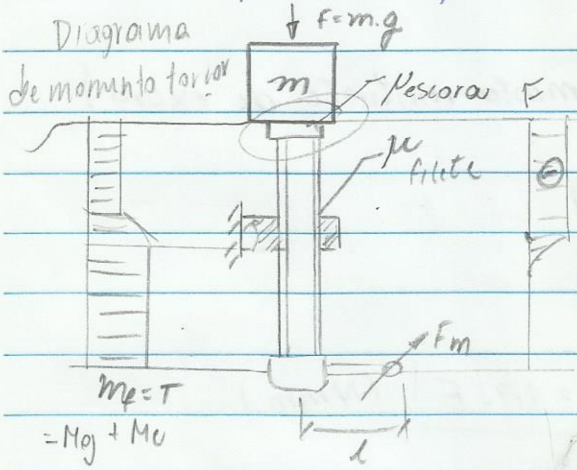
Elementos de máquina II 07/08/2015

Fusos ou parafusos de movimento Cap 4 Apostila

Fusos são parafusos que movimentam cargas, aplicando forças



Fm: força manual (N)

F = força axial

m = ?

Fuso TR 16×4 de aço

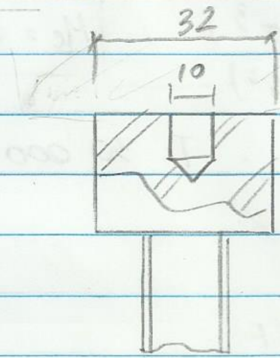
Porca (bucha) de aço

Ros trapezoidal

d = Ø nominal (externo)

P = P

lembrete



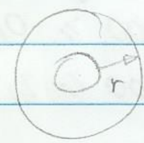
Para $F_m = 10\ kgf \cong 100N$

$l = 300\ mm$

$\mu_{filetes} = 0{,}05$

$\mu_{escora} = 0{,}15$

Qual o máximo valor da massa m a ser ~~levan~~ erguida?



$M_T = F_m \times l$

$M_{giro} = M_g$ = momento resistente entre filetes

$M_g = F \frac{d_2}{2} tg(\alpha + \varphi)$

$tg\,\alpha \simeq \frac{zP}{\pi d_2}$ $\quad tg\,\varphi = \frac{F_{at}}{N} = \mu_{filetes}$

$tg\,\varphi = 0{,}05 \therefore \boxed{\varphi = 2{,}86^\circ}$

TR 16×4 { $d_2 = 14$; $d_3 = 11{,}5$; $A_s = 127{,}7\ mm^2$; $H_1 = 2\ mm$

d_2: Ø de flanco

d_3: Ø interno

A_s: Area do parafuso

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$tg\,\alpha = \dfrac{z \cdot P}{\pi d_2}$ $\alpha = tg^{-1}\left(\dfrac{1 \times 4}{\pi \times 14}\right)$ $\therefore \alpha = 5{,}2^\circ$

$M_{giro} = F \times \dfrac{d^2}{2}\, tg\,(\alpha + \varphi)$ $Mg = F \times \dfrac{14^2}{2}\, tg\,(5{,}2 + 2{,}86)$

$\therefore$ $\boxed{Mg = 0{,}99\,F}$ (Nm)

$M_{escora} = \mu_{escora} \times F \times r$ (Me) {Momento resistente de escora}

$r = \dfrac{2\,(r_e^3 - r_i^3)}{3\,(r_e^2 - r_i^2)} = \dfrac{2\,(16^3 - 5^3)}{3\,(16^2 - 5^2)}$

$Me = 0{,}15 \times F \times \dfrac{2\,(16^3 - 5^3)}{3\,(16^2 - 5^2)}$ $\therefore$ $\boxed{Me = 1{,}72\,F}$ (Nmm)

$T = Fm\, l = 100 \times 300$ $\therefore$ $T = 30\,000$ Nmm

$T \geq Mg + Me$

$30\,000 \geq 0{,}99F + 1{,}72F$

$30\,000 \geq 2{,}71\,F$ $\therefore$ $\boxed{F \leq 11\,070{,}11\ N}$

$mg \leq 11070$; $g = 10\, m/s$ $\therefore$ $m \leq 1107$ Kg

Exercício 4.4

Fuso de rosca trapezoidal

TR $d \times P \times Z$

TR: Trapezoidal

d = Ø nominal, ou externo

P = passo

Z: nº de entradas

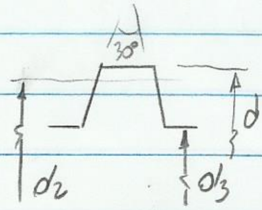
a: avanço = $Z \cdot P$

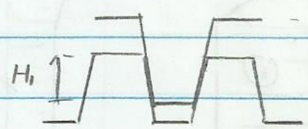
d_3 = Ø interno

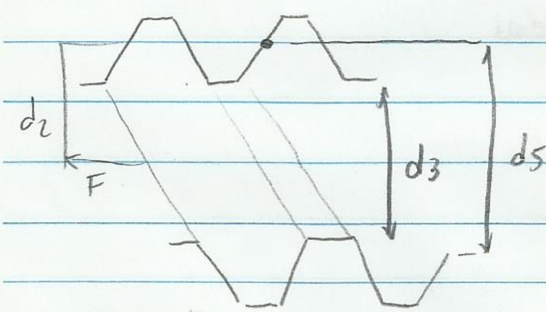
d_2 = Ø de flanco

$H = 1{,}866P$

$H_1 = 0{,}5P$

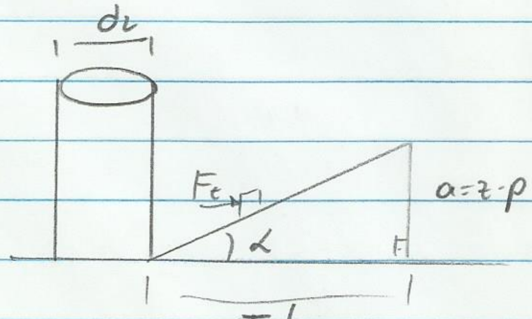






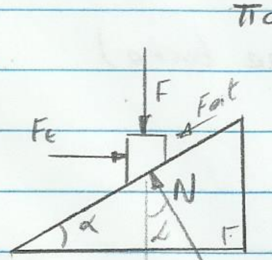
$$A_s = \frac{\pi}{4} d_s^2$$

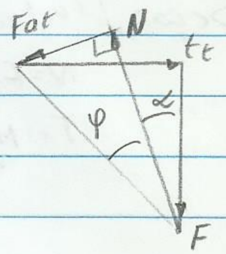
$$d_s = \frac{d_2 + d_3}{2}$$



α = ∡ de hélice

$$tg\,\alpha = \frac{Z P}{\pi d_2}$$





$tg\,\varphi = \frac{Fat}{N} = \mu_{\text{real}}$

φ: ∡ de atrito

$tg(\alpha + \varphi) = \frac{F_t}{F}$

$F_t = F\,tg(\alpha + \varphi)$

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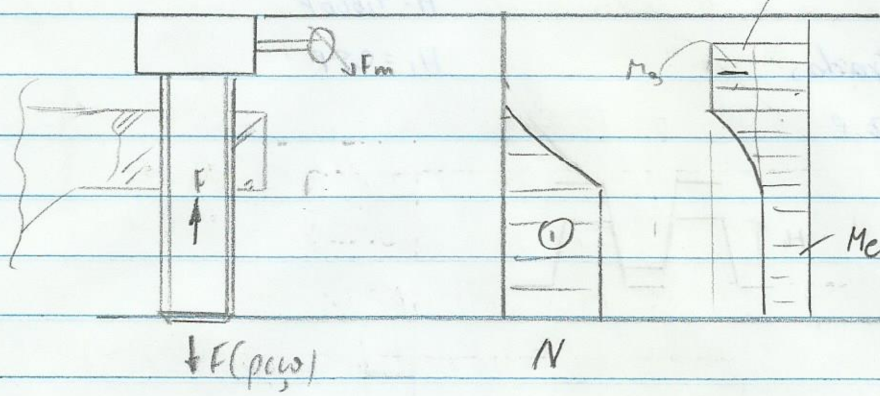
$M_g = F_t \cdot \frac{d_2}{2}$ $\quad M_g = F \frac{d_2}{2} tg(\alpha + \varphi)$

Dimensionamento 4.1 e 4.2

$T = F_m \cdot \ell \geq M_{giro} + M_e$



Torção e Compressão : Tensões combinadas

$\tau = \frac{T}{W_t} = \frac{T}{0{,}2 d_3^3}$ $\quad \sigma = \frac{F}{A_3}$

$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} \leq \frac{\sigma_e}{n}$

Exercício 4.4

TR 12x3: $d = 12$, $P = 3$, $d_2 = 10{,}5$, $d_3 = 8{,}5$, $A_s = 70{,}9$, $H_1 = 1{,}5$

Seção crítica (na bucha)

$N = F$

$T = M_g + M_e$

$\tau = \frac{T}{0{,}2 d_3^3} = \frac{M_g + M_e}{0{,}2 d_3^3}$

Classe:

$\sigma_r = 600$ MPa

$\sigma_e = 380$ MPa

$n = 4$

$M_g =$

$M_e =$

$F =$

Exercício 4.1

$N/mm^2 = MPa$

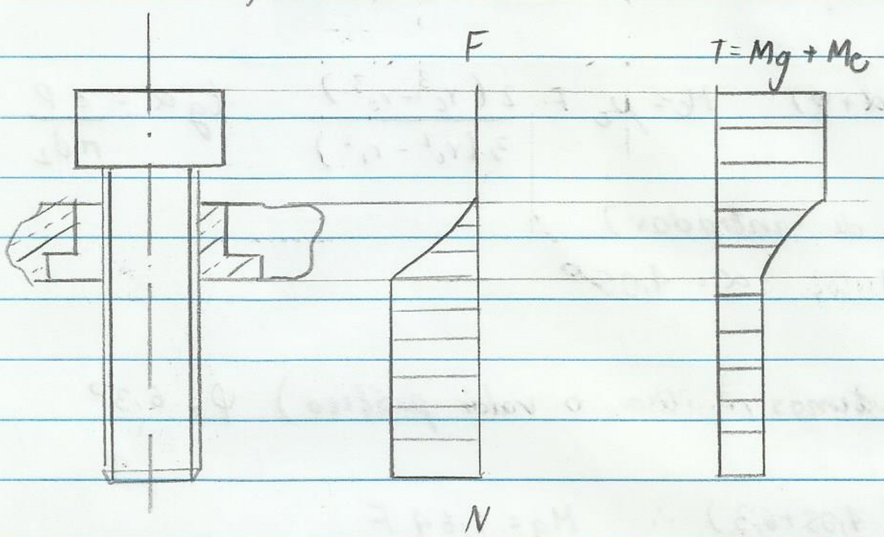
$n = 3$

$F_m = 100\ N$

Aço de classe 4.6 $\begin{cases} \sigma_e = 240\ N/mm^2 \\ \sigma_R = 400\ N/mm^2 \end{cases}$

