03-02-2015

EL 7810 - Controle e Servomecanismo I

Prof. Barbosa

Livro texto:

- Maya e Leonardi - Controle essencial 2ª edição, 2014
- Moodle

Bibliografia complementar:

- Ogata e Nise

Aula 1: x Cap 1 pg 1 a 9

x Cap 3 pg 34 - Revisão Laplace

LDR

Ponte de windstom

Revisão Laplace

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{+\infty} e^{-st} f(t)\, dt$$

$f(t),\ t \geq 0$	$F(s)$
$\delta(t)$	1
1	$1/s$
t	$1/s^2$
$\frac{1}{2}t^2$	$1/s^3$
$\frac{1}{(n-1)!}t^{n-1}$	$1/s^n$
e^{-at}	$1/(s+a)$

1.) $f(t) = 5e^{-t}$

$$\mathcal{L}[f(t)] = \frac{5}{s+1}$$

2.) $f(t) = 2e^{-0,5t}$

$$\mathcal{L}[f(t)] = \frac{2}{s+0.5}$$

3-) $f(t) = 11 e^{t}$ $F(s) = \frac{11}{s-1}$

4-) $f(t) = e^{-t} - e^{-2t}$

$F(s) = \frac{1}{s+1} - \frac{1}{s+2}$

5-) $f(t) = 6 e^{-(0.2+3t)} = 6 e^{-0.2} \cdot e^{-3t}$

$F(s) = 6 \cdot e^{-0.2} \cdot \frac{1}{s+3}$

x 7-) $\operatorname{senh}(at) = \frac{e^{at} - e^{-at}}{2}$ Sabendo que: $e^{-at} \operatorname{sen} bt \rightarrow \frac{b}{(s+a)^2 + b^2}$

$F(s) = \frac{a}{s^2 + a^2}$ "Resposta no final do livro é com (-)"

x 9-) $f(t) = e^{j2t} - e^{-j2t}$

$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[e^{j2t}] - \mathcal{L}[e^{-j2t}]$

$= \frac{1}{s-j2} - \frac{1}{s+j2} = \frac{s+j2-(s-j2)}{(s-j2)(s+j2)}$ "j4 é 2s"

$= \frac{j4}{s^2 + \cancel{js2} - \cancel{js2} + 4}$ $\therefore F(s) = \frac{j4}{s^2+4}$

10-) $f(t) = 7 e^{-3t} - 3 e^{7t}$ $F(s) = 7 \cdot \mathcal{L}[e^{-3t}] - 3 \mathcal{L}[e^{7t}]$

$F(s) = \frac{7}{s+3} - \frac{3}{s+7}$

$$\frac{s^2+7s+12}{(s+2)(s+4)(s+6)} = \frac{K_1}{s+2} + \frac{K_2}{s+4} + \frac{K_3}{s+6}$$

$$K_1 = \left.\frac{s^2+7s+12}{(s+4)(s+6)}\right|_{s \to -2} = \frac{(-2)^2+7(-2)+12}{(-2+4)(-2+6)} \qquad K_1 = \frac{1}{4}$$

$$K_2 = \left.\frac{s^2+7s+12}{(s+2)(s+6)}\right|_{s \to -4} \qquad K_2 = 0$$

$$K_3 = \left.\frac{s^2+7s+12}{(s+2)(s+4)}\right|_{s \to -6} \qquad K_3 = \frac{3}{4}$$

$$\frac{s^2+7s+12}{(s+2)(s+4)(s+6)} = \frac{1/4}{s+2} + \cancel{\frac{0}{s+4}} + \frac{3/4}{s+6}$$

$$\boxed{f(t) = \frac{1}{4}e^{-2t} + \frac{3}{4}e^{-6t}}$$

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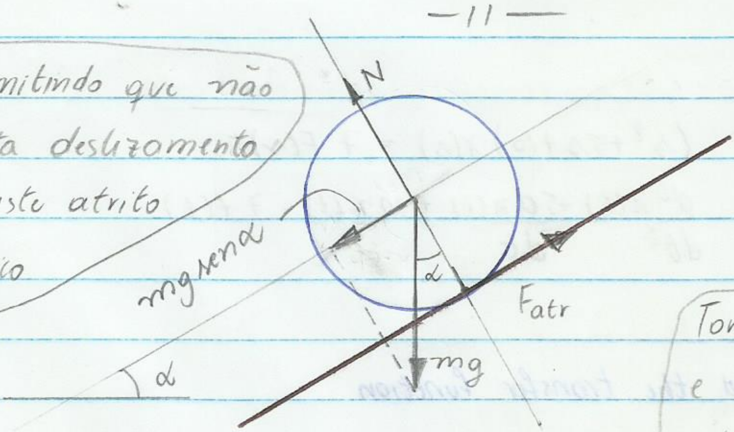
Admitindo que não exista deslizamento ∴ existe atrito estático

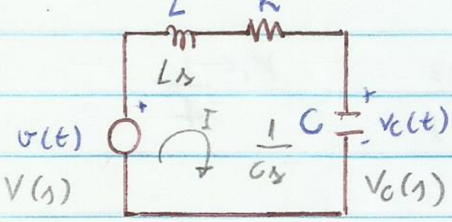


Ball on the beam:
massa: m
aceleração gravitacional: g
inclinação da barra: α

Torque relativo a força peso e força normal é nulo, pois atua na linha do centro de massa

$$m\,g \operatorname{sen}\alpha - F_{atr} = m \cdot a \quad (I)$$

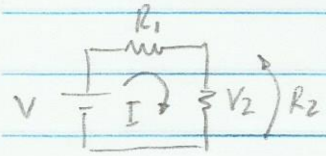
Modelagem matemática de sistemas mecânicos e elétricos



$$\frac{V_c(s)}{V(s)} = \frac{\frac{1}{Cs}}{Ls + \frac{1}{Cs} + R} = \frac{1}{LCs^2 + CRs + 1}$$

$$= \frac{1}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$\therefore \boxed{\frac{V_c(s)}{V} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$



$$\begin{cases} V = I(R_1 + R_2) \\ V_2 = I R_2 \end{cases}$$

$$V = \frac{V_2}{R_2}(R_1 + R_2)$$

$$\frac{V_2}{V} = \frac{R_2}{R_1 + R_2}$$

Componente	Impedance
Capacitor	$1/Cs$
Resitor	R
Indutor	Ls

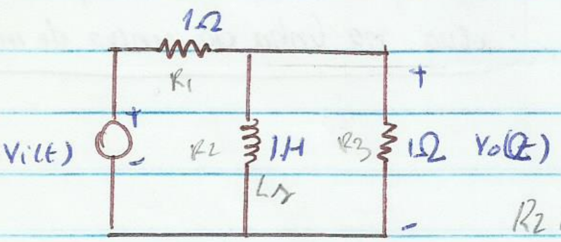
Exercise 8 - (Nise)

$$\frac{X(s)}{F(s)} = \frac{7}{s^2 + 5s + 10}$$

$$(s^2 + 5s + 10)\,X(s) = 7\,F(s)$$

$$\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 10\,x(t) = 7\,f(t)$$

Exercise 16. (Nise) Find the transfer function



$$V_o(s) = \frac{R_2 // R_3}{R_1 + R_2 // R_3}\,V_i(s)$$

$$R_2 // R_3 = \frac{Ls}{Ls + 1}$$

$$R_1 + R_2 \| R_3 = \frac{Ls}{Ls+1} + 1 = \frac{2Ls+1}{Ls+1}$$

$$V_o(s) = \frac{\frac{Ls}{Ls+1}}{\frac{2Ls+1}{Ls+1}} V_1(s) = \frac{Ls}{2Ls+1} V_1(s)$$

$$\frac{V_o(s)}{V_1(s)} = \frac{s}{2s+1}$$

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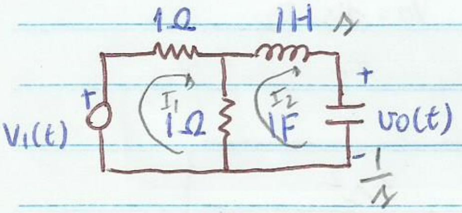
Usando análise Nodal

$$\frac{V_o - V_1}{1} + \frac{V_o}{s} + \frac{V_o}{1} = 0$$

$$(V_o - V_1)s + V_o + V_o s = 0$$

$$V_o(s + 1 + s) = V_1 s \rightarrow \frac{V_o}{V_1} = \frac{s}{2s+1}$$



$$2I_1 - I_2 = V_1(s)$$

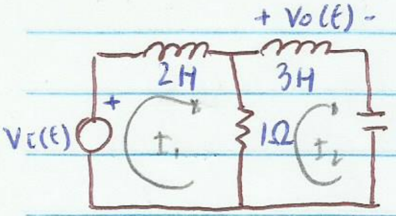
$$-I_1 + \left(s + 1 + \frac{1}{s}\right) I_2 = 0$$

$$\Delta = \begin{vmatrix} 2 & -1 \\ -1 & \frac{s^2+s+1}{s} \end{vmatrix} = \frac{2}{s}(s^2+s+1) - 1 = \frac{2s^2+2s+2-s}{s} = \frac{2s^2+s+2}{s}$$

$$\Delta_{I_2} = \begin{vmatrix} 2 & V_1(s) \\ -1 & 0 \end{vmatrix} = V_1(s) \quad ; \quad I_2 = \frac{\Delta_{I_2}}{\Delta} = \frac{s V_1(s)}{2s^2+s+2}$$

$$V_o(s) = \frac{1}{s} \cdot \frac{s}{2s^2+s+2} V_1(s) \quad \therefore \quad \frac{V_o(s)}{V_1(s)} = \frac{1}{s^2 + \frac{s}{2} + 1}$$

Exercise 18 (Nise) Find $G(s) = \frac{V_o(s)}{V_i(s)}$



$$(2s+1) I_1 - I_2 = V_i(s)$$
$$-I_1 + \left(3s + 1 + \frac{2}{s}\right) = 0$$

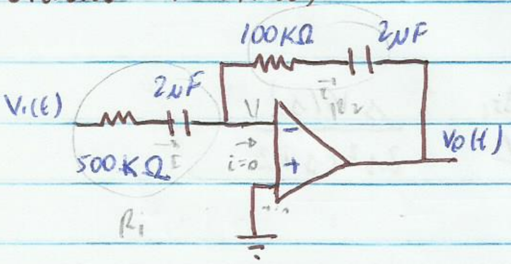
$$\Delta = \begin{vmatrix} 2s+1 & -1 \\ -1 & \frac{3s^2+s+2}{s} \end{vmatrix} = \frac{(2s+1)(3s^2+s+2)-1}{s}$$

$$\Delta = \frac{6s^3+2s^2+4s+3s^2+s+2-s}{s} \quad \therefore \Delta = \frac{6s^3+5s^2+4s+2}{s}$$

$$I_2 = \frac{\begin{vmatrix} 2s+1 & V_i(s) \\ -1 & 0 \end{vmatrix}}{\Delta} = \frac{V_i(s)}{\Delta} \quad ; \quad V_o = 3s\, I_2$$

$$V_o = \frac{3s \times V_i(s) \times s}{6s^3+5s^2+4s+2} \quad \therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{3s^2}{6s^3+5s^2+4s+2}}$$

Exercise 21 (Nise)



$$\frac{V_1 - \cancel{V}^{\,0}}{R_1} = \frac{\cancel{V}^{\,0} - V_o}{R_2}$$

$$V_1 R_2 = -V_o R_1$$

$$\frac{V_o}{V_1} = -\frac{R_2}{R_1}$$

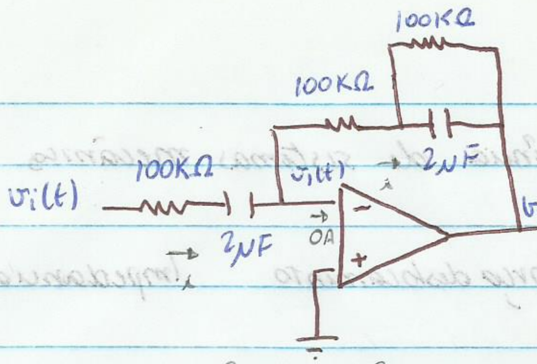
$$\frac{V_o}{V_1} = -\frac{100.10^3 + 500.10^3/s}{500.10^3 + 500.10^3/s} = -\frac{\frac{100.10^3 s + 500.10^3}{s}}{\frac{500.10^3 s + 500.10^3}{s}}$$

$$\frac{V_o}{V_1} = -\frac{s+5}{5s+5} \quad \therefore \boxed{\frac{V_o(s)}{V_i(s)} = -\frac{1}{5}\frac{(s+5)}{(s+1)}}$$

kajoma

$C: 1/sC$

$I: LC$



$$\frac{v_i - v_1}{Z_1} = \frac{v_1 - v_o}{Z_2} \quad \therefore \frac{v_o}{v_i} = -\frac{Z_2}{Z_1}$$

$$Z_1 = 100.10^3 + \frac{500.10^3}{s} = \frac{s+5}{s}.10^5$$

$$Z_2 = 100.10^3 + \frac{100.10^3 . 500.10^3}{s} \div 100.10^3 + \frac{500.10^3}{s}$$

$$= 100.10^3 + \frac{50.10^9}{s} \div \frac{s.10^5 + 5.10^5}{s}$$

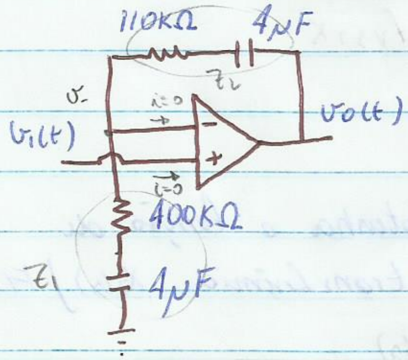
$$= 100.10^3 + \frac{50.10^{\cancel{9}\,4}}{(s+5).\cancel{10^5}} = \frac{100.10^3(s+5) + 50.10^4}{s+5}$$

$$= \frac{10.10^4 s + 50.10^4 + 50.10^4}{s+5} = \frac{s+10}{s+5}.10^5$$

$$\therefore \frac{v_o}{v_i} = -\left(\frac{s+10}{s+5}\right)\cancel{10^5} \div \frac{s+5}{s}.\cancel{10^5}$$

$$\boxed{\frac{v_o}{v_i} = -\frac{s(s+10)}{(s+5)^2}}$$

Exercise 22 (Nise) Determine $G(s) = V_o(s)/V_i(s)$



$$\frac{0 - v_-}{Z_1} = \frac{v_- - v_o}{Z_2} \Rightarrow Z_2 v_- = (v_- - v_o) Z_1$$

$$\Rightarrow -Z_2 v_- = Z_1 v_- - Z_1 v_o$$

$$\Rightarrow -(Z_2 + Z_1) v_- = -Z_1 v_o$$

$v_- = v_i(t)$

$$\therefore \frac{v_o}{v_i} = \frac{Z_1 + Z_2}{Z_1}$$

$$Z_1 = 400.10^3 + \frac{250.10^3}{s} = \frac{(40s+25)}{s}.10^4$$

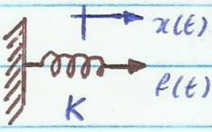
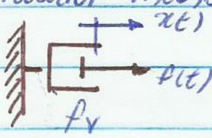
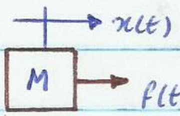
$$Z_2 = 110.10^3 + \frac{250.10^3}{s} = \left(\frac{11s+25}{s}\right).10^4$$

$$\therefore \frac{v_o}{v_i} = \left(\frac{51s+50}{\cancel{s}}\right).10^4 \div \left(\frac{40s+25}{\cancel{s}}\right).10^4$$

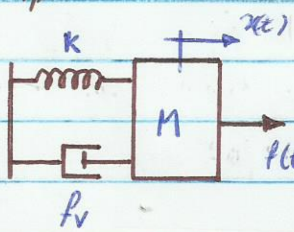
$$\frac{v_o}{v_i} = \frac{51(s + 50/51)}{40(s + \frac{25}{40})}$$

$$\boxed{\frac{v_o}{v_i} = 1{,}275\frac{(s+0{,}98)}{(s+0{,}625)}}$$

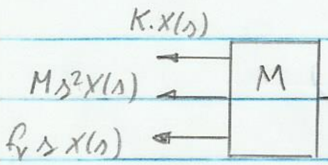
Funções de transferência de sistemas mecânicos

Componente	Força-velocidade	Força deslocamento	Impedância
mola 	$f(t)=K\int_0^t v(\tau)\,d\tau$	$f(t)=Kx(t)$	K
Amorteudor viscoso 	$f(t)=f_v\cdot v(t)$	$f(t)=f_v\dfrac{dx(t)}{dt}$	$f_v s$
Massa 	$f(t)=M\dfrac{dv(t)}{dt}$	$f(t)=M\dfrac{d^2x(t)}{dt^2}$	Ms^2

Exemplo:

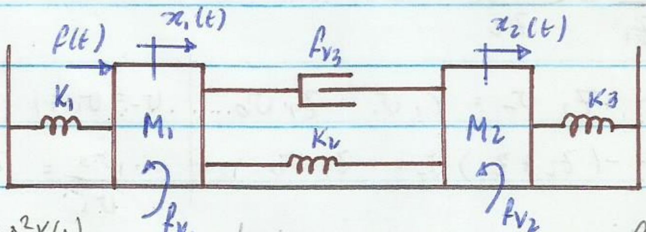


+ Obtenha função de transferência. $X(s)/F(s)$

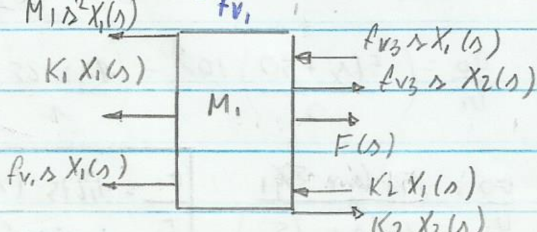


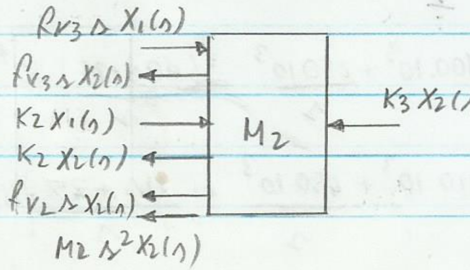
$$F(s)=(Ms^2+f_v s+K)\,X(s)$$

$$\frac{X(s)}{F(s)}=\frac{1}{Ms^2+f_v s+K}$$



+ Obtenha a função de transferência $X_2(s)/F(s)$

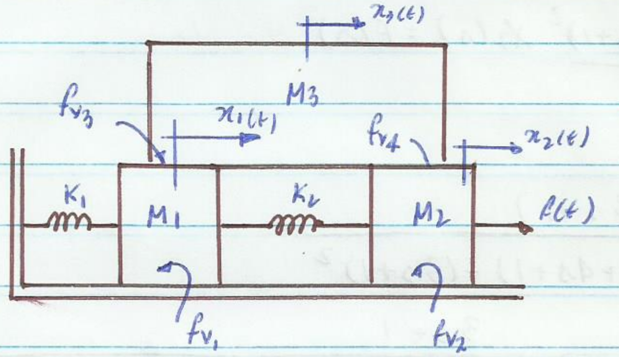




$$\begin{cases} (K_1 + f_{v_1} s + M_1 s^2 + f_{v_3} s + K_2) X_1(s) - (K_2 + f_{v_3} s) X_2(s) = F(s) \\ -(K_2 + f_{v_3} s) X_1(s) + (K_2 + f_{v_2} s + f_{v_3} s + M_2 s^2 + K_3) X_2(s) = 0 \end{cases}$$

$$\begin{cases} [M_1 s^2 + (f_{v_1} + f_{v_3}) s + (K_1 + K_2)] X_1(s) - (f_{v_3} s + K_2) X_2(s) = F(s) \\ -(K_2 + f_{v_3} s) X_1(s) + [M s^2 + (f_{v_2} + f_{v_3}) s + (K_2 + K_3)] X_2(s) = 0 \end{cases}$$

Resolvendo isso, temos: $\dfrac{X_2(s)}{F(s)} = G(s) = \dfrac{(f_{v_3} s + K_2)}{\Delta}$

