Estatística Básica

- Espaço amostral: Conjuntos dos resultados do experimento (Ω)

$\Omega = \{1,2,3,4,5,6\}$

pontos amostrais: elementos do espaço amostral

Exemplo das definições
· lançamento de um dado honesto de 1 a 6

evento aleatório: qualquer subconjunto do espaço amostral

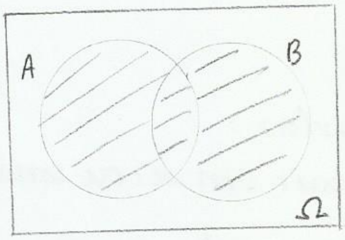
Evento A: saída de um número par.

$A = \{2,4,6\}$

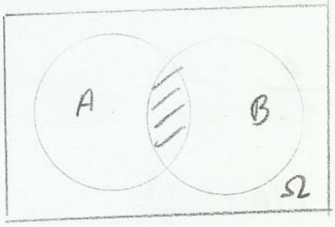
Função de probabilidade: É a função P que associa a cada evento da classe de eventos aleatórios, indicada por $F(\Omega)$, um único número real pertencente ao intervalo $[0,1]$.

Operações com eventos aleatórios

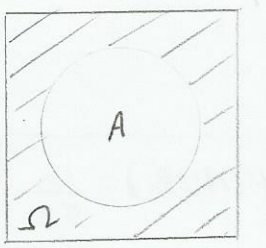
Reunião $A \cup B$



Interseção $A \cap B$



Complementação $\Omega - A = \bar{A}$



$P(\Omega) = 1$

$P(A \cup B) = P(A) + P(B)$

$P(A_1 \cup A_2 \cup \ldots \cup A_n) =$

$P(A_1) + P(A_2) + \ldots + P(A_n)$

Observação

$\sum$ eventos $= 1$

$P(\emptyset) = 0$

$P(A) + P(\bar{A}) = 1$

- Se $A \subset \Omega$ e $B \subset \Omega$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) \leq P(A) + P(B)$

Exemplos:

1-) Sendo $P(A) = x$, $P(B) = y$ $P(A \cap B) = z$

a) $P(\bar{A} \cup \bar{B})$; $P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - z$

b) $P(\bar{A} \cap \bar{B})$; $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - (x + y - z) = 1 - x - y + z$

c) $P(\bar{A} \cap B)$ ×

Cálculo da propabilidade de um evento

$$\boxed{P(A) = \frac{k}{n} \quad \begin{array}{l} k;\ \text{pontos amostrais} \\ n;\ \text{número de pontos} \end{array}}$$

Exemplos do livro

1-) Retira-se 1 carta de 52 cartas.
Qual a probabilidade de sair um rei ou uma carta de espada?

$\Omega = \{$"todas as cartas do baralho"$\}$

$A = \{Ro, Re, Rp, Rc\}$ $P(A) = \frac{4}{52}$

$B = \{1e, 2e, \cdots 10e, Je, De, Re\}$ $P(B) = \frac{13}{52}$

$\therefore P(A \cap B) = \frac{1}{52}$

ou

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$ $\therefore$ $\boxed{P(A \cup B) = \frac{16}{52}}$

2.) 5R, 4r, 6M, 3m

$A = \{5R, 6M\}$ $B = \{4r, 3m\}$ $C = \{5R, 4r\}$ $D = \{6M, 3m\}$

Calcular

a) $P(B \cup D)$

$$P(B \cup D) = P(B) + P(D) - P(B \cap D)$$
$$= \frac{7}{18} + \frac{9}{18} - \frac{3}{18} = \frac{13}{18}$$

b) $P(\bar{A} \cap \bar{C})$

$$P(\bar{A} \cap \bar{C}) = P(\overline{A \cup C}) = 1 - P(A \cup C) = 1 - (P(A) + P(C) - P(A \cap C))$$
$$= 1 - \left(\frac{11}{18} + \frac{9}{18} - \frac{5}{18}\right) = 1 - \frac{15}{18} = \frac{3}{18} = \frac{1}{6}$$

Exercícios

1)

a) $\Omega = \{1, 2, 3, 4, 5, 6\}$

b) $A = \{4\}$ $P(A) = \frac{1}{6}$

c) $B = \{2, 4, 6\}$ $P(B) = \frac{3}{6} = \frac{1}{2}$

d) $C = \{1, 2, 3, 4\}$ $P(C) = \frac{4}{6} = \frac{2}{3}$

e) $D = \{6\}$ $P(D) = \frac{1}{6}$

f) $E = \{1, 2, 3, 4, 5, 6\}$ $P(E) = \frac{6}{6} = 1$

g) $F = \{2, 4, 6\}$ $P(F) = \frac{3}{6}$

$G = \{1, 2, 3, 4\}$ $P(G) = \frac{4}{6}$

$$P(F \cup G) = P(F) + P(G) - P(F \cap G)$$
$$= \frac{3}{6} + \frac{4}{6} - \frac{2}{6} \quad \therefore \quad P(F \cup G) = \frac{5}{6}$$

4-) $\Omega = \{1,2,3,4,5,6,1,2,3,4,5,6\}$

a) $A = \{2,4,6,2,4,6\} = \frac{6}{12} = \frac{1}{2}$

b) $B = \{1,3,5,1,3,5\} = \frac{6}{12} = \frac{1}{2}$

c) $C = \{4,6,5,5,6,4\} = \frac{6}{12} = \frac{1}{2}$

Probabilidade condicional

$P(A|B) = \frac{P(A \cap B)}{P(B)}$, se $P(B) \neq 0$, seja $A \subset \Omega$ e $B \subset \Omega$. Definimos a probabilidade condicional de A, dado que B ocorre

$P(B|A) = \frac{P(B \cap A)}{P(A)}$, se $P(A) \neq 0$, Probabilidade condicional de B, dado que A ocorre

Exemplo:

$\Omega = \{2B, 3P, 4V\}$

a)

Estatística Básica

Aula 1

1-) a) $\Omega = \{1,2,3,4,5,6\}$

b) $P(A) = \frac{1}{6}$

c) $P(B) = \frac{3}{6} = \frac{1}{2}$

d) $P(C) = \frac{4}{6}$

e) $P(\emptyset) = \frac{0}{6} = 0$

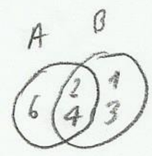
f) $P(E) = \frac{6}{6} = 1$

g) $\boxed{\text{"ou"} \rightarrow \text{significa união } (\cup)}$

$F = \{2,4,6\}$ $\quad G = \{1,2,3,4\}$

$P(F) = \frac{3}{6}$ $\quad P(G) = \frac{4}{6}$

$$P(F \cup G) = P(F) + P(G) - P(F \cap G)$$
$$= \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \frac{5}{6}$$



5 elementos

2-)

	H	M
+21	5	6
-21	4	3
	9	9

A) $P(A) = \frac{9}{18} = \frac{1}{2}$

B) $P(B) = \frac{3}{18} = \frac{1}{6}$

c) $P(C) = \frac{11}{18}$

3-) $\Omega = \{\text{"52 cartas"}\}$

a) $P(A) = \frac{4}{52} = \frac{1}{13}$

b) $P(B) = \frac{13}{52} = \frac{1}{4}$

c) $P(C) = \frac{1}{52}$

d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

4-)

	1	2	3	4	5	6
1						
2		(2,2)		(2,4)		(2,6)
3						
4		(4,2)		(4,4)		(4,6)
5						
6		(6,2)		(6,4)		(6,6)

$\Omega = \{\text{"36 resultados"}\}$

a) $P(A) = \frac{9}{36} = \frac{1}{4}$

b) (2,2); (2,3); (2,5); (3,2); (3,3); (3,5);
(5,2); (5,3); (5,5);

$P(B) = \frac{9}{36} = \frac{1}{4}$

c) (4,6); (5,5); (6,4)

$P(C) = \frac{3}{36} = \frac{1}{12}$

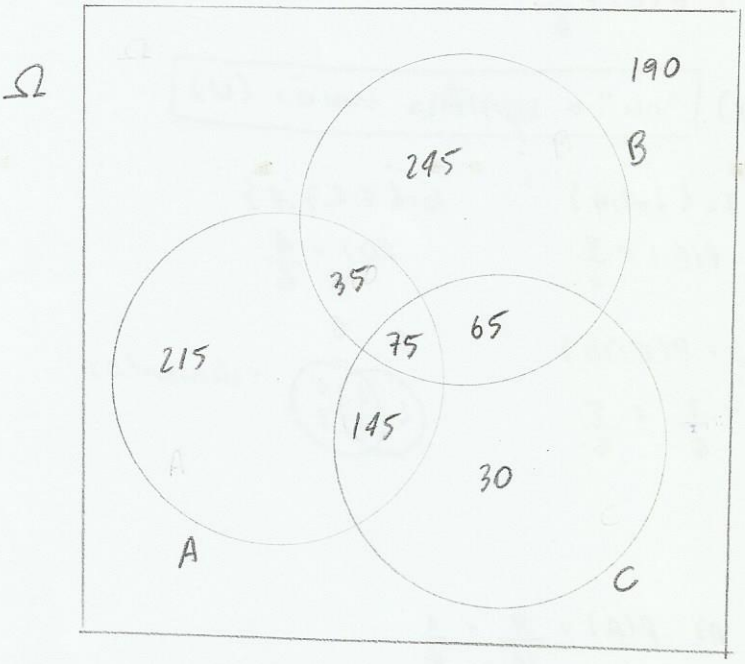
d) $P(D) = \frac{0}{36} = 0$

f) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{4} - \frac{1}{36} = \frac{17}{36}$

e) $P(E) = \frac{36}{36} = 1$

5)



$\Omega = \{$"1000 famílias"$\}$

A: 470   A e B: 110

B: 420   A e C: 220

C: 315   B e C: 140

A, B, C: 75   T = 1000

a) $P(A) = \frac{190}{1000} = \frac{19}{100}$

b) $P(B) = \frac{(215+245+30)}{1000} = \frac{49}{100}$

c) $P(C) = \frac{(35+75+65+145)}{1000} = \frac{8}{25}$

Probabilidade Condicional

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \; ; \; P(B) \neq 0$$

Prob. de A dado B ou Prob. de A sabendo que ocorreu B

Obs.: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

O que você quer calcular   O que você já sabe

Exercícios

1) 30 H e 20 M (+ 1,65)

Total: (80H e 120 M)

	H	M	
+1,65	30	20	50
-1,65	50	100	150
	80	120	200

a) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) = \frac{50}{200} = \frac{1}{4}$

$P(A \cap B) = \frac{30}{200} = \frac{3}{20}$

$\therefore P(A|B) = \frac{3}{20} \div \frac{1}{4}$

$\therefore P(A|B) = \frac{3}{5}$

b) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) = \frac{80}{200} = \frac{2}{5}$ $\quad P(A \cap B) = \frac{30}{200} = \frac{3}{20}$

$P(A|B) = \frac{3}{20} \div \frac{2}{5} = \frac{3}{8}$

c) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) = \frac{150}{200} = \frac{3}{4}$ $\quad P(A \cap B) = \frac{100}{200} = \frac{2}{4}$

$P(A|B) = \frac{2}{4} \div \frac{3}{4} = \frac{2}{3}$

d) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) = \frac{120}{200} = \frac{3}{5}$ $\quad P(A \cap B) = \frac{100}{200} = \frac{1}{2}$

$P(A|B) = \frac{1}{2} \div \frac{3}{5}$ $\therefore P(A|B) = \frac{5}{6}$

2) - 2000 lâmpadas

X, Y e Z : 2000

X: 500 (400 boas)

Y: 700 (600 boas)

Z: 800 (500 boas)

	Boas	Ruins	
X	400	100	500
Y	600	100	700
Z	500	300	800
	1500	500	2000

a) $P(B) = \frac{1500}{2000} = \boxed{\frac{3}{4}}$

b) $P(X/D) = \frac{P(X \cap D)}{P(D)}$

$P(D) = \frac{500}{2000} = \frac{1}{4}$ $\quad P(X \cap D) = \frac{100}{2000} = \frac{1}{20}$

$P(X/D) = \frac{1}{20} \div \frac{1}{4} = \boxed{\frac{1}{5}}$

c) $P(D/X) = \frac{P(D \cap X)}{P(X)}$

$P(X) = \frac{500}{2000} = \frac{1}{4}$ $\quad P(D \cap X) = \frac{1}{20}$ $\quad \therefore P(D/X) = \boxed{\frac{1}{5}}$

3) $P(A) = \frac{1}{3}$ $\quad P(B) = \frac{3}{4}$ $\quad P(A \cup B) = \frac{11}{12}$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$; $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

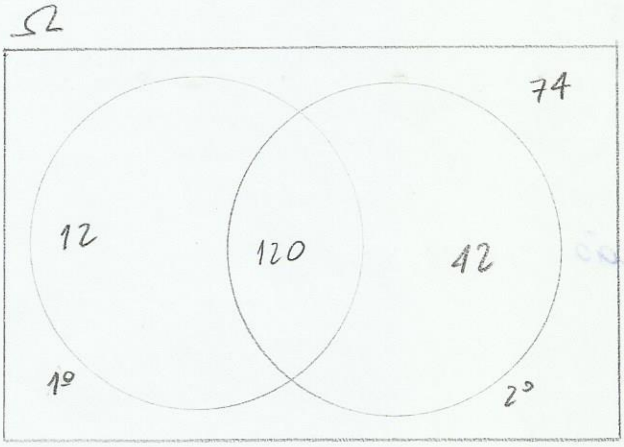
$P(A \cap B) = -P(A \cup B) + P(A) + P(B)$

$P(A/B) = \frac{-P(A \cup B) + P(A) + P(B)}{P(B)}$

$P(A/B) = \frac{-\frac{11}{12} + \frac{1}{3} + \frac{3}{4}}{\frac{3}{4}}$ $\quad \therefore P(A/B) = \boxed{\frac{2}{9}}$

