

$E_c = \frac{0{,}3(-3{,}098)^2}{2}$ $\quad v(0{,}06) = -3{,}098$

$E_c = 1{,}44\ J$ ou $v = -wA$

$\underline{v = -3{,}098}$

$E_c = \frac{1}{2} E_p \qquad E_m = E_c + E_p$

$E_m = \frac{1}{2} E_p + E_p \qquad E_m = \frac{3}{2} E_p$

$E_p = \frac{Kx^2}{2} = \frac{200 \cdot x^2}{2} \qquad E_p = 100x^2$

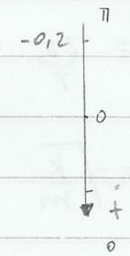
$E_m = \frac{3}{2} 100x^2 = 150x^2 \qquad E = \frac{1}{2} KA^2 = \frac{1}{2} 200 \cdot 0{,}12^2 = 1{,}44$

$1{,}44 = 150x^2 \quad \therefore \quad x = \pm 0{,}098\ m$

➡ $m = 0{,}2\ kg \qquad w = \frac{2\pi}{T} = \frac{2\pi}{2{,}4} = 2{,}62\ rad/s$

$A = -0{,}2\ m \quad (\theta = \pi)$

$T = 2{,}4\ s \qquad x(t) = 0{,}2 \cos(2{,}62t + \pi)$



$A = w^2 x = (2{,}62)^2 \cdot 0{,}05 = 0{,}34\ m/s^2$

$0{,}15 = 0{,}2 \cos(2{,}62t + \pi) \quad \therefore \quad t = 0{,}92\ s$

➡ $F = kx \qquad m = 0{,}22\ kg$

$k = \frac{F}{x} = \frac{24}{0{,}12} = 200\ N/m \qquad t = 0s \begin{cases} x = 0{,}015\ m \\ v = -1{,}2\ m/s \end{cases}$

$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0{,}22}} = 30{,}15\ rad/s$

$E = \frac{mv^2}{2} + \frac{Kx^2}{2} = \frac{0{,}22 \cdot (-1{,}2)^2}{2} + \frac{200 \cdot (0{,}015)^2}{2} = 0{,}1809\ J$

$E = \frac{1}{2} KA^2 \qquad A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2 \cdot 0{,}1809}{200}} = 0{,}04\ m$

$x(t) = 0{,}04 \cos(30{,}15t + \theta) \qquad x(t) = 0{,}04 \cos(30{,}15t + 1{,}21)$

$0{,}015 = 0{,}04 \cos \theta \qquad v(t) = -1{,}28\ sen\ (30{,}15t + 1{,}21)$

$\therefore \theta = 1{,}21\ rad/s$

$a = -w^2 x$

$a = -(30{,}15)^2 . 0{,}03 = -27{,}27 \ m/s^2$

$E = \frac{1}{2} KA^2 = \frac{1}{2} 200 . (0{,}042)^2 = 0{,}176 \ J$

$T = 0{,}5 s$ $\quad m = 0{,}2 \ kg$

$t = 0{,}1 s$ $\left\{ \begin{array}{l} x = 0{,}049 \ m \\ v = -1{,}8 \ m/s \end{array} \right.$

$w = \frac{2\pi}{T} = \frac{2\pi}{0{,}5} = 4\pi \ rad/s$

$w = \sqrt{\frac{K}{m}} \ \therefore \ K = w^2 . m = (4\pi)^2 . 0{,}2 = 31{,}58 \ N/m$

$a = -w^2 x$

$a = -(4\pi)^2 . 0{,}1 = -15{,}79 \ m/s^2$

$Ep = \frac{1}{2} kx^2 = \frac{1}{2} 31{,}58 . 0{,}1^2 = 0{,}158 \ J$

$Em = \frac{mv^2}{2} + \frac{1}{2} Kx^2 = \frac{0{,}2 . (-1{,}8)^2}{2} + \frac{1}{2} 31{,}58 . (0{,}049)^2 = 0{,}36$

$E = \frac{1}{2} KA^2 \ \therefore \ A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2 . 0{,}36}{31{,}58}} = 0{,}151 \ m$

$x(t) = 1{,}51 \cos(wt + \emptyset)$

$0{,}049 = 0{,}151 \cos(wt + \emptyset)$

$wt + \Theta = 1{,}24$

$\Theta = 1{,}24 - 4\pi . 0{,}1 = -0{,}016 \ rad$

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MHA

4-)

$$w = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$A(t) = A e^{\frac{-b}{2m} \cdot t}$

$b = 2\sqrt{mk}$

$m = 2000\ kg$

$A_F = \frac{1}{4} A_I$ $\quad x_{est} = 0{,}04\ m$

m em cada roda = 500 kg

$F = -kx$

$mg = kx$

$$k = \frac{mg}{x} = \frac{500 \cdot 10}{0{,}04} = \underline{125000\ N/m}$$

$$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{125000}{500}} = 15{,}81\ rad/s$$

$$w = \frac{2\pi}{T} \therefore T = \frac{2\pi}{w} = \frac{2\pi}{15{,}81} = 0{,}40\ s \qquad T = 0{,}4 \cdot 4 = 1{,}6$$

$$0{,}01 = 0{,}04\ e^{-\frac{b \cdot 1{,}6}{1000}}$$

$$\ln\left(\frac{0{,}01}{0{,}04}\right) = -\frac{b \cdot 1{,}6}{1000} \quad \therefore b = \underline{866{,}43\ kg/s}$$

$$E = \frac{kA^2}{2} = \frac{125000 \cdot 0{,}04^2}{2} = 100\ J$$

5-) $t = 0$ $\quad v_{máx} = 108\ km/h$

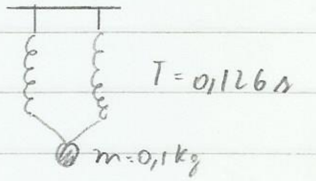
$$w = \frac{2\pi}{T} = \frac{2\pi}{4{,}9} = 1{,}28\ rad/s$$

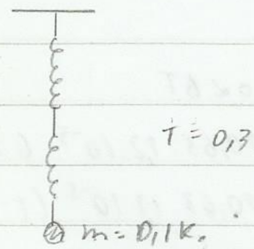
$m = 60\ kg$

$$T = \frac{9{,}8}{2} = 4{,}9\ s \qquad k = w^2 m = (1{,}28)^2 \cdot 60 =$$

4) $m = 0{,}1\ kg$

$T = 0{,}126\,s$





$$\frac{2\pi}{T} = \sqrt{\frac{K}{m}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{K}} \quad \therefore \quad T = 2\pi\sqrt{\frac{m}{K}}$$

$$0{,}126 = 2\pi\sqrt{\frac{0{,}1}{K}} \Rightarrow k = 248{,}67\,N \quad (\text{Paralelo})$$

$$0{,}314 = 2\pi\sqrt{\frac{0{,}1}{K}} \Rightarrow k = 40{,}04\,N \quad (\text{Série})$$

$$k_1 + K_2 = 248{,}67 \qquad k_1 = 248{,}67 - K_2$$

$$\frac{K_1 \cdot K_2}{K_1 + K_2} = 40{,}04 \qquad K_1 K_2 = 40{,}04\,K_1 + 40{,}04\,K_2$$

$$K_1 = \frac{40{,}04\,K_1}{K_2} + 40{,}04$$

$$K_1 = \frac{40{,}04\,(248{,}67 - K_2)}{K_2} + 40{,}04$$

$$K_1 = \frac{9956{,}7468 - 40{,}04\,K_2 + 40{,}04\,K_2}{K_2}$$

$$248{,}67\,K_2 - K_2^2 = 9956{,}7468 - \cancel{40{,}04\,K_2} + \cancel{40{,}04\,K_2}$$

$$-K_2^2 + 248{,}67\,K_2 - 9956{,}7468 = 0$$

$$K_2 = \frac{-248{,}67 \pm \sqrt{22009{,}78}}{-2}$$

$K_{2(I)} = \underline{198{,}51}$ ou $K_{2(II)} = \underline{50{,}16}$

$E\,\emptyset = 40{,}63\,mm \qquad T_0 = 30°C$

$P\,\emptyset = 40{,}61\,mm \qquad T_0 = 30°C$

$\Delta L = L_0 \alpha \Delta T$

$(40{,}61 - 40{,}63) = 40{,}63 \cdot 1{,}2 \cdot 10^{-5} \cdot (T - 30)$

$-0{,}02 = 40{,}63 \cdot 1{,}2 \cdot 10^{-5}\,(T-30)$

$\therefore T = -11{,}02°C$

$b = \frac{\Delta T^2}{\Delta L} = \frac{5{,}92}{3} = 1{,}97 \qquad T = 2\pi\sqrt{\frac{L}{g}} \qquad g = \left(\frac{2\pi}{T}\right)^2 \cdot L = \frac{4\pi^2}{T^2}L$

$b = \frac{4\pi^2}{g} \qquad g = \frac{4\pi^2}{b} = \frac{4\pi^2}{1{,}97} = 20{,}04\,cm/s^2$

$T = 2\pi\sqrt{\frac{L}{g}} \qquad \left(\frac{T}{2\pi}\right)^2 = \frac{L}{g} \qquad g = \frac{4\pi^2}{T^2} \cdot L$ b

$m = 0{,}1\,kg$ ou $100g$

$k = 1\,N/m$

$f = 0{,}4\,Hz$

$w = \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$

$w^2 = \frac{K}{m} - \frac{b^2}{4m^2}$

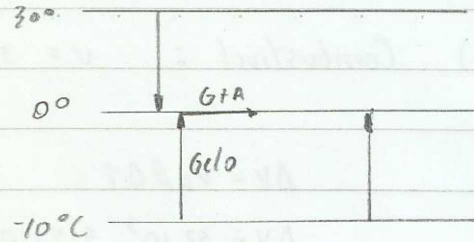
$w = 2\pi \cdot 0{,}4 = 0{,}8\pi$

$(0{,}8\pi)^2 = \frac{1}{0{,}1} - \frac{b^2}{4 \cdot (0{,}1)^2}$

$\therefore b = 0{,}38\ kg/s$

➡ Calorim : $m = 0{,}15$ kg
Gelo = $m = 0{,}1$ kg
Agua : $m = ?$ $T_0 = 30°C$

$T_0 = -10°C$



$Q_{EA} = m \cdot 4190\,(0-30) = -125700\,m$

$Q_{EG} = 0{,}1 \cdot 2100\,(0+10) = 2100$ J

$Q_{EC} = 0{,}15 \cdot 910\,(0+10) = 1365$ J

$Q_{LG} = 0{,}07 \cdot 334000 = 23380$

$\Sigma Q = 0$

$-125700\,m + 2100 + 1365 + 23380 = 0$

$m = 0{,}214$ kg

➡ $F = 20N$ $\quad T = 1{,}5s$ $\quad F = Kx$

$\bar{x} = 0{,}25$ m $\quad K = \frac{F}{x} = \frac{20}{0{,}25} = 80\,N/m$

$t = 0 \begin{cases} x = 0{,}25\,m \\ v = -2\,m/s \end{cases}$ $\quad w = \sqrt{\frac{K}{m}} = \frac{2\pi}{T}$

$\sqrt{\frac{80}{m}} = \frac{2\pi}{1{,}5}$ $\quad \therefore m = 4{,}56$ kg

$Em = \frac{4{,}56 \cdot (-2)^2}{2} + \frac{1}{2} \cdot 80 \cdot (0{,}25)^2 = 11{,}62$ J

$E = \frac{1}{2}KA^2$ $\quad A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2 \cdot 11{,}62}{80}} = 0{,}54$ m

$x(t) = 0{,}54\cos(4{,}19t + \phi)$

$0{,}25 = 0{,}54\cos(4{,}19 \cdot 0 + \phi)$

$\therefore \phi = 1{,}09$

$x(t) = 0{,}54\cos(4{,}19t + 1{,}09)$ (S.I)

1) Combustível : $V = 32.10^3 L$ $T_F = -18°C$

$\Delta V = V_0 \beta \Delta T$ $\Delta V = V - V_0$

$\Delta V = 32,10^3 . 9,5.10^{-4} (-18)$ $V = \Delta V + V_0$

$\Delta V = -547,2$

$V - V_0 = -547,2$ $V = -54,2 + 32.10^3 = 31452,8 L$

2) $b = \frac{\Delta T^2}{\Delta m} = \frac{0,90 - 0,30}{0,455 - 0,15} = 1,97 \, s^2/gk$

$\omega = \frac{2\pi}{T} = \sqrt{\frac{K}{m}} \Rightarrow \frac{4\pi^2}{T^2} = \frac{K}{m} \Rightarrow \frac{4\pi^2}{K} = \frac{T^2}{m}$

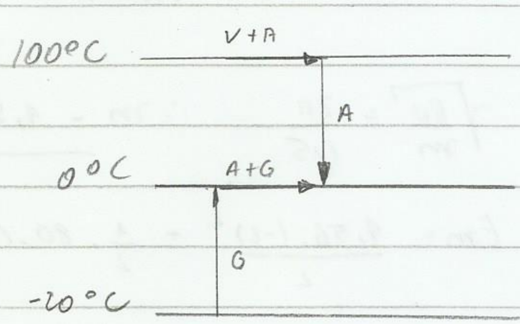
$\frac{4\pi^2}{K} = 1,97$ $\therefore K = 20,03 \, N/m$

$\frac{1}{K_1} + \frac{1}{K_2} = \frac{1}{K_F} \Rightarrow \frac{1}{K_1} + \frac{1}{30} = \frac{1}{20,03}$ $\therefore K_1 = 60,35 \, N/m$

3-) Gelo $m = 27 kg$ $T = -20°C$

Vapor $m_v = ?$ $T = 100°C$

$m_g = m_a$



$Q_{LV} = -m. 2,256.10^6$

$Q_{EV \to A} = m. 4190 (0 - 100) = -419000 m$

$Q_{EG} = 27. 2100 (0 + 20) = 1134000 J$

$Q_L = \frac{(-m + 27)}{2} . 3,34 . 10^5 = -167000 m + 4509000 J$

$\Sigma Q = 0$ / $-2,256.10^6 m - 419000 m + 1134000 - 167000 m + 4509000 = 0$

$-2842000 m + 5643000 = 0$

$m = 1,986 \, kg$

4) $m = 0{,}8$ kg

$\bar{x} = 10$ cm

$t = 0{,}05\,s \begin{cases} x = 5{,}6\text{ cm} \\ v = -57{,}4\text{ cm/s} \end{cases}$

$F = Kx \qquad K = \frac{F}{x} = \frac{0{,}8.10}{0{,}1} = 80\,N/m$

$w = \sqrt{\frac{K}{m}} = \sqrt{\frac{80}{0{,}8}} = 10$ rad/s

$E = \frac{0{,}8.(-0{,}574)^2}{2} + \frac{1}{2}\,80.(0{,}056)^2 = 0{,}257$ J

$E = \frac{1}{2} KA^2 \qquad A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2.\,0{,}257}{80}} = 0{,}08$ m

$x(t) = 0{,}08 \cos(10t + \emptyset)$

$t = 0{,}05\,s \rightarrow x = 0{,}0056$

$0{,}056 = 0{,}08 \cos(10.0{,}05 + \emptyset)$

$\therefore \emptyset = \underline{0{,}3\text{ rad}}$

ou

$\div \begin{cases} 5{,}6 = A \cos(0{,}5 + \emptyset) \\ 5{,}74 = A \sin(0{,}5 + \emptyset) \end{cases}$

$\text{arctg}\left(\frac{5{,}6}{5{,}74}\right) = 0{,}5 + \emptyset \quad \therefore \underline{\emptyset = 0{,}27\text{ rad}}$

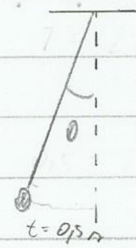
$-0{,}05 = 0{,}08 \cos(10t + 0{,}3)$

$\therefore t = 0{,}19459$

$v(t) = -0{,}8 \sin(10t + 0{,}3)$

$v(0{,}19459) = -0{,}62$ m/s

1) $\theta = 2{,}00°$



$\frac{T}{4} = 0{,}5 \quad T = 2\,s$

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \therefore \quad L = \frac{T^2}{4\pi^2} \cdot g = \frac{2^2}{4\pi} \cdot 9{,}8 = 0{,}99\ m$$

2) $A = 12\ cm$

$f = 0{,}4\ Hz$

$t = 0{,}3\,s \rightarrow x = 6{,}9\,cm$

$w = 2\pi f = 2\pi \cdot 0{,}4 = \underline{0{,}8\pi\ rad/s}$

$$A = -w^2 x(t) = -(0{,}8\pi)^2\,(-0{,}05) = \underline{0{,}316\ m/s^2}$$

$$x(t) = 0{,}12\cos(0{,}8\pi t + \theta)$$

$$0{,}069 = 0{,}12\cos(0{,}8\pi \cdot 0{,}3 + \theta)$$

$$\therefore \theta = 0{,}204\ rad$$

$$-0{,}08 = 0{,}12\cos(0{,}8\pi \cdot t + 0{,}209)$$

$$\therefore t = 0{,}839\,s$$

$$\Delta t = 0{,}839 - 0{,}3 = 0{,}539\,s$$

3-) Calorimetro = $m = 0{,}2\ kg$ $\Big\rangle\ T = 20°C$

Água = $m = 0{,}3\ kg$

Gelo = $m = 0{,}12\ kg$ $\quad T = -12°C$

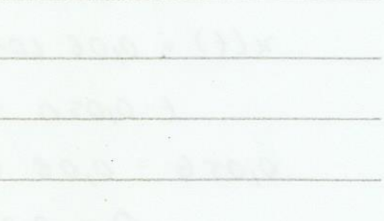
$T_E = 0°C$

$Q_{SC} = 0{,}2 \cdot 390\,(0-20) = -1560$

$Q_{EA} = 0{,}3 \cdot 4190\,(0-20) = -25140$

$Q_{EG} = 0{,}12 \cdot 2100\,(0+12) = 3024$

$Q_{LG} = 0{,}12 \cdot 3{,}34 \cdot 10^5 = 40{,}080$



$Q_g > Q_c + Q_a$

$\Sigma Q = 0$

$Q_{EA} + Q_{EC} + Q_{EG} + Q_{EG \to A}$

$-25140 - 1560 + 3024 + m\, 3{,}34 . 10^5 = 0$

$\therefore m = 0{,}071\ kg$

$m\ Água = 0{,}3 + 0{,}071 = 0{,}37\ kg$

$m\ Gelo = 0{,}12 - 0{,}071 = 0{,}049\ kg$

4-) $V_f = 1{,}0066\, V_0$ $\qquad \Delta V = V_0 \beta \Delta T$

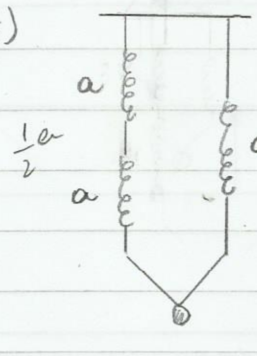
$T_0 = 22°C$ $\qquad V = V_0 (1 + \beta \Delta T)$

$3\alpha = \beta$ $\qquad 1{,}0066\, V_0 = V_0 (1 + 6.10^{-5} (T - 22))$

$\beta = 3.2{,}0 . 10^{-5}$ $\qquad 1{,}0066 = 1 + 6.10^{-5} (T - 22)$

$\beta = 6.10^{-5}$ $\qquad \therefore T = \underline{132\ °C}$

5)



$K = \frac{1}{3} a$

$a_{eq} = \frac{3}{2} a$

$\frac{2\pi}{T} = \sqrt{\frac{K}{m}}$ $\qquad \frac{4\pi^2}{T^2} = \frac{K}{m}$ $\qquad K = \frac{4\pi^2 . m}{T^2}$

$K = \frac{4\pi^2 . 2}{(0{,}5)^2} = 315{,}83\ N/m$

$315{,}83 = \frac{3}{2} a \quad \therefore a = 210{,}55$

$k = \frac{1}{3}\, 210{,}55 = 70{,}18\ N/m$

b) $K = 70{,}18$ $\qquad T = 2\pi\sqrt{\frac{m}{K}}$

$m = 2\ kg$

$$t = 2\pi\sqrt{\frac{2}{70{,}16}} = 1{,}06\ s$$

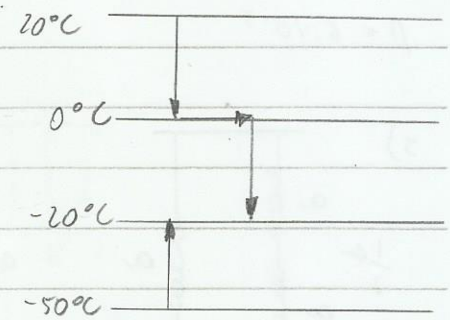
1-) $b = 2{,}5\ s^2/m$ $\qquad b = \frac{4\pi^2}{g}$ $\quad g = \frac{4\pi^2}{b} = \frac{4\pi^2}{2{,}5} = 15{,}79\ m/s^2$

- Como a gravidade da terra é maior que a gravidade da lua, o pêndulo oscilaria mais lentamente na lua

2-) Água = $m = 0{,}2\ kg$ $\quad T_0 = 20°C$ $\quad T_f = -20°C$

Gelo = $m = ?$ $\quad T_0 = -50°C$

Vapor = $T_0 = 100°C$



$Q_{SA} = 0{,}2 \cdot 4190\,(0-20) = -16760\ J$

$Q_{LA} = -0{,}2 \cdot 3{,}34 \cdot 10^5 = -66800\ J$

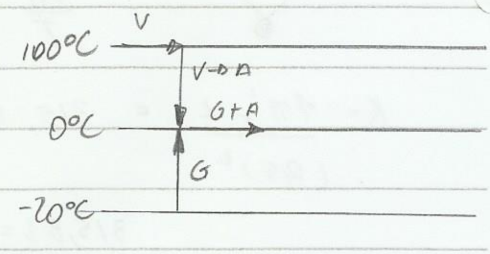
$Q_{SA \to G} = 0{,}2 \cdot 2100\,(-20-0) = -8400\ J$

$Q_{SG} = m \cdot 2100\,(-20+50) = 63000\,m$

$\Sigma Q = 0$

$-16760 - 66800 - 8400 + 63000\,m = 0$

$\therefore m = 1{,}46\ kg$



$m_{gelo} = 1{,}46 + 0{,}2 = 1{,}66$ $\quad T_0 = -20°C$

$m_{vapor} =$ $\quad T_0 = 100°C$

$\frac{m+g}{2} = \frac{m_v + 1{,}66}{2} = \frac{m_v}{2} + 0{,}83$

$Q_{LV} = -m \cdot 2{,}256 \cdot 10^6$

$Q_{SV \to A} = m \cdot 4190\,(0-100) = -419000\,m$

$Q_{SG} = 1{,}66 \cdot 2100\,(0+20) = 69720$

$Q_{LG} = \left(0{,}83 - \frac{m}{2}\right) \cdot 3{,}34 \cdot 10^5 = -167000m + 277220$

$\Sigma Q = 0$

$-2{,}256 \cdot 10^6 m - 419000 m - 167000 m + 277220 + 69720 = 0$

$2842000 m + 346940 = 0$

$m = 0{,}122 \text{ kg}$

3-) $x = 0{,}08 m$ $\begin{cases} E_p = 0{,}96 J \\ a = -120 m/s^2 \end{cases}$

$x = 0 \Rightarrow v = 4{,}65 m/s$

$E_p = \frac{1}{2} k x^2$ $\quad K = \frac{2E}{x^2} = \frac{2 \cdot 0{,}96}{0{,}08^2} = \underline{300 N/m}$

$a = -w^2 x$ $\quad w^2 = \frac{-a}{x} = \frac{120}{0{,}08} \therefore w = 38{,}73$

$w = \frac{2\pi}{T} = 38{,}73 \quad \therefore \underline{T = 0{,}162 s}$

$E = \frac{1}{2} KA^2 = \frac{1}{2} K \left(\frac{v_{max}}{-w}\right)^2$ $\quad v_{max} = -wA$ $\quad A = \frac{v_{ma}}{-w}$

$= \frac{1}{2} \cdot 300 \left(\frac{4{,}65}{38{,}73}\right)^2 = \underline{2{,}16223 J}$

$\begin{cases} -0{,}03 = A \cos\theta \\ 0{,}116 = A \sin\theta \end{cases}$ $\quad \theta = \operatorname{arctg}\left(\frac{0{,}116}{-0{,}03}\right) = -1{,}318 \text{ rad}$

$\theta = -1{,}318$

$A = \frac{-0{,}03}{\cos(-1{,}318)} < 0 \quad \therefore \theta = \theta + \pi = -1{,}318 + \pi = \underline{1{,}8235 \text{ rad}}$

a) $m = 1{,}8\ kg$

$k = 200\ N/m$

$A = 0{,}05\ m \quad t = 0$

$t = 6s \rightarrow A = 0{,}02\ m$

$A(t) = A\, e^{\frac{-b}{2m} t}$

$0{,}02 = 0{,}05\, e^{\frac{-b}{2 \cdot 1{,}8} \cdot 6}$

$\ln\left(\frac{0{,}02}{0{,}05}\right) = \frac{-b}{2 \cdot 1{,}8} \cdot 6$

$\therefore b = 0{,}55\ \frac{Kg}{s}$.

b) $w_a = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$w_a = \sqrt{\frac{200}{1{,}8} - \frac{(0{,}55)^2}{4 \cdot 1{,}8^2}} = 10{,}53$

$w_a = \frac{2\pi}{T} = 10{,}53 \quad \therefore T = 0{,}597s$

$1 os = 0{,}597 \qquad \therefore n = 10{,}05\ s$

$n = 6$

5) $L = L_1 + L_2$

$\Delta L = \Delta L_1 + \Delta L_2$

$\Delta L = (L_1 \alpha_1 + L_2 \alpha_2) \Delta T$

$\Delta L = (2 \cdot 2{,}4 \cdot 10^{-5} + 1{,}5 \cdot 14{,}2 \cdot 10^{-5})\, 90$

$\Delta L = 0{,}01099\ m$

1) $v = 20\ L$

Ondas Mecânicas

$v = \frac{\Delta s}{\Delta t}$ $\mu = \frac{m}{L}$ $v = \sqrt{\frac{F}{\mu}}$ $f = \frac{1}{T}$ ou $T = \frac{1}{f}$

λ: comprimento de onda

$v = \frac{\Delta s}{\Delta T}$ => quando $\Delta s = \lambda$ então $\Delta t = T$ => $v = \frac{\lambda}{T}$ ou $v = \lambda f$

Equação da onda: $y(x,t) = A\cos(kx - wt)$

Número de onda (k) $k = \frac{2\pi}{\lambda}$ (rad/m) ou $\frac{w}{v} = k$

posição: $A\cos(kx - wt)$
velocidade: $wA\,\text{sen}(kx - wt)$
aceleração: $-w^2 A\cos(kx - wt)$ ou $-w^2 y(x,t)$

Exercícios Ondas Mecanicas

Ex 15.1

$v = 344 \, m/s$ $\qquad v = \lambda f$

$f = 262 \, Hz$ $\qquad \lambda = \frac{v}{f} = \frac{344}{262} = 1,31 \, m$

Ex 15.2

$A = 0,075 \, m$ $\qquad t = 0 \begin{cases} d + (máx) \\ v = 0 \end{cases}$

$f = 2,0 \, Hz$

$v = 12,0 \, m/s$

a) amplitude: ? frequência angular? período? comprimento da onda:? número da onda:?

$v = \lambda f$ $\qquad T = \frac{1}{f} = \frac{1}{2} = 0,5 s$ $\qquad w = 2\pi f = 2\pi . 2 = 4\pi = 12,6 \, rad/m$

$\lambda = \frac{v}{f} = \frac{12}{2} = 6 m$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6} = 1,05 \, rad/m$

b) função da onda: ?

$\lambda(x,t) = A \cos(kx + wt)$

$\lambda(x,t) = 0,075 \cos(1,05x - 12,6t)$

c) função da onda quando $x = 3,0 \, m$

$\lambda(3,t) = 0,075 \cos(1,05.3 - 12,6t)$

$= 0,075 \cos(3,15 - 12,6t)$

/ /

Exercícios: Livro "Sears & Zemansky"

15,1-) $v = 344\ m/s$ a) Comprimento da onda = ? tempo (ms) = ?

$f = 784\ Hz$ b) " " " máxima

$v = \lambda f \quad \lambda = \frac{v}{f} = \frac{344}{784} = \underline{0,439\ m}$

$T = \frac{1}{f} = \frac{1}{784} = \underline{1,28\ ms}$

b) ?

15.3-) $\Delta s = 800\ km$ ou $800000\ m$

$T = 60s$ ou $1h$

$v = 288\ m/s$ ou $800\ km/h$

15.7-) $v = 8\ m/s$ sentido $-x$

$A = 0,07\ m$ $t = 0 \begin{cases} x = 0 \\ \text{sentido } + \end{cases}$

$\lambda = 0,320\ m$

a) $v = \lambda f$

$f = \frac{v}{\lambda} = \frac{8}{0,32} = \underline{25\ Hz}$

$T = \frac{1}{f} = \frac{1}{25} = \underline{0,04\ s}$

$w = 2\pi f = 2\pi . 25 = 50\pi = 157,08\ rad/m$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0,320} = \underline{19,63}\ rad/m$

b) $y(x,t) = 0,07 \cos(19,63x - 157,08t)$ (SI)

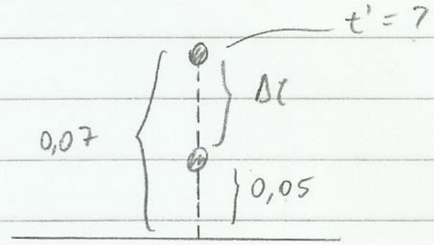
c) $x = 0{,}360\,m$

$t = 0{,}150\,s$

$y(x,t) = 0{,}07 \cos(19{,}63x - 157{,}08t)\ (SI)$

$y(0{,}360\ ;\ 0{,}15) = \underline{-0{,}05\ m}$

d) $x = 0{,}360$



$\left.\begin{array}{l} x = 0{,}360 \\ y = 0{,}07 \end{array}\right\} t' = ?$

$0{,}07 = 0{,}07 \cos(19{,}63 \cdot 0{,}360 + 157{,}08t)$

$\therefore t =$

Lista BASE

1.) $\mu = 0{,}03\ kg/m$

$(x,t) = 2{,}5 \cos\left(\frac{\pi x}{6} - \frac{3\pi}{2}t\right)$

a) $v = \lambda f$ $\qquad v = \lambda \cdot \frac{w}{2\pi} = \frac{\pi/6 \cdot 3\pi/2}{2\pi} =$

$w = 2\pi f \Rightarrow f = \frac{w}{2\pi}$

2-) $\mu = 0{,}04\ kg/m$ (direção $-x$)

$F = 4N$ $x = 0 \rightarrow y(máx)$ positivo

$A = 0{,}3\ m$

$t = 0 \begin{cases} x = 0{,}05\ m \\ y = 0{,}025\ m \end{cases}$

a) v. propagação da onda transversal

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{4}{0{,}04}} = 10\ m/s$$

número de onda

$$k = \frac{2\pi}{\lambda} \text{ ou } k = \frac{w}{v}$$

Exercícios treino

1º sem 2008 -)

$F = 18N$

$m = 0{,}002\ kg$

$l = 0{,}05\ m$

$\lambda = 0{,}48\ m$

$A = 0{,}0025\ m$

$v = \sqrt{\frac{F}{\mu}}$ $\mu = \frac{m}{L} = \frac{0{,}002}{0{,}5} = 0{,}004$

$v = \sqrt{\frac{18}{0{,}004}} = \underline{67{,}08\ m/s}$

$v = \lambda f \therefore f = \frac{v}{\lambda} = \frac{67{,}08}{0{,}48} = \underline{139{,}75}$

$P = \frac{1}{2}(\mu \cdot v\,(wA)^2)$

$w = 2\pi f = 2\pi \cdot 139{,}75 = 878{,}10$

$P = \frac{1}{2}\left(0{,}004 \cdot 67{,}08\,(878{,}10 \cdot 0{,}0025)^2\right)$

$\therefore P = \underline{0{,}647\,W}$

1º sem 2009 -)

$\lambda = 2{,}0\ m$

$f = \frac{200}{\pi}\ Hz$

$v = 95{,}5\ m/s$

$v_y = 6\ m/s$

a) $A = ?$

$|v_y^{max}| = wA \therefore A = \frac{|v_y^{max}|}{w} = \frac{6}{400} = \underline{0{,}015m}$

$w = 2\pi f = 2\pi \cdot \frac{200}{\pi} = 400$

$K = \frac{2\pi}{\lambda} = \frac{w}{v} = \frac{400}{95{,}5} = 4{,}19\,\frac{rad}{m}$

b) $y(x,t) = A\cos(kx - wt)$

$y(x,t) = 0{,}015\cos(4{,}19x - 400t)$ (SI)

/ /

1º sem 2010 $40 \frac{g}{m} = 0{,}04 \frac{kg}{m}$

$\mu = 40 g/m$

$y(x,t) = 7{,}6 \cdot 10^{-2} \cos(2\pi x + 40\pi t)$ [S.I.] $\begin{cases} A = 7{,}6 \cdot 10^{-2} \\ k = 2\pi \\ w = 40\pi \end{cases}$

a) $F = ?$

$v = \sqrt{\frac{F}{\mu}} \therefore F = v^2 \cdot \mu = 20^2 \cdot 0{,}04 = 16 N$

$v = \lambda f \quad k = \frac{2\pi}{\lambda} \therefore \lambda = \frac{2\pi}{K} = \frac{2\pi}{2\pi} = 1 m$

$v = \frac{40\pi}{2\pi} = 20 m/s$

b) $v(x,t) = -3{,}04\pi \sin(2\pi x + 40\pi t)$

$x = 0{,}19 \quad t = 0{,}33$

$v(0{,}19; 0{,}33) = -3{,}04\pi \sin(2\pi \cdot 0{,}19 + 40\pi \cdot 0{,}33)$

$\therefore v_y = 9{,}25 m/s$

c) $t = ?$ $x = 0{,}37 m$ $t = 0{,}13 s$

deslocamento máximo

$|\cos(2\pi x + 40\pi t)| = 1$

$2\cancel{\pi} x + 40\cancel{\pi} t = n\cancel{\pi}$

Em $x = 0{,}37$ e $t = 0{,}13$ $\Delta t = t' - t$

$2 \cdot 0{,}37 + 40 \cdot 0{,}13 = n \therefore n = 5{,}94 \Rightarrow n = 6$ $\Delta t = 0{,}1315 - 0{,}13$

$\underline{\Delta t = 0{,}0015 s}$

$2 \cdot 0{,}37 + 40 t' = 6 \therefore t' = 0{,}1315 s$

1 sem 2010 (1)

$\lambda = 0{,}2\ m$ $\qquad y(x,t) = A\cos(kx - wt)$

$T = 4s$

$|v_y^{max}| = 0{,}05\ m/s$ $\qquad k = \frac{2\pi}{\lambda} = \frac{2\pi}{0{,}2} = 10\pi \qquad w = \frac{2\pi}{T} = \frac{2\pi}{4} = 0{,}5\pi$

$v_y^{max} = wA \quad \therefore \quad A = \frac{v^{max}}{w} = \frac{0{,}05}{0{,}5\pi} = \frac{0{,}1}{\pi}$

$y(x,t) = \frac{0{,}1}{\pi}\cos(10\pi x - 0{,}5\pi t)$ [SI]

b) $a(x,t) = -w^2 y(x,t)$

$a(0{,}02;0{,}5) = -(0{,}5\pi)^2\, y(0{,}02;0{,}5)$

$\therefore a(0{,}02;0{,}5) = \underline{-0{,}0776\ m/s^2}$

2 sem 2006

$$y = 0{,}12\ sen\left[2\pi\left(\frac{t}{0{,}05} + \frac{x}{0{,}4}\right)\right] \text{ [SI]}$$

$t = ? \quad x = 0{,}42m \quad t = 0{,}33s$

No instante $t = 0{,}33s$ e $x = 0{,}42m$

$\alpha = 2\pi\left(\frac{0{,}33}{0{,}05} + \frac{0{,}42}{0{,}4}\right) = 15{,}3\pi$

O próximo deslocamento nulo é 16π

$\therefore 16\pi = 2\pi\left(\frac{t}{0{,}05} + \frac{0{,}42}{0{,}4}\right) \therefore \underline{t = 0{,}35s}$

$\Delta t = t' - t$

$\Delta t = 0{,}3475 - 0{,}33$

$\underline{\Delta t = 0{,}0175s}$

2ª sem 2006 (Noturno)

$$y = 5.10^{-3} \operatorname{sen}(314t)$$

a) $w = 2\pi f \therefore f = \frac{w}{2\pi} = \frac{314}{2\pi} = \underline{50\ Hz}$

$\lambda = ?$

$v = \lambda f \quad \therefore \quad \lambda = \frac{v}{f} = \frac{60}{50} = \underline{1,2\ m}$

b) $v = \sqrt{\frac{F}{\mu}} \quad \therefore \quad F = v^2 \mu = 60^2 \cdot 0,04 = \underline{144\ N}$

c) $y(x,t) = A \operatorname{sen}(Kx + wt)$ $\qquad K = \frac{2\pi}{\lambda} = \frac{2\pi}{1,2} = 5,24$

$y(x,t) = 5.10^{-3} \operatorname{sen}(5,24x - 314t)$

d)

Terminar Depois

Física II   Termodinâmica   AraKaki

Temperatura

→ Eq térmico

"Quente"

"Frio"

→ Quantificar "números"

Lei zero termodinamica

"Quando um corpo esta em equilibrio térmico com dois corpos, então ambos corpos estão em equibrio térmico".