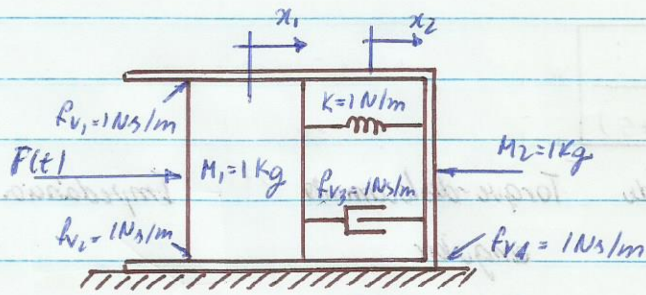


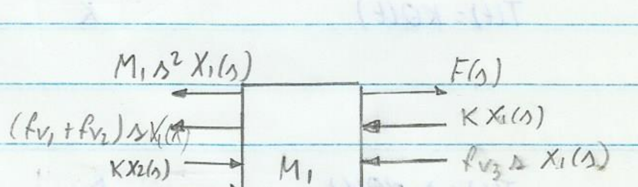
$$[M_1 s^2 + (f_{v_1} + f_{v_3}) s + (K_1 + K_2)] X_1(s) - K_2 X_2(s) - f_{v_3} s X_3(s) = 0$$

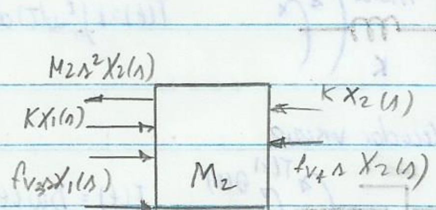
$$-K_2 X_1(s) + [M_2 s^2 + (f_{v_2} + f_{v_4}) s + K_2] X_2(s) - f_{v_4} s X_3(s) = F(s)$$

$$-f_{v_3} s X_1(s) - f_{v_4} s X_2(s) + [M_3 s^2 + (f_{v_3} + f_{v_4}) s] X_3(s) = 0$$

Determine $G(s) = X_2(s)/F(s)$







$$[M_1 s^2 + (f_{v_1} + f_{v_2} + f_{v_3}) s + K] X_1(s) - [(f_{v_1} + f_{v_2} + f_{v_3}) s + K] X_2(s) = F(s)$$

$$-[(f_{v_1} + f_{v_2} + f_{v_3}) s + K] X_1(s) + [M_2 s^2 + (f_{v_1} + f_{v_2} + f_{v_3} + f_{v_4}) s + K] X_2(s) = 0$$

$$\begin{cases} (s^2+3s+1)\,X_1(s) - (3s+1)\,X_2(s) = F(s) \\ -(3s+1)\,X_1(s) + (s^2+4s+1)\,X_2(s) = 0 \end{cases}$$

$$X_1(s) = \frac{(s^2+4s+1)}{(3s+1)}\,X_2(s)$$

$$(s^2+3s+1)\frac{(s^2+4s+1)}{(3s+1)}\,X_2(s) - (3s+1)\,X_2(s) = F(s)$$

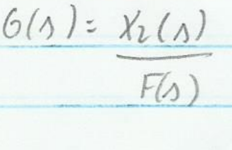
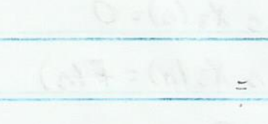
$$\frac{(s^2+3s+1)(s^2+4s+1) - (3s+1)^2}{(3s+1)}\,X_2(s) = F(s)$$

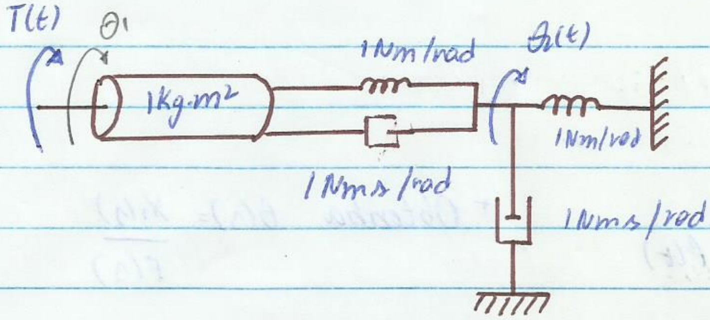
$$G(s) = \frac{X_2(s)}{F(s)} = \frac{(3s+1)}{(s^2+3s+1)(s^2+4s+1) - (3s+1)^2}$$

$$= \frac{3s+1}{s^4+4s^3+s^2+3s^3+12s^2+3s+s^2+4s+1-(9s^2+6s+1)}$$

$$= \frac{3s+1}{s^4+7s^3+5s}$$

$$\therefore \boxed{G(s) = \frac{3s+1}{s(s^3+7s^2+5)}}$$

Componente	Torque-velocidade angular	Torque-deslocamento angular	Impedância
	$T(t) = K\int_0^t \omega(\tau)\,d\tau$	$T(t) = K\theta(t)$	K
Amortecedor viscoso 	$T(t) = D\omega(t)$	$T(t) = D\,\frac{d\theta(t)}{dt}$	Ds
Inércia 	$T(t) = J\,\frac{d\omega(t)}{dt}$	$T(t) = J\,\frac{d^2\theta(t)}{dt^2}$	Js^2



+ Obtenha a função de transferência $G(s) = \frac{\theta_2(s)}{T(s)}$

$$\begin{cases} (s^2+s+1)\,\theta_1(s) - (s+1)\,\theta_2(s) = T(s) \\ -(s+1)\,\theta_1(s) + (s+1+s+1)\,\theta_2(s) = 0 \end{cases}$$

$$\theta_1(s) = \frac{(2s+2)}{(s+1)}\,\theta_2(s)$$

$$(s^2+s+1)\,\frac{(2s+2)}{(s+1)}\,\theta_2(s) - (s+1)\,\theta_2(s) = T(s)$$

$$\frac{(s^2+s+1)(2s+2) - (s+1)^2\,\theta_2(s)}{(s+1)} = T(s)$$

$$\frac{\theta_2(s)}{T(s)} = \frac{(s+1)}{(s^2+s+1)(2s+2) - (s+1)^2}$$

$$= \frac{(s+1)}{2s^3+2s^2+2s^2+2s+2s+2-s^2-2s-1}$$

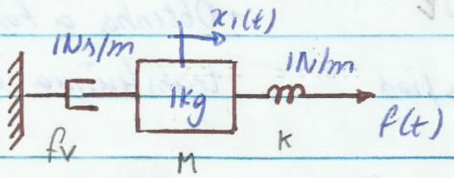
$$\frac{\theta_2(s)}{T(s)} = \frac{(s+1)}{2s^3+3s^2+2s+1}$$

	2	3	2	1
-1	2	1	1	0

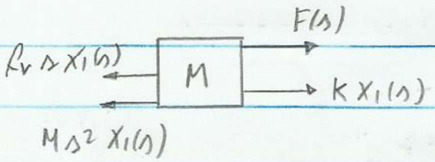
$$\frac{\theta_2(s)}{T(s)} = \frac{\cancel{(s+1)}}{\cancel{(s+1)}(2s^2+s+1)}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{1}{2s^2+s+1}}$$

Exercício 23) (Nise - 5ª ed)



+ Obtenha $G(s) = \dfrac{X_1(s)}{F(s)}$



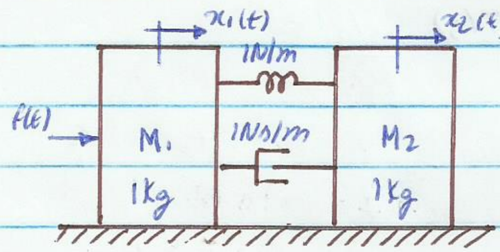
$$(Ms^2 + f_v s + K)\,X_1(s) = F(s)$$

$$G(s) = \frac{X_1(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

$$\therefore \boxed{G(s) = \frac{1}{s^2 + s + 1}}$$

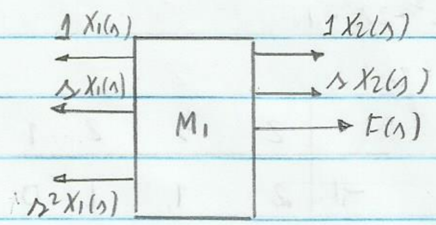
(Errada)

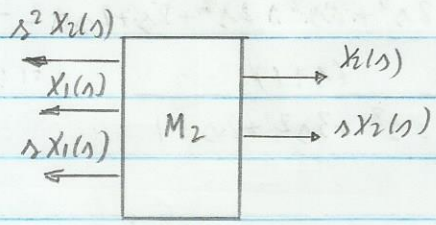
Exercício 24) (Nise - 5ª ed)



× Obtenha $G(s) = \dfrac{X_2(s)}{F(s)}$

(sem atrito)





$$\begin{cases} (s^2+s+1)X_1(s) - (s+1)X_2(s) = F(s) \\ -(s+1)X_1(s) + (s^2+s+1)X_2(s) = 0 \end{cases}$$

$$X_2(s) = \frac{\begin{vmatrix} (s^2+s+1) & F(s) \\ -(s+1) & 0 \end{vmatrix}}{\Delta} \quad ; \quad \Delta = \begin{vmatrix} (s^2+s+1) & -(s+1) \\ -(s+1) & (s^2+s+1) \end{vmatrix}$$

$\Delta = (s^2+s+1)^2 - (s+1)^2$

$X_2(s) = \frac{(s+1)F(s)}{\Delta}$

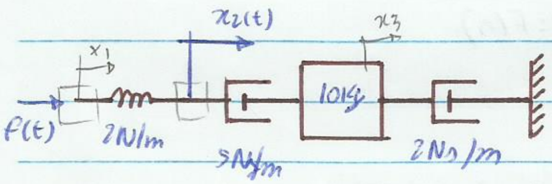
$\therefore X_2(s) = \frac{(s+1) \quad F(s)}{(s^2+s+1)^2 - (s+1)^2}$; $G(s) = \frac{X_2(s)}{F(s)}$

$G(s) = \frac{(s+1)}{(s^2+s+1)^2 - (s+1)^2} = \frac{(s+1)}{(s^2+s+1)(s^2+s+1) - (s^2+2s+1)}$

$= \frac{(s+1)}{s^4 + s^3 + \cancel{s^2} + s^3 + s^2 + \cancel{s} + s^2 + \cancel{s} + \cancel{1} - \cancel{s^2} - \cancel{2s} - \cancel{1}}$

$= \frac{(s+1)}{s^4 + 2s^3 + 2s^2}$ $\therefore$ $\boxed{G(s) = \frac{(s+1)}{s^2(s^2+2s+2)}}$

Exercício 25) (Nise 5ª) Obtenha $G(s) = X_2(s)/F(s)$



$X_3(s) = \frac{5s X_2(s)}{(10s^2+7s)}$

$2X_1(s) - 2X_2(s) = F(s)$

$-2X_1(s) + (5s+2)X_2(s) - 5X_3(s) = 0$

$0X_1(s) - 5s X_2(s) + (10s^2+7s)X_3(s) = 0$

$X_2(s) = \frac{\begin{vmatrix} 2 & F(s) & 0 \\ -2 & 0 & -5 \\ 0 & 0 & (10s^2+7s) \end{vmatrix}}{\Delta}$; $\Delta = \begin{vmatrix} 2 & -2 & 0 \\ -2 & (5s+2) & -5 \\ 0 & -5 & 10s^2+7s \end{vmatrix}$

$\Delta = [(10s^2+7s)(5s+2) - 25] \times 2 + 2 \times (-2)(10s^2+7s)$ ✓

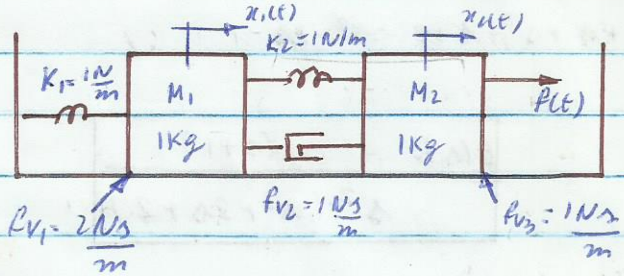
$= 2(10s^2+7s)(5s+2) - 50 + (10s^2+7s)(-4)$

$$\Delta = (10s^2+7s)(10s+4-4)-50$$
$$= 100s^3+70s^2-50$$

(As equações estão certa porém a função de transferência está errada)

$$X_2(s) = \frac{-F(s)\times(-2)(10s^2+7s)}{10\cdot(10s^3+7s^2-50)} \quad \therefore \quad \frac{X_2(s)}{F(s)} = \frac{10s^2+7s}{5(10s^3+7s^2-50)}$$

Exercício 26) (Nise 5ª ed)



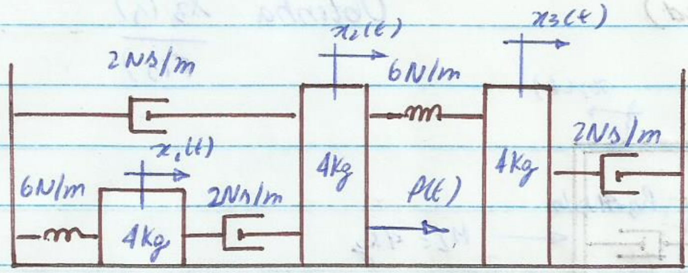
$$\begin{cases} (s^2+3s+2)X_1(s)-(s+1)X_2(s)=0 \\ -(s+1)X_1(s)+(s^2+2s+1)X_2(s)=F(s) \end{cases}$$

$$X_2 = \frac{\begin{vmatrix} 0 & -(s+1) \\ F(s) & (s^2+2s+1) \end{vmatrix}}{\begin{vmatrix} (s^2+3s+2) & -(s+1) \\ -(s+1) & (s^2+2s+1) \end{vmatrix}} = \frac{(s+1)F(s)}{(s^2+3s+2)(s^2+2s+1)-(s+1)^2}$$

$$G(s) = \frac{X_1(s)}{F(s)} = \frac{(s+1)}{(s^2+3s+2)(s^2+2s+1)-(s+1)^2}$$

Exercício 27) (Nise 5ª ed)

- Obtenha $\frac{X_3(s)}{F(s)}$



$$\begin{cases} (4s^2+2s+6)X_1(s) - 2s\,X_2(s) = 0 \\ -2s\,X_1(s) + (4s^2+4s+6)X_2(s) - 6X_3(s) = F(s) \\ -6X_2(s) + (4s^2+2s+6)X_3(s) = 0 \end{cases}$$

$$X_3(s) = \frac{\begin{vmatrix} (4s^2+2s+6) & -2s & 0 \\ -2s & (4s^2+4s+6) & F(s) \\ 0 & -6 & 0 \end{vmatrix} \begin{matrix} (4s^2+2s+6) & -2s \\ -2s & (4s^2+4s+6) \\ 0 & -6 \end{matrix}}{\begin{vmatrix} (4s^2+2s+6) & -2s & 0 \\ -2s & (4s^2+4s+6) & -6 \\ 0 & -6 & (4s^2+2s+6) \end{vmatrix} \begin{matrix} (4s^2+2s+6) & -2s \\ -2 & (4s^2+4s+6) \\ 0 & -6 \end{matrix}}$$

$$X_3(s) = \frac{6\,\cancel{(4s^2+2s+6)}\,F(s)}{(4s^2+2s+6)^{\cancel{2}}(4s^2+4s+6) - 36\,\cancel{(4s^2+2s+6)} - 4s\,\cancel{(4s^2+2s+6)}}$$

$$X_3(s) = \frac{6F(s)}{(4s^2+2s+6)(4s^2+4s+6) - 36 - 4s}$$

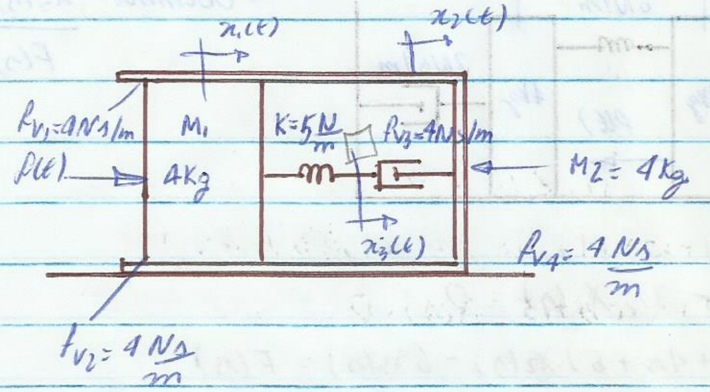
$$= \frac{6F(s)}{16s^4 + 16s^3 + 24s^2 + 8s^3 + 8s^2 + 12s + 24s^2 + 24s + 36 - 36 - 4s}$$

$$= \frac{6F(s)}{16s^4 + 24s^3 + 56s^2 + 32s}$$

$$\boxed{G(s) = \frac{X_3(s)}{F(s)} = \frac{3}{s(8s^3 + 12s^2 + 28s + 16)}}$$

28-) (Exercício - Nise 5ª ed)

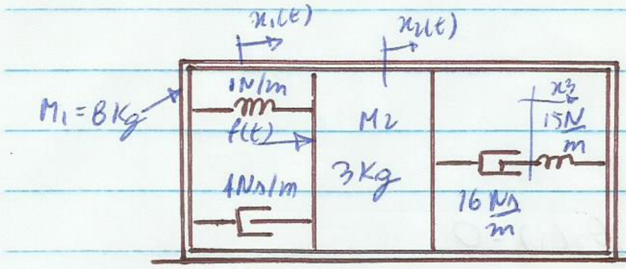
Obtenha $\frac{X_3(s)}{F(s)}$



$$\begin{cases} (4s^2+8s+5)X_1(s) - 8s X_2(s) - 5X_3(s) = F(s) \\ -8s X_1(s) + (4s^2+16s)X_2(s) - 4s X_3(s) = 0 \\ -5X_1(s) - 4s X_2(s) + (4s+5)X_3(s) = 0 \end{cases}$$

$$X_3 = \frac{\begin{vmatrix} (4s^2+8s+5) & -8s & F(s) \\ -8s & (4s^2+16s) & 0 \\ -5 & -4s & 0 \end{vmatrix}}{\begin{vmatrix} (4s^2+8s+5) & -8 & -5 \\ -8s & (4s^2+12s) & -4 \\ -5 & -4s & (4s+5) \end{vmatrix}}$$

$$\therefore \boxed{G(s) = \frac{X_3(s)}{F(s)} = \frac{13s+20}{4s(4s^3+25s^2+43s+15)}}$$



+ Obtenha $G(s) = \dfrac{X_3(s)}{F(s)}$

$$
\begin{cases}
(8s^2+4s+16)X_1(s) - (4s+1)X_2(s) - 15X_3(s) = 0 \\
-(4s+1)X_1(s) + (3s^2+20s+1)X_2(s) - 16s X_3(s) = F(s) \\
-15X_1(s) - 16s X_2(s) + (16s+15)X_3(s) = 0
\end{cases}
$$

$$
X_3 = \frac{\begin{vmatrix} (8s^2+4s+16) & -(4s+1) & 0 \\ -(4s+1) & (3s^2+20s+1) & F(s) \\ -15 & -16 & 0 \end{vmatrix}}{\begin{vmatrix} (8s^2+4s+16) & -(4s+1) & -15 \\ -(4s+1) & (3s^2+20s+1) & -16s \\ -15 & -16s & (16s+15) \end{vmatrix}}
$$

$$
\therefore \boxed{G(s) = \frac{X_3(s)}{F(s)} = \frac{128s^3+64s^2+316s+5}{384s^5+1064s^4+3476s^3+165s^2}}
$$

Exercício 29) (Nise 5ª edi)

$$
\begin{cases}
(s^2+2s+2)X_1(s) - X_2(s) - sX_3(s) = 0 \\
-X_1(s) + (s^2+s+1)X_2(s) - sX_3(s) = F(s) \\
-X_1(s) - X_2(s) + (s^2+2s+1)X_3(s) = 0
\end{cases}
$$

Exercício 29) (Nise 6ªed, ENG)

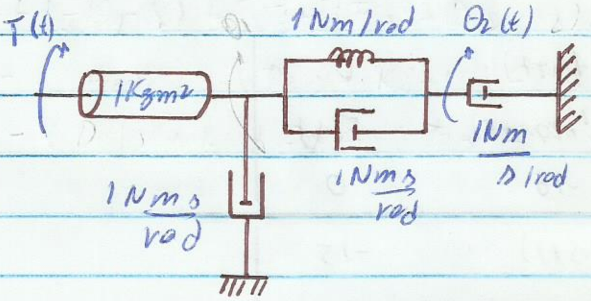
$$
\begin{cases}
(4s^2+4s+4)X_1(s) - 4X_2(s) - 2sX_3(s) = 0 \\
-4X_1(s) + (5s^2+3s+4)X_2(s) - 3sX_3(s) = F(s) \\
-2sX_1(s) - 3sX_2(s) + (5s^2+5s+5)X_3(s) = 0
\end{cases}
$$

Exercício 30) (Nise 5ª ed)