Exercícios do livro texto

2-7) 132 alunos — acertaram o 1º ✓
86 alunos — erraram o 2º ✓
120 alunos — acertaram os 2 ✓
54 alunos — apenas 1 acertaram ✓



a) $P(A) = \frac{74}{246} = \frac{37}{124} = 0,298$

b) $P(B) = \frac{42}{246} = \frac{21}{124}$

4.) se temos 100 pessoas

	A	B	AB	O
Prob. S	0,2			
Prob. N		0,9	0,95	

0,19
0,49
0,32 ✓

31.) A —> 4000 → 60% são 600s
B —> 6000 60% são 600s

	B	R	
A	3200	800	4000
B	3600	2400	6000
	6800	3200	10000

a) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(A) = \frac{40\cancel{00}}{10\,\cancel{000}} = \frac{2}{5}$ $\quad P(B \cap A) = \frac{32\cancel{00}}{100\cancel{00}} = \frac{8}{25}$

$P(B|A) = \frac{8}{25} \div \frac{2}{5} = \frac{4}{5} = 0,8$

$P(B|R)$

$P(R|B) = \frac{P(R \cap B)}{P(B)}$

condic

$\frac{2400/10000}{6000/10000} = \frac{2400}{6000}$

$P(B \cap R)$

b) $\frac{6800}{10000} = \underline{0{,}68}$

d) $P(R|B) = \frac{P(R \cap B)}{P(B)}$

$P(B) = \frac{32\cancel{00}}{100\cancel{00}} = \frac{8}{25}$ $P(R \cap B) = \frac{2400}{10000} = \frac{6}{25}$

$P(R|B) = \frac{6}{25} \div \frac{8}{25} = \underline{0{,}75}$

c) $P(R \cap B) = \frac{2400}{10000} = 0{,}24$

Aula 2 - Permutação

- Permutação simples: $P_n = n!$

SOL
SLO
LSO
LOS
OLS
OSL

6 resultados

$P_3 = 3! = \underline{6}$

- Permutação com repetição (PR)

$$\boxed{(PR)^{n}_{n_1, n_2, n_3 \dots n_n} \quad \frac{n!}{n_1! n_2! n_3! \dots n!}}$$

* AMA
AAM
MAA

$\underline{3}$ resultados

$(PR)^{3}_{2,1} = \frac{3!}{2!1!} = \underline{3}$

* MACACA

$(PR)^{6}_{3,2,1} = \frac{6!}{3!2!1!} = \frac{6.5.4.\cancel{3!}}{\cancel{3!}2!1!} = \underline{60}$

* ARARA

$(PR)^{5}_{3,2} = \frac{5!}{3!\,2!} = \underline{10}$

Lista

1) 5b, 4V, 3A — É extraído 3 bolas sem reposição

a) $P(\bar{V} \cap \bar{V} \cap \bar{V}) = \frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10} = \frac{14}{55}$ //

b) $P(V \cap \bar{V} \cap \bar{V}) \cdot (PR)^{3}_{1,2} = \frac{4}{12} \cdot \frac{8}{11} \cdot \frac{7}{10} \cdot \frac{3!}{1!\,2!} = \frac{28}{55}$ //

c) $P(B \cap B \cap B) + P(V \cap V \cap V) + P(A \cap A \cap A) =$

$= \frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} + \frac{4}{12} \cdot \frac{3}{11} \cdot \frac{2}{10} + \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10} = \frac{3}{44}$ //

2) $P(A) = \frac{2}{3}$ $P(B) = \frac{4}{5}$ $P(C) = \frac{7}{10}$

a) $P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= \frac{1}{3} \cdot \frac{1}{5} \cdot \frac{3}{10} = \frac{1}{50}$

Exercícios

1) $p(\text{def}) = 0,1$ ← Probabilidade fixa $p(\text{Boa}) = 0,9$

Retiram-se 5 peças

a) no máximo 2 boas?

1B, 4D e 2B e 3D e nenhuma

$p(\text{Max}) = (B \cap \bar{B} \cap \bar{B} \cap \bar{B} \cap \bar{B}) \cdot (PR)^{5}_{4,1} + (B \cap B \cap \bar{B} \cap \bar{B} \cap \bar{B}) \cdot (PR)^{5}_{3,2} + (\bar{B} \cap \bar{B} \cap \bar{B} \cap \bar{B} \cap \bar{B})$

$= 0,9 \cdot 0,1^{4} \cdot \frac{5!}{4!} + 0,9^{2} \cdot 0,1^{3} \cdot \frac{5!}{3!\,2!} + 0,1^{5}$ $\therefore$ $\boxed{p(\text{Max}) = 0,00856}$

b) pelo menos 4 sejam boas?

4B, 5B

$p(\text{pelo menos } 4) = (B \cap B \cap B \cap B \cap \bar{B}) \cdot (PR)^{5}_{4,1} + (B \cap B \cap B \cap B \cap B)$

$= 0,9^{4} \cdot 0,1 \cdot \frac{5!}{4!} + 0,9^{5}$ $\boxed{\therefore p(\text{pelo menos } 4) = 0,91854}$

c) exatamente 3 seja boas

$$p(ex.\ 3B) = (B\cap B\cap B\cap \bar{B}\cap \bar{B}).(PR)^{5}_{3,2}$$

$$= 0{,}9^3 . 0{,}1^2 . \frac{5!}{3!\,2!} \quad \therefore \quad P(ex.\ 3B) = 0{,}0729$$

d) pelos menos 1 seja defeituoso.

1D, 2D, 3D, 4D, 5D

$$P(\text{pelo menos } 1D) = 1 - p(\text{nenhum})$$

$$= 1 - P(B\cap B\cap B\cap B\cap B)$$

$$= 1 - 0{,}9^5 \quad \therefore \boxed{p(\text{pelo menos } 1D) = 0{,}40951}$$

2) (5V, 4A, 5B) "Urna" ; Retira-se 4 bolas da urna

a) Exatamente 3A

$$P(3A) = P(A\cap A\cap A\cap \bar{A}).(PR)^{4}_{3,1}$$

$$= \frac{4}{14} . \frac{3}{13} . \frac{2}{12} . \frac{10}{11} . \frac{4!}{3!} \quad \therefore \boxed{P(3A) = 0{,}03996}$$

b) Pelo menos 1V ⇒ V, 2V, 3V, 4V

0V – 4$\bar{V}$
1V – 3$\bar{V}$ ✓
2V – 2$\bar{V}$ ✓
3V – 1$\bar{V}$ ✓
4V – 0$\bar{V}$ ✓

$$P(\text{pelo menos } 1V) = 1 - p(\bar{V}\cap\bar{V}\cap\bar{V}\cap\bar{V})$$

$$= 1 - \frac{9}{14} . \frac{8}{13} . \frac{7}{12} . \frac{6}{11}$$

$$\therefore \boxed{p(\text{pelo menos } 1V) = 0{,}8741}$$

3) 6 lamp. de 40W
3 lamp. de 60W } caixa
1 lamp de 100W

Retira-se 5 lâmpadas com reposição

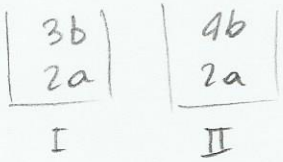
- Qual prob. 3 de 40W, 1 de 60W e 1 de 100W

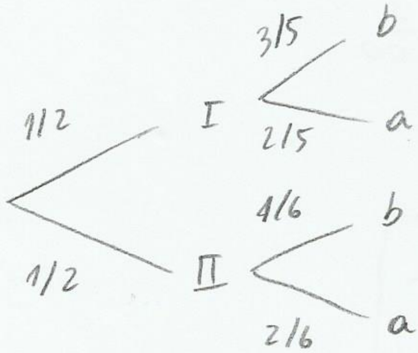
$$P(A) = ((3\text{ de }40W)\cap(1\text{ de }60W)\cap(1\text{ de }100W)).\ (PR)^{5}_{3,1,1}$$

$$= \left(\frac{6}{10}\right)^3 . \left(\frac{3}{10}\right) . \left(\frac{1}{10}\right) . \frac{5!}{3!\,1!\,1!} \quad \therefore \boxed{P(A) = 0{,}1296}$$

Teorema da Probabilidade total

Ex:





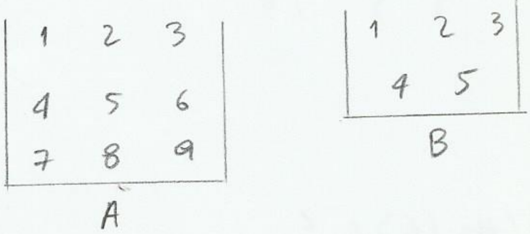
$p(b) = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$

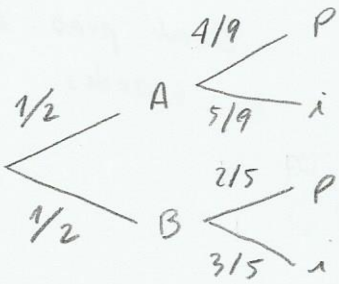
$p(b) = \frac{1}{2} \cdot \frac{4}{6} = \frac{1}{3}$

$p(b) = \frac{3}{10} + \frac{1}{3} = \frac{9+10}{30} \quad \therefore \quad \boxed{p(b) = \frac{19}{30}}$

Teorema de Bayes

1-)





$\rightarrow p(A \cap par) = \frac{1}{2} \cdot \frac{4}{9} = \frac{2}{9}$

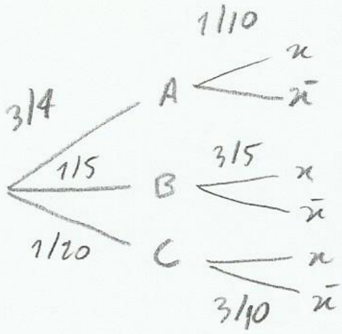
$\rightarrow p(B \cap par) = \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{5}$

$p(par) = \frac{2}{9} + \frac{1}{5} = \frac{10+9}{45} = \frac{19}{45}$

$p(A|par) = \frac{p(A \cap par)}{p(par)} = \frac{2}{9} \div \frac{19}{45}$

$\therefore p(A|par) = \frac{10}{19}$

2-)



$p(x \cap A) = \frac{3}{4} \cdot \frac{1}{10} = \frac{3}{40}$

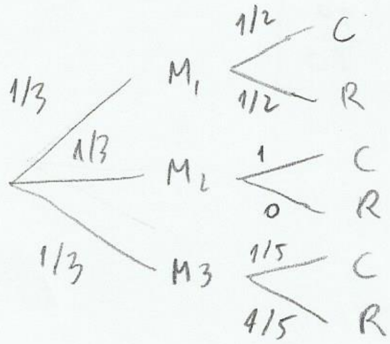
$p(x \cap B) = \frac{1}{5} \cdot \frac{3}{5} = \frac{3}{25}$

$p(x \cap C) = \frac{1}{20} \cdot \frac{3}{10} = \frac{3}{200}$

$p(x) = \frac{3}{40} + \frac{3}{25} + \frac{3}{200} = \frac{21}{100}$

$p(x/B) = \frac{3}{25} \div \frac{21}{100} \quad \therefore \quad \boxed{p(x/B) = \frac{4}{7}}$

3-)



$P(M_1 \cap C) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$P(M_2 \cap C) = 1 \cdot \frac{1}{3} = \frac{1}{3}$

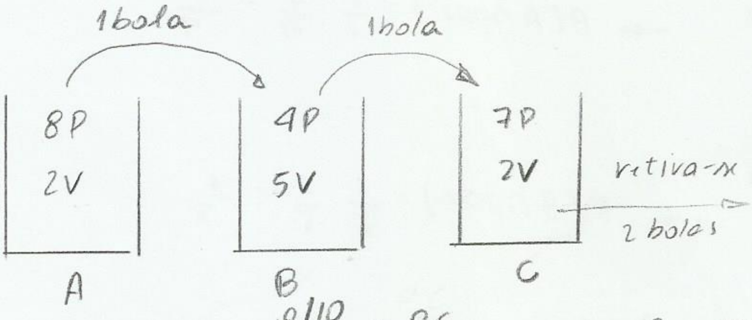
$P(M_3 \cap C) = \frac{1}{3} \cdot \frac{1}{5} = \frac{1}{15}$

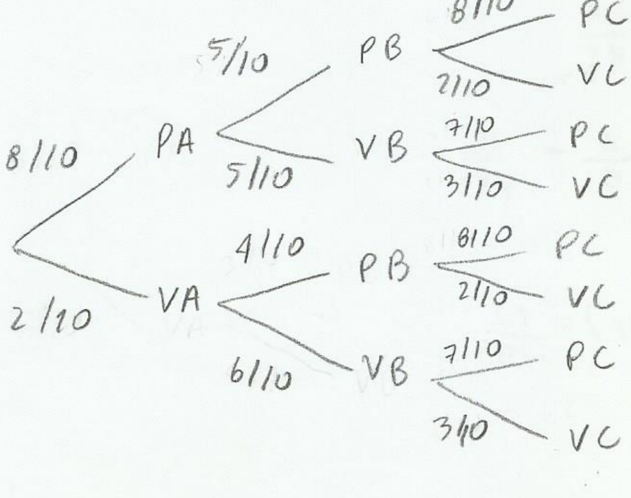
$P(C) = \frac{1}{6} + \frac{1}{3} + \frac{1}{15} = \frac{17}{30}$

$P(M_3/C) = \frac{1}{15} \div \frac{17}{30} \quad \therefore \quad P(M_3/C) = \frac{2}{17}$

4-)

Qual prob. de 2 verdes





$P(C) = \frac{8}{10} \cdot \frac{5}{10} \left(\frac{2}{10}\right)^2 + \frac{8}{10} \cdot \frac{5}{10} \left(\frac{3}{10}\right)^2 +$

$\frac{2}{10} \cdot \frac{4}{10} \left(\frac{2}{10}\right)^2 + \frac{2}{10} \cdot \frac{6}{10} \cdot \left(\frac{3}{10}\right)^2$

$\boxed{\therefore P(C) = 0{,}066}$

Exercícios resolvidos Cap. 2

3-)