Termometros

· Subst. termométrica

1) Resist. elétrica

2) Pressão de um gás

3) Dilatação térmica

Linearidade

Para construir um termometro, ~~bas~~ que seja linear, basta utilizar dois pontos e calibrar de acordo com os dados que você conhece

$$\frac{t_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} \Rightarrow T_F = \frac{9}{5} t_C + 32$$

$$T_C = \frac{5}{9}(T_F - 32)$$

/ /

No gráfico formado por gas x t(°C), foi descoberto a unidade Kelvin, cuja fórmula á seguir:

$$T_K = T_C + 273,15$$

Já no gráfico de gas x t(K), notamos que o gráfico é formado desde a origem, no qual foi observado que existe um coeficiente entre o gas e a temperatura em Kelvim $\Rightarrow P/T = K$

Expansão térmica tambem conhecida como dilatação térmica Linear

$$\Delta L = L - L_0 \Rightarrow \Delta L = L_0 \alpha \Delta T \quad ou \quad \Delta L = L_0 \alpha (L - L_0)$$

*coeficiente de dilatação linear:

$$\alpha = \frac{\Delta L}{L_0 \Delta T} \quad °C^{-1} \text{ ou } K^{-1}$$

$$L = L_0(1 + \alpha \Delta T)$$

* dilatação volumétrica

$$\Delta V = \beta V_0 \Delta T$$

$$\beta = \frac{\Delta V}{V_0 \Delta T} \quad °C^{-1} \text{ ou } K^{-1}$$

$$V = V_0(1 + \beta \Delta T)$$

relação entre α e β

$$L^3 = L_0^3 (1 + \beta \Delta T)$$

$$\cancel{L_0^3} (1 + \alpha \Delta T)^3 = \cancel{L_0^3} (1 + \beta \Delta T)$$

$$\cancel{1} + 3\alpha \Delta T + \cancel{3\alpha^2 \Delta T^2} + \cancel{\alpha^3 \Delta T^3} = \cancel{1} + \beta \Delta T \quad \Rightarrow \quad \beta = 3\alpha$$

$$3\alpha \Delta T = \beta \Delta T$$

Recapitulação breve

- Termometros
- escalas termométricas
- escala absoluta (Kelvin)
- dilatação linear (α)
- dilatação volumétrica $\beta = 3\alpha$

Calor:

Quantidade de calor

calor = energia

1cal = 4,186 J

Quantidade de calor

calor específico de uma substancia

É a quantidade de energia capaz de aumentar a temperatura de uma massa unitária de uma substancia de 1°C (1K)

$Q = mC\Delta t$, onde: Q = energia

m = massa

Δt = variação de temperatura

$dQ = mc\,dt$

$c = \frac{1}{m} \frac{dQ}{dt}$

Quantidade de calor específico - é a quantidade de energia capaz de aumentar a temperatura de uma massa unitária de uma substância de 1°C (1K) , $C = J/_{Kg°C}$ ou $K/_{KgK}$

Calor específico molar é a quantidade de energia para aumentar a temperatura de 1mol de substancia variar de 1°C (1K)

$C = Mc$

↳ massa molecular

↳ Calor específico molar

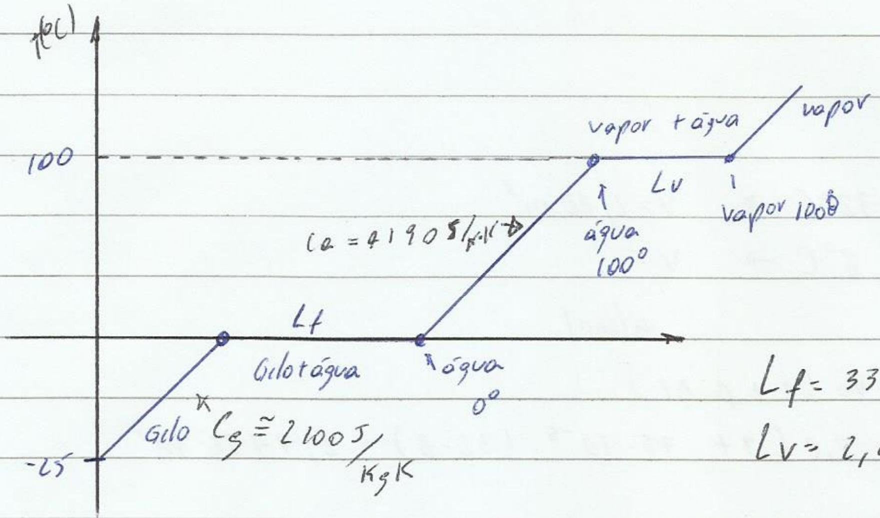
subst.	c J/kgK	C J/molK
Aluminio	910	24,6
Cobre	390	24,8
Alcool	2428	111,9
Gelo	2100	37,8
Ferro	470	26,3

Transição de fase

Fase: $\left\{ \begin{array}{l} \text{sólida} \\ \text{liquida} \\ \text{gasosa} \end{array} \right\}$ estado das substancias

Calor liquefação L_f
(latente)

Calor vaporização L_v
(latente)



$L_f = 334 \times 10^3\ J/kg$

$L_v = 2{,}256 \times 10^6\ J/kg$

Fórmulas

$$\frac{T_C}{100} = \frac{T_F - 32}{180}$$

$$\alpha = \frac{\Delta L}{L_0 \Delta T}$$

$$L = L_0 (1 + \alpha [T - T_0])$$

$$\beta = \frac{\Delta V}{L_0 \Delta T}$$

$$V = V_0 (1 + \beta [T - T_0])$$

$\left.\frac{L_0}{T_0}\right\}$ Frio $\quad \left.\frac{L}{T}\right\}$ Quente

$$\frac{°X-2}{96} = \frac{Tc}{100} \Rightarrow \frac{°X-2}{24} = \frac{Tc}{25}$$

(Ex. exemplo)

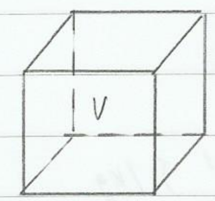
$25(x-2) = 24Tc$

$25(x-2) = 24x$

$25x-50 = 24x$

$x = 50$

17.24.208



$32°C \rightarrow V = 2,80 \, m^3$

$8°C \rightarrow V = ?$

álcool

$V = V_0(1 + \beta \Delta t)$

$V = 2,8(1 + 75 \cdot 10^{-5} \cdot (32-8) = 2,7496 \, m^3$

$\Delta V = 0,0504 \, m^3$

Aço

$V = V_0(1 + \beta \Delta t)$

$V = 2,8(1 + 3,6 \cdot 10^{-5} \cdot (-24)) = 2,797 \qquad \Delta V = 0,0029192$

R: $2,797 - 2,7496 = 0,0474 \, m^3$

Resumo

$Q = mc\Delta T$

Mudança de fase : $T = cte$

Água

$L_f : 334 \times 10^3 \ J/kg$

$L_v : 2{,}256 \times 10^6 \ J/kg$

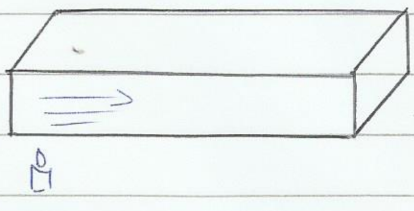
Calorimetria : $\Sigma Q_i = 0$

Calor perdido = Calor ganho

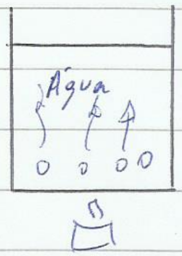
Mecanismo transferencia de Calor

- condução
- convenção
- radiação

condução metais



convenção



/ /

- Radiação "Ondas eletromagnéticas"

- As ondas magnéticas se propaga no vácuo

Formulário

Dilatação

$\Delta L = \alpha L_0 \Delta T$

$L = L_0 (1 + \alpha \Delta T)$

$\Delta V = \beta V_0 \Delta T$

$V = V_0 (1 + \beta \Delta T)$

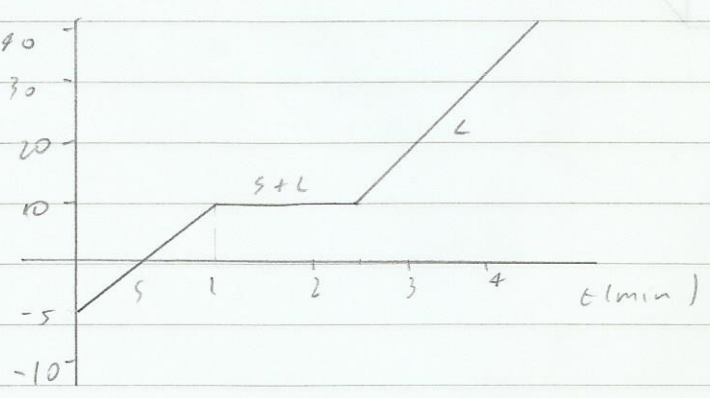
$Q = mc\Delta T$

mL_f = Energia p/ liquefazer uma subst. (solidificar)

mL_v = energia p/ vaporizar (liquefazer) 1 substancia

17.44

m = 500,0 g   10,0 kJ/min

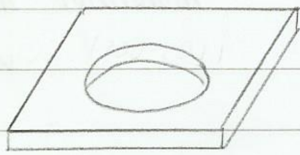


1-) $\varnothing = 3{,}5\,cm$

$T_0 = 20°C$

$T = 300°C$

$\alpha = 2{,}4.10^{-5}\,°C^{-1}$



$L = L_0(1+\alpha\Delta T)$

$L = 3{,}5(1+2{,}4.10^{-5}.280)$

$L = 3{,}52$ $\qquad$ $\underline{D = 9{,}75\,cm}$

2-) $T_0 = 16°C$ $\qquad$ $V_0 = x$ $\qquad$ $x = 100$ $\quad \therefore x' = 1{,}0023\,x$

$\beta = 6.10^{-5}$ $\qquad$ $V = 1{,}0023\,x$ $\qquad$ $x' = 100{,}23$

$\Delta V = V_0\,\beta\,\Delta T$

$\Delta T = \dfrac{\Delta V}{V_0\beta} \sim T - T_0 = \dfrac{\Delta V}{V_0\beta}$

$T = \dfrac{\Delta V}{V_0\beta} + T_0 \rightarrow T = \dfrac{1{,}0023x - x}{x\,6.10^{-5}} + 16 = \dfrac{\cancel{x}(1{,}0023-1)}{\cancel{x}.6.10^{-5}} + 16$

$\therefore T = \underline{54{,}3\,°C}$

3-) $L_0 = x$ $\qquad$ $x = 100$ $\quad \therefore x' = 1{,}009\,x$ $\qquad$ $\Delta L = L_0\,\alpha\,\Delta T$

$L = 1{,}009x$ $\qquad$ $x' = 100{,}9$ $\qquad$ $T - T_0 = \dfrac{\Delta L}{L_0\alpha}$

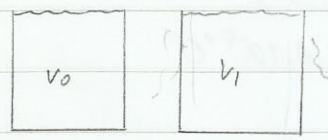
$T_0 = 12°C$

$\alpha = 17.10^{-6}$ $\qquad$ $T = \dfrac{L - L_0}{L_0\alpha} + T_0$

$\Rightarrow T = \dfrac{\cancel{x}(1{,}009-1)}{\cancel{x}\,17.10^{-6}} + 12 = \underline{541{,}4\,°C}$

4-) $V_0 = 1000\ cm^3$ Admitindo mercúrio $\Rightarrow 1$ $\beta_1 = 18.10^{-5}\ K^{-1}$

$T_0 = 10°C$ vidro $\Rightarrow 2$ $\beta_2 = 1{,}5.10^{-5}\ K^{-1}$



$$\begin{cases} \Delta V_1 = V_0 \beta_1 \Delta T \\ \Delta V_2 = V_0 \beta_2 \Delta T \end{cases}$$

$$\Delta V_1 - \Delta V_2 = V_0 \Delta T (\beta_1 - \beta_2)$$

$$\Delta T = \frac{\Delta V_1 - \Delta V_2}{V_0(\beta_1 - \beta_2)} \quad \therefore \quad T = \frac{\Delta V_1 - \Delta V_2}{V_0(\beta_1 - \beta_2)} + T_0$$

$$T = \frac{12}{1\,10^3(18.10^{-5} - 1{,}5.10^{-5})} + 10 = \underline{82{,}72\ °C}$$

5-) $\alpha = 2{,}0.10^{-5}$ $V = V_0(1 + \alpha \Delta t)$ $\therefore \Delta T = \left(\frac{V}{V_0} - 1\right) \div \alpha$

$\frac{V}{V_0} = 1{,}0048$ $\frac{V}{V_0} = 1 + \alpha \Delta T$

$\beta = 3\alpha = 6.10^{-5}$

$$\Delta T = \frac{1{,}0048 - 1}{6.10^{-5}} = \underline{80°C}$$

$$L = L_0(1 + \alpha \Delta T)$$

$$\frac{L}{L_0} = 1 + \alpha \Delta T \Rightarrow \frac{L}{L_0} = 1 + 2.10^{-5}.80 = \underline{1{,}0016}$$

6-) $\emptyset_1 = 4cm$ 1 = aço

$T_0 = 25°C$ 2 = bronze

$\emptyset_2 = 3{,}992\ cm$

$\alpha_1 = 11.10^{-5}$

$\alpha_2 = 19.10^{-6}$

$T = ?$

7-) $V_0 = 960\ cm^3$ $\quad \beta_m = 18.10^{-5}\ °C^{-1}$

$T_0 = 20°C$ $\quad \beta_v = 0,5.10^{-5}\ °C^{-1}$

$T = 180°C$ $\quad V_m = ?$

$$\begin{cases} \Delta V_m = V_0\, \alpha_m\, \Delta T \\ \Delta V_v = V_0\, \alpha_v\, \Delta T \end{cases}$$

$\Delta V_m - \Delta V_v = V_0 \Delta T (\alpha_m - \alpha_v)$

$V'_m = V_0 \Delta T (\alpha_m - \alpha_v)$

$\therefore V'_m = 960 . (180 - 20)(18.10^{-5} - 0,5.10^{-5})$

$= \underline{26,88\ cm^3}$

8-) $\alpha = 2,4.10^{-5}$ $\quad V_0 — 100$ $\quad \therefore V = 1,00756 V_0$

$\Delta T = ?$ $\quad V — 100,756$

$V = 1,00756 V_0$

$\beta = 3 . 2,4.10^{-5} =$ $\quad \Delta V = V_0\, \alpha\, \Delta T$

$= 7,2.10^{-5}$ $\quad \Delta T = \frac{\Delta V}{V_0 \beta} = \frac{V - V_0}{V_0 \beta}$

$$\Delta T = \frac{1,00756 V_0 - V_0}{V_0 . 7,2.10^{-5}} = \frac{\cancel{V_0}(1,00756 - 1)}{\cancel{V_0} . 7,2.10^{-5}} = 105°C$$

9-) $T_0 = 22°C$ $\quad V_0 — 100$ $\quad V = 1,0066 V_0$

$T = ?$ $\quad V — 100,66$

$\alpha = 2.10^{-5}\ °K^{-1}$

$V = 1,0066 V_0$ $\quad \Delta V = V_0\, \alpha\, \Delta T$ $\quad T - T_0 = \frac{V - V_0}{V_0 \alpha}$

$\beta = 6.10^{-5}\ °K^{-1}$ $\quad \Delta T = \frac{\Delta V}{V_0 \alpha}$

$T = \frac{V - V_0}{V_0 \alpha} + T_0$

$$T = \frac{1,0066 V_0 - V_0}{6.10^{-5} V_0} + 22 = \frac{\cancel{V_0}(1,0066 - 1)}{\cancel{V_0}\ 6.10^{-5}} + 22 = 132°C$$

/ /

10-) $T_0 = ?$

$T = T_0 - 16$

$V = 26.10^3$

$\beta = 9,5.10^{-4}$

$\Delta V = V_0 \beta \Delta T$

$V - V_0 = V_0 \beta \Delta T$

$V = V_0 (1 + \beta \Delta T)$

$V = 26.10^3 (1 + 9,5.10^{-4} . (-16))$

$V = 25.604,8 \ L$

Calorimetria

11-) $m_{água} = 500g$

$T_0 = 60°C$

$m_T = 500g_{(L)} + 120g_{(s)} + 100g_{(g)}$

$T = 60°$ ① $T = 0$ ② $T = 100°C$ ③

$P = 1 \ atm$

$C_{gelo} = 2,22 \ J/g \ K$

$C_{água} = 4,19 \ J/kg$

$L_f = 334 \ KJ/Kg$

$L_v = 2256 \ J/Kg$

$\Sigma Q = 0$

$500 . 4,19 (T - 60) + 120 . 2,22 (T - 0) + 100 . 2256 (T - 100) = 0$

$2095 T + 266,4 T + 225600 T = 125700 + 2,256.10^7$

$2,27961.4.10^5 T = 2,26857.10^7$

$T = 99,51°C$

b)

2-) $m_g = 50g$ $T_0 = -15°C$ $C_{gelo} = 2,22$ kJ/kg $L = 334$ kJ/kg

$m_a = 300g$ $T_0 = 25°C$ $C_{agua} = 4,19$ kJ/kg

$Q_g\ (0+15) = m.c\Delta T$

$= 50.2,22.15 = 1665$

$Q_g\ (0+15) = ml$

$= 50.334 = 16700$

$Q_T = 18365$ J

$\Sigma Q = 0$

3-) $m_g = 0,2$ kg $t_g = -12°C$

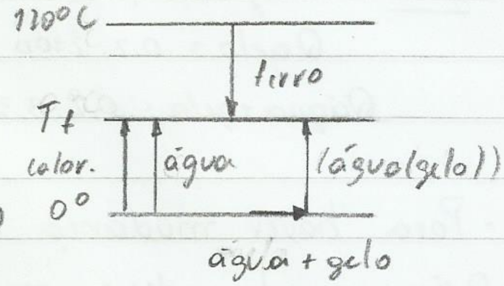
$m_a = 0,25$ kg $t_a = 32°C$

Ex-) 0,12 kg (calorimetro)
0,020 kg (gelo)
0,100 kg (água)
} 0°C

ferro: $m = 300\,g$

a.) $T_{eq.} = ?$; T_0 (ferro) = 120°C

$Q_{calor} + Q_{água} + Q_{gelo} + Q_{gelo \to água}) + Q_{ferro} = 0$



$0{,}12 . 910 (T-0) + 0{,}1 . 4190 (T-0) + 0{,}02 . 4190 \times$
$(T-0) + 0{,}02 . 3{,}34 . 10^5 + 0{,}3 . 470 (T-120) = 0$
$753T = 10240 \therefore T = 13{,}60°C$

b) $T_0 = ?$ (Ferro); se calorimetro tivesse: água ($T = 20°C$)

$Q_{gelo} + Q_{água+gelo} + Q_{água} + Q_{calo} + Q_{ferro} = 0$

$0{,}02 . 3{,}34 . 10^5 + 0{,}02 . 4190 (20-0) + 0{,}1 . 4190 (20-0) + 0{,}12 . 910 (20-0) +$
$0{,}3 . 470 (20 - T) = 0$
$21740 = 141T \therefore T = \underline{154{,}2°C}$

Ex:

Calorímetro: $m = 0{,}250\ kg \rightarrow T_0 = 80°C$

água: $m = 0{,}5\ kg \rightarrow T_0 = 80°C$

gelo: $m = ?$ $\quad T_0 = -30°C$

a) $m = 0{,}2\ kg \quad T_f = ?$

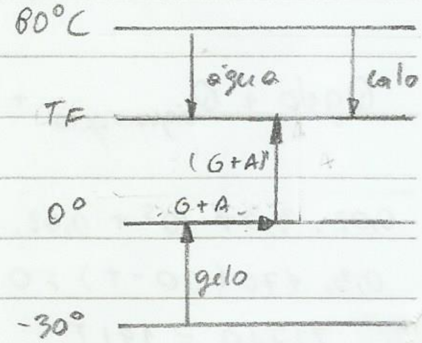
Teste: $Q_{água} = 0{,}5 \cdot 4190\,(0-80°) = |-167.600|\,J \rightarrow 167600$

$Q_{gelo} = 0{,}2 \cdot 2100\,(0+30) = 12600\,J$ (I) $\quad$ I + II $= 79400\,J$

$Q_{água+gelo} = 0{,}2 \cdot 3{,}34 \cdot 10^5 = 66800\,J$ (II)

- Para haver mudança de estado do sistema, o Q_{gelo} + $Q_{água+gelo}$ deve ser menor que o $Q_{água}$, ou seja:

$|Q_{água}| > Q_{gelo} + Q_{gelo+água}$



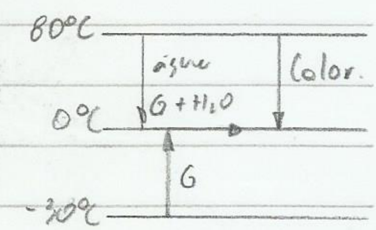
$Q_{gelo} + Q_{g+a} + Q_{(g+a)'} + Q_{água} + Q_{calori} = 0$

$0{,}2 \cdot 2100\,(0+30) + 0{,}2 \cdot 3{,}34 \cdot 10^5 + 0{,}2 \cdot 4190\,(T-0) +$

$0{,}5 \cdot 4190\,(T-80) + 0{,}250 \cdot 390\,(T-80) = 0$

$3030{,}5\,T = 96000 \quad \therefore \quad T = \underline{31{,}68°C}$

b) $m = ? \rightarrow T = -30°C$; somente água a 0°C



$Q_{gelo} + Q_{gelo+água} + Q_{calor} + Q_{água} = 0$

$m \cdot 2100\,(0+30) + m\,3{,}34 \cdot 10^5 + 0{,}250 \cdot 390\,(0-80) +$

$0{,}5 \cdot 4190\,(0-80) = 0$

$397000\,m = 175400$

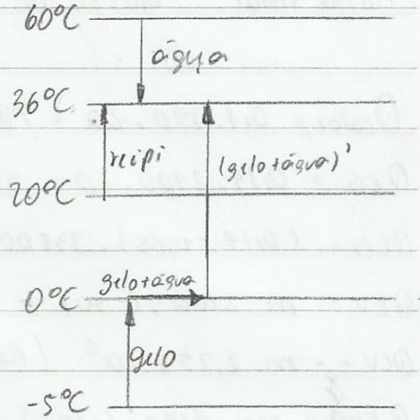
$m = 0{,}44\ kg = \underline{440\,g}$

19-) Recip.: $m = 0{,}2\ kg \rightarrow T_0 = 20°C$

água: $m = 0{,}3\ kg \rightarrow T_0 = 60°C$

gelo: $m = 0{,}05\ kg \rightarrow T_0 = -5°C$

$T = 36°C \quad c_{recip.} = ?$



$Q_{gelo} + Q_{(g+a)} + Q_{(g+a)'} + Q_{recip} + Q_{água} = 0$

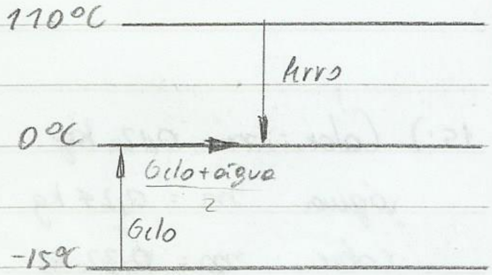
$0{,}05 . 2100 (0+5) + 0{,}05 . 334000 + 0{,}05 . 4190 (36-0) +$

$0{,}2 . c (36-20) + 0{,}3 . 4190 (36-60) = 0$

$-5401 + 3{,}2c = 0 \quad \therefore c = \underline{1687{,}8\ J/kg\,K}$

18-) gelo $m = 0{,}18\ kg \rightarrow T = -15°C$

ferro $m = ? \rightarrow T = 110°C$



$Q_{ferro} + Q_{gelo} + Q_{\frac{gelo+água}{2}} = 0$

$m . 470 . (0-110) + 0{,}18 . 2100 (0+15) + \frac{0{,}18}{2} . 334000 = 0$

$-51700m + 35730 = 0 \quad \therefore m = \underline{0{,}691\ kg}$

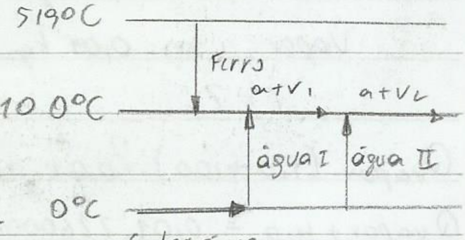
17-) Calor.: $m = 0{,}1\ kg \quad c = 390$

água: $m = 0{,}16\ kg \quad c = 4190$

gelo: $m = 0{,}018\ kg \quad c = 2100$

ferro: $m = 0{,}75\ kg \quad T = 519°C$

$T = 100°C$



$Q_{Ferro} + Q_{G+a} + Q_{a_I} + Q_{II} + Q_{a+v_1} + Q_{a+v_2} = 0$

$0{,}75 . 470 (100-519) + 0{,}018 . 334000 + 0{,}018 . 4190 (100-0) +$

$0{,}16 . 4190 (100-0) + m . 2 . 2{,}256 . 10^6 = 0$

$\therefore m = 0{,}015\ kg = \underline{15 g}$

/ /

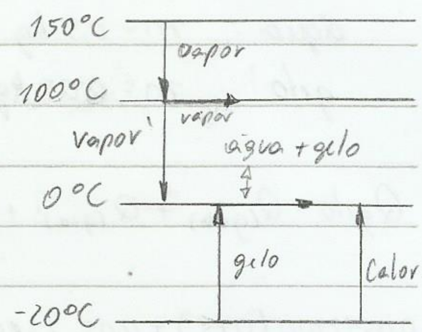
(conteúdo)

16-) Calor. $m = 0{,}1\ kg$ / $T = -20°C$ / $c = 390\ J/kg\ K$

gelo: $m = 0{,}14\ kg$ $c = 2100\ J/kg.K$ $T = -20°C$

vapor: $m = ?$ $T = 150°C$

Massa final: 0,05 kg de gelo



$Q_{calor} = 0{,}1 . 390 . 20 = 780\ J$

$Q_{EG} = 0{,}14 . 2100 . 20 = 5880\ J$

$Q_{LG} = (0{,}14 - 0{,}05) . 334000 = 30060\ J$ ∴ $780 + 5880 + 30060 - 105000m +$

$Q_{EV} = m . 2100 . (-50) = -105000\ m$ $-2{,}256.10^6 m - 419000m = 0$

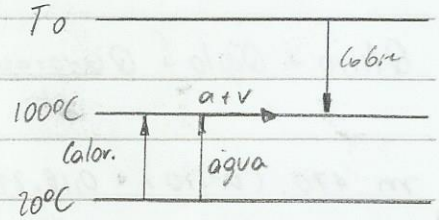
$Q_{LV} = -m . 2{,}256.10^6$ (Admt. ⊖ pois ele ta perdendo J) $36720 = 2780000\ m$

$Q_{EVA} = m . 4190 . (-100) = -419000m$ ∴ $m = 0{,}013\ kg$

15-) Calor. $m = 0{,}12\ kg$ $T_0 = 20°C$

água $m = 0{,}24\ kg$ $T_0 = 20°C$

cobre. $m = 0{,}32\ kg$ $T_0 = ?$ $T = 100°C$



$Q_{calor} + Q_{água} + Q_{cobre} + Q_{(a+v)} = 0$

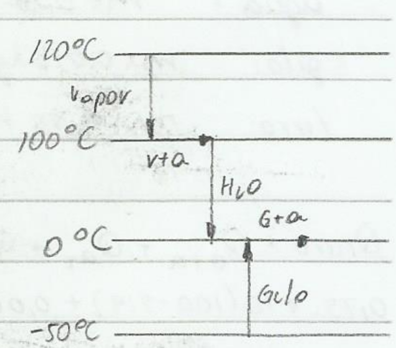
$0{,}12 . 910 (100-20) + 0{,}24 . 4190 (100-20) + 0{,}32 . 390 (100-T) + 0{,}006 . 2256 . 10^3 = 0$

$1{,}152.10^5 - 1{,}248\ t = 0$ ∴ $t = \underline{923°C}$

14-) Gelo : $m = 0{,}2\ kg$ → $T_i = -50°C$

Vapor: $m = 0{,}01\ kg$ → $T_i = 120°C$

$T = ?$



$Q_{vapor}\ (120 \to 100) = 0{,}01 . 2010 (100-120) = -402$ I

$Q_{vapor + H_2O} = 0{,}01 . 2260000 = 2{,}26.10^4$ II

$Q_{Gelo} = 0{,}2 . 2220 (0+50) = 22200$ III

$Q_{Gelo + H_2O} = 0{,}2 . 334000 = 66800$ IV $|I + II + IV| < |III + V|$

$Q_{água} = 0{,}01 . 4190 (0-100) = -4190$ V ∴ $T = 0°C$

Resolução de prova:

Gelo: $-50°C \rightarrow 0°C \Rightarrow m.c.\Delta T = 0{,}2 . 2220 . (0+50) = 22200\ J$

para fundir todo o gelo: $m L_f = 0{,}2 . 334000 = 66800\ J$

Vapor: $120°C \rightarrow 100°C \Rightarrow m.c.\Delta T = 0{,}01 . 2010 (100-120) = -402\ J$

para condensar a água: $m L_v = 0{,}01 . 2260000 = 22600\ J$

- Para passar da água para o vapor a água (proveniente do vapor) cede $\Rightarrow m.c\ \Delta T = 0{,}01 . 4190 (0-100) = -4190\ J$

$\therefore$ O vapor cede de 120 °C até 0 °C, 27192 J, que é menor que a energia para o gelo fundir, que é 89000 J.

