Circuitos III

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Capítulos: 16, 17, 13, 15

Cap. 16 → Transformada de Laplace
Cap 17 → Aplicação T. Laplace em circuitos
Cap. 13 → Redes magnéticas. Mutua
Cap. 15 → Analise de quadripolos.

Transformada de Laplace

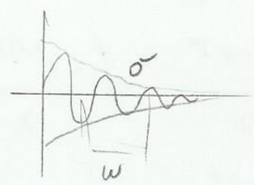
A transformada de Laplace, transforma o regime do domínio do tempo (t) para o domínio da frequência (s)

$$\boxed{\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt}$$

Onde $s = \sigma + j\omega$ $\begin{cases} \sigma < 0 \\ \sigma = 0 \\ \sigma > 0 \end{cases}$

s: é uma variável complexa

$\sigma < 0$ significa



Primitivas básicas. $f(t) \to F(s)$

1) Exemplo com $\delta(t-t_0)$ – Propriedade da amostragem

$f(t) = t e^{-at} \delta(t-1)$

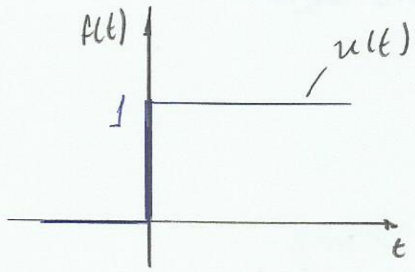
Basta substituir o valor de t por to

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} t e^{-at} \delta(t-1) e^{-st} dt$$

$$= \int_0^{\infty} t e^{-(s+a)t} \delta(t-1)\, dt$$

$$\mathcal{L}[f(t)] = F(s) = e^{-(s+a)}$$

2-) Função degrau: $u(t) \begin{cases} 0 \; ; & t<0 \\ 1 \; , & t \geq 0 \end{cases}$



$$\mathcal{L}[u(t)] = \int_0^{\infty} 1\, u(t)\, e^{-st}\, dt$$

$$\mathcal{L}[u(t)] = \int_0^{\infty} e^{-st}\, dt$$

$$\mathcal{L}[u(t)] = \int_0^{\infty} \frac{\overset{u}{\widetilde{e^{-st}}}}{-s}\, du = -\frac{e^{-st}}{s}\Bigg|_0^{\infty} = \frac{1}{s} \quad \therefore \quad \boxed{F(s) = \frac{1}{s}}$$

3.) $f(t) = t$ Determinar $F(s)$

$$\int u\, dv = uv - \int v\, du$$

$$\mathcal{L}[f(t)] = \int_0^{\infty} t\, e^{-st}\, dt$$

$$\text{para} \begin{cases} t = u & dt = du \\ v = \dfrac{-e^{-st}}{s} & dv = e^{-st}\, dt \end{cases}$$

$f(t) = t\, u(t)$

O $u(t)$ significa que é de 0 a ∞

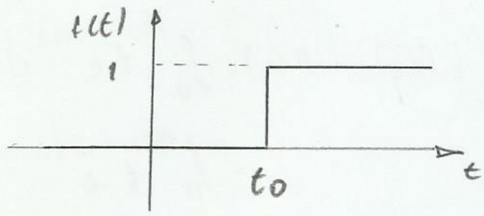
$$\int_0^{\infty} t\, e^{-st}\, dt = \frac{-t \cdot e^{-st}}{s} - \int -\frac{e^{-st}}{s}\, dt$$

$$= \frac{-t \cdot e^{-st}}{s} - \frac{1}{s} \times \frac{e^{-st}}{s}$$

$$= -\frac{t \cdot e^{-st}}{s} - \frac{e^{-st}}{s^2}\Bigg|_0^{\infty} \quad \therefore \quad \boxed{F(s) = \frac{1}{s^2}}$$

4-) Função degrau deslocada no tempo

$f(t) = u(t - t_0)$



$$\mathcal{L}[u(t-t_0)] = \int_0^{\infty} u(t-t_0) \cdot e^{-st}\, dt$$

$$= \int_{t_0}^{\infty} e^{-st}\, dt = \frac{-e^{-st}}{s}\Bigg|_{t_0}^{\infty}$$

$$\boxed{F(s) = \frac{e^{-t_0 s}}{s}}$$

5) $f(t) = \cos wt$

Notação de euler

$$\cos wt = \frac{e^{jwt} + e^{-jwt}}{2}$$

$$\mathcal{L}[\cos wt] = \int_0^\infty \cos wt \cdot e^{-st}\, dt$$

$$= \int_0^\infty \left(\frac{e^{jwt} + e^{-jwt}}{2} \right) e^{-st}\, dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{jwt - st}\, dt + \int_0^\infty e^{-jwt - st}\, dt \right]$$

$$= \frac{1}{2}\left[\int_0^\infty e^{(jw - s)t}\, dt + \int_0^\infty e^{(-jw - s)t}\, dt \right]$$

$$= \frac{1}{2}\left[\frac{e^{(jw-s)t}}{jw - s} + \frac{e^{(-jw-s)t}}{(-jw-s)} \right]\Bigg|_0^\infty$$

$$= \frac{1}{2}\left[-\frac{e^{-(s-jw)t}}{s-jw} - \frac{e^{-(s+jw)t}}{s+jw} \right]\Bigg|_0^\infty$$

$$= \frac{1}{2}\left[\frac{1}{s-jw} + \frac{1}{s+jw} \right] = \frac{s}{s^2 + w^2} \quad \therefore \quad \boxed{F(s) = \frac{s}{s^2 + w^2}}$$

6) $f(t) = \text{sen}\, wt$

$$\text{sen}\, wt = \frac{e^{jwt} - e^{-jwt}}{2j}$$

$$\mathcal{L}[\text{sen}\, wt] = \int_0^\infty \text{sen}\, wt\, e^{-st}\, dt$$

$$= \int_0^\infty \frac{e^{jwt} - e^{-jwt}}{2j} \cdot e^{-st}\, dt$$

$$= \frac{1}{2j}\left[\int_0^\infty e^{-(} \right.$$

Terminar

Depois

Propriedades úteis da transformada de Laplace

Auxiliam na obtenção da transformada desejada a partir das primitivas básicas

1°) $f(t)$ multiplicada por cte $A \neq 0$

$$\mathcal{L}[A f(t)] = A\, \mathcal{L}[f(t)]$$

exemplo: $f(t) = \underbrace{e^{-ax} \operatorname{sen} y}_{cte} \cos wt$

$$F(s) = e^{-ax} \operatorname{sen} y\, \mathcal{L}[\cos wt] \therefore F(s) = e^{-ax} \frac{\operatorname{sen} y \cdot s}{s^2 + w^2}$$

2°) Distributiva

$$\mathcal{L}[f_1(t) \pm f_2(t)] = \mathcal{L}[f_1(t)] \pm \mathcal{L}[f_2(t)]$$

exemplo: $f(t) = e^{-t} + e^{-2t}$

$$\mathcal{L}[e^{-t} + e^{-2t}] = \mathcal{L}\, e^{-t} + \mathcal{L}\, e^{-2t}$$

$$F(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

3°) Mudança de escala $\Rightarrow$ Variável "t" multiplicada por cte positiva

$$\mathcal{L}\, f(at) = \frac{1}{a} F\left(\frac{s}{a}\right) \quad a > 0$$

exemplo: $f(t) = \cos \frac{wt}{2}$

$$\mathcal{L}\left[\cos \frac{wt}{2}\right] = \frac{1}{\frac{1}{2}} \frac{\frac{s}{\frac{1}{2}}}{\left(\frac{s}{\frac{1}{2}}\right)^2 + w^2}$$

$$\therefore F(s) = 2 \cdot \frac{2s}{4s^2 + w^2}$$

$$F(s) = \frac{4s}{4s^2 + w^2}$$

4ª) Deslocamento no domínio do tempo por uma constante to

$$\mathcal{L}[f(t-t_0)]u(t-t_0) = e^{-t_0 \cdot s} \cdot F(s)$$

exemplo:

$$f(t) = (t-5)\,u(t-5)$$

$$F(s) = e^{-5s}\,\mathcal{L}.(t+5-5) \qquad \therefore \quad F(s) = \frac{e^{-5s}}{s^2}$$

$$= e^{-5s}\,\mathcal{L}(t)$$

5ª) Deslocamento no domínio do tempo "t" por uma constante to

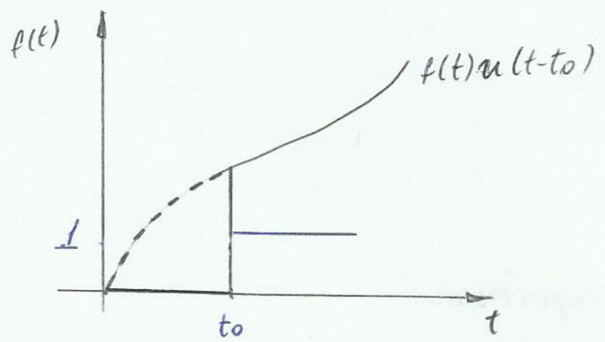
$$\mathcal{L}[f(t).u(t-t_0)] = e^{-t_0 s}\,\mathcal{L}\,f(t+t_0)$$

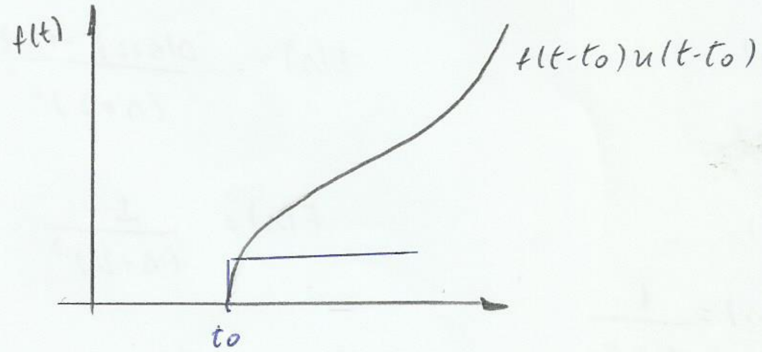
exemplo: $f(t) = t.u(t-5)$

$$F(s) = e^{-5s}\,\mathcal{L}[t+5)$$

$$= e^{-5s}\left[\mathcal{L}(t) + \mathcal{L}(5)\right]$$

$$= e^{-5s}\left(\frac{1}{s^2} + 5s\right)$$





6) Deslocamento no domínio da frequência

$$\mathcal{L}[e^{-at} \cdot f(t)] = F(s+a)$$

exemplo: $f(t) = e^{-2t} \underbrace{\cos wt}_{f_1(t)}$

p/ $f_1(t) = \cos wt \Rightarrow F_1(s) = \dfrac{s}{s^2+w^2}$

$f(t) = e^{-2t} f_1(t) \Rightarrow F(s) = F_1(s+2) \quad \therefore \quad F(s) = \dfrac{s+2}{(s+2)^2+w^2}$

7) Diferenciação: Derivar no domínio do tempo corresponde a multiplicar $F(s)$ por "s" e subtrair o valor inicial

$$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots s^0 f^{n-1}(0)$$

exemplo: $\dfrac{d^2 y(t)}{dt^2} + 3 = 0 \qquad y(0) = \quad y'(0) = 2$

$$s^2 Y(s) - s - s^0 (2) + 3 = 0$$

$$\therefore \quad s^2 \cdot y(s) - s + 1 = 0$$

$$\therefore \quad y(s) = \frac{s-1}{s^2}$$

8) Diferenciação no domínio da frequência

$$\mathcal{L}[t f(t)] = -\frac{d}{ds} F(s)$$

exemplo: $f(t) = t \cdot \underbrace{e^{-2t}}_{f_1(t)}$

p/ $f_1(t) = e^{-2t} \rightarrow F_1(s) = \dfrac{1}{s+2}$

$f(t) = t \cdot f_1(t) \Rightarrow F(s) = \dfrac{-d}{ds}\left(\dfrac{1}{s+2}\right)$

$$F(s) = -\frac{0(s+2) - 1(1)}{(s+2)^2}$$

$$\therefore \quad F(s) = \frac{1}{(s+2)^2}$$

9) <u>Integração</u>: Integrar no domínio do tempo corresponde a dividir por "s" no domínio da frequência.

$$\mathcal{L}\left[\int_0^t f(\lambda)\,d\lambda\right] \Rightarrow \frac{1}{s}\cdot F(s)$$

exemplo: $v_c(t) = \frac{1}{C}\int i(t)\,dt$ $\qquad V_c(s) = \frac{1}{sC} I(s)$

Exercícios - Determinar $F(s)$

16.4) $f(t) = \frac{1}{2}(t - 4e^{-2t})$

$$\mathcal{L}[f(t)] = F(s)$$

$$\therefore F(s) = \mathcal{L}\left[\frac{1}{2}(t - 4e^{-2t})\right]$$

$$= \frac{1}{2}\cdot \mathcal{L}(t - 4e^{-2t})$$

$$= \frac{1}{2}\cdot\left[\mathcal{L}(t) - 4\,\mathcal{L}(e^{-2t})\right]$$

$$= \frac{1}{2}\left[\frac{1}{s^2} - 4\cdot\frac{1}{s+2}\right]$$

$$\therefore F(s) = \frac{1}{2s^2} - \frac{2}{s+2}$$

16.5) $f(t) = t\cdot e^{-(t-1)} u(t-1) - e^{-(t-1)} u(t-1)$

$$F(s) = \mathcal{L}\left[t\, e^{-(t-1)} u(t-1) - e^{-(t-1)} u(t-1)\right]$$

$$= e^{-s}\,\mathcal{L}\left[t+1 \times e^{-(t-1+1)}\right] - e^{-s}\,\mathcal{L}\left[e^{-(t+1-1)}\right]$$

$$= e^{-s}\left[\mathcal{L}\,(t+1).e^{-t} - \mathcal{L}\,e^{-t}\right]$$

$$= e^{-s}\left[\mathcal{L}[te^{-t}] + \cancel{\mathcal{L}[e^{-t}]} - \cancel{\mathcal{L}[e^{-t}]}\right]$$

$$= e^{-s}.\,\mathcal{L}[te^{-t}]$$

$$\therefore F(s) = \frac{e^{-s}}{(s+1)^2}$$

16.8) $f(t) = \cos wt \cdot u(t-1)$

$\mathcal{L}[\cos wt] = \dfrac{s}{s^2+w^2}$ ....

$F(s) = e^{-s}\,\mathcal{L}[\cos w(t+1)]$

$\cos(wt+w) = \cos wt \cos w - \operatorname{sen} wt \operatorname{sen} w$

$F(s) = e^{-s}[\cos w\,\mathcal{L}[\cos wt] - \operatorname{sen} w\,\mathcal{L}[\operatorname{sen} wt]]$

$= e^{-s}\left[\dfrac{\cos w \cdot s}{s^2+w^2} - \dfrac{\operatorname{sen} w \cdot w}{s^2+w^2}\right]$

$= \dfrac{e^{-s}(s\cos w - w \operatorname{sen} w)}{s^2+w^2}$

Exercícios

(a) $f(t) = \underbrace{4e^{-3t}\cos 2t}_{g(t)} \cdot u(t-\pi)$

$\mathcal{L}[g(t)] = \mathcal{L}[e^{-3t}\underbrace{\cos 2t}_{s(t)}]$

$\cos(2t+2\pi) =$

$\cos 2t \cos 2\pi - \cancel{\operatorname{sen} 2t \operatorname{sen} 2\pi}$

Para $s(t) = \cos 2t$ $\quad S(s) = \dfrac{s}{s^2+a^2}$

$\cos(2t+2\pi) = \cos 2t$

logo $g(t) = e^{-3t}s(t)$

$G(s) = S(s+3)$

$= \dfrac{s+3}{(s+3)^2+4}$

$F(s) = 4.\ \mathcal{L}[e^{-3(t+\pi)}\cos 2(t+\pi)]\,.e^{-\pi s}$

$= 4.\ \mathcal{L}[e^{-3t}e^{3\pi}\cos 2t]\,e^{-\pi s}$

$= 4.\,e^{-(3+s)\pi}\,\mathcal{L}[e^{-3t}\cos 2t]$

$$\boxed{F(s) = \frac{4e^{-(3+s)\pi}(s+3)}{(s+3)^2+4}}$$

(b) $f(t) = e^{-3t}\, t\, \underbrace{\sin 2t}_{f_1(t)}$.

Lembrete:

$$t f(t) \Rightarrow -\frac{dF(s)}{ds}$$

$$F_1(s) = \frac{2}{s^2+4}$$

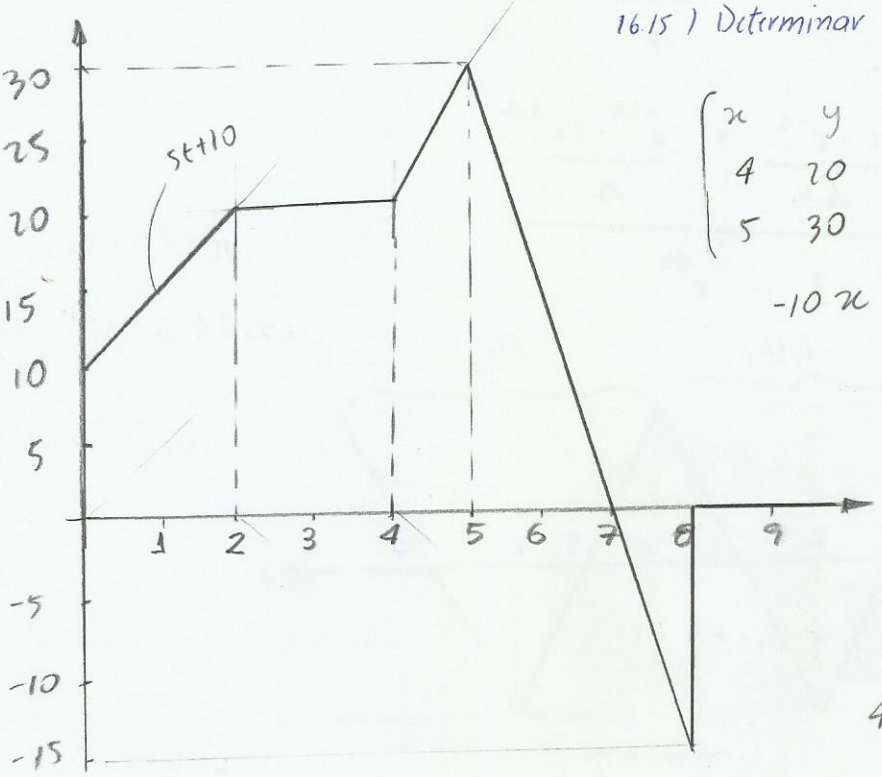
$f_2(t) = t\, f_1(t)$ $\qquad F_2(s) = \frac{-d}{ds} F_1(s) = \frac{-d}{ds}\left(\frac{2}{s^2+4}\right)$

$$F_2(s) = 2(s^2+4)^{-2} \times 2s = \frac{4s}{(s^2+4)^2}$$

$$F(s) = F_2(s+3)$$

$$\boxed{F(s) = \frac{4(s+3)}{[(s+3)^2+4]^2}}$$

16.15) Determinar $f(t)$ e $F(s)$



$$\begin{vmatrix} x & y & 1 \\ 4 & 20 & 1 \\ 5 & 30 & 1 \end{vmatrix} = 0$$

$$-10x + y + 20 = 0$$

$$y = 10x - 20$$

$$\begin{vmatrix} x & y & 1 \\ 5 & 30 & 1 \\ 8 & -15 & 1 \end{vmatrix} = 0$$

$$45x + 3y - 255 = 0$$

$$y = 85 - 15t$$

$f(t) = 10u(t) + 5t.u(t) - 5(t-2)\,u(t-2) + 10(t-4) - 10(t-5)\,u(t-5) - 15(t-5)\,u(t-5$

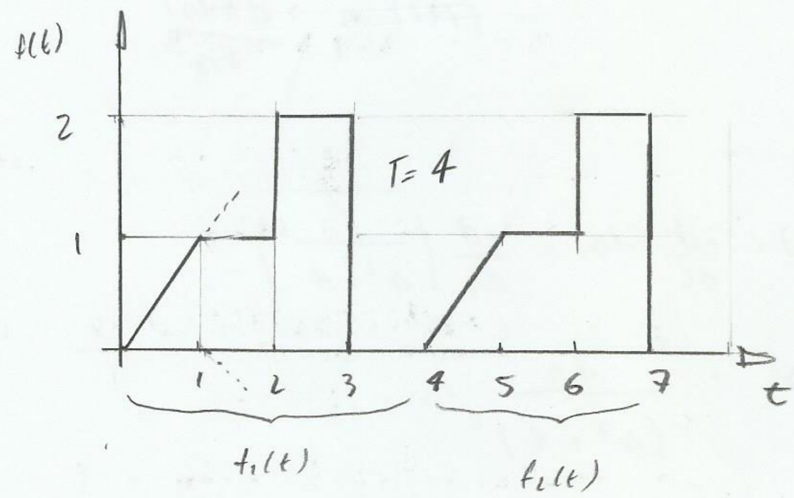
$+ 15(t-8)\,u(t-8) - 15\,u(t-8)$

$\therefore f(t) = 10\,u(t) + 5t\,u(t) - 5(t-2)u(t-2) + 10(t-4) - 25(t-5)u(t-5) + 15(t-8)$
$u(t-8) - 15u(t-8)$

logo $F(s) = \frac{10}{s} + \frac{5}{s^2} - \frac{5e^{-2s}}{s^2} + \frac{10e^{-4s}}{s^2} - \frac{25e^{-5s}}{s^2} + \frac{15e^{-8s}}{s^2} - \frac{15e^{-8s}}{s}$