$F_m \times l = 100 \times 300mm = 30\ Nm$

$M_t = F_m \cdot l \geq M_{giro} + M_{escora}$



Na seção crítica A: $\begin{cases} T = M_g + M_c \\ N = F \end{cases}$

$$\sigma = \frac{F}{A_s} \qquad \tau = \frac{M_g + M_c}{0{,}2 d_3^3} = \frac{30\,000}{0{,}2\, d_3^3}$$

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} \leq \frac{\sigma_e}{n}$$

$$\sigma_{eq} = \sqrt{\left(\frac{F}{A_s}\right)^2 + 3\left(\frac{30000}{0{,}2 d_3^3}\right)^2} < \frac{240}{3}$$

Desprezando, inicialmente σ

$$\sqrt{3\left(\frac{30\,000}{0{,}2 d_3^3}\right)^2} < 80$$

$$\therefore \quad d_3 > 14{,}81\ mm$$

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<u>1ª Tentativa</u> TR 20x4

$d_3 = 15,5$

$d = 20$

$p = 4$

$d_2 = 18$

$A_s = 220,4$

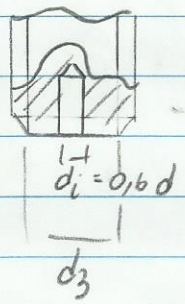
$$M_g = F\frac{d_2}{2}\,tg(\alpha+\varphi) \qquad M_e = \mu_e\, F \cdot \frac{2(r_e^3 - r_i^3)}{3(r_e^2 - r_i^2)} \qquad tg\,\alpha = \frac{z\,P}{\pi d_2}$$

$z = 1$ (número de entradas)

$\alpha = tg^{-1}\left(\frac{1\times 4}{\pi \cdot 18}\right) \quad \therefore \quad \alpha = 4,05°$

$\mu_{fil} = tg\,\varphi$ (não temos ∴ Usa o valor prático) $\varphi = 6,3°$

$$M_g = F \cdot \frac{18}{2}\,tg\,(4,05+6,3) \quad \therefore \quad M_g = 1,64\,F$$



$r_i = \frac{0,6\,d}{2} = 0,3\,d = 6\,mm \qquad \mu_e = 0,1$

$r_e = \frac{d_3}{2} = \frac{15,5}{2} = 7,75\,mm$

$$M_e = 0,1\,F\,\frac{2\,(7,75^3 - 6^3)}{3\,(7,75^2 - 6^2)} \quad \therefore \quad M_e = 0,691\,F$$

$M_g + M_e = 30\,000$

$1,64F + 0,691F = 30000 \quad \therefore \quad F = 12\,870\,N$

Verificação:

$$\sigma_{eq} = \sqrt{\left(\frac{12870}{220,4}\right)^2 + 3\left(\frac{30\,000}{0,2\times 15,5^3}\right)^2} \leq 80$$

$91 > 80 \quad \therefore$ Não Serve

2ª tentativa TR 22×5 $d = 22$ $d_3 = 16,5$

$P = 5$ $A_S = 254,5$

$d_2 = 19,5$ $H_1 = \frac{P}{2} = 2,5$

$\alpha = tg^{-1}\left(\frac{1 \times 5}{\pi \cdot 19,5}\right) \therefore \alpha = 4,67$

$\varphi = 6,3°$

$r_i = \frac{0,6}{2} \cdot d = 0,3d \therefore r_i = 6,6\ mm$

$r_e = \frac{d_3}{2} = \frac{16,5}{2} = 8,25$

$M_g = F \cdot \frac{19,5}{2}\ tg\,(4,67 + 6,3) \therefore M_g = 1,89F$

$M_e = 0,1 \cdot F \cdot \frac{2\,(6,6^3 - 8,25^3)}{3\,(6,6^2 - 8,25^2)} \therefore M_e = 0,75F$

$1,89F + 0,75F = 30000 \therefore F = 11\,386,2\ N$ 11 363,6 N

$\sigma_e = 73,1 < 80 \therefore$ $\boxed{TR\ 22 \times 5}$

Esmagamento de filetes

$p = \frac{F}{n \pi d_2 H_1} \leq p_{adm}$

$\frac{11\,363,6}{n \pi \cdot 19,5 \cdot 2,5} \leq 15$

$n \geq 4,95$

mas $n \geq 6$ $\Big\}\ n = 6$

$m = n \cdot p = 6 \cdot 5 = \boxed{30\ mm}$

Verificação $m \leq 2,5d$

$30 \leq 2,5 \times 22$

$30 \leq 55\ mm$

Altura da bucha de Bronze

Resposta: Esse parafuso serve e deve estar entre 30 e 55 mm

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Flombagem: (Porque tem compressão)

$$\sigma = \frac{F}{A_3} \leq \frac{\sigma_{FL}}{s_{FL}}$$

$$\frac{11\,363,6}{254,5} \leq \frac{\sigma_{FL}}{3} \quad \therefore \sigma_{FL} \geq 134 \text{ MPa}$$

Sem guia. $\lambda = \frac{8H}{d_3}$

$\lambda_{lim} = 105$
$\sigma_r = 400$

Admito (inicialmente) Euler

$$\sigma_{FL} = \frac{E\pi^2}{\lambda^2}$$

$$134 = \frac{210\,000\,\pi^2}{\lambda^2} \qquad \text{OK Vale Euler}$$

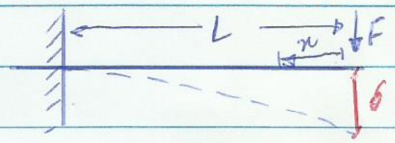
$$(\lambda = 124,37) > (\lambda_{lim} = 105)$$

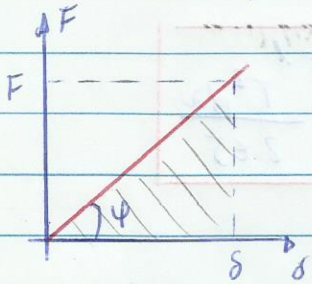
$$124,37 = \frac{8H}{16,5} \quad \boxed{H \leq 256,4 \text{ mm}}$$

$$\eta = \frac{FZP}{2\pi T} = \frac{11\,363,6 \cdot 1 \cdot 5}{2\pi \cdot 30\,000} \quad \therefore \boxed{\eta = 30,1\%}$$

Molas (Cap 14)

Definição





$$tg\,\varphi = \frac{F}{\delta} = K$$

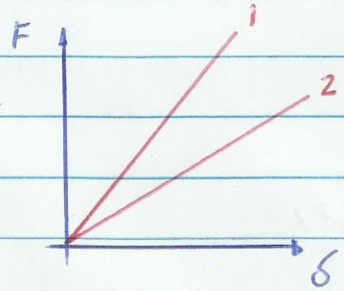
K = cte de rigidez (N/mm)

F: Força (N)

δ: Deformação linear (mm)

T: Torque (Nmm)

θ: Deformação angular (rad)

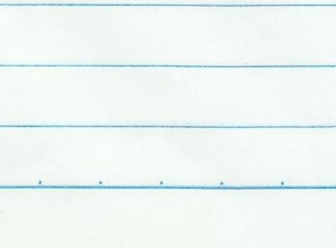


Mola (1) mais rígida que mola (2)

$K_1 > K_2$

$$U = \frac{F\delta}{2} \text{ (Trabalho)} \Rightarrow U = \frac{F\delta}{2} = \frac{T\theta}{2}$$

(Para a energia aplicada subitamente calculo através da área do retângulo)



δ = delta

∂ = del

$$\partial U = \delta\, \partial F$$

$$\delta = \frac{\partial U}{\partial F}$$

$$\theta = \frac{\partial U}{\partial T}$$

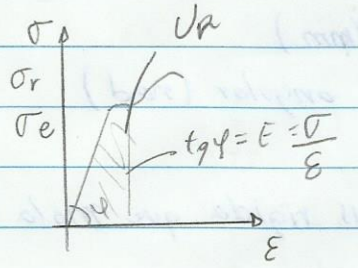
Teorema de Castigliano

$$\cancel{U = \int_0^l \frac{N^2 dx}{2EA}} \qquad \cancel{N} =$$

$$\boxed{U = \int_0^l \frac{N^2 dx}{2EA} + \cancel{X}\int_0^l \frac{V^2 dx}{2GA} + \int_0^l \frac{M^2 dx}{2EI} + \int_0^l \frac{T^2 dx}{2GJ}}$$

N: Força Normal (N)

E = Módulo de Elasticidade longitudinal $\left(\frac{N}{mm^2}\right)$



A = Área da seção transversal (mm^2)

V = força cortante (N)

G = Módulo de elasticidade transversal (MPa) (N/mm^2)

M = Momento fletor (Nmm)

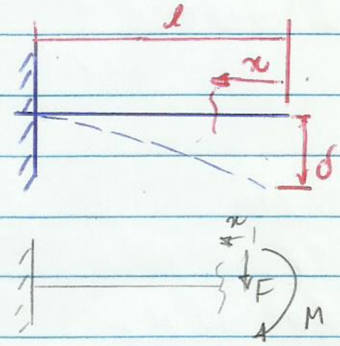
I = Momento de Inércia de área (mm^4)

T = Torque (Nmm)

J = momento polar de inércia de área (mm^4)

l = comprimento (mm)

U_R = Resiliência



$M = Fx$

$0 \le x \le l$

$N = 0$

$V = F$

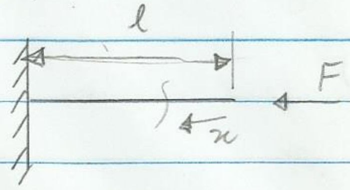
$T = 0$

$$U = \int_0^l \frac{M^2\,dx}{2EI} = \int_0^l \frac{(Fx)^2\,dx}{2EI} = \frac{F^2}{2EI}\,\frac{x^3}{3}\,(l-0)$$

$$\boxed{U = \frac{F^2 l^3}{6EI}} \qquad \frac{\partial U}{\partial F} = \delta = \frac{\partial}{\partial F}\left(\frac{F^2 l^3}{6EI}\right)$$

$$\boxed{\delta = \frac{F l^3}{3EI}} \qquad K = \frac{F}{\delta} \rightarrow K = \frac{F\,3EI}{F l^3}$$

$$\boxed{K = \frac{3EI}{l^3}}$$

② 

$$U = \int \frac{N^2\,dx}{2EA} = \int \frac{F^2}{2EA}\,dx = \frac{lF^2}{2EA}$$

$$\frac{\partial U}{\partial F} = \frac{\partial}{\partial F}\left(\frac{lF^2}{2EA}\right) \therefore \delta = \frac{lF}{EA}$$

$$K = \frac{F}{\delta} = \frac{EA}{l}$$

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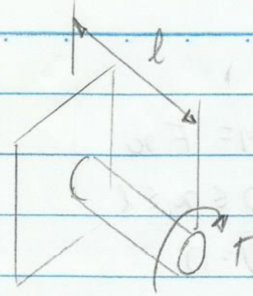
(3)