$\therefore\ \underline{T_f = 0°C}$

b) $Q_{vapor} + Q_{v+a} + Q_{agua} + Q_{gelo} + Q_{g+a} = 0$

Gelo: $mc\Delta T = 0{,}2 . 2220 (0+50) = 22200\ J$

Para fundir o gelo $m L_f = 0{,}2 . 33400 = 66800\ J$

Energia gasta $= \underline{89000\ J}$

Vapor: $mc\Delta T = 0{,}01 . 2010 (100-120) = -402\ J$

Para condensar água $m L_v = 0{,}01 . 2260000 = 22600\ J$

Vapor "virar" água $= mc\Delta T = 0{,}01 . 4190 . (100-0) = 4190\ J$

Energia gasta $= 23002\ J + 4190\ J = \underline{27{,}192\ J}$

$Q = 89000 - 27192 = 61808\ J$

Gelo: $Q = mL \Rightarrow m = \dfrac{Q}{L} = \dfrac{61808}{334000} = 0{,}185\ kg$

ou

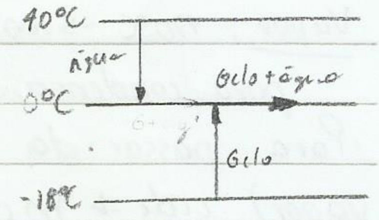
$Q = 27192 - 22200 = 4992\ J$

$Q = mL \Rightarrow m = \dfrac{Q}{L} = \dfrac{4992}{334000} = 0{,}0149 + 0{,}01 = 0{,}025\ kg$ (água)

$m = 0{,}2 - 0{,}0149 = 0{,}185\ kg$ (água)

13-) Gelo: $m = 0{,}28\ kg$ $T = -18°C$

Água: $T_0 = 40°C$ $m = ?$; para que $\frac{m_g}{2} \Rightarrow T = 0°C$



$Q_{água} + Q_{gelo} + Q_{a+g} = 0$

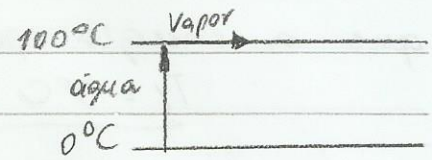
$m \cdot 4190(0-40) + 0{,}28 \cdot 2095(0+18) + 0{,}14 \cdot 334000 = 0$

$-167600m + 57318{,}8 = 0$

$\therefore m = \underline{0{,}342\ kg}$

12-) Água: $T_0 = 0°C$ $m = ?$ $T = 100°C$

Vapor: $m = 0{,}2\ kg$ $T_0 = 100°C$



$Q_{água} + Q_{vapor} = 0$

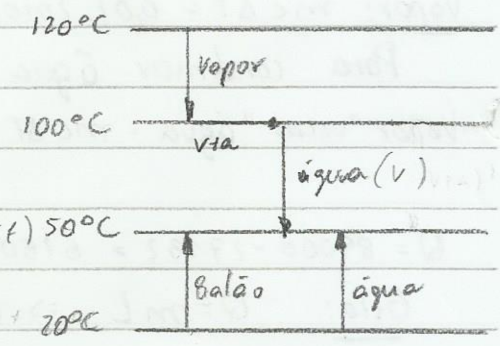
$m \cdot 4190(100-0) + 0{,}2 \cdot 2{,}256 \cdot 10^6 = 0$

$\therefore m = 1{,}08\ kg$

11-) Vapor: $T_0 = 120°C$ $m = ?$

Água: $m = 180g = 0{,}18\ kg$

Balão: $m = 0{,}08\ kg$ $T_0 = 20°C$ $T = 50°C$



$Q_{EV} = m \cdot 2010(100-120) = -40200$

$Q_{LV} = m \cdot 2{,}26 \cdot 10^6 \cdot (-1) = -2260000m$

$Q_{EVA} = m \cdot 4190(50-100) = -209500m$

$Q_{EB} = 0{,}08 \cdot 837(50-20) = 2008{,}8$

$Q_{EA} = 0{,}18 \cdot 4190(50-20) = 22625$

$\Sigma Q = Q_{EV} + Q_{LV} + Q_{EVA} + Q_{EB} + Q_{EA} = 0$

$-40200 - 2260000m - 209500m + 2008{,}8 + 22625 = 0$

$-2469500m - 24633{,}6 = 0$

$\therefore m = 0{,}00997\ kg = 9{,}98g$

1) $\varnothing = 2{,}725\ cm$ $\quad T_0 = 0°C$ $\quad l = l_0(1 + \alpha \Delta T)$

$T_f = 100°C$ $\quad L = 2{,}725(1 + 23.10^{-6}.(100-0))$

$L = 2{,}7312\ cm$

2) $m = 0{,}2\ kg$ $\quad$ a) $T = 0$ $\quad x_0 = 4\ cm$

$T = 8.2 = 16\ s$ $\quad x = A\cos\theta$

$4 = 10\cos\theta \quad \therefore \theta = 1{,}16\ rad$

b) $x(t) = A\cos(wt + \varnothing)$ $\quad w = \frac{2\pi}{T} = \frac{2\pi}{16} = 0{,}125\pi$

$x(t) = 10\cos(0{,}125\pi t + 1{,}16)\ (cm)$

$= 0{,}1\cos(0{,}125\pi t + 1{,}16)\ (m)$

c) $t = 10\ s$ $\quad A = -w^2 x(t)$

$A = -(0{,}125)\, x(10) = -5{,}64.10^{-3}\ m/s^2$

d) $Ec = \frac{mv^2}{2}$ $\quad v(t) = 3{,}92.10^{-2} \sin(0{,}125\pi t + 1{,}16)$

$v(3{,}5) = 2{,}24.10^{-2}$

$Ec = \frac{0{,}2\,(0{,}224)^2}{2} = 5{,}02.10^{-5}\ J$

e) $F = -Kx$ $\quad K = w^2.m$

$F = 3{,}08.10^{-2}.0{,}1$ $\quad K = (0{,}125\pi)^2.0{,}2 = 3{,}08.10^{-2}\ N/m$

$F = 3{,}08.10^{-3}\ N$

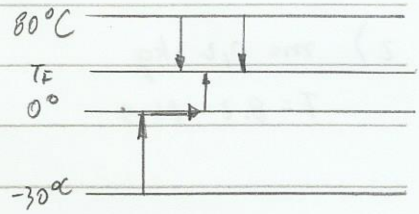
/ /

3) Cobre: $m = 0{,}250\ kg$ $T_0 = 80°C$

Água: $m = 0{,}5\ kg$ $T_0 = 80°C$

Gelo $m = ?$ $T_0 = -30°C$

a) se $m = 0{,}2\ kg$



$Q_{EA} = 0{,}5 . 4190 (0 - 80°) = -167600\ J$

$Q_{EG} = 0{,}2 . 2100 (0 + 30) = 12600\ J$

$Q_{LG} = 0{,}2 . 3{,}34 . 10^5 = 66800\ J$

$|Q_{EA}| > |Q_{EG} + Q_{LG}|$ Todo gelo sofre fusão

$Q_{EA} = 0{,}5 . 4190 (T - 80) = 2095T - 167600 (J)$

$Q_{EG} = 0{,}2 . 2100 (0 + 30) = 12600\ J$

$Q_{LG} = 0{,}2 . 3{,}34 . 10^5 = 66800\ J$

$Q_{EG \to H} = 0{,}2 . 4190 (T - 0) = 838\ T$

$Q_{EC} = 0{,}250 . 390 . (T - 80) = 97{,}5\ T - 7800$

$\Sigma Q = 0$

$2095T - 167600 + 12600 + 66800 + 838T + 97{,}5T - 7800 = 0$

$3030{,}5\ T - 96000 = 0$

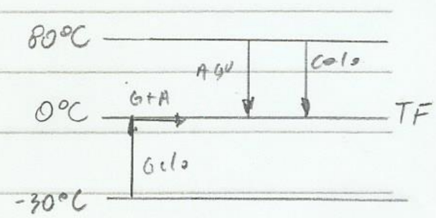
$\underline{T = 31{,}68\ °C}$

b) $Q_{EA} = 0{,}5 . 4190 (0 - 80) = -167600$

$Q_{EC} = 0{,}250 . 390 (0 - 80) = -7800$

$Q_{EG} = m\ 2100 (0 + 30) = 63000\ m$

$Q_{LG} = m . 3{,}34 . 10^5 = 334000\ m$



$\Sigma Q = 0$

$-167600 - 7800 + 63000m + 334000\ m = 0$ / $-175400 + 397000\ m = 0$

$\therefore m = 0{,}442\ Kg$

Formulário temperatura

dilatação térmica:

Linear → $\Delta L = l_0 . \alpha . \Delta T$; Onde: α: coeficiente de expansão linear

$\alpha = \frac{\Delta L}{l_0 \Delta T}$

Através disso: $L = l_0(1 + \alpha \Delta T)$

Volumétrica → $\Delta V = V_0 . \beta . \Delta T$; Onde: β: coeficiente de expansão volumétrica

$\beta = \frac{\Delta V}{V_0 \Delta T}$

Através disso: $V = V_0(1 + \beta \Delta T)$

Relação entre α e β: $\beta = 3\alpha$

$T_F = \frac{9}{5} T_C + 32$

$T_C = \frac{5}{9} T_F - 32$

$T_K = T_C + 273,15$

$\boxed{\text{Calor = energia} \therefore 1\,cal = 4,186\,J}$

Calor específico → $Q = mC\Delta T$

Calor específico molar → $C = Mc$

Calorimetria = $\sum Q_i = 0$

Calor latente → $Q = m.l$

$L_f = 334\ KJ/Kg$

$L_v = 2256\ KJ/Kg$

$c_{água} = 4.19\ KJ/KgK$

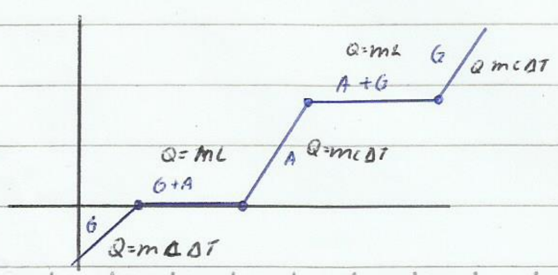
$c_{gelo} = 2.1\ KJ/KgK$

$L_f = 80\ cal/g$

$L_v = 539\ cal/g$

$c_{água} = 1\ cal/g°C$

$c_{gelo} = -0,5\ cal/g°C$



Equações do MHS

$x(t) = A\cos(wt+\phi)$ (Desloca...) $T = 2\pi\sqrt{\frac{m}{k}}$

$w = 2\pi f = \frac{2\pi}{T}$

$\boxed{v_x(máx) = \pm wA}$

$\boxed{a_x(máx) = \pm w^2 A}$

Para um sistema massa mola

$w = \sqrt{\frac{k}{m}}$ $E_m = \frac{kA^2}{2}$

eq. do deslocamento

$v_x = -Aw\,sen(wt+\phi)$ (velocidade)

$a_x(t) = -w^2 A\cos(wt+\phi)$ (aceleração) ou $a_x(t) = -w^2 x$

Energia no MHS

Energia Potencial (Elástica) => $E_p = \frac{1}{2}kx^2$

$E_m = E_p + E_c$

Energia Cinética $E_c = \frac{1}{2}mv^2$

$E_m = \frac{1}{2}kA^2$

x estático

$\frac{mg}{k}$

$x(t) = A\cos(\underbrace{wt+\phi}_{\text{fase no instante } t})$

w → frequencia angular

ϕ → fase inicial

A → amplitude do MHS

$F_x = -kx$

$F'_x = -bv_x$ F' = Força exercida pelo fluido

$b = 2\sqrt{mk}$ b: constante de amortecimento

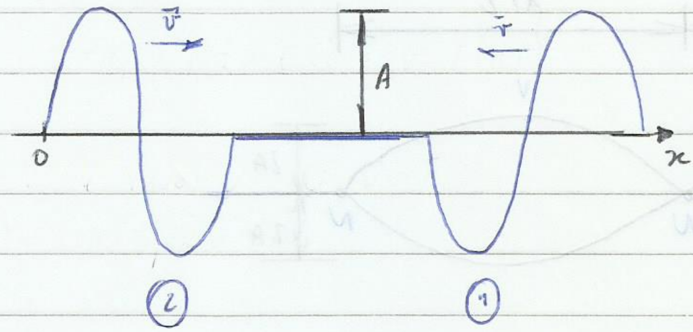
$x = A e^{\frac{-b}{2m}\cdot t}\cdot\cos(w_a t+\phi)$ $A(t) = A e^{\frac{-b}{2m}\cdot t}$ $w_a = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Nome: Erivelton Gualter dos Santos 11 210 368 . 4

1º semestre de 2011 2º ciclo Engenharia Básica

Física (Parte II)

Ondas Estacionárias em uma corda



1-) Onda incidente

$$y_1(x,t) = -A\cos(kx + wt)$$

2-) Onda refletida

$$y_2(x,t) = A\cos(kx - wt)$$

As ondas ① e ② sofrem interferência produzindo uma resultante

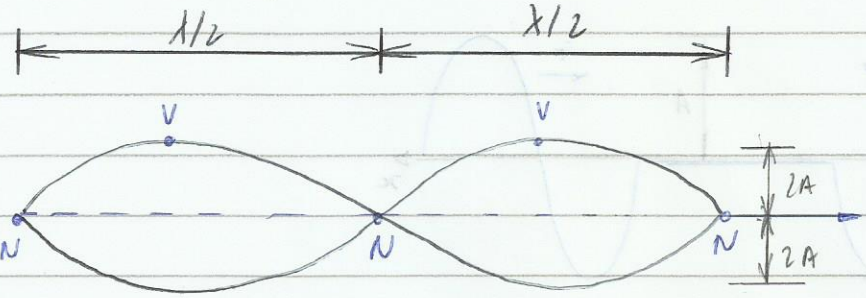
$$y(x,t) = y_2(x,t) + y_1(x,t)$$

$$y(x,t) = A\,[\cos\underbrace{(kx - wt)}_{a} - \cos\underbrace{(kx + wt)}_{b}]$$

Aplicando trigonometria para a diferença:
$\cos a - \cos b = \dots$ etc.
chega-se à função:

$$\boxed{y(x,t) = (2A\,\mathrm{sen}\,kx)\,\mathrm{sen}(wt)}$$ (Eq. da onda estacionária

Por exemplo, um padrão de onda estacionária é:



N = nós   V = ventre

Alguns pontos da corda não oscilam (nós) enquanto que outros oscilam com amplitude máxima (ventres).

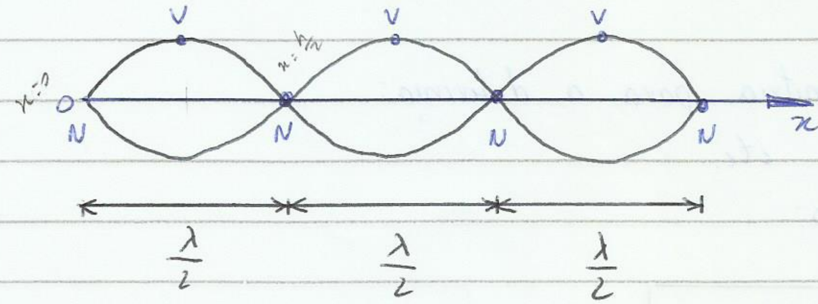
Cada ponto x da corda oscila (ou não) com amplitude:

$$\boxed{A(x) = 2A\,\text{sen}\,kx}$$ 1, quando A_{max}

A amplitude da onda estacionária é:

$$\boxed{A_{ES} = 2A}$$

Posições dos nós e dos ventres



Nós: x

$0 \cdot \frac{\lambda}{2}$ $\therefore$ nós $\boxed{x = n\,\frac{\lambda}{2}}$ $n = (0, 1, 2, 3, \ldots)$

$1 \cdot \frac{\lambda}{2}$

$2\,\frac{\lambda}{2}$

$\vdots$

$n\,\frac{\lambda}{2}$

ventre: x ou $(2n+1)\,\frac{\lambda}{2}$

$\frac{1}{2}\cdot\frac{\lambda}{2}$ $\therefore$ $\boxed{x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}}$

$\frac{3}{2}\,\frac{\lambda}{2}$

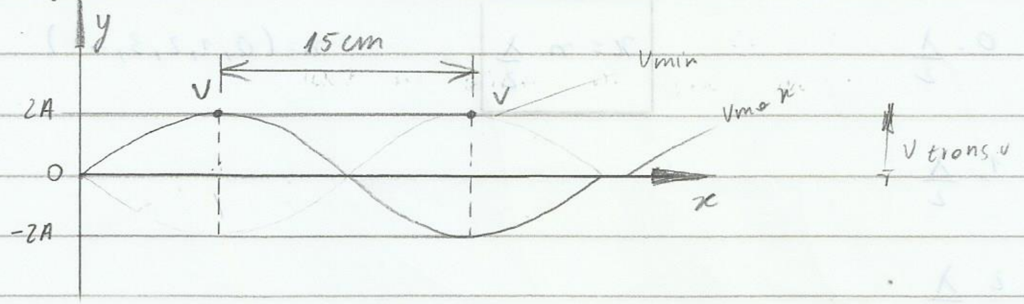
$(n = (0, 1, 2, 3 \ldots)$

$\frac{5}{2}\,\frac{\lambda}{2}$

$\vdots$

/ /

15.34 (p.34)



$A_{ES} = 2A = 0{,}85 cm$

$T = 0{,}075 s$

a) $d = ?$ (entre nós consecutivos) $\underline{d = 15 cm}$

b) comprimento de onda, amplitude e a velocidade das ondas que forma a equação da onda

$\frac{\lambda}{2} = 15 \therefore \lambda = \underline{30 cm}$

$2A = 0{,}85 \therefore \underline{A = 0{,}425 cm}$

$w = \frac{2\pi}{T} = \frac{2\pi}{0{,}075} = \underline{83{,}78 rad/s}$ $\quad K = \frac{2\pi}{\lambda} = \frac{2\pi}{0{,}30} = 20{,}94$

$v = \frac{83{,}78}{20{,}94} = \underline{4 m/s}$ ou $v = 30 \cdot \frac{1}{0{,}075} = \underline{400 cm/s}$

c) $v_y = ? \begin{cases} max \\ min \end{cases}$ $\quad y(x,t) = (2A sen(kx) sen(wt))$

$v_y = \frac{dy}{dt} = 2Aw sen(kx) cos(wt)$

$v_y^{min} = 0 \quad V_{max} = 2AW$

$= 0{,}85 \cdot \frac{2\pi}{0{,}075} = \underline{71{,}21 cm/s} \cong \underline{71 cm/s}$

$v_y \ min = ?$

$v_y \ min = 0$

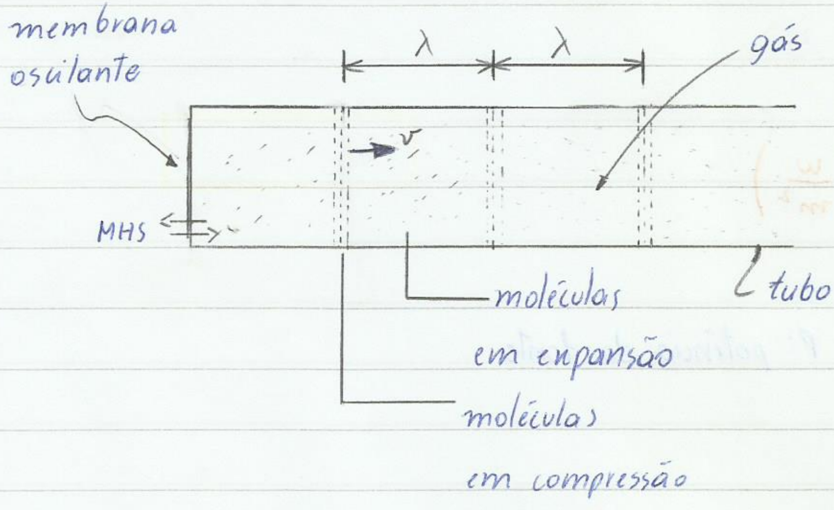
que é a veloc. do ponto de ventre $y = \pm 2A$

Se utilizarmos a eq. de v_y devemos colocar (por ex. para o 1º ventre): $x = \frac{1}{2} \cdot \frac{\lambda}{2} = \frac{\lambda}{4}$

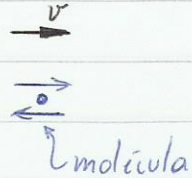
Portanto $\sin kx = \sin \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2 \cdot 2}$

$k = \frac{2\pi}{\lambda}$ $\quad = \sin \frac{\pi}{2} = 1$

Ondas sonoras

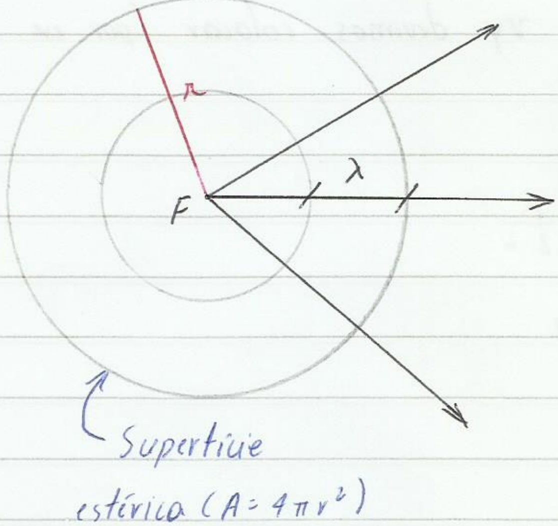


Cada molécula do gás osila paralelamente à velociadade v.



Por isso as ondas sonoras são do tipo longitudinais

As ondas sonoras propagam-se radialmente pelo espaço



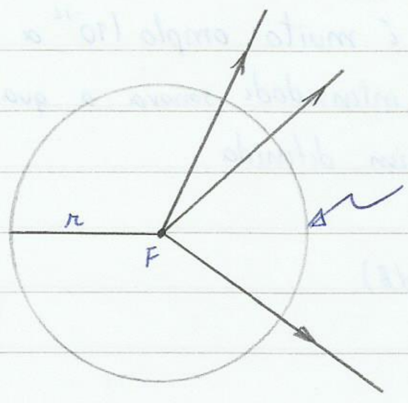
F: fonte sonora

Intensidade sonora:

$$I = \frac{\text{potência}}{\text{área}} \quad \left(\frac{W}{m^2}\right)$$

$$\boxed{I = \frac{P}{4\pi r^2}}$$ P: potência da fonte

Intensidade do som: I



Frente de onda esférica
(área $A = 4\pi r^2$)

F: fonte sonora

$$I = \frac{\text{potência}}{\text{área}} \left(\frac{W}{m^2}\right)$$

$$\boxed{I = \frac{P}{4\pi r^2}}$$ P: Potência da fonte

Se a potência da fonte sonora for mantida constante então $Ir^2 = \text{const.}$

$$\boxed{I_1 r_1^2 = I_2 r_2^2} = \text{const}$$

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} \quad \text{ou} \quad \frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

Para o ouvido humano: Mínima intensidade sonora audível:

$\boxed{I_0 = 10^{-12}\ W/m^2}$ (limiar de audibilidade)

Máxima intensidade sonora audível (sem causar dor):

$\boxed{I_{máx} = 1{,}0\ W/m^2}$ (limiar de dor)

Nível de intensidade sonora: β

Como o intervalo de intensidades do som é muito amplo (10^{-12} a $1{,}0\ W/m^2$) criou-se uma outra grandeza para medir a intensidade sonora a qual foi denominada nível de intensidade sonora (β) assim definida:

$$\boxed{\beta = 10 \log \frac{I}{I_0}} \qquad \beta \text{ em decibéis (dB)}$$

Exercício: Calcular os níveis de intensidade sonora (β) para o limiar de audibilidade e para o limiar de audibilidade (*) e para o limiar de dor (**)

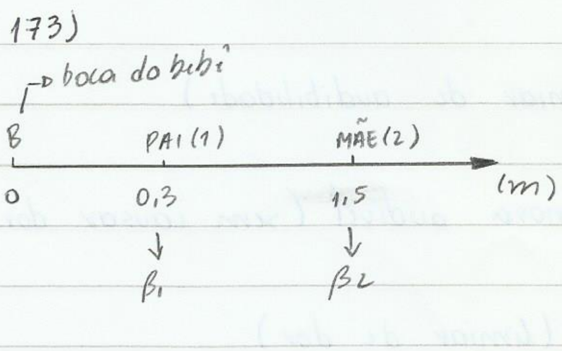
(*) $I = I_0 \Rightarrow \beta_0 = ?$

$$\beta_0 = 10 \log \cancel{\frac{I_0}{I_0}}^{\,1\ 0} \qquad \boxed{\beta_0 = 0}$$

(**) $I = I_{máx} = 1{,}0 \ \frac{W}{m^2}$

$$\beta_{máx} = 10 . \log \frac{1}{1.10^{-2}} \quad \therefore \quad \beta_{máx} = 120 \text{ dB}$$

<u>16.21</u> (p. 173)



$\beta_1 - \beta_2 = ?$

$$\begin{cases} \beta_1 = 10 \log \dfrac{I_1}{I_0} \\ \beta_2 = 10 \log \dfrac{I_2}{I_0} \end{cases}$$

$$\beta_1 - \beta_2 = 10 \left(\log \frac{I_1}{I_0} - \log \frac{I_2}{I_0} \right)$$

$$\beta_1 - \beta_2 = 10 \log \left(\frac{I_1}{I_0} : \frac{I_2}{I_0} \right)$$

$$\beta_1 - \beta_2 = 10 \log \left(\frac{I_1}{I_2} \right)$$

$$I_1 = \frac{P}{4\pi . 0{,}3^2} \qquad I_2 = \frac{P}{4\pi . 1{,}5^2}$$

$$\beta_1 - \beta_2 = 10 \log \left(\frac{P}{4\pi \, 0{,}3^2} : \frac{P}{4\pi . 1{,}5^2} \right)$$

$$\therefore \beta_1 - \beta_2 = 13{,}98 \text{ dB} \cong 14{,}00 \text{ dB}$$

A diferença entre a mãe e o pai é <u>14 dB</u>

16.23

a) Por qual fator deve a intensidade do som ser multiplicada para que o nível da intensidade sonora aumente em 13 dB?

$$\beta_2 = \beta_1 + 13$$
$$\beta_2 - \beta_1 = 13$$

$$10 \log \left(\frac{I_1}{I_0} : \frac{I_2}{I_0} \right) = 13$$

$$\log \left(\frac{I_1}{I_2} \right) = 1{,}3$$

$$\frac{I_1}{I_2} = 10^{1,3}$$

16.1 (p 172)

Dados: 344 m/s $B = 1{,}42 \cdot 10^5$ Pa

onda sonora

$f = 1000$ Hz

$A = 1{,}2 \cdot 10^{-8}$ m

$P_{max} = 3 \cdot 10^{-2}$ Pa

a) $\lambda = ?$

$v = \lambda f$

$\therefore \lambda = \frac{v}{f} = \frac{344}{1000} = 0{,}344$ m

b) $P_{máx} = 30$ Pa $\Rightarrow A = ?$

$K = \frac{2\pi}{0{,}344} = 18{,}265$

$P_{max} = BAK \quad \therefore A = \frac{30}{1{,}42 \cdot 10^5 \cdot 18{,}265}$

$A = 1{,}157 \cdot 10^{-5}$ m

c) $\lambda = ?$ $A = 1{,}2 \cdot 10^{-8}$ m

$f = ?$ $P_{máx} = 1{,}5 \cdot 10^{-3}$ Pa

$P_{máx} = BAK$

$1{,}5 \cdot 10^{-3} = 1{,}42 \cdot 10^5 \cdot 1{,}2 \cdot 10^{-8} K \quad \therefore K = 0{,}8803$

$K = \frac{2\pi}{\lambda} \quad \therefore \quad 0{,}8803 = \frac{2\pi}{\lambda} \quad \therefore \quad \lambda = 7{,}138$

$v = \lambda f \quad \therefore \quad f = \frac{v}{\lambda} = \frac{344}{7{,}138} = 48{,}2$ Hz

16,17 (pg 173)

a) $P_{máx} = ?$

b) $I = ?$ $\left(\frac{W}{m^2}\right)$

$f = 150$ Hz

$A = 5 \cdot 10^{-6}$ m

c) $\beta = ?$

a) $p_{máx} = BAK$

$= 1{,}42.10^{5} . 5.10^{-6} . 2{,}7398$

$\therefore p_{máx} = \underline{1{,}945}\ Pa$

$v = \lambda f$

$\lambda = \frac{v}{f} = \frac{344}{150} = 2{,}2933$

$K = \frac{2\pi}{\lambda} = \frac{2\pi}{2{,}2933} = 2{,}7398$

b) $\rho = \frac{B}{\sqrt{v}} = \frac{1{,}42105}{\sqrt{344}} = 7656{,}13$

$I = \frac{P^2_{máx}}{2\rho v}$ $\quad v = \sqrt{\frac{B}{\rho}} \therefore v^2\rho = B$

$v\rho = \frac{B}{v}$

$I = \frac{P^2_{max}}{\frac{2B}{v}} = \frac{P^2_{max} . v}{2B} = \frac{1{,}945^2 . 344}{2.1{,}42.10^5} = 4{,}58.10^{-3} \frac{W}{m^2}$

c) $\beta = 10 \log \left(\frac{I}{I_0} \right) = 10 \log \left(\frac{4{,}58.10^{-3}}{10^{-12}} \right) = \underline{96{,}61\ dB}$

15.14 (pg 132)

$F = ?$

$l = 2{,}5\ m$

$m = 0{,}12\ kg$

$f = 40\ Hz$

$\lambda = 0{,}75$

$v = \sqrt{\dfrac{F}{\mu}} \therefore F = v^2 \mu$

Porém

$\mu = \dfrac{m}{l} = \dfrac{0{,}12}{2{,}5} = 0{,}048\ kg/m$

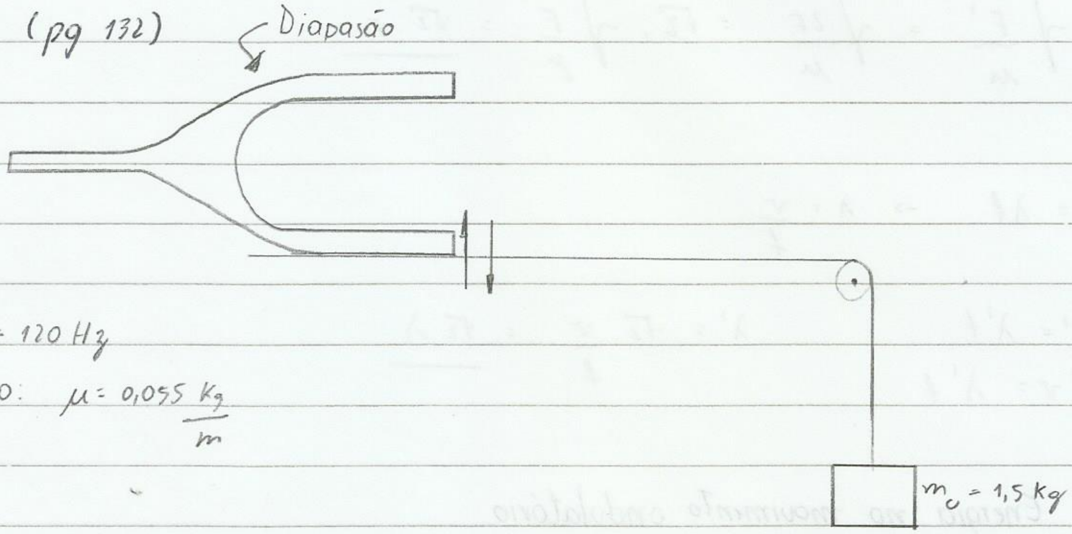
$v = \lambda f$

$v = 0{,}75 \cdot 40$

$v = 30$

$F = 30^2 \cdot 0{,}048 = \underline{43{,}2\ N}$

15.15 (pg 132)



$f = 120\ Hz$

Fio: $\mu = 0{,}055\ \dfrac{kg}{m}$

a) $v = ?$

$F = mg = 1{,}5 \cdot 10 = 15\ N$

$v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{15}{0{,}055}} = \underline{16{,}51\ m/s}$

b) $v = \lambda f$

$\therefore \lambda = \dfrac{v}{f} = \dfrac{16{,}51}{120} = \underline{0{,}14\ m}$

c) $v = \sqrt{\dfrac{3 \cdot 9{,}8}{0{,}055}} = 23{,}12$ $\qquad 1 = 16{,}4$ $\qquad \therefore x = \sqrt{2}$

$x = 23{,}12$

OU

Se for $m_c' = 3\,kg$

$\rightarrow m_c' = 2m_c \Rightarrow \begin{cases} v' = ? \\ \lambda' = ? \end{cases}$

$v = \sqrt{\dfrac{F}{\mu}}$ Se $m_c = 2m_c \Rightarrow F' = 2F$

$v' = \sqrt{\dfrac{F'}{\mu}} = \sqrt{\dfrac{2F}{\mu}} = \sqrt{2} \cdot \sqrt{\dfrac{F}{\mu}} = \underline{\sqrt{2}\, v}$

$v = \lambda f \rightarrow \lambda = \dfrac{v}{f}$

$v' = \lambda' f$

$\sqrt{2}\, v = \lambda' f$ $\qquad \lambda' = \sqrt{2}\,\dfrac{v}{f} = \underline{\sqrt{2}\,\lambda}$

Energia no movimento ondulatório

Uma onda não transporta matéria porém transporta ENERGIA

A potência é a taxa instantânea com que a energia é transportada sendo dada por:

$$\boxed{P(x,t) = \sqrt{\mu F} \cdot w^2 A^2 \sin^2(kx - wt)}$$

O valor de P será máximo quando $\sin^2(\dots) = 1$

Portanto:

$$\boxed{P_{máx} = \sqrt{\mu F} \cdot w^2 A^2}$$

Potência Média:

$$\boxed{P_{méd} = \frac{1}{2} P_{máx}}$$

15.20 (p 133)

FIO DE PIANO

onda:

$f = 120 Hz$

$A = 1,6 \, mm$

$m = 3g$

$L = 80 \, cm$

$F = 25N$

a) $P_{mid} = ?$ (transportada pela corda)

$$P_{mid} = \frac{1}{2} P_{máx}$$

$$\mu = \frac{m}{L} = \frac{0,003}{0,80} = 0,00375 \frac{kg}{m}$$

$$= \frac{1}{2} \sqrt{\mu F} \, w^2 A^2$$

$$w = 2\pi . 120 = 240\pi \ rad/m$$

$$= \frac{1}{2} \sqrt{0,00375 . 25} . (240 \pi . 0,0016)^2 = \underline{0,223W}$$

c) Se $A' = \frac{A}{2}$

$$P'_{mid} = \frac{1}{4} P_{mid}$$

Ondas Mecânicas

velocidade de propagação da onda:

$$v = \sqrt{\frac{F}{\mu}} = \lambda f = \frac{w}{K}$$

densidade linear: $\mu = \frac{m}{L}$ Frequência angular: $w = 2\pi f = \frac{2\pi}{T}$ (rad/s)

número de onda: $k = \frac{2\pi}{\lambda}$ (rad/m)

potência média $\bar{P} = \frac{\mu v w^2 A^2}{2}$ (w)

$A_{est} = 2A$

$f = \frac{f_{und}}{n}$

Deslocamento de uma partícula da Onda ou Equação da Onda

$$y(x,t) = A\cos(kx \pm wt)$$

$\hookrightarrow$ se "+" direção $-x$

se "−" direção $+x$

Velocidade de uma partícula da Onda ou Velocidade transversal ou de uma vibravação

$v(x,t) = -Aw\,sen(kx + wt)$ (direção $-x$)

$v(x,t) = +Aw\,sen(Kx - wt)$ (direção x)

A velocidade máximo: $v_{máx} = Aw$

Aceleração de uma partícula da Onda ou Aceleração transversal

$a(x,t) = -Aw^2\cos(Kx + wt)$ (direção $-x$)

$a(x,t) = -Aw^2\cos(Kx - wt)$ (direção $+x$)

A aceleração máxima $a_{max} = Aw^2$

Onda Estacionária

Equação da Estacionária

$$y(x,t) = \underbrace{A_{es}}_{\text{ventre}} \sin(kx) \sin(wt)$$

Amplitude em um x

$$A_{x\,(max)} = A_{est} \sin(kx)$$

Velocidade da Estacionária

$$v(x,t) = A w \sin(kx) \cos(wt)$$

Velocidade máxima em um x

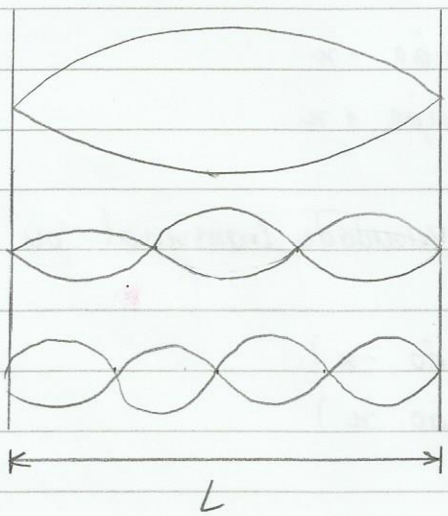
$$v_{max} = A_{est}\, w \sin(kx) \quad \text{ou}$$
$$= A_x w$$

Aceleração da Estacionária

$$a(x,t) = -A w^2 \sin(kx) \sin(wt)$$

Aceleração máxima em um x

$$a_{max} = A_{est}\, w^2 \sin(kx) \quad \text{ou}$$
$$= A_x w^2$$



$$L = n \cdot \frac{\lambda}{2}$$

$$f = \frac{n}{2L} \sqrt{\frac{F}{\mu}}$$

nós: $x = n\frac{\lambda}{2}$ $\quad y = 0 \rightarrow \sin(kx) = 0 \quad (0, \pi, 2\pi \ldots)$

ventre: $\left(n + \frac{1}{2}\right)\frac{\lambda}{2}$ $\quad y = \pm A_{est} \rightarrow \sin(kx) = \pm 1 \quad \left(\frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi \ldots\right)$

1) (P2 - 1º sem 03)

$\mu = 0{,}03\ kg/m$

$y(x,t) = 2{,}5 \cos(\pi x/6 - 3\pi t/2)$ [SI]

a) Velocidade transversal máxima = ?