Determinar $f(t)$ e $F(s)$

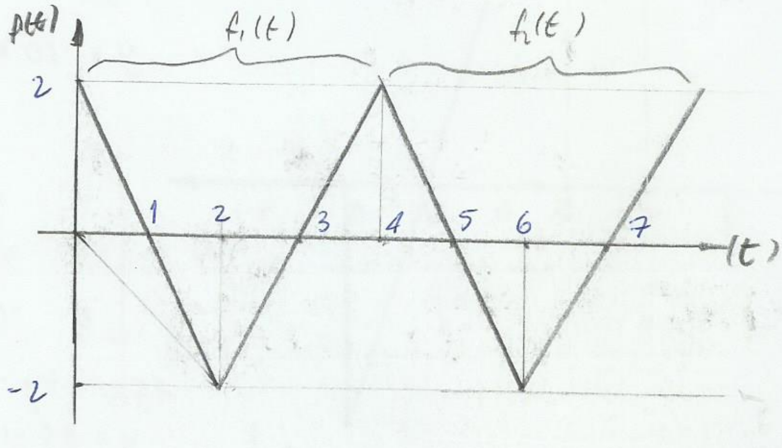
$$F(s) = \frac{F_1(s)}{1 - e^{-sT}}$$



$$f_1(t) = t\,u(t) - (t-1)\,u(t-1) + u(t-2) - 2u(t-3)$$

$$F_1(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s} - \frac{2e^{-3s}}{s}$$

$$F(s) = \frac{\dfrac{1-e^{-s}}{s^2} + \dfrac{e^{-2s} - 2e^{-3s}}{s}}{1 - e^{-4s}}$$



$$f_1(t) = 2u(t) - 2t\,u(t) + 2(t-2)\,u(t-2) + 2(t-2)\,u(t-2) - 2(t-4)\,u(t-4) - 2u(t-4)$$

$$= 2u(t) - 2t\,u(t) + 4(t-2)\,u(t-2) - 2(t-4)\,u(t-4) - 2u(t-4)$$

$$F_1(s) = \frac{2}{s} - \frac{2}{s^2} + \frac{4e^{-2s}}{s^2} - \frac{2e^{-4s}}{s^2} - \frac{2e^{-4s}}{s}$$

$$F(s) = \frac{1}{1-e^{-sT}}\left[\frac{2-2e^{-4s}}{s} + \frac{4e^{-2s} - 2 - 2e^{-4s}}{s^2}\right]$$

Dada $F(s)$, determinar $f(t)$

$$F(s) = \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)} = \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)} + \frac{k_3}{(s+3)}$$

$$k_1 = (s+1)F(s)\Big|_{s=-1} \Rightarrow k_1 = \frac{6(-1)^2+26(-1)+26}{(-1+2)(-1+3)} = \frac{6}{2} = 3$$

$$k_2 = (s+2)F(s)\Big|_{s=-2} \Rightarrow k_2 = \frac{6(-2)^2+26(-2)+26}{(-2+1)(-2+3)} = \frac{-2}{-1} = 2$$

$$k_3 = (s+3)F(s)\Big|_{s=-3} \Rightarrow k_3 = \frac{6(-3)^2+26(-3)+26}{(-3+1)(-3+2)} = \frac{2}{2} = 1$$

$\therefore$ $F(s) = \frac{3}{s+1} + \frac{2}{s+2} + \frac{1}{s+3}$ logo $\boxed{f(t) = (3e^{-t} + 2e^{-2t} + e^{-3t})u(t)}$

$$F(s) = \frac{s^2+3s+2}{s^2+2s+1} = \frac{\cancel{(s+1)}(s+2)}{(s+1)^{\cancel{2}}} = \frac{s+2}{s+1} = 1 + \frac{1}{s+1}$$

$$\boxed{f(t) = (\delta(t) + e^{-t})u(t)}$$

Exemplo:

$$F(s) = \frac{4}{s^2+2s+2}$$

$$s^2+2s+2=0$$

$$s = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$$

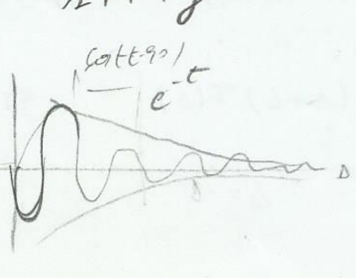
$$F(s) = \frac{4}{(s+1-j)(s+1+j)} = \frac{k_1}{(s+1-j)} + \frac{k_1^*}{(s+1+j)}$$

$$k_1 = (s+1-j)F(s)\Big|_{s=-1+j} = \frac{4}{(-1+j+1+j)} = \frac{4}{2j} = 2\angle{-90^\circ}$$

$$^*k_1 = 2\angle{90}$$

$$F(s) = \frac{2\angle{-90}}{s+1-j} + \frac{2\angle{90}}{s+1+j}$$

$$f(t) = 4e^{-t}\cos(t-90^\circ)u(t)$$



Dado $F(s)$ determinar $f(t)$

$$F(s) = \frac{10(s+3)}{(s+1)^3(s+2)} = \frac{K_{11}}{(s+1)} + \frac{K_{12}}{(s+1)^2} + \frac{K_{13}}{(s+1)^3} + \frac{K_2}{s+2}$$

$$K_2 = (s+2).F(s)\Big|_{s=-2} \qquad K_2 = \frac{10(-2+3)}{(-2+1)^3} = -10$$

$$K_{13} = (s+1)^3.F(s)\Big|_{s=-1} \qquad K_{13} = \frac{10(-1+3)}{(-1+2)} = 20$$

$$K_{12} = \frac{d}{ds}\left[(s+1)^3 F(s)\right]\Big|_{s=-1} \qquad K_{12} = \frac{d}{ds}\left[\frac{10(s+3)}{(s+2)}\right] \Rightarrow \frac{10(s+2)-1(10s+30)}{(s+2)^2}\Big|_{s=-1}$$

$$K_{12} = \frac{-10}{(s+2)^2}\Big|= -10$$

$$K_{11} = \frac{1}{2!}\left[\frac{d^2}{ds^2}(s+1)^3 F(s)\right]\Big|_{s=-1} \Rightarrow K_{11} = \frac{1}{2}\frac{d}{ds}\left(\frac{-10}{(s+2)^2}\right)\Big|_{s=-1}$$

$$K_{11} = \frac{1}{2}\left[\frac{20}{(s+2)^3}\right]\Big|_{s=-1} \quad \therefore \quad K_{11} = 10$$

$$F(s) = \frac{10}{s+1} - \frac{10}{(s+1)^2} + \frac{20}{(s+1)^3} - \frac{10}{s+2}$$

$$f(t) = 10e^{-t} - 10te^{-t} + \frac{20t^2e^{-t}}{2} - 10e^{-2t}$$

$$\therefore \boxed{f(t) = 10\left(e^{-t} - te^{-t} + t^2e^{-t} + e^{-2t}\right)u(t)}$$

Exercício 16.33 a) $F(s) = \frac{s+8}{s^2(s+6)}$ $\qquad f(t) = \left(\frac{4}{3}t - \frac{1}{18} + \frac{1}{18}e^{-6t}\right)u(t)$

$$F(s) = \frac{s+8}{s^2(s+6)} = \frac{K_{11}}{s} + \frac{K_{12}}{s^2} + \frac{K_2}{s+6}$$

$$K_2 = (s+6).F(s)\Big|_{s=-6} \qquad K_2 = \frac{(-6+8)}{(-6)^2} = \frac{1}{18}$$

$$K_{12} = s^2 F(s)\Big|_{s=0} \qquad K_{12} = \frac{0+8}{0+6} = \frac{4}{3}$$

$$K_{11} = \frac{1}{(2-1)!}\frac{d}{ds}\left[s^2F(s)\right] = \frac{d}{ds}\left[\frac{s+8}{s+6}\right] = \frac{(s+6)-(s+8)}{(s+6)^2}\Bigg|_{s=0}$$

$$K_{11} = \frac{-2}{36} = \frac{-1}{18}$$

$$F(s) = \frac{\frac{-1}{18}}{s} + \frac{\frac{4}{3}}{s^2} + \frac{\frac{1}{18}}{s+6} \quad \therefore \quad \boxed{f(t) = \frac{-1}{18} + \frac{4}{3}t + \frac{1}{18}e^{-6t}}$$

Aplicação de Transformada de Laplace em solução de equações integro-diferenciais

Exemplo: $\dfrac{d^2y(t)}{dt^2} + \dfrac{5\,dy(t)}{dt} + 6y(t) = e^{-t}$; $y(0)=1$ $y'(0)=1$

$$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right] = s^2Y(s) - s\,y(0) - y'(0) = s^2Y(s) - s - 1$$

$$\mathcal{L}\left[\frac{5\,dy(t)}{dt}\right] = 5.(sY(s) - s^0.\,y(0)) = 5(sY(s) - 1)$$

$$\mathcal{L}[6y(t)] = 6Y(s) \quad ; \quad \text{logo:}$$

$$s^2Y(s) - s - 1 + 5sY(s) - 5 + 6Y(s) = \frac{1}{s+1}$$

$$Y(s)(s^2+5s+6) = \frac{1}{s+1} + s + 6$$

$$Y(s)(s^2+5s+6) = \frac{s^2+s+6s+6+1}{s+1} = \frac{s^2+7s+7}{s+1}$$

$$Y(s) = \frac{s^2+7s+7}{(s^2+5s+6)(s+1)}$$

$$s^2+5s+6=0$$

$$s = \frac{-5 \pm \sqrt{25-24}}{2} \quad \begin{cases} s_1 = -2 \\ s_2 = -3 \end{cases}$$

$$Y(s) = \frac{s^2+7s+7}{(s+2)(s+3)(s+1)}$$

$$Y(s) = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = (s+1)\,Y(s)\Big|_{s=-1} \Rightarrow K_1 = \frac{(-1)^2 + 7(-1) + 7}{(-1+2)(-1+3)} = \frac{1}{2}$$

$$K_2 = (s+2)\,Y(s)\Big|_{s=-2} \Rightarrow K_2 = \frac{(-2)^2 + 7(-2) + 7}{(-2+3)(-2+1)} = 3$$

$$K_3 = (s+3)\,Y(s)\Big|_{s=-3} \Rightarrow K_3 = \frac{(-3)^2 + 7(-3) + 7}{(-3+1)(-3+2)} = \frac{-5}{2}$$

$$Y(s) = \frac{\frac{1}{2}}{s+1} + \frac{3}{s+2} - \frac{\frac{5}{2}}{s+3}$$

$$\boxed{y(t) = \left(\frac{1}{2}e^{-t} + 3e^{-2t} - \frac{5}{2}e^{-3t}\right)u(t)}$$

Exercício 16.57) $\frac{dy(t)}{dt} + 2y(t) + \int_0^t y(\lambda)\,d\lambda = 1 - e^{-2t}$ Para $y(0) = 0$

R: $f(t) = 2\left(e^{-2t} + te^{-t} - e^{-t}\right)u(t)$

$$\mathcal{L}\left[\frac{dy(t)}{dt} + 2y(t) + \int_0^t y(\lambda)\,d\lambda = 1 - e^{-2t}\right]$$

$$s\,F(s) - 0 + 2F(s) + \frac{1}{s}F(s) = 1 - \frac{1}{s+2}$$

$$F(s)\left(s + 2 + \frac{1}{s}\right) = \frac{s+1}{s+2}$$

$$F(s) = \frac{s+1}{s+2} \times \frac{s}{s^2 + 2s + 1}$$

(terminar)

Teorema do valor Inicial e Final

$$\boxed{\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) \qquad \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$

Exemplos:

16.51) $F(s) = \dfrac{2(s+2)}{s(s+1)}$ — Valor inicial (VI), Valor Final (VF)

$$VI = \lim_{s\to\infty} s.F(s) \Rightarrow \lim_{s\to\infty} \frac{\cancel{s}.2(s+2)}{\cancel{s}(s+1)} = 2$$

$$VF = \lim_{s\to 0} sF(s) \Rightarrow \lim_{s\to 0} \frac{2(s+2)}{s+1} = 4$$

$$F(s) = \frac{2(s^2+2s+6)}{(s+1)(s+2)(s+3)}$$

$$VI = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) \Rightarrow \lim_{s\to\infty} \frac{2s(s^2+2s+6)}{(s+1)(s+2)(s+3)} = 2$$

$$VF = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \Rightarrow \lim_{s\to 0} \frac{2s(s^2+2s+6)}{(s+1)(s+2)(s+3)} = 0$$

$$F(s) = \frac{10(s+1)}{s(s^2+2s+2)}$$

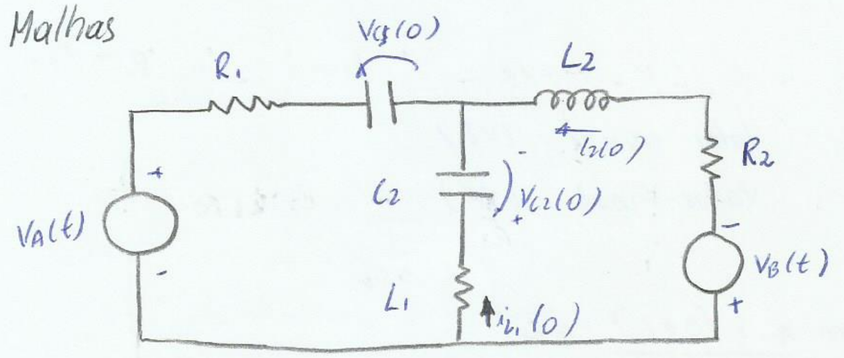
$$VI = \lim_{s\to\infty} sF(s) \Rightarrow \lim_{s\to\infty} \frac{10s(s+1)}{\cancel{s}(s^2+2s+2)} = 0$$

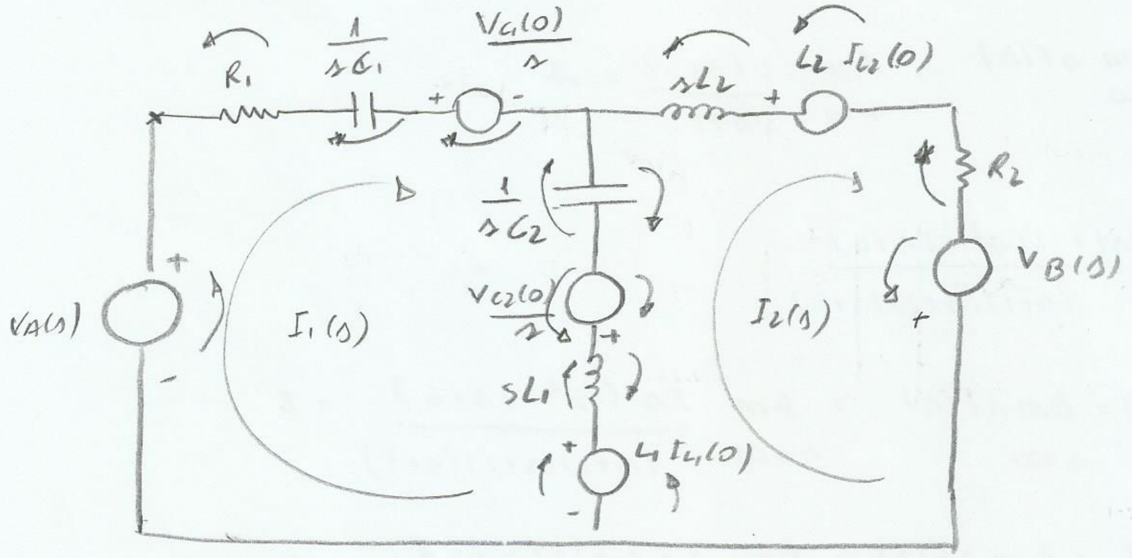
$$VF = \lim_{s\to 0} sF(s) \Rightarrow \lim_{s\to 0} \frac{10s(s+1)}{s(s^2+2s+2)} = 5$$

Aplicação da Transformada de Laplace na Análise de Circuitos

Técnicas de Análise

Malhas



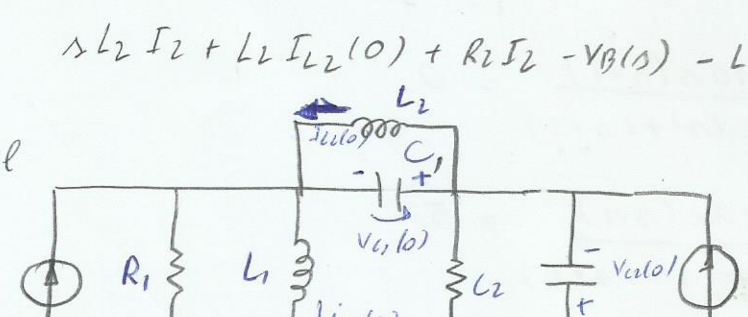


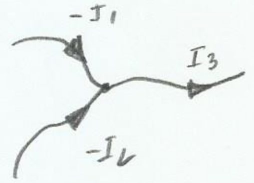
1: $R_1 I_1 + \frac{1}{sC_1} I_1 + \frac{V_{C_1}(0)}{s} + \frac{1}{sC_2}(I_1 - I_2) - \frac{V_{C_2}(0)}{s} + sL_1 (I_1 - I_2) +$

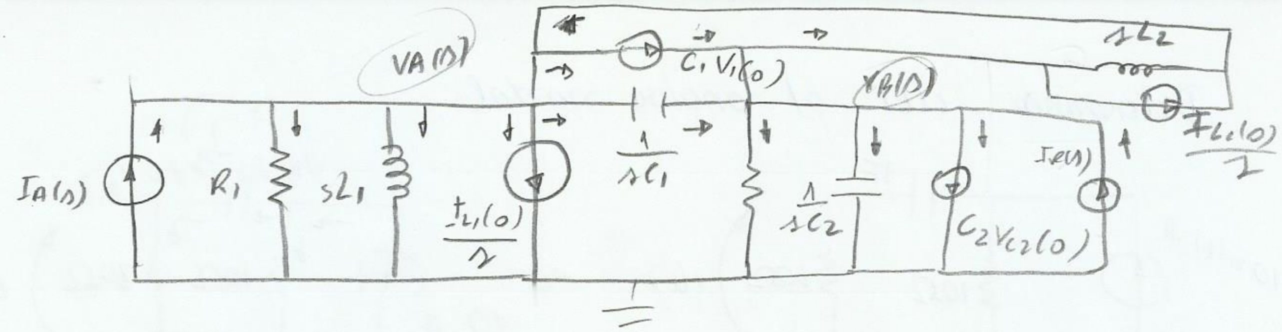
$+ L_1 I_{L_1}(0) - V_A(s) = 0$

2: $sL_2 I_2 + L_2 I_{L_2}(0) + R_2 I_2 - V_B(s) - L_1 I_{L_1} + sL_1 (I_2 - I_1) + \frac{V_{C_2}(0)}{s} + \frac{1}{sC_2}(I_2 - I_1) = 0$

Nodal







$$1: \quad -I_A(s) + \frac{V_A}{R_1} + \frac{V_A}{sL_1} + \frac{I_{L_1}(0)}{s} + \frac{V_A - V_B}{\frac{1}{sC_1}} + C_1V_{C1}(0) + \frac{V_A - V_B}{sL_2} - \frac{I_{L_2}(0)}{s} = 0$$

$$2: \quad \frac{I_{L_2}(0)}{s} - \frac{(V_A - V_B)}{sL_2} - C_1V_{C1}(0) - \frac{(V_A - V_B)}{\frac{1}{sC_1}} + \frac{V_B}{R_2} + \frac{V_B}{\frac{1}{sC_2}} + C_2V_{C2}(0) - I_B(s) = 0$$

Revisão de Regra de Crammer

coeficientes   incógnitas   termos independentes

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & \cdots & & a_{nn} \end{vmatrix} \times \begin{vmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix} \Rightarrow x_j = \frac{|A_j|}{|A|}$$

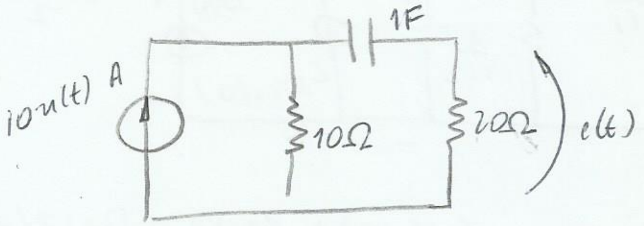
Exemplo:

$$\begin{cases} 3x + 4y = 8 \\ 2x + y = 4 \end{cases} \Rightarrow \begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} \times \begin{vmatrix} x \\ y \end{vmatrix} = \begin{vmatrix} 8 \\ 4 \end{vmatrix}$$

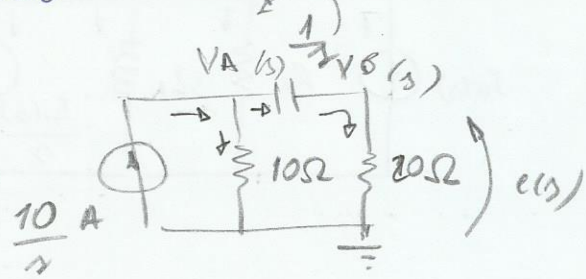
$$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} = -5 \qquad \Delta x = \begin{vmatrix} 8 & 4 \\ 4 & 1 \end{vmatrix} = -8 \qquad \Delta y = \begin{vmatrix} 3 & 8 \\ 2 & 4 \end{vmatrix} = -4$$

$$x = \frac{\Delta x}{\Delta} = \frac{-8}{-5} = \frac{8}{5} \qquad y = \frac{\Delta y}{\Delta} = \frac{-4}{-5} = \frac{4}{5}$$