(a)

$$\begin{cases} (s^2+9s+8)\,\theta_1(s) - (2s+8)\,\theta_2(s) = 0 \\ -(2s+8)\,\theta_1(s) + (s^2+2s+11)\,\theta_2(s) = T(s) \end{cases}$$

Exercício 31) (Nise 5ª ed) Obtenha $G(s) = \dfrac{\theta_2(s)}{T(s)}$



$$\begin{cases} (s^2+2s+1)\,\theta_1(s) - (s+1)\,\theta_2(s) = T(s) \\ -(s+1)\,\theta_1(s) + (2s+1)\,\theta_2(s) = 0 \end{cases}$$

$$\theta_2(s) = \frac{\begin{vmatrix} (s^2+2s+1) & T(s) \\ -(s+1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2+2s+1) & -(s+1) \\ -(s+1) & (2s+1) \end{vmatrix}} = \frac{(s+1)\,T(s)}{(s^2+2s+1)(2s+1)-(s+1)^2}$$

$$\theta_2(s) = \frac{(s+1)\,T(s)}{(s+1)^2(2s+1)-(s+1)^2}$$

	1	2	1
-1	1	1	0

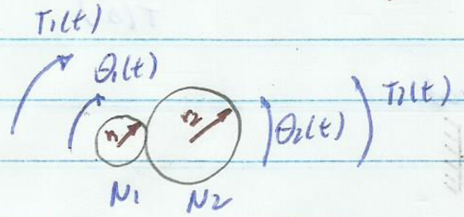
$(s+1)(s+1)$

$$G(s) = \frac{\theta_2(s)}{T(s)} = \frac{1}{(s+1)(2s+1)-(s+1)}$$

$$= \frac{1}{2s^2+s+2s+1-s-1}$$

$$\therefore\ \boxed{G(s) = \frac{\theta_2(s)}{T(s)} = \frac{1}{2s(s+1)}}$$

Funções de Transferência de Sistemas com Engrenagens



$r_1\theta_1 = r_2\theta_2$

$$\boxed{\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}} \qquad \boxed{\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}}$$

$$T_1 \rightarrow \boxed{\frac{N_2}{N_1}} \rightarrow T_2$$

$$(Js^2 + Ds + K)\,\theta_2 = T_1(s)\frac{N_2}{N_1}$$

$$(Js^2 + Ds + K)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

$$(Js^2 + Ds + K)\left(\frac{N_1}{N_2}\right)^2\theta_1(s) = T_1(s)$$

Exemplo 2.21

$$J_e = J_2 + J_1\left(\frac{N_2}{N_1}\right)^2 \qquad D_e = D_1\left(\frac{N_2}{N_1}\right)^2 + D_2 \qquad K_e = K_2$$

$$(J_e s^2 + D_e s + K_e)\,\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

$$\frac{\theta_2}{T_1} = \frac{N_2/N_1}{J_e s^2 + D_e s + K_e}$$

Exemplo 2.22

$$J_e = J_1 + J_2\left(\frac{N_1}{N_2}\right)^2 + J_3\left(\frac{N_1}{N_2}\right)^2 + J_4\left(\frac{N_1}{N_2}\frac{N_3}{N_4}\right)^2 + J_5\left(\frac{N_1 N_3}{N_2 N_4}\right)^2$$

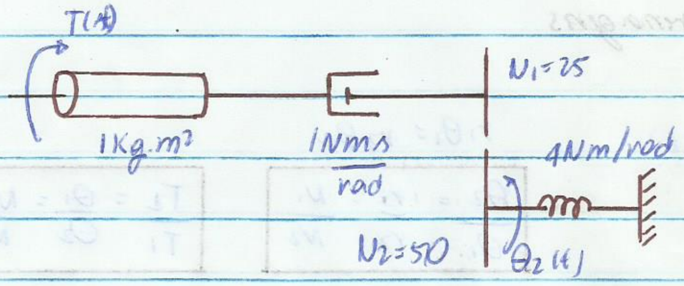
$$J_e = J_1 + (J_2 + J_3)\left(\frac{N_1}{N_2}\right)^2 + (J_4 + J_5)\left(\frac{N_1 N_3}{N_2 N_4}\right)^2$$

$$D_e = D_1 + D_2\cdot\left(\frac{N_1}{N_2}\right)^2$$

S T Q Q S S D
M T W T F S S

Exercício de Avaliação 2.10

$$G(s) = \frac{\Theta_2(s)}{T(s)}$$

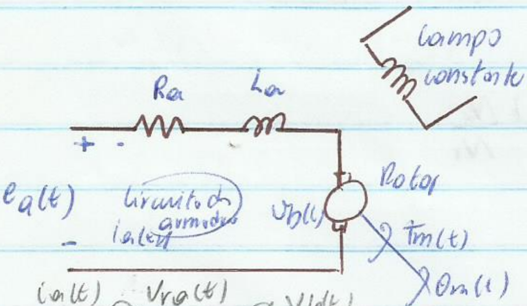


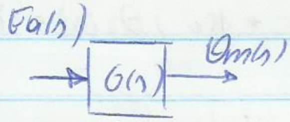
$$\left[(s^2+s)\left(\frac{50}{25}\right)^2 + 4\right]\Theta_2(s) = T(s)\,\frac{50}{25}$$

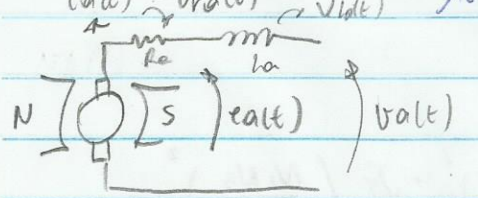
$$(4s^2+4s+4)\,\Theta_2(s) = 2\,(T(s))$$

$$\therefore \quad G(s) = \frac{\Theta_2(s)}{T(s)} = \frac{1/2}{s^2+4s+4}$$

Funções de transferência de Sistemas Eletromecânicos







$$v_a(t) = \left[R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_a(t)\right]$$

$$\boxed{V_a(s) = (R_a + L_a s)\,I_a(s) + E_a(s)}$$

$C_{motor}(t) = K_{motor}\, i_a(t)$ } Conjugado do Motor

$$\boxed{C_{motor}(s) = K_{motor} \cdot I_a(s)}$$

kajoma

$e_a(t) = K_{eletrico} \cdot w(t) = K_{eletrico} \times \frac{d\theta_m(t)}{dt}$

$$\boxed{E_a(s) = K_{eletrico} \cdot \Omega(s)}$$

$$C_{mecanico}(t) - C_{resistente}(t) = J \frac{dw_m(t)}{dt}$$

$$C_{mecanico}(t) = \frac{J \cdot dw_m(t)}{dt} + B w_m$$

$$\boxed{C_{mecanico}(s) = (Js + B)\,\Omega_m(s)}$$

Exercício 32) Obtenha a função de transferência $G(s) = \Theta_3(s) / T(s)$

$$J_e = J_5 + J_4 + (J_3 + J_2)\left(\frac{N_4}{N_3}\right)^2 + J_1\left(\frac{N_2}{N_1}\right)^2$$

$$D_e = D_5 + D_4 + (D_3 + D_2)\left(\frac{N_4}{N_3}\right)^2 + D_1\left(\frac{N_2}{N_1}\right)^2$$

$$T_e = T \cdot \frac{N_2}{N_1}\frac{N_4}{N_3}$$

$$(J_e s^2 + D_e s)\,\Theta_3(t) = T\,\frac{N_2 \cdot N_4}{N_1 \cdot N_3}$$

$$\boxed{\Theta_3(s) = \frac{\frac{N_2 \cdot N_4}{N_1 N_3}}{J_e s^2 + D_e s}}$$

S T Q Q S S D
M T W T F S S

Exercício 33) Determine $G(s) = \theta_2(s)/T(s)$

$$J_e = 2s^2(12/4)^2 + s^2 + 16s^2 \cdot (4/16)^2$$
$$J_e = 20s^2$$

$$D_e = s(12/4)^2 + 2s + 32 \times (4/16)^2 s$$
$$D_e = 13s$$

$$K_e = 64 \times (4/16)^2 \Rightarrow K_e = 4$$

$$(20s^2 + 13s + 4)\theta_2(s) = 3T$$

$$\boxed{G(s) = \frac{\theta_2(s)}{T(s)} = \frac{3}{20s^2 + 13s + 4}}$$

Exercício 34) Obtenha $G(s) = \theta_2(s)/T(s)$

$$3s^2\left(\frac{50}{5}\right)^2 + 3 \times \left(\frac{50}{5}\right)^2 s + 200s^2 + 250 + 200s^2\left(\frac{5}{25} \cdot \frac{50}{5}\right)^2 + 1000 \times \left(\frac{5}{25} \cdot \frac{50}{5}\right)^2$$

$$= T \cdot \frac{50}{5}$$

$$\left[3 \times \left(\frac{50}{5}\right)^2 + 200 + 200\left(\frac{50}{25}\right)^2\right]s^2 + \left[1000 \times \left(\frac{50}{25}\right)^2\right]s + \left[3 \times \left(\frac{50}{5}\right)^2 + 250\right] = T \times \frac{10}{5}$$

$$(1300s^2 + 400s + 550)\theta_2(s) = T \times 10$$

$$\boxed{G(s) = \frac{\theta_2(s)}{T(s)} = \frac{10}{1300s^2 + 400s + 500}}$$

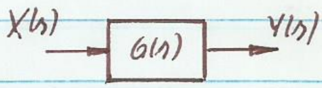
kajoma

Exercício 38) Obtenha $G(s) = \Theta_o(s) / \Theta_i(s)$

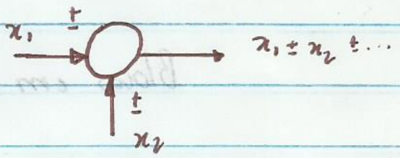
— // —

Diagramas de Bloco

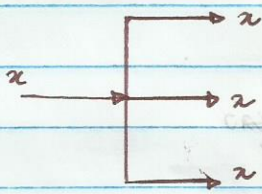
Bloco Operacional

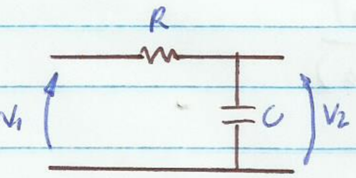


Bloco Somador



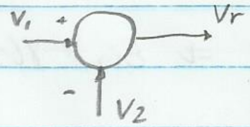
Ramificações



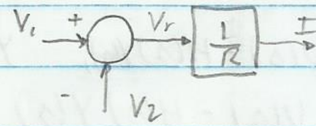


$V_R(s) = V_1(s) - V_2(s)$

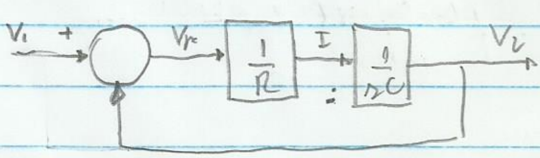
1º Passo



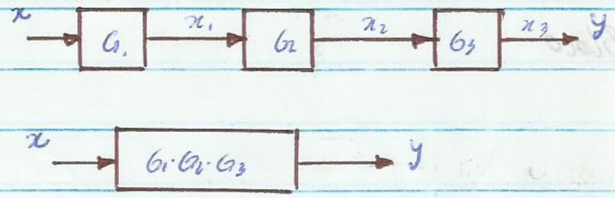
2º Passo



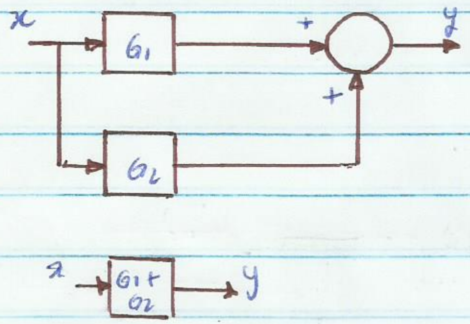
3º Passo



Reduções Básicas

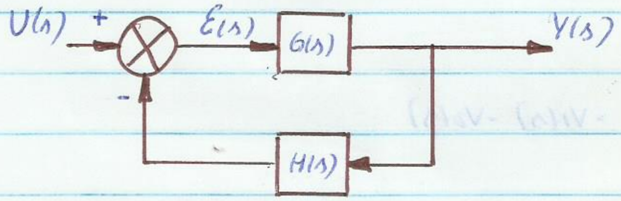


Blocos em série



Blocos em paralelo

Malhas de Realimentação



$$F(s) = \frac{Y(s)}{U(s)}$$

$\varepsilon(s) = U(s) - H(s)Y(s)$; $\quad Y(s) = \varepsilon(s)\,G(s) \Rightarrow \varepsilon(s) = Y(s)/G(s)$

$$\frac{Y(s)}{G(s)} = U(s) - H(s)\,Y(s)$$

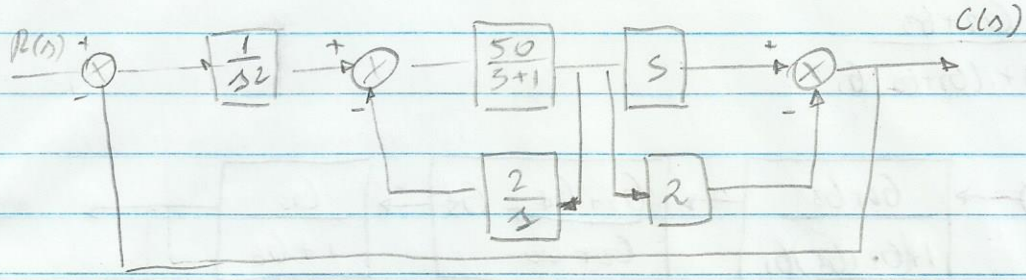
$$\frac{Y(s)}{G(s)} + H(s)\,Y(s) = U(s)$$

$$Y(s)\left(\frac{1}{G(s)} + H(s)\right) = U(s)$$

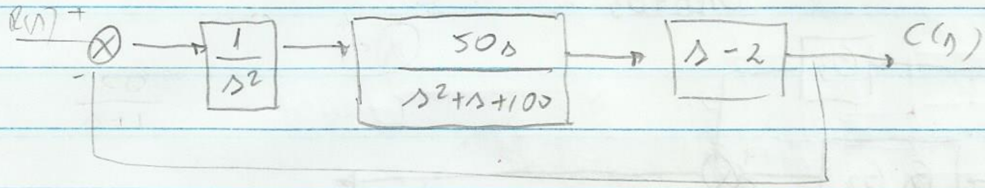
$$\boxed{F(s) = \frac{G(s)}{1 + H(s)\,G(s)}}$$

Equação Fundamental do controle

Exercício 1

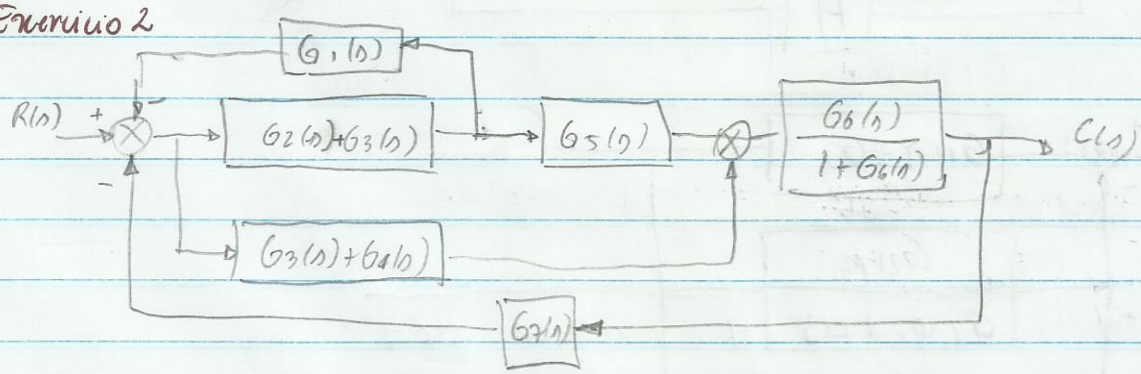


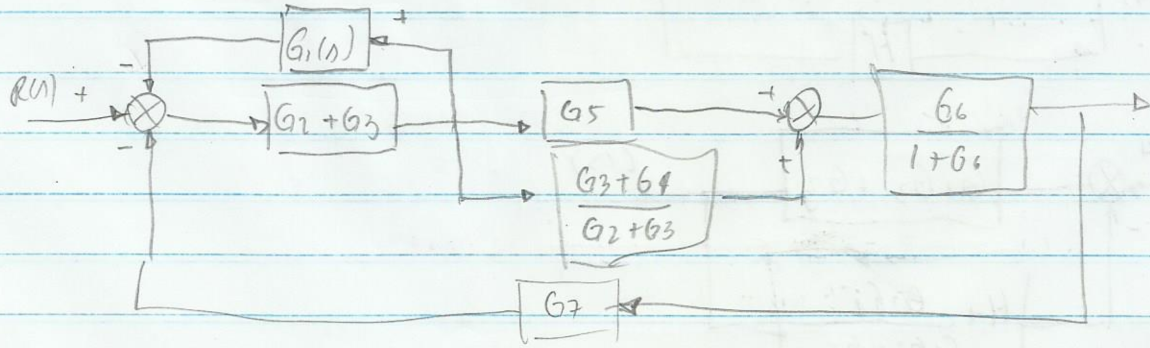
$$\frac{\frac{50}{s+1}}{1+\frac{50}{s+1}\times\frac{2}{s}} = \frac{\frac{50}{s+1}}{\frac{s(s+1)+100}{s(s+1)}} = \frac{50s}{s^2+s+100}$$

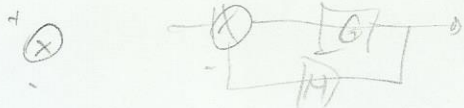


$$T(s) = \frac{50s}{s^2+s+100} \times \frac{1}{s^2} \times (s-2)$$

Exercício 2







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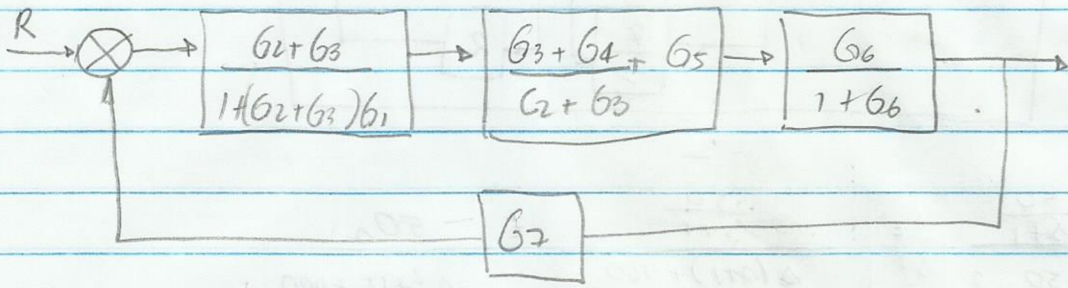
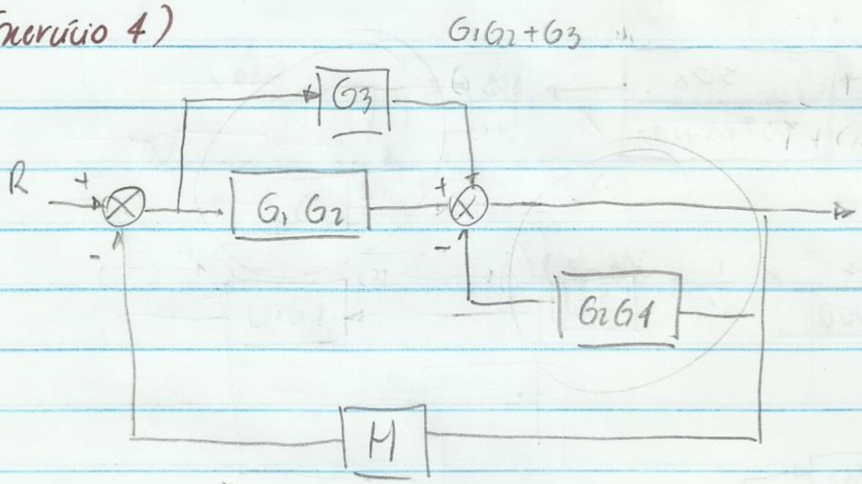
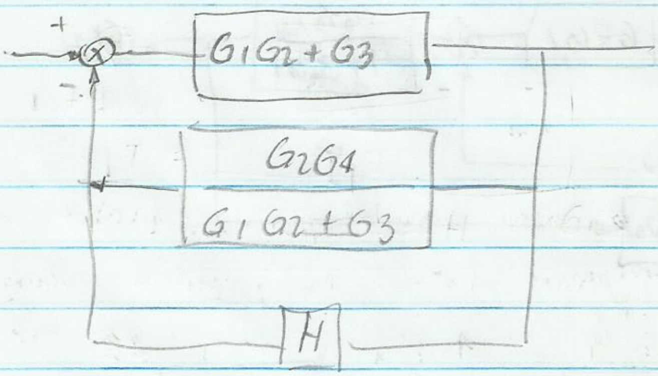
M T W T F S S

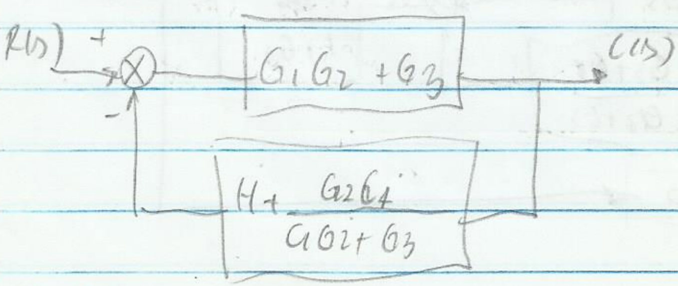
$$\frac{G_2+G_3}{1+(G_2+G_3)G_1}$$



Exercício 4)



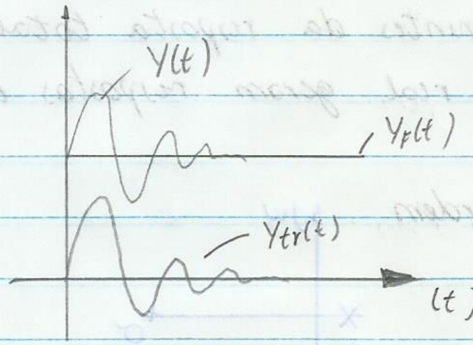




Resposta no domínio do Tempo

A resposta de um sistema é a soma da resposta transitória com a resposta forçada.