10 pessoas — $x > 20$

20 pessoas — $10 < x < 20$

70 pessoas — $x < 10$

Determine a prob. de pelo menos 3 pessoas ganhe menos 10

1p- ; 2p- ; 3p-

$$P(\text{pelo menos } 3) = P(P \cap \bar{P} \cap \bar{P}) \cdot (PR)^{3}_{2,1} + P(P \cap P \cap \bar{P}) \cdot (PR)^{3}_{2,1} + P(P \cap P \cap P)$$

$$= \frac{70}{100} \cdot \left(\frac{30}{100}\right)^2 \cdot \frac{3!}{2!} + \left(\frac{70}{100}\right)^2 \cdot \frac{30}{100} \cdot \frac{3!}{2!} + \left(\frac{70}{100}\right)^3$$

$$\therefore \boxed{P(\text{pelo menos } 3) = 0{,}973}$$

4-)

A ganha 60

B ganha 40

20 empates

A e B jogam 120 partidas

Eles jogam mais 3 vezes

a) $P(A) = P(A \cap A \cap A) = \left(\frac{60}{120}\right)^3 \quad \therefore \quad P(A) = \frac{1}{8}$

b) $P(B) = P(E \cap E \cap \bar{E}) \cdot (PR)^{3}_{2,1} = \left(\frac{20}{120}\right)^2 \cdot \frac{100}{120} \cdot \frac{3!}{2!\,1!} \quad \therefore \quad P(B) = \frac{5}{72}$

c) $P(C) = P(A \cap B \cap A) + P(B \cap A \cap B)$

$$= \left(\frac{60}{120}\right)^2 \cdot \frac{40}{120} + \left(\frac{40}{120}\right)^2 \cdot \frac{60}{120} \quad \therefore \quad \boxed{P(C) = \frac{5}{36}}$$

5-)

retira-se 1 bola

4A

6B

dobra-se as bolas azuis

1º T: 4A, 6B

2º T: 8A, 6B

3º T: 16A, 6B

$$P(B) = P(B_1) + P(\bar{B}_1 \cap B_2) + P(\bar{B}_1 \cap \bar{B}_2 \cap B_3)$$

$$= \frac{6}{10} + \frac{4}{10} \cdot \frac{6}{14} + \frac{4}{10} \cdot \frac{8}{14} \cdot \frac{6}{22}$$

$$\therefore P(B) = 0{,}8338$$

6.) - lote: 120 peças

- foi selecionado 5 peças

- há 20 defeitos

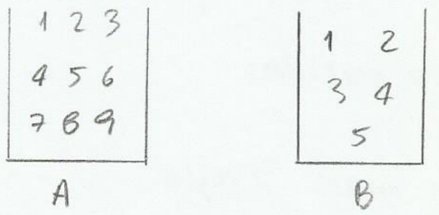
a) prob. do lote ser aceito (Aceitável 0 ou 1 defeito)

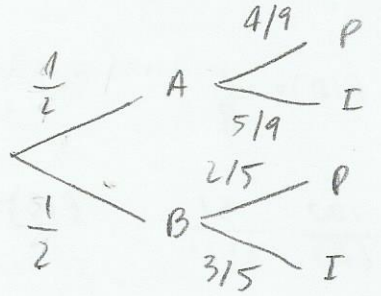
$$p(\text{aceito}) = p(A \cap A \cap A \cap A \cap A) + p(A \cap A \cap A \cap A \cap \bar{A}) \cdot (PR)^{5}_{4,1}$$

$$= \underbrace{\left(\frac{100}{120}\right)^5}_{0,4019} + \underbrace{\left(\frac{100}{120}\right)^4 \cdot \frac{20}{120} \cdot \frac{5!}{4!}}_{0,4019} \quad \therefore \boxed{P(\text{aceito}) = 0,8038}$$

b) $$P(1D/A) = \frac{P(1D \cap A)}{P(A)} = \frac{0,4019}{0,8038} \quad \therefore \boxed{P(1D/A) = 0,5}$$

7.)





$$P(Ap) = \frac{1}{2} \cdot \frac{4}{9} = \frac{2}{9}$$

$$P(Bp) = \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{5}$$

$$P(\text{nº seja Par}) = \frac{2}{9} + \frac{1}{5} = \frac{19}{45}$$

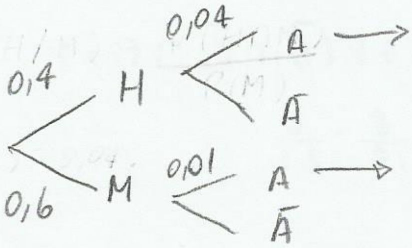
$$P(A/P) = \frac{P(A \cap P)}{P(P)} = \frac{2}{9} \div \frac{19}{45} \quad \therefore P(A/P) = \frac{10}{19}$$

8.) 4% de homens ⇒ +1,75

1% das mulheres ⇒ +1,75

60% do estudantes são mulheres

+ Um estudante é escolhido ao acaso e tem mais de 1,75. Qual a prob. de ser homem?



$$p(H \cap A) = 0,4 \cdot 0,04 = 0,016$$

$$p(M \cap A) = 0,6 \cdot 0,01 = 0,006$$

$$P(A) = 0,022$$

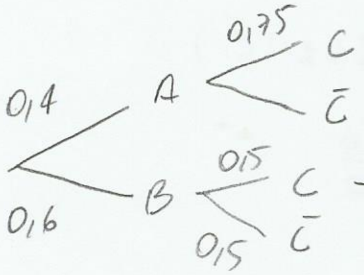
$$P(H/A) = \frac{0,016}{0,022} = \frac{8}{11}$$

9-) (Resolução no inicio da matéria)

10-) 11-) Foi resolvido pelo professor

12-)

$$P(A|C) = \frac{P(A\cap C)}{PC}$$



$C \to P(A\cap C) = 0,4 \cdot 0,75 = 0,3$

$C \to P(B\cap C) = 0,6 \cdot 0,5 = 0,3$

$P(C) = 0,3 + 0,3 = 0,6$

$P(A|C) = \dfrac{0,3}{0,6} = 0,5$

13-)

$$\left.\begin{matrix} 2A \\ 2B \\ 1C \end{matrix}\right\} X \qquad \left.\begin{matrix} 2A \\ 1B \\ 1C \end{matrix}\right\} Y$$

- Retira-se uma bola de cada urna. Qual é a prob. de saírem 2 bolas brancas da mesma cor

$P(\text{mesma cor}) = P(A\cap A) + P(B\cap B) + P(C\cap C)$

$= \frac{2}{5}\cdot\frac{2}{4} + \frac{2}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{4} = \frac{7}{20}$

$P(B\cap B \,|\, \text{mesma cor}) = \frac{1}{10} \div \frac{7}{20} = \frac{2}{7}$

14-)

a) $P(A) = \frac{60}{100} = 0,6$

b) $P(B) = \frac{40}{100} = 0,4$

c) $P(A_T \cap A_I) = \frac{24}{100} = 0,24$

$P(A_T \cup A_I) = P(A_T) + P(A_I) - P(A_I \cap P_I)$

$= \frac{60}{100} + \frac{40}{100} - \frac{24}{100} = 0,76$

b) $P(M) = \frac{20}{100} = 0,2$

$P(A) = \frac{60}{100} = 0,6$

$P(M)\cdot P(A) = 0,2 \cdot 0,6 = 0,12$

$P(M\cap A) = \frac{12}{100} = 0,12$

$\therefore$ Como $P(M\cap A) = P(M)\cdot P(A)$,

Logo eles são independentes

c) ×

0,6 A

0,4 B

$$p(\text{diferente}) = P(A \cap B) + P(A \cap B)$$
$$= 0{,}6 \cdot 0{,}4 + 0{,}6 \cdot 0{,}4$$
$$\therefore \boxed{p(\text{diferentes}) = 0{,}48}$$

×

$$P(C_1 \cap C_2) = P(C_1 \cap C_2) + P(C_1 \cap \bar{C_2}) \cdot 2! + P(C_2 \cap \bar{C_1}) \cdot 2!$$
$$= \frac{24}{60} \cdot \frac{16}{40} + \frac{24}{60} \cdot \frac{24}{40} + \frac{36}{60} \cdot \frac{16}{40}$$
$$= 0{,}64$$

ou

$C_1 \; C_2$

$C_1 \; \bar{C_2}$ ✓

$\bar{C_1} \; C_2$ ✓

$\bar{C_1} \; \bar{C_2}$ ✓

$$P(\text{pelo menos 1}) = 1 - P(\bar{C_1} \cap \bar{C_2})$$
$$= 1 - \frac{36}{60} \cdot \frac{24}{40} = 0{,}64$$

15-) prob (A ultrapasse 17,30) = 0,7 (em um saltro triplo)

$p(u) = 0{,}7 \quad p(\bar{u}) = 0{,}3$

p(pelos menos 1 salto de 4) = ?

$$p(\text{pelo menos 1}) = 1 - p(\bar{u} \cap \bar{u} \cap \bar{u} \cap \bar{u})$$
$$= 1 - 0{,}3^4 = 0{,}9919$$

16-) Dado A $\begin{cases} 3B \\ 3P \end{cases}$ Dado B $\begin{cases} 2B \\ 2P \\ 2V \end{cases}$ Dado C $\begin{cases} 2B \\ 4P \end{cases}$ Dado D $\begin{cases} 3b \\ 3p \end{cases}$

É lançado 4 dados

a) p(pelo menos uma seja branco)

$$= 1 - p(\bar{B_A} \cap \bar{B_B} \cap \bar{B_C} \cap \bar{B_D})$$
$$= 1 - \frac{3}{6} \cdot \frac{4}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} = \boxed{0{,}8889} = \boxed{\frac{8}{9}}$$

b) $p(\text{sejam pretos}) = ?$

$= P(P_A \cap P_B \cap P_C \cap \bar{P}_D) + P(P_A \cap P_B \cap \bar{P}_C \cap P_D) + P(P_A \cap \bar{P}_B \cap P_C \cap P_D) +$
$P(\bar{P}_A \cap P_B \cap P_C \cap P_D)$

$= \frac{3}{6}\cdot\frac{2}{6}\cdot\frac{4}{6}\cdot\frac{3}{6} + \frac{3}{6}\cdot\frac{2}{6}\cdot\frac{2}{6}\cdot\frac{3}{6} + \frac{3}{6}\cdot\frac{4}{6}\cdot\frac{4}{6}\cdot\frac{3}{6} + \frac{3}{6}\cdot\frac{2}{6}\cdot\frac{4}{6}\cdot\frac{3}{6}$

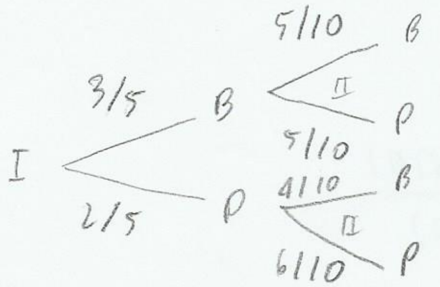
$\boxed{= \frac{1}{4}}$

17.) $I \begin{cases} 3B \\ 2P \end{cases}$ $II \begin{cases} 4B \\ 5P \end{cases}$ $III \begin{cases} 3B \\ 4P \end{cases}$

passa 1 bola

retira 1 bola

retiro 2 bolas



$P(B \cap B) = \frac{3}{5}\cdot\frac{5}{10} = \frac{3}{10}$

$P(P \cap B) = \frac{3}{5}\cdot\frac{5}{10} = \frac{3}{10}$

$P(P \cap B) = \frac{2}{5}\cdot\frac{4}{10} = \frac{4}{25}$

$P(P \cap P) = \frac{2}{5}\cdot\frac{6}{10} = \frac{6}{25}$

$P(P) = \frac{3}{10} + \frac{6}{25} = \frac{27}{50}$

$P(B) = \frac{3}{10} + \frac{4}{25} = \frac{23}{50}$

$P(B_{III}) = \frac{3}{7}\cdot\frac{2}{6} = \frac{1}{7}$

$P(P_{III}) = \frac{4}{7}\cdot\frac{3}{6} = \frac{2}{7}$

$P(\text{iguais}) = P(B \cap B_{III}) + P(P \cap P_{III})$
$= \frac{23}{50}\cdot\frac{1}{7} + \frac{27}{50}\cdot\frac{2}{7} = \boxed{\frac{11}{50}}$

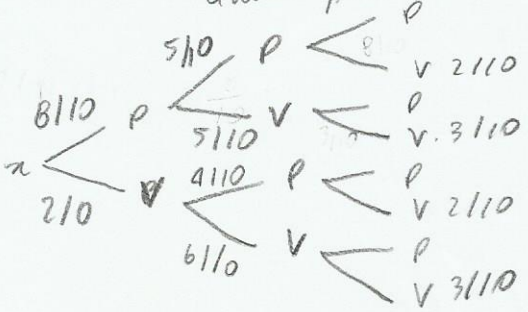
18.) $x \begin{cases} 8P \\ 2V \end{cases}$ $y \begin{cases} 4P \\ 5V \end{cases}$ $z \begin{cases} 2V \\ 7P \end{cases}$ $\longrightarrow$ retiro-se 2 bolas, com reposição

1 bola

1 bola

Qual prob. de 2 verdes?