Determinar $e(t)$ p/ analise nodal



$\rightarrow$



$$\begin{cases} \dfrac{-10}{s} + \dfrac{V_A}{10} + \dfrac{(V_A - V_B)}{\frac{1}{s}} = 0 \\[2ex] -\dfrac{(V_A - V_B)}{\frac{1}{s}} + \dfrac{V_b}{20} = 0 \end{cases}$$

$$\begin{cases} V_A\left(\dfrac{1}{10} + s\right) - s\,V_b = \dfrac{10}{s} \\[2ex] -s\,V_A + V_b\left(s + \dfrac{1}{20}\right) = 0 \end{cases}$$

$$\begin{vmatrix} \frac{1}{10} + s & -s \\ -s & s + \frac{1}{20} \end{vmatrix} =$$

$$\Delta = \left(\frac{1}{10} + s\right)\left(s + \frac{1}{20}\right) - s^2$$

$$= \frac{s}{10} + \cancel{s^2} + \frac{1}{200} + \frac{s}{20} - \cancel{s^2}$$

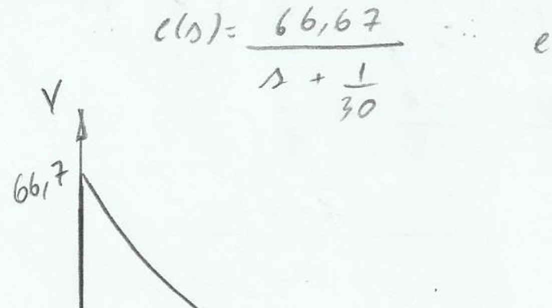
$$= \frac{30s + 1}{200}$$

$$\Delta V_B = \begin{vmatrix} \frac{1}{10} + s & \frac{10}{s} \\ -s & 0 \end{vmatrix} = 10$$

$$\therefore V_B = \left(\frac{30s+1}{200} \div 10\right)^{-1} = \frac{30s+1}{2000}$$

logo $$e(s) = \frac{2000}{30s+1} = \frac{2000}{30}\left(\frac{1}{s + \frac{1}{30}}\right)$$

$$e(s) = \frac{66{,}67}{s + \frac{1}{30}} \quad \therefore \quad e(t) = \left(66{,}67\, e^{\frac{-t}{30}}\right) u(t)\ \text{V}$$

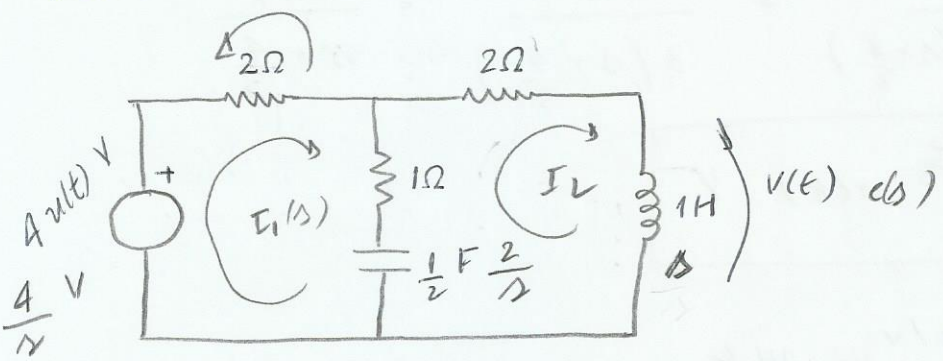


Através do teorema da condição inicial e final

$$V_I = \lim_{s \to \infty} sF(s) \Rightarrow \lim_{s \to \infty} \frac{s \cdot 66{,}7}{s + \frac{1}{30}} = 66{,}7$$

$$V_F = \lim_{s \to 0} sF(s) \Rightarrow \lim_{s \to 0} \frac{s\,66{,}7}{s + \frac{1}{30}} = 0$$

Determinar v(t) por analise de malhas C.I. nulas



$$\begin{cases} 2\,I_1(s) + (I_1(s) - I_2(s))\cdot 1 + \frac{2}{s}(I_1 - I_2) - \frac{4}{s} = 0 \\ 2\,I_2 + s I_2 + \frac{2}{s}(I_2 - I_1) + (I_2 - I_1)\cdot 1 = 0 \end{cases}$$

$$\begin{cases} \left(3 + \frac{2}{s}\right) I_1 - \left(1 + \frac{2}{s}\right) I_2 = \frac{4}{s} \\ -\left(1 + \frac{2}{s}\right) I_1 + \left(3 + \frac{2}{s} + s\right) I_2 = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} 3 + \frac{2}{s} & -\left(1 + \frac{2}{s}\right) \\ -\left(1 + \frac{2}{s}\right) & 3 + \frac{2}{s} + s \end{vmatrix} = 9 + \frac{6}{s} + 3s + \frac{6}{s} + \frac{4}{s^2} + 2 - \left(1 + \frac{2}{s}\right)^2$$

$$= \frac{12}{s} + 3s + 11 + \frac{4}{s^2} - \left(1 + \frac{4}{s} + \frac{4}{s^2}\right)$$

$$\Delta = \frac{\cancel{12}^{\,8}}{s} + 3s + 11 + \cancel{\frac{4}{s^2}} - 1 - \cancel{\frac{4}{s}} - \cancel{\frac{4}{s^2}}$$

$$\Delta = \frac{8 + 3s^2 + 10s}{s}$$

$$\Delta I_2 = \begin{vmatrix} 3 + \frac{2}{s} & \frac{4}{s} \\ -\left(1 + \frac{2}{s}\right) & 0 \end{vmatrix} = \frac{4}{s}\left(1 + \frac{2}{s}\right) = \frac{4}{s} + \frac{8}{s^2} = \frac{4s + 8}{s^2}$$

$$I_2 = \frac{4s + 8}{s^2} \div \frac{3s^2 + 10s + 8}{s} = \frac{4s + 8}{3s^3 + 10s^2 + 8s}$$

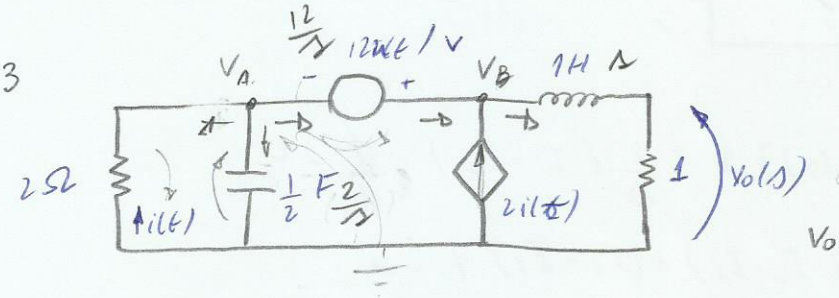
$$v(t) = I_2 \cdot s = \frac{4s + 8}{3s^2 + 10s + 8} = \frac{4}{3}\,\frac{(s + 2)}{\left(s^2 + \frac{10s}{3} + \frac{8}{3}\right)}$$

$$s = \frac{-\frac{10}{3} \pm \sqrt{\frac{100}{9} - \frac{32}{3}}}{2} = \frac{-\frac{10}{3} \pm \sqrt{\frac{4}{9}}}{2} = \frac{-\frac{10}{3} \pm \frac{2}{3}}{2} \quad \begin{cases} s_1 = -\frac{4}{3} \\ s_2 = -2 \end{cases}$$

logo $V(s) = \dfrac{4\cancel{(s+2)}}{3\cancel{(s+2)}(s+\frac{4}{3})} = \dfrac{4}{3(s+\frac{4}{3})} = \dfrac{4/3}{s+\frac{4}{3}}$

$\therefore \boxed{v(t) = \frac{4}{3} e^{-\frac{4t}{3}} u(t) \ V}$

17.3



Análise Nodal

$Vo(t) = \left(\frac{36}{5} + 7{,}59\, e^{-2t} \cos(t + 161{,}6°)\right) u(t)$

$\dfrac{VA}{2} + \dfrac{s\,VA}{2} + \dfrac{VA + \frac{12}{s}}{s+1} - 2i(s) = 0 \quad ; \; i(s) = \dfrac{-VA}{2}$

$\dfrac{VA}{2} + \dfrac{s\,VA}{2} + \dfrac{VA + \frac{12}{s}}{s+1} + VA = 0$

$VA\left(\dfrac{s}{2} - \dfrac{1}{2} + \dfrac{1}{s+1}\right) = \dfrac{-\frac{12}{s}}{s+1}$

$VA\left(\dfrac{s}{2} - \dfrac{1}{2} + \dfrac{1}{s+1}\right) = \dfrac{-12}{s(s+1)}$

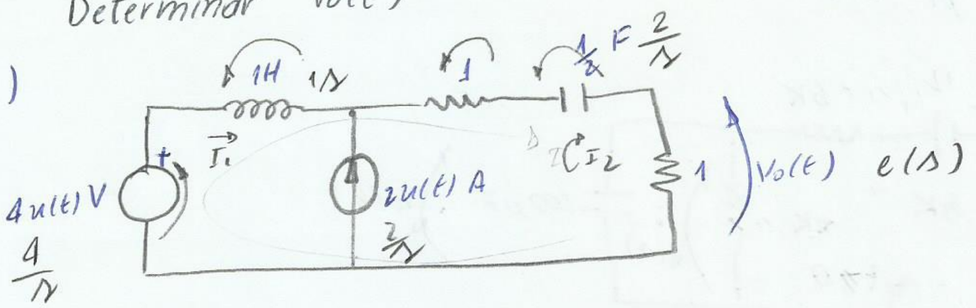
$VA\left(\dfrac{(s-1)(s+1)+4}{4(s+1)}\right) = \dfrac{-12}{s(s+1)}$

$VA = \dfrac{-12}{s\cancel{(s+1)}} \cdot \dfrac{4\cancel{(s+1)}}{(s-1)(s+1)+4} = \dfrac{-48}{s[(s-1)(s+1)+4]}$

(terminar depois)

Determinar $V_o(t)$

17.5)



$$-\frac{4}{s} + sI_1 + I_2 + \frac{2}{s} I_2 + I_2 = 0 \quad ; \quad I_2 - I_1 = \frac{2}{s}$$

$$e(s) = I_2$$

logo $I_1 = I_2 - \frac{2}{s}$

$$-\frac{4}{s} + s\left(I_2 - \frac{2}{s}\right) + \frac{2}{s} I_2 + 2I_2 = 0$$

$$-\frac{4}{s} + sI_2 - 2 + \frac{2}{s} I_2 + 2I_2 = 0$$

$$I_2\left(s + \frac{2}{s} + 2\right) = 2 + \frac{4}{s}$$

$$I_2 = \left(\frac{2s+4}{s}\right) \div \left(\frac{s^2+2s+2}{s}\right) = \frac{2s+4}{s^2+2s+2}$$

$$s^2+2s+2 = 0 \qquad s = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$$

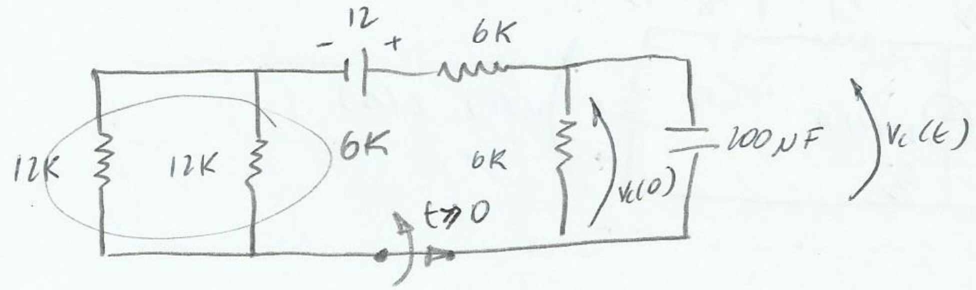
$$\frac{2(s+2)}{(s+1+j)(s+1-j)} = \frac{K_1}{s+1+j} + \frac{{}^*K_1}{s+1-j}$$

$$K_1 = (s+1-j)\,F(s)\Big|_{-1-j} = \frac{2(s+2)}{s+1-j} = \frac{2(-1-j+2)}{\cancel{-1}-j\ \cancel{+1}-j} = \frac{-j+1}{-j} = \frac{1-j}{j}$$

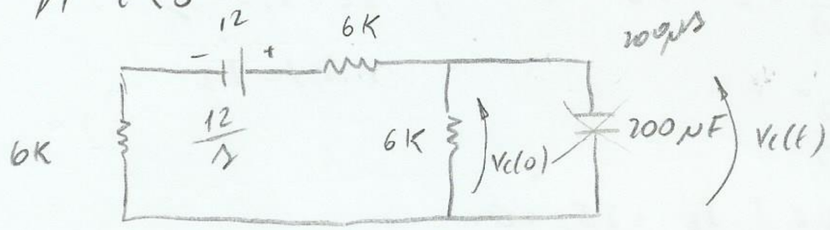
$$K_1 = \sqrt{2}\,\angle{-45^\circ} \qquad {}^*K_1 = \sqrt{2}\,\angle 45$$

$$V_o(s) = \frac{\sqrt{2}\,\angle{-45}}{s+1+j} + \frac{\sqrt{2}\,\angle 45}{s+1-j} \Rightarrow v_o(t) = 2\sqrt{2}\,e^{-t}\cos(t-45)\,u(t)\ \text{V}$$

Determinar v_c p/ $t>0$

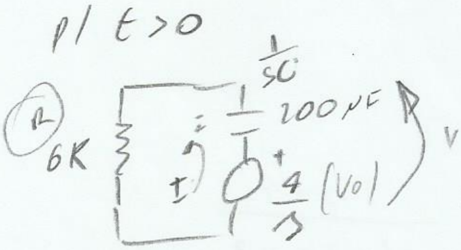


p/ $t<0$



$$V_c(0) = \frac{6}{18} \times 12 = 4\,V$$

p/ $t>0$



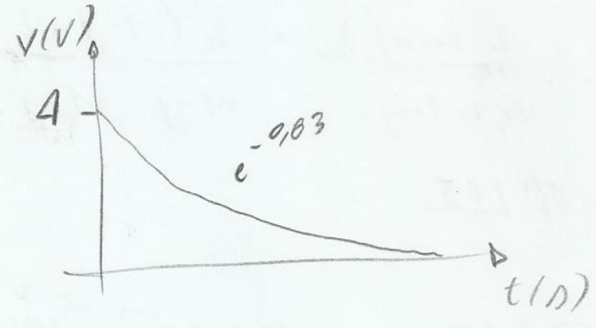
$$RI - \frac{4}{s} + \frac{1}{sC} I = 0$$

$$I\left(R + \frac{1}{sC}\right) = \frac{4}{s}$$

$$I = \frac{4}{\cancel{s}} \times \frac{\cancel{s}C}{RsC+1}$$

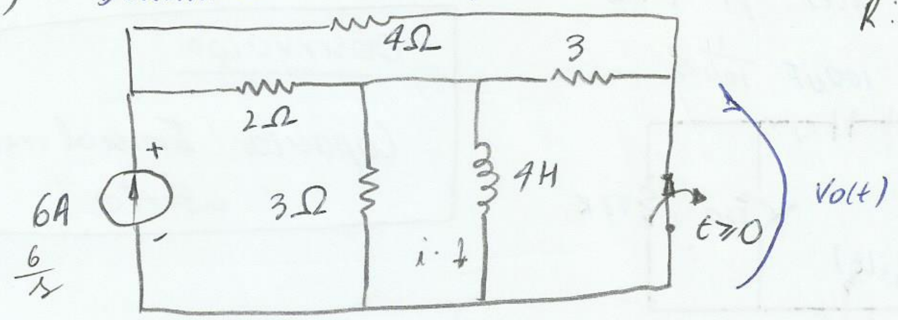
$$V_c(s) = \frac{4RC}{sRC+1} = \frac{\cancel{RC}(4)}{\cancel{RC}\left(s + \frac{1}{RC}\right)} = \frac{4}{s+0{,}63}$$

$$V_c(t) = 4e^{-0{,}63t}\,u(t)$$



17.30) Determinar $v_o(t)$ p/ $t>0$

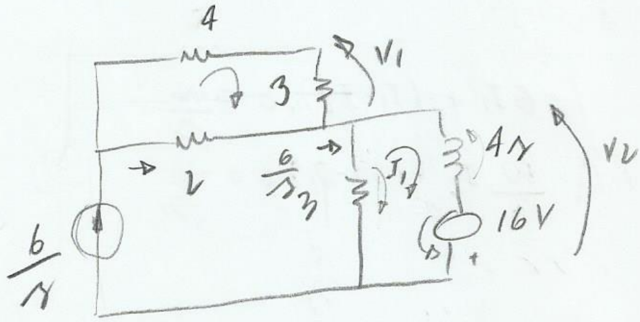
R: $V_o(t) = (4+6e^{-\frac{3}{4}t})u(t)$



Realizando o divisor de corrente

$i_L = \frac{4}{6} \times 6 = 4\ A$ (p/ $t<0$)

(p/ $t \geq 0$) temos:



$V_o(s) = V_1 + V_2$

Calculo de V_1

$V_1 = 3\ I_{R3}$; $I_{R3} = \frac{2}{2+4+3} \cdot \frac{6}{s} = \frac{12}{9s}$

$V_1 = 3 \cdot \frac{12}{9s} = \frac{4}{s}$

Calculo de V_2

$4s \cdot I_1 + 3\left(I_1 - \frac{6}{s}\right) = 16$

$I_1(4s+3) = 16 + \frac{18}{s}$

$I_1 = \frac{16s+18}{s(4s+3)}$

$V_{aux} = \frac{4\cancel{s} \cdot (16s+18)}{\cancel{s}(4s+3)}$

$V_2 = \frac{4(16s+18)}{4s+3} - 16$

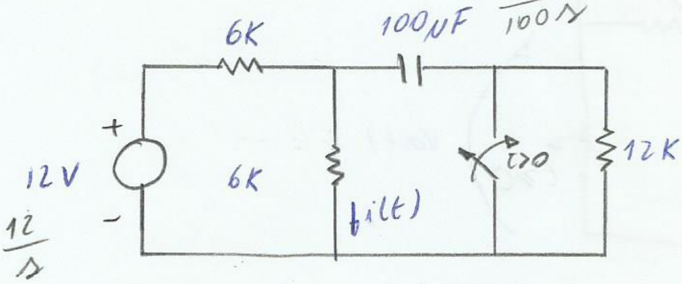
$= \frac{64s + 72 - 64s - 48}{4s+3}$

$= \frac{24}{4s+3} = \frac{6}{s+\frac{3}{4}}$

$V_o = \frac{4}{s} + \frac{6}{s+\frac{3}{4}}$

$V_o(t) = (4 + 6e^{-\frac{3}{4}(t)})\, u(t)$

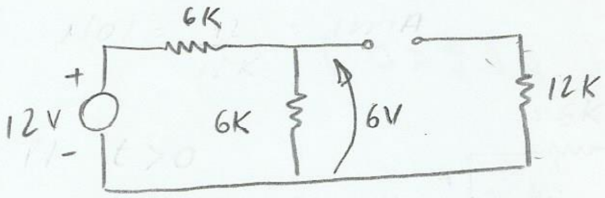
Calcular $i(t)$ p/ $t>0$



Observação:

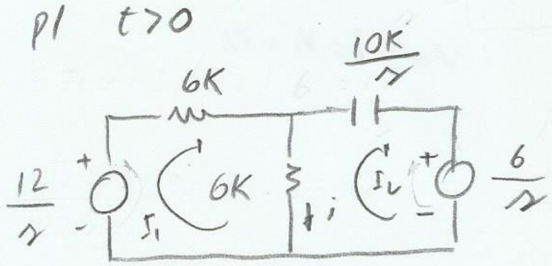
Capacitor Inicialmente é aberto

p/ $t<0$



$\therefore V(0) = 6V$

p/ $t>0$

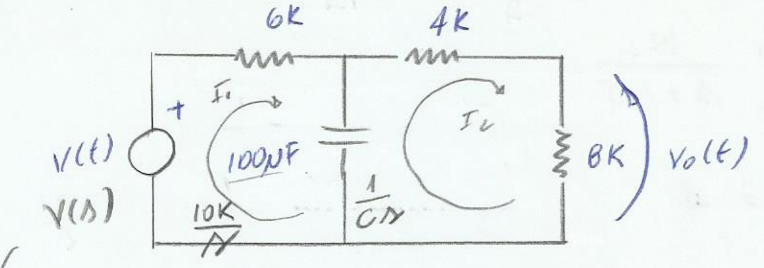


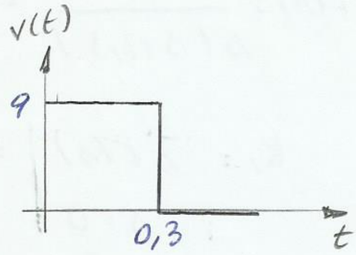
$$\begin{cases} 6I_1 + (I_1 - I_2).6 = \frac{12}{s} \\ \frac{10}{s} I_2 + (I_2 - I_1).6 = -\frac{6}{s} \end{cases}$$

$$\begin{cases} 6I_1 + 6I_1 - 6I_2 = \frac{12}{s} \\ \frac{10}{s} I_2 + 6I_2 - 6I_1 = \frac{-6}{s} \end{cases} \sim \begin{cases} 12I_1 - 6I_2 = \frac{12}{s} \\ -12I_1 + \frac{20I_2}{s} = \frac{-12}{s} \end{cases}$$

$I_2 = 0 \quad \therefore \quad I_1 = \frac{1}{s}$

$\therefore \underline{i(t) = 1\,mA}$

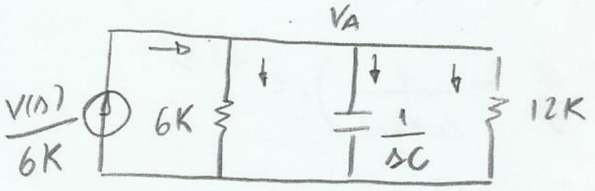




$$v(t) = 9u(t) - 9u(t-3)$$

$$V(s) = \frac{9}{s} - \frac{9e^{-0,3s}}{s} \quad \text{ou} \quad V(s) = \frac{9}{s}\left(1 - e^{-0,3s}\right)$$