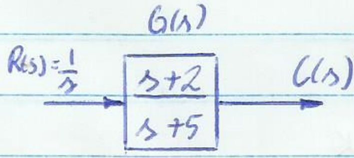
$$y(t) = y_{tr}(t) + y_f(t)$$

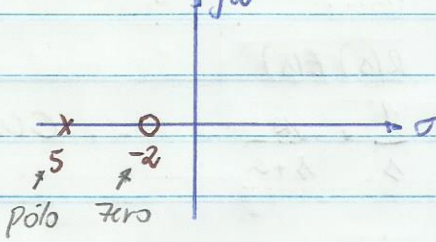


Pólos: São os valores da variável s, da transformada de laplace que levam o valor da função de transferencia a tender o infinito.

Zerós: São os valores da variável s, da transformada de laplace que anulam a função de transferência.

Exemplo:





Pólo: -5

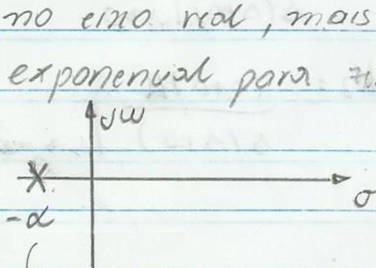
Zero: -2

1- Um pólo da função de entrada gera uma resposta forçada

2- Um pólo da função de transferencia gera uma resposta natural

3- Um pólo sobre o eixo real gera uma resposta exponencial (resposta transiente). ("Quando mais a esquerda no eixo real, mais rápido é o decaimento da resposta transiente exponencial para zero.")

4- Pólos e Zeros Geram as amplitudes tanto para a resposta forçada natural.

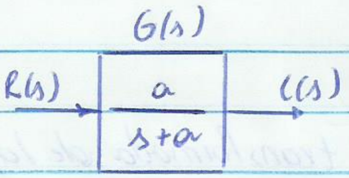


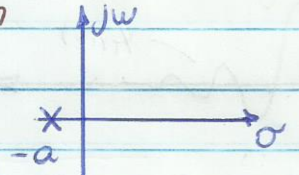
Gera: $Ke^{-\alpha t}$

Conclusão Sobre Pólos e Zeros

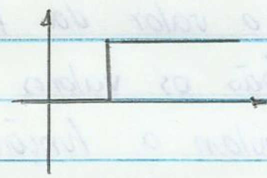
"Pólos determinam a natureza da resposta no domínio do tempo. Os pólos da função de entrada estabelecem a forma da resposta forçada, e os pólos da função de transferência estabelecem a forma da resposta natural. Os zeros e polos da entrada ou da função de transferência contribuem para as amplitudes partes componentes da resposta total. Finalmente, os polos sobre o eixo real geram respostas exponenciais".

Sistema de Primeira Ordem





Para função de Entrada: Degrau



$$R(s) = \frac{1}{s}$$

$$C(s) = R(s)\,G(s)$$

$$= \frac{1}{s} \times \frac{K}{s+a} \quad \therefore \quad C(s) = \frac{K}{s(s+a)}$$

$$\mathcal{L}^{-1}[C(s)] = \mathcal{L}^{-1}\left[\frac{K}{s(s+a)}\right] \quad ; \quad \frac{K}{s(s+a)} = \frac{K_1}{s} + \frac{K_2}{s+a}$$

$$K_1 = \left.\frac{Ks}{s(s+a)}\right|_{s \to 0} \quad \therefore \quad K_1 = \frac{K}{a}$$

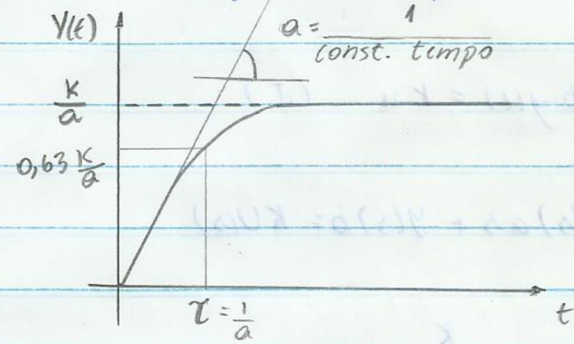
$$K_2 = \left.\frac{(s+a)K}{s(s+a)}\right|_{s \to -a} \quad \therefore \quad K_2 = -\frac{K}{a}$$

S T Q Q S S D
M T W T F S S

$$C(s) = \frac{K/a}{s} - \frac{K/a}{s+a}$$

$$\boxed{C(t) = C_f(t) + C_n(t) \Rightarrow C(t) = \frac{K}{a} - \frac{K}{a}e^{-at}}$$

- Pólo de entrada gerou a resposta forçada $c_f(t) = \frac{K}{a}$
- Pólo do sistema $(-a)$ gerou a resposta natural $c_n(t) = -\frac{K}{a}e^{-at}$



$$Y(t) = (1 - e^{-at})\frac{K}{a}$$

Para $t = \frac{1}{a} = (1 - e^{-\frac{a}{a}})\frac{K}{a} \Rightarrow y = 0{,}63\frac{K}{a}$

Constante de tempo τ

A constante de tempo da resposta determinada por $1/a$ é o tempo para a resposta ao degrau atingir 63% de seu valor

"frequência exponencial" $\Rightarrow a \left[\frac{1}{s}\right]$

Quanto maior a frequência exponencial a, maior a velocidade a resposta transiente.

Tempo de subida, T_r (Rise Time)

É o valor de subida o intervalo de tempo em que o sinal evolui de 10% a 90% do valor final da resposta.

$$\boxed{T_r = \frac{2{,}2}{a}} \qquad \boxed{T_s = T_{ac} = \frac{4}{a}}$$

Tempo de assentamento, T_s (ou "Tempo de acomodação") Setling time

É o tempo necessário para que a resposta se estabilize por 2% em torno do seu valor final

kajoma

S T Q Q S S D
M T W T F S S

Sistemas de Segunda Ordem

* Sistemas de 1ª Ordem altera apenas a velocidade da resposta.
* Variações dos parametros de 2ª Ordem podem alterar a forma da resposta.

$$\ddot{y}(t) + a\dot{y}(t) + b\,y(t) = k\,u \quad (I)$$

$$\mathcal{L}[(I)] = Y(s)s^2 + Y(s)as + Y(s)b = KU(s)$$

Função de transferência : $G(s) = \frac{Y(s)}{U(s)} = \frac{K}{s^2 + as + b}$

Sistema de Segunda Ordem Geral

$$\boxed{G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}} \quad s^2 + \alpha s + \omega d$$

(frequência de ressonância)
ou
ω_n: frequência natural : É a frequência de oscilação do sistema sem amortecimento

ξ: fração (ou grau) de amortecimento.

$\alpha = \xi\omega_n$: coeficiente de amortecimento

Pode ser definida a natureza da resposta transiente através do cálculo dos pólos do sistema.

$$\boxed{s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}}$$

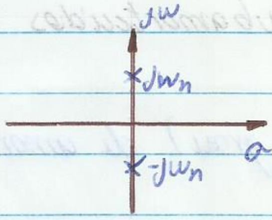
kajoma

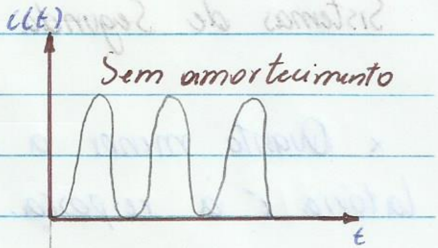
Fração (ou grau) de Amortecimento | Pólos | Resposta ao Degrau

(Dois polos Imaginários)

$0 \qquad \pm j\omega$

$\alpha = 0$

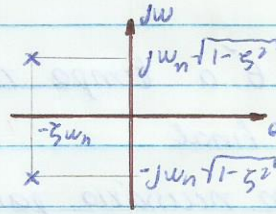


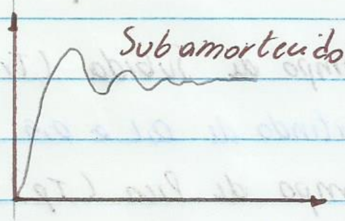


Pólos complexos

$0 < \zeta < 1$

$\alpha^2 < \omega_n^2$

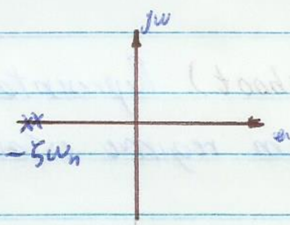


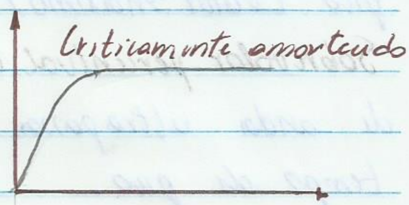


Polos reais em $-\sigma$

$\zeta = 1$

$\alpha^2 = \omega_n^2$

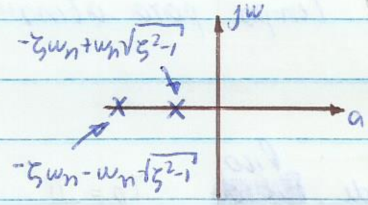


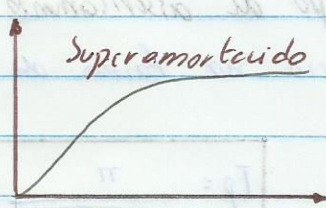


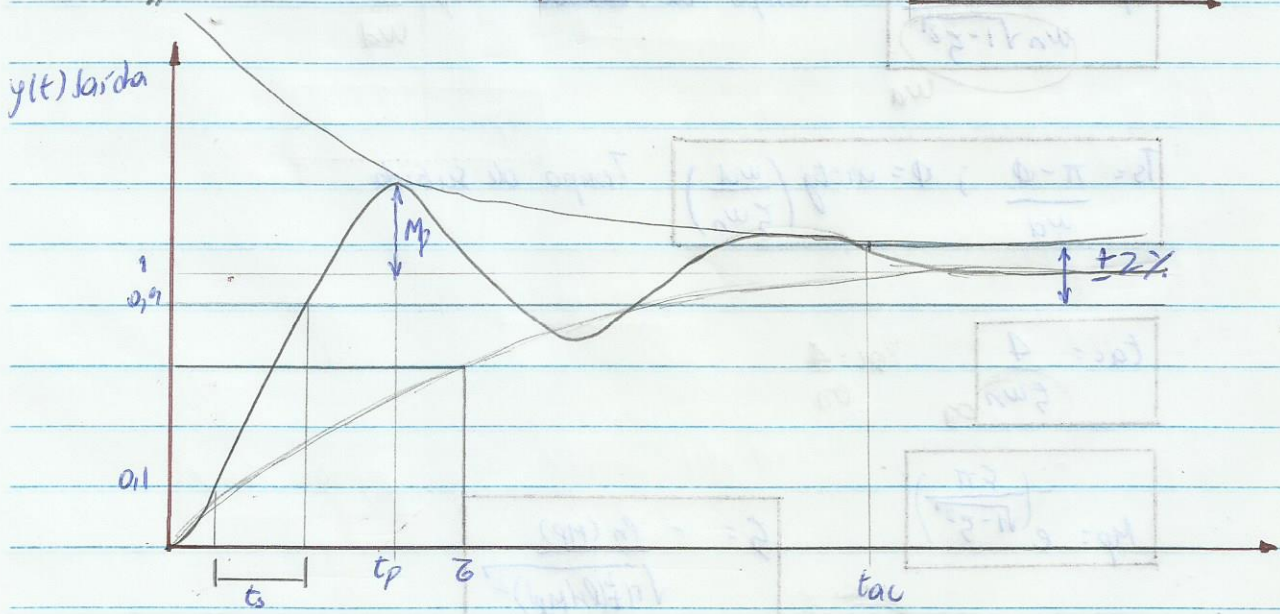
Dois polos Reais

$\zeta > 1 \qquad \sigma_1, \sigma_2$

$\alpha^2 < \omega_n^2$







Sistemas de Segunda Ordem Subamortecidos

× Quanto menor a fração (ou grau) de amortecimento, mais oscilatória é a resposta.

Tempo de Subida (Tr) ou (Ts) É o tempo necessário para a onda partindo de 0,1 a 0,9 de seu valor final

Tempo de Pico (Tp) É o tempo necessário para se atingir o primeiro pico (valor máximo).

Sobrevalor percentual (Mp) (overshoot) Representa o quanto a forma de onda ultrapassa o valor em regime estacionário, ou final, ~~do~~ no tempo de pico.

Tempo de assentamento (Ts ou Tac) Tempo para atingir o regime estacionário na faixa de ±2%

$$\boxed{T_p = \frac{\pi}{\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}}}$$ Tempo de Pico $T_p = \dfrac{\pi}{\omega_d}$

$$\boxed{T_s = \frac{\pi - \phi}{\omega_d} \; ; \; \phi = \operatorname{arctg}\left(\frac{\omega_d}{\zeta\omega_n}\right)}$$ Tempo de subida

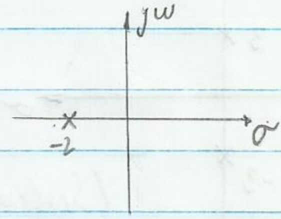
$$\boxed{t_{ac} = \frac{4}{\underbrace{\zeta\omega_n}_{\sigma_d}}}$$ $T_{ac} = \dfrac{4}{\sigma_d}$

$$\boxed{M_p = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)}}$$

$$\boxed{\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + (\ln(M_p))^2}}}$$

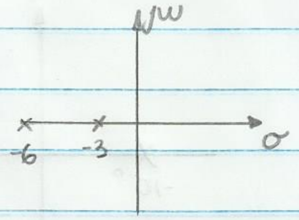
Exercício 8) (Nise 5ª)

(a) $T(s) = \frac{2}{s+2}$



Resposta de 1ª Ordem

(b) $T(s) = \frac{5}{(s+3)(s+6)}$



$(s+3)(s+6) = s^2 + 9s + 18$

$s^2 + 9s + 18 = 0$

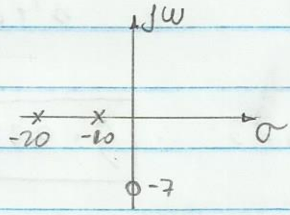
Sistema Superamortecido

$s = \frac{-9 \pm \sqrt{9^2 - 4 \times 18}}{2}$

$s_1 = -3$

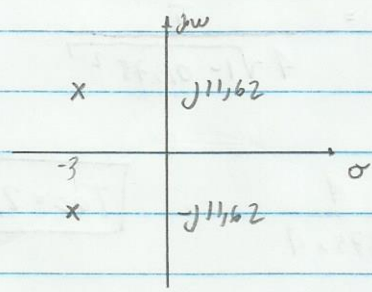
$s_2 = -6$

(c) $T(s) = \frac{10(s+7)}{(s+10)(s+20)}$



(Sistema Superamortecido)

(d) $T(s) = \frac{20}{s^2 + 6s + 144}$



$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

$\omega_n = \sqrt{144} = 12$

$2\zeta\omega_n = 6 \therefore \zeta = 0{,}25$

(Sistema subamortecido)

$s_{1,2} = -3 \pm 12\sqrt{0{,}25^2 - 1}$

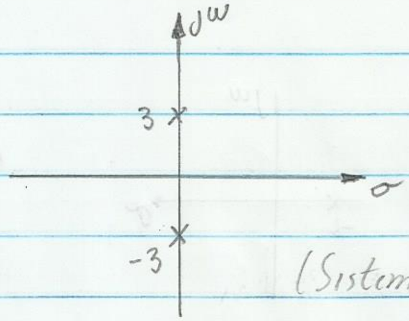
$s_{1,2} = -3 \pm j11{,}62$

S T Q Q S S D
M T W T F S S

e) $T(s) = \frac{s+2}{s^2+9}$

$s^2+9=0$

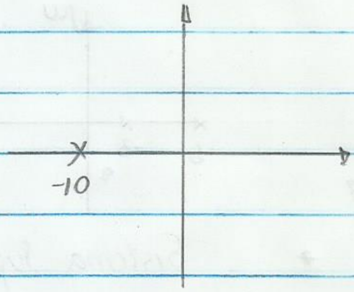
$s_{1,2} = \pm j3$



(Sistema sem amortecimento)

f) $T(s) = \frac{(s+5}{(s+10)^2}$

$s_{1,2} = -10$



(Sistema Criticamente amortecido)

Exercício 20)

(a) $T(s) = \frac{16}{s^2+3s+16}$ $\qquad \frac{wn^2}{s^2+2\zeta wn s+wn^2}$

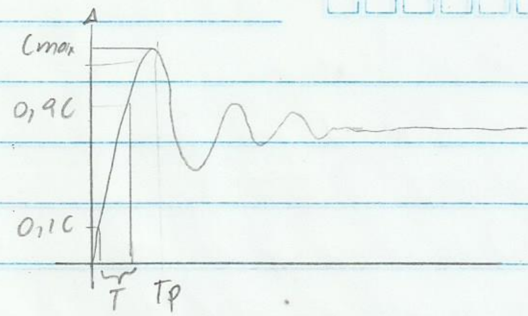
$\boxed{wn = 4}$ $\qquad 2\zeta wn = 3 \quad \therefore \boxed{\zeta = 0{,}375}$

$T_p = \frac{\pi}{w_n\sqrt{1-\zeta^2}} = \frac{\pi}{4\sqrt{1-0{,}375^2}} \quad \therefore \boxed{T_p = 0{,}8472\,s}$

$T_{ac} = \frac{4}{\zeta wn} = \frac{4}{0{,}375 \times 4} \quad \therefore \boxed{T_{ac} = 2{,}67\,s}$

$\%SP = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} = e^{-(0{,}375\pi/\sqrt{1-0{,}375^2})} \quad \therefore \boxed{\%SP = 28{,}06\%}$

b) $T(s) = \dfrac{0{,}04}{s^2 + 0{,}02s + 0{,}04}$



$s^2 + 2\zeta w_n s + w_n^2$

$w_n^2 = 0{,}04 \quad \therefore \quad w_n = 0{,}2 \text{ rad/s}$

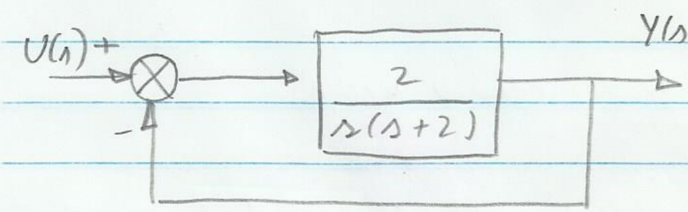
$2\zeta w_n = 0{,}02 \quad \therefore \quad \zeta = 0{,}05$

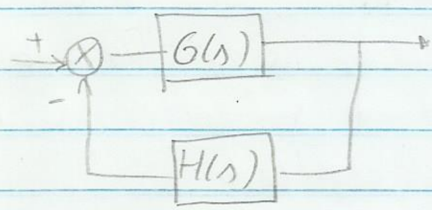
$T_p = \dfrac{\pi}{\underbrace{w_n\sqrt{1-\zeta^2}}_{w_d}} = \dfrac{\pi}{0{,}2\sqrt{1-0{,}05^2}} \quad \therefore \quad T_p = 15{,}73 s$

$T_{ac} = \dfrac{4}{\underbrace{\zeta w_n}_{\sigma_d}} = \dfrac{4}{0{,}05 \times 0{,}2} \quad \therefore \quad T_{ac} = 400 s$

$M_p\% = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} = e^{-(0{,}05\pi/\sqrt{1-0{,}05^2})} \quad \therefore \quad M_p\% = 85{,}45\%$

Exemplo 6.15) (livro)





$F(s) = \dfrac{\dfrac{2}{s(s+2)}}{1 + \dfrac{2}{s(s+2)}} = \dfrac{2}{s^2 + 2s + 2}$

$F(t) = \dfrac{G(s)}{1 + G(s)H(s)}$

(Sistema subamortecido)

$w_n = \sqrt{2}$

$\zeta = \sqrt{2}/2$

$T_p = \dfrac{\pi}{w_n\sqrt{1-\zeta^2}} = 3{,}14 s \qquad T_{ac} = \dfrac{4}{\frac{\sqrt{2}}{2} \times \sqrt{2}} = 4 s$

$T_s = \dfrac{\pi - \theta}{w_n\sqrt{1-\zeta^2}} \; ; \; \varphi = \left(\dfrac{w_n\sqrt{1-\zeta^2}}{\zeta w_n}\right) \quad T_s =$

Estabilidade

Um sistema linear invariante no tempo é:

estável se a resposta natural tende a zero quando o tempo tende a infinito

instável se a resposta natural aumenta sem limites na medida em que o tempo tende a infinito.

marginalmente instável quando a resposta forçada quando a resposta natural tender a zero.