$\boxed{P(V) = 0{,}060}$



$P(V_1) = \frac{8}{10}\cdot\frac{5}{10}\cdot\frac{2}{10}\cdot\frac{2}{10} = \frac{2}{125}$

$P(V_2) = \frac{8}{10}\cdot\frac{5}{10}\cdot\frac{3}{10}\cdot\frac{3}{10} = \frac{9}{250}$

$P(V_3) = \frac{2}{10}\cdot\frac{4}{10}\cdot\frac{2}{10}\cdot\frac{2}{10} = \frac{2}{625}$

$P(V_4) = \frac{2}{10}\cdot\frac{6}{10}\cdot\frac{3}{10}\cdot\frac{3}{10} = \frac{27}{2500}$

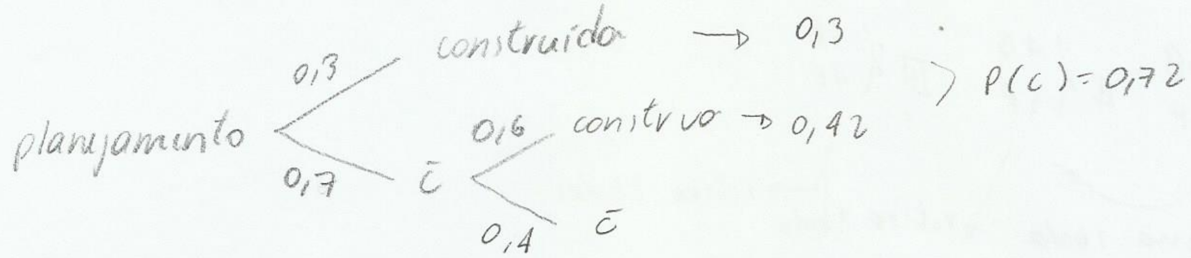
EB

19-) $p(\text{aluno}) = 30\% = 0{,}3$

$p(\text{aurto}) = \frac{1}{4} = 0{,}25$

$$p(S/A) = \frac{P(S \cap A)}{P(A)} = \frac{0{,}3}{}$$

20-) $p(\text{3anos}) = 0{,}30$



$$p(\text{entrar}) = \frac{0{,}3}{0{,}72} = \frac{5}{7}$$

21-) $X \begin{cases} 6B \\ 4A \end{cases}$ $Y \begin{cases} 3B \\ 5A \end{cases}$

$$P(A_x/A) = \frac{P(A_x \cap A)}{P(A)}$$

passam 2bolas

retira duas bolas com reposição

$$P(A_x/A) = \frac{\frac{49}{750}}{\frac{511}{1500}} = \boxed{0{,}1918}$$

Se passar 2 bolas (2 azuis):

$$p(\text{2bolas azuis}) = \frac{7}{10} \cdot \frac{7}{10} \cdot \frac{4}{10} \cdot \frac{3}{9} = \frac{49}{750}$$

Se passar 2bolas (2 brancas):

$$p(\text{2bolas brancas}) = \frac{5}{10} \cdot \frac{5}{10} \cdot \frac{6}{10} \cdot \frac{5}{9} = \frac{1}{12}$$

Se passar 2bolas (1branca e 1azul)

$$p(\text{2 bolas} \neq) = \frac{6}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} \cdot \frac{4}{9} \cdot 2! = \frac{24}{125}$$

$$P(\text{azul}) = \frac{511}{1500}$$

Exercícios propostos

9-)

$$\text{Urna} \begin{cases} 10V \\ 6A \end{cases} \quad \text{Retira-se 2 bolas}$$

a) sejam verdes:

$$P(2V) = \frac{10}{16} \cdot \frac{9}{15} = \frac{3}{8}$$

b) sejam iguais:

V V
V A
A V
A A

$$P(\text{iguais}) = P(V \cap V) + P(A \cap A)$$

$$= \frac{10}{16} \cdot \frac{9}{15} + \frac{6}{16} \cdot \frac{5}{15} = \frac{1}{2}$$

c) sejam diferentes)

$$P(\text{diferentes}) = P(A \cap V) + P(V \cap A)$$

$$= 2\, P(A \cap V)$$

$$= 2 \cdot \frac{6}{16} \cdot \frac{10}{15} = \frac{1}{2}$$

10)

$$\text{Caixa} \begin{cases} 6B \\ 4R \end{cases} \quad \text{Retiram-se 3 lampadas}$$

a) 3 boas

$$p(3\text{ boas}) = p(B \cap B \cap B)$$

$$= \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \frac{1}{6}$$

b) pelo menos 1 boa

$$p(\text{pelo menos 1 boa}) = 1 - p(\bar{B} \cap \bar{B} \cap \bar{B})$$

$$= 1 - \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \frac{29}{30}$$

11) Urna $\begin{cases} 5P \\ 3V \\ 3A \\ 2AM \end{cases}$ É extraída 5 bolas

Qual prob. que saiam 2P, 2A e 1 Amarelo?

$$P(A) = P(P \cap P \cap A \cap A \cap AM) \cdot (PR)^{5}_{2,2,1}$$

$$= \frac{5}{13} \cdot \frac{4}{12} \cdot \frac{3}{11} \cdot \frac{2}{10} \cdot \frac{2}{9} \cdot \frac{5!}{2!\,2!} = \boxed{\frac{20}{429}}$$

12) Urna I $\begin{cases} 4B \\ 4V \\ 2P \end{cases}$ Urna II $\begin{cases} 5B \\ 3V \\ 3P \end{cases}$ É extraída uma bola de cada urna. Qual a prob. de ser da mesma cor

$P(I \cap B) = \frac{4}{10}$ $P(II \cap B) = \frac{5}{11}$

$P(I \cap V) = \frac{4}{10}$ $P(II \cap V) = \frac{3}{11}$

$P(I \cap P) = \frac{2}{10}$ $P(III \cap P) = \frac{3}{11}$

$$P(\text{iguais}) = P(B_1 \cap B) + P(V_1 \cap V_2) + P(P_1 \cap P_2)$$

$$= \frac{4}{10} \cdot \frac{5}{11} + \frac{4}{10} \cdot \frac{3}{11} + \frac{2}{10} \cdot \frac{3}{11}$$

$$= \frac{19}{55}$$

13) Caixa $\begin{cases} 6 \text{ de } 40W \\ 3 \text{ de } 60W \\ 1 \text{ de } 100W \end{cases}$ Retira-se 5 lampadas com reposição

a) Saia 3 de 40W, 1 de 60W, 1 de 100W

$$P(A) = P(40 \cap 40 \cap 40 \cap 60 \cap 100) \cdot (PR)^{5}_{3,1,1}$$

$$= \left(\frac{6}{10}\right)^{3} \cdot \frac{3}{10} \cdot \frac{1}{10} \cdot \frac{5!}{3!} = \frac{81}{625}$$

b) Saiam 4 de 40W e 1 de 60W

$$P(B) = P(40 \cap 40 \cap 40 \cap 40 \cap 60)\,(P.R)^{5}_{4,1}$$

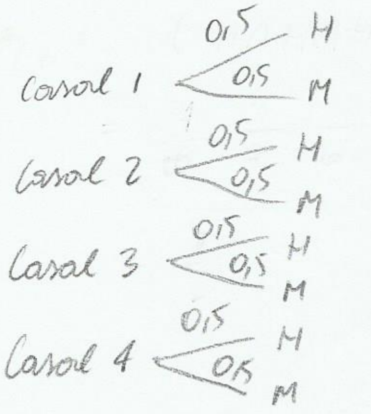
$$= \left(\frac{6}{10}\right)^{4} \cdot \frac{3}{10} \cdot \frac{5!}{4!} = \frac{243}{1250}$$

c) Não saia nenhuma de 00w?

$P(C) = P(\bar{b0} \cap \bar{b0} \cap \bar{b0} \cap \bar{b0} \cap \bar{b0})$

$= \left(\frac{7}{10}\right)^5 = 0{,}16807$

14) Sala { 4 casais   . 1 é escolhido (1 componente)



$P(3H \text{ ou } 4M) =$

$= P(H \cap H \cap H \cap M) \cdot (PR)^4_{3,1} + P(M \cap M \cap M \cap M)$

$= \left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} \cdot \frac{4!}{3!} + \left(\frac{1}{2}\right)^4 = \boxed{\frac{5}{16}}$

15) $P(M) = 0{,}5$ $P(F) = 0{,}3$ $P(E) = 0{,}2$ É selecionado 3 estudantes. Qual a prob. de pelo menos 1 escolha estatístico.

$P(\text{pelo menos } 1E) = 1 - (\bar{E} \cap \bar{E} \cap \bar{E})$

$= 1 - 0{,}8^3 = \underline{0{,}488}$

17) grupo { 12H, 8M   Retira-se 4 pessoas

a) pelo menos 1 mulher

$P(\text{pelo menos 1 mulher}) = 1 - P(\bar{M} \cap \bar{M} \cap \bar{M} \cap \bar{M})$

$= 1 - \frac{12}{20} \cdot \frac{11}{19} \cdot \frac{10}{18} \cdot \frac{9}{17} = \underline{0{,}8978}$

b) 1 mulher

$P(1 \text{ mulher}) = P(M \cap \bar{M} \cap \bar{M} \cap \bar{M}) \, (PR)^4_{3,1}$

$= \frac{8}{20} \cdot \frac{12}{19} \cdot \frac{11}{18} \cdot \frac{10}{17} \cdot \frac{4!}{3!} = \underline{0{,}3633}$

c) $P(2 \text{ sexos}) = 1 - P(M \cap M \cap M \cap M) - P(H \cap H \cap H \cap H)$

$= 1 - \frac{8}{20} \cdot \frac{7}{19} \cdot \frac{6}{18} \cdot \frac{5}{17} - \frac{12}{20} \cdot \frac{11}{19} \cdot \frac{10}{18} \cdot \frac{9}{17}$

$= \underline{0{,}88338}$

20)

urna I $\begin{cases} 1A \\ 9B \end{cases}$ Urna II $\begin{cases} xA \\ yB \end{cases}$ $(10-x)B$ — 10 bolas

- retiro um bola de cada urna
- junto tudo e retiro 2 bolas

- calcular o valor mínimo de x, sendo $p_2(2A) > p_1(2A)$

$$p_1(2A) = \frac{1}{10} \cdot \frac{x}{10} = \frac{x}{100}$$

$$p_2(2A) = \frac{1+x}{20} \cdot \frac{x}{19} = \frac{x^2+x}{380}$$

$$\frac{x^2+x}{380} > \frac{x}{100}$$

$$100x^2 + 100x > 380x$$

$$100x^2 - 280x > 0$$

$$x(100x - 280) > 0 \quad \therefore x > 0 \quad \boxed{x > 3}$$

22) $p(def) = 10\% = 0{,}1$ Retira 5 peças

a) no máximo 2 sejam boas?

$$p(A) = p(d \cap d \cap d \cap d \cap d) + p(\bar{d} \cap d \cap d \cap d \cap d) \cdot (PR)^{5}_{4,1} + p(\bar{d} \cap \bar{d} \cap d \cap d \cap d)(PR)^{5}_{3,2}$$

$$= 0{,}1^5 + 0{,}9 \cdot 0{,}1^4 \cdot \frac{5!}{4!} + 0{,}9^2 \cdot 0{,}1^3 \cdot \frac{5!}{3!2!} = \underline{0{,}00856}$$

b) pelo menos 4 boas? (5B ou 4B)

$$p(B) = p(B \cap B \cap B \cap B \cap B) + (B \cap B \cap B \cap B \cap D) \cdot (PR)^{5}_{4,1}$$

$$= 0{,}9^5 + 0{,}9^4 \cdot 0{,}1 \cdot \frac{5!}{4!} = \underline{0{,}91854}$$

23)

Estudantes $\begin{cases} 12P \\ 18S \end{cases}$; 4 bolsas

a) $P(1P) = P(P \cap \bar{P} \cap \bar{P} \cap \bar{P}) \cdot (PR)^{4}_{3,1}$

$$= \frac{12}{30} \cdot \frac{18}{29} \cdot \frac{17}{28} \cdot \frac{16}{27} \cdot \frac{4!}{3!} = 0{,}3573$$

b) $p(\text{no max } 1S) = p(P\cap P\cap P\cap P) + p(S\cap P\cap P\cap P).(PR)_{3,1}^{4} +$

$= \frac{12}{30}\cdot\frac{11}{29}\cdot\frac{10}{28}\cdot\frac{9}{27} + \frac{18}{30}\cdot\frac{12}{29}\cdot\frac{11}{28}\cdot\frac{10}{27}\cdot\frac{4!}{3!}$

$= \frac{11}{609} + \frac{88}{609} = \underline{0,1626}$

c) $p(\text{um de cada colo}) = 1 - p(P\cap P\cap P\cap P) - p(S\cap S\cap S\cap S)$

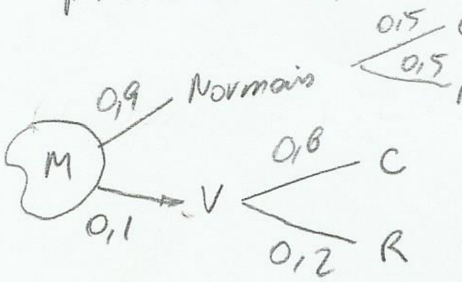
$= 1 - \frac{12}{30}\cdot\frac{11}{29}\cdot\frac{10}{28}\cdot\frac{9}{27} - \frac{18}{30}\cdot\frac{17}{29}\cdot\frac{16}{28}\cdot\frac{15}{27}$

$= \underline{0,8703}$

27) 1 moeda em 10 é viciada | 5 lançamentos com 1 moeda obtendo 3C e 2R

$p(\text{cara } v) = 0,8$

$p(\text{viciada}) = 0,1$



$P(N\cap C) = 0,45$

$P(N\cap R) = 0,45$

$P(V\cap C) = 0,1\cdot 0,8 = 0,08$

$P(V\cap R) = 0,1\cdot 0,2 = 0,02$

$P(V/3C\,e\,2R) = \dfrac{P(V\cap \text{"}3C\,e\,2R\text{"})}{P(3C\,e\,2R)}$

$P(3C\,e\,2R) = P_N(C\cap C\cap C\cap R\cap R) + P_V(C\cap C\cap C\cap R\cap R)$

$= 0,45^3\cdot 0,45^2\cdot\frac{5!}{3!\,2!} + 0,08^3\cdot 0,02^2\cdot\frac{5!}{3!\,2!} = 0,1845$

$P(V\cap \text{"}3C\,e\,2R\text{"}) = P(C\cap C\cap C\cap R\cap R)$

$= 0,08^3\cdot 0,02^2\cdot\frac{5!}{3!\,2!} = 0,002\cdot 10^{-3}$

$P(V/3C\,e\,2R) = \dfrac{0,002\cdot 10^{-3}}{0,1845} =$

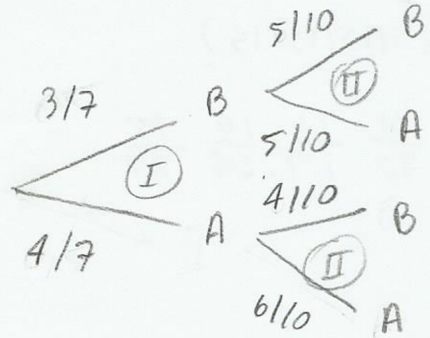
28-)

Urna I $\begin{cases} 3B \\ 4A \end{cases}$ Urna II $\begin{cases} 4B \\ 5A \end{cases}$

passa 1 bola

extrai 1 bola

Qual a prob. de ser branca?