Transformação de fonte



$$V_o(s) = \frac{8K}{12K} \cdot V_A$$

$$-\frac{V(s)}{6K} + \frac{V_A}{6K} + sCV_A + \frac{V_A}{12K} = 0$$

$$-2V(s) + 2V_A + 12K\,sC\,V_A + V_A = 0$$

$$V_A(2 + 12K \cdot sC + 1) = 2V(s)$$

$$V_A = \frac{2V(s)}{3 + 12K \cdot sC}$$

$$V_o = \frac{8}{12} \cdot \frac{2V(s)}{(3 + 12K\,sC)} \quad \therefore \quad V_o = \frac{4V(s)}{3(3 + 12K\,sC)}$$

$$V_o(s) = \frac{4V(s)}{3(3 + 1{,}2s)}$$

$$= \frac{4}{\cancel{3}(3 + 1{,}2s)} \cdot \frac{\overset{3}{\cancel{9}}}{s}\left(1 - e^{-0,3s}\right)$$

$$= \frac{12}{(3 + 1{,}2s)s}\left(1 - e^{-0,3s}\right)$$

$$= \frac{10\left(1 - e^{-0,3s}\right)}{s(s + 2{,}5)} = \underbrace{\frac{10}{s(s + 2{,}5)}}_{F(s)} - \frac{10e^{-0,3s}}{s(s + 2{,}5)}$$

$$F(s) = \frac{10}{s(s+2,5)} = \frac{K_1}{s} + \frac{K_2}{s+2,5}$$

$$K_1 = sF(s)\Big|_{s=0} = \frac{10}{0+2,5} = 4$$

$$K_2 = (s+2,5)F(s)\Big|_{s=-2,5} = \frac{10}{-2,5} = -4$$

$$F(s) = \frac{4}{s} - \frac{4}{s+2,5}$$

logo $V_o(s) = \left(\frac{4}{s} - \frac{4}{s+2,5}\right) - \left(\frac{4}{s} - \frac{4}{s+2,5}\right) \cdot e^{-0,3s}$

$\therefore V_o(t) = \left(4 - 4e^{-2,5t}\right)u(t) - \left(4 - 4e^{-2,5(t-0,3)}\right)u(t-0,3)$

Exercício 17.57) Se a entrada da rede é $x_1(t) = u(t)$ e a resposta ao impulso unitário é $y_2(t) = e^{-t} - e^{-2t}$. Determine a saída $y_1(t)$.

p/ $x_2(t) = \delta(t) \rightarrow X_2(s) = 1 \Rightarrow Y_2(s) = H(s)$

$$H(s) = \frac{1}{s+1} - \frac{1}{s+2} = \frac{s+2-s-1}{(s+2)(s+1)} = \frac{1}{(s+1)(s+2)}$$

p/ $x_1(t) = u(t)$ $\quad X_1(s) = \frac{1}{s}$

$$Y_1(s) = H(s)\,X_1(s) = \frac{1}{s(s+1)(s+2)} = \frac{\frac{1}{2}}{s} - \frac{1}{s+1} + \frac{1/2}{s+2}$$

$$K_1 = sY_1(s)\Big|_{s=0} = \frac{1}{(s+1)(s+2)} = \frac{1}{2}$$

$$K_2 = (s+1)Y_1(s)\Big|_{s=-1} = \frac{1}{s(s+2)} = \frac{1}{(-1)(-1+2)} = -1$$

$$K_3 = (s+2)Y_1(s)\Big|_{s=-2} = \frac{1}{s(s+1)} = \frac{1}{(-2)(-2+1)} = \frac{1}{2}$$

$$y_1(t) = \left(\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right)u(t)$$

Exercício 17.58) Se $y_1(t) = 4[1 - te^{-t} - e^{-t}]u(t)$ p/ $x_1(t) = e^{-2t} - e^{-6t}$

Determinar a resposta da rede p/ o impulso unitário.

$$Y_1(s) = 4\,\mathcal{L}[1 - te^{-t} - e^{-t}]$$

$$= 4\left[\frac{1}{s} - \frac{1}{(s+1)^2} - \frac{1}{s+1}\right] \qquad H(s) = \frac{Y(s)}{X(s)}$$

$$X_1(s) = \mathcal{L}[e^{-2t} - e^{-6t}]$$

$$= \frac{1}{s+2} - \frac{1}{s+6}$$

$$Y_1(s) = 4\left(\frac{(s+1)^2 - s - s(s+1)}{s(s+1)^2}\right)$$

$$= 4\left(\frac{\cancel{s^2} + \cancel{2s} + 1 - \cancel{s} - \cancel{s^2} - \cancel{s}}{s(s+1)^2}\right) = \frac{4s}{s(s+1)^2}$$

$$X_1(s) = \frac{s+6-(s+2)}{(s+2)(s+6)} = \frac{4}{(s+2)(s+6)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{4s}{s(s+1)^2} \div \frac{4}{(s+2)(s+6)} = \frac{\overbrace{(s+2)(s+6)}^{s^2+8s+12}}{s(s+1)^2}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{(s+1)^2}$$

$y_2(t)$ p/ $\delta(t) \Rightarrow Y_2(s) = H(s)\,1$

$$K_1 = s\,H(s)\Big|_{s=0} = 12$$

$$K_{21} = \frac{d}{ds}\left[(s+1)^2 \cdot Y(s)\right]\Big|_{s=-1} = \frac{d}{ds}\left(\frac{s^2+8s+12}{s}\right) = \frac{(2s+8)s - (s^2+8s+12)}{s^2}$$

$$= \frac{2s^2+8s-s^2-8s+12}{s^2} = \frac{s^2-12}{s^2}\Big|_{-1} = -11$$

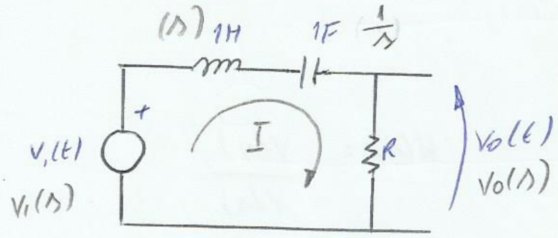
$$K_{22} = (s+1)^2\,Y_2(s)\Big|_{s=-1} = \frac{(s+2)(s+6)}{s}\Big|_{-1} = -5$$

$$Y_2(s) = \frac{12}{s} - \frac{11}{s+1} - \frac{5}{(s+1)^2} \qquad y_2(t) = (12 - 11e^{-t} - 5te^{-t})\,u(t)$$

Exercício 17.14) Determinar R p/ que a rede tenha $\xi = \frac{\sqrt{2}}{2}$ e calcular $V_o(t)$ p/ $V_i(t) = u(t)$

xi

$$\boxed{s^2 + 2\xi w_o s + w_o^2}$$



$$V_o = \frac{R}{s + \frac{1}{s} + R} V_i(s) \quad ; \quad H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{s + \frac{1}{s} + R}$$

$$H(s) = \frac{Rs}{s^2 + 1 + Rs}$$

$$s^2 + Rs + 1$$

$$s^2 + 2\xi w_o s + w_o^2$$

$$w_o^2 = 1 \quad \therefore \quad w_o = 1$$

$$2\xi w_o = R \quad ; \quad \text{Para } \xi = \frac{\sqrt{2}}{2}$$

$$2 \cdot \frac{\sqrt{2}}{2} \cdot 1 = R \quad \therefore \quad R = \sqrt{2}$$

$$H(s) = \frac{\sqrt{2}\, s}{s^2 + \sqrt{2}\, s + 1} \quad ; \quad p/ \ V_i(t) = u(t) \quad \therefore \quad V_i(s) = \frac{1}{s}$$

$$V_o = \frac{1}{\cancel{s}} \cdot \frac{\sqrt{2}\, \cancel{s}}{(s^2 + \sqrt{2}\, s + 1)} = \frac{\sqrt{2}}{s^2 + \sqrt{2}\, s + 1}$$

$$s = \frac{-\sqrt{2} \pm \sqrt{2 - 4}}{2} = \frac{-\sqrt{2} \pm j\sqrt{2}}{2}$$

$$V_o = \frac{K}{s + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}} + \frac{K^*}{s + \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}}$$

$$K = \left(s + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) V_o(s) \Big|_{s = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{s + \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}}$$

$$K = \frac{\sqrt{2}}{-\cancel{\frac{\sqrt{2}}{2}} + j\frac{\sqrt{2}}{2} + \cancel{\frac{\sqrt{2}}{2}} + j\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{j\sqrt{2}} = \frac{1}{j} = 1 \angle -90$$

$$K^* = 1 \angle 90$$

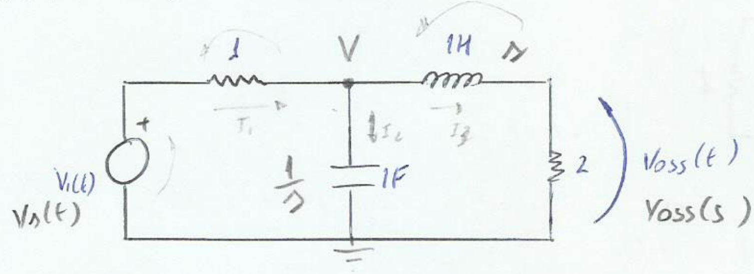
$$V_o(s) = \frac{1\angle -90}{s + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}} + \frac{1 \angle 90}{s + \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}} \quad \therefore \quad \boxed{V_o(s) = 2e^{-\frac{\sqrt{2}}{2}t} \cos\left(\frac{\sqrt{2}}{2}t - 90\right) u(t)}$$

Observação:

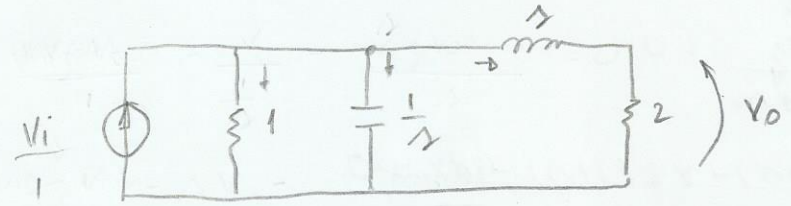
$$V_{ss}(t) = X_{max}\, |H(j\omega_0)| \cos(\omega_0 t + \phi(j\omega_0) + \theta)$$

$$V_i = X_{max} \cos(\omega_0 t + \theta)$$

Exercício 17.13)



$V_i(t) = 12 \cos 2t \ (V)$



$$V_i - V - sV - \frac{V}{s+2} = 0$$

$$V_i(s+2) - V(s+2) - sV(s+2) - V = 0$$

$$V_i(s+2) = V(s+2 + s(s+2) + 1)$$

$$V_i(s+2) = V(s+2 + s^2 + 2s + 1)$$

$$V_i(s+2) = V(s^2 + 3s + 3)$$

$$V_i\cancel{(s+2)} = \frac{\cancel{(s+2)}}{2} V_o (s^2 + 3s + 3)$$

$$V_o = \frac{2}{s+2} V$$

$$V = \frac{s+2}{2} V_o$$

$$\frac{V_o}{V_i} = \frac{2}{s^2+3s+3} = H(s)$$

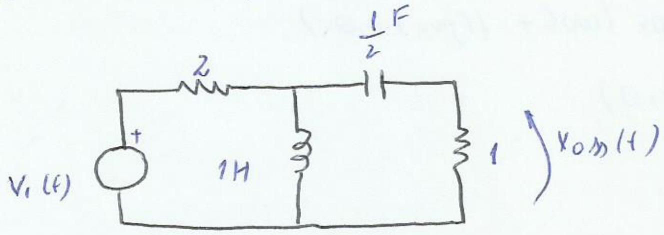
$$Y(s) = H(s)\, X(s)$$

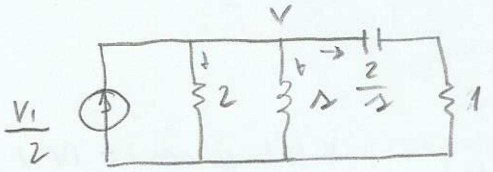
$$H(j2) = \frac{2}{(j2)^2 + 3j2 + 3} = \frac{2}{j6 - 1} = \frac{2 \angle 0}{6{,}08 \angle 99{,}46} = 0{,}33 \angle -99{,}5$$

$$V_{oss}(t) = 12 \cdot (0{,}33) \cos(2t - 99{,}5) \quad \therefore \quad \boxed{V_{oss}(t) = 3{,}95 \cos(2t - 99{,}5°) \ (V)}$$

Determinar $v_{oss}(t)$ p/ $t>0$

$v_1(t) = 10\cos 2t$





$$\frac{V_1}{2} - \frac{V}{2} - \frac{V}{s} - \frac{V}{\frac{2}{s}+1} = 0$$

$$\frac{V_1}{2} - \frac{V}{2} - \frac{V}{s} - \frac{Vs}{2+s} = 0$$

$$V_1 s(2+s) - Vs(2+s) - V2(2+s) - 2s^2V = 0$$

$$V_1 s(2+s) = V(s(2+s) + 2(2+s) + 2s^2)$$

$$V_0 = \frac{1}{1+\frac{2}{s}} V \quad \therefore \quad V = \left(1+\frac{2}{s}\right) V_0 \qquad V = \left(\frac{s+2}{s}\right) V_0$$

$$V_1 s\cancel{(2+s)} = \left(\frac{\cancel{s+2}}{s}\right) V_0 \left(s(2+s) + 2(2+s) + 2s^2 \right.$$

$$\frac{V_0}{V_1} = \frac{s^2}{s(2+s)+2(2+s)+2s^2)} = \frac{s^2}{2s+s^2+4+2s+2s^2} = \frac{s^2}{3s^2+4s+4}$$

$$Y(s) = H(s)X(s)$$

$$H(j2) = \frac{(j2)^2}{3(j2)^2 + 4(j2)+4} = \frac{-4}{-8+j8} = \frac{4\angle 180}{11{,}31\angle 135} = 0{,}35\angle 45$$

$$V_{oss}(t) = 3{,}5\cos(2t+45)\,(V)$$

Função de Transferência

Função de transferência: $H(s) = \dfrac{\overset{\text{saída}}{Y_o(s)}}{\underset{\text{entrada}}{X_i(s)}}$

Mudar

Caso:

Entrada ($X_i(s)$)	Saída ($Y_o(s)$)	$H(s)$
Tensão	Tensão	Ganho de tensão
Corrente	Corrente	Ganho de corrente
Tensão	Corrente	Admitância de transferência
Corrente	Tensão	Impedância de transferência

Obs.: se $X_i = \delta(t) \Rightarrow X_i(s) = 1$ logo $Y(s) = H(s)$
"Resposta de um sistema ao impulso unitário é a própria função de transferência"

Equação característica

$$s^2 + 2\xi . w_0 s + w_0^2 = 0$$

$$s_1 s_2 = \frac{-2\xi . w_0 \pm \sqrt{4\xi^2 w_0^2 - 4w_0^2}}{2}$$

$$s_1 s_2 = -\xi w_0 \pm w_0\sqrt{\xi^2 - 1}$$

; ξ = coeficiente de amortecimento
w_0 = frequência natural não amortecida

Quando $\xi > 1$ (rede sobreamortecida) (raízes reais)

$$s_1 s_2 = -\xi w_0 \pm w_0\sqrt{\xi^2 - 1}$$

$$X(t) = K_1 e^{-(\xi w_0 + w_0\sqrt{\xi^2-1})t} + K_2 . e^{-(\xi . w_0 - w_0\sqrt{\xi^2-1})t}$$

Quando $\xi < 1$ (rede subamortecida) (raízes imaginárias)

$$s_1 s_2 = -\xi w_0 \pm j w_0\sqrt{1-\xi^2} \rightarrow p/ \sqrt{\xi^2-1} < 1$$

$$x(t) = K_1 e^{-\xi w_0 t} \cos[w_0\sqrt{(\xi^2-1)}\, t + \phi]$$

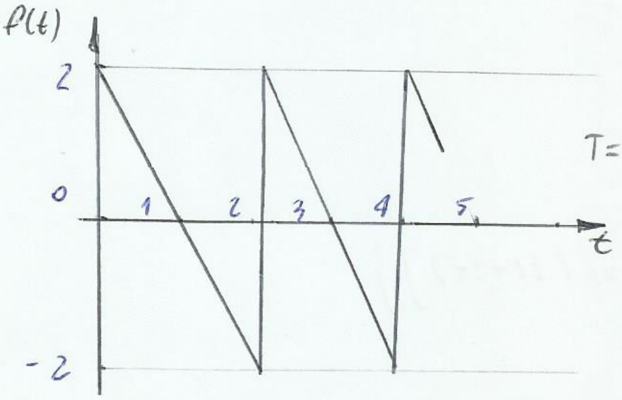
Quando $\xi = 1$ (rede criticamente amortecida) raízes reais iguais

$s_1 s_2 = -w_0 \rightarrow \xi = 1$

$x(t) = K_1 t e^{-w_0 t} + K_2 e^{-w_0 t}$

Prova P1 - 2sem. 2011 (Diurno)

1) Determinar a transformada de Laplace da onda periódica



$T = 2$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}}$$

$$f(t) = 2u(t) - 2t\,u(t) + 2(t-2)u(t-2) + 4u(t-2) - 2u(t-2)$$

$$= 2u(t) - 2t\,u(t) + 2(t-2)u(t-2) + 2u(t-2)$$

$$F_1(s) = \left[\frac{2}{s} - \frac{2}{s^2} + \frac{2e^{-2s}}{s^2} + \frac{2e^{-2s}}{s}\right]$$

$$F_1(s) = \frac{2}{s^2}\left(s - 2 + e^{-2s} + e^{-2s}s\right)$$

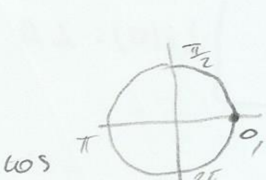
$$F(s) = \frac{2}{s^2}\left[\frac{s - 2 + e^{-2s} + e^{-2s}s}{1 - e^{-2s}}\right]$$

2-) Determinar a transformada de Laplace das funções abaixo

$$f(t) = 2e^{-t}\underbrace{\cos(2t)\,u(t-\pi)}_{f_1(t)}$$

$$F(s) = \mathcal{L}[f(t)] = 2\,F_1(s+1)$$

Determinando $F_1(s)$



$$F_1(s) = \mathcal{L}[\cos(2t)\,u(t-\pi)]$$

$$= \mathcal{L}[\cos(2(t+\pi))] \cdot e^{-\pi s}$$

$$= \mathcal{L}[\cos(2t + 2\pi)] \cdot e^{-\pi s}$$

$$= \mathcal{L}[\cos 2t \underbrace{\cos 2\pi}_{1} - \sin 2t \underbrace{\sin 2\pi}_{0}]\, e^{-\pi s}$$

$$= \mathcal{L}[\cos 2t]\, e^{-\pi s}$$

$$= \frac{s}{s^2 + 4} \cdot e^{-\pi s} \quad \therefore \quad F_1(s+1) = \frac{(s+1)}{(s+1)^2 + 4}\, e^{-\pi(s+1)}$$

$$\boxed{\text{logo } F(s) = \frac{2(s+1)\,e^{-\pi(s+1)}}{(s+1)^2 + 4}}$$

$f(t) = e^{-2t}\,\sin^2(2t)\,u(t) \quad f_1(t)$

$F(s) = \mathcal{L}[f(t)] = F_1(s+2)$

$F_1(s) = \mathcal{L}[f_1(t)]$

$= \mathcal{L}[\sin^2(2t)]$

$= \mathcal{L}[\sin(2t)\sin(2t)]$

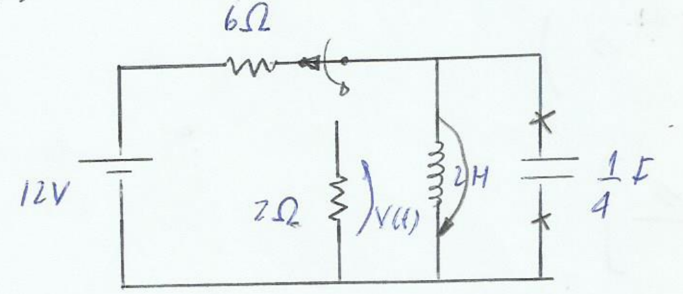
$= \mathcal{L}\left\{ \frac{1}{2}\left[\overset{1}{\cos(2t-2t)} - \cos(2t+2t)\right]\right\}$

$= \frac{1}{2}\,\mathcal{L}[1 - \cos 4t]$

$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+16}\right]$

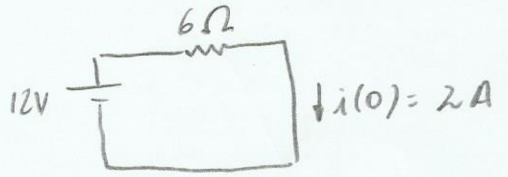
$F(s) = F_1(s+2)$ logo $F(s) = \frac{1}{2}\left[\frac{1}{(s+2)} - \frac{s+2}{(s+2)^2+16}\right]$

3-)

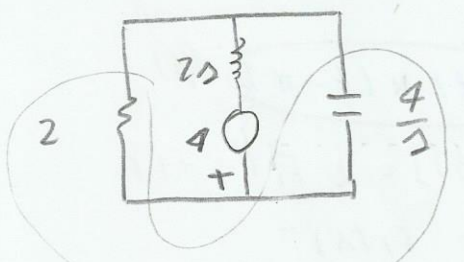


- Determine $V(t)$ para $t>0$

Para $t<0$



Para $t>0$



$2 \parallel \frac{4}{s} = \dfrac{\frac{8}{s}}{\frac{2s+4}{s}} = \frac{8}{2s+4}$

$V(s) = \dfrac{\frac{8}{\cancel{2s+4}}}{\frac{8+4s^2+6s}{\cancel{2s+4}}} \cdot 4 = \frac{32}{4(s^2+2s+2)}$

$$V(s) = \frac{8}{s^2+2s+2} = \frac{K_1}{s+1-j} + \frac{{}^*K_1}{s+1+j}$$

$s^2+2s+2=0$

$$s = \frac{-2 \pm \sqrt{4-8}}{2}$$

$s = -1 \pm j$

$$K_1 = (s+1-j)F(s)\Big|_{s=-1+j} = \frac{8}{s+1+j} = \frac{8}{-1+j+1+j} = \frac{8\angle 0}{2\angle 90} = 4\angle -90$$