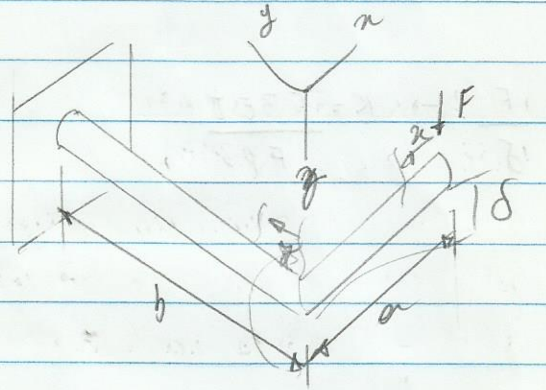


$$U = \int \frac{T^2}{2GJ}\,dx = \frac{T^2 x}{2GJ}$$

$$\theta = \frac{\partial U}{\partial T} = \frac{\partial}{\partial T}\left(\frac{T^2 x}{2GJ}\right) = \frac{Tx}{GJ}$$

$$K = \frac{GJ}{T}$$

(4)



Trecho 1: (x)

$V = F$

$M = Fx$

$0 \le x \le a$

Trecho 2: (y)

$V = F$

$T = Fa$

$M = Fy$, $0 \le y \le b$

$U = U_1 + U_2$

$$U = \int_0^a \frac{M_1^2}{2EI}\,dx + \int_0^b \frac{M_2^2\,dy}{2EI} + \int_0^b \frac{T^2\,dy}{2GJ}$$

$$= \frac{F^2}{2EI}\frac{a^3}{3} + \frac{F^2}{2EI}\cdot\frac{b^3}{3} + \frac{F^2a^2}{2GJ}\cdot b$$

$$= \frac{F^2a^3}{6EI} + \frac{F^2b^3}{6EI} + \frac{F^2a^2 b}{2GJ}$$

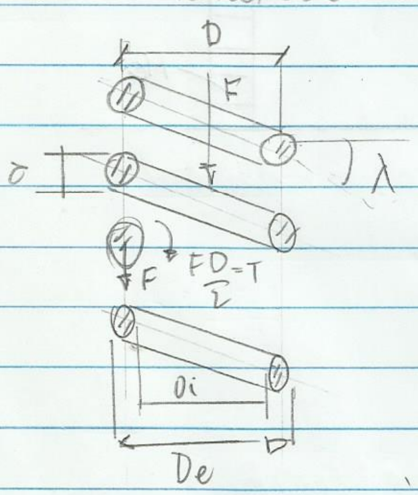
$$\delta = \frac{\partial U}{\partial F} = \frac{\partial}{\partial F}\left(\frac{F^2}{6EI}(a^3+b^3) + \frac{F^2a^2b}{2GJ}\right)$$

$$\delta = \frac{F}{3EI}(a^3+b^3) + \frac{Fa^2b}{GJ}$$

$$K = \frac{F}{\delta} = \frac{F}{\frac{F(a^3+b^3)}{3EI} + \frac{Fa^2b}{GJ}} = \frac{\cancel{F}\cdot 3EI\cdot GJ}{\cancel{F}(a^3+b^3)GJ + \cancel{F}a^2b.3EI}$$

$$\therefore K = \frac{3EIGJ}{(a^3+b^3)GJ + a^2b.3EI}$$

(5) Mola helicoidal cilíndrica da seção circular, sob compressão



$d = \phi$ do fio (arame)

$D = \phi$ mola (médio)

$De = D + d$ diametro externo

$Di = D - d$ diametro interno

$C = \frac{D}{d}$ indice da mola

$\lambda =$ angulo de hélice

$Na =$ nº de espiras

$N =$ nº total de espiras

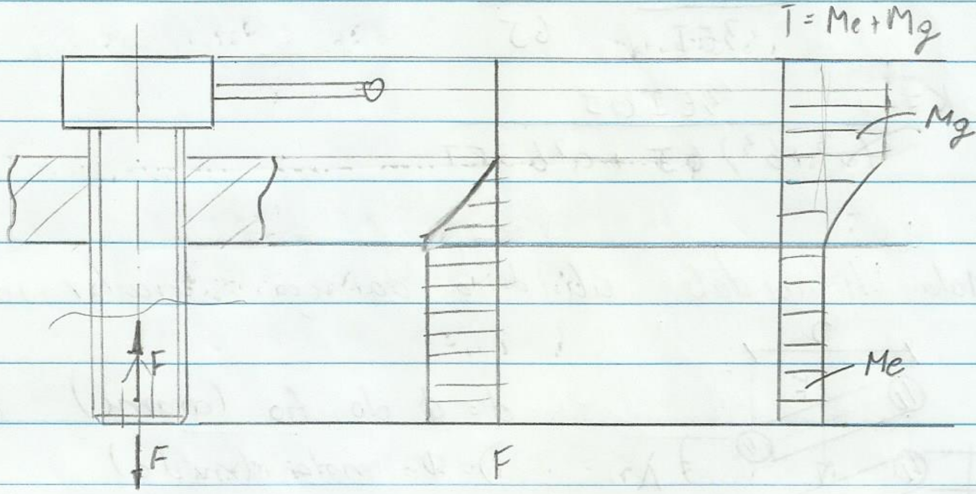
Calcule $K =$

$$K = \frac{Gd}{8NaC^3}$$

Capítulo 4 - Parafusos de movimento

Exercício 4.1

Dados: - $F_m = 100N$ com $\ell = 300mm$
- Aço de classe 4.6 $\begin{cases} \sigma_r = 400\ MPa \\ \sigma_e = 240\ MPa \end{cases}$
- $n = 3$



(a) Dimensões do Fuso
(b) Força F
(c) curso H do fuso
(d) altura h da bucha de bronze
(e) rendimento do sistema

$$M_t = F_m \cdot \ell \geq M_{giro} + M_{escora}$$

Existe compressão e tração na peça

$$\sigma = \frac{F}{A_s} \leq adm \quad e \quad \tau_t = \frac{M_t}{W_t} \leq \tau adm$$

$$\sigma_e = \sqrt{\sigma^2 + 3\tau^2} \leq \frac{\sigma_e}{n}$$

$\frac{N}{mm^2} = MPa$

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Através dos dados do problema temos:

$$\tau_t = \frac{100 \times 300}{0{,}2\, d_3^3} \leq \tau_{adm} \Rightarrow \frac{150\,000}{d_3^3} \leq \tau_{adm}$$

$$\sigma_{eq} = \sqrt{\sigma^2 + 3 \times \left(\frac{150000}{d_3^3}\right)^2} < \frac{240}{3}$$

Para σ desprezível, temos: $d_3 > 14{,}81$ mm

Para $d_3 > 14{,}81$ podemos usar um fuso TR 20 x 4

1ª tentativa)

TR 20 x 4 $\quad d = 20 \quad d_2 = 18 \quad A_s = 220{,}4$

$p = 4 \quad d_3 = 15{,}5$

Verificação das tensões no corpo do parafuso

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} < \frac{\sigma_e}{n} \quad \text{(Tem que ser satisfeito)}$$

Sabendo que $M_t = F_m \cdot l \geq M_{giro} + M_{esc}$

$$M_{giro} = F \cdot \frac{d_2}{2} \, tg(\alpha + \varphi)$$

A partir da tab 5.13: $\varphi = 6{,}3$

$$tg\,\alpha = \frac{z\,P}{\pi d_2} = \frac{1 \times 4}{\pi \cdot 18} \therefore \alpha = 4{,}05°$$

$$M_g = F \times \frac{18}{2}\, tg(6{,}3 + 4{,}05)$$

$\therefore M_g = 1{,}64\,F$

$M_{esc} = \mu_{escora} \cdot F \cdot r$

$\mu_{escora} = 0{,}1$

$r_{ex} = \frac{d_3}{2} = \frac{15{,}5}{2} = 7{,}75$

$r_i = \frac{0{,}6\,d}{2}$

$r_i = \frac{0{,}6 \cdot 20}{2}$

$r_i = 6\,m$

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$r = \dfrac{2(r_e^3 - r_i^3)}{3(r_e^2 - r_i^2)}$ $\quad M_{esc} = 0{,}1\,F.\,\dfrac{2(7{,}75^3 - 6^3)}{3(7{,}75^2 - 6^2)}$

$M_{esc} = 0{,}691F$

$M_t = F_m . l > M_{giro} + M_{escora}$

$100 \times 300 > 1{,}64F + 0{,}691F \quad \therefore \quad F < 12852\,N$

Verificação:

$$\sigma_{eq} = \sqrt{\left(\frac{12\,852}{220{,}4}\right)^2 + 3\times\left(\frac{300\times100}{0{,}2\times15{,}5^3}\right)^2} < \frac{240}{3}$$

$91 < 80$ (Falso)

2ª tentativa $\quad$ TR 22x5 $\quad d = 22 \quad d_2 = 19{,}5 \quad A_s = 254{,}5$

$p = 5 \quad d_3 = 16{,}5$

$\varphi = 6{,}3 \quad tg\,\alpha = \dfrac{1.5}{\pi . 19{,}5} \quad \therefore \alpha = 4{,}67°$

$M_{giro} = F \times \dfrac{19{,}5}{2}\,tg(4{,}67 + 6{,}3) \quad \therefore \quad M_{giro} = 1{,}89F$

$M_{escora} = 0.1 \times F.\,\dfrac{2(8{,}25^3 - 6{,}6^3)}{3(8{,}25^2 - 6{,}6^2)} \quad \therefore \quad M_{escora} = 0{,}746F$

$300 \times 100 > 1{,}89F + 0{,}746F \quad \therefore \quad F = 11386\,N$

$$\sigma_{eq} = \sqrt{\left(\frac{11386}{254{,}5}\right)^2 + 3.\left(\frac{300\times100}{0{,}2\times16{,}5^3}\right)^2} < \frac{240}{3}$$

$73{,}12 < 80 \quad \therefore$ Válido pelo critério das tensões.