" de propagação da onda = ?

$v = A\omega \Rightarrow v = 2{,}5 \cdot \frac{3\pi}{2} = \underline{11{,}78\ m/s}$ (velocidade transversal máxima)

$v = \sqrt{\frac{F}{\mu}} = \lambda f = \frac{\omega}{k}$

$v = \frac{\omega}{k} = \frac{3\pi}{2} : \frac{\pi}{6} = \underline{9\ m/s}$ (velocidade de propagação da onda)

b) Potência média = ? Tração do Fio

$v = \sqrt{\frac{F}{\mu}} \therefore F = v^2 \cdot \mu = 9^2 \cdot 0{,}03 = \underline{2{,}43\ N}$

$\bar{P} = \frac{\mu \cdot v\,(\omega A)^2}{2} = \frac{0{,}03 \cdot 9 \cdot (3\pi/2 \cdot 2{,}5)^2}{2} \therefore \bar{P} = \underline{18{,}74\ W}$

c) $k = \frac{2\pi}{\lambda} \therefore \lambda = \frac{2\pi}{k} = 2\pi : \frac{\pi}{6} = 12\ m$

$x = n\frac{\lambda}{2}$; Quando $n = 1$

$x = 1 \cdot \frac{12}{2} = \underline{6\ m}$

/ /

d) Qual equação estacionária da "c"

$y(x,t) = A_{est} \cdot sen(kx)\, sen(wt)$

$A_{est} = 2A$

$A_{est} = 2 \cdot 2,5 = 5\ m$

$y(x,t) = 5\ sen(\pi x/6)\, sen(3\pi t/2)$

2) (P2 - 2º sem 03)

$\mu = 0,04\ kg/m$

$F = 4N$

$\frac{\lambda}{2} = 0,30 \therefore \lambda = 0,6$

$t = 0s \begin{cases} x = 0,05\ m \\ y = 0,025\ m \end{cases}$

$x = 0 \rightarrow$ desl. transv. máx e positivo

a) velocidade de propagação da onda transversal e o número de onda

$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{4}{0,04}} = \underline{10\ m/s}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0,6} = \underline{10,47\ rad/m}$

b) Equação da onda transversal

$y(x,t) = A\cos(kx - wt)$

$v = \frac{w}{K} \therefore w = v \cdot k = 10 \cdot 10,47 = 104,7\ rad/s$

$y(x,t) = A\cos(10,47x - 104,7t)$

Quando $t = 0s$ $\begin{cases} x = 0{,}05\ m \\ y = 0{,}025\ m \end{cases}$

$0{,}025 = A \cos(10{,}47 \cdot 0{,}05 - 104{,}7 \cdot 0)$

$A = 0{,}0289$

$y(x,t) = 0{,}0289 \cos(10{,}47\pi - 104{,}7t)$ [S.I]

c) $y(x,t) = 0{,}0577 \,\text{sen}\,(10{,}47\pi)\,\text{sen}\,(104{,}7t)$ [SI]

d) $\frac{\lambda}{2} \cdot 3 = L \therefore L = \frac{0{,}6 \cdot 3}{2} = 0{,}9\ m$

3) (1º sem 1)

$m = 0{,}2\ kg$ (corda)

$m = 3\ kg$ (bloco)

$\frac{\lambda}{2} = 10\ m \therefore \lambda = 20\ m$

$f = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$

/ /

4) Aço $\mu_1 = 7{,}8\, g/cm$ Prata $\mu_2 = 10{,}6\, g/cm$

L L

$$f^1 = \frac{n}{2L}\sqrt{\frac{F}{\mu_1}} = 600 \qquad f^2 = \frac{n}{2l}\sqrt{\frac{F}{y_2}}$$

$$\frac{n}{2L}\cdot\sqrt{F} = 600\sqrt{y_1} \qquad \frac{n}{2l}\cdot\sqrt{F} = f^2\sqrt{y_2}$$

$$600\sqrt{y_1} = f\sqrt{y_2} \quad \therefore \quad f = 600\cdot\sqrt{\frac{y_1}{y_2}} = \underline{514{,}70\ Hz}$$

5) (P2 - 1º sem 05 - diurno)

$A = 0{,}007\ m$ a) $y(x,t) = 7.10^{-3}\, sen\left(\frac{\pi}{0{,}2}x\right) sen\,(30\pi t)$

$f = 15\ Hz$

$\frac{\lambda}{2} = 0{,}20\ m$ $K = \frac{2\pi}{\lambda} = \frac{2\pi}{0{,}4} = \frac{\pi}{0{,}2}$ $w = 2\pi f = 2\pi.15 = 30\pi$ rad/s

b) $y = A\, sen\,(Kx)$ ventre $y = \frac{0{,}2}{2} = 0{,}1 \therefore x' = 0{,}1 - 0{,}03 = 0{,}07$

$y = 7.10^{-3}\, sen\left(\frac{\pi}{0{,}2}x\right)$

$v^{max} = 6{,}23.10^{-3}.30\pi$

$y(0{,}07) = 6{,}23.10^{-3}\ m$ $\therefore v^{max} = \underline{0{,}588\ m/s}$

6) (P2 - 2º sem 05 - noturno)

(MHS) $A = 0{,}9\ cm = 9 \cdot 10^{-3} m$

$T = 0{,}02\ s$

$\lambda = 0{,}28 m = 28 cm$

$w = \frac{2\pi}{T} = \frac{2\pi}{0{,}02} = 100\pi\ rad/s$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{28} = \frac{\pi}{14}\ rad/m$

a) $v = 28 \cdot \frac{1}{0{,}02} = \underline{1400\ cm/s}$

b) $v(x,t) = 0{,}9 \cdot 100\pi \operatorname{sen}\left(\frac{\pi}{14} \cdot x\right) \cos(100\pi t)\ (cm/s)$

c) $v = \frac{\lambda}{4} = \frac{28}{4} = 7$ $\quad x = 7 - 2 = 5$

$A(x) = 0{,}9 \operatorname{sen}\left(\frac{\pi}{14} \cdot 5\right) \quad \therefore \quad A(5) = \underline{0{,}81\ cm}$

7-) (P2 - 2º sem 2005 - diurno)

$y(x,t) = 0{,}2 \operatorname{sen}(\pi/0{,}2\, x) \operatorname{sen}(32\pi t)\ [SI]$

a) $k = \frac{2\pi}{\lambda} \therefore \lambda = \frac{2\pi}{k} = 2\pi : \frac{\pi}{0{,}2} = 0{,}4\ m$

$d = \frac{\lambda}{2} = \frac{0{,}4}{2} = \underline{0{,}2\ m}$

b) $1 \cdot 10^{-3} = 0{,}2 \operatorname{sen}(\pi/0{,}2\, x)$

$\therefore x = \underline{3{,}18 \cdot 10^{-4}\ m}$ ou $\underline{0{,}000318\ m}$

c) $v = \frac{w}{k} = \frac{32\pi}{(\pi/0{,}2)} = \underline{6{,}4\ m/s}$

8) (P3 - 2º sem 05 - diurno)

$v = 20\, m/s$

$f = 25\, Hz$

$A = 8.10^{-2}\, m$

$x = \frac{\lambda}{8}$

a) $v = \lambda f$

$\therefore \lambda = \frac{v}{f} = \frac{20}{25} = 0{,}8$

$x = \frac{\lambda}{8} = \frac{0{,}8}{8} = 0{,}1$

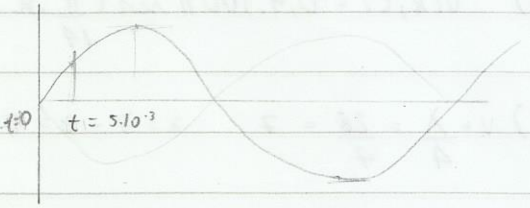
$v^{máx}(x,t) = 8.10^{-2}.50\pi \, sen\left(\frac{\pi}{0{,}4} x\right)$

$\omega = 2\pi f = 2\pi.25 = 50\pi\, rad/s$

$v^{máx}(0{,}1) = \underline{8{,}89\, m/s}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0{,}8} = \frac{\pi}{0{,}4}\, rad/m$

b) $x = \frac{\lambda}{8} = \frac{0{,}8}{8} = 0{,}1\, m$



$y(x,t) = 4.10^{-2} \cos\left(\frac{\pi}{0{,}4} x - 50\pi t\right)$

$\frac{\pi}{0{,}4} . 0{,}1 - 50\pi t = 0 \quad \therefore t = 5.10^{-3}\, s$

$\frac{\pi}{0{,}4} . 0{,}4 - 50\pi t' = \quad \therefore t' = 0{,}02\, s$

9) (P2 - 1º sem 06 - diurno)

$v = 20\, m/s$

$f = 60\, Hz$

$A = 4.10^{-2}\, m$

$\omega = 2\pi f = 2\pi.60 = 120\pi$

$v = \lambda f \therefore \lambda = \frac{v}{f} = \frac{20}{60} = \frac{1}{3}$

$k = \frac{2\pi}{(1/3)} = 6\pi$

$v(x,t) = 4.10^{-2}.120\pi \, sen(6\pi x) \cos(120\pi t)$

$v^{máx}(0{,}12) = 4.10^{-2}\, 120\pi \, sen(6\pi.0{,}12)$

$\therefore v^{máx} = \underline{11{,}62\, m/s}$

10) (P2 - 1º sem 06 - noturno)

4º harmônico

$L = 1,2\,m$ a) $\lambda = ?$ e $f = ?$

$m = 4.10^{-3}\,kg$

$F = 4N$ $L = \frac{4\lambda}{2}$ ∴ $\lambda = \frac{L}{2} = \frac{1,2}{2} = \underline{0,6\,m}$

$A = 3.10^{-3}$

$f = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \Rightarrow f = \frac{4}{2.1,2}\sqrt{\frac{4}{(4.10^{-3})/1,2}} \Rightarrow \underline{f = 57,74\,Hz}$

b) $A = ? \rightarrow x = 0,2m$ $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0,6} = \frac{\pi}{0,3}$ $w = 2\pi f = 362,76$

$y(x,t) = 6.10^{-3}\,sen\left(\frac{\pi}{0,3}x\right)sen(362,76t)$

$y^{máx}(0,2) = 6.10^{-3}\,sen\left(\frac{\pi}{0,3}.0,2\right)$ ∴ $\underline{y^{máx} = 5,196.10^{-3}\,m}$

c) $v(x,t) = 6.10^{-3}.362,76\,sen\left(\frac{\pi}{0,3}x\right)cos(362,76t)$

∴ $\underline{v(5,196.10^{-3};\ 4) = 0,0141\,m/s}$

11-) (P3 - 1º sem 06 - noturno)

$\frac{\lambda}{4} = 24\,cm$ ∴ $\underline{\lambda = 0,96\,m}$

$f = 100\,hz$ $v = \lambda f = 0,96.100 = 96\,m/s$

$F = 0,03\,N$ $v = \sqrt{\frac{F}{\mu}}$ ∴ $\mu = \frac{F}{v^2} = \frac{0,03}{96^2} = 3,25.10^{-6}\,kg/m$

12-) (P2 - 2º sem 06 - diurno)

$y(x,t) = 0{,}12 \cos[2\pi(x/0{,}4 + t/0{,}05)]$ [SI]

- Calcular $t = ?$ em $x = 0{,}42\,m$ e $t = 0{,}33\,s$

$2\pi\left(\frac{x}{0{,}4} + \frac{t}{0{,}05}\right) = 2\pi n$; $x = 0{,}42$ e $t = 0{,}33$

$\therefore n = 7{,}65 \Rightarrow n = 8$

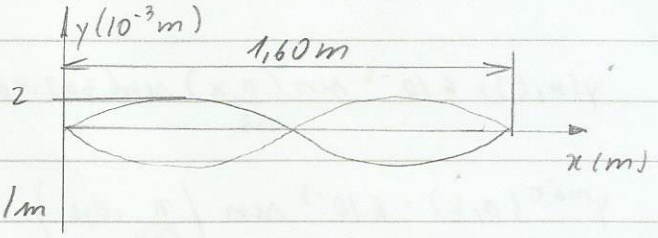
$\Delta t = 0{,}3475 - 0{,}33$

$\Delta t = 0{,}0175$

$2\pi\left(\frac{0{,}42}{0{,}4} + \frac{t'}{0{,}05}\right) = 2\pi \cdot 8 \quad \therefore \quad t' = 0{,}3475$

13-) (P2 - 2º sem 06 diurno)

$v = 180\,m/s$



a)

$\lambda = 1{,}60 \quad \therefore \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{1{,}6} = \frac{\pi}{0{,}8}$ rad/m

$w = v \cdot k = 180 \cdot \frac{\pi}{0{,}8} = 225\pi$ rad/s

b) $y(x,t) = 2 \cdot 10^{-3} \sin\left(\frac{\pi}{0{,}8}x\right)\sin(225\pi t)$ [SI]

$v^{máx}(x) = 2 \cdot 10^{-3} \cdot 225\pi \sin\left(\frac{\pi}{0{,}8} \cdot x\right)$

$v^{máx}(0{,}6) = 1\,m/s$

$0 = 2 \cdot 10^{-3} \sin\left(\frac{\pi}{0{,}8} \cdot 0{,}4\right)\sin(\underbrace{225\pi t}_{=\pi})$

$\therefore t = \frac{\pi}{225\pi} = 4{,}44 \cdot 10^{-3}$

14-) (P2 - 2º sem 06 - noturno)

$y = 5.10^{-3} \cos(314t)$ [SI]

$v = 60\,m/s$ $\qquad w = 2\pi f \therefore f = \frac{w}{2\pi} = \frac{314}{2\pi} = \underline{49,97\,Hz}$

$k = \frac{w}{v} = \frac{314}{60} = 5,23\,rad/s$ $\qquad \lambda = \frac{2\pi}{k} = \frac{2\pi}{5,23} = \underline{1,2\,m}$

b) $v = \sqrt{\frac{F}{\mu}} \therefore F = v^2 \mu = 60^2 \cdot 0,04 = \underline{144\,N}$

c) $y(x,t) = 5.10^{-3} \cos(5,23x - 314t)$ [SI]

d) $v(x,t) = -5.10^{-3} \cdot 314 \sin(5,23x - 314t)$

$v(1,5;2) = -1,57\,m/s$

e) $f = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \Rightarrow f = \frac{3}{2.2}\sqrt{\frac{144}{0,04}} = \underline{45\,Hz}$

$\frac{\lambda}{2} = \frac{L}{3} \therefore \lambda = \frac{2.2}{3} = \underline{1,33\,m}$

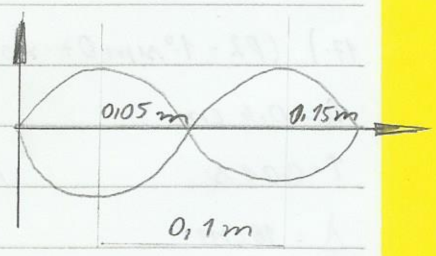
15-) (P2 - 1º sem 07 - diurno)

2º harmônico $\qquad w = \frac{2\pi}{T} = \frac{2\pi}{0,05} = 40\pi\ rad/s$ $\qquad \lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \frac{1}{5} = \underline{0,2\,m}$

$A = 0,03\,m$

$T = 0,05\,s$ $\qquad k = \frac{w}{v} = \frac{40\pi}{4} = 10\pi\ rad/m$

$v = 4\,m/s$



b) $y = 0,03 \sin(10\pi x)$

$y(3.10^{-2}) = \underline{0,024\,m}$

$v^{máx} = 0,024 \cdot 40\pi = \underline{3,02\,m/s}$

c) $y(x,t) = 0{,}03 \cos(10\pi x - 40\pi t)$

$10\pi x - 40\pi t = 2\pi n$; $x = 0{,}12\ m$ e $t = 0{,}26$

$\therefore n = 4{,}6 \Rightarrow n = 5$

$10\pi x - 40\pi t' = 2\pi . 5$; $x = 0{,}12$

$\therefore t' = 0{,}22 s$ $\qquad \Delta t = 0{,}22 - 0,$

16-) (P2 - 1º sem 2007 - noturno)

10m em $\frac{1}{4} . T$ $\qquad v = \frac{10}{(10/4)} = 4 m/s$

$T = 10 s$ $\qquad w = \frac{2\pi}{T} = \frac{2\pi}{10} = \frac{\pi}{5}$ rad/s

$k = \frac{w}{v} = \frac{4}{(\pi/5)} = 4 : \frac{\pi}{5} = \frac{20}{\pi}$ rad/m

b) 5s

c) $F = 0{,}16 N$ $\qquad v = \sqrt{\frac{F}{\mu}} = \mu = \frac{F}{v^2} = \frac{0{,}16}{4^2} = 0{,}01\ kg/m$

d) $y(x,t) = 0{,}2 \cos\left(\frac{20x}{\pi} - \frac{\pi}{5} t\right)$ [SI]

17-) (P2 - 1º sem 07 - noturno) $\qquad v = \frac{18}{2} = 9 cm \qquad x = 9 - 2 = 7 cm$

$A = 0{,}6\ cm$ $\qquad \lambda = 36 \qquad k = \frac{2\pi}{36} = \frac{\pi}{18}$

$T = 0{,}04 s$ $\qquad A(x) = 0{,}6 \operatorname{sen}\left(\frac{\pi}{18} x\right)$

$\frac{\lambda}{2} = 18 cm$

$A(7) = 0{,}56 cm$

18.) (P3 - 1º sem 07 - noturno)

$f = 20 Hz$

a) $T = \frac{1}{f} = \frac{1}{20} = 0{,}05\,s$ $\quad w = \frac{2\pi}{T} = \frac{2\pi}{0{,}05} = \underline{40\pi \text{ rad/s}}$

$\lambda = 10 - 2 = 8\,m$ $\quad k = \frac{2\pi}{8} = \frac{\pi}{4}$ rad/m

b) $y(x,t) = 0{,}3 \cos\left(\frac{\pi}{4}x + 40\pi\right)$ [SI]

c) $v^{máx} = 0{,}3 \cdot 40\pi = 12\pi$ m/s

$v(x,t) = 0{,}3 \cdot 40\pi \operatorname{sen}\left(\frac{\pi}{4} \cdot x\right)$; se $v = 12\pi$

$\therefore x = 2$; Porem a velocidade aqui é negativa

d) $\frac{w}{k} = \sqrt{\frac{F}{\mu}}$ $\therefore$ $\mu = \frac{F}{(w/k)^2} = \frac{2048}{(40\pi/\frac{\pi}{4})^2} = \underline{0{,}08 \text{ kg/m}}$

19.) (P2 - 2º Sem 2007 / Diurno)

23) (P2 - 1º sem 2006 - diurno)

$f = 20\ Hz$

$A = 0{,}036\ m$

$v = 4\ m/s$

a) número de onda: $k = ?$

$v = \frac{w}{K} \quad \therefore \quad k = \frac{w}{v} \qquad w = 2\pi f$

$= 2\pi \cdot 20 = 40\pi\ rad/s$

$k = \frac{40\pi}{4} = \underline{10\pi\ rad/m}$

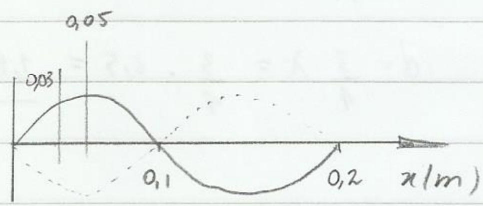
b) equação do deslocamento transversal

$s(x,t) = 0{,}036\ sen(10\pi x)\,sen(40\pi t)\ [SI]$

c) velocidade transversal de um ponto $x = 0{,}02\ m$ no $t = 0{,}315\ s$

$v(x,t) = 1{,}44\pi\ sen(10\pi x)\cos(40\pi t)$

$k = \frac{2\pi}{\lambda} \quad \therefore \quad \lambda = \frac{2\pi}{K} = \frac{2\pi}{10\pi} = 0{,}2$



$v(0{,}03;\ 0{,}315) = \underline{-1{,}13\ m/s}$

"Como determinar o nº harmônico"

24.) (P2 - 1º sem 2006 - diurno)

$F = 18N \qquad \lambda = 0{,}48\ m$

$m = 0{,}002\ kg \qquad A = 0{,}0025\ m$

$L = 0{,}5\ m$

a) velocidade de propação

$v = \sqrt{\frac{F}{\mu}} \quad ; \quad \mu = \frac{m}{L} = \frac{0{,}002}{0{,}5} = 0{,}004$

$v = \sqrt{\frac{18}{0{,}004}} = \underline{67{,}08\ m/s}$

$v = \lambda f \quad \therefore \quad f = \frac{v}{\lambda} = \frac{67,08}{0,48} = \underline{139,75\ Hz}$

$w = 2\pi f = 2\pi \cdot 139,75$

$w = 878,07$

b) $\bar{P} = \frac{\mu v (wA)^2}{2} = \frac{0,004 \cdot 67,08 \cdot (878,07 \cdot 0,0025)^2}{2}$

$\bar{P} = \underline{0,647\ W}$

25) (P2 - 1º sem 08 - noturno)

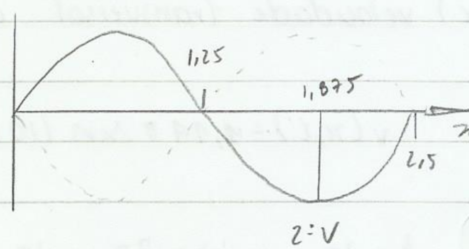
$y(x,t) = 3 \cdot 10^{-3} \operatorname{sen}(0,8\pi x) \operatorname{sen}(20\pi t)$ [SI]

a) velocidade de propagação da corda

$v = \frac{w}{k} = \frac{20\pi}{0,8\pi} = \underline{25\ m/s}$

b) $k = \frac{2\pi}{\lambda} \quad \therefore \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{0,8\pi} = 2,5$

$d = \frac{3}{4}\lambda = \frac{3}{4} \cdot 2,5 = \underline{1,875\ m}$



c) módulo da vel. transv. máx no $x = 0,5\ m$

$y^{max} = A \operatorname{sen}(kx)$

$= 3 \cdot 10^{-3} \operatorname{sen}(0,8\pi x) \ ; \ x = 0,5$

$y^{max} = 0,00285$

$v_{máx} = w\, y^{max}$

$= 20\pi \cdot 0,00285$

$v_{máx} = \underline{0,18\ m/s}$

26) (P3 - 1º sem 2006 - diurno)

$L = 8,4 m$

$m = 0,12 kg$

$F = 96 N$

a) Velocidade escalar das ondas na corda

$v = \sqrt{\frac{F}{\mu}}$ $\mu = \frac{m}{L} = \frac{0,12}{8,4} = 0,014$

$v = \sqrt{\frac{96}{0,014}} = \underline{81,98 m/s}$

b) o x^{max} e frequência

?

27.) $y_1(x,t) = 0,13 \cos(0,79x - 15t)$ [S.I]

a) velocidade de propagação e comp. da onda

$v = \frac{w}{k} = \frac{15}{0,79} = \underline{18,99 m/s}$

$v = \lambda f \therefore \lambda = \frac{v}{f} = \frac{v}{\frac{w}{2\pi}} = \frac{2\pi v}{w} = \frac{2\pi . 18,99}{15} = \underline{7,95 m}$

b) $y(x,t) = 0,26 \sin(0,79x) \sin(15t)$

$y_2(x,t) = -0,13 \cos(0,79x + 15t)$ [SI]

/ /

c) Velocidade transv. no inst. $t = 0{,}18s$ e $x = 2{,}3m$

$$v(x,t) = 3{,}9 \,sen(0{,}79x)\cos(15t)$$
$$v(2{,}3;\, 0{,}18) = \underline{-3{,}42\ m/s}$$

28) (P2 - 2º sem - diurno)

$\lambda = 1{,}25\ m$ $\qquad$ $y(x,t) = 0{,}045\, sen(4\pi x)\, sen(503t)$ [SI]

$m = 0{,}005\ kg$

a) Força tensora na corda

$$\sqrt{\frac{F}{\mu}} = \frac{w}{k} \quad \therefore \quad F = \frac{w^2}{k^2}\cdot\mu \quad \Rightarrow \quad \mu = \frac{m}{L} =$$

$$F = \frac{(503)^2}{(4\pi)^2} \cdot \frac{0{,}005}{1{,}25} = \underline{6{,}41\ N}$$

b) Veloc. transv. máx $x = 0{,}2m$ e $t = 2s$

$$v^{max}(x,t) = Aw\, sen(kx)$$
$$= 0{,}045 \cdot 503\, sen(4\pi \cdot x)\cos(503t)$$
$$v^{max}(0{,}2,\, 2) = \underline{10{,}26\ m/s}$$

29) (P2 - 2º sem 2006 - noturno)

$L = 5m$ $\quad$ $y(x,t) = 0{,}07\cos(6{,}25\pi x + 50\pi t)$

$m = 0{,}2\ kg$

$t=0 \rightarrow x\ máx$ $\quad$ $\lambda = ?$ $\quad v = ?$

a)

$\mu = \frac{0{,}2}{5} = 0{,}04\ kg/m$ $\quad$ $k = \frac{2\pi}{\lambda}$ $\quad$ $\lambda = \frac{2\pi}{6{,}25\pi} = \underline{0{,}32\,m}$

$v = \frac{w}{k} = \frac{50\pi}{6{,}25\pi} = \underline{8\ m/s}$

b) $\bar{P} = \frac{\mu v (wA)^2}{2} = \frac{0{,}04 \cdot 8 \cdot (50\pi \cdot 0{,}07)^2}{2} = \underline{19{,}34\ W}$

c) $x = 0{,}36\ m$ $\quad$ $t = 0{,}15\ s$

$6{,}25\pi x + 50\pi t = \alpha$

$6{,}25\pi \cdot 0{,}36 + 50\pi \cdot 0{,}15 = \alpha$ $\quad \therefore \alpha = 9{,}75\alpha \Rightarrow \alpha = 10\pi$

$6{,}25\pi \cdot 0{,}36 + 50\pi t' = 10\pi$

$\therefore t' = 0{,}155$

$\Delta t = 0{,}155 - 0{,}15 = 0{,}005$

d)

30) (P3 - 2º sem 08 - diurno)

$t_1 = 0 \quad t_2 = 60.10^{-3}$

$v = \frac{1,5}{60.10^{-3}} = \underline{25\ m/s}$

$v = \lambda f$

$\therefore f = \frac{25}{2} = 12,5$

$T = \frac{1}{f} = \frac{1}{12,5} = \underline{0,08\ s}$

b) $v^{max} = -Aw$

$w = 2\pi f$

$w = 2\pi . 12,5$

$w = 78,54$

$A = 6.10^{-3}$

$v^{max} = -6.10^{-3} . 78,54$

$v^{max} = \underline{0,47\ m/s}$

31-) $\mu = 0,02\ kg/m$

$F = 50N$

$L = 0,50\ m$

$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{50}{0,02}} = 50\ m/s$

a) $n = 5$

$f = \frac{5}{2.0,5}\sqrt{\frac{50}{0,02}} = \underline{250\ Hz}$

b) $\frac{\lambda}{2} = \frac{0,5}{5} \quad \therefore \lambda = 0,2 \qquad Vent. = \frac{\lambda}{2} = 0,1 + 0,03 = 0,13\ m \qquad A = 0,04\ m$

$A(x) = 2A\ sen(Kx)$

$A(0,13) = 2.0,04\ sen\left(\frac{\pi}{0,1} . 0,13\right)$

$K = \frac{2\pi}{\lambda} = \frac{2\pi}{0,2} = \frac{\pi}{0,1}$

$\therefore A_{est} = \underline{0,065\ m}$

$$L = \frac{(nV).\lambda}{2} \therefore nV = \frac{2L}{\lambda}$$

32) (P2 - 1º sem 2009 - diurno)

5º harmônico

$y(x,t) = 3{,}8.10^{-2} . sen(\pi x/3)\, sen(30\pi . t)$ [SI]

a) Amplitude, comprimento de onda (Ondas progressivas)

$A_{est} = 2A$ $\quad A = \frac{A_{est}}{2} = \frac{3{,}8.10^{-2}}{2} = \underline{1{,}9.10^{-2}\ m}$

$\frac{w}{K} = \lambda f \Rightarrow \frac{30\pi}{(\pi/3)} = \lambda . \frac{30\pi}{2\pi} \therefore \lambda = \underline{6\ m}$

b) $L = n(\text{ventres}).\frac{\lambda}{2} \therefore L = \frac{5.6}{2} = \underline{15\ m}$

c) 3º harmônico

$L = n\frac{\lambda}{2} \therefore \lambda = \frac{2L}{3}$, $L = 15$ $\quad \lambda = \underline{10\ m}$

$k = \frac{2\pi}{10} = 0{,}2\pi$ $\quad \frac{w}{k} = v \Rightarrow w = v.K$

$w = 90 . 0{,}2\pi = 18\pi$

$v = 30\pi : \frac{\pi}{3} = 90\ m/s$

$\therefore y(x,t) = 3{,}8.10^{-2}\, sen(0{,}2\pi x)\, sen(18\pi t)$ [SI]

33-) (P2 - 1º sem 09 - diurno)

$L = 2m$

$f = 200/\pi$ Hz

$v = 95{,}5\ m/s$ (sent. +)

$|v\ máx| = 6\ m/s$

a) Amplitude

$v^{max} = Aw$

$w = 2\pi f = 2\pi . \frac{200}{\pi} = 400\ rad/s$

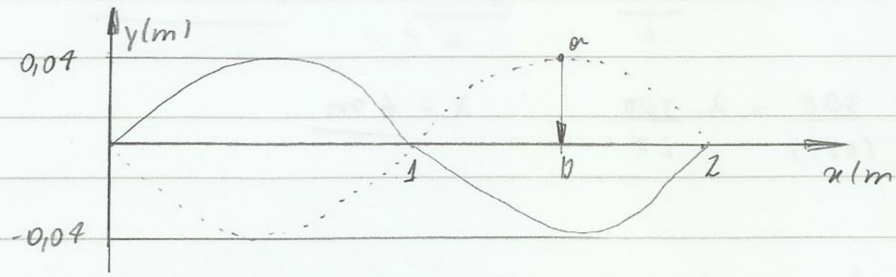
$\therefore A = \frac{v^{max}}{w} = \frac{6}{400} = 0{,}015\ m = 15.10^{-3}\ m$

b) $y(x,t) = A\cos(kx - wt)$

$v = \frac{w}{K}$ $\therefore$ $k = \frac{w}{v} = \frac{400}{95,5} = 4,19$ rad/m

$y(x,t) = 0,015\cos(4,19x - 400t)$ [SI]

34-) (P2 - 1º sem 09 - noturno)



$x = 1,5\,m \rightarrow t = 0,010\,s$

a) Periodo e núm. de onda

$A = 0,04\,m$ - $k = \frac{2\pi}{2} = \pi$ rad/m

$\lambda = 2\,m$

$\Delta T = \frac{1}{4}T$; $\Delta T = 0,01$ $\therefore$ $T = 0,04\,s$

b) $y(x,t) = A_{est}\sin(kx)\sin(wt)$

$w = \frac{2\pi}{T} = \frac{2\pi}{0,04} = 50\pi$

$y(x,t) = 0,04\sin(\pi x)\sin(50\pi t)$ [SI] c-) $A(x) = A_{cst}\, sen(kx)$

$A(1,8) = 0{,}04\, sen(\pi \cdot 1{,}8)$

$A(1,8) = \pm\, \underline{0{,}0235\ m}$

d) $y(x,t) = 0{,}04\, sen(\pi x)\, sen(50\pi t)$

$\frac{dy}{dt} = v(x,t) = 0{,}04 \cdot 50\pi\, sen(\pi x)\cos(50\pi t)$

$v(0{,}7;\, 0{,}03) = \underline{0\ m/s}$

e) $f = \frac{n}{2L} v$ $L = \frac{n\lambda}{2} = \frac{2 \cdot 2}{2} = 2m$

$f = \frac{3}{2 \cdot 2} \cdot \frac{50\pi}{\pi} = \underline{37{,}5\ Hz}$

35-) (P3 - 1º sem 2009 - noturno)

$A = 4{,}6 \cdot 10^{-3}\ m$

$f = 16\ Hz$

$v = 32\ m/s$ (sent. -)

$\mu = 15\ g/m = 0{,}015\ \frac{kg}{m}$

a) potência média

$\bar{P} = \frac{\mu \cdot v\, (wA)^2}{2}$

$\bar{P} = \frac{0{,}015 \cdot 32\, (2\pi \cdot 16 \cdot 4{,}6\ 10^{-3})^2}{2}$

$\therefore \bar{P} = \underline{0{,}0513\ W}$

b) $y(x,t) = A\cos(kx + wt)$

$y(x,t) = 4{,}6 \cdot 10^{-3}\cos(\pi x + 32\pi t)$ [SI]

$w = 2\pi f = 2\pi \cdot 16 = 32\pi$

$v = \frac{w}{k} \therefore k = \frac{w}{v} = \frac{32\pi}{32} = \pi\ \frac{rad}{m}$

$\frac{dy}{dt} = v(x,t) = -0{,}462\, sen(\pi x + 32\pi t)$

$v(1{,}3;\, 0{,}26) = \underline{-0{,}337\ m/s}$

/ /

d) $y(x,t) = 4{,}6 \cdot 10^{-3} \cos(\pi x + 32\pi t)$ [SI]

$\pi x + 32\pi t = \alpha$; $x = 0{,}7$ e $t = 0$

$\pi \cdot 0{,}7 + 32\pi \cdot 0 = \alpha$ $\therefore$ $\alpha = 0{,}7\pi \Rightarrow y = 0 \to \pi$

$\pi \cdot 0{,}7 + 32\pi t' = \pi$ $\quad \Delta T = T' - T = 0{,}009 - 0{,}7 =$?