$v(t) = 8e^{-t}\cos(t-90)\,u(t)$ (V)

4-) Resposta do impulso unitário: $v_o(t) = [1 + te^{-2t}]\,u(t)$ (V)

Determinar a resposta para $v_i(t) = e^{-4t}u(t)$

$$H(s) = \frac{Y(s)}{X(s)}$$

Para $x(t) = \delta(t) \rightarrow X(s) = 1$

$\therefore H(s) = X(s)\,Y(s) \quad \therefore H_1(s) = Y(s)$

$H(s) = \mathcal{L}[1 + te^{-2t}]$

$= \mathcal{L}[1] + \mathcal{L}[\,t\,\overset{f(t)}{e^{-2t}}\,]$

$= \mathcal{L}[1] + F_1(s+2) \quad ; \quad F_1(s) = \frac{1}{s^2}$

$$= \frac{1}{s} + \frac{1}{(s+2)^2}$$

$$= \frac{(s+2)^2 + s}{s(s+2)^2}$$

$$X(s) = \frac{1}{s+4} \qquad Y_1(s) = \frac{1}{(s+4)} \cdot \frac{(s+2)^2+s}{s(s+2)^2}$$

$$Y_1(s) = \frac{s^2+4s+4+s}{s(s+4)(s+2)^2} = \frac{s^2+5s+4}{s(s+4)(s+2)^2} = \frac{(s+1)\cancel{(s+4)}}{s\cancel{(s+4)}(s+2)^2}$$

$s^2+5s+4=0$

$$s = \frac{-5 \pm \sqrt{25-16}}{2} \quad s_1 = \frac{-5+3}{2} = -1 \qquad s_2 = \frac{-5-3}{2} = -4$$

$$V_1(s) = \frac{s+1}{s(s+2)^2} = \frac{K_1}{s} + \frac{K_{21}}{s+2} + \frac{K_{22}}{(s+2)^2}$$

$$K_1 = sF(s)\Big|_{s=0} = \frac{s+1}{(s+2)^2} = \frac{1}{4}$$

$$K_{21} = \frac{d}{ds}\left[(s+2)^2 F(s)\right]\Big|_{s=-2} = \frac{d}{ds}\left(\frac{s+1}{s}\right) = \frac{s-(s+1)}{s^2}\Big|_{s=-2}$$

$$= \frac{s-s-1}{s^2}\Big|_{s=-2} = \frac{-1}{(-2)^2} = \frac{-1}{4}$$

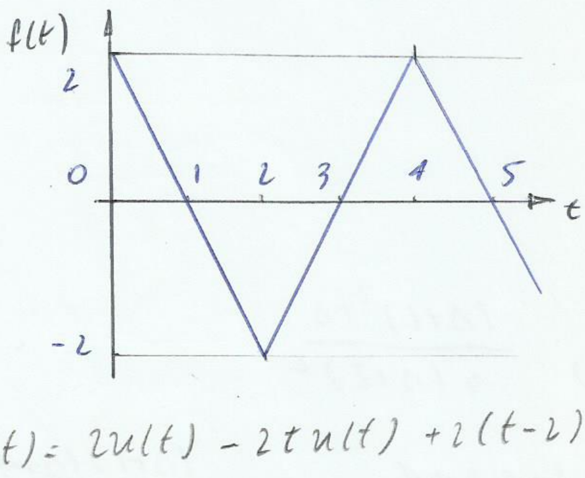
$$K_{22} = (s+2)^2 F(s)\Big|_{s=-2} = \frac{s+1}{s}\Big|_{s=-2} = \frac{-2+1}{-2} = \frac{-1}{-2} = \frac{1}{2}$$

$$V_1(s) = \frac{\frac{1}{4}}{s} + \frac{\left(-\frac{1}{4}\right)}{s+2} + \frac{\frac{1}{2}}{(s+2)^2}$$

$$\therefore V_1(t) = \left(\frac{1}{4} - \frac{1}{4}e^{-2t} + \frac{1}{2}te^{-2t}\right)u(t)\ (V)$$

Prova P1 - 1º sem 2011 (Noturno)

1-) Determinar a transformada de Laplace da forma de onda periódica



$$F(s) = \frac{F_1(s)}{1-e^{-sT}}$$

$T = 4$

(Preguiça de arrumar!)

$$f_1(t) = 2u(t) - 2tu(t) + 2(t-2)u(t-2) + 2(t-2)u(t-2) - 2u(t-4)$$

$$= 2u(t) - 2tu(t) + 4(t-2)u(t-2) - 2u(t-4)$$

$$F_1(s) = \frac{2}{s} - \frac{2}{s^2} + \frac{4e^{-2t}}{s^2} - \frac{2e^{-4t}}{s}$$

$$= \frac{2}{s^2}\left(s - 1 + 2e^{-2t} - e^{-4t}\right)$$

$$F(s) = \frac{2}{s^2}\left[\frac{s-1+2e^{-2t}-e^{-4t}}{1-e^{-4t}}\right]$$

2-) Determinar a transformada de Laplace das funções abaixo

a) $f(t) = 4 e^{-3t} \underbrace{\cos 2t \; u(t-\pi)}_{f_1(t)}$

$F(s) = \mathcal{L}\{f(t)\} = 4 . F_1(s+3)$

Determinando $F_1(s)$

$F_1(s) = \mathcal{L}[\cos 2(t+\pi)] e^{-\pi s}$

$= \mathcal{L}[\cos(2t + 2\pi)] e^{-\pi s}$

$= \mathcal{L}[\cos 2t \overset{\to 1}{\cos 2\pi} - \operatorname{sen} 2t \overset{0}{\operatorname{sen} 2\pi}] e^{-\pi s}$

$= \mathcal{L}[\cos 2t] e^{-\pi s}$

$= \dfrac{s}{s^2+4} \cdot e^{-\pi s}$

$$\boxed{F(s) = \frac{4 \times (s+3) e^{-\pi(s+3)}}{(s+3)^2 + 4}}$$

b) $f(t) = e^{-3t} . \underbrace{t . \operatorname{sen} 2t \; u(t)}_{f_1(t)}$

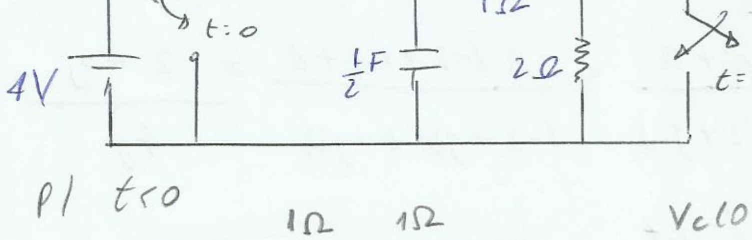
$F(s) = \mathcal{L}[f(t)] = F_1(s+3)$

$F_1(s) = \mathcal{L}[t \underbrace{\operatorname{sen} 2t}_{f_2(t)}] = -\dfrac{d F_2(s)}{dt}$

$F_2(s) = \dfrac{2}{s^2+4}$ $\quad F_1(s) = \dfrac{4s}{(s^2+4)^2}$

$F(s) = \dfrac{4(s+3)}{[(s+3)^2+4]^2}$

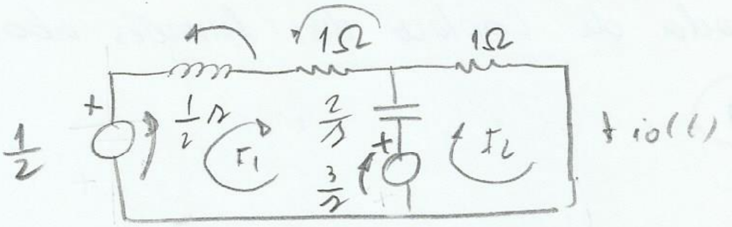
3-) Determine $i_o(t)$ para $t>0$



p/ t<0

$V_c(0) = \dfrac{3}{3+1} . 4 = 3V$

$I_L(0) = \dfrac{4}{4} = 1 A$



$$\begin{cases} \left(\frac{s}{2}+1+\frac{2}{s}\right) I_1 - \frac{2}{s} I_2 = \frac{1}{2} - \frac{3}{s} \\ -\frac{2}{s} I_1 + \left(1+\frac{2}{s}\right) = \frac{3}{s} \end{cases}$$

$$\begin{vmatrix} \frac{s}{2}+1+\frac{2}{s} & -\frac{2}{s} \\ -\frac{2}{s} & 1+\frac{2}{s} \end{vmatrix} =$$

$$= \frac{s}{2}+1+\frac{2}{s}+1+\frac{2}{s}+\cancel{\frac{4}{s^2}}-\cancel{\frac{4}{s^2}} = \frac{s}{2}+2+\frac{4}{s} = \frac{s^2+4s+8}{2s}$$

$$\begin{vmatrix} \frac{s}{2}+1+\frac{2}{s} & \frac{1}{2}-\frac{3}{s} \\ -\frac{2}{s} & \frac{3}{s} \end{vmatrix} =$$

$$+\frac{3}{2}+\frac{3}{s}+\cancel{\frac{6}{s^2}}+\frac{1}{s}-\cancel{\frac{6}{s^2}} = \frac{3}{2}+\frac{4}{s} = \frac{3s+8}{2s}$$

$$I_2 = \frac{\frac{3s+8}{\cancel{2s}}}{\frac{s^2+4s+8}{\cancel{2s}}} = \frac{3s+8}{s^2+4s+8}$$

$$s^2+4s+8=0$$

$$s = \frac{-4 \pm \sqrt{16-32}}{2}$$

$$s = -2 \pm j2$$

$$I_o(s) = \frac{K_1}{s+2-j2} + \frac{{}^*K_1}{s+2+j2}$$

$$K_1 = (s+2-j2)F(s)\Big|_{s=-2+j2} = \frac{3s+8}{(s+2+j)} = \frac{3(-2+j2)+8}{(-2+j2+2+j2)} = \frac{2+6j}{4j}$$

$$K_1 = \frac{3{,}61 \angle 71{,}6}{4 \angle 90} = 1{,}80 \angle -33{,}7$$

$$\therefore \boxed{I(t) = 3{,}6\, e^{-2t} \cos(t-18{,}4)}$$

4. Resposta do impulso unitário : $v_o(t) = u(t)\,[1 - e^{-2t}]\ (V)$

Determinar a resposta para $v_i(t) = e^{-t}\,u(t)\ (V)$

$$H(s) = \mathcal{L}[1 - e^{-2t}] = \frac{1}{s} - \frac{1}{s+2} \qquad V_i(s) = \frac{1}{s+1}$$

$$Y(s) = \left(\frac{1}{s} - \frac{1}{s+2}\right)\cdot\frac{1}{s+1}$$

$$= \frac{\cancel{s}+2-\cancel{s}}{s(s+2)(s+1)} = \frac{2}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$$K_1 = sF(s)\Big|_{s=0} = \frac{2}{(s+1)(s+2)} = 1$$

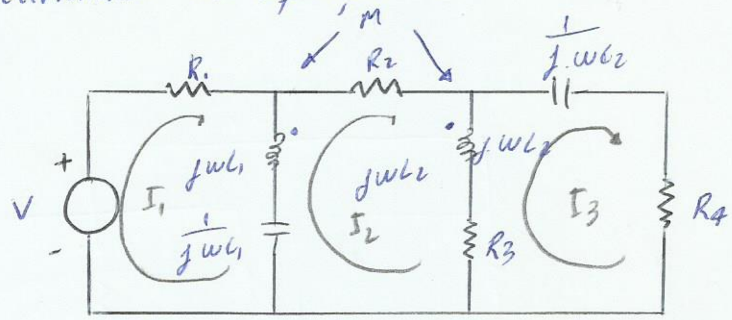
$$K_2 = (s+1)F(s)\Big|_{s=-1} = \frac{2}{s(s+2)} = \frac{2}{(-1)(-1+2)} = -2$$

$$K_3 = (s+2)F(s)\Big|_{s=-2} = \frac{2}{s(s+1)} = \frac{2}{(-2)(-2+1)} = 1$$

$$\therefore\ Y(s) = \frac{1}{s} + \frac{(-2)}{s+1} + \frac{1}{s+2}$$

$$\therefore\ y(t) = (1 - 2e^{-t} + e^{-2t})\,u(t)\ V$$

13.6 Determinar as equações de malha

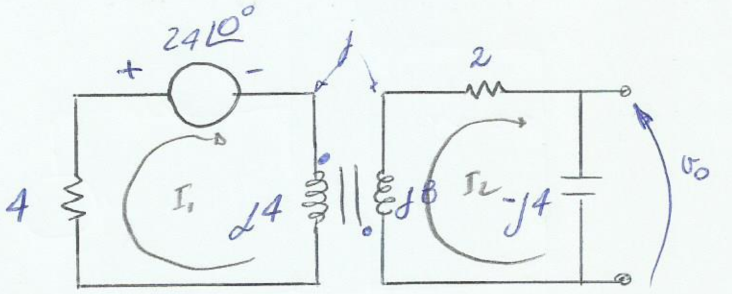


I) $R_1 I_1 + j\omega L_1 (I_1 - I_2) + j\omega M (I_2 - I_3) + \frac{1}{j\omega C_1} (I_1 - I_2) - V = 0$

II) $R_2 \cdot I_2 + j\omega L_2 (I_2 - I_3) + j\omega M (I_1 - I_3) + R_3 (I_2 - I_3) + \frac{1}{j\omega C_1} (I_2 - I_1) + j\omega L_1 (I_2 - I_1) + j\omega M (I_3 - I_2) = 0$

III) $\frac{1}{j\omega C} I_3 + R_4 I_3 + R_3 (I_3 - I_2) + j\omega L_2 (I_3 - I_2) - j\omega M (I_1 - I_2) = 0$

13.2) Determinar I_1, I_2 e a tensão V_o



$I_1 = 4{,}29 \angle 137{,}13$

$= -4{,}29 \angle -42°$

$I_2 = 0{,}96 \angle -16{,}3$

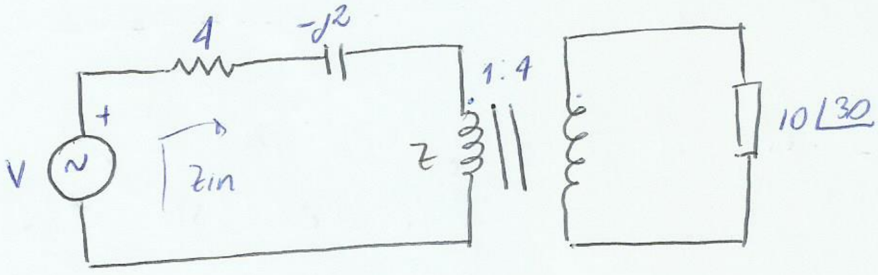
$V_o =$

(I) $4 I_1 + j4 \cdot I_1 + j I_2 + 24 \angle 0 = 0$

(II) $2 I_2 - j4 I_2 + j8 I_2 + j I_1 = 0$

Circuitos elétricos
Coleção show

Calcular Z_{in}

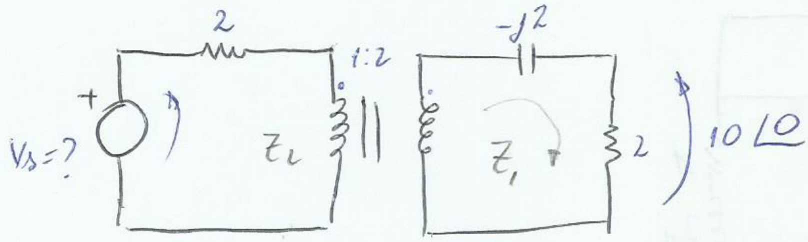


$Z = 10 \angle 30 \cdot \left(\frac{1}{4}\right)^2 \quad \therefore \quad Z = 0{,}625 \angle 30 = 0{,}54 + j\,0{,}31$

$Z_{in} = 4 - j2 + 0{,}625 \angle 30 \quad \therefore \quad Z_{in} = 4{,}54 - 1{,}69j$

$= 4{,}84 \angle -20{,}4$

Determinar V_s na rede ; $V_0 = 10\angle 0°$



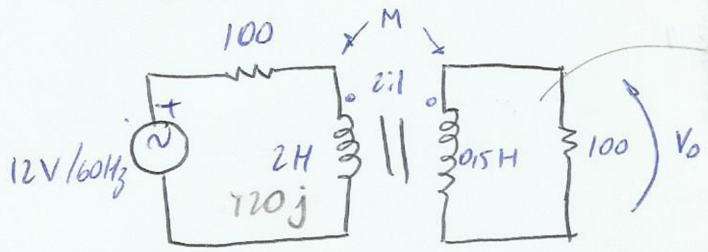
$Z_1 = 2 - j2$ $\quad\therefore\quad$ $Z_2 = (2-j2).\left(\frac{1}{2}\right)^2 = 0{,}5 - j0{,}5$

$Z_{in} = 2{,}5 - j0{,}5$

$I_1 = \dfrac{10\angle 0}{2} = 5\angle 0$ $\qquad$ $I_1 = 2 \times 5\angle 0 = 10\angle 0$

$V_s = (2{,}5 - j0{,}5).\,10\angle 0 = (2{,}55\angle -11{,}31) \times 10\angle 0 = 25{,}5\angle -11{,}3°\ V$

Determinar V_0 considerando trafo ideal e Real

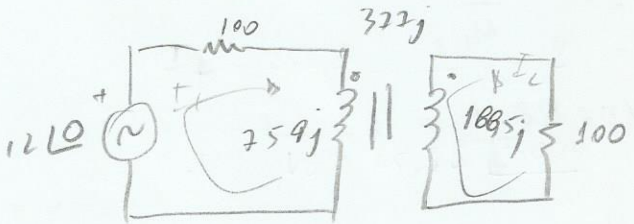


(trafo ideal)

$Z_{in} = 100.\left(\frac{2}{1}\right)^2 = 400\Omega$

$V_1 = \dfrac{400}{400+100} \cdot 12 = 9{,}6\ V$ $\qquad\therefore\ V_0 = 9{,}6\left(\frac{1}{2}\right) = 4{,}8\ V$

$\dot{V}_0 = 4{,}8\angle 0\ V$



$$\Delta = \begin{vmatrix} 754j + 100 & -377j \\ -377j & 188{,}5j + 100 \end{vmatrix}$$

$$\begin{cases} 100 I_1 + 754j I_1 - 377j I_2 - 12 = 0 \\ 100 I_2 + 188{,}5j I_2 - 377j I_1 = 0 \end{cases}$$

$\Delta = 94779\angle 83{,}9°$

$$\begin{cases} (754j + 100) I_1 - 377j I_2 = 12 \\ -377j I_1 + (100 + 188{,}5j) I_2 = 0 \end{cases}$$

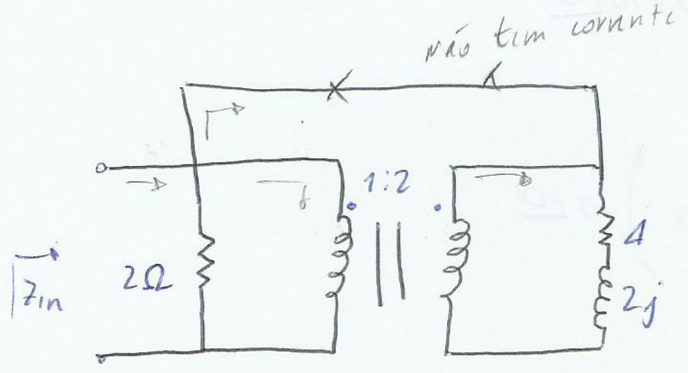
$$\begin{vmatrix} 754j + 100 & 12 \\ -377j & 0 \end{vmatrix} = 4524j$$

$I_2 = 0{,}047\angle 6{,}1°$

$I_2 = \dfrac{4524j}{94779\angle 83{,}9}$

$V_0 = 4{,}77\angle 6{,}1°\ V$

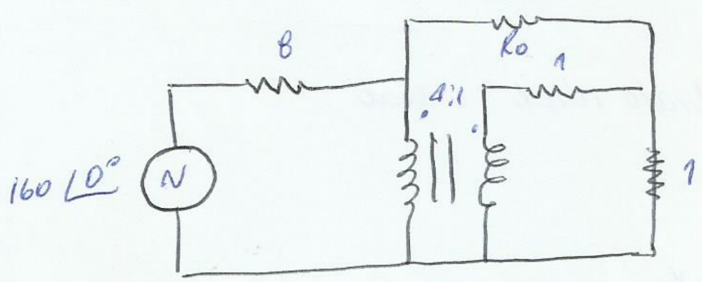
13.53) (Irwin) Determine a impedancia da entrada



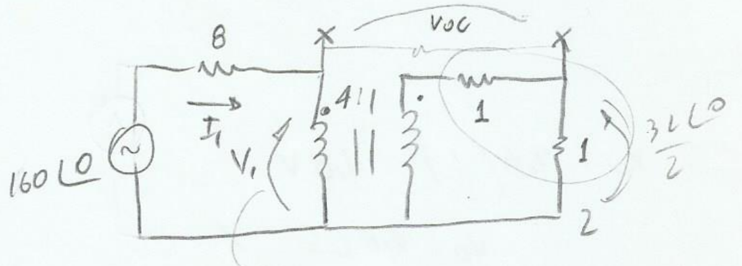
$$Z = (4+2j)\cdot\left(\frac{1}{2}\right)^2 = 1{,}12 \angle 6{,}64$$

$$Z_{in} = \frac{2 \times 1{,}12 \angle 6{,}64}{2 + 1{,}12 \angle 6{,}64} \qquad Z_{in} = 0{,}74 \angle 17{,}1^\circ$$