Critério do esmagamento nos filetes

$$p = \frac{F}{x \pi d_2 H_1} \leq p_{adm}$$

$$\frac{11\,386}{x . \pi . 19,5 . 0,5 . 5} \leq 15 \quad \therefore \quad x \geq 4,96$$

Verificação

$m \leq 2,5\, d$

$m = 6 \times 5 = 30$

$30 \leq 2,5 \times 22$

$30 \leq 55 \quad \therefore$ Válido pelo critério do esmagamento

Pelo Critério de Flambagem

$$\sigma = \frac{11\,386}{254,5} \leq \frac{\sigma_{FL}}{3} \quad \therefore \quad \sigma_{FL} \geq 134,22 \text{ MPa}$$

Sem guia: $\lambda = \frac{8\, H}{16,5}$ $\quad \lambda_{lim} = 105$

$\sigma_r = 400$

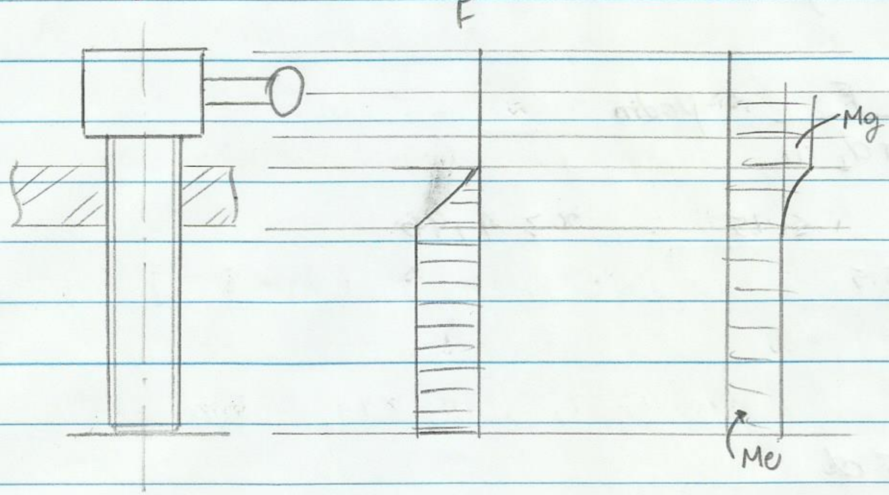
Hipótese de Euler $\quad \lambda \geq \lambda_{lim}$

$$134,22 = \frac{210 . 10^3 . \pi^2}{\lambda^2} \quad \therefore \quad \lambda = 124,27 > 105 \quad \therefore$$

$$124,27 = \frac{8 H}{16,5} \quad \therefore \quad H = 256,3 \text{ mm}$$

/ /

Exercicio 4.4)



Fuso TR 12 x 3 $\left\{ \begin{array}{lll} p = 3 & d_2 = 10{,}5 & A_s = 70{,}9 \\ d = 12 & d_3 = 8{,}5 & \end{array} \right.$

Fuso:

aço SAE 1040 (St60) $\left\{ \begin{array}{ll} \sigma_r = 600\,MPa & n = 4 \\ \sigma_e = 380\,MPa & S_H = 3 \end{array} \right.$

Parafusos de fixação:

- aço, classe 12.9 $\left\{ \begin{array}{ll} \sigma_r = 1200\,MPa & n = 1{,}1 \\ \sigma_e = 1080\,MPa & \end{array} \right.$
- M6

$$M_t = F_m \cdot l > M_{giro} + M_{esc}$$

$$M_{giro} = F \frac{d_2}{2} tg(\alpha + \varphi) \qquad \alpha = tg^{-1}\left(\frac{1 \times 3}{\pi \times 10{,}5}\right) \therefore \alpha = 5{,}20°$$

Como não tem μ filetes $\therefore P = 6{,}3°$

$$M_{giro} = F \times \frac{10{,}5}{2} tg(5{,}20 + 6{,}3) \therefore M_{giro} = 1{,}02\,F$$

$$M_{escora} = 0{,}1\,F \times \frac{2 \times 4{,}25^3}{3 \times 4{,}25^2} \therefore M_{escora} = 0{,}283\,F$$

$r_e = 8{,}5/2 = 4{,}25$ $\therefore M_t = 1{,}02F + 0{,}283F$

$r_i = 0$ logo: $M_t = 1{,}3\,F$

Analise pelas tensões no corpo do parafuso

$$\sigma_{eq} = \sqrt{\left(\frac{F}{A_s}\right)^2 + 3\left(\frac{M_t}{W_t}\right)^2} \leq \frac{\sigma_e}{n}$$

$$\sqrt{\left(\frac{F}{70{,}9}\right)^2 + 3\left(\frac{1{,}3F}{0{,}2 \times 8{,}5^3}\right)^2} \leq \frac{380}{4}$$

$$\frac{F^2}{70{,}9^2} + 3 \times \frac{1{,}3^2 F^2}{(0{,}2 \times 8{,}5^3)^2} \leq \left(\frac{380}{4}\right)^2$$

$$\therefore F \leq 4107{,}18 \text{ N}$$

Analise pela pressão de esmagamento nos filetes

$25 = x\,3 \quad \therefore n = 8{,}33$

$$\frac{F}{8{,}33\pi \cdot 10{,}5 \cdot 0{,}5 \cdot 3} \leq 7{,}5 \quad \therefore F \leq 3092{,}51 \text{ N}$$

Analise pela Flambagem

$$\sigma = \frac{F}{70{,}9} \leq \frac{\quad}{3}$$

/ /

Exercício em sala de aula)

Dado uma peça de aço com $S_n = S_{n\,real} = 500$ MPa

$K_T = 1$ $K_F = 1$, $\sigma_R = 1\,000$ MPa $\sigma_e = 800$ MPa

Calcule por Goodman, a vida N

1) $\sigma_{max} = 900$ MPa, $\sigma_{min} = 300$ MPa

$R = 0{,}33$ $b = 3{,}2095$ $N = 8522$

$\sigma_m = 600$ MPa $m = 0{,}085$

$\sigma_a = 300$ MPa $\sigma'_a = 750$

2) $\sigma_m = 400$ MPa $\sigma_a = 450$ MPa

$R =$ $b =$

$\sigma_{max} =$ $m =$

$\sigma_{min} =$ $\sigma'_a =$

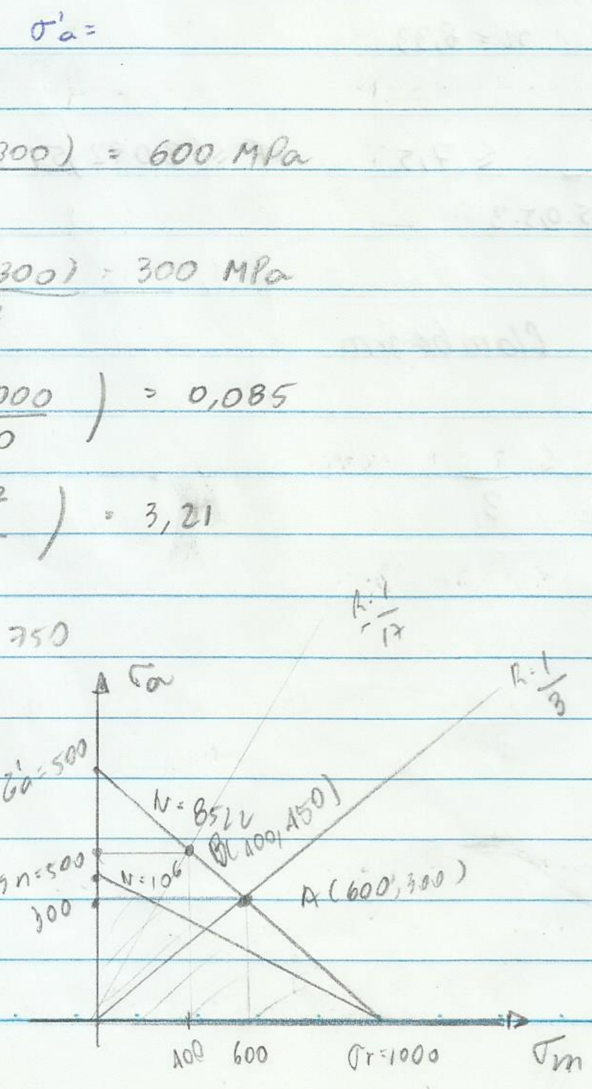
1) $\sigma_m = 1 \cdot \dfrac{(900+300)}{2} = 600$ MPa

$\sigma_a = 1 \times \dfrac{(900-300)}{2} = 300$ MPa

$m = \dfrac{1}{3} \log\left(\dfrac{0{,}9 \cdot 1000}{500}\right) = 0{,}085$

$b = \log\left(\dfrac{0{,}81 \cdot 1000^2}{500}\right) = 3{,}21$

$\sigma'_a = \dfrac{1000 \cdot 300}{1000 - 600} = 750$



2) $R = 0{,}58 = 1/1{,}7$

$\sigma_m = 850$ $\quad b = $ Osmesmo $\quad \sigma_e' = 750$

$\sigma_{min} = -50$ $\quad m =$ Osmesmo $\quad N = 8522$

— // —

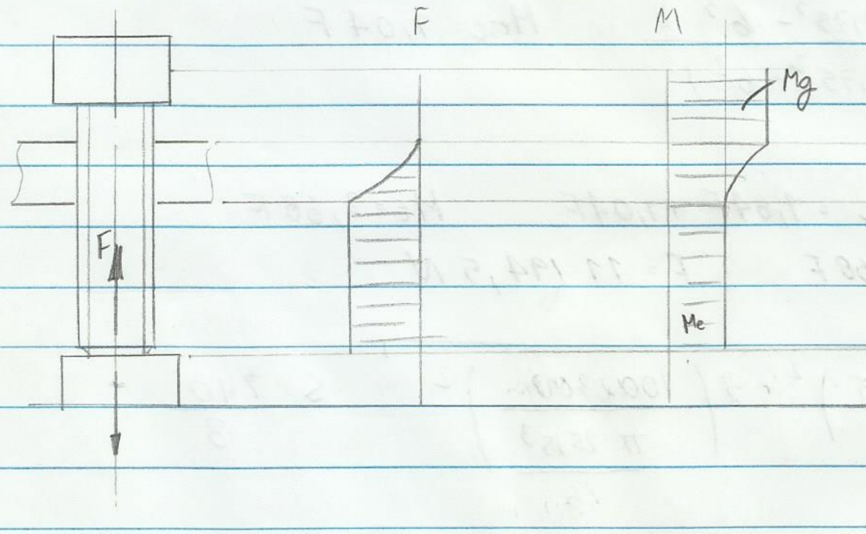
Exercícios pré Prova

Exercício 4-1)

$F_m = 100N$ $\quad$ aço classe 4.6 $\begin{cases} \sigma_r = 400\ MPa \\ \sigma_e = 240\ MPa \end{cases}$

$l = 300mm$

$n = 3$



$M_t = F_m l \geq M_{giro} + M_{escora}$

Torção aplicada na peça

$$\tau_t = \frac{M_t}{W_t} \leq \tau_{adm} \Rightarrow \frac{10 \times 300}{\frac{\pi d_3^3}{16}} < \tau_{adm}$$

$$(\sigma^2 + 3\tau^2)^{0,5} \leq \sigma_e / n$$

Admitindo $\sigma = 0 \rightarrow \left[3 \times \left(\frac{100 \times 300}{\frac{\pi d_3^3}{16}} \right)^2 \right]^{0,5} < \frac{240}{3}$