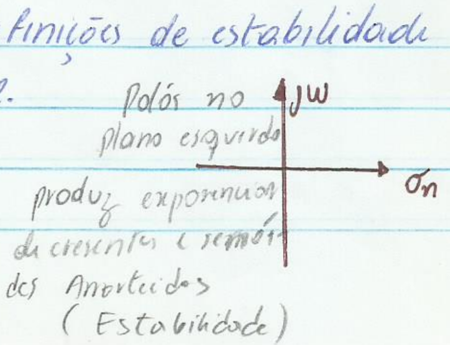
Um sistema é estável de forma marginal se a resposta natural não decair nem crescer, mas permanecer constante ou oscilar.

Estabilidade BIBO (Bounded-Input, Bounded Output) - Entrada limitada, saída limitada

+ Um sistema é estável se toda entrada limitada gerar uma saída limitada
+ Um sistema é instável se alguma entrada limitada gerar uma saída ilimitada.

Como determinar se um sistema é estável?

- As definições de estabilidade são obtidas com base na resposta natural.



- ~~Sistemas estáveis~~ possuem função de transferência em malha fechada com pólos somente no semiplano da esquerda.
- Sistemas instáveis possuem função de transferência em malha fechada com pelo menos um pólo no semiplanos da direita e/ou pólos de multiplicidade maior que um eixo no imaginário
- Sistemas marginalmente estáveis apresentam função de transferência em malha fechada somente com pólos de multiplicidade 1 no eixo imaginário e pólos no semiplanos da esquerda.

+ Caso a função de transferencia possui variação dos sinais no polinômio do denominador, significa que é instável. Pois deve ser todos com mesmo sinal.

Critério de Routh-Hurwitz

Método para ~~determinar~~ verificar se o sistema é estavel ou não. Ele apenas verifica atraves dos polinômios sem precisar obter os pólos do sistema. (Na malha fechada)

$$R(s) \longrightarrow \boxed{\dfrac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}} \longrightarrow C(s)$$

Tabela de Routh Completa

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_2} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

S T Q Q S S D
M T W T F S S

Obs.: Podemos multiplicar por qualquer valor a linha da tabela, pois não mudará o valor final.

Interpretação da tabela de Routh Básica

O número de raízes do polinômio que se situam no semi-plano direito é igual ao número de mudanças de sinal da primeira coluna.

Casos especiais: (1) Um zero apenas na primeira coluna de uma linha
(2) Linha inteira com valores nulos

Zero Apenas na Primeira Coluna

- Caso exista algum zero na primeira coluna, a próxima linha iria ter um valor dividido por zero.
- Para o valor de zero, admite-se um ϵ (épsilon)
- Para determinar se o sistema é estável ou não, faz este valor tender a valor infinitamente negativo e positivo.

Linha Completa de Zeros

- Acontece quando a linha inteira é formada por zeros
- Para evitar isso devemos fazer a derivada do polinômio daquela linha

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(P1 - 2º Sem2012 Noturno)

$$E(s) = \frac{1}{s} \longrightarrow \boxed{} \longrightarrow C(t) = 3(1 - e^{-t/8})$$

$$C(t) = 3 - 3e^{-t/8}$$

$$\mathcal{L}[C(t)] = \frac{3}{s} - \frac{3}{s+\frac{1}{8}}$$

$$F(s) = \frac{C(s)}{E(s)} = \frac{\dfrac{3(s+1/8) - 3s}{\cancel{s}(s+1/8)}}{\dfrac{1}{\cancel{s}}} \quad \therefore \quad F(s) = \frac{3/8}{s+1/8}$$

Exercício 4) Determine: τ, T_r, T_s

$$G(s) = \frac{20}{s+25} \qquad \frac{1}{s} \longrightarrow \boxed{G(s)} \longrightarrow \frac{20}{s(s+25)}$$

$$\frac{20}{s(s+25)} = \frac{K_1}{s} + \frac{K_2}{s+25}$$

$$K_1 = \left.\frac{20}{s+25}\right|_{s\to 0} \therefore K_1 = 0{,}8 \qquad K_2 = \left.\frac{20}{s}\right|_{s\to -25} \therefore K_2 = -0{,}8$$

$$C(s) = \frac{0{,}8}{s} - \frac{0{,}8}{s+25} \Rightarrow C(t) = 0{,}8(1 - e^{-25t})$$

$$\tau = \frac{1}{25} \quad \therefore \quad \boxed{\tau = 0{,}04\,s}$$

$$T_r = \frac{2{,}2}{a} = \frac{2{,}2}{25} \quad \therefore \quad \boxed{T_r = 0{,}088\,s} \qquad T_s = \frac{4}{a} = \frac{4}{25} \quad \therefore \quad \boxed{T_s = 0{,}16\,s}$$

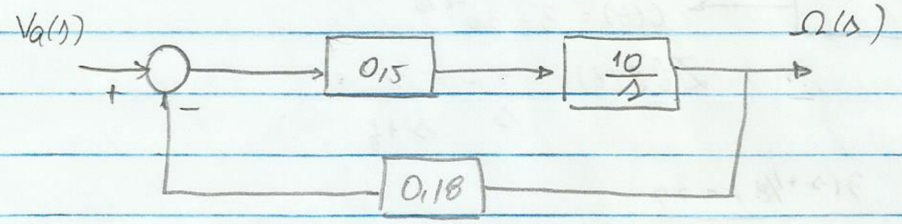
Ganho estático $\boxed{\frac{20}{25} = 0{,}8}$

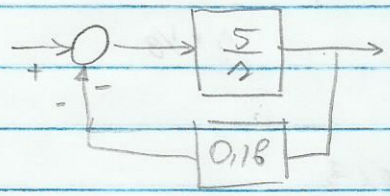
$$\boxed{y(\infty) = 0{,}8}$$

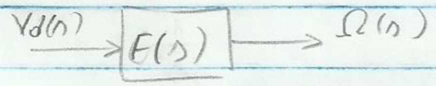
S T Q Q S S D
M T W T F S S

$\tau = ?$ Resposta ao degrau 20

Exercício 5)







$$F(s) = \frac{\frac{5}{s}}{1 + \frac{0,9}{s}} = \frac{5}{s+0,9}$$

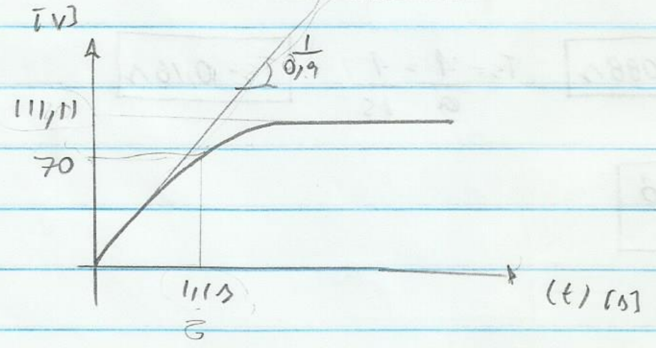
$\therefore \boxed{\tau = 1,1\,s}$

(b) 20V

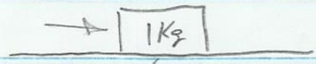
$$C(s) = \frac{100}{s(s+0,9)} = \frac{K_1}{s} + \frac{K_2}{s+0,9}$$

$$K_1 = \left.\frac{100}{s+0,9}\right|_{s\to 0} \quad \therefore K_1 = 111,11 \qquad K_2 = \left.\frac{100}{s}\right|_{s\to -0,9} = -111,11$$

$$\therefore \boxed{C(t) = 111,11\left(1 - e^{-0,9t}\right)\ [s]}$$



Exercício 6) $\qquad 10\,000N \rightarrow 0{,}01s$

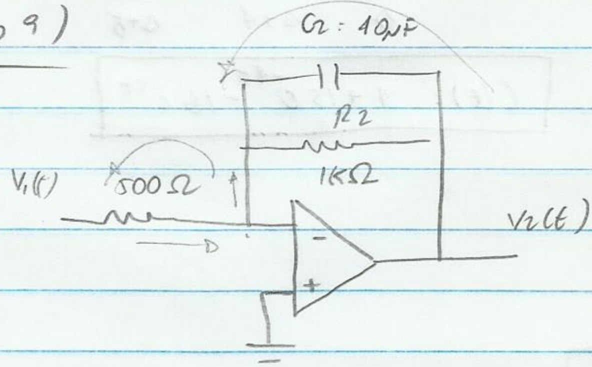


$f = 0{,}1 \frac{Ns}{m}$ $\qquad$ Impulso ?

$v(t) = ?$

$$\text{Impulso} = F \cdot \Delta t$$
$$= 10 \cdot 10^3 \cdot 0{,}01 \quad \therefore \quad \text{Impulso} = 100\ Ns$$

Exercício 9)



$$G_s = \frac{V_2}{V_1}$$

$$\frac{V_1(s) - 0}{Z_1} = \frac{0 - V_2}{Z_2} \quad \therefore \quad \frac{V_2}{V_1} = -\frac{Z_2}{Z_1}$$

$$Z_2 = \frac{\frac{1}{Cs} \times R_2}{\frac{1}{Cs} + R_2} = \frac{R_2}{1 + R_2 C s} = \frac{10^3}{1 + 10^3 \times 40 \cdot 10^{-6} s} \quad \therefore \quad Z_2 = \frac{10^3}{1 + 0{,}04s}$$

$$G(s) = -\frac{10^3}{1 + 0{,}04s} \times \frac{1}{500} = \frac{-10^3}{500(1 + 0{,}04s)} \quad \therefore \quad \boxed{G(s) = \frac{2}{1 + 0{,}04s} = \frac{-50}{s + 25}}$$

$$K_g = \frac{50}{25} \quad \therefore \quad \boxed{K_g = 2}$$

$$V_2(s) = \frac{1}{s} \cdot \frac{-50}{s+25} \quad \therefore \quad V_2(s) = \frac{-50}{s(s+25)} = \frac{K_1}{s} + \frac{K_2}{s+25}$$

$$K_1 = \left.\frac{-50}{s+25}\right|_{s \to 0} = -2 \qquad K_2 = \left.\frac{-50}{s}\right|_{s \to -25} = 2$$

$$v_2(t) = -2(1 - e^{-25t}) \quad [V]$$

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M T W T F S S

Exercício 13)

$$G(s) = \frac{20(s+1)}{(s+4)(s+5)}$$

$$C(s) = \frac{20(s+1)}{s(s+4)(s+5)} = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+5}$$

$$K_1 = \left.\frac{20(s+1)}{(s+4)(s+5)}\right|_{s=0} = 1$$

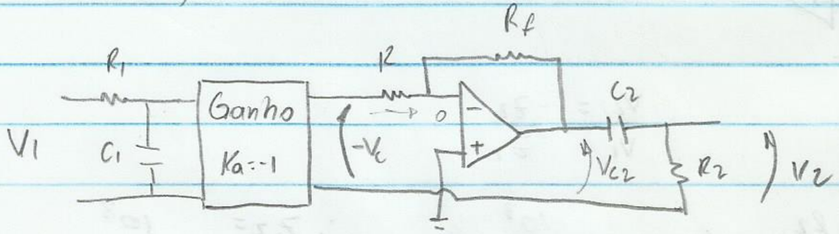
$$K_2 = \left.\frac{20(s+1)}{s(s+5)}\right|_{s\to -4} = 15$$

$$C(s) = \frac{1}{s} + \frac{15}{s+4} - \frac{16}{s+5}$$

$$K_3 = \left.\frac{20(s+1)}{s(s+4)}\right|_{s\to -5} = -16$$

$$\boxed{C(t) = 1 + 15e^{-4t} - 16e^{-5t}}$$

Exercício 14)



$$V_c = \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} \cdot V_1 = \frac{1}{sR_1C_1 + 1} V_1$$

$$\frac{-V_c - 0}{R} = \frac{0 - V_{C2}}{R_f} \quad \therefore \quad V_{C2} = \frac{R_f}{R} \cdot V_c$$

$$V_2 = \frac{R_2}{\frac{1}{sC_2} + R_2} \cdot V_{C2} \quad \therefore \quad V_2 = \frac{sC_2 \cdot R_2}{(1 + sC_2R_2)} \times \frac{R_f}{R} \times \frac{V_1}{(sR_1C_1 + 1)}$$

$$\therefore \frac{V_2}{V_1} = \frac{1}{(1+s)} \times \frac{10^5}{2.10^4} \times \frac{1}{(0{,}1s+1)}$$

$$\frac{V_2}{V_1} = \frac{500}{(1+s)(s+10)0{,}1} \quad \therefore \quad \frac{V_2}{V_1} = \frac{50}{(s+1)(s+10)}$$

Exercício 27)

$T_p = 0,5\,s$ $\quad T_p = \dfrac{\pi}{w_n\sqrt{1-\xi^2}} = \dfrac{\pi}{w_d}$ $\therefore w_d = 2\pi$ $\quad T_{ac} = \dfrac{4}{\sigma_D} = \dfrac{4}{w_n \xi}$

$M_p = e^{-(\pi\xi/\sqrt{1-\xi^2})}$ $\quad T_{ac} = 1,4\,s$

$$\begin{cases} w_n \xi = 20/7 \\ w_n\sqrt{1-\xi^2} = 2\pi \end{cases}$$

$\rightarrow w_n^2(1-\xi^2) = (2\pi)^2$

$\left(\dfrac{20/7}{\xi}\right)^2 (1-\xi^2) = (2\pi)^2$

$\dfrac{(20/7)^2}{\xi^2} - (20/7)^2 = (2\pi)^2$

$\therefore$

$M_p = 27,85\,\%$ $\quad \xi = 0,414$

$w_n = 6,9$

$\boxed{M_p = 23,97\%}$

$F(t) = \dfrac{47,64}{s^2 + 5,71\,s + 47,64}$

$$\text{Função de Transferência} = \frac{\text{Pólos}}{\text{Zeros}} \quad \frac{\times}{O}$$

S T Q Q S S D

M T W T F S S

Cap 12 -

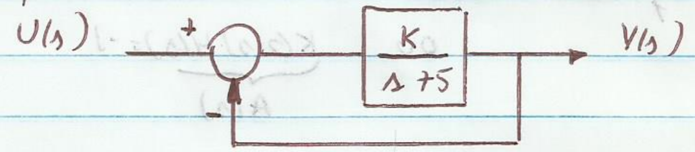
Diagrama do Lugar das Raízes (LR)

↳ Tem a função de adicionar polos para que o sistema fique mais afastado do eixo $j\omega$

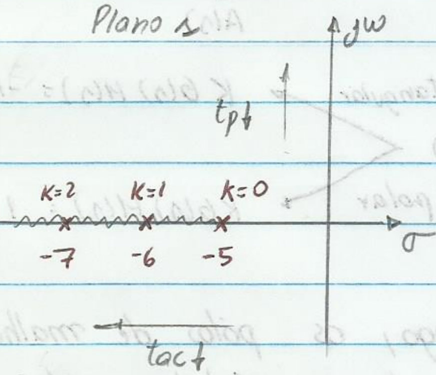
12.1 O método

Trata-se de um método que permite localizar no plano s as raízes da equação característica (pólos) de um sistema de malha fechada, em função da variação de um parametro (geralmente o ganho K).

Exemplo 12.1



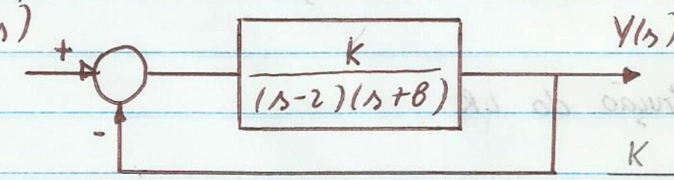
$$F(s) = \frac{G}{1+GH}$$



$$GH = \frac{K}{s+5}$$

$$F(s) = \frac{G}{1+GH} = \frac{K}{s+(5+K)}$$

Exemplo 12.2

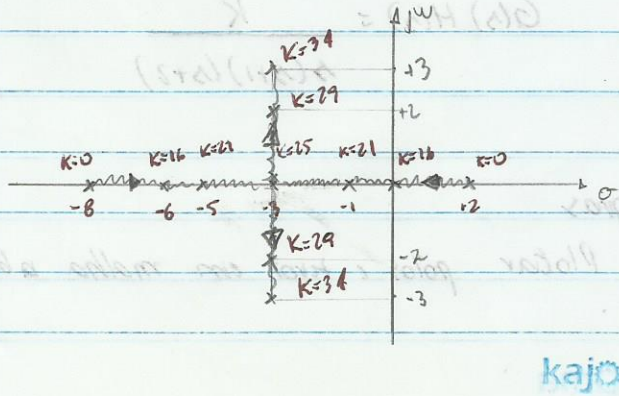


K	0	16	21	25	29	34
s_1	2	0	-1	-3	$-3+j2$	$-3+j3$
s_2	-8	-6	-5	-3	$-3-j2$	$-3-j3$

$$GH = \frac{K}{(s-2)(s+8)}$$

$$F(s) = \frac{G}{1+GH} = \frac{K}{s^2+6s-16+K}$$

$\hookrightarrow -3 \pm \sqrt{25-K}$



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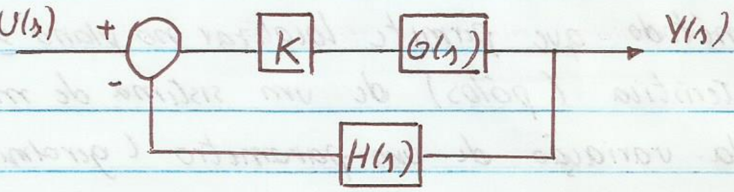
12-2. Princípios do método do LR

Adotaremos a representação do ganho separado do $G(s)$



$$F(s) = \frac{K\,G(s)}{1 + \underbrace{K\,G(s)H(s)}_{A(s)}}$$

Eq. característica: $1 + K\,G(s)\,H(s) = 0$ ou $\underbrace{K\,G(s)\,H(s)}_{A(s)} = -1$

$A(s)$ — retangular → $K\,G(s)\,H(s) = -1 + 0j$

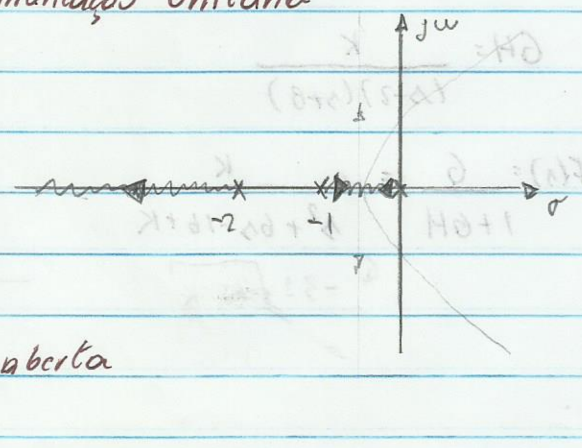
$A(s)$ — polar → $K\,G(s)\,H(s) = 1\ \underline{/-180° \pm N360°}$