13.54) Irwin Determinar Ro p/ maxima transf. de Potencia



Determinando 0



$$Z = 2\cdot(4^2) = 32$$

$$V_1 = \frac{32}{32+8}\cdot 160 = 128 \angle 0$$

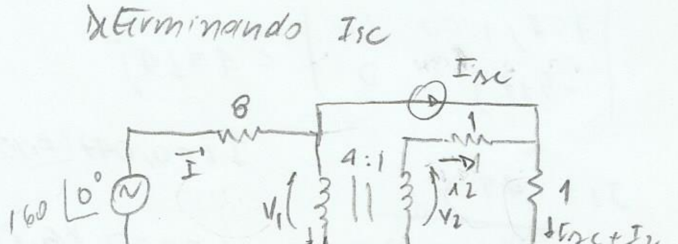
Determinando Voc

$$V_2 = V_1 \cdot \frac{N_2}{N_1}$$

$$V_2 = 128 \cdot \frac{1}{4} = 32 \angle 0$$

$$V_{oc} = 128\angle 0 - \frac{32}{2}\angle 0 = 112 \angle 0 \ \text{V}$$

Determinando Isc



$8.I + V_1 = 160$

* $8I + 1(I_{sc} + I_2) = 160$

* $I_1 = I_{sc} + I_1$

$I_2 + 1(I_{sc} + I_2) = V_2$

$V_1 = 1(I_{sc} + I_2)$

* $V_2 = \frac{V_1}{4} \quad I_1 = \frac{I_2}{4}$

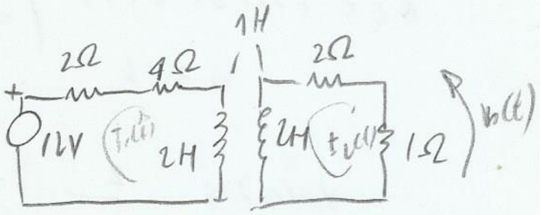
$$8I_1 + 1(I_{sc} + I_2) = 160$$
$$8(I_{sc} + I_1) + I_{sc} + I_2 = 160$$
$$8\left(I_{sc} + \frac{I_2}{4}\right) + I_{sc} + I_2 = 160 \rightarrow 8I_{sc} + 2I_2 + I_{sc} + I_2 = 160$$
$$9I_{sc} + 3I_2 = 160$$

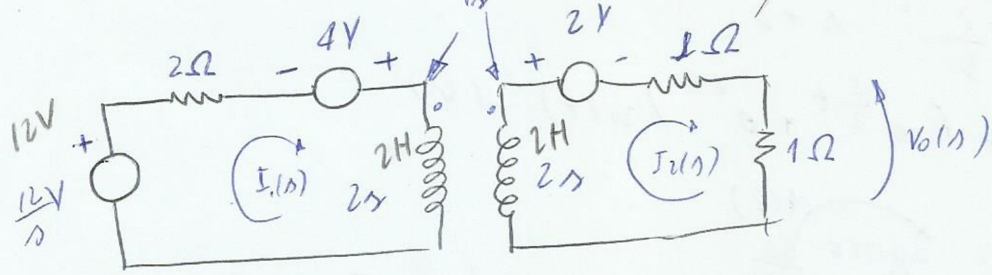
$$1I_2 + 1(I_{sc} + I_2) = V_2$$
$$1I_2 + I_{sc} + I_2 = \frac{V_1}{4}$$
$$I_2 + I_{sc} + I_2 = \frac{I_{sc} + I_2}{4}$$
$$4I_2 + 4I_{sc} + 4I_2 = I_{sc} + I_2 \rightarrow 3I_{sc} + 7I_2 = 0$$

$$\begin{cases} 9 \cdot I_{sc} + 3I_2 = 160 \\ 3I_{sc} + 7I_2 = 0 \end{cases} \qquad \Delta = \begin{vmatrix} 9 & 3 \\ 3 & 7 \end{vmatrix} = 54$$

$$\Delta I_s = \begin{vmatrix} 160 & 3 \\ 0 & 7 \end{vmatrix} = 1120 \qquad I_s = \frac{1120}{54} = \underline{20{,}74\ A}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{112}{20{,}74} = \boxed{5{,}4\,\Omega}$$





$$I_1(2+2s) + I_2 \cdot 2s - sI_2 = \frac{12}{s} + 4$$
$$I_2(2+2s) - sI_1 = -2$$

$$\Delta = \begin{vmatrix} 2+2s & -s \\ -s & 2+2s \end{vmatrix} = 3s^2 + 8s + 4$$

$$\Delta I_2 = \begin{vmatrix} 2+2s & \frac{12+4s}{s} \\ -s & -2 \end{vmatrix} = \cancel{-4s} - 4 + 12 + \cancel{4s} = 8$$

$$I_2(s) = \frac{8}{3s^2 + 8s + 4} \qquad V_0(s) = \frac{8}{3s^2 + 8s + 4}$$

$$\frac{8}{3s^2+8s+4} = \frac{(8/3)}{s^2+\frac{8}{3}s+\frac{4}{3}}$$

$$s^2+\frac{8}{3}s+\frac{4}{3}=0$$

$$s = \frac{-\frac{8}{3} \pm \sqrt{\frac{64}{9}-\frac{16}{3}}}{2} = \frac{-\frac{8}{3} \pm \sqrt{\frac{64-48}{9}}}{2}$$

$$= \frac{-\frac{8}{3} \pm \frac{4}{3}}{2} \quad \therefore \quad s_1 = \frac{-2}{3} \quad s_2 = -2$$

$$\frac{8/3}{s^2+\frac{8}{3}s+\frac{4}{3}} = \frac{K_1}{(s+\frac{2}{3})} + \frac{K_2}{(s+2)}$$

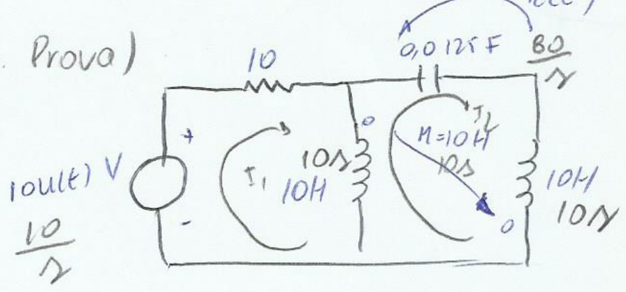
$$K_1 = (s+\tfrac{2}{3}).F(s)\Big|_{s=-\frac{2}{3}} = \frac{\frac{8}{3}}{s+2} = \frac{\frac{8}{3}}{\frac{-2}{3}+2} = 2$$

$$K_2 = (s+2)\,F(s)\Big|_{s=-2} = \frac{\frac{8}{3}}{(s+\frac{2}{3})} = \frac{\frac{8}{3}}{-2+\frac{2}{3}} = -2$$

$$Vo(s) = \frac{2}{s+\frac{2}{3}} - \frac{2}{s+2}$$

$$Vo(t) = 2\left(e^{-\frac{2}{3}t} - e^{2t}\right)u(t) \quad (V)$$

(Ex. Prova)



logo $I_2 = \dfrac{s}{2(s^2+2s+2)}$

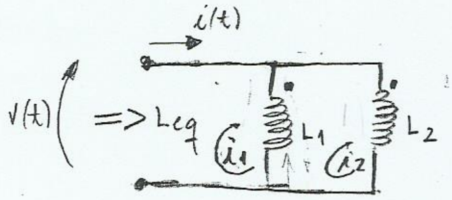
$V_c(t) = \dfrac{80}{s} I_2 = \dfrac{40}{s^2+2s+2}$

$$\begin{cases} I_1(10+10s) - 10sI_2 = \frac{10}{s} \\ I_2(20s+\frac{80}{s}) + 10s(I_2-I_1) + 10s(I_2-I_1) = 0 \end{cases}$$

$$\begin{cases} (10+10s)I_1 - 10s\,I_2 = \frac{10}{s} \\ -20s\,I_1 + (40s+\frac{80}{s})I_2 = 0 \end{cases}$$

CIRCUITOS ELETRICOS III

28/09/12



$i(t) = i_1(t)$

$$j\omega L_1 (i_1 - i_2) + j\omega M i_2 - V_1 = 0$$

$$j\omega L_2 i_2 - j\omega M (i_2 - i_1) + j\omega L_1 (i_2 - i_1) - j\omega M i_2 = 0$$

$$\begin{cases} (j\omega L_1) \cdot i_1 + (j\omega M - j\omega L_1) i_2 = V_1 \\ (j\omega M - j\omega L_1) i_1 + (j\omega L_1 + j\omega L_2 - 2 j\omega M) \cdot i_2 = 0 \end{cases}$$

$$\begin{bmatrix} (j\omega L_1) & (j\omega M - j\omega L_1) \\ (j\omega M - j\omega L_1) & (j\omega L_1 + j\omega L_2 - 2j\omega M) \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix}$$

$$\Delta = \cancel{j^2 \omega^2 L_1^2} + j^2\omega^2 L_1 L_2 - \cancel{2 j^2 \omega^2 L_1 M} - j^2 \omega^2 M^2 + \cancel{2 j^2 \omega^2 L_1 M} - \cancel{j^2 \omega^2 L_1^2}$$

$$\underline{\Delta = j^2 \omega^2 L_1 L_2 - j^2 \omega^2 M^2}$$

$$\Delta i_1 = \begin{vmatrix} V & (j\omega M - j\omega L_1) \\ 0 & (j\omega L_1 + j\omega L_2 - 2j\omega M) \end{vmatrix} \Rightarrow \Delta i_1 = j\omega (L_1 + L_2 - 2M) \cdot V$$

$$i_1 = \frac{\Delta i_1}{\Delta} \Rightarrow i_1 = \frac{j\omega (L_1 + L_2 - 2M) \cdot V}{j^2 \omega^2 (L_1 L_2 - M^2)} \Rightarrow \frac{\cancel{j}\omega^{\cancel{2}} (L_1 L_2 - 2M)}{\cancel{j\omega} (L_1 + L_2 - M^2)} = \frac{V}{i_1}$$

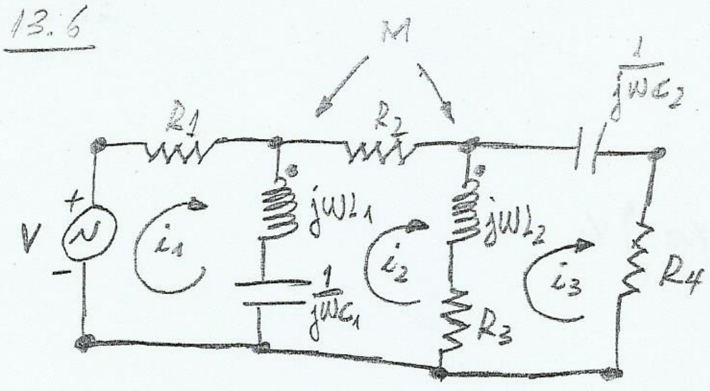
$$L_{eq} = \frac{V}{i_1} \Rightarrow L_{eq} = \frac{j\omega (L_1 L_2 - M^2)}{(L_1 + L_2 - 2M)}$$

$$L_{eq} = \frac{L_1 \cdot L_2 - M^2}{L_1 + L_2 + 2M}$$

$L_1 = L_2 = L \quad \text{y } k \simeq 1$

$$L_{eq} = \frac{L^2 - M^2}{2L + 2M} = \frac{0}{4L} = 0$$

13.6



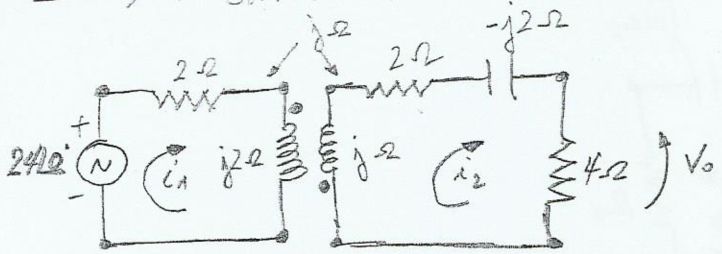
$$i_1\cdot\left(R_1+j\omega L_1+\frac{1}{j\omega C_1}\right)-i_2\cdot\left(j\omega L_1+\frac{1}{j\omega C_1}\right)+i_2\cdot j\omega M-i_3 j\omega M=V$$

$$-i_1\cdot\left(j\omega L_1+\frac{1}{j\omega C_1}\right)+i_2\cdot\left(j\omega L_1+R_2+j\omega L_2+R_3+\frac{1}{j\omega C_1}\right)+j\omega M i_1-j\omega M i_2-j\omega M i_2$$

$$+j\omega M i_3=0$$

$$-i_2\left(j\omega L_2+R_3\right)+i_3\left(R_3+j\omega L_2+\frac{1}{j\omega C_2}+R_4\right)+j\omega M i_2-j\omega M i_1=0$$

13.1) Problemas:



$$\begin{cases} i_1 \cdot (2+j2) + i_2 \cdot (j) = 24 \\ i_2 \cdot (j - j2 + 4 + 2) + i_1 \cdot (j) = 0 \end{cases}$$

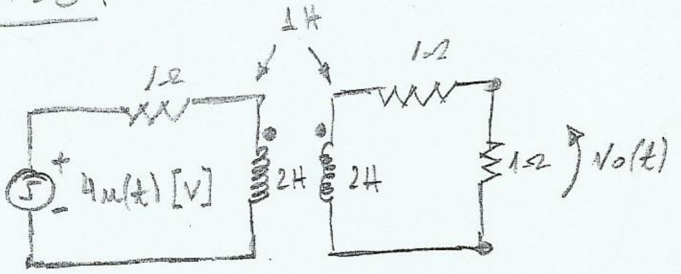
$$\Delta = \begin{vmatrix} 2+j2 & j \\ j & 6-j \end{vmatrix} \Rightarrow \Delta = 12 - j2 + j12 + 2 + 1 \Rightarrow \Delta = 15 + j10 = 18{,}03 \angle 33{,}7°$$

$$\Delta i_2 = \begin{vmatrix} 2+j2 & 24 \\ j & 0 \end{vmatrix} \Rightarrow \Delta i_2 = -j24 = 24 \angle -90°$$

$$i_2 = \frac{24 \angle -90°}{18{,}03 \angle 33{,}7°} \Rightarrow i_2 = 1{,}33 \angle -123{,}7° \ [A]$$

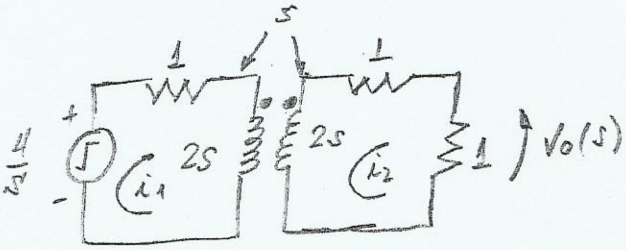
$$\dot{V}_0 = (4 \angle 0°) \cdot (1{,}33 \angle -123{,}7°) \Rightarrow \dot{V}_0 = 5{,}32 \angle -123{,}7° \ [V]$$

17.38:



determinar Vo(t) para t>0.

Condições iniciais é nula na bobina.



$$i_1 \cdot (1+2s) - i_2 \cdot (s) = \frac{4}{s}$$
$$-i_1 \cdot (s) + i_2 \cdot (2+2s) = 0$$
$$\Rightarrow \begin{bmatrix} (1+2s) & -(s) \\ -(s) & (2+2s) \end{bmatrix} \cdot \begin{bmatrix} i_1(s) \\ i_2(s) \end{bmatrix} = \begin{bmatrix} \frac{4}{s} \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} (1+2s) & -(s) \\ -(s) & (2+2s) \end{vmatrix} \Rightarrow \Delta = 2+2s+4s+4s^2-s^2 \Rightarrow$$
$$\Delta = 3s^2+6s+2$$

$$\Delta i_2 = \begin{vmatrix} (1+2s) & \frac{4}{s} \\ -(s) & 0 \end{vmatrix} \Rightarrow \Delta i_2 = 4$$

$$i_2(s) = \frac{4}{3s^2+6s+2} \quad \therefore \quad V_o(s) = 1 \cdot \frac{4}{3s^2+6s+2} \Rightarrow V_o(s) = \frac{4}{3} \cdot \frac{1}{s^2+2s+2/3}$$

$$V_o(s) = \frac{4}{3} \cdot \frac{1}{(s+0,423)\cdot(s+1,577)} = \frac{K_1}{s+0,423} + \frac{K_2}{s+1,577}$$

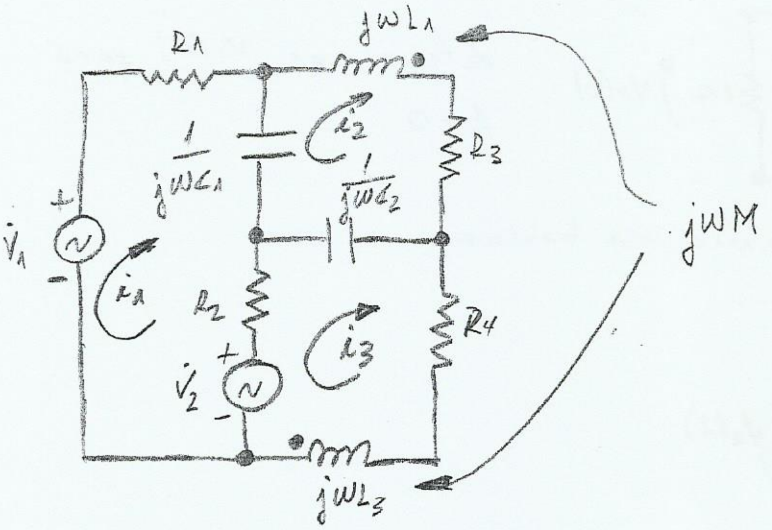
$$K_1 = \frac{4}{3} \cdot \frac{1}{s+1,577}\Bigg|_{s=-0,423} \Rightarrow K_1 = \frac{4}{3} \cdot \frac{1}{-0,423+1,577} \Rightarrow \boxed{K_1 = 1,155}$$

$$K_2 = \frac{4}{3} \cdot \frac{1}{s+0,423}\Bigg|_{s=-1,577} \Rightarrow K_2 = \frac{4}{3} \cdot \frac{1}{-1,577+0,423} \Rightarrow \boxed{K_2 = -1,155}$$

$$V_o(s) = \frac{1,155}{s+0,423} - \frac{1,155}{s+1,577} \quad \therefore \quad V_o(t) = 1,155 \cdot e^{-0,423t} - 1,155\, e^{-1,577t}\, u(t)$$

$$\boxed{V_o(t) = 1,155 \cdot \left(e^{-0,423t} - e^{-1,577t}\right) \cdot u(t) \ [V]}$$

Problema 13.4:



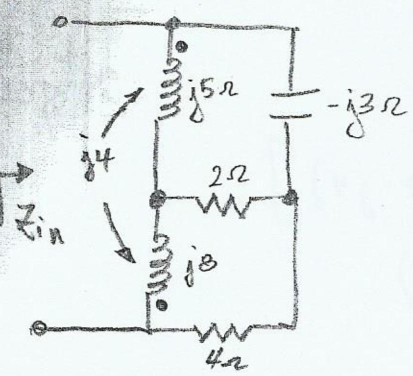
$$i_1\cdot\left(R_1+R_2+\frac{1}{j\omega C_1}\right) - i_2\cdot\left(\frac{1}{j\omega C_1}\right) - i_3\cdot(R_2) = \dot{V}_1 - \dot{V}_2$$

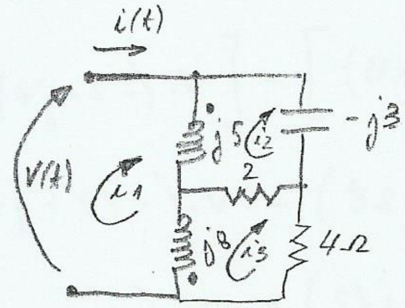
$$-i_1\cdot\left(\frac{1}{j\omega C_1}\right) + i_2\cdot\left(R_3 + j\omega L_1 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right) - i_3\cdot\left(\frac{1}{j\omega C_2}\right) + i_3\cdot(j\omega M) = 0$$

$$-i_1\cdot(R_2) - i_2\cdot\left(\frac{1}{j\omega C_2}\right) + i_3\cdot\left(R_2+R_4+\frac{1}{j\omega C_2}+j\omega L_3\right) + i_2\cdot(j\omega M) = \dot{V}_2$$

$$\begin{bmatrix} \left(R_1+R_2+\frac{1}{j\omega C_1}\right) & -\left(\frac{1}{j\omega C_1}\right) & -(R_2) \\ -\left(\frac{1}{j\omega C_1}\right) & \left(R_3+j\omega L_1+\frac{1}{j\omega C_1}+\frac{1}{j\omega C_2}\right) & \left(j\omega M-\frac{1}{j\omega C_2}\right) \\ -(R_2) & \left(j\omega M-\frac{1}{j\omega C_2}\right) & \left(R_2+R_4+\frac{1}{j\omega C_2}+j\omega L_3\right) \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \dot{V}_1-\dot{V}_2 \\ 0 \\ \dot{V}_2 \end{bmatrix}$$