$\alpha = x$

1 / 1

$$\frac{\sqrt{3} \times 100 \times 300}{\frac{\pi d_3^3}{16}} < \frac{240}{3} \quad \therefore \quad d_3 > 14,9 \text{ mm}$$

Para TR 20x4 $\begin{cases} d = 20 \quad d_2 = 18 \quad As = 220,4 \\ P = 4 \quad d_3 = 15,5 \end{cases}$

$$M_{giro} = F \times \frac{18}{2} tg(\alpha + \varphi) \; ; \; \varphi = 6,3 \quad tg\alpha = \frac{1 \times 4}{\pi \times 18} \therefore \alpha = 4,05$$

$$M_{giro} = F \times \frac{18}{2} tg(6,3 + 4,05) \therefore M_{giro} = 1,64F$$

$$M_{esc} = 0,1 F \times \frac{2(7,75^3 - 6^3)}{3(7,75^2 - 6^2)} \therefore M_{esc} = 1,04 F$$

$$M_t = M_{giro} + M_{esc} = 1,64F + 1,04F \therefore M_t = 2,68 F$$

$$100 \times 300 = 2,68 F \therefore F = 11\,194,5 \text{ N}$$

$$\sqrt{\left(\frac{11\,194,5}{220,4}\right)^2 + 3\left(\frac{100 \times 300}{\frac{\pi \, 15,5^3}{16}}\right)^2} \leq \frac{240}{3}$$

$$87,35 \leq 80 \quad \text{(Falso)}$$

Para TR 22x5 $\begin{cases} d = 22 \quad d_2 = 19,5 \quad As = 254,5 \\ P = 5 \quad d_3 = 16,5 \end{cases}$

$$M_{esc} = 0,1 \times F \times \frac{2(8,25^3 - 6,6^3)}{3(8,25^2 - 6,6^2)} \therefore M_{esc} = 0,746 F$$

$$M_{giro} = F \times \frac{19,5}{2} tg(\alpha + \varphi) \; ; \; \varphi = 6,3^\circ \quad tg\alpha = \frac{1 \times 5}{\pi \, 19,5} \therefore \alpha = 4,67$$

$$M_{giro} = F \times \frac{19,5}{2} tg(6,3 + 4,67) \therefore M_{giro} = 1,89 F$$

$$300 \times 100 = 0{,}746F + 1{,}89F \quad \therefore \quad F = 11\,386{,}22\ N$$

$$\sqrt{\left(\frac{11386{,}22}{254{,}5}\right)^2 + 3\left(\frac{100 \times 300}{\frac{\pi \times 16{,}5^3}{16}}\right)^2} \leq \frac{240}{3}$$

$$73{,}97 \leq 80 \quad \therefore \quad \text{Satisfaz}$$

Critério pelo esmagamento nos filetes

$$p = \frac{F}{n \pi d_2 H_1} \leq p_{adm}$$

$m \leq 2{,}5d$ $\quad$ $m = n P$

$m \leq 2{,}5 \times 22$ $\quad$ $55 = n \times 5$

$m \leq 55$ $\quad$ $n = 11 \geq 6$ (Válido)

$$\frac{11\,386{,}22}{11 \times \pi \times 19{,}5 \times 0{,}5 \times 5} < 15$$

$$6{,}76 < 15 \quad (\text{Válido})$$

Critério de Flambagem

$\lambda_{lim}(400\,MPa) = 105$

$$\frac{11\,386{,}22}{254{,}5} \leq \frac{\sigma_{FL}}{3} \quad \therefore \quad \sigma_{FL} \geq 134{,}22$$

$$134{,}22 = \frac{210 \times 10^3 \cdot \pi^2}{\lambda^2} \quad \lambda = 124{,}266 > 105 \quad \text{Válido}$$

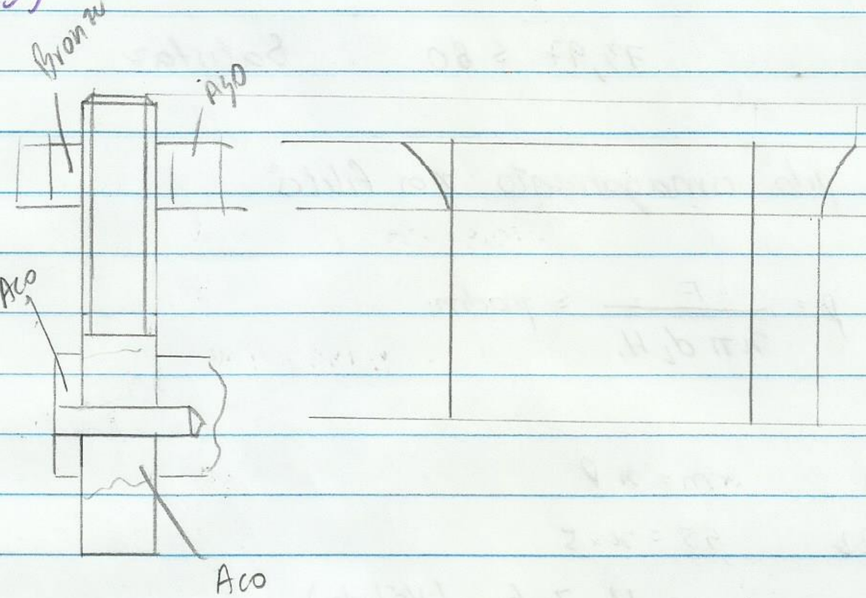
$$124{,}266 = \frac{8H}{16{,}5} \quad \therefore \quad H = 256{,}3\ mm$$

/ /

Rendimento

$$\eta = \frac{11386{,}22 \times 1 \times 5}{2\pi \times 300 \times 100} \therefore \eta = 0{,}30 \Rightarrow 30\%$$

Exercício 4.3)



$n = 1{,}5 \quad S_{el} = 3$

Cavilha classe 8.8 $\begin{cases} \sigma_r = 800 \text{ MPa} \\ \sigma_e = 640 \text{ MPa} \end{cases}$

Fuso aço classe 5.8 $\begin{cases} \sigma_r = 500 \text{ MPa} \\ \sigma_e = 400 \text{ MPa} \end{cases}$

TR 12x3 $\begin{cases} d = 12 & d_2 = 10{,}5 & A_s = 70{,}9 \\ P = 3 & d_3 = 8{,}5 \end{cases}$

Para F estrutura

$$M_{escora} = 0{,}08 \times F \times \frac{2}{3} b \therefore M_{esc} = 0{,}32\, F$$

$$M_{giro} = F \times \frac{10{,}5}{2} tg(\alpha + \varphi) \;;\; \varphi = 6{,}3 \text{ e } \alpha = tg^{-1}\left(\frac{1 \times 3}{\pi \times 10{,}5}\right) = 5{,}2$$

$$\therefore M_{giro} = F \times \frac{10{,}5}{2} \times tg(6{,}3 + 5{,}2) \;\downarrow\; M_{giro} = 1{,}07\, F$$

$Mt = 1{,}39F$

$$\sqrt{\left(\frac{F}{70{,}9}\right)^2 + 3\left(\frac{1{,}12F}{\frac{\pi \times 8{,}5^3}{16}}\right)^2} \leq \frac{400}{1{,}5} \quad \therefore F < 12\,456{,}91\ N$$

Por esmagamento

$25 \leq x \times 3$

$x = \frac{25}{3}$

$$\frac{F}{\frac{25}{3} \times \pi \times 10{,}5 \times 0{,}5 \times 3} < 15$$

$$\therefore F \leq 6\,185{,}01\ N$$

Por Flambagem:

$\lambda lim\,(500) = 89$

$\lambda = \frac{2{,}8 \times 150}{8{,}5} = 49{,}41$

Como $\lambda < \lambda lim$

$\sigma_{FL} = 335 - 0{,}62 \times 49{,}41 \quad \therefore \sigma_{FL} = 304{,}36\ MPa$

$$\frac{F}{70{,}9} \leq \frac{304{,}36}{3} \quad \therefore F \leq 7\,193{,}15$$

$\therefore$ Resposta final: $F < 6185\ N$

$M_T = 1{,}12F = 8\,583{,}5\ Nmm \quad \therefore Fm = 122{,}62\ N$

/ /

Exercício 4.2)

TR 30x6 $\begin{cases} d = 30 & d_2 = 27 & A_s = 490{,}9 \\ P = 6 & d_3 = 23 \end{cases}$

$n = 40$ rpm $\qquad P = 2\pi n T$

Pot = 0,5 KW $\qquad 0{,}5 \cdot 10^3 = 2\pi \times \frac{40}{60} T \therefore T = 119{,}37$ Nm

$\varphi = 6{,}3 \qquad tg\alpha = \frac{1 \times 6}{\pi \times 27} \therefore \alpha = 4{,}046$

$T = 2 \, Mgiro \therefore Mgiro = \frac{119{,}37}{2} \therefore Mgiro = 59{,}68 \cdot 10^3$

$59{,}68 \cdot 10^3 = F \times \frac{27}{2} tg(6{,}3 + 4{,}046) \therefore F = 24\,216$ N

Tensões no corpo do parafuso

$$\sqrt{\left(\frac{24\,216}{490{,}9}\right)^2 + 3\left(\frac{59{,}68 \cdot 10^3}{0{,}2 \times 23^3}\right)^2} < \frac{\sigma e}{3} \therefore \sigma e \geq 195{,}3$$

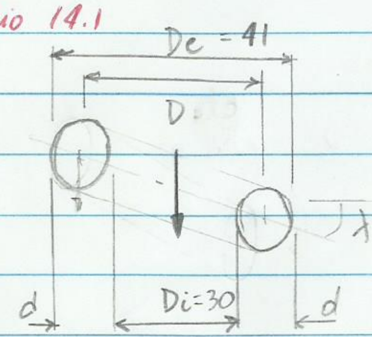
Esmagamento

$H_1 = 0{,}5 P = 0{,}5 \cdot 6 = 3 \qquad \frac{24\,216}{x \pi \times 27 \times 3} < 15$

$p_{adm} = 15$ MPa $\qquad \therefore x > 6{,}34$ (OK)

$m = x p = 6 \times 6 = 36$ mm $< 2{,}5 \times 30 = 75$ (OK)

Exercício 14.1



$F_i = 50N$ $F_f = 800N$

$De = 41mm$ $Di = 30mm$

$e = 4{,}9mm$ $H = 104{,}6mm$

ABNT 6150 Serviço médio Extremidades em esquadro e retificados

Teorema de Castigliano

$$U = \int_0^l \frac{N^2}{2EA}dx + X\int_0^l \frac{V^2}{2GA}dx + \int_0^l \frac{M^2 dx}{2EI} + \int_0^l \frac{T^2 dx}{2GJ}$$