$\therefore t' = 0{,}009$

36-) (P2-2º sem 09- diurno)

$L = 3{,}0\,m$

$y(x,t) = 0{,}02 \sin(\pi x)\sin(100\pi t)$ [SI]

$L = \frac{n\lambda}{2}$ $\quad k = \frac{2\pi}{\lambda}$ $\therefore$ $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} = 2m$

$n = \frac{2L}{\lambda} = \frac{2 \cdot 3}{2} =$ 3 ventres

b) $w = 2\pi f$ $\therefore$ $f = \frac{w}{2\pi} = \frac{100\pi}{2\pi} = 50\,Hz$

$f' = \frac{50}{3} = 16{,}67\,Hz$

c) $A(x) = A_{est} \sin(kx)$

$A(x) = 0{,}02 \sin(\pi x)$

$A(1{,}26) = 0{,}02 \sin(\pi \cdot 1{,}26)$

$\therefore$ $A(1{,}26) = 0{,}0154\,m$

d) $f = 50\ Hz$

$F' = 9F$

$v_{en} = ?$

$$\sqrt{\frac{F'}{\mu}} = \sqrt{\frac{9F}{\mu}} = 3\sqrt{\frac{F}{\mu}} \ ; \ \sqrt{\frac{F}{\mu}} = v$$

$$v = \frac{w}{K} = \frac{100\pi}{\pi} = 100\ m/s$$

$$3\sqrt{\frac{F}{\mu}} = \frac{w}{k} \Rightarrow 3.100 = \frac{100\pi}{k} \quad \therefore \quad k = \frac{\pi}{3}\ rad/m$$

$$L = n\frac{\lambda}{2} \qquad k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = 2\pi : \frac{\pi}{3} = 6$$

$$n = \frac{2L}{\lambda} = \frac{2.3}{6} = 1 \text{ ventre}$$

37) (P2 - 2º sem 2009 - noturno)

$\mu = 0{,}01\ kg/m$ a) velocidade da onda

$F = 40N$

$x = 0 \begin{cases} A = 0{,}3\ m \\ T = 0{,}1\ s \end{cases}$

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{40}{0{,}01}} = \underline{63{,}25\ m/s}$$

b) $$w = \frac{2\pi}{T} = \frac{2\pi}{0{,}1} = 20\pi$$

$$v = \frac{w}{K} \quad \therefore \quad k = \frac{w}{v} = \frac{20\pi}{63{,}25} = 0{,}99\ rad/m$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0{,}99} = \underline{6{,}325\ m}$$

c) $y(x,t) = 0{,}03 \cos(0{,}99t + 20\pi t)$ [SI]

38) (P2 - 2º sem 2009 - noturno)

3º harmônico

$y(x,t) = 0{,}10 \operatorname{sen}(\pi x/4) \operatorname{sen}(100\pi t)$ [SI]

a) $k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k} = 2\pi : \frac{\pi}{4} = \underline{8\,m}$

$L = n \cdot \frac{\lambda}{2} = 3 \cdot \frac{8}{2} = \underline{12\,m}$

b) $w_3 = 3w_1$ $\qquad w_3 = \frac{2\pi}{T_3} = 3w_1$ $\qquad w_1 = \frac{2\pi}{3T_3}$

$w_2 = 2w_1$

$w_2 = \frac{2\pi}{T_2} = 2w_1$ $\qquad w_1 = \frac{\pi}{T_2}$

$w_1 = w_1 \Rightarrow \frac{2\pi}{3T_3} = \frac{\pi}{T_2} \Rightarrow T_2 = \frac{3T_3}{2}$

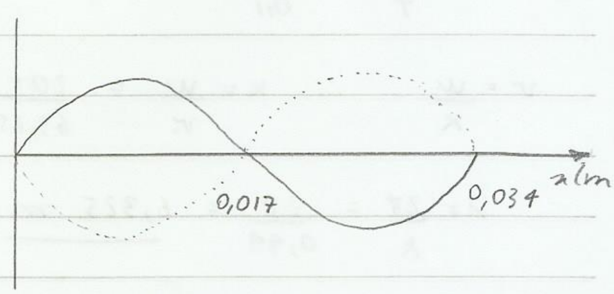
$T_3 = \frac{2\pi}{w_3} = \frac{2\pi}{100\pi} = 0{,}02\,s$ $\qquad T_2 = \frac{3 \cdot 0{,}02}{2} = \underline{0{,}03\,s}$

39) (P3 - 2º sem 2009 - diurno)

$\lambda = 0{,}034\,m$

$f = 440\,Hz$ (1º harmônico)

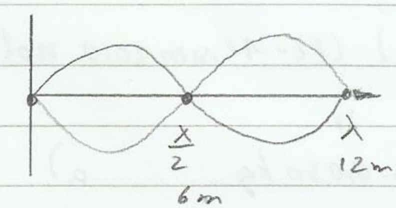
$v = 340\,m/s$



40) (P3 - 2º sem. diurno)

$y(x,t) = 0{,}08 \, \text{sen}((\pi/6)x) \, \text{sen}(20\pi t)$ [S.I]

a) $v = \dfrac{20\pi}{(\pi/6)} = \underline{120 \, m/s}$



b) $k = \dfrac{2\pi}{\lambda} \quad \therefore \lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{(\pi/6)} = \underline{12 \, m}$

$d = \dfrac{\lambda}{2} = \dfrac{12}{2} = \underline{6 \, m}$

c) $v(x,t) = -20\pi . 0{,}08 \, \text{sen}((\pi/6)x) \cos(20\pi t)$

$v(1{,}8 \, ; 1{,}17) = -20\pi . 0{,}08 \, \text{sen}((\pi/6) . 1{,}8) \cos(20\pi . 1{,}17)$

$v = \underline{-1{,}26 \, m/s}$

41) (P2 - 1º sem 2010 - noturno)

$\mu = 0{,}05 \, kg/m$

$y(x,t) = 8{,}2 . 10^{-2} \cos(4\pi x + 60\pi t)$ [SI]

a) $\dfrac{w}{k} = \sqrt{\dfrac{F}{\mu}} \quad \therefore \quad F = \left(\dfrac{w}{k}\right)^2 . \mu = \left(\dfrac{60\pi}{4\pi}\right)^2 . 0{,}05$

$\therefore \quad F = \underline{11{,}25 N}$

b) $\frac{dy}{dt} = v(x,t) = -8{,}2 \cdot 10^{-2} \cdot 60\pi \, sen(4\pi x + 60\pi t)$

$v(0{,}13;\, 0{,}35) = \underline{15{,}43 \, m/s}$

c) $4\pi x + 60\pi T = \alpha$

$4\pi \cdot 0{,}33 + 60\pi \cdot 0{,}17 = \alpha \quad \therefore \alpha = 11{,}52\pi \cong 12\pi$

$4\pi \, 0{,}33 + 60\pi T' = 12\pi \quad \therefore T' = 0{,}178 s$

$\Delta T = 0{,}178 - 0{,}17 = \underline{0{,}008 s}$

42) (P2 - 1º sem 2010 noturno)

$m = 0{,}030 \, kg$
$L = 1{,}5 \, m$
$F = 200N$

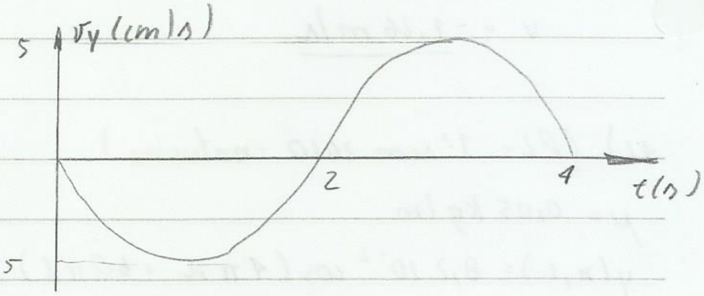
a) $v = \sqrt{\frac{F}{\mu}}$; $\mu = \frac{m}{L} = \frac{0{,}03}{1{,}5} = 0{,}02$

$v = \sqrt{\frac{200}{0{,}02}} = \underline{100 m/s}$

$L = 1{,}5 \frac{\lambda}{2} \quad \therefore \lambda = \frac{2L}{1{,}5} = \frac{2 \cdot 1{,}5}{1{,}5} = \underline{2m}$

43) (P2 - 1º sem 10 diurno)

$\lambda = 0{,}2 \, m$



a) $y(x,t) = A\cos(kx - wt)$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0{,}2} = 10\pi \, rad/m$

$v^{max} = -Aw$

$\therefore A = \frac{v^{max}}{w} = \frac{0{,}05}{0{,}5\pi} = \frac{\pi}{10}$

$w = \frac{2\pi}{f} = \frac{2\pi}{4} = \frac{\pi}{2} \, rad/s$

$y(x,t) = \frac{\pi}{10} \cos\left(10\pi x - \frac{\pi}{2} t\right) \, [SI]$

b) $a(x,t) = -A\omega^2 \operatorname{sen}(kx)\operatorname{sen}(\omega t)$

$a(x,t) = -\frac{\pi}{10} \cdot \left(\frac{\pi}{2}\right)^2 \operatorname{sen}(10\pi x)\operatorname{sen}\left(\frac{\pi}{2} t\right)$

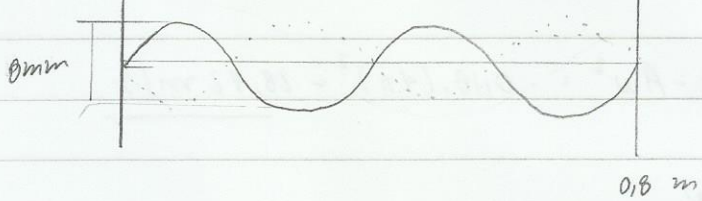
$a(0{,}02; 0{,}5) =$

44-) (P2 - 1º sem diurno)

$m = 0{,}0025$ kg

$l = 0{,}80$ m

$T = 325$ N



a) comprimento da corda - ? frequencia das ondas transv.

$L = n\frac{\lambda}{2}$, $\lambda = \frac{2L}{n} = \frac{2 \cdot 0{,}8}{4} = \underline{0{,}4\ m}$

$f = \frac{n}{2L}\sqrt{\frac{F}{\mu}} = \frac{4}{2 \cdot 0{,}8}\sqrt{\frac{325}{(0{,}0025/0{,}8)}} = \underline{806{,}23\ Hz}$

b) módulo da velocidade máxima na abcissa 0,18m

$\omega = 2\pi f \Rightarrow \omega = 2\pi \cdot 806{,}23 = 1612{,}45\pi$ rad/s

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0{,}4} = 5\pi$

$y(x,t) = 0{,}004 \operatorname{sen}(5\pi x)\operatorname{sen}(1612{,}45\pi t)$

$v(x,t) = 0{,}004 \cdot 1612{,}45\pi \operatorname{sen}(5\pi x)\cos(1612{,}45\pi t) = 1$, pois é velocidade max

$v(0{,}18) = \underline{6{,}26\ m/s}$

45-) (P3 - 1º sem 10 diurno)

$y(x,t) = 0{,}18 \cos((\pi x/0{,}6) - 4\pi t)$ [S.I]

a) velocidade no ponto $x = 0{,}051 m$ e $t = 0{,}23 s$

$v(x,t) = 0{,}18 . 4\pi \sin(\pi x/0{,}6 - 4\pi t)$

$v(0{,}051; 0{,}23) = \underline{-1{,}1207 \ m/s}$

b) aceleração máx em qualquer ponto

$a^{máx} = -A\omega^2 = -0{,}18.(4\pi)^2 = \underline{28{,}42 \ m/s}$

c) $F = 2{,}3 N$

$\lambda = \frac{2\pi}{k} \Rightarrow \lambda = \frac{2\pi}{(\pi/0{,}6)} = 1{,}2 m$ $\quad f = \frac{\omega}{2\pi} \Rightarrow f = \frac{4\pi}{2\pi} = 2 Hz$

$v = \lambda f = 1{,}2 . 2 = 2{,}4$

$\therefore v = \sqrt{\frac{F}{\mu}} \quad \therefore \mu = \frac{F}{v^2} \Rightarrow \mu = \frac{2{,}3}{(2{,}4)^2} = \underline{0{,}399 \ kg/m}$

46-) (P3 - 1º sem 2010 - noturno)

$y(x,t) = 0{,}28 \cos(0{,}7x - 16t)$ [SI]

a) aceleração transversal no $x = 0{,}31 m$ e $t = 0{,}23 s$

$a(x,t) = -0{,}28 . 16^2 \cos(0{,}7x - 16t)$

$a(0{,}31; 0{,}23) = \underline{68{,}01 \ m/s^2}$

b) $y(x,t) = -0{,}28 \cos(0{,}7x + 16t)$

c) $v(x,t) = -0{,}28 \cdot 16 \operatorname{sen}(0{,}7x + 16t)$

$v = 0{,}5318 \quad \therefore \quad v = Aw \quad \therefore \quad A = \dfrac{v}{w} = \dfrac{0{,}5318}{16} = 0{,}0332\ m$

$A_{est} = 2A \quad \therefore \quad A_{est} = 2 \cdot 0{,}0332 = \underline{0{,}0665\ m}$

47.) (P2 - 2º sem 2010 diurno)

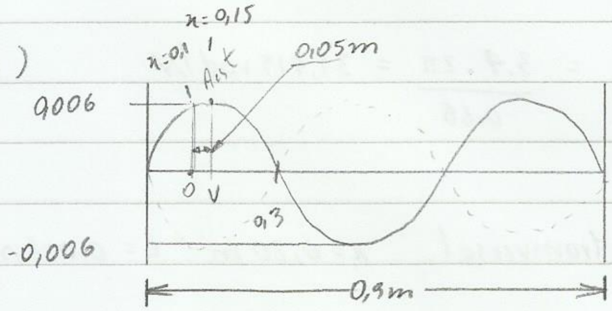
3º harmônico

$A = 0{,}006\ m$

$f = 20\ Hz$

$L = 0{,}90\ m$

$\mu = 0{,}005\ kg/m$



$\dfrac{\lambda}{2} = \dfrac{0{,}9}{3} \quad \therefore \quad \lambda = 0{,}6$

a) $v^{máx} = Aw \quad \therefore \quad v^{máx} = 0{,}006 \cdot 2\pi \cdot 20 = \underline{0{,}754\ m/s}$

b) $y(x,t) = A_{est} \operatorname{sen}(kx) \operatorname{sen}(wt)$

$y(x,t) = 0{,}00693 \operatorname{sen}\left(\dfrac{\pi}{0{,}3} x\right) \operatorname{sen}(40\pi t) \quad [SI]$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0{,}6} = \dfrac{\pi}{0{,}3} \qquad w = 2\pi f = 2\pi \cdot 20 = 40\pi$

$y_{máx} = A \operatorname{sen}(kx)$

$A = \dfrac{0{,}006}{\operatorname{sen}\left(\frac{\pi}{0{,}3} \cdot 0{,}1\right)} = 0{,}00693\ m$

c) $\sqrt{\dfrac{F}{\mu}} = \dfrac{w}{k} \quad \therefore \quad F = \left(\dfrac{w}{k}\right)^2 \cdot \mu = \left(\dfrac{40\pi}{\pi/0{,}3}\right)^2 \cdot 0{,}005 = \underline{0{,}72\ N}$

48) (P3 - 2º sem 10 - diurno)

$A = 0{,}085 m$ — a) equação da onda

$\lambda = 0{,}68 m$

$v = 3{,}4 m/s$ $\xrightarrow{+}$ — $y(x,t) = 0{,}085 \cos(9{,}24x - 31{,}415t)$ [S.I]

$\bar{p} = 0{,}12 W$ — ou $y(x,t) = 0{,}085 \sin(\pi x/0{,}34 - 10\pi t)$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0{,}68} = 9{,}24 rad/m$ $v = \lambda f \therefore f = \frac{v}{\lambda}$ $\frac{w}{2\pi} = \frac{v}{\lambda}$

$\therefore w = \frac{v \cdot 2\pi}{\lambda} = \frac{3{,}4 \cdot 2\pi}{0{,}68} = 31{,}415 rad/s$

b) velocidade transversal $x = 0{,}20 m$ $t = 0{,}053 s$

$v(x,t) = 0{,}085 \cdot 10\pi \sin(\pi x/0{,}34 - 10\pi t)$

$v(0{,}2; 0{,}053) = \underline{0{,}456 m/s}$

c) $\bar{p} = \frac{\mu . v (wA)^2}{2} \therefore \mu = \frac{2\bar{p}}{v(wA)^2} = \frac{2 . 0{,}12}{3{,}4(10\pi . 0{,}085)^2} = 9{,}9 . 10^{-3} kg/m$

49-) (P2 - 2º sem 10 noturno)

$y(x,t) = 4{,}2 . 10^{-3} \cos(3{,}2x - 20\pi t)$ [S.I]

$m = 8{,}5 . 10^{-3} kg$

$l = 1{,}2 m$

a) $v = \frac{w}{k} = \frac{20\pi}{3{,}2} = \underline{6{,}25\pi m/s}$

b) $\mu = \frac{m}{l} = \frac{8{,}5 . 10^{-3}}{1{,}2} = 0{,}007$ $F = v^2 \mu = (6{,}25)^2 \, 0{,}007 = \underline{2{,}73 N}$

c) $\bar{p} = \dfrac{\mu . v (A\omega)^2}{2} = \dfrac{0{,}007 . 6{,}25\pi (4{,}2.10^{-3} . 20\pi)^2}{2}$

$\therefore \bar{p} = 9{,}89 . 10^{-3}\ W$

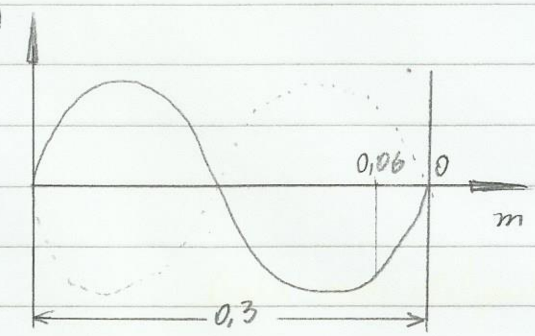
50-) (P3 - 2º sem10 - noturno)

2º harmônico

$L = 0{,}3\ m$

$m = 4.10^{-3}\ kg$

$F = 70N$



a) calcular frequencia, comprimento de onda

$\dfrac{\lambda}{2} = \dfrac{0{,}3}{2} \quad \therefore \lambda = 0{,}3 m$

$f = \dfrac{n}{2l}\sqrt{\dfrac{F}{\mu}} = \dfrac{2}{2.0{,}3}\sqrt{\dfrac{70}{(4.10^{-3}/0{,}3)}} = 241{,}5\ Hz$

b) $A_{est} = 8.10^{-3}\ m$

$y(x,t) = 8.10^{-3} \operatorname{sen}\left(\dfrac{2\pi}{0{,}3} x\right) \operatorname{sen}(483{,}09\pi t)$ [SI]

$k = \dfrac{2\pi}{0{,}3} \qquad \omega = 2\pi f = 2\pi . 241{,}5 = 483\pi$

c) $y^{max} = A \operatorname{sen}(kx)$

$y^{max} = 8.10^{-3} \operatorname{sen}\left(\dfrac{2\pi}{0{,}3} . 0{,}06\right)$

$\therefore y^{max} = 7{,}6 . 10^{-3}\ m$

Equações de propagação de ondas sonoras

Eq. do deslocamento

$$y(x,t) = A.\cos(kx - wt) \quad ; \quad A = y_{máx}$$

Eq. da pressão

$$p = p_{máx} . \sin(kx - wt)$$

$$P_{máx} = \underset{\text{módulo de compressão}}{B} \; \underset{y_{máx}}{A} \; \underset{\frac{2\pi}{\lambda}}{K} \quad ; \quad B = \frac{-\Delta P}{\frac{\Delta V}{V}}$$

$$v = \sqrt{\frac{B}{\rho}} \quad \rho = \frac{m}{V} \qquad I = \frac{P^2_{máx}}{2\rho v}$$

Dados mais utilizados

$v = 344 \, m/s \quad B = 1,42 \cdot 10^5 Pa$

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

$$I = \frac{\bar{P}}{4\pi r^2}$$

/ /

Ondas Sonoras

$I = \frac{P}{4\pi r^2}$ $\beta = 10 \log \frac{I}{I_0}$

1) (P2-2º sem 2005)

$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$

$I = 0{,}003 \, W/m^2$ $I = \frac{P}{4\pi r^2}$ $\therefore$ $P = I . 4\pi r^2$

$r = 16m$ $P = 0{,}003 . 4\pi . 16^2 = \underline{9{,}65 \, W}$

$\beta = 10 \log \frac{I}{I_0} \Rightarrow 80 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 1.10^{-4} \, w/m^2$

$I = \frac{P}{4\pi r^2} \Rightarrow 1.10^{-4} = \frac{9{,}65}{4\pi r^2} \therefore r = \underline{87{,}63m}$

2) (P3 - 1º sem 09 - diurno)

d, I, β $\beta_1 - \beta_2 = 12$

$$\begin{cases} \beta_1 = 10 \log\left(\frac{P}{4\pi r_0^2} : I_0\right) \\ \beta_2 = 10 \log\left(\frac{P}{4\pi r_1^2} : I_0\right) \end{cases}$$

$\beta_t = \beta_1 - \beta_2 = 10 \log\left(\frac{\cancel{P}}{4\pi r_0^2 \cancel{I_0}} : \frac{\cancel{P}}{4\pi r_1^2 \cancel{I_0}}\right)$

$12 = 10 \log\left(\frac{r_1^2}{r_0^2}\right) \therefore \left(\frac{r_1}{r_0}\right) = 3{,}88$

$R = 3{,}88 . d$

3) (P2 - 1º sem 01)

I) I, β, D II) $I', \beta', 0{,}3D$

$I = \frac{P}{4\pi r^2}$ $I' = \frac{P}{4\pi r_1^2}$ $\frac{I}{I'} = \left(\frac{d}{1{,}3d}\right)^2 = \left(\frac{1}{1{,}3}\right)^2 = 0{,}59$

$\beta_1 - \beta_2 = 10 \log\left(\frac{I_0}{I_1}\right.$

?

4-) (P3 - 2º xm02 , P3 1º xm05)

$f = 150\ Hz$

$I = 10^{-3}\ W/m^2$

$d = 40\ m$

a) $\beta = 10 \log\left(\frac{10^{-3}}{10^{-12}}\right) = \underline{90\ dB}$

b) $I = \frac{P}{4\pi r^2}$ $\therefore$ $10^{-3} = \frac{P}{4\pi . 40^2}$ $\therefore$ $P = \underline{20,1\ W}$

c) $110 = 10 \log\left(\frac{I}{10^{-12}}\right)$ $\therefore$ $I = 0,1$

$I = \frac{P}{4\pi r^2}$ $\Leftrightarrow$ $0,1 = \frac{20,1}{4\pi r^2}$ $\therefore$ $r = \underline{4\ m}$

5) (P3 - 1º xm04)

$d = 6m$

$I = 0,3 W/m^2$

a) $I = 0,012\ W/m^2$

$P = 0,3 . 4\pi . 6^2 = 135,7168$

$0,012 = \frac{135,7168}{4\pi r^2}$ $\therefore$ $r = 30\ m$

b) $\beta_2 = \beta_1 + 29$ $\therefore$ $\beta_2 - \beta = 29$

$\beta_2 = 10 \log\left(\frac{I_2}{I_0}\right)$ $\quad$ $\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_0} : \frac{I_1}{I_0}\right)$

$\beta_1 = 10 \log\left(\frac{I_1}{I_0}\right)$ $\quad$ $29 = 10 \log\left(\frac{I_2}{I_1}\right)$ $\therefore$ $\frac{I_2}{I_1} = \underline{794,33}$

6) (P3 - 2º sem 09)

$d = 100 m$

$\beta_1 = 150 dB$

$\beta_2 = 120 dB$

$$\beta_1 - \beta_2 = 10 \log \left(\frac{I_1}{I_0} : \frac{I_2}{I_0} \right)$$

$$3 = \log \left(\frac{I_1}{I_2} \right) \therefore \frac{I_1}{I_2} = 1000 = \left(\frac{r_2}{r_1} \right)^2$$

$$\left(\frac{r_2}{100} \right)^2 = 1000 \therefore r_2 = 3162,28 m$$

7) (P2 - 2º sem 08 - noturno)

$d = 1500 m$

$\beta = 100 dB$

$$100 = 10 \log \left(\frac{I}{10^{-12}} \right) \therefore I = 0,01$$

$$0,01 = \frac{P}{4\pi (1500)^2} \therefore P = 282,7 . 10^{-3} W$$

8) (P3 - 2º sem 06 - diurno)

$f = 200 Hz$

$d = 30 m$

$W = 5,4 10^{-3} W/m^2$

$$I = \frac{5,4.10^{-3}}{4\pi . (30)^2} = 4,77.10^{-7}$$

$$\beta = 10 \log \left(\frac{4,77.10^{-7}}{10^{-2}} \right) = 56,8 dB$$

9) (P3-2º sem 06-noturno)

$P = 1{,}5W$ a) $d = 2m$

$f = 120Hz$

$$I = \frac{1{,}5}{4\pi . 2^2} = 0{,}0298 \qquad \beta_1 = 10.\log\left(\frac{0{,}0298}{10^{-12}}\right)$$

$$\therefore \beta_1 = \underline{104{,}75\ dB}$$

$\beta_1 - \beta_2 = 10$

$104{,}75 - \beta_2 = 10 \quad \therefore \beta_2 = 94{,}75$

$$94{,}75 = 10 \log\left(\frac{I}{10^{-12}}\right) \quad \therefore \quad I = 2{,}98.10^{-3}$$

$$2{,}98.10^{-3} = \frac{1{,}5}{4\pi r^2} \quad \therefore r = \underline{6{,}32m}$$

10.) (P2-1º sem 07-diurno)

$d = 12m$

$\beta = 100dB$

$\beta_f = 0{,}8.100 = 80\ dB$

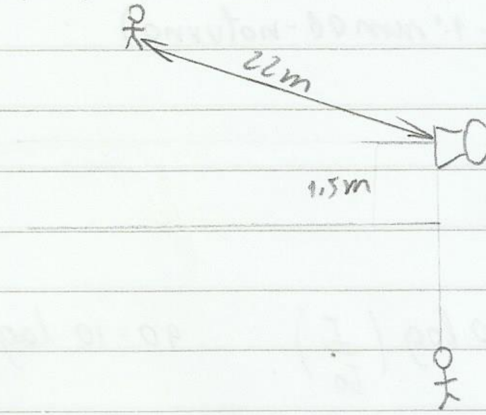
$$100 = 10 \log\left(\frac{I}{10^{-12}}\right) \quad \therefore \quad I = 0{,}01$$

$$0{,}01 = \frac{P}{4\pi . 12^2} \quad \therefore \quad P = 18{,}096$$

$$80 = 10 \log\left(\frac{I}{10^{-12}}\right) \quad \therefore \quad I = 1.10^{-4}$$

$$1.10^{-4} = \frac{18{,}096}{4\pi r^2} \quad \therefore \quad r = \underline{120\ m}$$

11-) (P2- 2º sem 2007/Diurno)



$T = 20°C = 293\,K$

$\gamma = 1{,}4$

$M = 28{,}8\,g/mol \qquad v = 1482\,m/s$

12-) (P3 - 2º sem 07/Diurno)

$f = 160\,Hz \qquad \beta = 10 \log\left(\frac{I}{I_0}\right)$

$\beta = 85\,dB$

$d = 20\,m \qquad 85 = 10 \log\left(\frac{I}{1.10^{-12}}\right) \quad \therefore \quad I = 3{,}16.10^{-4}$

a) $I = \frac{P}{4\pi r^2} \qquad 3{,}16.10^{-4} = \frac{P}{4\pi.20^2} \quad \therefore \quad \underline{P = 1{,}59\,W}$

b)

c) $\frac{I_1}{I_2} = \frac{d_2^2}{d_1^2} \Rightarrow \frac{1,5849 . 10^{-9}}{1 . 10^{-7}} = \frac{d^2}{8^2}$ $\therefore d = \underline{1,007 m}$

$50 = 10 \log \left(\frac{I}{1 . 10^{-12}} \right)$ $\therefore I = 1 . 10^{-7}$

15-) (P2 - 2º sem 09 - diurno)

$A = 0,020 mm = 0,02 . 10^{-3} m$ $\rho = 1,29 kg/m^3$

$f = 300 Hz$ $v_{ar} = 340 m/s$

$\bar{P} = 0,245 W$

$v = \sqrt{\frac{B}{\rho}}$ $\therefore B = v^2 . \rho$

$P_{máx} = BAK$

$= v^{\cancel{2}} \rho . A . \frac{2\pi f}{\cancel{v}}$ $K = \frac{2\pi}{\lambda}$ $\therefore v = \lambda F \therefore \lambda = \frac{v}{F}$

$= v \rho . A . 2\pi f$ $K = 2\pi : \frac{v}{F} \therefore \frac{2\pi f}{v}$

$= 340 . 1,29 . 0,02 . 10^{-3} . 2\pi . 300$

$\therefore p_{máx} = \underline{16,53 Pa}$

b) $I = \frac{0,245}{4\pi . 5^2} = 7,8 . 10^{-4}$

$\beta = 10 \log \left(\frac{7,8 . 10^{-4}}{1 . 10^{-12}} \right)$ $\therefore \beta = \underline{88,92 dB}$

16-) (P2 - 2º sem 10 - diurno) $P_{max} = \sqrt{2 \rho v I}$

$f = 524 Hz$ $\rho = 1,2 kg/m^3$ $= \sqrt{2 . 1,2 . 344 . 1 . 10^{-7}}$

$\beta = 50 dB$ $50 = 10 \log \left(\frac{I}{10^{-12}} \right)$ $\therefore P_{max} = \underline{9,09 . 10^{-3} Pa}$