Lista de exercícios professor Benko . Redes magneticamente acoplado





$$i_1\cdot(j5+j8) - i_2\cdot(j5) - i_3\cdot(j8) - i_1\cdot(2\cdot j4) + i_2\cdot(j4) + i_3(j4) = V(t)$$

$$i_1\cdot(j5+j8-j8) + i_2\cdot(-j5+j4) + i_3\cdot(-j8+j4) = V(t)$$

$$i_1\cdot(j5) + i_2\cdot(-j) + i_3\cdot(-j4) = V(t)$$

$$-i_1\cdot(j5) + i_2(j5+2-j3) - i_3\cdot(2) + i_1\cdot(j4) - i_3\cdot(j4) = 0$$

$$i_1\cdot(-j5+j4) + i_2\cdot(j5-j3+2) + i_3\cdot(-2-j4) = 0$$

$$i_1\cdot(-j) + i_2\cdot(2+j2) + i_3\cdot(-2-j4) = 0$$

$$-i_1\cdot(j8) - i_2\cdot(2) + i_3\cdot(j8+2+4) + i_1\cdot(j4) - i_2\cdot(j4) = 0$$

$$i_1\cdot(j8+j4) + i_2\cdot(-2-j4) + i_3\cdot(6+j8) = 0$$

$$i_1\cdot(j12) + i_2\cdot(-2-j4) + i_3\cdot(6+j8) = 0$$

$$\Delta = \left|\begin{matrix} (j5) & (-j) & (-j4) \\ (-j) & (2+j2) & (-2-j4) \\ (j12) & (-2-j4) & (6+j8) \end{matrix}\right| \begin{matrix} (j5) & (-j) \\ (-j) & (2+j2) \\ (j12) & (-2-j4) \end{matrix}\Bigg| \Rightarrow \Delta = 184{,}3\angle{-154{,}2^\circ}$$

[A]

$$\Delta = \left[(j5)\cdot(2+j2)\cdot(6+j8) + (-j)\cdot(-2-j4)\cdot(j12) + (-j4)\cdot(-j)\cdot(-2-j4)\right] - [B]$$

$$[A] = [(-140-j20) + (-24-j48) + (8+j16) = -156-j52 = 164{,}44\angle{-161{,}5}$$

$$[B] = [(j12)\cdot(2+j2)\cdot(j4) + (-2-j4)(-2-j4)(j5) + (6+j8)(-j)(-j)]$$

$$[B] = (96+j96) + (-80-j60) + (-6-j8) = 10+j28 = 29{,}73\angle{70{,}34^\circ}$$

$$\Delta i_1 = \begin{vmatrix} V & -j & -j4 \\ 0 & 2+j2 & -2-j4 \\ 0 & -2-j4 & 6+j8 \end{vmatrix} \begin{matrix} V & -j \\ 0 & 2+j2 \\ 0 & -2-j4 \end{matrix} \Rightarrow \Delta i_1 = 14{,}42 \angle 56{,}31°$$

$$\Delta i_1 = \left[V \cdot (2+j2) \cdot (6+j8)\right] - \left[V \cdot (-2-j4)(-2-j4)\right]$$

$$\Delta i_1 = V \cdot (-4+j28) - V \cdot (-12+j16)$$

$$\Delta i_1 = V \cdot (8+j12) = V \cdot (14{,}42 \angle 56{,}31°$$

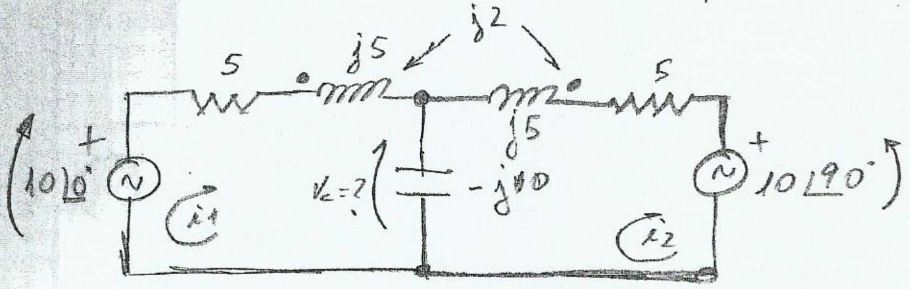
$$i_1 = \frac{V \cdot (14{,}42 \angle 56{,}31°)}{184{,}3 \angle -154{,}2°} \Rightarrow \frac{V}{i_1} = \frac{184{,}3 \angle -154{,}2°}{14{,}42 \angle 56{,}31°}$$

$$Z_{in} = 12{,}78 \angle -210{,}51°$$

?

$$Z_{in} = -11 + j6{,}5 \ \Omega$$

4) Calcular a tensão no capacitor V_c.



$$i_1 \cdot (5 + j5 - j10) - i_2 \cdot (-j10) - i_2 \cdot (j2) = 10$$

$$-i_1 \cdot (-j10) + i_2 \cdot (j5 + 5 - j10) - i_1 \cdot (j2) = -10\angle 90°$$

$$\begin{bmatrix} (5-j5) & (j8) \\ (j8) & (5-j5) \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 10\angle 90° \end{bmatrix}$$

$$\Delta = (5-j5)^2 - (j8)^2 \Rightarrow \Delta = (-50) - (-64) \Rightarrow \underline{\Delta = 14}$$

$$\Delta i_1 = 10 \cdot (5-j5) - (j10) \cdot (j8) \Rightarrow \Delta i_1 = (50 - j50) - (-80)$$

$$\underline{\Delta i_1 = 130 - j50}$$

$$\Delta i_2 = (j10) \cdot (5-j5) - (j8)(10) \Rightarrow \Delta i_2 = (50 + j50) - j80$$

$$\underline{\Delta i_2 = 50 - j30}$$

$$i_1 = 9,28 - j3,57 \qquad i_2 = 3,57 - j2,14$$

$$i_c = i_1 - i_2 \Rightarrow \underline{i_c = 5,71 - j1,43}$$

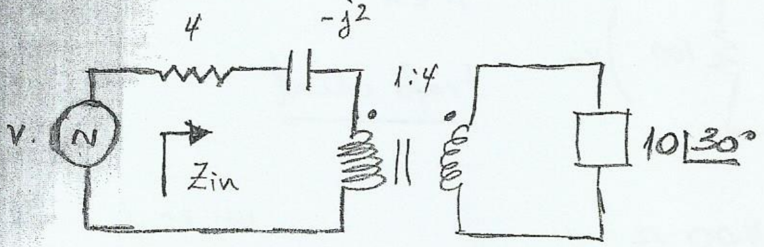
$$\dot{V}_c = i_c \cdot (-j10) \Rightarrow \dot{V}_c = (5,89\angle -14,1°) \cdot (10\angle -90°)$$

$$\dot{V}_c = 58,9\angle 104,1° \text{ (V)}$$

?

CIRCUITOS ELÉTRICOS III

05/10/12



Calcular Zin

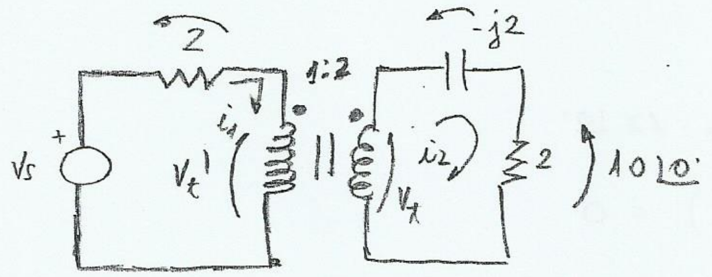
$$\dot{Z}in_A = (10\angle 30^\circ)\cdot\left(\frac{1}{4}\right)^2 \Rightarrow \dot{Z}in_A = \frac{10\angle 30^\circ}{16\angle 0^\circ} \Rightarrow \dot{Z}in_A = 0{,}625\angle 30^\circ$$

$$\therefore\ \dot{Z}in = 4 - j2 + \dot{Z}in_A \Rightarrow \dot{Z}in = 4 - j2 + 0{,}625\angle 30^\circ$$

$$\dot{Z}in = 4 - j2 + 0{,}54 + j0{,}31$$

$$\dot{Z}in = 4.54 - j1{,}69 \Rightarrow \dot{Z}in = 4{,}84\angle -20{,}4^\circ\ (\Omega)$$

Determinar Vs:



$$\dot{Z}_A = (2-j2)\cdot(0{,}5)^2 \Rightarrow \dot{Z}_A = 0{,}5 - j0{,}5 \Rightarrow \dot{Z}_A = 0{,}71\angle -45^\circ\ \Omega$$

$$i_2 = \frac{10\angle 0^\circ}{2} \Rightarrow i_2 = 5\angle 0^\circ\ A \Rightarrow V_c = (5\angle 0^\circ)\cdot(-j2) \Rightarrow V_c = -j10\ (V)$$

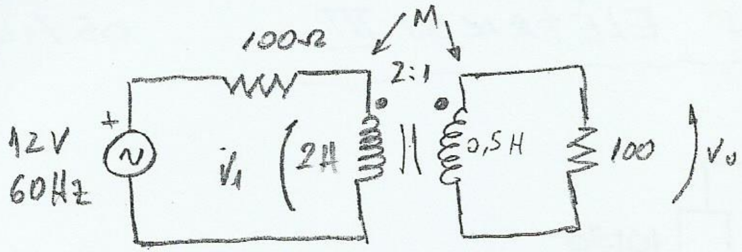
$$V_t = (10\angle 0^\circ) + (-j10) \Rightarrow V_t = 10 - j10 \Rightarrow V_t = 14{,}14\angle -45^\circ\ (V)$$

$$V_t' = \frac{14{,}14\angle -45^\circ}{2} \Rightarrow V_t' = 7{,}07\angle -45^\circ\ (V)$$

$$i_1 = 5\angle 0^\circ\cdot\left(\frac{2}{1}\right) \Rightarrow i_1 = 10\angle 0^\circ\ (A)$$

$$\dot{V}_R = 20\angle 0^\circ\ (V) \Rightarrow \dot{V}_s = 20\angle 0^\circ + 7{,}07\angle -45^\circ$$

$$\dot{V}_s = 25{,}4\angle -11{,}3^\circ\ (V)$$



$k = 1$

trafo ideal:

$\omega = 2\pi f$

$\omega =$

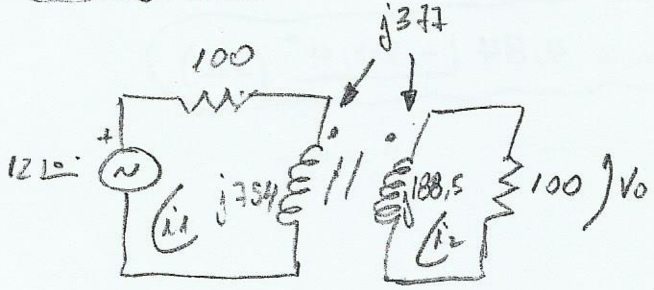
$$\dot Z in = 100\left(\frac{2}{1}\right)^2 \Rightarrow \dot Z in = 400\ \Omega$$

$$V_1 = \frac{400}{400+100}\cdot 12\angle 0^\circ \Rightarrow \dot V_1 = 9{,}6\angle 0^\circ$$

$$V_0 = 9{,}6\angle 0^\circ \cdot \frac{1}{2} \Rightarrow \boxed{V_0 = 4{,}8\angle 0^\circ}$$

trafo Real: $M = 1\cdot\sqrt{2\cdot 0{,}5} \Rightarrow M = 1H \Rightarrow M = j\omega M$

$$M = j377\ \Omega$$



$$\begin{cases} i_1\cdot(100+j754) - i_2\cdot(j377) = 12\angle 0^\circ \\ -i_1\cdot(j377) + i_2\cdot(100+j188{,}5) = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} 100+j754 & -j377 \\ -j377 & 100+j188{,}5 \end{vmatrix} \Rightarrow \Delta = 10000 + j75400 + j18850 - \cancel{142129} + \cancel{142129}$$

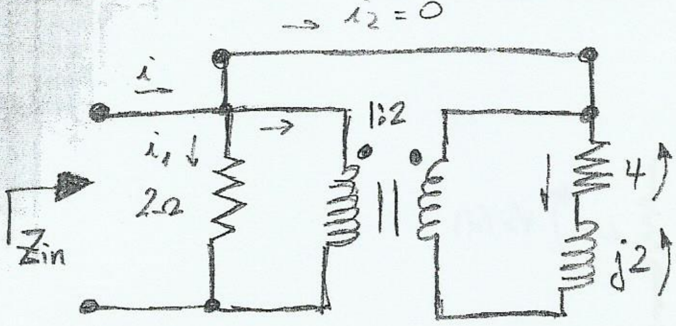
$$\Delta = 10000 + j94250 = 94779\angle 83{,}9^\circ$$

$$\Delta i_2 = \begin{vmatrix} 100+j754 & 12\angle 0^\circ \\ -j377 & 0 \end{vmatrix} \Rightarrow \Delta i_2 = j4524 = 4524\angle 90^\circ$$

$$i_2 = \frac{4524\angle 90}{94779\angle 83{,}9^\circ} \Rightarrow i_2 = 0{,}048\angle 6{,}1^\circ\ (A)$$

$$\dot V_0 = i_2\cdot 100 \Rightarrow V_0 = 4{,}8\angle 6{,}1^\circ\ (V)$$

13.53: IRWIN



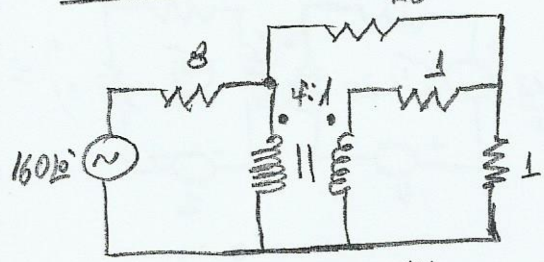
$$\dot{Z}in = 2 \;//\; (4+j2)\cdot\left(\frac{1}{2}\right)^2 = 2 \;//\; 1 + j0{,}5$$

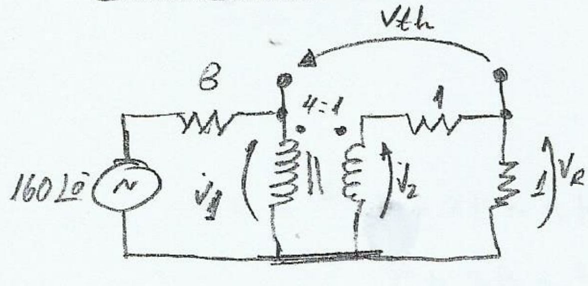
$$Zin = \frac{2\cdot(1+j0{,}5)}{3+j0{,}5} \Rightarrow \dot{Z}in = \frac{2+j}{3+j0{,}5} \Rightarrow \dot{Z}in = 0{,}73\,\angle 17{,}1^\circ\ \Omega$$

13.54: IRWIN

$\boxed{\text{Resp: } 5{,}4\,\Omega}$

Determinar Ro para máxima transferência de potência.



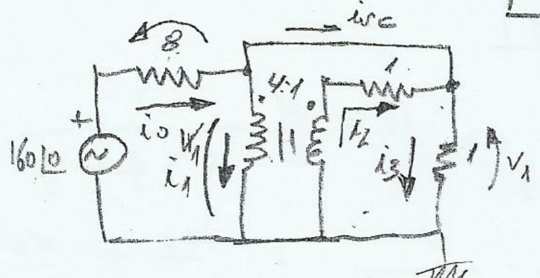


$$\dot{Z} = 2\cdot(4^2) \Rightarrow \dot{Z} = 32\,\Omega$$

$$\dot{V}_1 = \frac{32}{40}\cdot 160\angle 0^\circ \Rightarrow \dot{V}_1 = 128\,V$$

$\dot{V}_2 = 32\,V$ ∴ $\dot{V}_R = \frac{1}{2}\cdot 32 \Rightarrow \boxed{\dot{V}_R = 16\,V}$

∴ $Vth = 128 - 16 \Rightarrow \boxed{Vth = 112\,V}$



$i_0 = i_1 + isc$

$i_3 = i_2 + isc$

$i_0 = i_1 + isc$

$i_3 = 4i_1 + isc$

$V_2 = \frac{V_1}{4}$ $\quad i_2 = 4i_1$

$8i_0 + V_1 = 160$

$i_2 + i_2 + isc + V_2 = 0$

$8(i_1 + isc) + V_1 = 160$

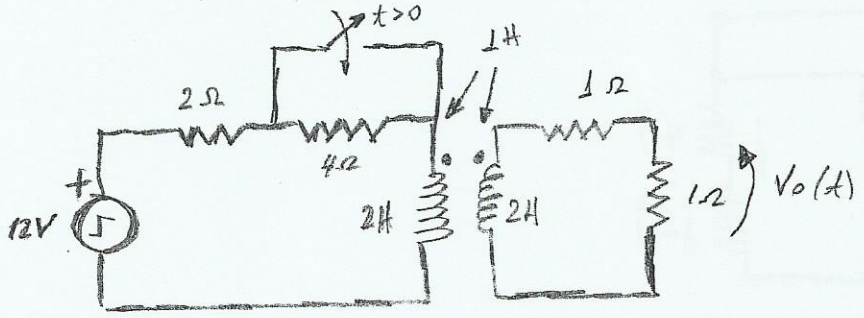
$4i_1 + 4i_1 + isc + \frac{V_1}{4} = 0$

$8i_1 + 8isc + 4i_1 + isc = 160$

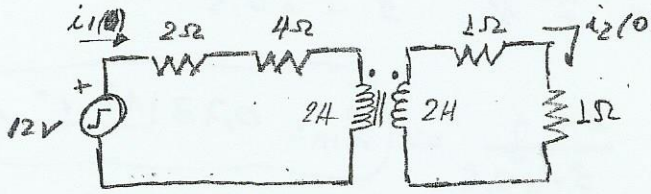
$4i_1 + 4i_1 + isc + \frac{4i_1 + isc}{4} = 0$

CIRCUITOS ELÉTRICOS III

10/10/2012



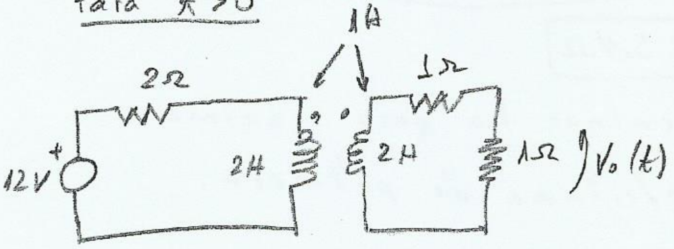
Para $t<0$



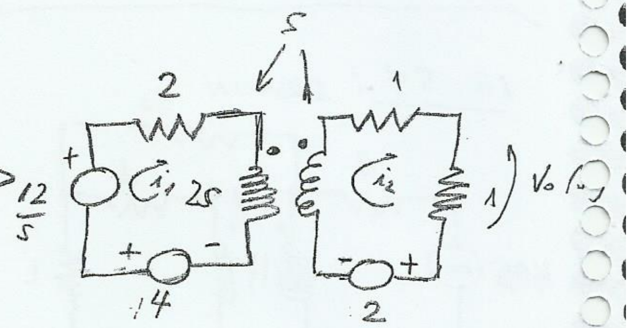
$i_1(0) = \frac{12}{2+4} \Rightarrow \boxed{i_1(0) = 2\ A}$

$\boxed{i_2(0) = 0\ A}$

Para $t>0$



$\Longrightarrow$



$$\begin{cases} i_1(2+2s) - i_2(s) = \frac{12}{s} + 4 \\ -i_1(s) + i_2(2+2s) = -2 \end{cases}$$

$$\Delta = \begin{vmatrix} (2+2s) & -s \\ -s & (2+2s) \end{vmatrix} \Rightarrow \Delta = 4 + 4s + 4s + 4s^2 - s^2$$

$$\Delta = 3s^2 + 8s + 4$$

$$\Delta i_2 = \begin{vmatrix} 2+2s & \frac{12}{s}+4 \\ -s & -2 \end{vmatrix} \Rightarrow \Delta i_2 = -4 - \cancel{4s} + 12 + \cancel{4s} \Rightarrow$$

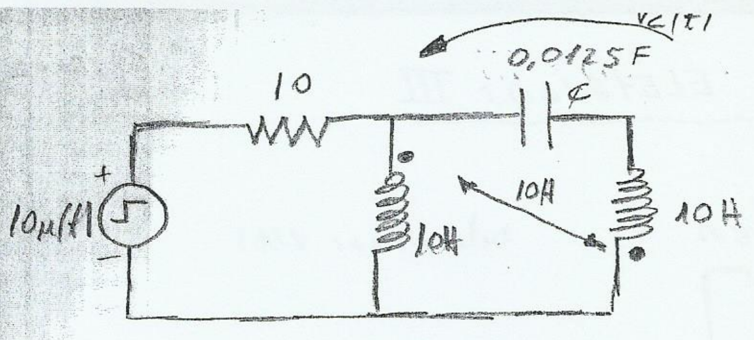
$$\boxed{\Delta i_2 = 8}$$

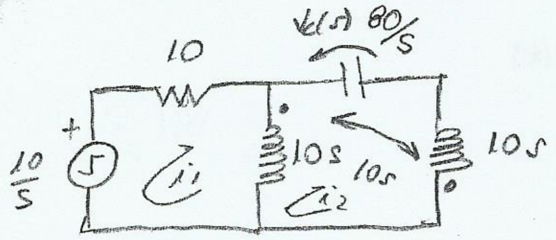
$$i_2 = \frac{8}{3s^2+8s+4} \Rightarrow i_2 = \frac{8}{3\left(s^2+\frac{8}{3}s+\frac{4}{3}\right)} \Rightarrow i_2 = \frac{8}{3}\frac{1}{s^2+\frac{8}{3}s+\frac{4}{3}}$$

$$i_2 = \frac{8}{3} \cdot \frac{1}{(s+2)(s+\frac{2}{3})} \Rightarrow V_o(s) = \frac{8}{3} \cdot \frac{1}{(s+2)(s+\frac{2}{3})} = \frac{K_1}{s+2} + \frac{K_2}{s+\frac{2}{3}}$$