Logo, os pólos de malha fechada são pontos do plano s onde $A(s)$ tem módulo unitário (condição de ganho) e angulo de fase $\theta = -180° + N360°$, sendo $N = 0, 1, 2, \ldots$ (condição de fase)

12.3- Regras básicas de construção do LR

Consideremos um sistema c/ realimentação unitária

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+2)}$$



Regras

1- Plotar pólos e zeros em malha aberta

2-

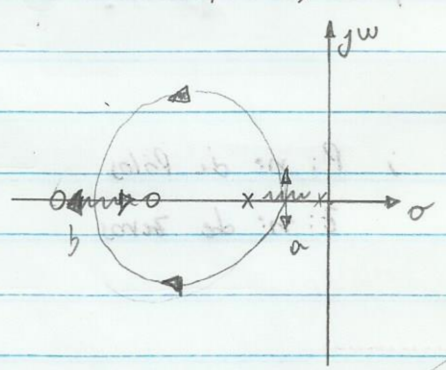
3- Determinar os segmentos do lugar das Raízes (LR) pertinentes no eixo real. Tais segmentos situam-se a esquerda de um número ímpar mais polos e zeros (Sentido: começo no polo termina no zero)

4- Determinar o número de ramos que vai para o infinito, P-Z

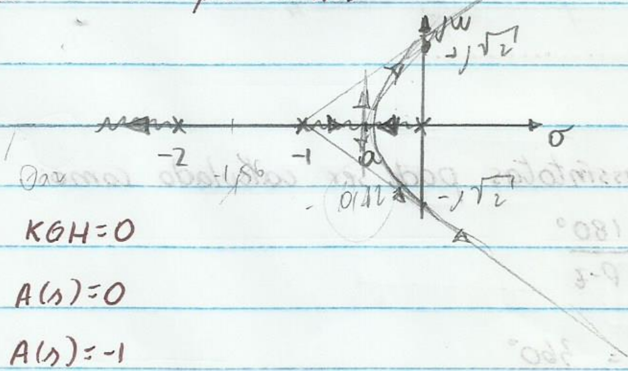
5- Determinar os pontos de separação sobre o eixo real

a) pela derivada

A medida que K aumenta pode acontecer de dois segmentos se encontrar, com sentidos opostos, sobre o eixo real



Para o exemplo temos



$1 + KGH = 0$

$1 + A(s) = 0$

$A(s) = -1$

$\therefore \frac{K}{s(s+1)(s+2)} = -1 \Rightarrow \frac{K}{s(s^2+3s+2)} = -1$

p/ $s = \sigma + j\cancel{\omega}^{0}$ (ramo no eixo real)

$K = -\sigma(\sigma^2 + 3\sigma + 2)$

$\frac{dK}{d\sigma} = -(3\sigma^2 + 6\sigma + 2)$; $\sigma^2 + 2\sigma + \frac{2}{3} = 0$ $\begin{cases} \sigma_1 = -0,42 & \text{pto de separação} \\ \sigma_2 = -1,58 & \text{ignorar pois está fora do LR} \end{cases}$

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M T W T F S S

b) por tentativas sucessivas

$$\left| \frac{K}{s(s^2+3s+2)} \right| = |-1|$$

$$K = |\sigma(\sigma^2+3\sigma+2)|$$

σ	K
-0,5	0,3750
-0,4	0,3840
→ Chute: -0,42	0,3848
-0,3	0,357

6- Comportamento no infinito

Os ramos que se dirigem para o infinito ficam localizados por assíntotas.

$$\sigma = \frac{\Sigma polos_{m.a} - \Sigma zeros_{m.a}}{P - Z}$$; P: nº de Pólos; Z: nº de zeros

Para nosso exemplo:

$$\sigma = \frac{0-1-2}{3-0} = -1$$ (ponto de radiação)

Já a direção das assíntotas pode ser calculado como:

- 1ª assíntota $\theta_a = \frac{180°}{P-Z}$
- entre assíntota $\Delta\theta_a = \frac{360°}{P-Z}$

Para nosso exemplo

$$\theta_{a_1} = \frac{180°}{3-0} = 60° \qquad \theta_{a_2} = \theta_{a_1} + \Delta\theta_a = 180°$$

$$\Delta\theta_a = \frac{360}{3-0} = 120° \qquad \theta_{a_3} = \theta_{a_2} + \Delta\theta_a = 300°$$

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7- Pontos de cruzamento do LR com o eixo $j\omega$

Pelo critério de Routh

$$F(s) = \frac{K}{s(s+1)(s+3)+K} = \frac{K}{s^3+3s^2+2s+K}$$

s^3	1	2
s^2	3	K
s^1	6-K	0
s^0	K	

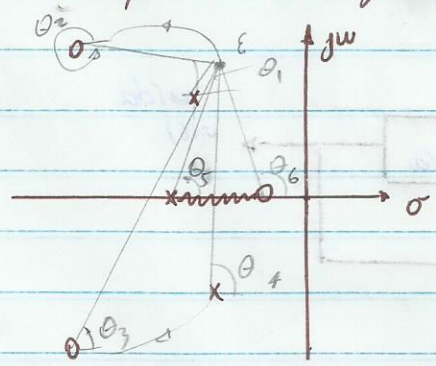
Para o sistema ser estável: $0 < K < 6$

Para $K=6$: linha nula

$\therefore \quad 3s^2 + K = 0 \; ; \; K=6$

$3s^2 + 6 = 0 \quad \begin{cases} s_1 = +j\sqrt{2} \\ s_2 = -j\sqrt{2} \end{cases}$

8- Angulos de partida e chegada p/ polos e ou zeros complexos



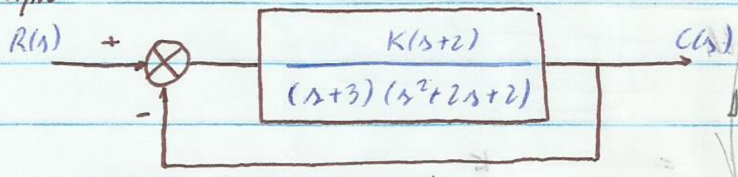
Critério de módulo $K = \frac{1}{GH}$

Critério de fase $\sum \theta_p - \sum \theta_z = -180°$

$\theta_1 + \theta_4 + \theta_5 - \theta_2 - \theta_3 - \theta_6 = -180$

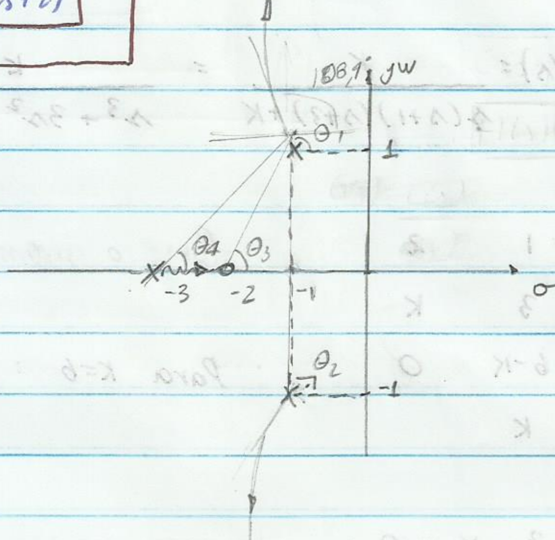
$\theta_1 = -180 - \theta_4 - \theta_5 + \theta_2 + \theta_3 + \theta_6$

Exemplo:



$s^2+2s+2=0$

$s = \frac{-2 \pm \sqrt{4-8}}{2} \therefore s = -1 \pm j1$



$\Sigma\theta_p - \Sigma\theta_z = -180$

$\theta_1 + \theta_2 + \theta_4 - \theta_3 = -180$

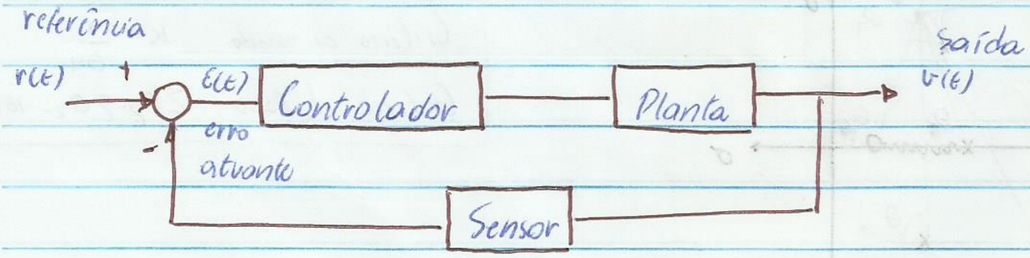
$\theta_2 = 90°$

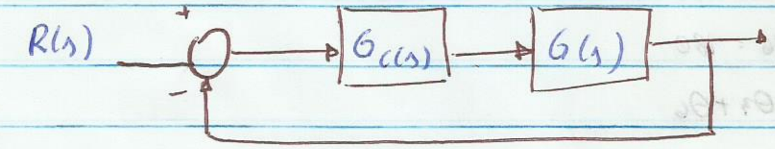
$\theta_3 = 45$

$\theta_4 = tg^{-1}\frac{1}{2} = 26{,}56°$

$\theta_1 + 90 + 26{,}56 - 45 = -180 \therefore \theta_1 = -251{,}56°$ ou $180{,}43°$

Cap 13 - Projeto de compensadores (Controlador ou Filtro)

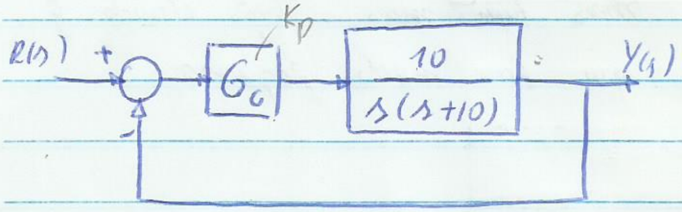


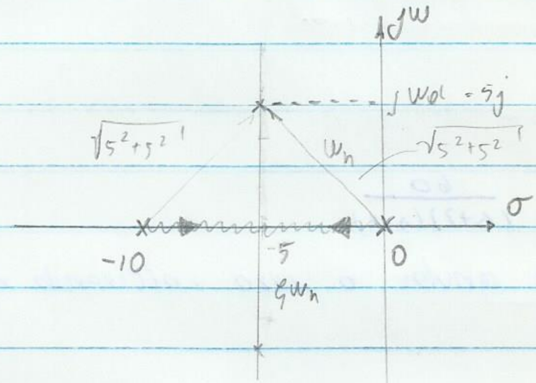


Controlador proporcional (P)

Vantagens:
Sistemas lento

Exemplo: Dado o sistema a seguir, vamos projetar um controlador proporcional de ganho K_p de forma que o sistema tenha um grau de amortecimento 0,707





$$\operatorname{sen}\theta = \frac{\zeta\omega_n}{\omega_n} = \zeta$$

$$\therefore \theta = \operatorname{sen}^{-1}\zeta = \operatorname{sen}^{-1} 0{,}707$$

$$\theta = 45^\circ$$

Pelo Critério de módulo

$$K = \frac{\prod \text{Comprimento dos Pólos}}{\prod \text{Comprimento dos Zeros}}$$

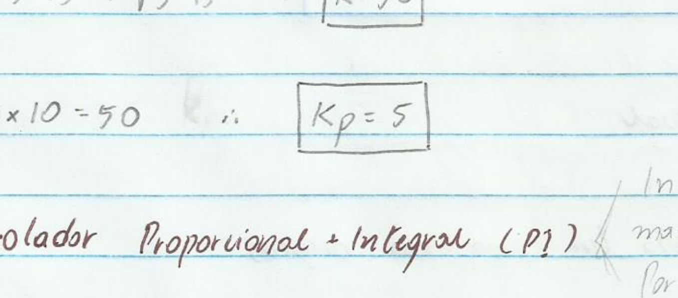
$$K = \sqrt{5^2+5^2} \times \sqrt{5^2+5^2} \quad \boxed{K = 50}$$

$$K = K_p \times 10 = 50 \quad \therefore \quad \boxed{K_p = 5}$$

Controlador Proporcional + Integral (PI)

Integral: Muito bom para reduzir erro, mas altera ζ mudando tempos de pico
Por isso usamos PI

Utilizado para aumentar a precisão do sistema, sem alterar significamente o LR.



$$G_c(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} = \frac{K_p\left(s + \frac{K_i}{K_p}\right)}{s}$$

$$\boxed{G_c(s) = \frac{K_p\,(s + z_i)}{s}}$$

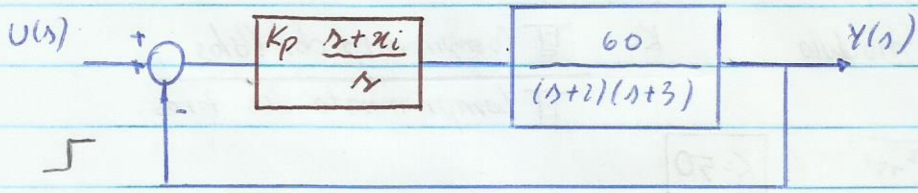
Aumentar ganho diminui o erro, mas tem mais oscilação devido ξ que fica muito pequeno.

tabela de erros

	Posição	Velocidade	Aceleração
0	$\frac{1}{1+K_g}$	∞	∞
1	0	$\frac{1}{K_g}$	∞
2	0	0	$\frac{1}{K_a}$

Exemplo: Dado o sistema $G(s) = \frac{60}{(s+2)(s+3)}$

Calcular o com PI de forma a anular o erro, alterando minimamente ξ.



- Determine o erro atual

Erro $= \frac{1}{1+K_g}$ Pois não tem polo na origem

$K_g = \frac{60}{2 \times 3} = 10 \qquad e_{st,p} = \frac{1}{1+K_g} = 0{,}091 = 9{,}1\%$

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- Determinar ξ s/ compensação

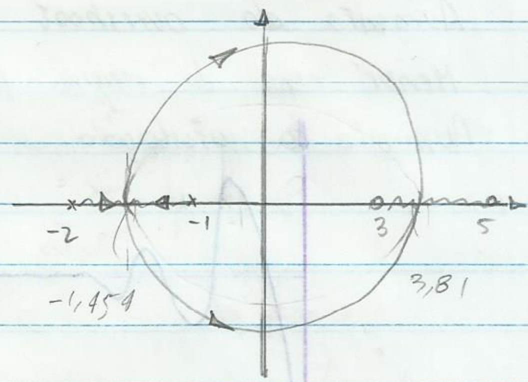
$$F(s) = \frac{G(s)}{1+G(s)} = \frac{60}{s^2+5s+66} = \frac{K}{s^2+2\xi w_n s + w_n^2}$$

$w_n = 8,124$ e $\xi = 0,308$

Exercício

Para o sistema a seguir, desenhe o LR determinando os ptos de saída e chegada no eixo real

$$G(s) = \frac{K(s-3)(s-5)}{(s+2)(s+1)}$$



Critério de módulo

$$F(s) = \frac{KG(s)}{1+KG(s)H(s)}$$

$$1+KG(s)H(s) = 0$$

$$K = \frac{-1}{G(s)H(s)} \Rightarrow K = \frac{-(s+2)(s+1)}{(s-3)(s-5)}$$

Queremos saber o cruzamento com o eixo real $\therefore s = \sigma + j0$

$$K = \frac{-(\sigma+2)(\sigma+1)}{(\sigma-3)(\sigma-5)} = -\frac{\sigma^2+3\sigma+2}{\sigma^2-8\sigma+15}$$

$$\frac{dK}{d\sigma} = -\left(\frac{(2\sigma+3)(\sigma^2-8\sigma+15)-(\sigma^2+3\sigma+2)(2\sigma^2-8)}{(\sigma^2-8\sigma+15)^2}\right) = 0$$

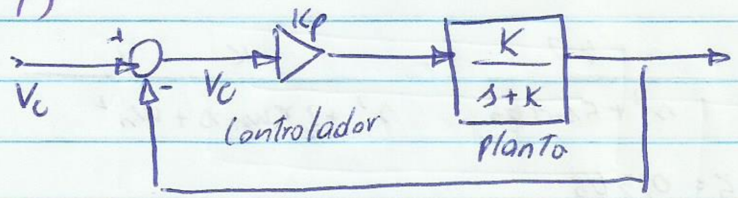
$$11\sigma^2 - 26\sigma - 61 = 0 \quad \begin{cases} \sigma_1 = 3,81 \\ \sigma_2 = -1,454 \end{cases}$$

I love you amor da minha vida ♥

S T Q Q S S D
M T W T F S S

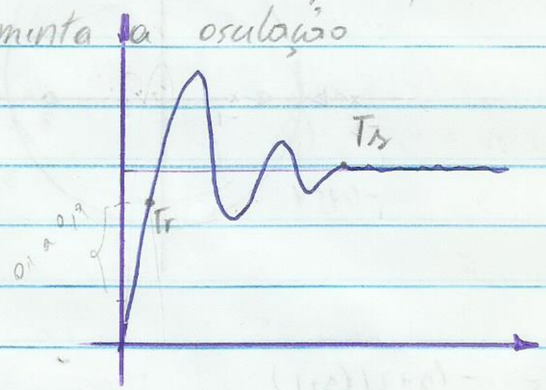
Prova laboratório

Questão 1)



Controlador Proporcional

- É a adição de um ganho para amplificar o erro
- Menor constante de tempo do sistema
- Resposta do sistema mais rápida
- Aumento do overshoot
- Menor erro do regime permanente
- Aumenta a oscilação



↑ Ganho ⇒ aumenta a precisão (Diminui o erro estático).
⇒ menor constante de tempo.
⇒ mais rápida a resposta.
⇒ maior esforço de controle.

I love you my love of my life

S	T	Q	Q	S	S	D
M	T	W	T	F	S	S

Exercícios Propostos

Exemplo 6.5

$$T(s) = \frac{20}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20}$$

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	-1	-2	1	2	0
s^5	1	3	2	0	0
(Par) s^4	1	3	2	0	0
s^3	2	3	0	0	0
s^2	3	4	0	0	0
s^1	1/3	0	0	0	0
s^0	4	0	0	0	0

$\rightarrow P(s) = s^4 + 3s^2 + 2$

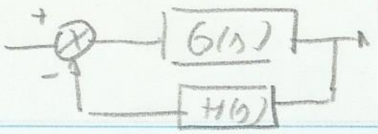
$\frac{dP(s)}{ds} = 4s^3 + 6s + 0$

Polinômio par: Surge na linha imediatamente acima da linha de zeros.

Numero de raízes do semiplano da direita: Número de mudanças de sinal desde a linha do polinômio par até o final da tabela é igual ao número de raízes no semiplano da direita do polinômio par.

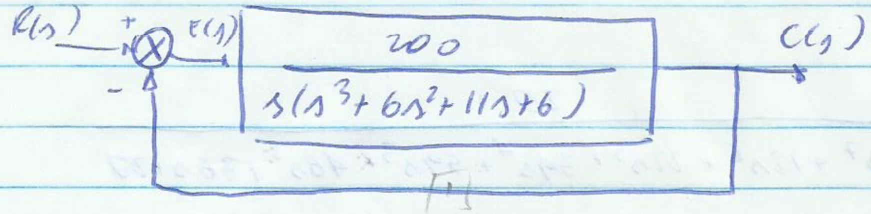
Simetria das raízes: O numero de raízes do polinômio par no semiplano da esquerda deve ser idêntico ao número de raízes do polinômio no semiplano da direita.