$d = 8 m$

$v = 344 m/s$ $\therefore I = 1 . 10^{-7}$

/ /

$$P_{máx} = BAK \qquad v = \sqrt{\frac{B}{\rho}} \therefore B = v^2 \rho$$

$$A = \frac{P_{máx}}{BK} \qquad K = \frac{2\pi}{\lambda} \; ; \; v = \lambda f \therefore \lambda = \frac{v}{f}$$

$$A = \frac{P_{máx}}{v^2 \rho \cdot \frac{2\pi f}{v}} \qquad K = 2\pi \div \frac{v}{f} = \frac{2\pi f}{v}$$

$$\therefore A = \frac{9{,}09 \cdot 10^{-3}}{344 \cdot 1{,}2 \cdot 2\pi \cdot 524} \quad \therefore A = 6{,}686 \cdot 10^{-9} \text{ m}$$

b)

$$60 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 1 \cdot 10^{-6} \text{ W/m}^2$$

$$I = \frac{P}{4\pi r^2} \qquad 10^{-7} = \frac{P}{4\pi 8^2} \therefore P = 8{,}04 \cdot 10^{-5}$$

$$\therefore r^2 = \frac{8{,}04 \cdot 10^{-5}}{4\pi \cdot 1 \cdot 10^{-6}} \quad \therefore r = \underline{2{,}53 \text{ m}}$$

17-) (P2 - 2º sem. 10 - noturno)

$$\beta_1 = 68 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I_1 = 6{,}3095 \cdot 10^{-6}$$

$$\beta_2 = 54 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I_2 = 2{,}5119 \cdot 10^{-7}$$

$$\beta_3 = 50 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I_3 = 1 \cdot 10^{-7}$$

$$\frac{6{,}3095 \cdot 10^{-6}}{1 \cdot 10^{-6}} = \frac{r_1^2}{r_3^2} \quad \therefore r_3 = r_1 \sqrt{63{,}096}$$

$$\underline{r_3 = 7{,}94 \cdot d}$$

Ondas Sonoras

40-) (P2 2°08)

$r = 1{,}5.10^3\ m$

$I = 100\,dB$

$\bar{P} = ?$

$100 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 0{,}01$

$0{,}01 = \frac{\bar{P}}{4\pi.(1{,}5.10^3)^2} = \underline{282{,}7.10^3\ W}$

41-) (P2 1°08)

$\bar{P} = 20W$

$d = ?$

$I = 90\,dB$

$90 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 0{,}001\ W/m^2$

$0{,}001 = \frac{20}{4\pi r^2} \therefore r = \underline{39{,}89\ m}$

42) (P3 - 2°07)

$f = 180\,Hz$

$I = 85\,dB$

$d = 20m$

$T = 25°C$

a) $k = ?$ $\quad K = \frac{2\pi}{\lambda}$, $\lambda = \frac{v}{f}$

$\therefore K = 2\pi \div \frac{v}{f} = \frac{2\pi f}{v} = \frac{2\pi . 180}{344}$

$\therefore k = 3{,}29\ rad/m$

b) $85 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 3{,}162\,10^{-4}\ W/m^2$

$3{,}162.10^{-4} = \frac{\bar{P}}{4\pi.20^2} \therefore \bar{P} = \underline{1{,}59\ W}$

c) $\begin{cases} \beta_2 = 10 \log(I_2/I_0) \\ \beta_1 = 10 \log(I_1/I_0) \end{cases}$ $\quad \beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$

$-17 = 10 \log\left(\frac{r_1}{r_2}\right)^2$; $r_1 = 20$

$\therefore r_2 = \underline{141{,}59\ m}$

43(P3 1º07)

$d_1 = 18m \rightarrow \beta_1 = 100dB$

$d_2 = ? \rightarrow \beta_2 =$

$\beta_1 = 10 \log \left(\frac{I_1}{I_0} \right)$ $\beta_1 - \beta_2 = 10 \log \left(\frac{I_1}{I_0} \cdot \frac{I_2}{I_0} \right)$

$\beta_2 = 10 \log \left(\frac{I_2}{I_0} \right)$ $\beta_1 - \beta_2 = 10 \log \left(\frac{I_1}{I_2} \right)$

$\beta_1 - \beta_2 = 10 \log \left(\frac{r_2}{r_1} \right)^2$

$30 = 10 \log \left(\frac{r_2}{18} \right)^2 \therefore r_2 = \underline{592,21\ m}$

44) (P2 2º05)

$I = 0,0028\ W/m^2$ $0,0028 = \frac{\bar{P}}{4\pi 14^2} \therefore \bar{P} = \underline{6,9\ W}$

$d = 14m$

$70 = 10 \log \left(\frac{I}{10^{-12}} \right) \therefore I = 1.10^{-5}$

$1.10^{-5} = \frac{6,9}{4\pi\ d^2} \therefore d = \underline{234,3\ m}$

45.) (P2-2º05) $18 = 10 \log \left(\frac{I_1}{10^{-12}} \right) \therefore I_1 = 6,31.10^{-11}$

$\beta_1 = 18dB$

$d_1 = 60m$ $6,31.10^{-11} = \frac{\bar{P}}{4\pi.60^2} \therefore \bar{P} = 2,85.10^{-6}$

$d_2 = 120m$

$I_2 = \frac{2,65 \cdot 10^{-6}}{4\pi \cdot 120^2} = 1,577 \cdot 10^{-11}\ W/m^2$

$\beta_2 = 10 \log\left(\frac{1,577 \cdot 10^{-11}}{1 \cdot 10^{-12}}\right) \therefore \underline{\beta_2 = 11,98\ dB}$

(P2 - 1º sem 2008)

$P = 40W$

$\beta_1 = 90\ dB$

$90 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 1 \cdot 10^{-3}$

$1 \cdot 10^{-3} = \frac{40}{4\pi \cdot r^2} \therefore \underline{r = 56,42\ m}$

(P2 - 2º sem 2008)

$d = 8m \quad \beta_1 = 32\ dB \qquad \gamma = 1,4$

$f = 250\ Hz \quad e \quad T = 25°C \quad p = 1 \cdot 10^5\ Pa \qquad M = 28,8\ \frac{g}{mol} = 28,8 \cdot 10^{-3}\ \frac{kg}{mol}$

a) velocidade de propagação nestas condições

$v = \sqrt{\frac{\gamma R T}{M}} = \sqrt{\frac{1,4 \cdot 8,315 \cdot 298}{28,8 \cdot 10^{-3}}} = \underline{347,06\ m/s}$

b) $p(x) = ?$; $x = 8m$

$v = \sqrt{\frac{B}{\rho}} \therefore \boxed{B = 347,06^2 \cdot 1,42 \cdot 10^5 \quad \therefore B = 1,68 \cdot 10^{10}}$

$32 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 1,5849 \cdot 10^{-9}$

$I = \frac{p^2_{max}}{2 \cdot \rho v}$

Duvida

(/ /)

(P2 - 2º sem 08)

$d = 1{,}2 . 10^3\ m$ $\quad 90 = 10 \log \left(\frac{I}{10^{-12}} \right)$ $\therefore I = 1 . 10^{-3}$

$\beta_1 = 90\ dB$

$1 . 10^{-3} = \frac{\bar{P}}{4\pi . (1{,}2 . 10^3)^2}$ $\quad \therefore \underline{\bar{P} = 18095{,}57\ W}$

(P2 2º 2009)

$A = 0{,}03 . 10^{-3}\ m$ $\quad \rho ar = 1{,}29\ kg/m^3$

$f = 300\ Hz$ $\quad$ Vsom no ar $= 340\ m/s$

$\bar{P} = 0{,}320\ W$

a) $p(x) = p_{max} \sin (kx - wt)$

$I = \frac{p^2 max}{2\rho . v}$ $\quad p_{máx} = BAK$ $\quad v = \sqrt{\frac{B}{\rho}}$

$v = \lambda f$ $\quad K = \frac{2\pi}{\lambda}$ $\quad \therefore K = 2\pi : \frac{v}{f} = \frac{2\pi f}{v}$ $\quad$ ou $\quad v = \frac{w}{k}$ $\therefore k = \frac{2\pi f}{v}$

$\lambda = \frac{v}{f}$

$B = v^2 \rho$

$\therefore p_{máx} = v^{\cancel{2}} \rho . A . \frac{2\pi f}{\cancel{v}} = 340 . 1{,}29 . 0{,}03 . 10^{-3} . 2\pi . 300$

$\therefore p_{máx} = 24{,}80\ pa$

b) Intensidade e o nível de Intensidade, com $d = 7m$

$I = \frac{0{,}320}{4\pi . 7^2} = \underline{5{,}1969 . 10^{-4}\ w/m^2}$ $\quad \beta_1 = 10 \log \left(\frac{5{,}197 . 10^{-4}}{10^{-12}} \right)$

$\therefore \beta_1 = 87{,}16\ dB$

(P2 - 2º sem 2010)

$f = 524\ Hz$

$\beta_1 = 50\ dB \rightarrow d = 8m$

$v = 344 m/s \quad \rho = 1,2\ kg/m^3$

a) amplitude de pressão e amplitude do deslocamento a 8m

$$I = \frac{p^2_{máx}}{2\rho . v} \quad \therefore p = \sqrt{2 I \rho v} \qquad p_{máx} = BAK$$

$$50 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 1.10^{-7}\ W/m^2$$

$$\therefore p = \sqrt{2.10^{-7}.1,2.344} = \underline{9,09.10^{-3}}$$

$$A = \frac{p_{máx}}{BK} \qquad B = v^2\rho \quad K = \frac{2\pi f}{v}$$

$$A = \frac{p_{máx}}{v\rho 2\pi f} = \frac{9,09.10^{-3}}{344.1,2.2\pi.524} = 6,686.10^{-9}\ m$$

b) $\beta_1 = 50\ dB \quad \beta_2 = 60\ dB$

$$\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right) = 10 \log\left(\frac{r_1}{r_2}\right)^2$$

$$10 = 10 \log\left(\frac{r_1}{r_2}\right)^2 \quad \therefore \frac{r_1}{r_2} = \sqrt{10} \quad \therefore r_2 = \frac{8}{\sqrt{10}} = \underline{2,53\ m}$$

(P2 - 2º 2010)

a)

$$\begin{cases} \beta_2 = 10 \log \left(\frac{I_2}{I_0} \right) \\ \beta_1 = 10 \log \left(\frac{I_1}{I_0} \right) \end{cases}$$

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right)$$

$$18 = 10 \log \left(\frac{r}{d} \right)^2 \quad \therefore \frac{r}{d} = 7{,}94$$

$$\therefore \underline{r = 7{,}94\, d}$$

b) trompete ⇒ 68 dB (β_1)

violino ⇒ 54 dB (β_2)

$$\beta_1 - \beta_2 = 10 \log \frac{I_T}{I_V} = 10 \log \left(\frac{P_T}{4\pi d^2} : \frac{P_V}{4\pi d^2} \right)$$

$$14 = 10 \log \left(\frac{P_T}{P_V} \right) \quad \therefore \frac{P_T}{P_V} = 25{,}12$$

$$\therefore \underline{P_T = 25{,}12\, P_V}$$

(P3 - 1º sem 2009)

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_0} : \frac{I_1}{I_0} \right)$$

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right) \Rightarrow \beta_2 - \beta_1 = 10 \log \left(\frac{r_1}{r_2} \right)^2$$

$$8 = 10 \log \left(\frac{r_1}{r_2} \right)^2 \quad \therefore \frac{r_1}{r_2} = 2{,}5119$$

$$\underline{r_1 = 2{,}5119\, r_2}$$

(P3 - 2º sem 2006)

$f = 200 Hz$

$d = 30 m$

$I = 5{,}40 \cdot 10^{-3}\ W/m^2$

a)

$\beta_1 = 10 \log \left(\frac{5{,}40 \cdot 10^{-3}}{10^{-12}} \right) \therefore \underline{\beta_1 = 97{,}32\ dB}$

b) $\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_0} : \frac{I_1}{I_0} \right)$

$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right)$

$60 - 97{,}32 = 10 \log \left(\frac{30}{r_2} \right)^2 \quad \therefore r_2 = \underline{2204{,}54\ m}$

c) $5{,}40 \cdot 10^{-3} = \frac{\bar{P}}{4\pi \cdot 30^2} \therefore \bar{P} = \underline{61{,}07\ W}$

(P3 - 2º sem 2006)

$\bar{P} = 1{,}5\ W$

$f = 120\ Hz$ a) $I = \frac{1{,}5}{4\pi \cdot 2^2} = \underline{0{,}0298\ W/m^2}$

ou

$\beta = 10 \log \left(\frac{0{,}0298}{10^{-12}} \right) \therefore \underline{\beta = 104{,}75\ dB}$

b) $\beta_2 = \beta_1 - 10 \quad \therefore \beta_2 = 94{,}75 \qquad 94{,}75 = 10 \log \left(\frac{I}{10^{-12}} \right) \therefore I = 2{,}98 \cdot 10^{-3}$

$2{,}98 \cdot 10^{-3} = \frac{1{,}5}{4\pi \cdot d^2} \quad \therefore d = \underline{6{,}32\ m}$

(P3 - 2º sem 2007)

$f = 180\,Hz$

$\beta = 85\,dB$

a) $K = \dfrac{2\pi f}{v} = \dfrac{2\pi . 180}{346} = \underline{3,27\ rad/m}$

b) $85 = 10 \log\left(\dfrac{I}{10^{-12}}\right) \therefore I = 3,162.10^{-4}$

$3,162.10^{-4} = \dfrac{P}{4\pi . 20^2} \therefore \underline{P = 1,59\,W}$

c) $\beta_2 - \beta_1 = 10 \log\left(\dfrac{I_2}{I_1}\right)$

$\beta_2 - \beta_1 = 10 \log\left(\dfrac{r_1}{r_2}\right)^2$

$17 = 10 \log\left(\dfrac{r}{20}\right)^2 \therefore r = \underline{141,59\,m}$

(P3 - 1º sem 2003)

$f = 587\,Hz$

$\beta = 76,6\,dB$

$d = 5m$

a) $k = \dfrac{2\pi f}{v} = \dfrac{2\pi . 587}{344} = 10,72\ rad/m$

$76,6 = 10 \log\left(\dfrac{I}{10^{-12}}\right) \therefore I = 4,57.10^{-5}$

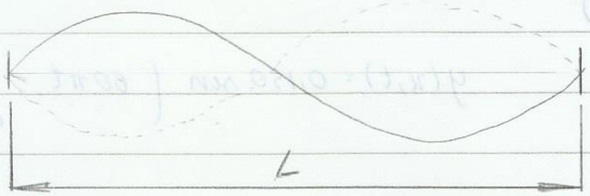
$4,57\,10^{-5} = \dfrac{\bar{P}}{4\pi . 5^2} \therefore \bar{P} = 0,014\,W$

b) $I = \dfrac{p^2_{max}}{2\rho . v} \therefore p_{max} = \sqrt{2 I \rho v}$

$= \sqrt{2 . 4,57.10^{-5} . 1,2 . 344} = \underline{0,194\,Pa}$

Oscilações Amortecidas e Ondas Mecânicas

$L = n \frac{\lambda_n}{2}$



Velocidade de propoção da onda

$$\boxed{v = \sqrt{\frac{F}{\mu}} = \lambda f = \frac{w}{k}}$$

Densidade linear $\boxed{\mu = \frac{m}{L} \ \left(\frac{kg}{m}\right)}$ Número de onda $\boxed{K = \frac{2\pi}{\lambda} \ \frac{rad}{m}}$

Potência Média $\boxed{\bar{P} = \frac{\mu v w^2 A^2}{2}}$

Equação da onda: $y(x,t) = A\cos(kx \pm wt)$

Equação da Velocidade transversal $v(x,t) = -Aw\sin(kx + wt)$

Equação da Aceleração transversal $a(x,t) = -Aw^2\cos(kx + wt)$

Obs.: Velocidade máxima $v_{máx} = Aw$

Aceleração máxima $A_{máx} = Aw^2$

Frequência: $\boxed{f = \frac{n}{2L}\sqrt{\frac{F}{\mu}}}$

Exercícios de Fixação

1) (P2 2°08)

$$y(x,t) = 0{,}150 \operatorname{sen}\left(80\pi t - \frac{\pi}{0{,}8} x\right) \text{ [SI]}$$

$L = 5{,}00\,m$

$m = 0{,}2\,kg$

a) $\lambda = ?$ $v = ?$

$v = \frac{w}{K} = \frac{\pi}{0{,}8} : 80\pi = \underline{64\,m/s}$ $\qquad k = \frac{2\pi}{\lambda} \therefore \lambda = \frac{2\pi}{(\pi/0{,}8)} = \underline{1{,}6\,m}$

b) $\bar{P} = \frac{\mu . v (wA)^2}{2} = \frac{0{,}04 . 64 . (0{,}15 . 80\pi)^2}{2} = \underline{1819\,W}$

$\mu = \frac{0{,}2}{5} = 0{,}04$

c) $80\pi t - \frac{\pi}{0{,}8} x = n\pi$; $t = 0{,}1$ e $x = 0{,}5$

$7{,}375 = n \therefore n' = 8$

$80\pi . t' - \frac{\pi}{0{,}8} . 0{,}5 = 8\pi$

$\therefore t' = 0{,}108$

$\Delta t = t' - t$

$= 0{,}108 - 0{,}1 \therefore \Delta t = \underline{0{,}08\,s}$

d) $x = 0,1\ m$

$v_{max} = A\omega$

$= 0,150 \cdot 80\pi = \underline{37,70\ m/s}$

2 (P3 1°07)

a) $\lambda = 8m$ $\qquad$ $f = 20\ Hz$

$k = \frac{2\pi}{8} = \frac{\pi}{4}\ rad/m$ $\qquad$ $T = \frac{1}{f} = \frac{1}{20} = \underline{0,05s}$

$\omega = 2\pi f = \underline{40\pi\ rad/s}$

b) $y(x,t) = 0,3 \sin\left(\frac{\pi}{4}x + 40\pi t\right)$ (SI)

c) Os pontos que possui velocidade transversal positiva é 0 e 8m

$v(x,t) = 12\pi \cos\left(\frac{\pi}{4}x + 40\pi t\right)$

$v(0,0) = 12\pi\ m/s$ $\qquad$ $v(8,0) = 12\pi\ m/s$

d) $F = 2048\ N$ $\quad$ $\mu = ?$

$v = \sqrt{\frac{F}{\mu}}$ $\therefore$ $\mu = \frac{F}{v^2} = \frac{2048}{\left(40\pi \div \frac{\pi}{4}\right)^2}$

$\therefore \mu = 0,08\ kg/m$

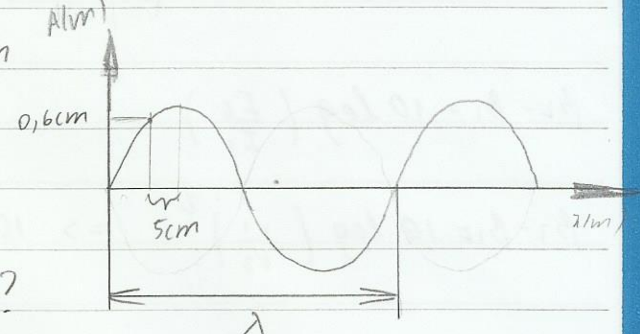
P2 - 2º 2010

1) $x = 0{,}05\,m$ (3º harmônico)

$A = 6.10^{-3}\,m$ $\quad \mu = 0{,}005\ kg/m$

$f = 20\ Hz$

$L = 0{,}9\,m$



a) Velocidade transversal máxima?

$|v^{máx}| = 6.10^{-3}.\,2\pi.\,20$ $\qquad K = \frac{2\pi}{\lambda} = \frac{2\pi}{(\frac{2}{3}.0{,}9)} = \frac{\pi}{0{,}3}\ rad/m$

$\therefore v_{máx} = 0{,}75\ m/s$

$\frac{\lambda}{4} = 0{,}15$

$\therefore A = 6{,}93.10^{-3}\,m$

$V = 6{,}93.10^{-3}.\,2\pi.\,20 = 5{,}25\,\pi\ m/s$ ou $16{,}50\ m/s$

$Y(x,t) = 6{,}93.10^{-3} \operatorname{sen}\left(\frac{\pi x}{0{,}3}\right) \operatorname{sen}(40\pi t)$ (SI)

c) $\frac{w}{K} = \sqrt{\frac{F}{\mu}} \quad \therefore \quad F = \left(\frac{w}{K}\right)^2 . \mu$

$F = \left(\frac{40\pi}{(\pi/0{,}3)}\right)^2 . 0{,}005 \quad \therefore \quad F = 0{,}72\,N$

2-) $f = 524\ Hz$ $\qquad$ a) $I = \frac{p^2_{máx}}{2\rho v}$ $\qquad 50 = 10 \log\left(\frac{I}{10^{-12}}\right) \therefore I = 1.10^{-7}$

$\beta_1 = 50\,dB$

$d = 8\,m$ $\qquad 1.10^{-7} = \frac{p^2_{máx}}{2.\,1{,}2.\,344} \quad \therefore \quad p_{máx} = 9{,}08.10^{-3}\ Pa$

$v = 344\ m/s$

$\rho = 1{,}2\ kg/m^3$

b) $\beta_2 = \beta_1 + 10$

$\beta_2 - \beta_1 = 10$

$\beta_2 = 10 \log\left(\frac{I_2}{I_0}\right)$

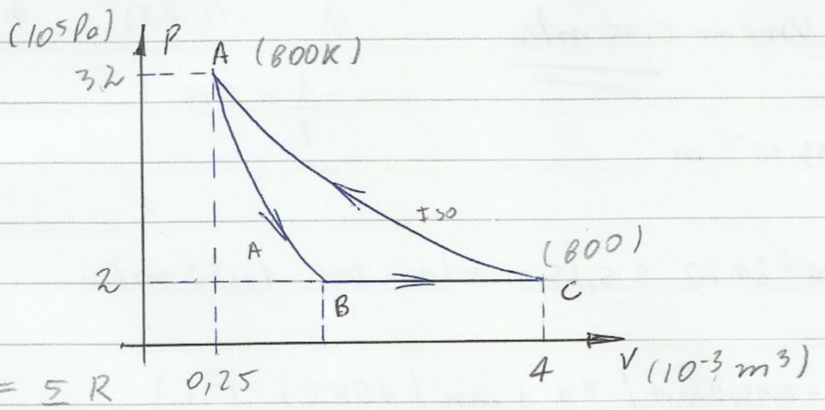
$\beta_1 = 10 \log\left(\frac{I_1}{I_0}\right)$

$\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$

$\beta_2 - \beta_1 = 10 \log\left(\frac{r_1}{r_2}\right)^2 \Rightarrow 10 = 10 \log\left(\frac{8}{r_2}\right)^2$

$\therefore r_2 = \underline{2{,}53\ m}$

3)



$W_{CA} = -2218\ J$

$C_v = \frac{3}{2} R$

$C_p - C_v = R$

$C_p = R + \frac{3}{2} R = \frac{5}{2} R$

$W_{ca} = pv \ln\left(\frac{V_f}{V_i}\right) \qquad -2218 = 2.4.100 \ln\left(\frac{V_f}{4}\right) \therefore V_f = 0{,}25$

$\therefore \underline{V_A = 0{,}25.10^{-3}\ m^3}$

$\gamma = \frac{C_p}{C_v} = \frac{5}{2} R \cdot \frac{3}{2} R$

$p_a v_a = p_c v_c$

$p_a = \frac{2.4}{0{,}25} \therefore \underline{p_a = 32.10^5\ Pa}$

$\therefore \gamma = 1{,}67 = (5/3)$

$p_a v_a^{\gamma} = p_b v_b^{\gamma} \Rightarrow 32.0{,}25^{1{,}67} = 2.\ v_b^{1{,}67}$

$\therefore \underline{v_b = 1{,}32.10^{-3}\ m^3}$

$\frac{Vb}{Tb} = \frac{Vc}{Tc}$ ∴ $\frac{1,32}{Tb} = \frac{4}{800}$ ∴ $tb = 263,03\ K$

b) $W_{AB} = ?$ $Q_{BC} = ?$

$W_{AB} = -nc_v \Delta T$

$= \frac{-3}{2} nR \frac{(1,32.2 - 32.0,15)}{nR} .100$

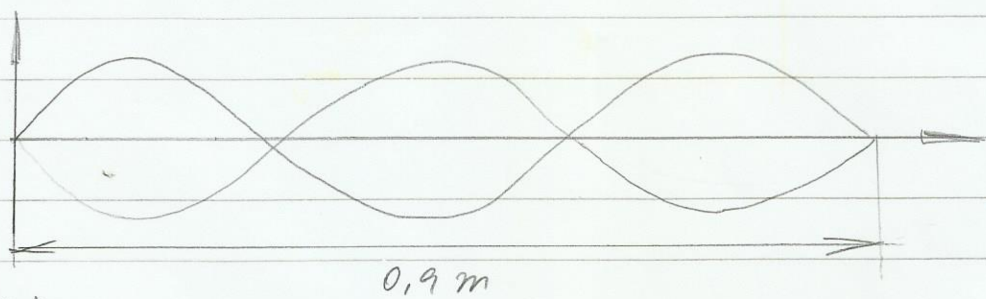
∴ $W_{AB} = \underline{804\ J}$

$Q_{BC} = nc_p \Delta T$

$= \frac{5}{2} nR \frac{(2.4 - 2.1,32)}{nR} .100$

∴ $Q_{BC} = \underline{1340\ J}$

4)



$m = 0,2\ kg$

$\emptyset = 0,5\ mm = 0,5.10^{-3}\ m$

$\rho = 3,5.10^2\ kg/m^3$

$f = \frac{n}{2L} . \sqrt{\frac{F}{\mu}}$

$f = \frac{3}{2.0,9} \sqrt{\frac{2}{6,87\ 22}}$

∴ $f = \underline{284,32\ Hz}$

$\rho = \frac{m}{V}$; $\mu = \frac{m}{L}$

$\mu = \frac{\rho . (\frac{\emptyset}{2})^2 . \cancel{L}}{\cancel{L}} = 3,5.10^2 . (\frac{0,5}{2} . 10^{-3})^2 \pi$

∴ $\mu = 6,87.10^{-0,5}$

5) $R = 3\,\Omega$

$m_a = 180\,g$

$c = 1\,cal/g.°C$

$C = 32{,}6\,cal/°C$

$I = 2{,}5\,A$

$\Delta t = ?$

$\Delta\theta = 22{,}4°C$

$J = 4{,}35\,J/cal$

$$J = \frac{W}{Q} = \frac{RI^2 \cdot \Delta T}{(m_a c + C) \cdot \Delta\theta}$$

$$\therefore \Delta T = \frac{J\,(m_a c + C)\,\Delta\theta}{R I^2}$$

$$\Delta T = \frac{4{,}35\,(180 + 32{,}6) \cdot 22{,}4}{3 \cdot 2{,}5^2}$$

$$\Delta T = 1109{,}8\,s = \underline{18{,}41\,min}$$

P2 (2º sem 2010 Noturno) Início 10:10

1) $C_v = \frac{3}{2} R$ $\quad p \cdot V^{\alpha} = k$, $\alpha = 1{,}5$

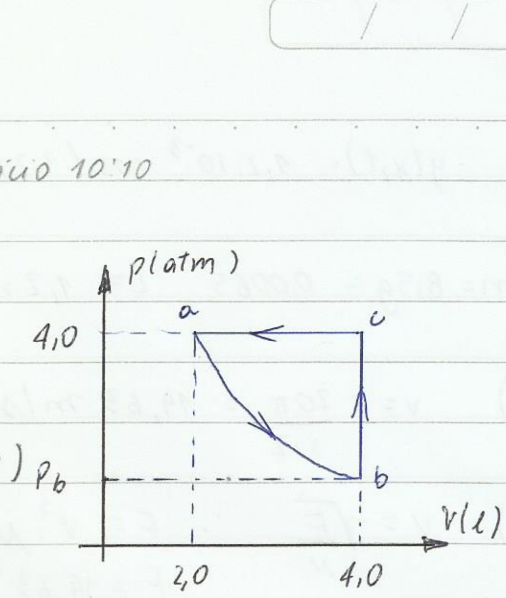
a)

$p_a v_a^{\alpha} = p_b v_b^{\alpha}$

$4 \cdot 2^{1,5} = p_b \cdot 4^{1,5}$

$\therefore p_b = 1{,}41 \text{ atm} = 1{,}41 \cdot 1{,}013 \cdot 10^5$

$\therefore p_b = 1{,}43 \cdot 10^5 \text{ Pa}$



$C_p - C_v = R$

$C_p = R + \frac{3}{2} = \frac{5}{2} R$

b) $W_{ciclo} = W_{ab} + \cancel{W_{bc}} + W_{ca}$

$= \int p\,dv + 4 \cdot (2-4) \cdot 101{,}3$

$= \int p\,dv - 810{,}4$

$\gamma = \frac{5}{3} = 1{,}67$

$\int_2^4 11{,}31\, v^{-1,5}\, dv = 11{,}31 \cdot \left(\frac{v^{-0,5}}{-0{,}5} \right) \Big|_2^4$

$k = 4 \cdot 2^{1,5} = 11{,}31$

$\therefore W_{ab} = 474{,}57 \text{ J}$

$p v^{\alpha} = k$

$p = k\, V^{-\alpha}$

$W_{ciclo} = 474{,}57 - 810{,}4$

$\therefore W_{ciclo} = -335{,}83 \text{ J}$

c) $\Delta U_{\cancel{ciclo}}^{\,0} = Q_{ciclo} - W_{ciclo}$

$Q_{ciclo} = W_{ciclo} = -335{,}83 \text{ J}$

d) O calor do ciclo é rejeitado pois o $Q < 0$

e) O ciclo representa um refrigerador, pois o trabalho é negativo ($W < 0$).