Resistencia maior

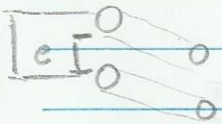
Mola retificada seção quadrada

Indice da Mola $C = \frac{D}{d}$

$d = \frac{De - Di}{2} = \frac{41-30}{2}$ ∴ $\boxed{d = 5{,}5mm}$

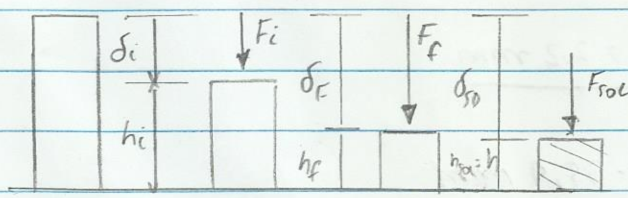
$D = Di + d = 30 + 5{,}5$ ∴ $\boxed{D = 35{,}5mm}$

$C = \frac{D}{d} = \frac{35{,}5}{5{,}5}$ $\boxed{C = 6{,}4545}$ Indice $\Big\{$ Geralmente $3 < C < 12$



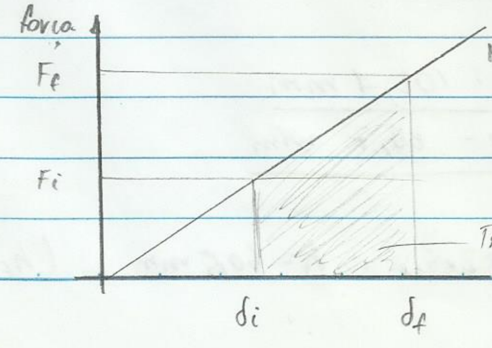
$P = e + d$

$P = 4{,}9 + 5{,}5$ ∴ $\boxed{P = 10{,}4mm}$



$h_F > h$

$h_F > h$



$K = \frac{F}{\delta} = \frac{F_i}{\delta_i} = \frac{F_f}{\delta_F} = \frac{F_F - F_i}{\delta_F - \delta_i} = \frac{F_{sol}}{\delta_{sol}}$

$\delta_F - \delta_i = f_u$ curso útil

/ /

Tab 14.1

$H = PNa + 2d = h_f + d_f$

$h = dNa + 2d$

$N = Na + 2$

Sabendo $H = PNa + 2d$

$104{,}6 = 10{,}4\, Na + 2 \times 5{,}5 \therefore \boxed{Na = 9}$ espiras ativas

$N = Na + 2 \quad \boxed{N = 11}$ espiras

$N = 9 + 2$

Para Aço ABNT 6150 (tab 14.10)

$4 < d < 6 \rightarrow \begin{cases} \tau_{adm} = 600\ MPa \\ E = 210\ GPa \\ G = 78{,}4\ GPa \\ S_n = S_{nreal} = 390\ MPa \\ \tau_e = 770\ MPa \end{cases}$

$K = \dfrac{Gd}{8 \cdot Na\, C^3} \Rightarrow K = \dfrac{78{,}4 \cdot 10^3 \times 5{,}5}{8 \times 9 \times 6{,}4545^3} \therefore \boxed{K = 22{,}272\ \dfrac{N}{mm}}$

$K = \dfrac{F_f - F_i}{\delta_f - \delta_i} = \dfrac{F_f - F_i}{f_u} \therefore f_u = \dfrac{800 - 50}{22{,}272} \quad \underline{f_u = 33{,}675}$

$\delta_i = \dfrac{50}{22{,}272} \therefore \underline{\delta_i = 2{,}2\ mm}$

$\delta_f = \dfrac{800}{22{,}272} \therefore \underline{\delta_f = 35{,}9\ mm}$

$h_i = 104{,}6 - 2{,}2 \therefore \underline{h_i = 102{,}4\ mm}$

$h_f = 104{,}6 - 35{,}9 \therefore \underline{h_f = 68{,}7\ mm}$

$h = dNa + 2d = 5{,}5 \times 9 + 2 \times 5{,}5 \therefore h = 60{,}5\ mm \quad (h_f > h)$ OK

Livro WAHL
Mechanical Springs

Acima de 10^3 ciclos considera carga dinâmica

Para carga estática

$$\tau = \frac{8 F C K_s}{\pi d^2} \leq \tau_{adm} \quad ; \quad K_s = 1 + \frac{1}{2C} = 1{,}08$$

$$\frac{8 \cdot F \cdot 6{,}4545 \cdot 1{,}08}{\pi \times 5{,}5^2} \leq 600 \qquad K_a = K_w = \frac{4C-1}{4C-4} + \frac{0{,}615}{C} = 1{,}23$$

$F \leq 1022{,}47\ N$ Sendo $F_f < F$ OK
por resistência

Para não atingir a altura sólida

$H = h + \delta_{sol} \therefore \delta_{sol} = 44{,}1\ mm$

$F_{sol} = 44{,}1 \times 22{,}3 \therefore F_{sol} = 982{,}2\ N$

$\therefore$ Encosta antes de quebrar

Para achar f (16,2)

Apartir da massa da mola é possível determinar a frequência natural da mola ω_n

$$\omega_n = \sqrt{\frac{k}{m}}$$

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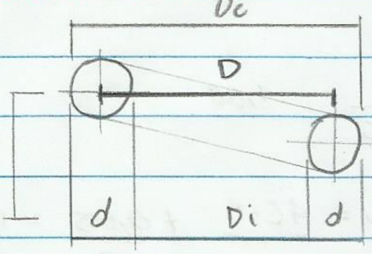
Exercícios de Molas

Exercício 14.1)



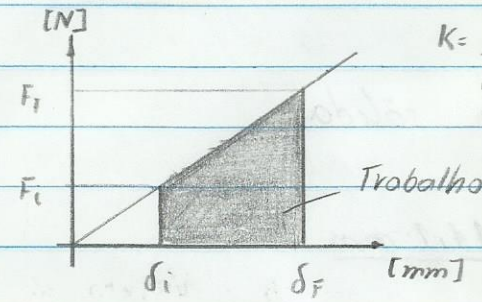
$F_i = 50N$ $D_e = 41mm$

$F_f = 800N$ $e = 4{,}9mm$ (Mola Descarregada)

$D_i = 30\ mm$

$h_{livre} = 104{,}6mm$

$d = \frac{De - Di}{2} = \frac{41-30}{2} \therefore d = 5{,}5\ mm$



$K = \frac{F}{\delta} = \frac{F_i}{\delta_i} = \frac{F_f}{\delta_f} = \frac{F_f - F_i}{\delta_F - \delta_i}$

Principais dimensões da mola

$D_e = 41mm$ $D_i = 30\ mm$ $d = 5{,}5\ mm$ $D = 35{,}5\ mm$

$D = De - d$

Índice da Mola

$C = \frac{D}{d} = \frac{35{,}5}{5{,}5} \therefore C = 6{,}45$

Fator de Concentração de tensões estático

$K_S = 1 + \frac{d}{2D} = 1 + \frac{1}{2C} \therefore K_S = 1 + \frac{5{,}5}{2 \cdot 35{,}5} \therefore K_S = 1{,}08$

Fator dinâmico de Wahl

$$K_A = \frac{4C-1}{4C-4} + \frac{0,615}{C} \therefore K_A = 1,23$$

$$P = e + d = 4,9 + 5,5 \therefore P = 10,4\ mm$$

De acordo com Tabela 14.1

$H = P N_a + 2d = hf + \delta f$
$h = d N_a + 2d$
$N = N_a + 2$

Sabendo $H = P N_a + 2d$
$104,6 = 10,4 N_a + 2.5,5 \therefore N_a = 9$ (Numero de espiras ativos)

$h = d N_a + 2d$
$h = 5.5 \times 9 + 2 \times 9 \therefore h = 60,5\ mm$ ($h = h_s$ altura sólida)

$N = N_a + 2$
$N = 9 + 2 \therefore N = 11N$ (Força Normal)

Para Aço ABNT 6150
$\tau_{adm} = 600\ MPa$
$E = 210\ GPa$
$G = 78,4\ GPa$
$S_{nrd} = S_n = 390\ MPa$
$\tau_e = 770\ MPa$

$$K = \frac{Gd}{8 N_a C^3} \therefore K = 22,4\ N/mm$$

/ /

Forças e deltas

$\delta_i = 2{,}23\ mm$ $\delta_F = 35{,}74\ mm$

$h_i = 102{,}37$ $h_F = 68{,}4\ mm$ $h_F > h$ (OK)

$K = \dfrac{F_F - F_i}{F_u}$ ∴ $F_u = 33{,}50\ mm$

Para cargas repetidas com mais de 10^3 ciclos, por fadiga

$\tau_m = \dfrac{8 F_m C K_s}{\pi d^2} = \dfrac{8 \cdot F \cdot 6{,}45 \cdot 1{,}08}{\pi \cdot 5{,}5} \leq \tau_{adm}$

Para $\tau_{adm} = 600\ MPa$ ∴ $F \leq 1023{,}2\ N$

$(F_F < F)$ OK por resistência

$H = h\ \delta sol$

$104{,}6 = 60{,}4 + \delta_{sol}$ ∴ $\delta_{sol} = 44{,}2$ ∴ $F_{sol} = 384{,}N < F$

Para achar γ ∴ $\gamma = 78{,}5\ \dfrac{N}{dm^3}$ ∴ $\gamma = \dfrac{78{,}5}{10^{12}}\ N/mm$

$1\ dm = 0{,}1 \cdot 10^3\ mm$

$1\ dm^3 \quad 0{,}1 \cdot 10^3)^3\ mm$

$tg\,\lambda = \dfrac{P}{\pi D}$ ∴ $tg\,\lambda = \dfrac{10{,}4}{\pi\, 35{,}5}$ ∴ $\lambda = 5{,}33°$

$f_n = \dfrac{1}{2\pi \cdot 6{,}45 \times 35{,}5 \cdot 9} \sqrt{\dfrac{10 \cdot 10^3\ 78{,}4 \cdot 10^3}{2 \times 78{,}5 \cdot 10^{6}}}$ ∴ $f_n = 172{,}6\ Hz$

$A = \dfrac{F_u (F_F + F_i)}{2} = \dfrac{33{,}50\,(800 - 50)}{2}$ ∴ $A = 14{,}2\ Nm$