$P(B \cap B) = \frac{3}{7} \cdot \frac{5}{10} = \frac{3}{14}$.

$P(A \cap B) = \frac{4}{7} \cdot \frac{4}{10} = \frac{8}{35}$

$P(B) = \frac{3}{14} + \frac{8}{35} = \boxed{\frac{31}{70}} = \underline{0{,}4429}$

32-)

Aula 3

- **variável aleatória**: é a função que associa todo evento pertencente a uma partição do espaço amostral um único número real.

- **esperança matemática**: é um número real, também é uma média aritmética ponderada.

$$E(x) = \sum_{i=1}^{n} x_i \, p(x_i)$$

- **variância** é a medida que dá o grau de dispersão (ou de concentração) de probabilidade em torno da média.

$$VAR(X) = E(X^2) - \{E(X)\}^2$$

- **Desvio padrão**: $\sigma_x = \sqrt{VAR(x)}$

Exemplo:

x	$P(x)$
0	0,2
1	0,1
2	0,1
3	a
4	0,3
Σ	1

1º) $a = 1 - 0{,}2 - 0{,}1 - 0{,}1 - 0{,}3 \quad \therefore \quad a = 0{,}3$

2º) $E(x)$; $VAR(x)$ e σ_x

x	$P(x)$	$x \cdot P(x)$	$x^2 \, P(x)$
0	0,2	0	0
1	0,1	0,1	0,1
2	0,1	0,2	0,4
3	0,3	0,9	2,7
4	0,3	1,2	4,8
Σ	1	2,4	8,0
		$E(x)$	$E(x^2)$

$$VAR(x) = E(x^2) - \{E(x)\}^2$$
$$= 8 - 2{,}4^2 \Rightarrow \therefore VAR(x) = \underline{2{,}24}$$

$$\sigma_x = \sqrt{VAR(x)} = \sqrt{2{,}24} = \underline{1{,}4967}$$

Ex 2

Do ponto de vista do jogador A	A paga	A recebe	Lucro líquido A
face 1 somente 1 dado	20,00	20,00	0
face 1 em 2 dados	20,00	50,00	30,00
face 1 nos 3 dados	20,00	80,00	60,00
nenhuma face	20,00	0	-20,00

x Calcular o lucro médio de A em uma jogada.

x: lucro médio de A

x	$P(x)$	$xP(x)$
0	$\frac{25}{72}$	0
30	$\frac{5}{72}$	$\frac{25}{12}$
60	$\frac{1}{216}$	$\frac{5}{18}$
-20	$\frac{125}{216}$	$\frac{-625}{54}$
Σ	1	-9,21

$$P(1) = \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{3!}{2!} = \frac{25}{72}$$

$$P(1 \text{ em } 2D) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{3!}{2!} = \frac{5}{72}$$

$$P(1 \text{ em } 3D) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

$$P(0) = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$\therefore \boxed{E(x) = -9,21}$$

Ex 3

$$f(x) = \begin{cases} k(2x - x^2) & \text{se } 0 \leq x \leq 1 \\ 0, & \text{se } x < 0 \text{ ou } x > 1 \end{cases}$$

a) Determinar k

b) Calcular $E(x)$ e $VAR(x)$

c) Calcular $P(0 \leq x \leq 1/2)$

a) Determinar k

$f(x) \geq 0$ e $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} \cancel{f(x)}\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} \cancel{f(x)}\,dx = 1$ $\quad (\to 0)$

$\int_0^1 K(2x - x^2)\,dx = 1$

$K \cdot \left(x^2 - \frac{x^3}{3}\right)\Big|_0^1 = 1 \to K\left[1 - \frac{1}{3} - 0\right] = 1 \quad \therefore \boxed{K = \frac{3}{2}}$

$\therefore f(x) = \begin{cases} \frac{3}{2}(2x - x^2) & \text{se } 0 \leq x \leq 1 \\ 0, & \text{se } x < 0 \text{ e } x > 1 \end{cases}$

b) Calcular $E(x)$ e $VAR(x)$

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x \cdot \frac{3}{2}(2x - x^2)\,dx$

$E(x) = \frac{3}{2}\int_0^1 2x^2 - x^3\,dx = \frac{3}{2}\left(\frac{2}{3}x^3 - \frac{x^4}{4}\right)\Big|_0^1$

$= \frac{3}{2}\cdot\left[\frac{2}{3} - \frac{1}{4}\right] = \frac{5}{8} \quad \boxed{E(x) = \frac{5}{8}}$

$VAR(x) = E(x^2) - \{E(x)\}^2$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^1 x^2 \frac{3}{2}(2x - x^2)\,dx$

$E(x^2) = \frac{3}{2}\int_0^1 2x^3 - x^4\,dx = \frac{3}{2}\left(\frac{x^4}{2} - \frac{x^5}{5}\right)\Big|_0^1$

$E(x^2) = \frac{9}{20}$

$VAR(x) = \frac{9}{20} - \left(\frac{5}{8}\right)^2 \quad \therefore VAR(x) = \frac{19}{320}$

c) Calcular $P(0 \leq x \leq 1/2) = \int_0^{1/2} f(x)\,dx$

$= \int_0^{1/2} \frac{3}{2}(2x - x^2)\,dx = \frac{3}{2}\cdot\left(x^2 - \frac{1}{3}x^3\right)\Big|_0^{1/2} = \boxed{\frac{5}{16}}$

Exercícios

1-) Urna $\begin{cases} 4b \\ 6p \end{cases}$ $\longrightarrow$ retiro-se 3 bolas com reposição

X Construir a distribuição de X e calcular $E(x)$

x: número de bolas brancas

x	$P(x)$	$xP(x)$
0	27/125	0
1	54/125	54/125
2	36/125	72/125
3	8/125	24/25
Σ		6/5

$P(0) = \left(\frac{6}{10}\right)^3 = \frac{27}{125}$

$P(1) = \frac{4}{10}\left(\frac{6}{10}\right)^2 \cdot \frac{3!}{2!} = \frac{54}{125}$

$P(2) = \left(\frac{4}{10}\right)^2 \cdot \frac{6}{10} \cdot \frac{3!}{2!} = \frac{36}{125}$

$P(3) = \left(\frac{4}{10}\right)^3 = \frac{8}{125}$

$\therefore \boxed{E(x) = \frac{6}{5} = 1{,}2}$

2-) $P(\text{cara}) = 4x$ $4x + x = 1$

$P(\text{coroa}) = x$ $x = \frac{1}{5}$; x: número de caras

x	$P(x)$	$xP(x)$	$x^2P(x)$
0			
1			
2			
3			
4			

$P(0) = \left(\frac{1}{5}\right)^4 = \frac{1}{625}$

$P(1) = \frac{4}{5} \cdot \left(\frac{1}{5}\right)^3 \cdot \frac{4!}{3!} = \frac{16}{625}$

$P(2) = \left(\frac{4}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^2 \cdot \frac{4!}{2!} = \frac{192}{625}$

$P(3) = \left(\frac{4}{5}\right)^3 \cdot \frac{1}{5} \cdot \frac{4!}{3!} = \frac{256}{625}$

$P(4) = \left(\frac{4}{5}\right)^4 = \frac{256}{625}$

3) x: tempo usado em carga máxima

$$f(x) = \begin{cases} \dfrac{x}{1500^2}, & \text{se } 0 \leq x < 1500 \\ \dfrac{3000 - x}{1500^2}, & \text{se } 1500 \leq x \leq 3000 \end{cases}$$

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int_{-\infty}^{0} x f(x) dx + \int_{0}^{1500} x f(x) dx + \int_{1500}^{3000} x f(x) dx + \int_{3000}^{\infty} x f(x) dx$$

$$= \int_0^{1500} x \cdot \frac{x}{1500^2}\, dx + \int_{1500}^{3000} \frac{x \cdot (3000 - x)}{1500^2}\, dx$$

$$= \frac{1}{1500^2}\left(\frac{x^3}{3}\right)\Bigg|_0^{1500} + \frac{1}{1500^2}\left(\frac{3000x^2}{2} - \frac{x^3}{3}\right)\Bigg|_{1500}^{3000}$$

$$= \frac{1500^2 \cdot 1500}{1500^2 \cdot 3} + \frac{1}{1500^2}\left(\frac{3000 \cdot 3000^2}{2} - \frac{3000^3}{3} - \frac{3000 \cdot 1500^2}{2} + \frac{1500^3}{3}\right)$$

$$\boxed{E(x) = 1500}$$

Exercícios resolvidos do livro

2- Disco: (4m, 3b, 2p, 1l) são 2disco

situação:	paga	ganha	lucro
2 maçãs	80	40	-40
2 banana	80	80	0
2 peras	80	140	60
2 laranjas	80	180	100
0	80	0	-80

$p(2M) = \frac{4}{10} \cdot \frac{4}{10} = 0{,}16$

$p(2B) = \frac{3}{10} \cdot \frac{3}{10} = 0{,}09$

$p(2p) = \frac{2}{10} \cdot \frac{2}{10} = 0{,}04$

$p(2l) = \frac{1}{10} \cdot \frac{1}{10} = 0{,}01$

$p(0) = 1 - 0{,}16 - 0{,}09 - 0{,}04 - 0{,}01 \therefore p(0) = 0{,}7$

x	$P(x)$	$x\,P(x)$
-40	0,16	-6,4
0	0,09	0
60	0,04	2,4
100	0,01	1,0
-80	0,70	-56
Σ	1	-59

$$E(x) = \sum_{i=1}^{n} x_i P(x_i)$$

$\therefore \boxed{E(x) = -59}$

(custo: 25)

3-) 1ª 50 por un. — 90% perfeita; 10% defeituosa — 60% recuperados; $\bar{n}$ recuperados

Venda: 90 (Perfeita)
20 (defeituoso)

Custo	Venda	x (lucro)	$P(x)$	$x\,P(x)$	
50	90	40	0,9	36	1º Maq (Perf)
50+25	90	15	0,06	0,9	2º Maq (Perf)
50+25	20	-55	0,04	-2,2	Pecar Def.
Σ				34,7	

$\therefore \boxed{E(x) = 34,7}$

4-)
6 → 30% (1/6)
5 → 20% (1/6)
4 → 10% (1/6)
1,2,3 → 5% (3/6)

a) 5 clientes, pelo menos 1 consiga > 10%

$P(\text{pelomenos } 1 > 10) = 1 - P(\text{nenhum} > 10\%)$

$= 1 - \left(\frac{4}{6}\right)^5 = 0,8683$

b) A: conseguir 30%

$P(\bar{A} \cap \bar{A} \cap \bar{A} \cap A) = \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} = 0,0965$

c) X: Desconto

X	P(X)	$x \cdot P(x)$
30%	1/6	5
20%	1/6	10/3
10%	1/6	5/3
5%	3/6	5/2
	1	25/2

$\therefore E(x) = 12,5\%$

Exercícios propostos

1) Urna $\begin{cases} 4b \\ 3p \end{cases}$ ⟶ retira-se 3 bolas sem reposição

X: número de bolas brancas

x	$P(x)$
0	1/35
1	12/35
2	18/35
3	4/35

$P(0B) = \frac{3}{7} \cdot \frac{2}{6} \frac{1}{5} = \frac{1}{35}$

$P(1B) = \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{3!}{2!} = \frac{12}{35}$

$P(2B) = \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{3}{5} \cdot \frac{3!}{2!} = \frac{18}{35}$

$P(3B) = \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} = \frac{4}{35}$

2)

$P(0B) = \left(\frac{3}{7}\right)^3 = 27/343$

$P(1B) = \frac{4}{7} \cdot \left(\frac{3}{7}\right)^2 \frac{3!}{2!} = 108/343$

$P(2B) = \left(\frac{4}{7}\right)^2 \frac{3}{7} \cdot \frac{3!}{2!} = 144/343$

$P(3B) = \left(\frac{4}{7}\right)^3 = 64/343$

x	$P(x)$
0	27/343
1	108/343
2	144/343
3	64/343

4)

x	$P(x)$	$xP(x)$
1	0,05	0,05
2	0,20	0,40
3	0,40	1,20
4	0,25	1,00
5	0,1	0,5
Σ		3,15

$E(x) = 3,15$

4000 carros / hora

em 10 horas, quantas pessoas?

3,15 pessoas — 1 carro

x — 4000 carros

$x = 12\,600$ pessoas / hora

em 10 horas, temos 126 000 pessoas

3.

x	0	1	2	3	4	5
$p(x)$	0	p^2	p^2	p	p	p^2

$$p^2+p^2+p+p+p^2=1$$

$$3p^2+2p=1 \rightarrow 3p^2+2p-1=0$$

a) $$p=\frac{-2\pm\sqrt{4+12}}{6} \therefore p_1=\frac{-2+4}{6}=\frac{2}{6}=\frac{1}{3}$$

$$p_2=\frac{-2-4}{6}=\cancel{\frac{-6}{6}=-1}$$

$\therefore \boxed{p=\frac{1}{3}}$

b) $P(X\geq 4)$ e $P(X<3)$

$$P(X\geq 4)=\frac{1}{3}+\frac{1}{9}=\frac{4}{9}$$

$$P(x<3)=0+\frac{1}{9}+\frac{1}{9}=\frac{2}{9}$$

c) $P(|X-3|<2)=\ ?$

5) Urna $\begin{cases} 1\ 2\ 3 \\ 4\ 5\ 6 \end{cases}$

Situação	Paga	Recebe
retirar a 6	600	1500
" 2,3,4,5	600	0
" 1	600	6 → 3600; (1,2,3,4,5) – 0

× Calcular a esperança.