2) $y(x,t) = 4{,}2 \cdot 10^{-3} \cos(3{,}2x - 20\pi t)$ [SI]

$m = 8{,}5g = 0{,}0085 \quad L = 1{,}2m$

a) $v = \frac{20\pi}{3{,}2} = 19{,}63 \, m/s$

b) $v = \sqrt{\frac{F}{\mu}} \quad \therefore \quad F = v^2 \cdot \mu$

$F = 19{,}63^2 \cdot \frac{8{,}5 \cdot 10^{-3}}{1{,}2} \quad \therefore \quad \underline{F = 2{,}73 \, N}$

c) $\bar{p} = \frac{7{,}08 \cdot 10^{-3} \cdot 19{,}63 \cdot (20\pi \cdot 4{,}2 \cdot 10^{-3})^2}{2}$

$\therefore \bar{p} = 4{,}89 \cdot 10^{-3} \, W$

3) trompete: $\beta_1 = 68dB$
violino: $\beta_2 = 59dB$

$\beta - \beta_2 = 1B$

$\beta - \beta_2 = 10 \log\left(\frac{I}{I_2}\right)$

$1B = 10 \log\left(\frac{r_1}{d'}\right)^2 \quad \therefore \quad r_1 = 7{,}94d$

Lista 1

1-) (P3 2º 2010)

$n = 1$ mol

$\gamma = 1{,}4$

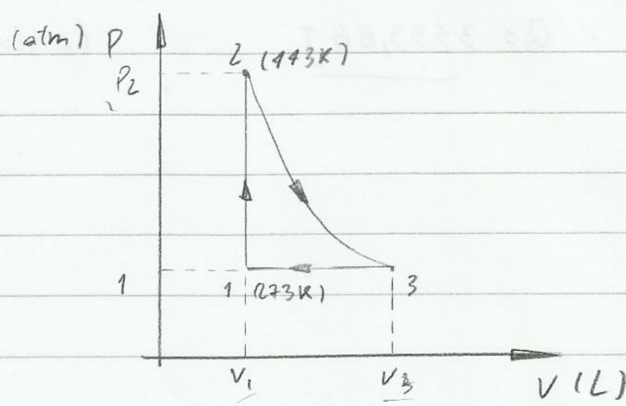
$p_1 = 1$atm $t = 0°C = 273K$ (estado 1)

Isocoricamente

$T = 170°C = 443K$

expandido adiabaticamente até o início do ciclo

a)



b) $p_1 V_1 = nRT$

$$V_1 = \frac{nRT}{p_1} = \frac{1 \cdot 0{,}082 \cdot 273}{1} = 22{,}386\ L$$

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1} \quad \therefore \quad \frac{p_2}{443} = \frac{1}{273} \quad \therefore p_2 = 1{,}6227$$

$$p_2 V_2^{\gamma} = p_3 V_3^{\gamma} \Rightarrow 1{,}6227 \cdot 22{,}386^{1,4} = 1 \cdot V_3^{1,4} \quad \therefore V_3 = \underline{31{,}63 L}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_3 V_3}{T_3} \Rightarrow \frac{22{,}386}{273} = \frac{31{,}63}{T_3} \quad \therefore T_3 = \underline{385{,}78\ K}$$

c) $Q_1 = n c_p \Delta T$ (Isobárico) $C_v = \frac{5}{2}R$ $C_p - C_v = R$

$C_p = R + C_v$

$C_p = \frac{7}{2} \cdot 8{,}315 = 29{,}1025$ $C_p = R + \frac{5}{2}R = \frac{7R}{2}$

$Q = 1 \cdot 29{,}1025\,(273 - 385{,}78)$

$\therefore Q = \underline{-3282{,}18\,J}$

Isocórico $C_v = \frac{5}{2}R$ $C_p = \frac{7}{2}R$ (Monoatomico)

$\gamma = 1{,}4 = \frac{C_p}{C_v}$

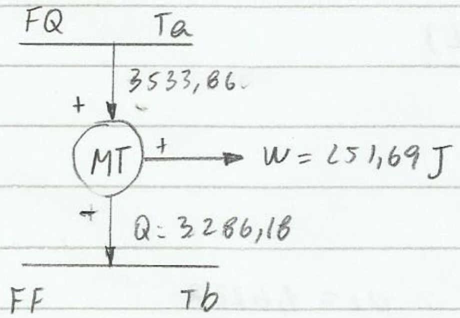
$Q = n c_v \Delta T$

$Q = 1 \cdot \frac{5}{2} \cdot 8{,}315 \cdot (443 - 273)$ $\therefore Q = \underline{3533{,}86\,J}$ $C_p - C_v = R$

Adiabático

$Q = 0\,J$

d)



$e = \frac{251{,}69}{3533{,}86} \cong 0{,}071$

$e\% \cong 7{,}12\%$

2) (P3 - 2=10)

$\gamma = 1,67$

$p_1 = 16$ atm e $V_1 = 1L$ e $T_1 = 600 K$

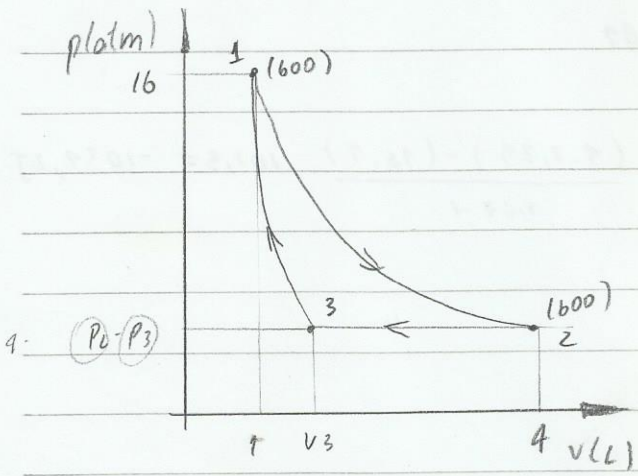
o gás se expandiu isotérmicamente

$V_2 = 4L$

o gás restriou isobaricamente

$V_3 = ?$ $t_3 = ?$

compressão adiabática



b) $\frac{p_1 V_1}{\cancel{T_1}} = \frac{p_2 V_2}{\cancel{T_2}}$ $16 . 1 = p_2 . 4$ $\therefore p_2 = 4$ atm

$p_1 V_1^{\gamma} = p_3 V_3^{\gamma}$ $16 . 1^{1,67} = 4 . V_3^{1,67}$ $\therefore V_3 = 2,29 L$

$\frac{\cancel{p_2} V_2}{T_2} = \frac{\cancel{p_3} V_3}{T_3}$ $\frac{4}{600} = \frac{2,29}{T_3}$ $\therefore t_3 = 349,6 K$

$t = \frac{pv}{nR}$

		Q(J)	W(J)	ΔU(J)
T	1-2	2246,9	2246,9	0
P	2-3	-1727,06	-692,89	
ADI	3-1	0	-1034,2	
	ciclo	519,84	519,84	0

Isotérmica $Q = W$

$\Delta U = Q - W$

$$Q_{1-2} = W_{1-2} = pv \ln\left(\frac{v_f}{v_i}\right) = nRT \ln\left(\frac{v_f}{v_i}\right)$$

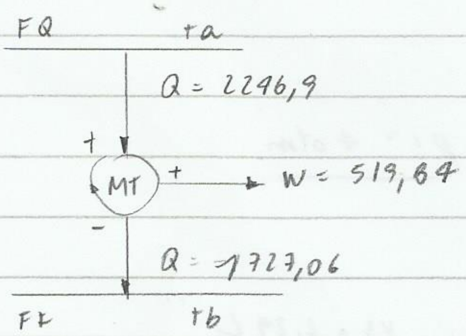
$$= 16 \cdot 1 \cdot \ln\left(\frac{4}{1}\right) = 22{,}181 \cdot 101{,}3 = 2246{,}9$$

$$W = \int_{v_i} p\,dv = 4(2{,}29 - 4) \cdot 101{,}3 = -692{,}89$$

$$W_{3-1} = -\Delta U = -n c_v \Delta T = \frac{p_3 v_3 - p_1 v_1}{\gamma - 1} = \frac{(4 \cdot 2{,}29) - (16 \cdot 1)}{1{,}67 - 1} \cdot 101{,}3 = -1034{,}2\,J$$

$$W_{ciclo} = \Sigma W = \underline{519{,}84\,J}$$

d)



$$e = \frac{|W|}{|Q_q|} = \frac{519{,}84}{2246{,}9} = 0{,}23$$

$$e\% = 23{,}13\%$$

3) (P2-2°10)

a) $p_a v_a^{\alpha} = p_b v_b^{\alpha}$

$4 \cdot 2^{1,5} = p_b \cdot 4^{1,5} \quad \therefore \quad p_b = 1{,}41 \cdot 1{,}013 \cdot 10^5 = 1{,}43 \cdot 10^5 \ Pa$

b) $W_{c-a} = 4 \cdot (2-4) = -8 \cdot 101{,}3 = -810{,}4 \ J$

$W_{b-c} = 0$

$W_{ab} = \int p \, dv \qquad\qquad p v^{\alpha} = K \quad \therefore \quad p = K v^{-\alpha}$

$= \int K v^{-\alpha} dv$

$= K \int v^{-1,5} dv = K \cdot \left(\frac{v^{-0,5}}{-0,5} \right) \Big|_2^4 = 4 \cdot 2^{+1,5} \cdot \left(\frac{v^{-0,5}}{-0,5} \right) \Big|_2^4 = 4{,}686 \ atm \, L$

$W_{ab} = 474{,}72 \ J$

$W = 474{,}72 - 810{,}4 = \underline{-335{,}28 \ J}$

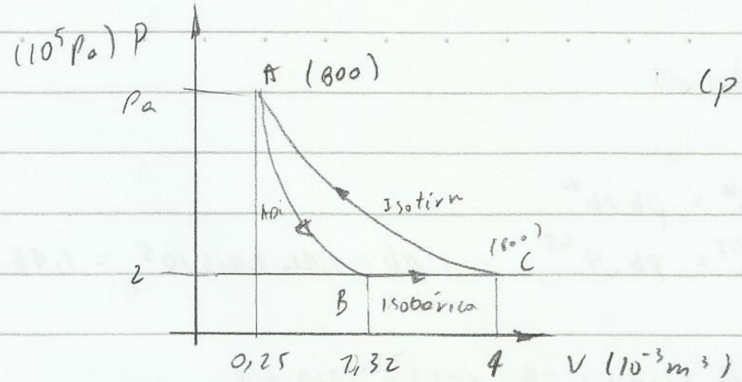
c) $\Delta U = Q - W = n c_v \Delta T$

$\Delta U = 0 \ ; \quad \therefore \quad Q = W \quad \therefore \quad Q = \underline{-335{,}28 \ J}$

d) O sistema rejeita calor, pois o calor do ciclo é negativo

e) O ciclo representa um refrigerador pois o trabalho do ciclo é negativo

4) (P2 2º10)



$Cp - Cv = R$

n

$pv = nRT$

$nR = \frac{pV}{T} = \frac{4.2}{800} . 100 = 1 \frac{J}{K}$

$W_{ca} = nRT \ln\left(\frac{V_a}{V_c}\right) = 1.800 \ln\left(\frac{V_a}{4}\right) = -2218 \quad \therefore \quad \underline{V_a = 0,25 . 10^{-3} m^3}$

$\frac{p_a v_a}{\cancel{T_a}} = \frac{p_c v_c}{\cancel{T_c}} \qquad p_a . 0,25 = 2.4 \quad \therefore \quad \underline{p_a = 32 . 10^5 Pa}$

$Cp - Cv = R$

$Cp = R + Cv$

$Cp = R + \frac{3}{2} R = \frac{5}{2} R$

$\gamma = \frac{Cp}{Cv} = \frac{5}{2} R : \frac{3}{2} R = 1,667$

$p_a v_a^{\gamma} = p_b v_b^{\gamma}$

$32 . 0,25^{(5/3)} = 2 . v_b^{5/3}$

$\therefore \underline{v_b = 1,32 . 10^{-3} m^3}$

$\frac{\cancel{p_B} v_B}{T_B} = \frac{\cancel{p_C} v_C}{T_C}$

$\frac{1,32}{T_B} = \frac{4}{800} \quad \therefore \quad \underline{T_B = 263,9 K}$

c) $W_{AB} = ? \quad Q_{BC} = ?$

$W_{AB} = -\Delta U = -n C_v \Delta T = -n . \frac{3}{2} R (T_B - T_A)$

$= -1 . \frac{3}{2} (263,9 - 800) = \underline{804,15 J}$

$Q_{BC} = n c_p \Delta T$

$= n . \frac{5}{2} R (T_C - T_B)$

$= \frac{5}{2} (800 - 263,9)$

$= \underline{1340,25 J}$

5) (P3 - 1º 2010)

a) A transformação isotérmica é $a \to b$, pois é menos inclinada que $a \to c$

b) $p_c v_c^\gamma = p_a v_a^\gamma$

$\frac{p_c}{p_a} = \left(\frac{v_a}{v_c}\right)^\gamma$

$\ln\left(\frac{p_c}{p_a}\right) = \gamma \ln\left(\frac{v_a}{v_c}\right)$

$\therefore \gamma = \ln\left(\frac{p_c}{p_a}\right) : \ln\left(\frac{v_a}{v_c}\right)$

$= \ln(10{,}08) : \ln 4 = \underline{1{,}6667}$

c) $W_{c \to a} = \frac{p_c v_c - p_a v_a}{\gamma - 1} = \frac{10{,}08 - 4}{1{,}667 - 1} = 9{,}12\,atm\,l = \underline{912\,J}$

d) $W = W_{ab} + \overset{0}{\cancel{W_{bc}}} + W_{ca}$

$W_{ab} = pv \ln\left(\frac{v_f}{v_i}\right) = 4.1 \ln\left(\frac{1}{4}\right) = -5{,}545 \cdot 100 = -554{,}52\,J$

$W = -554{,}52 + 912 = \underline{357{,}48\,J}$

$Q_{ab} = W_{ab} = -554{,}52$

$Q_{bc} = 912\,J$

	Q	W	ΔU
ab	-554,52	-554,52	0
bc	912	0	
ca	0	912	
ciclo	357,48	357,48	0

$e = \frac{357{,}48}{912} = \underline{0{,}392}$ $\quad e\% = \underline{39{,}2\%}$

e) $e = 1 - \frac{T_c}{T_h} = 1 - \frac{4.1}{10.08.1} = 0{,}6032$ $\quad e\% = 60{,}32\%$

/ /

6) (P3 - 1º10)

Calor: $Q_{ab} = 0$ — O calor fornecido para a mistura é no $\underline{Q_{bc}}$

$Q_{cd} = 0$

$Q_{bc} = n c_v \Delta T$ (+)

$Q_{da} = n c_v \Delta T$ (-)

Trabalho $W_{bc} = W_{da} = 0$

$W_{ab} = -\Delta U$ (compressão) $W -$

$W_{cd} = -\Delta U$ (expande) $\therefore W +$

$\therefore$ O trabalho realizado pela mistura é o $\underline{W_{cd}}$

b)

$$e = 1 - \frac{1}{r^{\gamma - 1}}$$

$$C_p - C_v = R$$

$$C_p = R + \frac{5}{2} R = \frac{7}{2} R$$

$$\gamma = \frac{7}{2} R : \frac{5}{2} R$$

$$\gamma = 1{,}4$$

$$e = 1 - \frac{1}{8{,}8^{1{,}4 - 1}} = 0{,}58 \quad e\% = 58{,}1\%$$

c) $0{,}18 \ \frac{kg}{min}$

$0{,}18 \ kg - 1 \ min$

$x - 20 \ min$

$\therefore x = 3{,}6 \ kg$

$$Q = mL = 3{,}6 \cdot 43{,}7 \cdot 10^6 = \underline{157{,}32 \cdot 10^6 \ J}$$

d) $P = \frac{W}{\Delta T}$ $\quad e = \frac{W}{Q}$ $\quad \therefore W = 0{,}581 \cdot 157{,}32 \cdot 10^6 = 91{,}4 \cdot 10^6 \ J$

$$P = \frac{91{,}4 \cdot 10^6}{20 \cdot 60} = \underline{76169{,}1 \ W}$$

7-) (P2 - 1°2010) $pv = nRT$

	Q	W	ΔU
AB		3000 J	
BC	-4700		
CA			0 (Isotérmico)
ciclo			

$$W_{ab} = \frac{(9+6).4}{2} \; 10^5 . 10^{-3} = 30.10^2 = 3000 \text{ J}$$

$$\Delta U_{(AB)} = nC_v \Delta T = \frac{pv}{RT} . \frac{3}{2} R \; (T_B - T_A)$$

8) (P2 · 1°10)

$c_v = \frac{3}{2} R$

$T_a = 400\,K$

$p_a v_a = nRT$

$= 8{,}315 . 400 = 3326\,J$

$W_{ca} = p_a(3V_a - V_a) = 2V_aP_a$

$= 2 . 3326 = -6652\,J$

	Q	W	U
AB	8315	2078,75	6236,25
BC	9978	6236,25	3791,75
CA	-16630	-6652	-9978
ciclo	1663	1663	0

$T_b = \frac{p_b V_b}{R} = \frac{1{,}5^2 . 3326}{8{,}315} = 900\,K$

$T_c = \frac{p_c v_c}{R} = \frac{3 . 3326}{8{,}315} = 1200\,K$

$W_{ab} = \frac{(1{,}5P_a + P_a).(1{,}5V_a - V_a)}{2} = \frac{2{,}5 . 0{,}5\, P_aV_a}{2} = \frac{2{,}5 . 0{,}5 . 3326}{2} = 2078{,}75\,J$

$W_{BC} = \frac{(1{,}5P_a + P_a).(3V_a - 1{,}5V_a)}{2} = \frac{2{,}5 . 1{,}5\, P_aV_a}{2} = \frac{2{,}5 . 1{,}5 . 3326}{2} = 6236{,}25\,J$

$\Delta U_{BC} = n c_v \Delta T$

$= \frac{3}{2} nR(T_B - T_C) = \frac{3}{2}(p_BV_B - p_CV_C) = +3791{,}75\,J$

$\Delta U_{AB} = n c_v \Delta T$

$= \frac{3}{2}(T_B - T_A) = \frac{3}{2}(p_BV_B - p_AV_A) = 6236{,}25\,J$

$e = \frac{1663}{(8315 + 9978)} = 0{,}091 \qquad e\% = 9{,}1\%$

/ /

$e_c = 1 - \frac{T_b}{T_a} = 1 - \frac{400}{1200} = 0{,}667$ $\quad e_{c\%} = 66{,}7\,\%$

9.) (P3 - 1°08)

$W_{23} = -644\,J$ (Isotérmico)

	Q	W	ΔU
12			
23	644	-644	0
31			

$\frac{p_1}{v_1} = \frac{p_2}{v_2}$

$\frac{p_3v_3}{T_3} = \frac{p_2v_2}{T_2}$

$\frac{p_1}{1} = \frac{p_2}{v_2}$ $\quad \therefore p_1 = p_2/v_2$

$p_2v_2 = 4$ $\quad 4 = p_2v_2$

$W_{23} = pv\,ln\left(\frac{v_1}{v_1}\right) = 4\,ln\left(\frac{1}{v_2}\right) \cdot 100 = -644 \quad \therefore v_2 = \underline{5\,L}$

$p_2 = \frac{4}{5} = \underline{0{,}8\,atm}$

b) $p_1 = \frac{0{,}8}{5} = \underline{0{,}16\,atm}$

$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2} \Rightarrow \frac{0{,}16 \cdot 1}{T_1} = \frac{0{,}8 \cdot 5}{300} \quad \therefore t_1 = \underline{12\,K}$

c) $pv = nRT$ $\quad C_p - C_v = R$

$0{,}8 \cdot 5 = n \cdot 0{,}082 \cdot 300 \quad \therefore n = \underline{0{,}163\,mol}$ $\quad C_v = C_p - R$

$C_v = \frac{5}{2}R - R = \frac{3R}{2}$

d) $W_{12} = \frac{(0{,}8 + 0{,}16) \cdot 4}{2} = 1{,}92 \cdot 100 = \underline{192\,J}$

$\Delta U = nC_v\Delta T = \frac{3}{2}nR(T_2 - T_1) = 1{,}5(p_2v_2 - p_1v_1) = 1{,}5(0{,}8 \cdot 5 - 0{,}16) = 5{,}76\,atm\,L =$

$\Delta U = \underline{576\,J}$

$Q = 576 + 192 = \underline{768\,J}$

10 (P2 - 1°09)

1-2 adiabático

$\gamma = 1,67$

$T_1 = 100K$ e $C_v = \frac{3}{2}R$

a) $p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$

$0,5 . 0,4^{1,67} = p_2 . 0,2^{1,67}$ ∴ $p_2 = \underline{1,59\,atm}$

$T v^{\gamma-1} = cte$

$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$

$100 . 0,4^{0,67} = T_2 . 0,2^{0,67}$ ∴ $T_2 = \underline{159,11\,K}$

$\frac{p_1 \cancel{v_1}}{T_1} = \frac{p_3 \cancel{v_3}}{T_3}$ ∴ $\frac{0,5}{100} = \frac{1,59}{T_3}$ ∴ $T_3 = \underline{318,2\,K}$

b) $\Delta U = -W$

$\Delta U = n c_v \Delta T$

$= \frac{3}{2} nR (T_2 - T_1) = \frac{3}{2}(p_2 v_2 - p_1 v_1) = \frac{3}{2}(1,59 . 0,2 - 0,5 . 0,4) . 100 = \underline{17,7}\,J$

$W = -\Delta U = \underline{-17,7\,J}$

c) $\Delta U = Q = n c_v \Delta T$

$= \frac{3}{2} nR (T_1 - T_3) = \frac{3}{2}(p_1 v_1 - p_3 v_3)$

$= \frac{3}{2}(0,5 . 0,4 - 1,59 . 0,4) . 100 = \underline{-65,4\,J}$

d) $W_{23} = 1,59 . 0,2 . 100 = 31,8$

$W_T = 31,8 + 0 - 17,7 = \underline{14,1\,J}$

$\Delta U = Q - W$

$Q + 200 = -100$

11-) (P2-1°09)

$C_v = \frac{3}{2} R$

$n = 0{,}1 mol$

a) T_1: $pV = nRT$

$1.2.100 = 0{,}1.8{,}315\,T \quad \therefore T = 240{,}53K$

T_2: $3.3.100 = 0{,}1.8{,}315.T \quad \therefore T = 1082{,}36K$

T_3: $4.1.100 = 0{,}1.8{,}315\,T \quad \therefore T = 481{,}06K$

b) $W_{31} = -1.2.100 = -200J$

c) $W_{12} = \frac{(3+1).1}{2}.100 = 200J$

$W_{23} = \frac{(3+1).1}{2}.100 = 200J$

	Q	W	ΔU
12	1250	200	1050
23	-750	200	-950
31	-300	-200	-100
ciclo	200	200	0

$\Delta U_{12} = nc_v\Delta T$

$= \frac{3}{2} nR(T_2 - T_1) = \frac{3}{2}(p_2V_2 - p_1V_1) = \frac{3}{2}(3.3 - 1.2).100 = 1050J$

$\Delta U_{23} = nc_v\Delta T$

$= 0{,}1.\frac{3}{2}.8{,}315(481{,}06 - 1082{,}36) = -750J$

$e = \frac{200}{1250} = 0{,}16 \quad \therefore e\% = 16\%$

d) $e_c = 1 - \frac{240{,}53}{1082{,}36} = 0{,}78 \quad e_c\% = 78\%$

/ /

12.) (P3 1º 09) $C_v = \frac{5}{2}R$ $C_p - C_v = R$

$C_p = R + \frac{5}{2}R = \frac{7}{2}R$

Estado 1

$V_1 = 1L$ $T_1 = 27°C = 300K$ $p_1 = 5\ atm$

expande isobaricamente

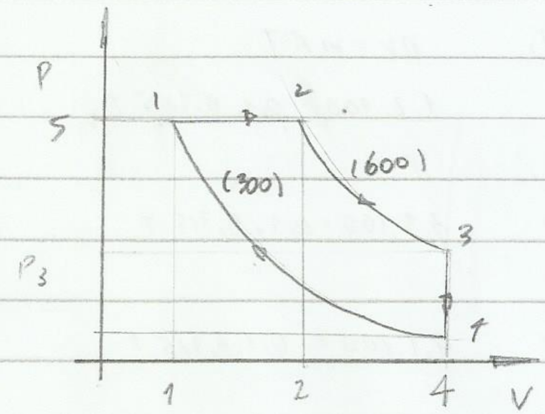
$V_2 = 2L$

expande isotérmica

$V_3 = 4L$

Isocórica

$T_4 = 27°C$ $T = 300K$



$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ $\frac{1}{300} = \frac{2}{T_2}$

b) $\therefore T_2 = \underline{600\ K}$

c) $p_3 V_3 = p_2 V_2$

$4p_3 = 2p_2$ $\therefore$ $p_3 = \frac{1}{2}p_2$ $\therefore$ $p_3 = \underline{2{,}5 . 10^5\ Pa}$

d) $\frac{p_3}{T_3} = \frac{p_4}{T_4}$ $\Rightarrow$ $\frac{2{,}5}{600} = \frac{p_4}{300}$ $\therefore$ $p_4 = \underline{1{,}25 . 10^5\ Pa}$

e) $Q_{41} = pv \ln\left(\frac{v_4}{v_1}\right) = 1{,}25 . 4 \ln\left(\frac{1}{4}\right) . 100 = -693{,}15\ J$

$Q_{23} = 5.2 \ln\left(\frac{4}{2}\right) . 100 = 693{,}15\ J$

$Q_{12} = n c_p \Delta T$

$= \cancel{n} . \frac{7}{2}\cancel{R} . (5.2 - 5) = 17{,}5 . 100 = 1750\ J$

$Q_q = 17500 + 693,15 = 18193,15 \text{ J}$

$W_{12} = 5.1.100 = 500 \text{J}$

$W_{23} = 693,15 \text{ J}$

$W_{41} = p_i v_i \ln\left(\frac{v_1}{v_4}\right) = 5.1 \ln\left(\frac{1}{1}\right).100 = -693,15 \text{J}$

$W = \sum W = 500$

$e = \frac{500}{2443,15} = 0,2046 \qquad e\% = \underline{20,40\%}$

(Victor)

$\gamma = 1{,}4$ $\quad n = 1$

$p_1 = 1atm$ $\quad t_1 = 0°C = 273\ K$ $\quad$ (Estado 1)

Gás aquecido isocoricamente

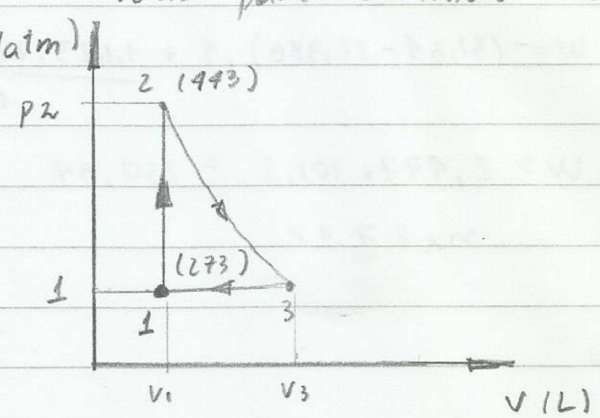
$t_2 = 170°C = 443\ K$ $\quad$ (Estado 2)

expandido adiabaticamente

$p_3 = 1atm$ $\quad$ Estado 3)

volta para o inicio (resfriado isobaricamente)

a)



b) $t_3 = ?$ $\quad V_3 = ?$

$p_1 V_1 = nRT$

$V = 1 \cdot 0{,}082 \cdot 273$ $\quad \therefore V_1 = \underline{22{,}386\ L}$

$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow \frac{1}{273} = \frac{p_2}{443}$ $\quad \therefore p_2 = \underline{1{,}623\ atm}$

$p_2 V_2^{\gamma} = p_3 V_3^{\gamma}$

$1{,}623 \cdot 22{,}386^{1,4} = 1 \cdot V_3^{1,4}$ $\quad \therefore V_3 = \underline{31{,}64\ L}$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ $\quad \therefore \frac{22{,}366}{273} = \frac{31{,}64}{T_2}$ $\quad \therefore t_2 = \underline{385{,}85\ K}$

c) $Q_{12} = n c_v \Delta T$

$= 1 \cdot \frac{5}{2} R \Delta T = \frac{5}{2} \cdot 8{,}315 \cdot (443 - 273) = \underline{3533{,}875 \text{ J}}$

$Q_{31} = n c_p \Delta T$

$= 1 \cdot \frac{7}{2} R \Delta T = \frac{7}{2} \cdot 8{,}315 \cdot (273 - 385{,}85) = \underline{-3284{,}22 \text{ J}}$

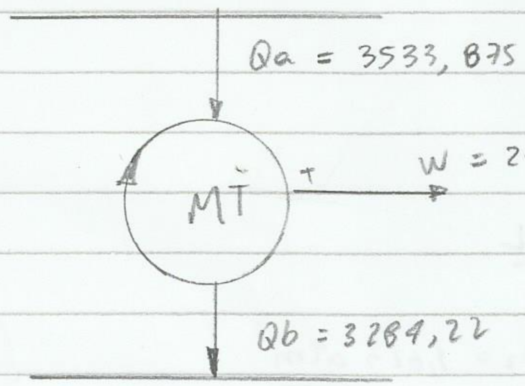
$Q_{23} = 0$

d) $n = \frac{W}{Q_a}$ $\therefore$ $W = -(31{,}64 - 22{,}386) \cdot 1 + \frac{1{,}623 \cdot 22{,}386 - 31{,}64}{0{,}4}$

$W = 2{,}477 \cdot 101{,}3 = 250{,}94$

$n = \frac{250{,}94}{3533{,}875} = \underline{0{,}07}$ $n\% = 7{,}1\%$

ou



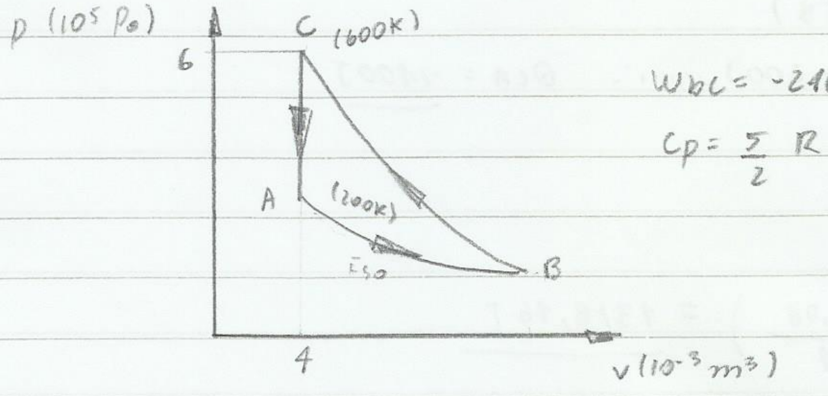
$n = \frac{249{,}655}{3533{,}875} = 0{,}071$

$n\% = 7{,}1\%$

$\Delta U = Q - W$

$\therefore W = -n c_v \Delta T$

1-) (P2-2009-)



$W_{bc} = -2400 J$

$c_p = \frac{5}{2} R$

$c_p - c_v = R$

$c_v = c_p - R$

$c_v = \frac{5}{2} - R = \frac{3}{2} R$

a) $p_A = ?$ $p_B = ?$ $t_A = ?$ $T_B = ?$ $v_A : ?$ $v_B ?$

$\gamma = \frac{5}{2} R : \frac{3}{2} R = 5/3$

$W_{bc} = -n c_v \Delta T$

$= -\frac{3}{2} nR (T_C - T_B) = -2400$ $\qquad nR = \frac{p_C v_C}{T_C} = \frac{6.4.10^2}{600} = 4$

$= -\frac{3}{2} .4 (600 - T_B) = -2400 \therefore \underline{T_B = 200 K = T_A}$

$\frac{p_C \cancel{v_C}}{T_C} = \frac{p_A \cancel{v_A}}{T_A} \Rightarrow \frac{6}{600} = \frac{p_A}{200} \therefore p_A = \underline{2.10^5 Pa}$

$(T v^{\gamma-1})_C = (T v^{\gamma-1})_B$

$600 . 4^{2/3} = 200 . v_B^{2/3} \therefore \underline{v_B = 20{,}78 . 10^{-3} m^3}$

$\frac{p_A v_A}{\cancel{T_A}} = \frac{p_B v_B}{\cancel{T_B}}$

$2.4 = p_B . 20{,}78 \therefore p_B = \underline{0{,}385 . 10^5 Pa}$

b) $Q_{CA} = n c_V \Delta T$

$= \frac{3}{2} nR (T_A - T_B)$

$= \frac{3}{2} \cdot 4 \, (200 - 600) \quad \therefore \quad Q_{CA} = \underline{-2400 \, J}$

$Q_{AB} = pV \ln\left(\frac{V_f}{V_i}\right)$

$= 2.4.10^2 \ln\left(\frac{20{,}76}{4}\right) = \underline{1318{,}16 \, J}$

c) $W = 1318{,}16 - 2400 = \underline{-1081{,}84 \, J}$

2-) (P2 - 2º09)

$c_p - c_v = R$

$c_p - \frac{3}{2} R = R \quad \therefore \quad c_p = \frac{5}{2} R$

$\frac{256}{600} = \frac{1}{t_1} \quad \therefore \quad t_1 = \underline{2{,}3 \, K}$

$\gamma = 5/3 = 1{,}67$

$\frac{1}{V_1} = \frac{p_2}{6} \qquad V_1 p_2 = 6 \qquad p_2 = \frac{6}{V_1} \quad e \quad V_1 = \frac{6}{p_2} \quad (I)$

$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

$\frac{V_1}{2{,}3} = \frac{6 p_2}{150} \quad \therefore \quad V_1 = 0{,}092 \, p_2 \quad (II)$

$\frac{6}{p_2} = 0{,}092 \, p_2 \quad \therefore \quad p_2 = 8{,}076 \text{ atm}$

$p_2 = \underline{8{,}18.10^5 \, Pa}$

$V_1 = 0{,}092 \cdot 8{,}18.10^5 = 0{,}74.10^{-3} \, m^3$

b) $Q_{23} = 0$ $\quad nR = \frac{p_3 v_3}{T_3} = \frac{256 \cdot 0,74}{600}$

$W_{23} = -n c_v \Delta T$

$= -\frac{3}{2} nR (T_3 - T_2)$ $\quad \therefore nR = 0,3157$

$= -\frac{3}{2} \cdot 0,3157 (600 - 150)$

$\therefore W_{23} = -213,12$ J

c) $W = \int p\,dv$

$= \frac{(8,18 + 1) \cdot (6 - 0,74)}{2} \cdot 10^2 = \underline{241,43\text{ J}}$

3.) (P3-1:09)

$c_v = \frac{5}{2} R$

a) $\frac{1}{2} = \frac{3}{V_b}$ $\quad \therefore \underline{V_b = 6L = 6 \cdot 10^{-3}\ m^{-3}}$

$3 \cdot 6 = V_c$ $\quad \therefore \underline{V_c = 18L \text{ ou } 18 \cdot 10^{-3}\ m^3}$

$W_{ab} = \frac{(3+1) \cdot (6-2)}{2} = 8 \cdot 100 = 800$ J

$W_{ac} = (2 - 18) \cdot 1 = -16 \cdot 100 = -1600$ J

$W_{bc} = pv \ln\left(\frac{v_f}{v_i}\right)$

$= 3 \cdot 6 \ln\left(\frac{18}{6}\right) = 19,775 \cdot 100 = 1977,5$ J

$W = \underline{1177,5\text{ J}}$

$\Delta U = Q - W$

$Q_{ca} = n c_p \Delta T$ $\qquad C_p - C_v = R$

$= \frac{7}{2} nR \left(\frac{2-16}{nR} \right)$ $\qquad C_p = R + C_v = R + \frac{5}{2}R = \frac{7}{2}R$

$= -5600 \text{ J}$

$Q_{bc} = 1977{,}5 \text{ J}$

$Q_{ab} = \Delta U + W_{ab}$

$= \frac{5}{2}(3{,}6 - 2) \cdot 100 + 800$

$Q_h = 4600 \text{ J}$

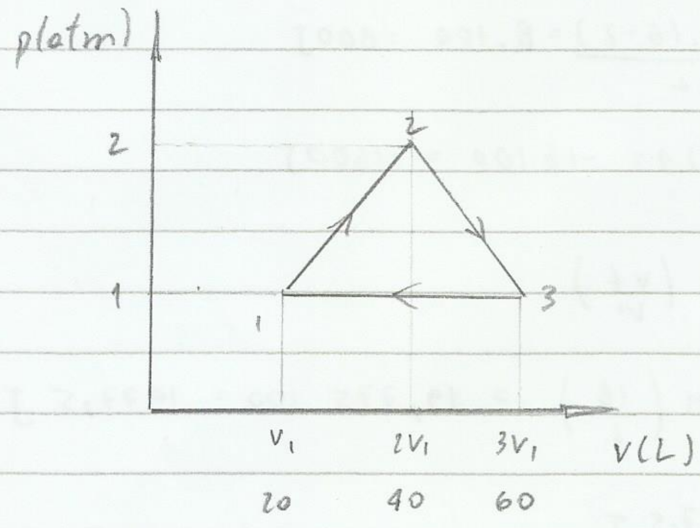
$n = \frac{1977{,}5}{6777{,}5} = 0{,}17 \qquad n_\% = 17{,}37\%$

d) $e_c = 1 - \frac{1.2}{3.6} = 0{,}89 \qquad e_c = 89\%$

4-) (P2-2°08)