$F_m = \frac{F_f + F_i}{2} = \frac{600+50}{2} \therefore F_m = 425\,N$

$F_a = \frac{F_f - F_i}{2} = \frac{800-50}{2} \therefore F_a = 375\,N$

$\tau_a = \frac{8 F_a C K_a}{\pi d^2} \therefore \tau_a = 250,4\ MPa$

$\tau_m = \frac{8 F_m C K_s}{\pi d^2} \therefore \tau_m = 249,22$

Exercício 14.2

$N_a = 5$ espiras ativas $\qquad \tau_{adm} = 400\,MPa$

$D = 40\,mm \qquad G = 85\,GPa$

$d = 6\,mm$

Calcule a Máxima Carga e a Flecha

$\tau = \frac{8FCK_s}{\pi d^2}$; $C = \frac{D}{d} \therefore C = 6,\bar{6}$

$K_s = 1 + \frac{1}{2C} \therefore K_s = 1,075$

$\frac{8 F \cdot 6,6 \cdot 1,075}{\pi \cdot 6^2} \leq 400 \therefore \boxed{F = 789,05\,N}$

$\delta = \frac{8FC^3 N_a}{Gd} = \frac{8 \cdot 789,05 \cdot 6,6^3 \cdot 5}{85 \cdot 10^3 \cdot 6} \therefore \boxed{\delta = 18,3\,mm}$

/ /

Exercício 14.3

$Na = 16$ Determine a nova rigidez

$K = 100\ N/mm$

$N_1 = 4$ e $N_2 = 12$

$$K = \frac{Gd}{8C^3 Na}$$ Constante para as novas molas

$$K = \frac{\Delta}{Na}$$ ∴ Para $Na = 16$ e $K = 100$ ∴ $\Delta = 1600$

Para Molas helicoidais de compressão com extremidades em ponta. $N = Na$

∴ $K_1 = \frac{\Delta}{N_1}$ e $K_2 = \frac{\Delta}{N_2}$ ∴ $K_1 = 400\ N/mm$

$K_2 = 133\ N/mm$

Exercício 14.4

$Fm = 200\ N$ e $hm = 57\ mm$ $De = 30\ mm$

$400\ N \rightarrow h = 47\ mm$ $C = 6{,}6$

$C = \frac{D}{d}$ $De = D + d$ $D + d = 30$; $D = 6{,}6d$

∴ $6{,}6d + d = 30$

∴ $d \approx 3{,}95$

$d_{normalizado} = 3{,}5$, logo $D = 23{,}1\ mm$

Mola { $D = 23{,}1\ mm$ $De = 26{,}6\ mm$
$d = 3{,}5\ mm$ $Di = 19{,}6\ mm$

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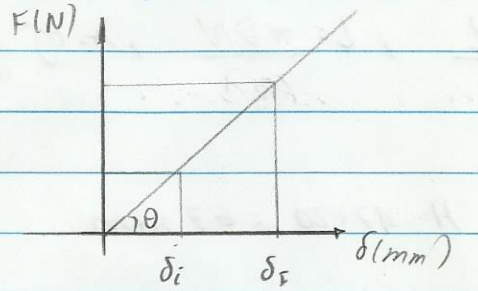

$\delta = H - h$

$200 \rightarrow H - 57$

$400 \rightarrow H - 47$

$K = \dfrac{400 - 200}{(H-47)-(H-57)}$ $\therefore$ $\boxed{K = 20 \frac{N}{mm}}$

Para mola helicoidal cilíndrica com extremidades retificadas e em esquadro

$H = PNa + 2d = hf + \delta f$

$h = dNa + 2d$

$N = Na + 2$

Para aço SAE 1065

$\tau_{adm} = 640$ MPa

E = 210 GPa

G = 74,8 GPa

$\sigma_r = 1300$ MPa

$\sigma_e = 1000$ MPa

$\tau_e = 600$ MPa

$S_n = S_{nreal} = 330$ MPa

$\dfrac{8FCK_s}{\pi d^2} < \tau_{adm}$, $K_s = 1 + \dfrac{1}{2C}$; $C = 6,6$ logo $K_s = 1,08$

$\dfrac{8F \cdot 6,6 \cdot 1,08}{\pi \cdot 3,5^2} < 640$ $\therefore$ $\boxed{F < 434\ N}$

$K = \dfrac{Gd}{8 N_a C^3}$ $\quad 20 = \dfrac{74,8 \cdot 10^3 \cdot 3,5}{8 N_a \cdot 6,6^3}$ $\therefore$ $N_a = 5,69$ espiras ativas

$N = Na + 2$ $\therefore$ $\boxed{N = 7,69 \text{ espiras}}$

γ (De acordo com tabela) $= 78,5 \cdot 10^{-6}\ N/mm^3$ $\quad \gamma = \dfrac{\text{massa} \cdot \text{aceleração}}{\text{Volume}}$

/ /

$\gamma = \frac{P \cdot g}{v}$; $v = \pi r^2 \cdot L = \frac{\pi d^2 L}{4}$, $L = \frac{\pi D N}{\cos\lambda}$, $\lambda = tg^{-1} \frac{P}{\pi D}$

$H = h_f + \delta_F$; $\delta_F = \frac{400}{20} = 20$ $\therefore$ $H = 47 + 20 = 67$ mm

$H = P Na + 2d$ $\quad 67 = P \cdot 5{,}69 + 2 \cdot 3{,}5$ $\therefore$ $P = 10{,}54$ mm

$\lambda = tg^{-1}\left(\frac{10{,}54}{\pi \cdot 23{,}1}\right)$ $\therefore$ $\lambda = 8{,}27^\circ$

$L = \frac{\pi \cdot 23{,}1 \cdot 7{,}69}{\cos 8{,}27}$ $\therefore$ $L = 564{,}03$ mm

$v = \frac{\pi \cdot 3{,}5^2 \cdot 564{,}03}{4}$ $\therefore$ $v = 5\,426{,}6$ mm^3

$78{,}5 \cdot 10^{-6} = \frac{Peso}{5426{,}6}$ $\therefore$ $\boxed{Peso = 0{,}426 \text{ N}}$

$A = \frac{(\delta_F - \delta_i)(400 + 200)}{2}$ $\therefore$ $\boxed{A = 3000 \text{ Nmm}}$

$h = h_s = d Na + 2d = 3{,}5 \cdot 5{,}69 + 2 \cdot 3{,}5$ $\therefore$ $h_s = 26{,}92$ mm

$\delta_s = H - h_s = 67 - 26{,}92$ $\therefore$ $\boxed{\delta_s = 40{,}1 \text{ mm}}$

Exercício 14.5)

Força variando dinamicamente de 200N → F

$n_f = 2$

Soderberg: $\frac{\tau_a}{S_n} + \frac{\tau_m}{\tau_e} = \frac{1}{n_s}$ Goodman: $\frac{\tau_a}{S_n} + \frac{\tau_m}{\tau_R} = \frac{1}{n_g}$

$F_m = \frac{F_f + F_i}{2} = \frac{F + 200}{2}$ e $F_a = \frac{F_f - F_i}{2} = \frac{F - 200}{2}$

$\tau_a = \frac{8 F_a C K_a}{\pi d^2}$ e $\tau_m = \frac{8 F_m C K_s}{\pi d^2}$

$K_a = \frac{4C-1}{4C-4} + \frac{0{,}615}{C}$ Para $C = 6{,}6$ ∴ $K_a = 1{,}23$

$K_s = 1 + \frac{1}{2C}$, Para $C = 6{,}6$ ∴ $K_s = 1{,}08$

$$\frac{\frac{8 \cdot \left(\frac{F-200}{2}\right) \cdot 6{,}6 \cdot 1{,}23}{\pi \cdot 3{,}5^2}}{330} + \frac{\frac{8\left(\frac{F+200}{2}\right) \cdot 6{,}6 \cdot 1{,}08}{\pi \cdot 3{,}5^2}}{600} = \frac{1}{2}$$

$2{,}55 \cdot 10^{-3} F - 0{,}51 + 1{,}23 \cdot 10^{-3} F + 0{,}25 = 0{,}5$

∴ $\boxed{F = 202{,}12\ N}$

Por Goodman:

$2{,}55 \cdot 10^{-3} F - 0{,}51 + 7{,}096 \cdot 10^{-4} F + 0{,}142 = 0{,}5$

∴ $\boxed{F = 266{,}3\ N}$

/ /

Exercício 14.6)

$F = 5\,KN$ $\quad G = 75\,GPa$

$\tau_{max} = 750\,MPa$ $\quad \lambda = 8°$

$K = 25\,kN/m$ $\quad C = 7$

Para extremidades em esquadro e retificadas, temos:

$H = PNa + 2d = h_f + \delta_f$

$h = dNa + 2d$

$N = Na + 2$

$$\tau_{max} = \frac{8FCK_s}{\pi d^2} \; ; \; K_s = 1 + \frac{1}{2C}$$

Para $C = 7 \Rightarrow K_s = 1{,}07$ $\quad \dfrac{8.5.10^3.7.1{,}07}{\pi d^2} < 750$

$\therefore d > 11{,}28$ $\quad \boxed{d_{norm} = 12\ mm}$ $\quad \boxed{D = 84\,mm}$

Para $\begin{cases} F = 5\,KN \\ K = 25\,KN/m \end{cases}$ $\quad \delta = 200\ mm$

$$\delta = \frac{8FC^3Na}{Gd} \qquad 200 = \frac{8.5.10^3.7^3\,Na}{75.10^3.12} \quad \therefore \boxed{\begin{matrix} Na = 13{,}12 \\ N = 15{,}12 \end{matrix}}$$

$h = dNa + 2d = 12 \times 13{,}12 + 2 \times 12$ $\quad \therefore \boxed{h = 181{,}43\ mm}$

$tg\,\lambda = \dfrac{P}{\pi D}$ $\quad tg\,8 = \dfrac{P}{\pi.84}$ $\quad \therefore \boxed{P = 37{,}09\ mm}$

$H = PNa + 2d$ $\quad H = 37{,}09.13{,}12 + 2.12$ $\quad \therefore \boxed{H = 510{,}58\ mm}$

$P = e + d$ $\quad 37{,}09 = e + 12$ $\quad \therefore \boxed{e = 25{,}09\ mm}$

Exercício 14.7

$F = 600N$ $\quad G = 80GPa$ $\quad$ Extremidades em esquadro esmerilhadas

$D = 40\ mm$ $\quad Na = 7$ $\quad H = PNa + 2d = hf + \delta f$

$d = 5\ mm$ $\quad \lambda = 8°$ $\quad hs = h = dNa + 2d$

$N = Na + 2$

$C = \frac{D}{d} \therefore C = 8 \quad Ks = 1 + \frac{1}{2C} \therefore Ks = 1,06$

$\tau = \frac{8FCKs}{\pi d^2} = \frac{8 . 600 . 8 . 1,06}{\pi . 5^2} \therefore \boxed{\tau = 519,5\ MPa}$

$\delta = \frac{8FC^3Na}{Gd} = \frac{8 . 600 . 8^3 . 7}{80 . 10^3 . 5} \therefore \boxed{\delta = 43\ mm}$