	Par	Resto	total
Semiplano da direita	0	2	2
Semiplano da esquerda	0	2	2
$j\omega$	4	0	4



$$\frac{G(s)}{1+GH}$$

Exemplo 6.6



$$T(s) = \frac{200}{s^4 + 6s^3 + 11s^2 + 6s + 200}$$

s^4	1	11	200
s^3	$\cancel{6}$ 1	$\cancel{6}$ 1	0
s^2	$\cancel{10}$ 1	$\cancel{200}$ 20	0
s^1	−19	0	0
s^0	20	0	0

2 raízes pólo esquerdo e 2 raízes no pólo direito

Exemplo 6.7

$$T(s) = \frac{1}{2s^5 + 3s^4 + 2s^3 + 3s^2 + 2s + 1}$$

				$\epsilon +$	$\epsilon -$
s^5	2	2	2	+	+
s^4	3	3	1	+	+
s^3	$\cancel{0}\,\epsilon$	$4/3$	0	+	−
s^2	$\frac{3\epsilon-4}{\epsilon}$	1	0	+	+
s^1	$\frac{3\epsilon^2-12\epsilon+12}{9\epsilon-12}$	0	0	+	−
s^0	1			+	+

Exemplo 6.8

$$T(s) = \frac{128}{s^8 + 3s^7 + 10s^6 + 24s^5 + 48s^4 + 96s^3 + 128s^2 + 192s + 128}$$

s^8	1	10	48	128	128	
s^7	$\cancel{3}\ 1$	$\cancel{24}\ 8$	$\cancel{96}\ 32$	$\cancel{192}\ 64$	0	
s^6	$\cancel{2}\ 1$	$\cancel{16}\ 8$	$\cancel{64}\ 32$	$\cancel{128}\ 64$	0	
s^5	$\cancel{0}\ \cancel{6}\ 3$	$\cancel{0}\ \cancel{32}\ 16$	$\cancel{0}\ \cancel{64}\ 32$	0	0	(Par)
s^4	$\cancel{8/3}\ 1$	$\cancel{64/3}\ 8$	$\cancel{64}\ 24$	0	0	
s^3	$\cancel{-8}\ -1$	$\cancel{-40}\ -5$	0	0	0	
s^2	$\cancel{3}\ 1$	$\cancel{24}\ 8$	0	0	0	
s^1	3	0	0	0	0	
s^0	8					

$P(s) = s^6 + 8s^4 + 32s^2 + 64$

$\dfrac{dP(s)}{ds} = 6s^5 + 32s^3 + 64$

Lista Base 1)

s^3	1	31
s^2	$\cancel{10}\ 1$	$\cancel{1030}\ 103$
s^1	-72	0
s^0	103	0

Instável

S T Q Q S S D

M T W T F S S

b)

s^5	1	3	5
s^4	2	6	3
s^3	$\cancel{0}$ ϵ	7/2	0
s^2	$\frac{-7+6\epsilon}{\epsilon}$	3	ϵ
s^1	$\frac{6\epsilon^2-49-12\epsilon}{-14\epsilon}$	0	0
s^0	3		

(Prova)

$$T(s) = \frac{K(s+1)}{s^3+4s^2+(K-5)s+K}$$

s^3	1	$K-5$
s^2	4	K
s^1	$\frac{-5+3K}{4}$	0
s^0	K	

$$\begin{cases} K > 0 \\ \frac{-5+3K}{4} > 0 \end{cases} \therefore \quad K > 5/3$$

	P	V	A
0	$\frac{1}{1+K_g}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$

Capítulo - Realimentação

Erro de estado estacionário

Tipos de sinal de entrada

step	posição constante	1	$1/s$
rampa	velocidade constante	t	$1/s^2$
parabola	aceleração constante	$\frac{t^2}{2}$	$1/s^3$

Tipo

	0	1	2
step	$\frac{1}{1+Kp}$	∞	∞
rampa	0	$\frac{1}{Kv}$	∞
parabola	0	0	$\frac{1}{Ka}$

$$e(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)}$$

$$e(\infty) = \frac{1}{\lim_{s\to 0} s\,G(s)}$$

$$e(\infty) = \frac{1}{1 + \lim_{s\to 0} s^2 G(s)}$$

Exemplo

	Posição	Velocidade	Aceleração
0	$\frac{1}{1+Kg}$	∞	∞
1	0	$\frac{1}{Kv}$	∞
2	0	0	$\frac{1}{Ka}$

$$G(s) = \frac{10(s+20)(s+30)}{s(s+25)(s+35)}$$

$15u(t)$, $15t\,u(t)$, $15t^2u(t)$

$e_{degrau}(\infty) = 0$

$e_{rampa}(\infty) = 2,1875$

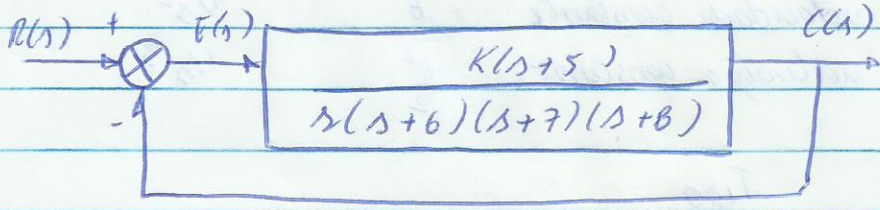
$e_{parabola}(\infty) = \infty$

	P	V	A
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$

Problema $\quad G(s) = \dfrac{1000(s+8)}{s^0(s+7)(s+9)}$

tipo 0 , $K_p = 126{,}98 \quad K_v = 0 \quad K_a = 0$

Exemplo 7.6



Tipo 1 $\quad$ Para $e = 0{,}1$

$$e(\infty) = \frac{1}{K_v}$$

	P	V	A
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$

$$e(\infty) = \frac{1}{1 + \lim\limits_{s\to 0} s\,G(s)} \rightarrow K_v$$

$$10 = \lim_{s\to 0} s\,G(s)$$

$$K_v = \frac{1}{e(\infty)} = \frac{1}{0{,}1} = 10$$

$$\lim_{s\to 0} \frac{\cancel{s}\,K(s+5)}{\cancel{s}(s+6)(s+7)(s+8)} = \frac{5K}{336} = 10 \;\therefore\; K = 672$$

Ex 7.3

$e(\infty) = 0{,}1$, tipo 0

$$e(\infty) = \frac{1}{1+K_g} = \frac{1}{1 + \lim\limits_{s\to 0} G(s)} = 0{,}1$$

$0{,}1(1 + K_g) = 1$

$0{,}1 + 0{,}1K_g = 1 \qquad \lim\limits_{s\to 0} \dfrac{K(s+12)}{(s+14)(s+18)} = \dfrac{12K}{252} = 9 \;\therefore\; K = 189$

$\therefore K_g = 9$

Construção do LR

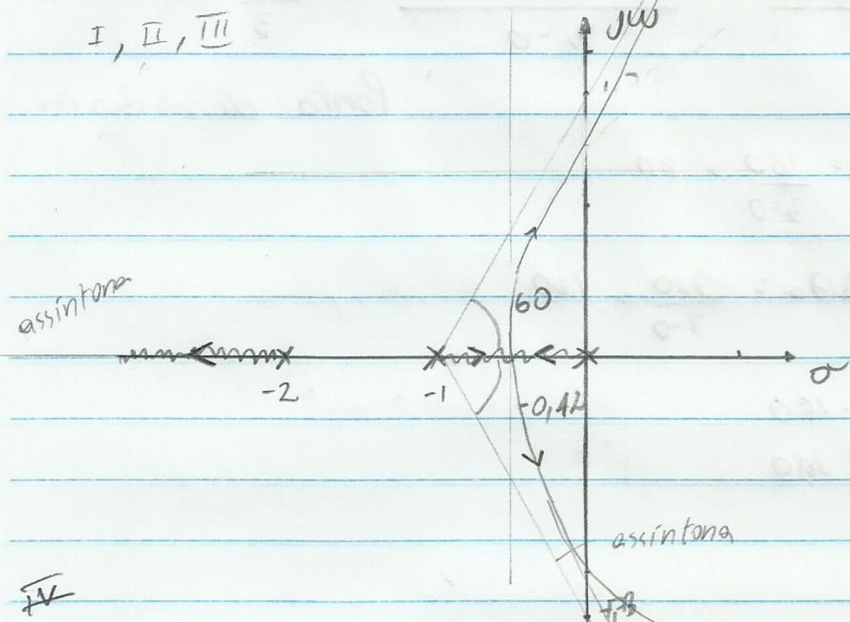
$A(s) = G(s) = \dfrac{K}{s(s+1)(s+2)}$ Assíntota

Função de Transferência

Zero (o)

pólo (x)

I, II, III



IV

número de ramos para o infinito $p - z = 3 - 0 = \underline{3}$

V

$1 + KGH = 0$; $KGH = A(s)$ $\quad \dfrac{K}{s(s+1)(s+2)} = -1$

$1 + A(s) = 0$

$A(s) = -1$

Analisando para o eixo real

$\sigma(\sigma^2 + 3\sigma + 2) = \sigma^3 + 3\sigma^2 + 2\sigma$

$\therefore \quad K = -\sigma^3 - 3\sigma^2 - 2\sigma$

$\dfrac{dK}{d\sigma} = -3\sigma^2 - 6\sigma - 2 = 0 \qquad \sigma = \dfrac{6 \pm \sqrt{36-24}}{-6}$ $\quad \sigma_1 = \cancel{-1{,}58}$; $\sigma_2 = -0{,}42$

12	2
6	2
3	3
1	$2^2\ 3$

$\sqrt{12} = 2\sqrt{3}$ $\qquad \sigma = \dfrac{6 \pm 2\sqrt{3}}{-6}$

S T Q Q S S D
M T W T F S S

VI - Comportamento no infinito

$$\sigma_a = \frac{\Sigma\, polos - \Sigma\, zeros}{p-z} = \frac{(0-1-2)-0}{3-0} = \frac{-3}{3} = -1$$

Ponto de radiação

1ª assíntota: $\theta_a = \frac{180}{3-0} = 60$

Entre assíntota $\Delta\theta_a = \frac{360}{3-0} = 120$

$\theta_{a2} = 60+120 = 180$

$\theta_{a3} = 180+120 = 300$

VII - Routh

$$T(s) = \frac{K}{s(s+1)(s+2)} \div \left(1 + \frac{K}{s(s+1)(s+2)}\right)$$

$$= \frac{K}{\cancel{s(s+1)(s+2)}} \div \frac{s(s+1)(s+2)+K}{\cancel{s(s+1)(s+2)}}$$

$$T(s) = \frac{K}{s(s+1)(s+2)+K} = \frac{K}{s^3+3s^2+2s+K}$$

s^3	1	2	0
s^2	3	K	0
s^1	$\frac{6-K}{3}$	0	
s^0	K		

Para $K=6$ linha nula

$P(s) = 3s^2 + K = 0$, Para $K=6$

$P(s) = 3s^2 + 6 = 0 \quad \therefore s = \pm\sqrt{-2} = s = \pm j\sqrt{2}$

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Exercício 12)

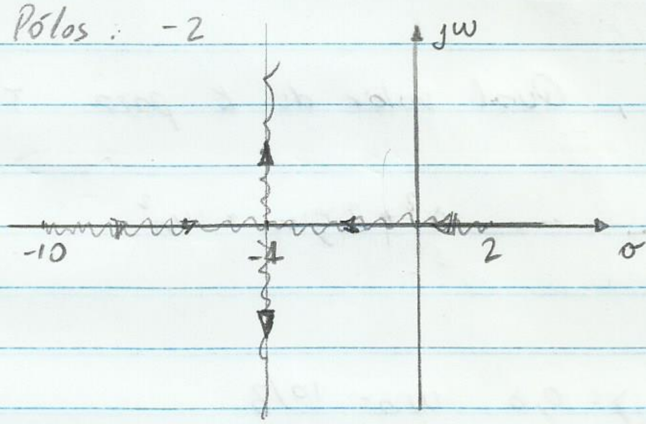
(a) $A(s) = \dfrac{K}{s^2+8s-20} = \dfrac{K}{(s-2)(s+10)}$

$s^2+8s-20=0$ $\qquad$ $s^2+8s-20=(s-2)(s+10)$

$s = \dfrac{-8 \pm \sqrt{8^2+4\times 20}}{2}$ $<$ $s_1 = 2$, $s_2 = -10$

I-) Pólos e Zeros em malha aberta

Pólos : -2



Ramos para o infinito $\quad p-z = 2-0 = 2$

$1 + \underbrace{KGH}_{A} = 0 \quad \therefore \quad A(s) = -1$

$\dfrac{K}{s^2+8s-20} = -1$; Para o eixo real $s=\sigma$

$\dfrac{K}{\sigma^2+8\sigma-20} = -1 \qquad -\sigma^2-8s+20 = K$

$\dfrac{dK}{d\sigma} = -2\sigma - 8 = 0 \quad \therefore \quad \sigma = \dfrac{8}{-2} = -4$

$K(\sigma) = -(\sigma^2+8\sigma-20)$

$K(-4) = 36$ (Ponto de separação)

Comportamento para o infinito

$\sigma_1 = \dfrac{(-10+2)-0}{2-0} = \dfrac{-8}{2} = -4$ (Ponto de irradiação)

1ª assint: $\quad \theta a = \dfrac{180}{2-0} = 90 \qquad \Delta\theta a = \dfrac{360}{2-0} = 180$

2ª assint $\quad \theta a_2 = \theta a_1 + \Delta\theta a = 270$

$$T(s) = \frac{K}{s^2+8s-20+K}$$

s^2	1	$-20+K$
s^1	8	0
s	$-20+K$	

$-20+K > 0$

$K > 20$

(b) $A(s) = \dfrac{K}{s^2+4s+8}$, Qual valor de K para $\xi = 0{,}6$

$$T(s) = \frac{K}{s^2+4s+8+K} \qquad s^2 + 2\xi w_n + w_n^2$$

$2\xi w_n = 4$

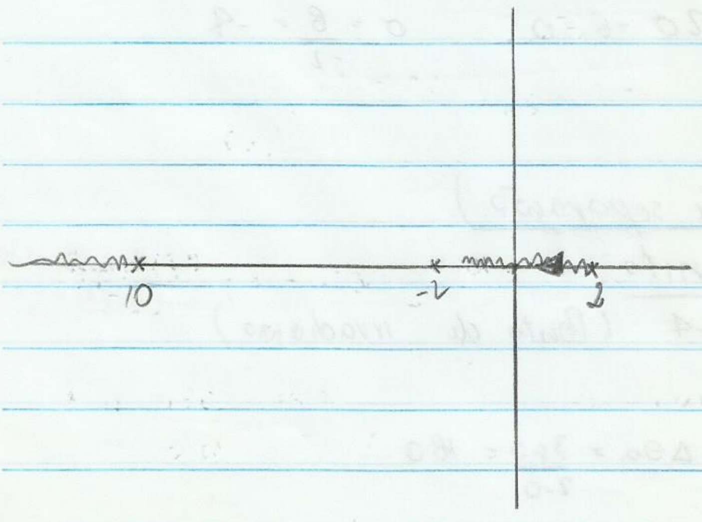
$8+K = w_n^2$ Para $\xi = 0{,}6$ $w_n = 10/3$

$\therefore$ $K = w_n^2 - 8$ $\therefore$ $K = 3{,}11$

(c) $A(s) = \dfrac{K(s+2)}{s^2+8s-20}$; Qual o valor de K para um polo na posição -1

$$T(s) = \frac{K(s+2)}{s^2+8s-20+K(s+2)} = \frac{K(s+2)}{s^2+(8+K)s+(2K-20)}$$

$(s^2+8s-20) = (s-2)(s+10)$



Ramos para o infinito

$1 + \underbrace{KGH}_{A} = 0 \quad \therefore \quad A(s) = -1$

$$\frac{K(s+2)}{s^2+8s-20} = -1$$

$K(s+2) = -s^2 - 8s + 20$ para $s = -1$

$K = 27$

$27(s+2) = -s^2 - 8s + 20$

$27s + 54 = -s^2 - 8s + 20$

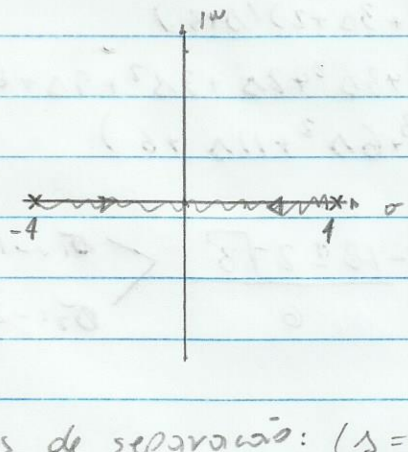
$s^2 + 35s + 34 = 0$

$$s = \frac{-35 \pm \sqrt{35^2 - 4 \times 34}}{2} \quad \begin{cases} s = -1 \\ s = -34 \end{cases}$$

(d) $A(s) = \dfrac{K(s+6)}{s^2-16}$, Determine K para - Origem

- Pontos de separação

$(s-4)(s+4)$ Qual o valor para maior wd



$1 + \underbrace{KGH}_{A(s)} = 0 \quad \therefore \quad A(s) = -1$

$$\frac{K(s+6)}{s^2-16} = -1$$

$K(s+6) = -(s^2-16)$ Para $s = 0 + 0j$

$$K = \frac{-(0^2-16)}{(0+6)} = 2{,}67$$

Pontos de separação: $(s = \sigma)$

$$K = \frac{-(\sigma^2-16)}{\sigma+6} = \frac{-\sigma^2+16}{\sigma+6}$$

$$\frac{dK}{d\sigma} = \frac{-2\sigma(\sigma+6) - (-\sigma^2+16)}{(\sigma+6)^2} = 0$$

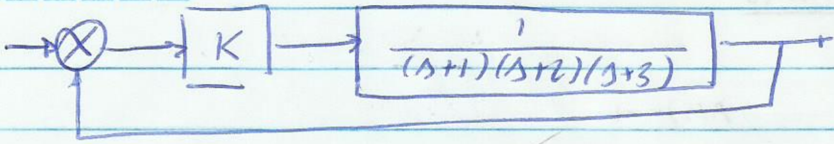
$-2\sigma^2 - 12\sigma + \sigma^2 - 16 = 0 \quad \therefore \quad -\sigma^2 - 12\sigma - 16 = 0$

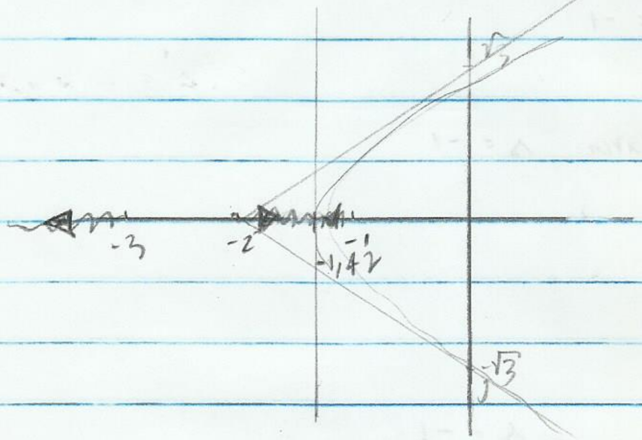
$\sigma^2 + 12\sigma + 16 = 0$

$$\sigma = \frac{-12 \pm \sqrt{12^2 - 4 \times 16}}{2}$$

$\sigma_1 = -1{,}528 \rightarrow K = 3{,}06$

$\sigma_2 = -10{,}47 \rightarrow$





número de ramos $3-0=3$

$\underbrace{1+KGH}_{A}=0 \qquad 1+A(s)=0 \qquad A(s)=-1$

$$\frac{K}{(s+1)(s+2)(s+3)}=-1 \quad \therefore \quad K=-(s+1)(s+2)(s+3)$$
$$=-(s^2+3s+2)(s+3)$$
$$=-(s^3+3s^2+2s+3s^2+9s+6)$$
$$=-(s^3+6s^2+11s+6)$$

$$\frac{dK}{ds}=-(3s^2+12s+11)=0$$

$$s=\frac{-12\pm\sqrt{12^2-4\times3\times11}}{2\times3}=\frac{-12\pm2\sqrt{3}}{6}\quad \begin{cases} s_1=-1{,}42 \\ s_2=\cancel{-2{,}58} \end{cases}$$

$K(-1{,}42)=0{,}38$

$$\sigma_1=\frac{\Sigma\,polos-\Sigma\,zeros}{p-z}=\frac{-3-2-1-0}{3-0}=-2 \text{ (Ponto de irradiação)}$$

$$\theta_{a1}=\frac{180}{3-0}=60 \qquad \theta_{a2}=60+120=180$$
$$\theta_{a3}=180+120=300$$

$$\Delta\theta_{a1}=\frac{360}{3-0}=120$$

S T Q Q S S D
M T W T F S S

$$T(s) = \frac{K}{s^3+6s^2+11s+6+K}$$

$$\begin{array}{c|ccc} s^3 & 1 & 11 & 0 \\ s^2 & 6 & 6+K & 0 \\ s^1 & \frac{60-K}{6} & 0 & \\ s^0 & 6+K & & \end{array}$$

$6+K>0 \therefore K>-6$

$\frac{60-K}{6} > 0 \therefore K < 60$

$\therefore -6 < K < 60$

$$\frac{(6+K)-6\times 11}{-6} = \frac{6+K-66}{-6} = \frac{K-60}{-6} = \frac{60-K}{6}$$

$$\frac{-(6+K)\times\left(\cancel{\frac{60-K}{6}}\right)}{-\left(\cancel{\frac{60-K}{6}}\right)} = 6+K$$

Para linha s_1 se $K=60$ (linha 0)

$6s^2+6+K=0$, $K=60$

$6s^2+66=0 \therefore s = \pm j\sqrt{60}$

Para $F(s) = \frac{N(s)}{s^3+3s^2+(K+2)s+10K}$

$F(s) = \frac{6}{1+GH} =$

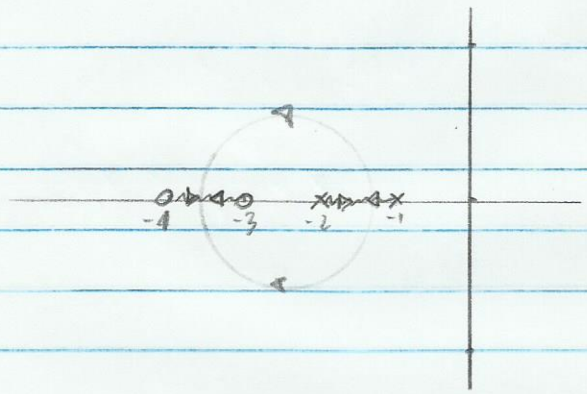
kajoma

wherever you are, whatever you are doing...
Just know that I love you!