$C_v = \frac{5}{2}R$

$V_1 = 20L$



a) $w = \frac{(40-20).1}{2} + \frac{(60-40).1}{2}$

$\therefore w_{ciclo} = 20\ atmL = \underline{2000\ J}$

$Cp - Cv = R$

$Cp = R + Cv$

$Cp = R + \frac{5}{2}R$

$\therefore Cp = \frac{7}{2}R$

b) $Q_{31} = n c_p \Delta T$

$= \frac{7}{2} \cancel{nR} \left(\frac{20-60}{\cancel{nR}} \right) = -140.100 = \underline{-14000\ J}$

c) $\Delta U = Q - W$

$\therefore Q = \Delta U + W \quad (2 \to 3)$

$Q = n c_v \Delta T + w$

$Q = \frac{5}{2} \cancel{nR} \left(\frac{60-60}{\cancel{nR}} \right).100 + \frac{(2+1)(60-40)}{2}.100$

$\therefore Q = \underline{-2000\ J}$

d) $Q_{12} = \Delta U + W$

$= \frac{5}{2} \cancel{nR} \frac{(80-20)}{\cancel{nR}}.100 + \frac{3.20}{2}.100$

$= 18000\ J$

$Q_{23} = -2000\ J$

$Q_{31} = n c_p \Delta T$

$= \frac{7}{2} nR \left(\frac{20-60}{nR} \right) = -14000\ J$

$e = \frac{2000}{18000} = 0{,}11 \qquad e\% = 11{,}11\%$

5) (P2 2°08)

$C_v = \frac{5}{2} R$

$C_p - C_v = R$

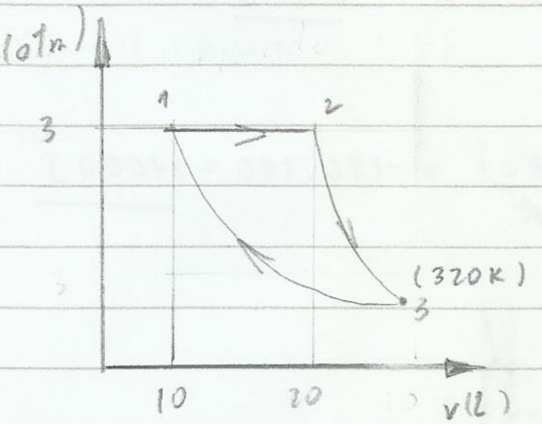
$C_p = R + C_v$

$= R + \frac{5}{2} R$

$\therefore C_p = \frac{7}{2} R$

$\gamma = 1{,}4$

a)



b) $\frac{\cancel{p_1} v_1}{T_1} = \frac{\cancel{p_2} v_2}{T_2}$

$\frac{10}{T_1} = \frac{20}{T_2}$ $\quad T_1 = \frac{1}{2} T_2$ e $T_2 = 2T_1$

Como $T_1 = T_3 = 320$

$T_2 = 2.320 = \underline{640K}$

c)

$(T v^{\gamma - 1})_2 = (T v^{\gamma - 1})_3$

$640 . 20^{0,4} = 320 . v^{0,4}$ $\quad \therefore v_3 = \underline{113{,}14\ L}$

d) $W_{1-3} = W_{12} + W_{23}$

$= (20-10) . 3 . 100 + \frac{5}{2} \cancel{nR} \frac{(30-60) . 100}{\cancel{nR}}$

$W_{1-3} = \underline{10500\ J}$

b) (P3 2º 08)

$n = 0{,}1\ mol$

$C_v = \frac{3}{2}R$

$C_p - C_v = R$

$C_p = R + \frac{3}{2}R = \frac{5}{2}R$

$\gamma = 1{,}67$

$p_b \cdot 0{,}3^{1{,}67} = 1 \cdot 1^{1{,}67}$

$\therefore p_b = \underline{7{,}47\ atm}$

$\frac{1}{1} = \frac{7{,}47}{V_c} \therefore V_c = 7{,}47\ L$

$W = -\Delta U$

$W = -n C_v \Delta T$

$= -\frac{3}{2} nR \frac{(7{,}47 \cdot 0{,}3 - 1 \cdot 1)}{nR} \therefore W_{a-b} = -1{,}86\ atm\,L$

$= -186{,}06\ J$

c) $W = W_{ab} + W_{bc} + W_{ca}$

$= -186{,}06 + 7{,}47 \cdot (7{,}47 - 0{,}3) \cdot 100 - \frac{(7{,}47+1)(1-7{,}47)}{2} \cdot 100$

$\therefore W = 2429{,}88$

$Q_{b \to c} = n C_p \Delta T$

$= \frac{5}{2} nR \frac{(7{,}47^2 - 7{,}47 \cdot 0{,}3)}{nR} \cdot 100 \therefore Q_{b \to c} = 13\,369{,}975\ J$

$Q_{c \to a} = \Delta U + W_{ca}$

$= \frac{3}{2} \cancel{nR} \frac{(1 - 7{,}47^2)}{\cancel{nR}} \cdot 100 + \frac{(7{,}47+1)(1-7{,}47)}{2}$

$\therefore Q = -8247{,}535\ J$

$e = \frac{2429{,}88}{13\,369{,}975} = 0{,}18 \qquad e\% = \underline{18\%}$

7-) (P3 - 2º 2008)

$n = 1\,mol$

$C_v = 20{,}8\ J/mol\,K \qquad \gamma = 1{,}4$

$C_p - C_v = R$

$C_p = 8{,}315 + 20{,}8 = 29{,}115$

$pV = nRT$

$1 . V\,100 = 1 . 8{,}315 . 283$

$\therefore V_A = 23{,}53\,L$

$\frac{p_B}{T_B} = \frac{p_A}{T_A} \Rightarrow \frac{p_B}{423} = \frac{1}{283} \quad \therefore \underline{p_B = 1{,}4947}$

$31{,}35 . 1 . 100 = 1 . 8{,}315 . T$

$\therefore T_c =$

$(pV^{\gamma})_B = (pV^{\gamma})_c$

$1{,}4947 . 23{,}53^{1,4} = 1 . V^{1,4} \quad \therefore \underline{V_c = 31{,}35\,L}$

$\left(\frac{V}{T}\right)_A = \left(\frac{V}{T}\right)_B \Rightarrow \frac{23{,}53}{283} = \frac{31{,}35}{T} \quad \therefore \underline{T_c = 377{,}11\,K}$

b) $Q_{A\to B} = n\,c_v\,\Delta T$

$= 1 . 20{,}8\,(423 - 283) \quad \therefore \underline{Q_{A\to B} = 2912\,J}$

$Q_{C\to A} = n\,c_p\,\Delta T$

$= 1 . 29{,}115\,(283 - 377{,}11) \quad \therefore \underline{Q_{C\to A} = -2740{,}01}$

c) $W_{ciclo} = -(31{,}35 - 23{,}53)\,100 + 20{,}8 . (377{,}11 - 423)$

$= 954{,}512 - 782 = 172{,}512$

$e = \frac{172{,}512}{2912} = 0{,}059 \qquad e\% = 5{,}9\,\%$

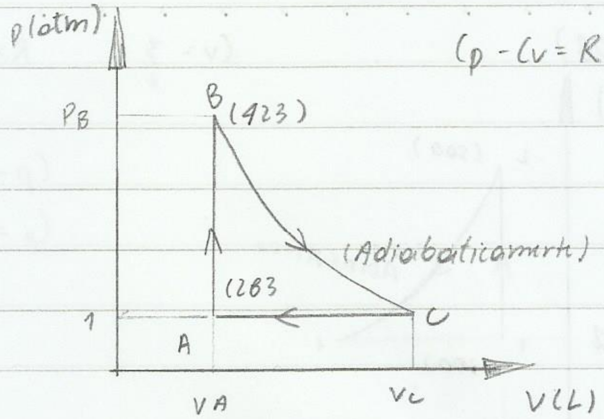
7-) (P3 - 2°08)

$n = 1\,mol$

$C_v = 20{,}8\ J/(mol.K)$

$\gamma = 1{,}4$

$C_p - C_v = R$



$$\frac{p_A \cancel{V_A}}{T_A} = \frac{p_B \cancel{V_B}}{T_B}$$

$$\frac{1}{283} = \frac{p_B}{423} \quad \therefore \ p_B = 1{,}4947\ atm \quad (A)$$

$$pV = nRT$$

a)

$$1 \cdot V = 1 \cdot 0{,}082 \cdot 283 = \underline{23{,}206\ L}$$

$$(pV^{\gamma})_B = (pV^{\gamma})_A$$

$$1{,}4947 \cdot 23{,}206^{1,4} = 1 \cdot V_c^{1,4} \quad \therefore \ V_c = \underline{30{,}923\ L} \ (B)$$

$$\frac{V_A}{T_A} = \frac{V_c}{T_c} \Rightarrow \frac{23{,}206}{283} = \frac{30{,}923}{T_c} \quad \therefore \ T_c = \underline{377{,}1098\ K}\ (C)$$

b)

$$Q_{a \to B} = n c_v \Delta T$$

$$= 1 \cdot 20{,}8\,(423 - 283) \quad \therefore \ Q_{a \to b} = \underline{2912\ J}$$

$$Q_{c \to a} = n c_p \Delta T$$

$$= 1 \cdot (8{,}315 + 20{,}8) \cdot (283 - 377{,}1098) = -2740{,}01\,J$$

c)

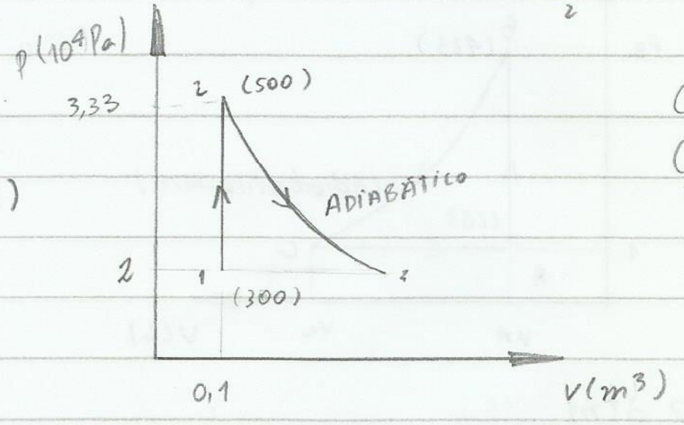
$$W = \overset{0}{\cancel{W_{aB}}} + W_{BC} + W_{CA}$$

$$= -1 \cdot 20{,}8\,(377{,}1098 - 423) + (23{,}206 - 30{,}923) \cdot 100 = 182{,}816\ J$$

$$e = \frac{182{,}816}{2912} = 0{,}0628 \qquad e\% = 6{,}28\,\%$$

8.) (P2 1°08) $c_v = \frac{3}{2}R$ $R = 8{,}315$ J/mol.K

a)



$C_p - C_v = R$

$C_p = R + C_v = R + \frac{3}{2}R = \frac{5}{2}R$

$\gamma = 5/3$

$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ $\frac{2}{300} = \frac{p_2}{500}$ $\therefore p_2 = 3{,}33 \cdot 10^4$ Pa

$pV = nRT$

c) $3{,}33 \cdot 0{,}1 \cdot 10^4 = n \cdot 8{,}315 \cdot 500$ $\therefore n = 0{,}8$ mol

b) $(pV^\gamma)_2 = (pV^\gamma)_3$

$3{,}33 \cdot 0{,}1^{5/3} \cdot 2V^{5/3}$ $\therefore V = 0{,}136\ m^3$

$(TV^{\gamma-1})_2 = (TV^{\gamma-1})_3$

$500 \cdot 0{,}1^{2/3} = T \cdot 0{,}136^{2/3}$ $\therefore T = 407{,}60$ K

d) $W_{13} = W_{12} + W_{23}$ (ADIAB.) $\Delta U = \cancel{Q} - W$

$\Delta U = -W$ $\therefore W = -\Delta U$

$W_{12} = 0$ $W = -nc_v\Delta T$

$W_{23} = -nc_v\Delta T$

$= -\frac{3}{2}\cancel{nR}\frac{(2 \cdot 10^4 \cdot 0{,}136 - 3{,}33 \cdot 10^4 \cdot 0{,}1)}{\cancel{nR}} = 920$ J

$W_{13} = 0 + 920 = 920$

$\Delta U = \cancel{Q} - W$ $\therefore \Delta U = -W = -920$ J

9) (P3 - 1°08)

$n = 0{,}0963\ mol \qquad c_v = 15\ J/mol.K$

$Q_{AB} = 1082J$

a) $Pv = nRT$

$1.10^5 . 2.10^{-3} = 0{,}0963 . 8{,}315\ T \quad \therefore \underline{T_A = 249{,}77\ K}$

$\frac{1}{2} = \frac{P_c}{4} \quad \therefore \underline{P_c = 2.10^5\ Pa}$

$\frac{p_a v_a}{T_a} = \frac{p_c v_c}{T_c} \Rightarrow \frac{1.10^5 . 2.10^{-3}}{249{,}77} = \frac{2.10^5 . 4.10^{-3}}{T} \quad \therefore \underline{T_c = 999{,}08\ K}$

$\frac{p_a}{T_a} = \frac{p_B}{T_D} \Rightarrow \frac{1}{249{,}77} = \frac{p_B}{999{,}08} \quad \therefore \underline{p_B = 4.10^5\ Pa}$

b) $W_{ciclo} = \cancel{W_{AB}} + W_{BC} + W_{CA}$

$= pv \ln\left(\frac{V_f}{V_i}\right)(BC) + \frac{(P_C + P_A) 2.10^{-3}}{2}$

$= 4.10^5 . 2.10^{-3} \ln\left(\frac{4}{2}\right) - \frac{(2.10^5 + 1.10^5) . 2.10^{-3}}{2}$

$\therefore W_{ciclo} = 554{,}518 - 300 = \underline{254{,}52\ J}$

c) $\Delta U = Q - W \quad \therefore \quad Q = \Delta U + W$

$Q = n c_v \Delta T - 300$

$= 0{,}0963 . 15 (249{,}77 - 999{,}08) - 300 = \underline{-1382{,}38\ J}$

10-) (P3 2°07)

$n = 0{,}0782$ mol

$c_p - c_v = R$

$24{,}93 - 16{,}62 = R \quad \therefore \quad R = 8{,}31$

$c_p / c_v = \gamma \quad \therefore \quad \gamma = 1{,}5$

a) $pv = nRT$

$300.10^3 . 1 . 10^{-3} = 0{,}0782 . 8{,}31\, T \quad \therefore \quad T_3 = 461{,}65\ k$

$(pv^\gamma)_1 = (pv^\gamma)_2$

$300 . 1^{1,5} = 100 . v^{1,5} \quad \therefore \quad v = 2{,}08\, L = 2{,}08 . 10^{-3}\, m^3$

$pv = nRT$

$100 . 2{,}08 = 0{,}0782 . 8{,}31\, T \quad \therefore \quad T_1 = 320{,}09\ k$

$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow \frac{100}{320{,}09} = \frac{p_2}{639{,}8} \quad \therefore \quad p_2 = 199{,}88\ kPa$

	Q(J)	W(J)	ΔU(J)
a	415,52	0	415,52
b	-501,47	-269,94	-232,54
c	0	183,98	-183,98
ciclo	-85,95	-85,96	0

$Q_a = n c_v \Delta T$

$= 0{,}0782 . 16{,}62 (639{,}8 - 320{,}09) = 415{,}52\ J$

$W_b = \frac{(300 + 199{,}88) . 10^3}{2} . (2{,}08 - 1) . 10^{-3} = -269{,}94\ J$

$Q_b = -269{,}94 + n c_v \Delta T$

$= -269{,}94 + 0{,}0782 . 16{,}62 . (461{,}65 - 639{,}8) = -501{,}47\ J$

11. (P2 - 1°08)

$\Delta U = Q - W \quad ; \; Q = 0$

$\therefore \; W = -\Delta U$

a) $= -n c_v \Delta T \qquad \therefore W_{1\to 2} = \frac{-5}{2}(p_2 v_2 - p_1 v_1)$

$= -\frac{5}{2} \cancel{nR} \frac{(p_2 v_2 - p_1 v_1)}{\cancel{nR}}$

$c_p - c_v = R$

$c_p = R + c_v \qquad \gamma = \frac{7}{2}R : \frac{5}{2}R = 1{,}4$

$c_p = R + \frac{5}{2}R = \frac{7}{2}R$

$(pv^\gamma)_1 = (pv^\gamma)_2$

$2 \cdot 1^{1,4} = p\, 4^{1,4} \quad \therefore p_2 = 0{,}2871$

$W_{1\to 2} = -\frac{5}{2}(0{,}2871 \cdot 4 - 2 \cdot 1) = 2{,}128 \text{ atmL} = \underline{215{,}59 \text{ J}}$

b) $W = W_{1\to 2} + W_{3\to 4} \qquad \frac{2}{600} = \frac{p_4}{300} \therefore p_4 = 1$

$= 215{,}59 - \frac{5}{2}\cancel{nR}\frac{(1 \cdot 1 - 4 \cdot 0{,}1436)}{\cancel{nR}}100$

$1 \cdot 1^{1,4} = p \cdot 4^{1,4} \quad \therefore p_1 = 0{,}1436$

$W = 109{,}177 \text{ J}$

$Q_{4\to 1} = n c_p \Delta T \qquad Q_{3\to 2} = n c_p \Delta T$

$= \frac{7}{2}\cancel{nR}\frac{(2 \cdot 1 - 1 \cdot 1)}{\cancel{nR}} \cdot 100 = 350 \text{ J} \qquad = \frac{7}{2} nR \frac{(0{,}1436 \cdot 4 - 0{,}2871 \cdot 2)}{nR}$

$Q_{1\to 2} = 0$

$\therefore e = \frac{109{,}177}{350} = 0{,}31$

12-) (P2 - 2°07)

$n = 1\,mol$

$c_v = \frac{3}{2} R$

$c_p - c_v = R$

$c_p = R + c_v = R + \frac{3}{2} R = \frac{5}{2} R$

$\gamma = c_p / c_v = 5/3$

$T_a V_a^{2/3} = T_c\, 5V_a^{2/3}$

$300\, V_a^{2/3} = T_c (5V_a)^{2/3}$ $\therefore T_c = 102,6\ K$

$p_a V_a^{\gamma} = p_c V_c^{\gamma}$

$10^5\, 1,013 . V_a^{5/3} = p_c . (5 V_a)^{5/3}$

$p_c = \frac{1,013 . 10^5\ V_a^{5/3}}{5^{5/3} . V_a^{5/3}} = \underline{6928,8\ Pa}$

b) $Q = pV \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{V_f}{V_i}\right)$

$Q = 1 . 8,315 . 300 \ln\left(\frac{5 . 6928,8}{6928,8}\right) = \underline{4015,74\ J}$

c) $Q = n c_v \Delta T$

$= \frac{3}{2} . 1 . 8,315 (102,6 - 300) = \underline{-2462,09\ J}$

Retirou-se 2462,09 J

d) $W_{ciclo} = W_{12} + \cancel{W_{23}}^{0} + W_{31}$

$= nRT \ln\left(\frac{V_f}{V_i}\right) - n c_v \Delta T$

$= 1 . 8,315 . 300 \ln(5) - 1 . \frac{3}{2} . 8,315 (300 - 102,6)$

$= \underline{1552,67}\ J$

13-) (P3 - 1°07)

$C_p = \frac{5}{2} R$

$C_p - C_v = R$

$C_v = C_p - R = \frac{5}{2} R - R = \frac{3}{2} R$

$W_{CA} = -499,4 \, J$

$W = pV \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{V_f}{V_i}\right)$

$-499,4 = 4,5 \cdot 10^2 \ln\left(\frac{4,5}{V_i}\right) \therefore V_i =$

Calor específico

Capacidade Calorífica ou Capacidade Térmica: É a quantidade de calor necessária para que um corpo ou sistema varie sua temperatura em 1 grau.

$$\boxed{C = \frac{\Delta Q}{\Delta \theta}}$$

ΔQ = Calor trocado

$\Delta \theta$ = Variação de temperatura

Obs: A capacidade térmica é proporcional a massa.

No [SI] a unidade utilizada é J/K mas tambem pode utilizar cal/°C

Calor específico É a capacidade térmica de um corpo por unidade de massa

$$\boxed{c = \frac{C}{m} \Rightarrow c = \frac{\Delta Q}{m \Delta \theta}}$$

Unidade: [SI] => J/kg.K, sendo a mais utilizada cal/g°C

Observação: A energia contida em um sistema fechado se conserva

$\Delta Q = mc\,\Delta\theta$ 1 cal = 4,186 J

Efeito Joule

- É a transformação de energia elétrica em calor, que ocorre em condutores

$W = P.\Delta T$ sendo $P = U.I = RI^2$

O calor (Q) recebido pela água e pelo calorímetro é dado por:

$$Q = m_a c_a \Delta\theta + C\Delta\theta$$

m_a = massa de água colocada no interior do recipiente (calorímetro)
c_a = 1,00 (cal/g°C) é o calor específico da água
C = capacidade térmica do calorímetro
$\Delta\theta$ = é a variação de temperatura sofrida pelo sistema

Equivalente mecânico do calor (J) é:

$$\boxed{J = \frac{W}{Q} \quad \left(\frac{\text{Joule}}{\text{cal}}\right)}$$

1-) (P2 2º10)

650 cal

4,15 J/cal

50 cal

Sem perda:

$Q = 650\,cal$ e $J = 4,15\,J/cal$

Equivalente mecânico $\therefore$ $J = \dfrac{W}{Q}$

$W = J.Q$

$= 4,15 . 650$

$\therefore W = 2692,5$

Considerando a perda:

$J = \dfrac{W}{Q} \therefore J = \dfrac{2692,5}{700} = \underline{4,132\ J/cal}$

2-) (P2-2010)

$R = 3\,\Omega$

$m_a = 180\,g$

$c_a = 1\,cal/g°C$

$C = 32,6\,cal/°C$

$I = 2,5\,A$

$\Delta\theta = 22,4°C$

$J = 4,35\,J/cal$

$Q = m_a c\Delta\theta + C\Delta\theta$

Calor recebido pela água e o calorímetro

$W = P . \Delta T$

$= U . I\,\Delta T$

$= R . I^2 \Delta T \quad \therefore \quad W = R . I^2 \Delta T$

$J = \dfrac{W}{Q} = \dfrac{R . I^2 \Delta T}{(m_a c + C)\Delta\theta}$

$\Delta T = \dfrac{J . (m_a c + C)\Delta\theta}{R . I^2} = \dfrac{4,35\,(180 . 1 + 32,6) . 22,4}{3 . 2,5^2}$

$\therefore \Delta T = 1104,63\,s = 18,41\,min$

3-) (P2 1°10)

$C = 32,6 \; cal/°C$

$m_a = 250 g$

$T_{a_0} = 20°C$

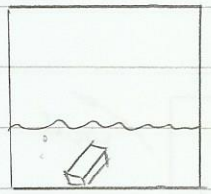
$U = 3,5 V$

$I = 1,5 A$

$\Delta T = 25 min = 1500 s$

$m_l = 80 g \quad T_{l_0} = 120°C$

$J = ?$



$$J = \frac{W}{Q} \; ; \quad W = UI . \Delta T$$

$$W = 3,5 . 1,5 . 1500 = 7875 J$$

$$J = \frac{W}{Q} \therefore Q = \frac{W}{J} = \frac{7875}{4,186} = 1881,27 \; cal$$

$$Q = m c_a \Delta T + C \Delta T + m_L c_L \Delta T$$

$$1881,27 = 250.(T_f - 20) + 32,6 (T_f - 20) + 80 . 0,22 (T_f - 120°)$$

$$9695,27 = 300,2 T_f \quad \therefore \quad T_f = 32,13 °C$$

$$J = \frac{7875}{(250 + 32,6).(32,13 - 20)} = 2,2974 \; J/cal$$

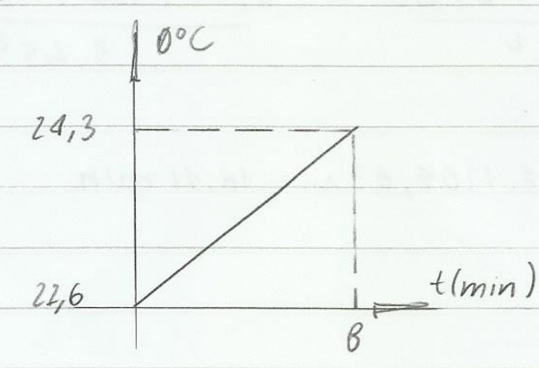
4-) (P2-2009)

água: $m_a = 250 g$ calorímetro: $C = 32,6 \; cal/°C$

$c_a = 1 cal/g°C$

$I = 2,0 A$

$J = 4,186 J/cal$



a) $E_x = \frac{|J_{teo} - J_{exp}|}{J_{teo}} . 100$

$3,82 = \frac{|4,186 - J|}{4,186} . 100 \quad \therefore \quad J_{exp} = \underline{4,0261 \text{ J/cal}}$

b) $J = \frac{W}{Q}$; $W = U.I$ e $Q = (m_a c_a + C)\Delta T$.

$Q = (250.1 + 32,6).(24,3 - 22,6) \quad \therefore \quad Q = 480,42 \text{ cal}$

$\therefore W = J.Q = 4,0261 . 480,42$

$\therefore W = 1934,22 \text{ J}$

$W = U.I.\Delta T \quad \therefore U = \frac{W}{I \Delta T} = \frac{1934,22}{2.(8.60 - 0)} \quad \therefore U = \underline{2,01 V}$

5) (P2 . 1º09)

$J = 5,581$ J/cal

$J_{tab} = 4,186$ J/cal

a) Houve perda, pois o equivalente mecanico é inversamente proporcional a energia térmica, E tambem o W do ciclo, não varia, e para o equivalente mecânico aumentar, a unica manuvra, é diminuindo o valor da energia térmica.

$W = J.Q$ (teoria) $\quad W = J'.Q'$

$JQ = J'Q'$

$\frac{J}{J'} = \frac{Q'}{Q} = 0,75 \quad \therefore \quad Q' = 0,75Q \quad \therefore \quad Q' < Q$

/ /

b) $Q' = 0{,}75Q$

$$\left|\frac{Q'-Q}{Q}\right| \cdot 100 = \left|\frac{0{,}75Q - Q}{Q}\right| \cdot 100 = \left|\frac{0{,}75-1}{1}\right| \cdot 100 = \underline{25\%}$$

6) (P3 - 1°09)

$R = 3\,\Omega$

$m_a = 150g$

$C = 40 cal/°C$

$I = 1{,}8 A$

$\Delta t = ?$

$T_0 = 18°C$

$T_f = 33{,}5°C$

$J_{exp} = 5{,}10\ J/cal$

$$J = \frac{W}{Q} \Rightarrow J = \frac{R.I^2\ \Delta T}{(m_a c + C)\Delta\theta}$$

$$\therefore \Delta T = \frac{J(m_a C + C)\Delta\theta}{R.I^2}$$

$$= \frac{5{,}10\ (150.1 + 40)(33{,}5-18)}{3.\ 1{,}8^2}$$

$\therefore \Delta T = 1545{,}2\ s \quad \therefore \Delta T = 25{,}75\ min$

7) (P2 2°08)

$m_a = 150g$

$C = 30 cal/°C$

$\Delta\theta = 15°C$

$J_{exp} = 4{,}52\ J/cal$

$J_{teo} = 4{,}186\ J/cal$

$$J_{exp} = \frac{W}{Q}$$

$W = 4{,}52 . (150 + 30). 15 = 12204$

$4{,}186 = \frac{12204}{Q'} \quad \therefore Q' = 2915{,}43$

$\Delta Q = Q' - Q$

$= 2915{,}43 - 2700 = \underline{215{,}43\ cal}$

8) (P3 - 1º07)

$R = 2\Omega$

$m_a = 150g$

$C = 30 cal/°C$

$I = 2A$

$\Delta T = 20min = 1200 s$

perda de 8% de energia elétrica

$T_0 = 26°C$ $T_f = ?$

$$J = \frac{RI^2 \Delta T . 0,92}{(m_a c + C)\Delta\theta}$$

$$\Delta\theta = \frac{RI^2 \Delta T . 0,92}{J(m_a c + C)}$$

$$= \frac{2.2^2 . 1200 . 0,92}{4,186(150+30)}$$

$\therefore \Delta\theta = 11,72$

$\Delta\theta = \theta_f - \theta_i$

$11,72 = \theta_f - 26$ $\therefore$ $\theta_f = 37,72 cal$

9) (P2 2º06) (Sem Resolução)

10) (P2 2º02)

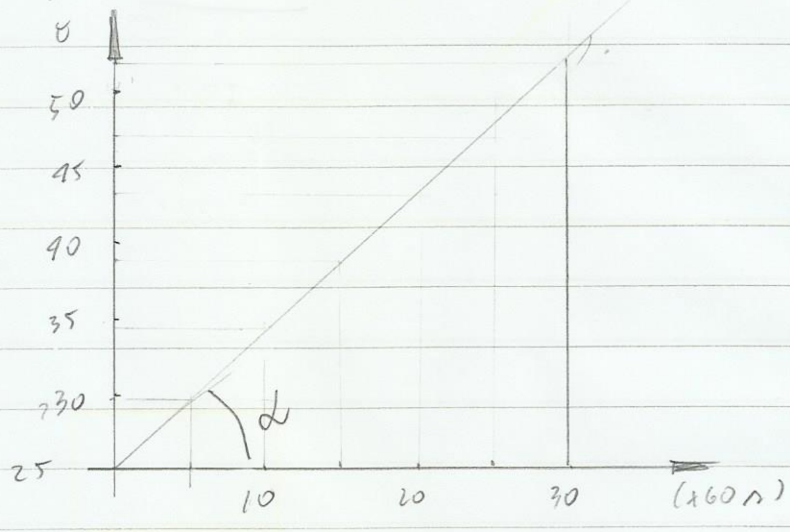
$m_a = 100g$

$T_0 = 25°C$

$C = 25 cal/°C$

$U = 4V$

$I = 2A$



$$\alpha = \frac{52,55 - 25}{30 - 0} = \frac{\Delta\theta}{\Delta t}$$

$\therefore \alpha = 0,01531 °C/s$

b) $J = \frac{U.I.\Delta T}{(m_a c_a + C).\Delta\theta}$; $\frac{\Delta T}{\Delta\theta} = \alpha^{-1} = \frac{1}{\alpha}$

$$J = \frac{4.2}{(100+25).0,01531}$$

$\therefore$ $J = 4,174 cal/g°C$

Apostila

3-) $J = 4{,}35\ J/cal$ $\qquad W = 2697$

$Q = 620$

$$J = \frac{2697}{620} = \underline{4{,}21\ J/cal}$$

4-) $J = 4{,}38\ J/cal$ $\qquad W = 4{,}38 \cdot 700 = 3066\ J$

$Q = 700\ cal$

$$Q' = \frac{3066}{4{,}186} = 732{,}44$$

$\Delta Q = Q' - Q$

$= 732{,}44 - 700 = \underline{32{,}44\ cal}$ Calor perdido

Gráficos monolog

Serve para linearizar funções do tipo $\boxed{y = a \cdot e^{\pm bx}}$

a: coeficiente linear

b: coeficiente angular da reta no papel monolog

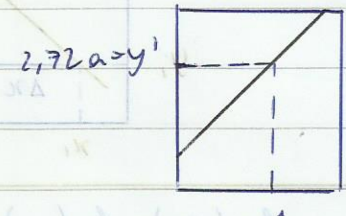
b>0 reta crescente

b<0 reta decrescente

$$\boxed{b = \frac{\ln(y_2) - \ln(y_1)}{x_2 - x_1}}$$

(Para $x = 0$ temos $y = a$)

Para $b > 0$



$bx' = 1 \therefore x' = \frac{1}{b}$ ou $b = \frac{1}{x'}$

Para x'

$y' = a e^1$ par ordenado $(x', 2{,}72a)$

$y' = 2{,}72a$

Para achar o "a" $\boxed{a = \frac{y}{e^{\pm bx}}}$

Para $b < 0$

$|b| = \frac{1}{x'}$

Para $x' \Rightarrow y' = a e^{-1} = 0{,}368a$

Gráfico Dilog

Serve para linearizar funções do tipo: $\boxed{y = a x^{\alpha}}$

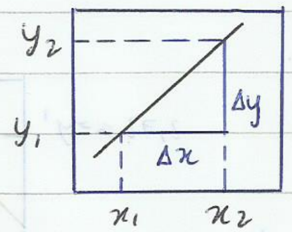
$\alpha > 0$ Crescente

$\alpha < 0$ Decrescente

α: coef. angular (α é adimensional)

a: coef. linear (pode ou não ter unidade)

Para obter α



$$\boxed{\alpha = \frac{\Delta y}{\Delta x} = \frac{\ln(y_2) - \ln(y_1)}{\ln(x_2) - \ln(x_1)} = \frac{\ln(y_2/y_1)}{\ln(x_2/x_1)}}$$

Monolog

11-) (P2-2005)

$$I = I_0 e^{\alpha t}$$

a) Em anexo

b) $y = a e^{-bx}$ $\qquad$ $y' = a e^{-1}$

$\therefore y' = 50 . e^{-1} = 18,39$

$\therefore a = 50$ $\qquad$ $\therefore x' = 15$

$$|b| = \frac{1}{x^{-1}} = \frac{1}{15} = 0,0667$$

$$\therefore I = 50 e^{-0,0667 t} \; (\mu A)$$

ou

$$|b| = \frac{\ln(y_2) - \ln(y_1)}{x_2 - x_1} = \frac{\ln(36) - \ln(2)}{5 - 57} = -0,06$$

$$a = \frac{y}{e^{bx}} = \frac{36}{e^{-0,06 . 5}} \quad \therefore \underline{a = 47,53} \qquad I = 47,53 . e^{-0,06 t}$$

c) $I(232) = 50 e^{-0,0667 . 232}$ $\therefore$

$$I(232) = 8,87 . 10^{-6} \; (\mu A)$$