$hs = dNa + 2d = 5 . 7 + 2 . 5 \therefore \boxed{hs = 45\ mm}$

$tg\lambda = \frac{P}{\pi D} \quad tg 8 = \frac{P}{\pi 40} \therefore P = 17,66\ mm$

$H = PNa + 2d = 17,66 . 7 + 2 . 5 \therefore \boxed{H = 133,63\ mm}$ Altura livre

$H = hf + \delta f \quad 133,63 = hf + 43 \therefore \boxed{hf = 90,63\ mm}$

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Exercício 14.8

$N = 7,2$ espiras

$\lambda = 8°$

$C = 6$

Carga dinâmica $50 \rightarrow 300\,N$

Aço tipo corda de Piano

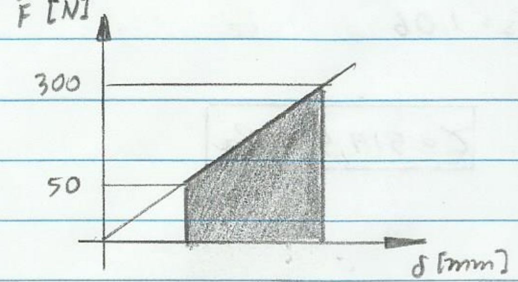
Serviço Leve

Extremidades em esquadro e retificadas

$H = PNa + 2d = hf + \delta_f$

$h = dNa + 2d$

$N = Na + 2$



$Na = N - 2$; $N = 7,2$ ∴ $Na = 5,2$ espiras

Temos : $G = 84\,GPa$ $\sigma_r = 2000\,MPa$ $\tau_e = 1050\,MPa$

$E = 210\,GPa$ $\sigma_e = 1750\,MPa$

Para cargas repetidas com mais de 10^3 ciclos

$$Fm = \frac{F_f + F_i}{2} = \frac{300+50}{2} = 175\ N$$

$$K_s = 1 + \frac{1}{26} \therefore K_s = 1,08$$

$$Fa = \frac{F_f - F_i}{2} = \frac{300-50}{2} = 125\ N$$

$$Ka = \frac{4 \cdot 6 - 1}{4 \cdot 6 - 4} + \frac{0,615}{6} \therefore Ka = 1,25$$

$$\tau m = \frac{8 \cdot 175 \cdot 6 \cdot 1,08}{\pi d^2} = \frac{2896,62}{d^2}$$

$$\tau a = \frac{8 \cdot 125 \cdot 6 \cdot 1,25}{\pi d^2} = \frac{2392,10}{d^2}$$

Pelo critério de Soderberg (crítico $n_s = 1$)

$$\frac{\tau_a}{S_n} + \frac{\tau_m}{\tau_e} = \frac{1}{n_s}$$

$$\frac{\frac{2\,392,10}{d^2}}{560} + \frac{\frac{2\,896,62}{d^2}}{1050} \leq 1$$

$d > 2,65\ mm \therefore \boxed{d = 2,80\ mm}$ (Soderberg)

Para Goodman: $\frac{\tau_a}{S_n} + \frac{\tau_m}{\tau_r} = \frac{1}{n_g}$ (Crítico $n_g = 1$)

$$\frac{\frac{2\,392,10}{d^2}}{560} + \frac{\frac{2\,896,62}{d^2}}{0,8 \cdot 2000} < \frac{1}{1}$$

$\therefore d > 2,47\ mm \therefore \boxed{d = 2,50\ mm}$ (Goodman)

Exercício 11.9

Para Associação em Série $F = F_1 = F_2$ e $\delta = \delta_1 + \delta_2$

$K = \frac{F}{\delta}$; $\delta = \delta_1 + \delta_2 \Rightarrow \frac{F}{K} = \frac{F_1}{K_1} + \frac{F_2}{K_2}$

$\frac{F}{K} = \frac{F}{K_1} + \frac{F}{K_2} \therefore \boxed{\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}}$

Para Associação em Paralelo $F = F_1 + F_2$ e $\delta = \delta_1 = \delta_2$

$F = F_1 + F_2$; $K \cdot \delta$

$K \cdot \delta = K_1 \delta_1 + K_2 \delta_2$

$K \cdot \delta = K_1 \delta + K_2 \delta \therefore \boxed{K = K_1 + K_2}$

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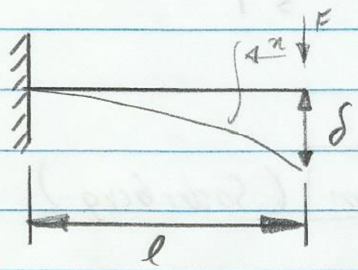
Exercícios : Teorema de Castigliano



$N = 0$

$V = F$

$M = Fx \quad ; \quad 0 < x < \ell$

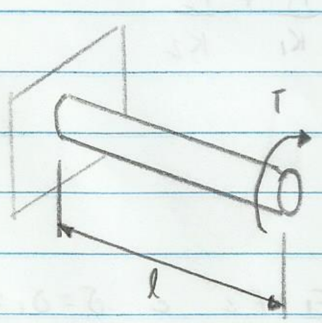
$T = 0$

$$U = \int \frac{M^2 dx}{2EI} = \int_0^{\ell} \frac{(Fx)^2}{2EI} dx = \frac{F^2}{2EI} \int_0^{\ell} x^2 dx$$

$$\therefore U = \frac{F^2}{2EI} \frac{x^3}{3} (\ell - 0) \qquad \therefore U = \frac{F^2 \ell^3}{6EI}$$

$$\delta = \frac{dU}{dF} = \frac{d}{dF}\left(\frac{F^2 \ell^3}{2EI}\right) \qquad \therefore \boxed{\delta = \frac{F\ell^3}{3EI}}$$

$$K = \frac{F}{\delta} = \frac{F}{\frac{F\ell^3}{3EI}} \qquad \therefore \boxed{K = \frac{3EI}{\ell^3}}$$



$N = 0$

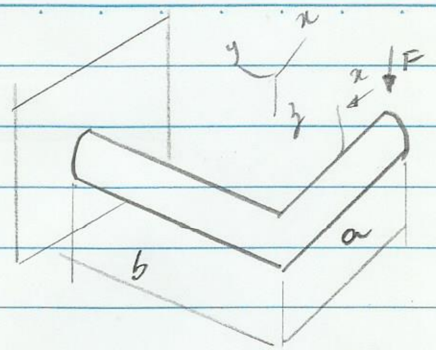
$V = 0$

$M = 0$

$T = T$

$$U = \int_0^{\ell} \frac{T^2 dx}{2GJ} \quad \therefore \quad U = \frac{T^2 \ell}{2GJ}$$

$$\theta = \frac{dU}{dT} = \frac{d}{dT}\left(\frac{T^2 x}{2GJ}\right) \quad \therefore \boxed{\theta = \frac{T\ell}{GJ}} \qquad \therefore \boxed{K = \frac{GJ}{\ell}}$$



$U = U_1 + U_2$

Trecho U_1 $\quad N = 0$ $\quad U_1 = \int_0^a \frac{(Fx)^2}{2EI}dx \quad \frac{F^2a^3}{6EI}$

$V = F$ (Desprezível)

$M = Fx$

$T = 0$

Trecho U_2 $\quad N = 0$

$V = F$ $\quad U_2 = \int_0^b \frac{(Fy)^2}{2EI}dy + \int_0^b \frac{(Fa)^2}{2GJ}dy$

$M = Fy$

$T = Fa$ $\quad U_2 = \frac{F^2b^3}{6EI} + \frac{F^2a^2 \cdot b}{2GJ}$

$$\therefore U = \frac{F^2a^3}{6EI} + \frac{F^2b^3}{6EI} + \frac{F^2a^2b}{2GJ}$$

$$\delta = \frac{dU}{dF} = \frac{d}{dF}\left(\frac{F^2a^3}{6EI} + \frac{F^2b^3}{6EI} + \frac{F^2a^2b}{2GJ}\right)$$

$$\delta = \frac{Fa^3}{3EI} + \frac{Fb^3}{3EI} + \frac{Fa^2b}{GJ} \quad \therefore \boxed{\delta = \frac{F}{3EI}(a^3+b^3) + \frac{Fa^2b}{GJ}}$$

$$K = \frac{F}{\delta} \qquad K = \frac{F}{\frac{F(a^3+b^3)}{3EI} + \frac{Fa^2b}{GJ}} = \frac{F}{\frac{GJ\,F(a^3+b^3) + 3EI\,Fa^2b}{3EIGJ}}$$

$$\therefore \boxed{K = \frac{3EIGJ}{GJ(a^3+b^3) + 3EI\,a^2b}}$$

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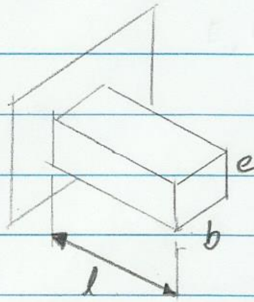
Exercício 14.10)

E, G



a) Com força cortante F

$$U = \int_0^l \frac{(Fx)^2}{2EI}\,dx = \frac{F^2 l^3}{6EI}$$

$$\delta = \frac{dU}{dF} = \frac{d}{dF}\left(\frac{F l^3}{6EI}\right) \therefore \delta = \frac{F l^3}{3EI} \; ; \; I = \frac{b e^3}{12} \therefore \boxed{\delta = \frac{4 F l^3}{E b e^3}}$$

$$K = \frac{F}{\delta} = \frac{F}{\frac{4 F l^3}{E b e^3}} \therefore \boxed{K = \frac{E b e^3}{4 l^3}}$$

b)

Exercício 14.7)

$F=600N$	$\tau = \frac{8FC\,K_s}{\pi d^2}$
$D=40mm$	
$d=5\ mm$	$C=\frac{40}{5}=8$
$G=80\ GPa$	
$Na=7$	$K_s=1+\frac{1}{2.8}\ \therefore K_s=1{,}0625$
$\lambda=8$	

$\tau=\frac{8.600\cdot 8\cdot 1{,}0625}{\pi.5^2}\ \therefore\ \tau=519{,}48\ MPa$

$\delta=\frac{8FC^3Na}{Gd}\ \therefore\ \delta=\frac{8\cdot 600\cdot 8^3\cdot 7}{80.10^3\cdot 5}\ \therefore\ \delta=43\ mm$

$h=dNa+2d=5.7+2.5\ \therefore\ h=45\ mm$

$H=PNa+2d \qquad tg\lambda=\frac{P}{\pi D} \qquad tg\,8=\frac{P}{\pi.40}\ \therefore\ P=17{,}66\ mm$

$H=17{,}66\cdot 7+2\cdot 5 \qquad H=133{,}63\ mm$

$H=h_f+\delta_s \qquad 133{,}63=h_f+43\ m\ \therefore\ h_f=90{,}63\ mm$