X	$p(x)$	$x\,p(x)$
900	1/6	150
-600	4/6	-400
3000	1/30	100
-600	2/15	-80
Σ	1	-230

$\therefore \boxed{E(x)=-230{,}00}$

6-) Pacotes: 15 sementes

Caso + 2 sementes sem germinar → "Indenizados"

$p(\text{germ}) = 95\%$

a) $p(\text{Ind}) = 0{,}95.$

7-) A paga 100 para B

$P(C_1) = 0{,}3$ $\quad P(C_2) = 0{,}2$

M_1 — 0,3 — C — 0,2 — C (+200) → $P(C \cap C) = 0{,}2 \cdot 0{,}3 = 0{,}06$

M_2 — 0,8 — R (+100) → $P(C \cap R) = 0{,}3 \cdot 0{,}8 = 0{,}24$

M_1 — 0,7 — R → (0) $\quad P(R) = 0{,}7$

x	$p(x)$	$x \cdot p(x)$
100	0,06	6
0	0,24	0
-100	0,7	-70
Σ	1,0	-64

$\boxed{E(x) = -64{,}00}$

8) A paga 5,00 para B

3 ⇒ ganha 20

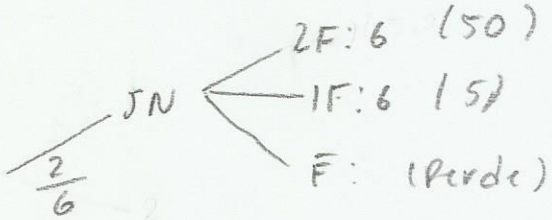
4,5 ou 6 ⇒ perde

1 ou 2 ⇒ joga novamente ⇒ Se sair 2F: 6 ganha 50

1F: 6 recebe 5,00

F̄: perde

x	$p(x)$	$x \cdot p(x)$
15	1/6	15/6
-5	3/6	-15/6
45	1/108	45/108
0	5/54	0
-5	25/108	-125/108
Σ	1	-20/27



$\therefore \boxed{E(x) = -0{,}74}$

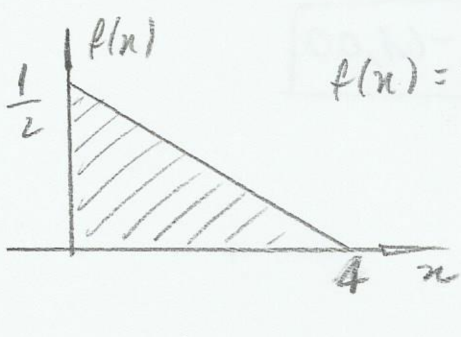
$$P(6 \cap 6) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{2}{6} = \frac{1}{108}$$

$$P(6 \cap \bar{6}) = \frac{1}{6} \cdot \frac{5}{6} \cdot 2! \cdot \frac{2}{6} = \frac{5}{54}$$

$$P(\text{outros}) = \frac{2}{6} \cdot \left[1 - \left(\frac{1}{6}\right)^2 - \frac{1}{6} \cdot \frac{5}{6} \cdot 2!\right] = \frac{25}{108}$$

Exercícios da pág. 146 a 149

2.)



$$f(x) = \frac{-x}{8} + \frac{1}{2} = \frac{-x+4}{8}, \quad p/ \; 0 < x < 4$$

a) $P(X > 2)$

$$P(x) = \int_2^4 \left(\frac{-x+4}{8}\right) dx = \frac{1}{8}\left(\frac{-x^2}{2} + 4x\right)\Big|_2^4$$

$$= \frac{1}{8}\left[\frac{-4^2}{2} + 4 \cdot 4 + \frac{2^2}{2} - 4 \cdot 2\right] \therefore \boxed{P(x) = \frac{1}{4}}$$

b) m tal que $P(X>m) = \frac{1}{8}$

$$\int_m^4 \left(\frac{-x+4}{8}\right)dx = \frac{1}{8} = \frac{1}{8}\left(\frac{-x^2}{2}+4x\right)\Big|_m^4$$

$$1 = -8+16+\frac{m^2}{2}-4m \quad \therefore \quad m^2-8m+14=0$$

$$m = \frac{8 \pm \sqrt{64-56}}{2} \quad \therefore \quad \boxed{m_1 = 2,586} \; m_2 = \cancel{5},41$$

c) $E(x) = \int_0^4 x \cdot \left(\frac{-x+4}{8}\right)dx = \frac{1}{8}\int_0^4 (-x^2+4x)\,dx$

$$= \frac{1}{8}\left(\frac{-x^3}{3}+\frac{4x^2}{2}\right)\Big|_0^4 = \frac{1}{8}\left[\frac{-4^3}{3}+\frac{4\cdot 4^2}{2}\right] = \boxed{\frac{4}{3}}$$

d) $VAR(x) = E(x^2) - \{E(x)\}^2$

$$E(x^2) = \int_0^4 x^2 \cdot \left(\frac{-x+4}{8}\right)dx = \frac{1}{8}\int_0^4 -x^3+4x^2\,dx$$

$$= \frac{1}{8}\left(\frac{-x^4}{4}+\frac{4x^3}{3}\right)\Big|_0^4 = \frac{1}{8}\left(\frac{-4^4}{4}+\frac{4\cdot 4^3}{3}\right) = \frac{8}{3}$$

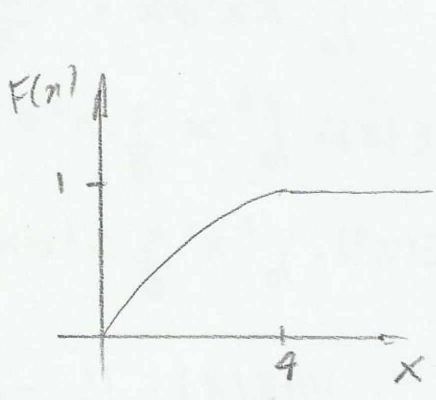
$$VAR(x) = \frac{8}{3} - \left(\frac{4}{3}\right)^2 = \boxed{\frac{8}{9}}$$

e) $F(x)$ e seu gráfico

$$F(x) = \int_0^x \frac{1}{8}(4-s)\,ds = \frac{1}{8}\left(4s-\frac{s^2}{2}\right)\Big|_0^x$$

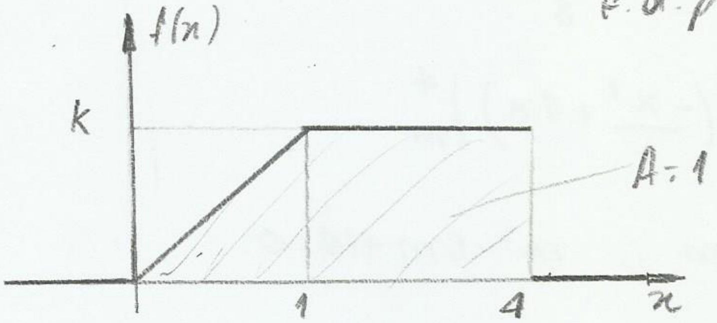
$$= \frac{1}{8}\left(4x-\frac{x^2}{2}\right) = \frac{-x^2}{16}+\frac{x}{2}$$

$$\therefore F(x) = \begin{cases} 0 & \text{se } x \le 0 \\ \frac{-x^2}{16}+\frac{x}{2} & \text{se } 0 < x \le 4 \\ 1 & \text{se } x > 4 \end{cases}$$



6)

f.d.p



a) Determinar K

$K.3 + \frac{K}{2} = 1 \rightarrow \frac{6K+K}{2} = 1 \therefore \boxed{K = \frac{2}{7}}$

b) $P(0 \leq x \leq 2)$

$= \frac{2}{7} \cdot 1 \cdot \frac{1}{2} + \frac{2}{7} \cdot 1 = \boxed{\frac{3}{7}}$

c) $E(x) = \int_0^1 x \cdot \frac{2}{7} x \, dx + \int_1^4 x \cdot \frac{2}{7} \, dx$

$= \frac{2}{7} \cdot \frac{x^3}{3} \Big|_0^1 + \frac{2}{7} \cdot \frac{x^2}{2} \Big|_1^4$

$= \frac{2}{21} + \frac{16}{7} - \frac{1}{7} \therefore \boxed{E(x) = \frac{47}{21}}$

Exercícios propostos

1)

a) $f(x) = \begin{cases} Kx^2 & \text{se } 0 \leq x \leq 2 \\ 0 & \text{se } x < 0 \text{ ou } x > 2 \end{cases}$

Para a função ser considerada f d p a $\Sigma p(x_i) = 1$

$\int_0^2 Kx^2 dx = \frac{Kx^3}{3} \Big|_0^2 = \frac{K}{3} \cdot 2^3 = 1 \therefore \boxed{K = \frac{3}{8}}$

$E(X) = \int_0^2 x \cdot \frac{3}{8} x^2 dx = \frac{3}{8} \cdot \frac{x^4}{4} \Big|_0^2 \boxed{\frac{3}{2}}$

$E(x^2) = \int_0^2 x^2 \cdot \frac{3}{8} x^2 dx = \frac{3}{8} \cdot \frac{x^5}{5} \Big|_0^2 = \frac{3 \cdot 2^5}{40} = \frac{12}{5}$

$VAR(X) = \frac{12}{5} - \left(\frac{3}{2}\right)^2 = \boxed{\frac{3}{20}}$

b) $f(x) = \begin{cases} K(2-x) & \text{se } 0 \leq x \leq 1 \\ 0 & \text{se } x < 0 \text{ ou } x > 1 \end{cases}$

$$1 = \int_0^1 K(2-x)\,dx = K\left(2x - \frac{x^2}{2}\right)\Big|_0^1$$

$$K \cdot \left(2 - \frac{1}{2}\right) = 1 \quad \therefore \boxed{K = \frac{2}{3}}$$

$$E(x) = \int_0^1 \frac{2}{3}(2-x)\cdot x\,dx = \frac{2}{3}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]\Big|_0^1$$

$$= \frac{2}{3}\cdot\left(\frac{2}{2} - \frac{1}{3}\right) = \boxed{\frac{4}{9}}$$

$$E(x^2) = \int_0^1 \frac{2}{3}(2-x)\cdot x^2\,dx = \frac{2}{3}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]\Big|_0^1$$

$$= \frac{2}{3}\left(\frac{2}{3} - \frac{1}{4}\right) = \frac{5}{18}$$

$$VAR(X) = \frac{5}{18} - \left(\frac{4}{9}\right)^2 = \boxed{\frac{13}{162}}$$

c) $f(x) = \begin{cases} Ke^{-2x} & \text{para } x \geq 0 \\ 0 & \text{para } x < 0 \end{cases}$

$$1 = \int_0^\infty K\cdot e^{-2x}\,dx = \int_0^\infty Ke^{-2x}\,\frac{d(-2x)}{-2}$$

$$-2 = K\int_0^\infty e^{(-2x)}\,d(-2x)$$

$$-2 = K\,e^{-2x}\Big|_0^\infty \rightarrow -2 = K\,(0-1) \quad \therefore K = 2$$

Distribuição Binomial

$\boxed{X: B(n,p)}$ → A variável X tem distribuição binomial, com parametros n e p.

X: número de sucesso

n: tentativas

$$\boxed{P(X=k) = \binom{n}{k} \cdot p^k \cdot q^{n-k}}$$

sucesso fracasso

Exemplo:

- 1 moeda é lançada 20 vezes. Qual a prob. de sair 8 caras?

$p(C) = \frac{1}{2} \quad p(R) = \frac{1}{2}$

$X: B(20, \frac{1}{2})$

$p(X=8) = \binom{20}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{12} = \underline{0{,}12013}$

> Observação importante: Para verificar se as variáveis são independentes basta verificar se $P(x=x_i, y=y_i) = P(x=x_i) \cdot P(y=y_i)$ p/ toda distribuição

<u>Exemplos "Distribuição Binomial"</u>

P(Acertar) = 0,2 $\quad$ n = 30 Alunos

a) $X=4 \qquad X: B(30; 0{,}2)$

$P(X=4) = \binom{30}{4} \cdot 0{,}2^4 \cdot 0{,}8^{26} = \boxed{0{,}13252}$

b) pelo menos 3 acertem $\quad X \geq 3$

$P(X \geq 3) = 1 - P(X<3)$

$= 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - \binom{30}{0} \cdot 0{,}2^0 \cdot 0{,}8^{30} - \binom{30}{1} 0{,}2^1 \cdot 0{,}8^{29} - \binom{30}{1} \cdot 0{,}2^2 \cdot 0{,}8^{28}$

$\boxed{P(X \geq 3) = 0{,}9558}$

2)

$n = 50$ questões

· sucesso (em 1 tentativa): acertar $p = \frac{1}{5}$; $k = 25$

· fracasso (em 1 tentativa): errar $q = \frac{4}{5}$

$X: B(50; \frac{1}{5})$

$P(X=25) = \binom{50}{25} \cdot \left(\frac{1}{5}\right)^{25} \left(\frac{4}{5}\right)^{25} = 1{,}60 \cdot 10^{-6}$

3) $n = 20$ aparelhos

Rejeitado: pelo menos 4 rejeitados

$p(\text{def}) = 0{,}01$ $\quad p(\text{boa}) = 0{,}99$

$X: B(20; 0{,}1)$

$P(X \geq 4) = 1 - P(X < 4)$

$= 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$

$= 1 - \binom{20}{0} 0{,}01^0 \cdot 0{,}99^{20} - \binom{20}{2} 0{,}01^1 \cdot 0{,}99^{19} - \binom{20}{3} 0{,}01^2 \cdot 0{,}99^{18} -$

$\binom{20}{3} 0{,}01^3 \cdot 0{,}99^{17} = \boxed{0{,}0000 4}$

4) $n = 20$ $\quad p = 0{,}3$ $\quad \therefore \quad X: B(20; 0{,}3)$

Esperança: $E(x) = n \cdot p$

Variança: $VAR(X) = n \cdot p \cdot q$

$E(x) = 20 \cdot 0{,}3 = \underline{6}$

$VAR(X) = 20 \cdot 0{,}3 \cdot 0{,}7 = \underline{4{,}2}$

$\lambda = np$

Book

Distribuição de Poisson

$$P(X=K) = \frac{e^{-\lambda} \cdot \lambda^K}{K!} \quad \begin{cases} \lambda: \text{valor esperado} \\ K: \text{nº de sucessos} \end{cases}$$

(1) 720 mensagens em 8 horas

a) prob. de que em 4min não receba mensagem

1º) λ

nº mensagens	tempo (min)
720	8.60 min
λ	4min

$\therefore \lambda = 6$

2º) $P(X=K) = \frac{e^{-\lambda} \cdot \lambda^K}{K!}$

$$P(X=0) = \frac{e^{-6} \cdot 6^0}{0!} \cong \underline{0{,}002479}$$

b) em 6min receba pelo menos 4 mensagem

$$P(X \geq 4) = 1 - P(X<4)$$
$$= 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$$

1º) λ

nº mensagem	tempo (min)
720	8.60
λ	6

$\therefore \lambda = 9$

$$P(X \geq 4) = 1 - \frac{e^{-9} \cdot 9^0}{0!} - \frac{e^{-9} \cdot 9^1}{1!} - \frac{e^{-9} \cdot 9^2}{2!} - \frac{e^{-9} \cdot 9^3}{3!}$$

$$\therefore P(X \geq 4) = \underline{0{,}978774}$$

2) Em 800 páginas há 800 erros

Prob. de que uma página contenha pelo menos 1 erro?