S T Q Q S S D
M T W T F S S

Para $G(s)H(s) = \dfrac{K}{s(s+2)^2(s+4)}$

Para $G(s) = \dfrac{(s+3)(s+4)}{(s+1)(s+2)}$ Polos: $-1, -2$
Zeros: $-3, -4$



$p - z = 2 - 2 = 0$ (Raios que tende ao infinito)

$A(s) = -1$

$K \dfrac{(s+3)(s+4)}{(s+1)(s+2)} = -1$ $\quad K = \dfrac{-(s+1)(s+2)}{(s+3)(s+4)} = \dfrac{-(s^2+3s+2)}{(s^2+7s+12)}$

Para $s = \sigma$ $\therefore$ $K = \dfrac{-(\sigma^2+3\sigma+2)}{\sigma^2+7\sigma+12}$

$$\frac{dK}{d\sigma} = -\left[\frac{(2\sigma+3)(\sigma^2+7\sigma+12) - (\sigma^2+3\sigma+2)(2\sigma+7)}{(\sigma^2+7\sigma+12)^2}\right] = 0$$

$= 2\sigma^3 + 14\sigma^2 + 24\sigma + 3\sigma^2 + 21\sigma + 36 - (2\sigma^3 + 6\sigma^2 + 4\sigma + 7\sigma^2 + 21\sigma + 14)$

$= \cancel{2\sigma^3} - \cancel{2\sigma^3} + 14\sigma^2 + 3\sigma^2 - 6\sigma^2 - 7\sigma^2 + 24\sigma + 21\sigma - 4\sigma - 21\sigma + 36 - 14$

$= 7\sigma^2 + 20\sigma + 22 = 0$

$\sigma = \dfrac{-20 \pm \sqrt{20^2 - 4 \times 7 \times 22}}{2 \times 7}$ $\quad \sigma_1 =$
$\quad \sigma_2 =$

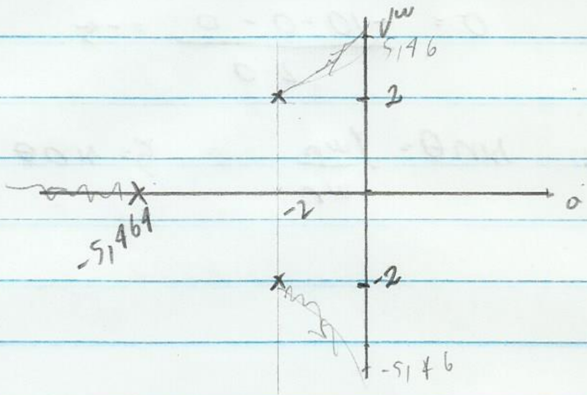
kajoma

$$A(s) = \frac{K}{(s+5,464)(s^2+4s+8)}$$

$s^2+4s+8=0$

$$s = \frac{-4 \pm \sqrt{4^2-4\times 8}}{2} = \frac{-4 \pm j\sqrt{14}}{2} \therefore s = -2 \pm j2$$

$$A(s) = \frac{K}{(s+5,464)(s+2-j2)(s+2+j2)}$$



$$\sigma = \frac{-5,464-2-2-j2+j2}{3-0} = -3,15$$

$$F(s) = \frac{K}{(s+5,464)(s^2+4s+8)+K} =$$

$= s^3+9,464s^2+29,856s+43,712+K$

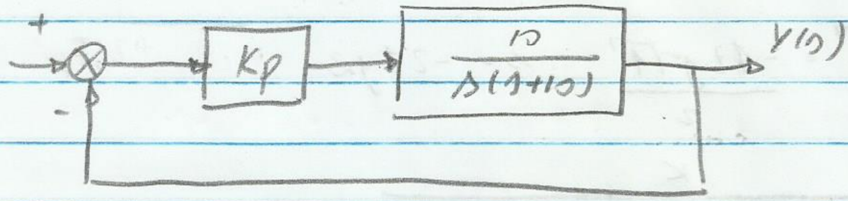
s^3	1	29,856	0
s^2	9,464	43,712+K	0
s^1	A	0	
s^0	43,712+K		

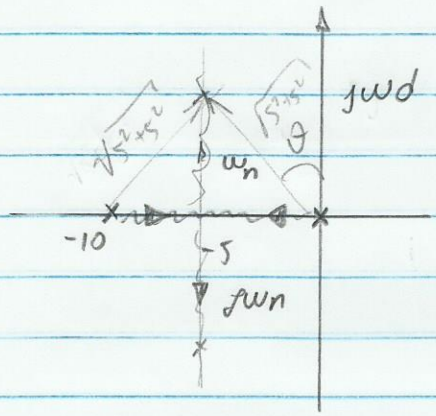
$$\frac{43,712+K-282,557}{-9,464} = A \qquad \text{Para } K = 238,85 \text{ (Zera Linha)}$$

$D(s) = 9,464s^2 + 43,712+K = 0 \quad \text{Para } K = 238,85$

$s = \pm 5,46j$

Controlador Proporcional P





$$\sigma = \frac{-10-0-0}{2-0} = -5$$

$$\operatorname{sen}\theta = \frac{\xi w_n}{w_n} \quad \therefore \quad \xi = \operatorname{sen}\theta$$

Para $\xi = 0{,}707 \therefore \theta = 45 \therefore jwd = j5$

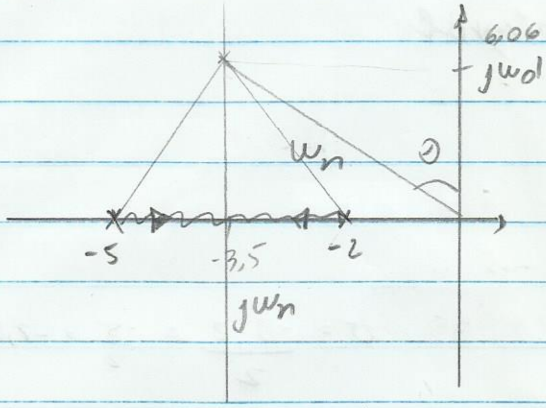
Critério de módulo

$$K = \frac{\Pi \text{ comprimento dos pólos}}{\Pi \text{ comprimento dos zeros}}$$

$$K = \sqrt{5^2+5^2} \cdot \sqrt{5^2+5^2} = 50$$

$10K_p = K \quad \therefore \quad K_p = 5$

Exercício 1

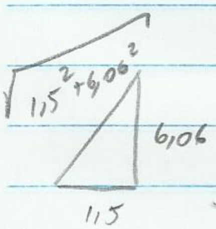


$G(s) = \dfrac{10}{(s+2)(s+5)}$

$\sigma = \dfrac{-5-2}{2-0} = -3,5$

Para $\zeta = 0,5$ $\quad \sin\theta = \dfrac{j\omega_n}{\omega_n}$ $\quad \therefore \theta = 30°$

$tg\,\theta = \dfrac{3,5}{\omega d}$ $\quad \therefore \omega d = 6,06$



$\therefore K = \dfrac{\prod \text{dist Polos}}{\prod \text{dist Zeros}} = \sqrt{1,5^2+6,06^2} \cdot \sqrt{1,5^2+6,06^2}$

$\therefore K = 39$

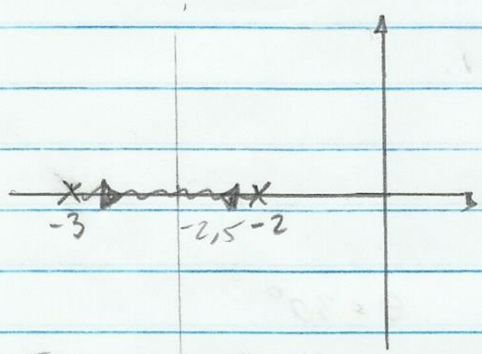
$K = 10\,Kp \quad \therefore \quad Kp = 3,9$

$F(s) = \dfrac{39}{s^2+7s+10+39} \quad \therefore \quad F(s) = \dfrac{39}{s^2+7s+49}$

S T Q Q S S D
M T W T F S S

Controlador Proporcional Integral

$$G(s) = \frac{60}{(s+2)(s+3)}$$



$$\sigma = \frac{-3-2}{2} = \frac{-5}{2} = -2{,}5$$

	P	V	A
0	$\frac{1}{1+Kg}$	∞	∞
1	0	$\frac{1}{Kv}$	∞
2	0	0	$\frac{1}{Ka}$

1º) Erro atual

Aplicando um degrau

$$e(\infty) = \frac{1}{1+\frac{60}{6}} = \frac{1}{1+10} \therefore e(\infty) = 0{,}091$$

$Kg = 10$

2º) ξ sem compensadores

$$F(s) = \frac{60}{s^2+5s+6+60} \therefore F(s) = \frac{60}{s^2+5s+66}$$

$s^2 + 2\xi w_n s + w_n^2$

$s = -2{,}5 \pm j\,7{,}73$

$2\xi w_n = 5$ $\therefore$ $\xi = 0{,}308$

$w_n^2 = 66$

3º) Projetar compensador sem alterar ξ

$$G_c(s) = K_p + \frac{K_i}{s} = \frac{sK_p + K_i}{s} = K_p\frac{(s+z_i)}{s}$$

kajoma

Para $z_1 = 0{,}1$

$$G_c(s) = \frac{K_p(s+0{,}1)}{s} \qquad \therefore K_i = 0{,}1K_p$$

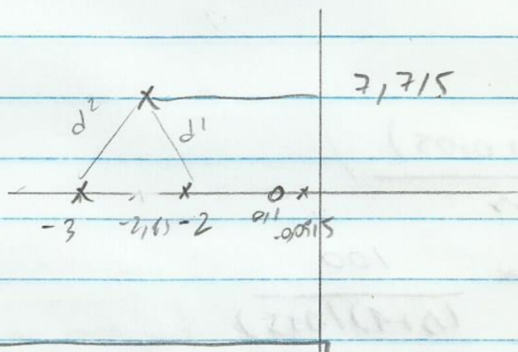
$$G_c(s)\,G(s) = \frac{K_p(s+0{,}1)}{s} \cdot \frac{60}{(s+2)(s+3)} =$$

$$F(s) = \frac{60(s+0{,}1)}{s(s^2+5s+6)+60(s+0{,}1)}$$

$$F(s) = \frac{60(s+0{,}1)}{s^3+5s^2+66s+6}$$

$s_1 = -0{,}0915$

$s_{1,2} = -2{,}45 \pm 7{,}715$



$$d_2 = \sqrt{(3-2{,}45)^2 + 7{,}715^2} = 7{,}735$$
$$d_1 = \sqrt{(2{,}45-2)^2 + 7{,}715^2} = 7{,}728$$
$$d_0 = \sqrt{(2{,}45-0{,}1)^2 +}$$

$$\therefore K = 7{,}735 \times 7{,}728 \quad \therefore K = 59{,}77$$

$$K_p = \frac{59{,}77}{60} = 0{,}996$$

	P	V	A
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$

S	T	Q	Q	S	S	D
M	T	W	T	F	S	S

2) $G(s) = \dfrac{100}{(s+4)(s+5)}$

erro de posição

$$e(\infty) = \frac{1}{1+\frac{100}{20}} = \frac{1}{1+5} = \frac{1}{6} = 0{,}167$$

K_p

ζ sem compensador

$s^2 + 9s + 20$

$2\zeta w_n = 9 \quad \therefore \quad \zeta = 1{,}006$

$w_n^2 = 20$

Compensador

$$G_c = \frac{K_p(s+z_i)}{s} = \frac{K_p(s+0{,}05)}{s}$$

$$G_c G_c = \frac{K_p(s+0{,}05)}{s} \times \frac{100}{(s+4)(s+5)}$$

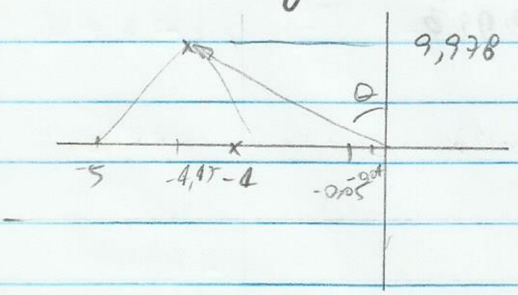
$$F(s) = \frac{(s+0{,}05)100}{s^3+9s^2+20s+100s+5} = \frac{100s+5}{s^3+9s^2+120s+5}$$

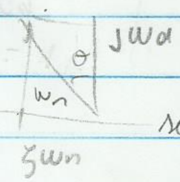
$s_1 = -0{,}041797$

$s_2 = -4{,}479 \pm j9{,}978$

$d_1 = \sqrt{(5-4{,}478)^2 + 9{,}978^2} = 9{,}99$

$d_2 =$





$\sin\theta = \dfrac{\zeta w_n}{w_n}$

$$\theta = tg^{-1}\left(\frac{4{,}479}{9{,}978}\right) = 0{,}42$$

$\sin 0{,}42 = \zeta \quad \therefore \quad \zeta = 0{,}4095$

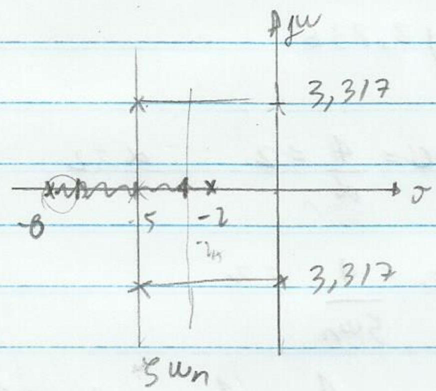
kajoma

Controlador de atraso de fase (LAG)

$$G(s) = \frac{20}{(s+2)(s+8)} = \frac{20}{s^2+10s+16}$$

$$t_p = \frac{\pi}{\omega_d} \qquad \tau = \frac{1}{\alpha}$$

$$F(s) = \frac{20}{s^2+10s+36} \rightarrow s_{1,2} = -5 \pm j\,3{,}3166$$



Para $2t_p$ então $\frac{\xi\omega_n}{2} = \frac{-5}{2} = -2{,}5$

$(\xi\omega_n)_{novo} = -2{,}5$

Novo denominador $(s+2{,}5+j3{,}317)(s+2{,}5-j\,3{,}317)$

$= (s+2{,}5)^2 + 3{,}317^2$

$= s^2 + 5s + 17{,}25$

$$F'(s) = \frac{K}{(s+2)(s+3)+K} = \frac{K}{s^2+5s+6+K} =$$

$\therefore 6 + K = 17{,}25 \therefore K = 11{,}25$

$$\therefore G_c(s)\,G(s) = \frac{11{,}25}{20}\frac{(\cancel{s+8})}{(s+3)} \times \frac{20}{(s+2)\cancel{(s+8)}} = \frac{11{,}25}{(s+3)(s+2)}$$

$20\,K_c = 11{,}25$

$\therefore K_c = 0{,}563$

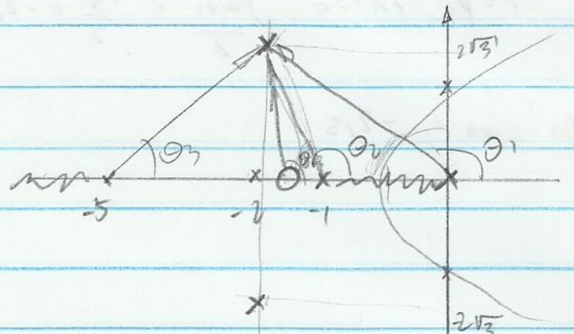
Controlador Proporcional Derivativo (PD)

$G_c(s) = K_c\,(s+b)$ $\qquad \xi = 0{,}5 \quad t_{ac} = 2s$

$G(s) = \dfrac{30}{s(s+1)(s+5)} = \dfrac{30}{s^3+6s^2+5s}$

$F(s) = \dfrac{30}{s^3+6s^2+5s+30}$ $\qquad s_1 = -6$

$s_{2,3} = \pm j\,2{,}236$



$t_{ac} = \dfrac{4}{\alpha} = 2 \quad \therefore \alpha = 2$

$t_{ac} = \dfrac{4}{\xi\omega_n} = 2$

$\therefore \omega_n = \dfrac{4}{\xi 2} = \dfrac{4}{0{,}5\times 2} \quad \therefore \omega_n = 4$

$\omega_d = \sqrt{\omega_n^2 - \alpha^2}$

logo Polos dominantes

$s_{1,2} = -2 \pm j\,2\sqrt{3}$

$\Sigma\theta_z - \Sigma\theta_p = -180$

$\theta_4 - \theta_1 - \theta_2 - \theta_3 = -180$

$\theta_4 - \left(180 - tg^{-1}\left(\dfrac{2\sqrt{3}}{2}\right)\right) - \left(180 - tg^{-1}\left(\dfrac{2\sqrt{3}}{1}\right)\right) - tg^{-1}\left(\dfrac{2\sqrt{3}}{3}\right) = -180$

$\theta_4 - 120 - 106{,}102 - 49{,}106 = -180$

$\therefore \theta_4 = 95{,}21$

$180 - \theta_4 = tg^{-1}\left(\dfrac{2\sqrt{3}}{x}\right) \quad \therefore x = 0{,}3158$

$\therefore\ -2 + 0{,}3158 = -1{,}68$ (Posição do zero)

Exercício 4

$$G(s) = \frac{100}{s(s+5)(s+8)}$$

$T_{ac} = 1s$

$\xi = 0,5$

$$F(s) = \frac{100}{s^3 + 13s^2 + 40s + 100}$$

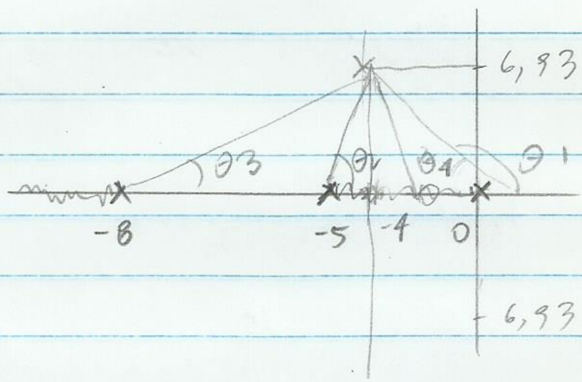
$T_{ac} = \frac{4}{\xi w_n}$; $\xi = 0,5$ e $T_{ac} = 1$ $\therefore$ $w_n = 8$ rad/s

σ

$\therefore$ $\alpha = \frac{4}{1} = 4$

$W_d = \sqrt{8^2 - 4^2} = \sqrt{48}$

Pólos dominantes $-4 \pm \sqrt{48}$



$$\theta_4 - \theta_1 - \theta_2 - \theta_3 = -180$$

$$\theta_4 - \left(180 - tg^{-1}\left(\frac{\sqrt{48}}{4}\right)\right) - tg^{-1}\left(\frac{\sqrt{48}}{1}\right) - tg^{-1}\left(\frac{\sqrt{48}}{3}\right) = -180$$

$\theta_4 - 120 - 81,79 - 66,59 = -180$ $\therefore$ $\theta_4 = 88,37$

$180 - \theta_4 = tg^{-1}\left(\frac{\sqrt{48}}{x}\right)$ $\therefore$ $x =$

$(s+1)^2(s+1)$

$(s^2+2s+1)s + (s^2+2s+1)$

S T Q Q S S D

M T W T F S S

$s^3+2s^2+s + s^2+2s+1$

s^3+3s^2+3s+1

$K_c \dfrac{(s+b)}{(s+a)}$ $\qquad \dfrac{K_{pN}}{K_{pD}} = \dfrac{z_c}{p_c} = \dfrac{b}{a}$

$G(s) = \dfrac{20}{(s+2)(s+8)}$ $\qquad t_p = \dfrac{\pi}{\omega_d} \qquad \tau = \dfrac{1}{a}$

$\dfrac{K_{pn}}{K_{po}} = \dfrac{b}{a} = 0$

kajoma