1º) λ

nº páginas	nº erros
800	800
λ	1

$\therefore \lambda = 1$

2º) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - \frac{e^{-1} \cdot 1^{0}}{0!}$

$\boxed{P(X \geq 1) = 0{,}63212}$

3) 2 acidentes para 100 km. Qual prob. de ocorrer pelo menos 3 acidentes em 250

1º) λ

acidentes	distancia
2	100
λ	250

$\therefore \lambda = 5$

2º) $P(X = K) = \frac{e^{-\lambda} \lambda^{K}}{K!}$

$P(X \geq 3) = 1 - P(X < 3)$

$= 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - \frac{e^{-5} \cdot 5^{0}}{0!} - \frac{e^{-5} \cdot 5^{1}}{1!} - \frac{e^{-5} \cdot 5^{2}}{2!}$

$\therefore P(X \geq 3) = \underline{0{,}8753}$

4) 400 lampadas 2 queimam

Qual prob. de 900 lampadas, 8 queimarem

1º) λ

lampadas	queimam
400	2
900	λ

$\therefore \lambda = 4{,}5$

2º) $P(X=K) = \dfrac{e^{-\lambda} \cdot \lambda^{K}}{K!}$

$P(X=8) = \dfrac{e^{-4{,}5} \cdot 4{,}5^{8}}{8!} = \underline{0{,}046330}$

Aproximação da distribuição binomial pela distribuição de Poisson

$E(x) = np$ (na binomial)

$E(x) = \lambda$ (na Poisson)

$\therefore \lambda = np$

<u>Exemplo:</u> $X: B(200; 0{,}01)$ Calcular $P(X=10)$.

Usando: a) Binomial

b) Aproximação por Poisson

a) $X: B(n, p)$

$P(X=10) = \binom{200}{10} \cdot 0{,}01^{10} \cdot 0{,}99^{190} = 0{,}000033$

b) $\lambda = n.p = 200.0{,}01 = 2$

$P(x=10) = \dfrac{e^{-2} \cdot 2^{10}}{10!} = 0{,}000038$

$\boxed{\text{Revisão}}$

1) urna $\begin{cases} 1\ 2\ 3 \\ 4\ 5 \end{cases}$

a) $p(\text{primos}) = \frac{3}{5} \cdot \frac{2}{4} = \frac{3}{10}$ (sem reposição)

b) $p(\text{primos}) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$

5-) Sala $\begin{cases} 12\ C \\ 68\ \bar{C} \end{cases}$ $\longrightarrow$ É escolhido 5 sem reposição

a) $p(\text{nenhum contrário}) = ?$

$p(\text{ñ contrário}) = \frac{68}{80} \cdot \frac{67}{79} \cdot \frac{66}{78} \cdot \frac{65}{77} \cdot \frac{64}{76} = \underline{0{,}4336}$

b) $p(B) = p(C \cap \bar{C} \cap \bar{C} \cap \bar{C} \cap \bar{C})$

$= \frac{12}{80} \cdot \frac{68}{79} \cdot \frac{67}{78} \cdot \frac{66}{77} \cdot \frac{65}{76} = 0{,}0813$

c) $p(C) = p(C \cap \bar{C} \cap \bar{C} \cap \bar{C} \cap \bar{C}) \cdot (PR)^{5}_{4,1}$

$= \frac{12}{80} \cdot \frac{68}{79} \cdot \frac{67}{77} \cdot \frac{66}{77} \cdot \frac{65}{76} \cdot \frac{5!}{4!} = 0{,}4065$

10) falhas do equipamento: 10%

" " " e operador: 5%

operador: 40%

a) $p(\text{falha ou erro do operador}) = p(\text{falha}) + p(\text{operador})$

$= 10\% + 40\% = 50\%$

b) num falho do operador e num falha do operador

$p(B) = 1 - 0,5 = 50\%$

2-) Com reposição

Urna $\begin{cases} 20P \\ 30B \end{cases}$ ⟶ 25 bolas

Distribuição Binomial

$P(X=K) = \binom{n}{K} \cdot p^K \cdot q^{n-K}$

Distribuição de Poisson

$P(X=k) = \frac{e^{-\lambda} \cdot \lambda^K}{k!}$

a) 2 sejam pretas?

$P(X=k) = \binom{n}{k} \cdot p^k \cdot q^{n-k}$

p: prob. de sucesso

q: prob. de falha

$P(X=2) = \left[\begin{matrix} 25 \\ 2 \end{matrix}\right] \cdot \left(\frac{20}{50}\right)^2 \left(\frac{30}{50}\right)^{23}$ $\therefore$ $p(X=2) = 0,00038$

b) $P(X \geq 3) = 1 - P(X<3)$

$= 1 - [P(x=0) + P(X=1) + P(X=2)]$

$= 1 - \binom{25}{0} \cdot \left(\frac{20}{50}\right)^0 \left(\frac{30}{50}\right)^{25} - \binom{25}{1} \cdot \left(\frac{20}{50}\right)^1 \left(\frac{30}{50}\right)^{24} -$

$\binom{25}{2} \left(\frac{20}{50}\right)^2 \left(\frac{30}{50}\right)^{23}$

$P(X \geq 3) = 1 - 2,84 \cdot 10^{-6} - 4,74 \cdot 10^{-5} - 3,79 \cdot 10^{-4}$

$\therefore$ $P(X \geq 3) = 0,99957$

3-) 2 acidentes para 100 km

a) 250 km pelo menos 3 acidentes

acidentes	distancia
2	100
λ	250

$\therefore \lambda = 5$

$$p(x=k) = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$$

$$p(x \geq 3) = 1 - p(x=0) - p(x=1) - p(x=2)$$

$$= 1 - \frac{e^{-5} \cdot 5^0}{0!} - \frac{e^{-5} \cdot 5^1}{1!} - \frac{e^{-5} \cdot 5^2}{2!}$$

$$p(x \geq 3) = 0,8753$$

b) 300 km ocorram 5 acidentes?

2	100
λ	300

$\therefore \lambda = 6$

$$p(x=k) = \frac{e^{-\lambda} \cdot \lambda^k}{k}$$

$$p(k=5) = \frac{e^{-6} \cdot 6^5}{5!} = 0,1606$$

4) p(autor lucho) = 0,2 $n = 30$

a) $p(x=4) = \left[\binom{30}{4}\right] \cdot 0,2^4 \cdot 0,8^{26} = 0,13252$

b) $p(x \geq 3) = 1 - p(x < 3)$

$$= 1 - p(x=0) - p(x=1) - p(x=2)$$

$$= 1 - \binom{30}{0} \cdot 0,2^0 \cdot 0,8^{30} - \binom{30}{1} \cdot 0,2^1 \cdot 0,8^{29} - \binom{30}{2} \cdot 0,2^2 \cdot 0,8^{28}$$

$\therefore$ $p(x \geq 3) = 0,95582$

E22

5) 20 aparelhos

Ele é rejeitado se pelo menos 4 forem defeituosos

p(defeito) = 1%

× Determinar a prob. de rejeitar o lote

$$p(X \geq 4) = 1 - p(X < 4)$$

$$= 1 - p(X=0) - p(X=1) - p(X=2) - p(X=3)$$

$$= 1 - \binom{20}{0} \cdot 0{,}01^0 \cdot 0{,}99^{20} - \binom{20}{1} \cdot 0{,}01^1 \cdot 0{,}99^{19} - \binom{20}{2} \cdot 0{,}01^2 \cdot 0{,}99^{18} -$$

$$\binom{20}{3} \cdot 0{,}01^3 \cdot 0{,}99^{17}$$

$$\therefore \quad p(X \geq 4) = 0{,}0004$$

6) tratamento A: 20% (não sobrevivem)

a) $X : B(20; 0{,}2)$

b) $E(x) = 20 \cdot 0{,}2 = 4$

$VAR(X) = npq = 20 \cdot 0{,}2 \cdot 0{,}8 = 3{,}2$

c) $P(2 < X \leq 4) = P(X=3) + P(X=4)$

$$= \binom{20}{3} \cdot 0{,}2^3 \cdot 0{,}8^{17} + \binom{20}{4} 0{,}2^4 \cdot 0{,}8^{16}$$

$$P(2 < x \leq 4) = \underline{0{,}42356}$$

d) $P(X \geq 2) = 1 - P(X < 2)$

$= 0{,}93082$

7-)

lampadas	queimam
400	2
600	λ

$\therefore \lambda = 3$

a) no minimo 3

$$p(X \geq 3) = 1 - p(X < 3)$$

$$= 1 - \frac{e^{-3} \times 3^0}{0!} - \frac{e^{-3} \cdot 3^1}{1!} - \frac{e^{-3} \cdot 3^2}{2!}$$

$$\therefore p(X \geq 3) = 0{,}577$$

b) $X = 4$

$$\begin{array}{l} 400 - 2 \\ 900 - \lambda \end{array} \quad \therefore \lambda = 4{,}5$$

$$p(X=4) = \frac{e^{-4,5} \cdot 4{,}5^8}{8!} = 0{,}0463$$

8)

$$\begin{array}{l} 60 - 30 \\ 3 - \lambda \end{array} \quad \therefore \lambda = 1{,}5$$

$$p(X \geq 3) = 1 - p(X=0) - p(X=1) - p(X=2)$$

$$= 1 - \left\{ \frac{e^{-1,5} \cdot 1{,}5^0}{0!} + \frac{e^{-1,5} \cdot 1{,}5^1}{1!} + \frac{e^{-1,5} \cdot 1{,}5^2}{2!} \right\}$$

$$\therefore p(X \geq 3) = 0{,}191154$$

Prova antiga

1)

Caixa c/ 10 produtos

+1 produto (são indenizados)

p(def) : 5%

Caso indenizado : -32

Caso N̄ 11   +X

E(x) : 50,00 , determine X

para +1 defeito   -32

para 0 defeito   X

X	p(X)	x P(x)
-32	0,086	-2,756
X	0,914	0,914x
Σ	1	0,914x - 2,756

$$p(\text{mais que } 1) = p(x>1)$$

$$= 1 - p(x \leq 1)$$

$$= 1 - [p(x=0) + p(x=1)]$$

$$= 1 - \left(0,95^{10} + 0,05 \cdot 0,95^{9} \cdot \frac{10!}{9!}\right)$$

$$= 0,086$$

$$\therefore \quad 50 = 0,914x - 2,756 \quad \therefore \quad \boxed{x = 57,73}$$

2) a) 10 reclamações por hora

Determine a prob. receber pelo menos 2 reclamações em 15min

$$10 - 60$$
$$\lambda - 15 \quad \therefore \quad \lambda = 2,5$$

$$P(x \geq 2) = 1 - p(x<2)$$

$$= 1 - p(x=0) - p(x=1)$$

$$= 1 - \frac{e^{-2,5}\, 2,5^{0}}{0!} - \frac{e^{-2,5} \cdot 2,5^{1}}{1!} \quad \therefore \quad p(x \geq 2) = 0,7127$$

b) $p(def) = 0{,}02$ $n = 20$ apresente no máximo 2 defeitos

$X : (20 ; 0{,}02)$

$P(X \leq 2) = p(X=0) + p(X=1) + p(X=2)$

$= \binom{20}{0} \cdot 0{,}02^0 \cdot 0{,}98^{20} + \binom{20}{1} 0{,}02^1 \cdot 0{,}98^{19} + \binom{20}{2} \cdot 0{,}02^2 \cdot 0{,}98^{18}$

$\therefore P(X \leq 2) = 0{,}99293$

3) $P(A|B) = 0{,}2$ Determine $P(A \cup B) = ?$

$P(\bar{A}) = 0{,}52 \rightarrow P(A) = 1 - 0{,}52 = 0{,}48$

$\frac{P(A)}{P(B)} = 16$

$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow 0{,}2 = \frac{P(A \cap B)}{\left(\frac{0{,}48}{16}\right)} \therefore P(A \cap B) = 0{,}006$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0{,}48 + \frac{0{,}48}{16} - 0{,}006 \therefore P(A \cup B) = 0{,}504$

4) Chaves $\begin{cases} 2 \text{ abre} \\ 4 \text{ fecha} \end{cases}$

$P(x=1) = \frac{2}{6}$

$P(X=2) = \frac{4}{6} \cdot \frac{2}{5} = \frac{4}{15}$

$P(X=3) = \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} = \frac{1}{5}$

$P(X=4) = \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} = \frac{2}{15}$

$P(X=5) = \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{2} = \frac{1}{15}$

x	$p(x)$	$x\,p(x)$
1	2/6	2/6
2	4/15	8/15
3	1/5	3/5
4	2/15	8/15
5	1/15	5/15
		7/3

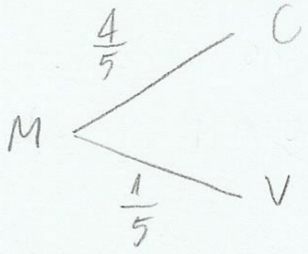
$\boxed{\therefore E(x) \cong 2{,}3333}$

E24

5)

Caixa $\begin{cases} 4\,M_c \\ 1\,M_v \rightarrow (2\text{ coroas}) \end{cases}$ $\longrightarrow$ Retira 1 moeda $\rightarrow$ e joga 4 vezes

x Determinar a prob. de ser a viciada, sabendo que ocorreu 4 coroas $\therefore$ $p(V/C) = \frac{p(V \cap C)}{p(C)}$



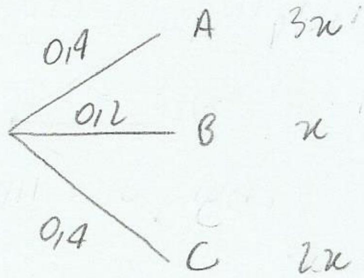
$p(C \cap C) = \frac{4}{5}\left(\frac{1}{2}\right)^4 = \frac{1}{20}$

$p(V \cap C) = \frac{1}{5} \cdot 1^4 = \frac{1}{5}$

$$p(V/C) = \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{20}} \quad \therefore \quad p(V/C) = \frac{4}{5} = 0{,}8$$

6) $p(def) = 0{,}11$

$p(defA) = 3\,p(defB)$; $p(defB) = \frac{p(defC)}{2}$

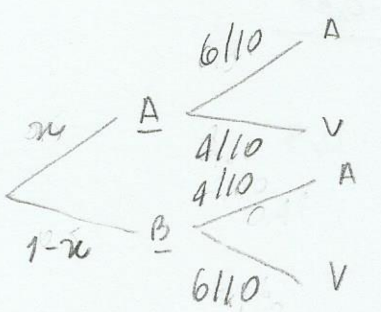


$1{,}2x + 0{,}2x + 0{,}8x = 0{,}11 \quad \therefore \quad x = 0{,}05$

$p(A) = 0{,}15 \quad p(B) = 0{,}05 \quad p(C) = 0{,}1$

P1 antigo

1.)
$A \begin{cases} 6A \\ 4V \end{cases} \quad B \begin{cases} 4A \\ 6V \end{cases}$ $p(A) = 0{,}44$ Qual a prob. de ser corno A



$$\frac{6}{10}x + \frac{4}{10} - \frac{4x}{10} = 0{,}44$$

$$\boxed{x = 0{,}2}$$

2.)

atendimentos	hora
15	60
λ	10

$\lambda = 2{,}5$

$$P(x \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-2,5} \cdot 2{,}5^0}{0} + \frac{e^{-2,5} \cdot 2{,}5^1}{1!} + \frac{e^{-2,5} \cdot 2{,}5^2}{2!}$$

$$P(X \leq 2) = 0{,}5438$$

3.)

Urna X $\begin{cases} 6B \\ 4A \end{cases}$ Urna Y $\begin{cases} 3B \\ 5A \end{cases}$ → saíram 2 Bola azuis

2 bolas

sem reposição

- Qual prob de sair 2 azuis de x

$$P(Ax/Ay) = \frac{P(Ax \cap Ay)}{P(Ay)}$$

$$P(\text{2azuis}) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{7}{10} \cdot \frac{6}{9} = \frac{14}{225}$$

$$P(Ay) = \frac{14}{225} + \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{5}{10} \cdot \frac{4}{9} + \frac{6}{10} \cdot \frac{4}{9} \cdot \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{21}{11} = \frac{212}{675}$$

$$P(Ax/Ay) = 0{,}1981$$

4) $X: B(20,p)$

$VAR(X)=4,2$ e $p<0,5$

a) $p=?$

$VAR(X)=n p . q$

$4,2 = 20 . p(1-p)$

$0,21 = p - p^2$

$\underline{p=0,3}$

$-p^2+p-0,21=0$

$p = \frac{-1 \pm \sqrt{1-0,84}}{-2}$

$\cancel{p_1 = 0,7}$ $p_2 = 0,3$

b) $p(X \geq 1) = 1 - p(X<1)$

$= 1 - p(X=0)$

$= 1 - \binom{20}{0} . 0,3^0 \, 0,7^{20}$

$p(X \geq 1) = 0,9992$

5) 2 moedas viciadas

X: número de caras

$p(0) = (1-p)^2$

$p(1) = p.(1-p).2$

$p(2) = p^2$

a)

X	p(x)	x p(x)
0	$(1-p)^2$	0
1	$2p(1-p)$	$2p-2p^2$
2	p^2	$2p^2$
		$2p$

$2p = 1,3$

$\therefore p = 0,65$ e $q = 0,35$

b) $p(X>1) = 1 - p(X=0) - p(X=1)$

$= 1 - (1-0,65)^2 - 2 . 0,65 . (1-0,65)$

$= 0,4225$

ou $p(X=2) = 0,65^2 = 0,4225$

6)

$$f(x) = \begin{cases} \frac{2}{9}, \text{se } 0 \le x < 3 \\ \frac{2(6-x)}{27}, \text{se } 3 < x < 6 \\ 0, \text{se } x < 0 \text{ ou } x > 6 \end{cases}$$

Determine $E(x)$

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \cancel{\int_{-\infty}^{0} x f(x) dx}^{\to 0} + \int_{0}^{3} x f(x)\, dx + \int_{3}^{6} x f(x)\, dx + \cancel{\int_{6}^{\infty} x f(x) dx}^{\to 0}$$

$$= \int_0^3 \frac{2}{9} x\, dx + \int_3^6 \frac{12}{27} x - \frac{2x^2}{27}\, dx$$

$$= \left.\frac{x^2}{9}\right|_0^3 + \left.\frac{6}{27}x^2 - \frac{2x^3}{81}\right|_3^6$$

$$= \underbrace{\left(\frac{3^2}{9} - \frac{0}{9}\right)}_{1} + \left(\frac{6 \cdot 6^2}{27} - \frac{2 \cdot 6^3}{81} - \frac{6 \cdot 3^2}{27} + \frac{2 \cdot 3^3}{81}\right)$$

$$= \underline{2{,}3333}$$

Prova antiga

1-) $n: 30$ $\begin{cases} a & b \\ c & d \end{cases}$; Qual prob. de acertar no máximo 3

X: número de acertos

$$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \binom{30}{0} \cdot \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{30} + \binom{30}{1} \cdot \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{29} + \binom{30}{2} \cdot \left(\frac{1}{4}\right)^2 \cdot \left(\frac{3}{4}\right)^{28} +$$

$$\binom{30}{3} \left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^{27} = 0{,}03745$$

2) hora atendimento x - 1h

60	x
6	11,99096

$\therefore \boxed{x = 119{,}90}$

$p(X \geq 1) = 1 - p(X < 1)$

$0{,}9999938 = 1 - p(x=0) = 1 - \dfrac{e^{-\lambda}\lambda^0}{0!}$

$0{,}9999938 = 1 - e^{-\lambda} \therefore \lambda = 11{,}99096$

6) Urna A $\begin{cases} 4B \\ 6V \end{cases}$ Urna B $\begin{cases} 3B \\ 6V \end{cases}$ → retira-se 5 bolas

sem rep.

1 bola

* Qual a prob. de ocorrer 3b e 2V, sabendo que a bola retirada urna é branca A

$P(A/1B) = \dfrac{P(A \cap 1B)}{P(1B)}$

$\frac{4}{10}$ B − (3B, 6V) $P_1(3B,6V) = \frac{4}{10} \cdot \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} \cdot \frac{6}{7} \cdot \frac{5}{6} \cdot \frac{5!}{3!2!} = \frac{2}{21}$

$\frac{6}{10}$ V − (3B, 6V) $P_2(3B,6V) = \frac{6}{10} \cdot \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} \cdot \cancel{\frac{7}{7}} \cdot \frac{6}{6} \cdot \frac{5!}{3!2!} = \frac{1}{20}$

$P(A/1B) = \frac{2}{21} \div \left(\frac{2}{21} + \frac{1}{20} \right) = \frac{40}{61} = \underline{0{,}6557}$

Exercício prova antiga

$p(\text{curva}) = 0{,}15 \qquad X : B(30; 0{,}15)$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - \binom{30}{0} \cdot 0{,}15^0 \cdot 0{,}85^{30} - \binom{30}{1} \cdot 0{,}15^1 \cdot 0{,}85^{29}$$

$$\therefore \; P(X \geq 2) = \underline{0{,}952}$$

Exercício de prova

Urna I $\begin{cases} 5V \\ 5A \end{cases}$ Urna II $\begin{cases} 3V \\ 6A \end{cases}$ → 2 bolas

1 bola

- sabendo que saiu 2 amarelas
- Qual a prob. das bolas vermelha de I para II

$$P(V_{I \to II} \mid A_{II \to}) = \frac{P(V \cap A)}{P(A)}$$

$$P(A) = \frac{5}{10} \cdot \frac{6}{10} \cdot \frac{5}{9} + \frac{5}{10} \cdot \frac{7}{10} \cdot \frac{6}{9} = \frac{2}{5}$$

$$P(V \cap A) = \frac{5}{10} \cdot \frac{6}{10} \cdot \frac{5}{9} = \frac{1}{6}$$

$$\therefore \; P(V \mid A) = \frac{1}{6} \div \frac{2}{5} = \frac{5}{12} = \underline{0{,}4167}$$

E27

Exercício Prova antiga

Urna X { 6 B, 4 A }   Urna Y { 3 B, 5 A }   → 2 bolas Azuis

sem reposi. 2 Bolas

x Qual prob. de ter sido 2 bolas x para Y

$$P(AA_{x \to y} \mid AA_y) = \frac{P(AA_{x \to y} \cap AA_y)}{P(AA_y)}$$

$$P(AA_{x \to y} \cap AA_y) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{7}{10} \cdot \frac{6}{9} = \frac{14}{225}$$

$$P(AA_y) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{7}{10} \cdot \frac{6}{9} + \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{5}{10} \cdot \frac{4}{9} + \frac{6}{10} \cdot \frac{4}{9} \cdot \frac{6}{10} \cdot \frac{5}{9} \cdot 2! = \frac{212}{675}$$

$$\therefore P(AA_{x \to y} \mid AA_y) = \underline{0,1981}$$

4)

Erros	Páginas
510	X
λ	1

$$P(X \geq 1) = 1 - P(X < 1)$$

$$0,817316 = 1 - P(X = 0)$$

$$0,817316 = 1 - \frac{e^{-\lambda} \cdot \lambda^0}{0!}$$

$$\therefore \lambda = 1,7$$

$$X = \frac{510}{1,7} = \underline{300}$$

Exercício prova antiga

Equipamentos	defeito
400	2
900	λ

$\therefore \lambda = 4,5$

$$P(x \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

Obs; $P(X=k) = \dfrac{e^{-\lambda} \lambda^k}{k!}$ $\therefore P(x \leq 3) = 0,3423$

Exercício de prova

$1x + 2x + 3x + 4x + 5x + 6x = 1$

$21x = 1 \therefore x = \frac{1}{21}$

Se a soma for 7

$P(7) = P(1 e 6) + P(2 e 5) + P(3 e 4)$

$= \frac{1}{21} \cdot \frac{6}{21} \cdot 2! + \frac{2}{21} \cdot \frac{5}{21} \cdot 2! + \frac{3}{21} \cdot \frac{4}{21} \cdot 2!$

$= \frac{8}{63}$

Se a soma for 11

$P(11) = P(5 e 6)$

$= \frac{5}{21} \cdot \frac{6}{21} \cdot 2! = \frac{20}{147}$

Se a soma for 2

$P(2) = P(1 e 1)$

$= \frac{1}{21} \cdot \frac{1}{21} = \frac{1}{441}$

X	P(x)	x P(x)
-50	$\frac{8}{63}$	$-\frac{400}{63}$
0	$\frac{20}{147}$	0
100	$\frac{1}{441}$	$\frac{100}{441}$
-100	$\frac{324}{441}$	$-\frac{32400}{441}$
	1	$\boxed{-79,59}$

Exercício de prova

1 defeito — 250 m

λ — 625 m

$\lambda = 2,5$

625 m — 1 dia

$P(K=0) = \frac{e^{-2,5} \cdot 2,5^0}{0!} = 0,082 \therefore P(K=0) = 6,56 \cdot 69 \cong$ 7 dias

Exercício prova

$p(def) = 10\%$

$p(x \geq 2) = 1 - p(x=0) - p(x=1)$

$= 1 - \binom{20}{0} \cdot 0{,}1^0 \cdot 0{,}9^{20} - \binom{20}{1} \cdot 0{,}1^1 \cdot 0{,}9^{19} = \underline{0{,}6083}$

Exercício prova

Urna X { 2A, 2B, 1V } Urna Y { 2A, 1B, 1V }

Retiro-se 1 bola

- Calcule a prob. de 2 brancas, sendo que saíram 2 bolas iguais

$p(BB / Iguais) = \dfrac{P(BB \cap Iguais)}{P(Iguais)}$

$P(A) = \frac{2}{5} \cdot \frac{2}{4} = \frac{4}{20}$ $P(B) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$ $P(C) = \frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$

$P(Iguais) = \frac{4}{20} + \frac{2}{20} + \frac{1}{20} = \frac{7}{20}$

$P(BB / Iguais) = \frac{2}{20} \div \frac{7}{20} = \frac{2}{7}$

Revisão de Distribuição

- Distribuição Binomial

$X: (n; p)$ $P(X=K) = \binom{n}{K} p^K \cdot q^{n-K}$

- Distribuição de Poisson

~~X: (n;p)~~ $P(X=K) = \dfrac{e^{-\lambda} \lambda^K}{K}$

Aproximação Distribuição Binomial pela distribuição de Poisson

- Quando o valor do $n >$ que o da tabela
- $p < 0{,}1$; $\lambda = np \therefore E = np$