$\boxed{K_1 = -2}$ e $\boxed{K_2 = 2}$

$$V_o(t) = 2\left(e^{-\frac{2}{3}t} - e^{-2t}\right)u(t)\ [V]$$





$$i_1(10+10s) - i_2(10s) - i_2\cdot(10s) = \frac{10}{s}$$

$$-i_1\cdot(10s) + i_2\cdot\left(10s+\frac{80}{s}+10s\right) - i_1\cdot(10s) + i_2\cdot(10s) + i_2(10s) = 0$$

$$i_1\cdot(10+10s) + i_2\cdot(-20s) = \frac{10}{s}$$

$$i_1\cdot(-20s) + i_2\cdot\left(\frac{40s^2+80}{s}\right) = 0$$

$$\Delta = \begin{vmatrix} 10+10s & -20s \\ -20s & \frac{40s^2+80}{s} \end{vmatrix} \Rightarrow \Delta = \left(\frac{400s^2+800}{s} + \cancel{400}s^2+800\right) - \cancel{400}s^2$$

$$\Delta = \frac{400s^2+800+800s}{s}$$

$$\Delta i_2 = \begin{vmatrix} 10+10s & \frac{10}{s} \\ -20s & 0 \end{vmatrix} \Rightarrow \Delta i_2 = +\frac{200}{s}$$

$$i_2 = \frac{200}{\cancel{s}}\cdot\frac{s}{400s^2+800s+800} \Rightarrow i_2 = \frac{2\cancel{00}}{4\cancel{00}}\cdot\frac{s}{s^2+2s+2}$$

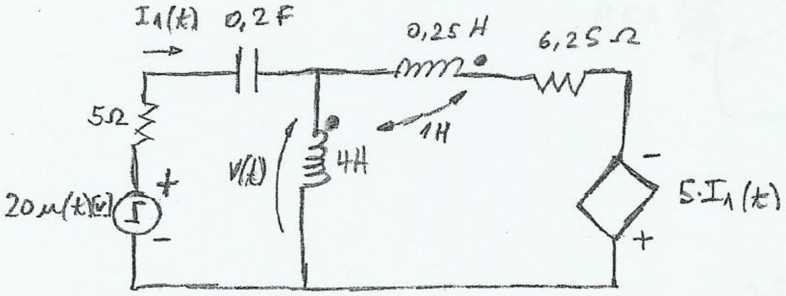
$$i_2 = \frac{1}{2}\cdot\frac{s}{(s+1-j)\cdot(s+1+j)} \; ; \; V_c(s) = \frac{1}{2}\cdot\frac{\cancel{s}}{(s+1-j)(s+1+j)}\cdot\frac{80}{\cancel{s}}$$

$$V_c(s) = \frac{40}{\cancel{1}(s+1-j)(s+1+j)} = +\frac{K_1}{s+1-j} + \frac{K_1^*}{s+1+j} \; ; \; K_1 = 20\angle{-90} \; , \; K_1^* = 20\angle{+90}$$

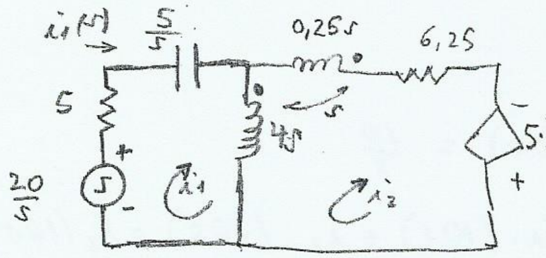
$$\boxed{v_c(t) = 40\cdot e^{-t}\cdot\cos(t-90)\,u(t)\;[V]}$$

CIRCUITOS ELÉTRICOS III

19/10/2012



Determinar V(t)



$$i_1 \cdot \left(5 + \frac{5}{s} + 4s\right) - i_2 \cdot (4s) - i_2 \cdot (s) = \frac{20}{s}$$

$$i_2 \cdot (0{,}25s + 6{,}25 + 4s) - i_1 \cdot (4s) - i_1 \cdot (s) + i_2(s) + i_2 \cdot (s) = 5 i_1$$

$$i_1 \cdot \left(5 + \frac{5}{s} + 4s\right) - i_2 \cdot (5s) = \frac{20}{s}$$

$$-i_1 \cdot (5s) + i_2 \cdot (0{,}25s + 6{,}25 + 4s + 2s) = 5 i_1$$

$$-i_1 \cdot (5s) + i_2 \cdot (6{,}25s + 6{,}25) = 5 i_1$$

$$-i_1 \cdot (5 + 5s) + i_2 \cdot (6{,}25 + 6{,}25s) = 0$$

$$i_1 \cdot \left(\frac{5s + 5 + 4s}{s}\right) - i_2 \cdot (5s) = \frac{20}{s}$$

$$-i_1 \cdot (5 + 5s) + i_2 (6{,}25 + 6{,}25s) = 0$$

$$\Delta = \begin{vmatrix} \frac{5+9s}{s} & -(5s) \\ -(5+5s) & (6{,}25 + 6{,}25s) \end{vmatrix} \Rightarrow \Delta = 31{,}25\left(\frac{s^2 + 2s + 1}{s}\right)$$

$$\Delta i_1 = \begin{vmatrix} \frac{20}{s} & -5s \\ 0 & (6{,}25s + 6{,}25) \end{vmatrix} \Rightarrow \Delta i_1 = 125\left(\frac{s+1}{s}\right)$$

$$\Delta i_2 = \begin{vmatrix} 5 + \frac{5}{s} + 4s & \frac{20}{s} \\ -(5s+5) & 0 \end{vmatrix} \Rightarrow \Delta i_2 = 100\left(\frac{s+1}{s}\right)$$

$$i_1(s) = \frac{\Delta i_1}{\Delta} \Rightarrow i_1(s) = \frac{125(s+1)}{s} \cdot \frac{s}{31{,}25(s^2+2s+1)}$$

$$i_1(s) = \frac{4(s+1)}{s^2+2s+1} \Rightarrow$$

$$i_2(s) = \frac{\Delta i_2}{\Delta} \Rightarrow i_2(s) = \frac{100(s+1)}{s} \cdot \frac{s}{31{,}25(s^2+2s+1)}$$

$$i_2(s) = \frac{3{,}2(s+1)}{(s^2+2s+1)} \Rightarrow$$

$$i_1(s) = \frac{4 \cdot \cancel{(s+1)}}{(s+1)^{\cancel{2}}} \Rightarrow \boxed{i_1(s) = \frac{4}{s+1}}$$

$$i_2(s) = \frac{3{,}2\,\cancel{(s+1)}}{(s+1)^{\cancel{2}}} \Rightarrow \boxed{i_2(s) = \frac{3{,}2}{s+1}}$$

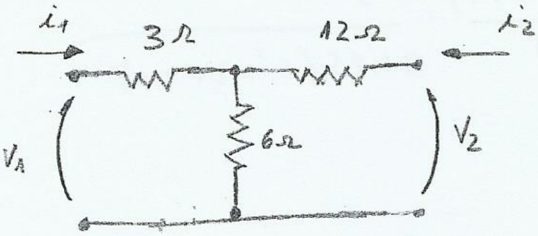
$$V(s) = 4 \cdot s \left[\frac{4-3{,}2}{s+1}\right] - s\left(\frac{3{,}2}{s+1}\right)$$

$$V(s) = \frac{4 \cdot s\,(0{,}8)}{s+1} - \frac{3{,}2\,s}{s+1}$$

$$V(s) = \frac{3{,}2\,s}{s+1} - \frac{3{,}2\,s}{s+1} \Rightarrow \boxed{V(s) = 0}$$

$$\therefore\ V(t) = 0\,V$$

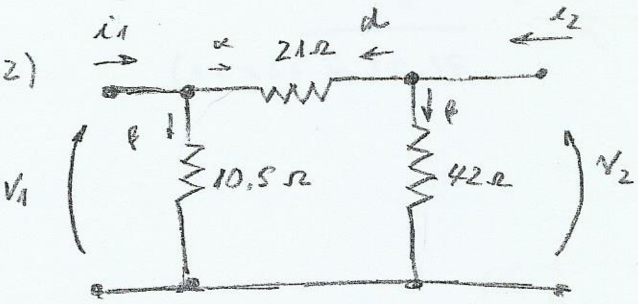
1)



$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$

$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

2)



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1) $Z_{11} = 6+3 \Rightarrow \boxed{Z_{11} = 9\Omega}$

$\boxed{Z_{12} = 6\Omega}$ $\boxed{Z_{21} = 6\Omega}$ $\boxed{Z_{22} = 18\Omega}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 6 & 18 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

2) $Z_{11} = ? \Rightarrow i_2 = 0 \Rightarrow Z_{11} = \dfrac{(10{,}5)\cdot(21+42)}{10{,}5+21+42} \Rightarrow \boxed{Z_{11} = 9\Omega}$

$Z_{12} = ? \Rightarrow i_1 = 0 \Rightarrow Z_{12} = \dfrac{V_1}{i_2}$

$\alpha = \dfrac{42}{10{,}5+21+42} \cdot i_2 \Rightarrow \alpha = 0{,}571 \cdot i_2$; $V_1 = \alpha \cdot 10{,}5$

$V_1 = 0{,}571 \cdot i_2 \cdot 10{,}5 \Rightarrow V_1 = 6 \cdot i_2 \Rightarrow \dfrac{V_1}{i_2} = 6\Omega \Rightarrow \boxed{Z_{12} = 6\Omega}$

$Z_{21} = ? \Rightarrow i_2 = 0 \Rightarrow Z_{21} = \dfrac{V_2}{i_1}$;

$\alpha = \dfrac{10{,}5}{21+42+10{,}5} \cdot i_1 \Rightarrow \alpha = 0{,}143 \cdot i_1 \Rightarrow V_2 = \alpha \cdot 42 \Rightarrow V_2 = 0{,}143 \cdot i_1 \cdot 42$

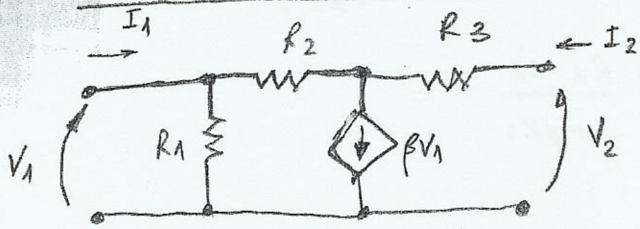
$V_2 = 6 \cdot i_1 \Rightarrow \dfrac{V_2}{i_1} = 6\Omega \Rightarrow \boxed{Z_{21} = 6\Omega}$

$Z_{22} = ? \Rightarrow i_1 = 0 \Rightarrow Z_{22} = \dfrac{V_2}{i_2} \Rightarrow \dfrac{V_2}{i_2} = \dfrac{(42)\cdot(10{,}5+21)}{42+10{,}5+21} \Rightarrow \dfrac{V_2}{i_2} = 18 \Rightarrow \boxed{Z_{22} = 18\Omega}$

CIRCUITOS ELÉTRICOS III

24/10/2012

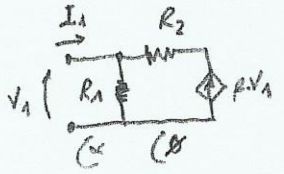
- **Parâmetros de Impedância (Z):**



$V_1 = Z_{11} \cdot i_1 + Z_{12} \cdot i_2$

$V_2 = Z_{21} \cdot i_1 + Z_{22} \cdot i_2$

$Z_{11} = \dfrac{V_1}{i_1}$; para $i_2 = 0$:



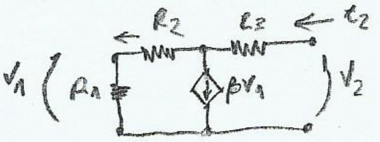
$\alpha \cdot R_1 - \varnothing \cdot R_1 = V_1$ e $\varnothing = \beta V_1$

$i_1 \cdot R_1 - (\beta V_1) \cdot R_1 = V_1 \Rightarrow i_1 \cdot R_1 - \beta V_1 \cdot R_1 = V_1$

$i_1 \cdot R_1 = V_1 + \beta V_1 R_1 \Rightarrow i_1 \cdot R_1 = V_1 (1 + \beta R_1)$

$\dfrac{V_1}{i_1} = \dfrac{R_1}{1 - \beta \cdot R_1} \Rightarrow Z_{11} = \dfrac{R_1}{1 + \beta \cdot R_1}\ \Omega$

$Z_{12} = \dfrac{V_1}{i_2}$; para $i_1 = 0$:

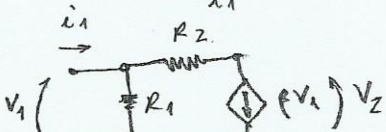


$i_2 = \beta V_1 + \dfrac{V_1}{R_1} \Rightarrow i_2 = V_1 \left(\beta + \dfrac{1}{R_1}\right)$

$i_2 = V_1 \left(\dfrac{\beta R_1 + 1}{R_1}\right) \Rightarrow \dfrac{V_1}{i_2} = \dfrac{R_1}{\beta R_1 + 1}\ \Omega$

$Z_{12} = \dfrac{R_1}{\beta R_1 + 1}\ \Omega$

$Z_{21} = \dfrac{V_2}{i_1}$; para $i_2 = 0$:



$V_2 = V_1 - \beta V_1 \cdot R_2 \Rightarrow$

$V_1 = R_1 \cdot (i_1 - \beta V_1)$

$V_2 = R_1 \cdot (i_1 - \beta V_1) - \beta V_1 \cdot R_2$

$V_1 = V_2 + \beta V_1 R_2$

$V_1 (1 - \beta R_2) = V_2$

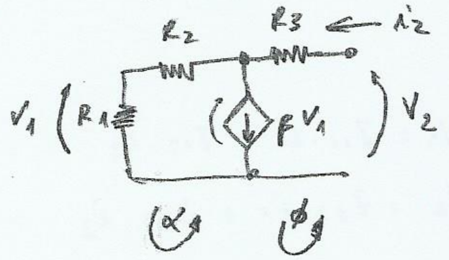
$V_1 = \dfrac{V_2}{1 - \beta R_2}$

$i_1 = \dfrac{V_1}{R_1} + \beta V_1$

$i_1 = \dfrac{V_2}{1 - \beta R_2} \cdot \dfrac{1}{R_1} + \beta \cdot \dfrac{V_2}{1 - \beta R_2} \Rightarrow i_1 = \dfrac{V_2}{1 - \beta R_2} \cdot \left(\dfrac{1}{R_1} + \beta\right)$

$i_1 = \dfrac{V_2}{1 - \beta R_2} \cdot \dfrac{1}{R_1} \cdot (1 + \beta R_1) \Rightarrow \dfrac{V_2}{i_1} = \dfrac{R_1 (1 - \beta R_2)}{1 + \beta R_1} \Rightarrow Z_{21} = \dfrac{R_1 (1 - \beta R_2)}{1 + \beta R_1}$

$$Z_{22} = \frac{V_2}{i_2} \; ; \; i_1 = 0 \; :$$



$$V_2 = R_3 \cdot i_2 + R_2 \cdot \left(\frac{V_1}{R_1} \right) + V_1$$

$$V_1 = \frac{R_1 \cdot i_2}{1 + \beta R_1}$$

$$V_2 = R_3 \cdot i_2 + \frac{R_2}{\cancel{R_1}} \cdot \frac{\cancel{R_1} \cdot i_2}{1 + \beta R_1} + \frac{R_1 \cdot i_2}{1 + \beta R_1}$$

$$V_2 = R_3 \cdot i_2 + \frac{R_2 \cdot i_2}{1 + \beta R_1} + \frac{R_1 \cdot i_2}{1 + \beta R_1}$$

$$V_2 = i_2 \cdot \left(R_3 + \frac{R_2 + R_1}{1 + \beta R_1} \right)$$

$$\frac{V_2}{i_2} = R_3 + \frac{R_1 + R_2}{1 + \beta R_1} \Rightarrow Z_{22} = R_3 + \frac{R_1 + R_2}{1 + \beta V_1} \, \Omega$$

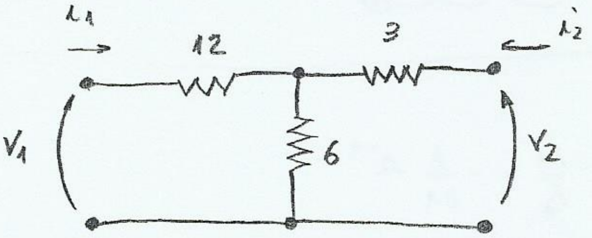
$$Z = \begin{bmatrix} \frac{R_1}{1 + \beta R_1} & \frac{R_1}{1 + \beta R_1} \\ \frac{R_1 (1 - \beta R_2)}{1 + \beta R_1} & R_3 + \frac{R_1 + R_2}{1 + \beta V_1} \end{bmatrix} [\Omega]$$

• Parâmetros de Admitância:

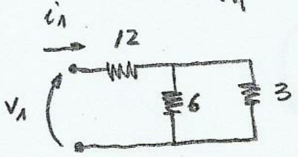
$$I_1 = y_{11}\cdot V_1 + y_{12}\cdot V_2$$

$$I_2 = y_{21}\cdot V_1 + y_{22}\cdot V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



$y_{11} = \frac{I_1}{V_1}$, para $V_2 = 0$:

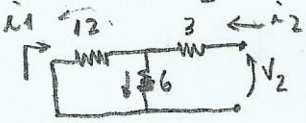


$Req = \frac{6\cdot 3}{6+3} + 12 \Rightarrow Req = 14\,\Omega \Rightarrow \frac{V_1}{i_1} = 14\,\Omega \Rightarrow \frac{i_1}{V_1} = \frac{1}{14}\,\Omega$

9

$y_{11} = \frac{1}{14}\,\Omega^{-1}$

$y_{12} = \frac{I_1}{V_2}$, para $V_1 = 0$:



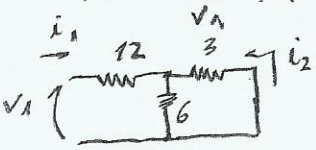
$V_2 = +12\cdot i_1 + 3\cdot i_2$

$i_1 = \frac{6}{12+6}\cdot i_2 \Rightarrow i_2 = \frac{18}{6}\cdot i_1 \Rightarrow i_2 = 3\, i_1$

$V_2 = +12\cdot i_1 + 3\cdot 3\cdot i_1 \Rightarrow V_2 = i_1\cdot(12+9) \Rightarrow \frac{i_1}{V_2} = -\frac{1}{21}\,\Omega^{-1}$

$y_{12} = -\frac{1}{21}\,\Omega$

$y_{21} = \frac{i_2}{V_1}$, para $V_2 = 0$:

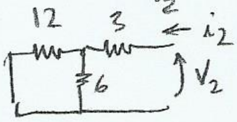


$i_2 = \frac{6}{6+3}\cdot i_1 \Rightarrow i_2 = \frac{2}{3}\cdot i_1 \Rightarrow i_1 = \frac{3}{2}\, i_2$

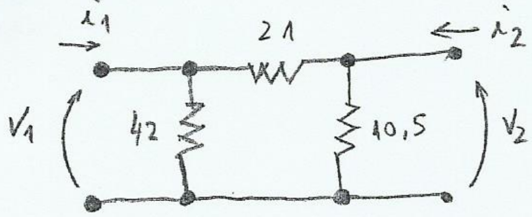
$V_1 = 12\, i_1 + 3\, i_2 \Rightarrow V_1 = 12\cdot\frac{3}{2}\cdot i_2 + 3\, i_2$

$V_1 = 18\, i_2 + 3\, i_2 \Rightarrow V_1 = i_2\,(21) \Rightarrow \frac{i_2}{V_1} = -\frac{1}{21}$ $\quad Z_{21} = -\frac{1}{21}\,\Omega^{-1}$

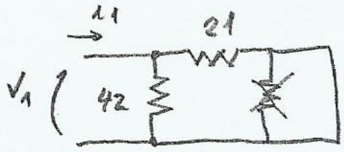
$y_{22} = \frac{i_2}{V_2}$, para $V_1 = 0$:



$Req = \frac{12\cdot 6}{12+6} + 3 \Rightarrow Req = 7\,\Omega$ $\quad Z_{22} = \frac{1}{7}\,\Omega^{-1}$



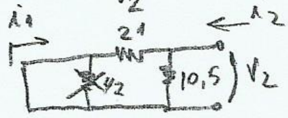
$Y_{11} = \frac{i_1}{V_1}$, para $V_2 = 0$:



$R_{eq} = \frac{21 \cdot 42}{21+42} \Rightarrow R_{eq} = 14\,\Omega$

$\frac{i_1}{V_1} = \frac{1}{14}\,\Omega^{-1} \Rightarrow Y_{11} = \frac{1}{14}\,\Omega^{-1}$

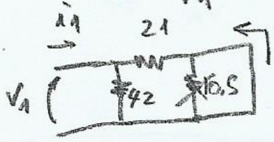
$Y_{12} = \frac{i_1}{V_2}$, para $V_1 = 0$:



$V_2 = i_1 \cdot 21 \Rightarrow \frac{i_1}{V_2} = -\frac{1}{21}\,\Omega^{-1}$

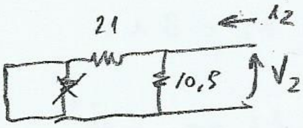
$Y_{12} = -\frac{1}{21}\,\Omega^{-1}$

$Y_{21} = \frac{i_2}{V_1}$, para $V_2 = 0$:



$V_1 = i_2 \cdot 21 \Rightarrow \frac{i_2}{V_1} = \frac{1}{21} \Rightarrow Y_{21} = -\frac{1}{21}\,\Omega^{-1}$

$Y_{22} = \frac{i_2}{V_2}$, para $V_1 = 0$



$V_2 = i_2 \left(\frac{21 \cdot 10,5}{21 + 10,5} \right) \Rightarrow V_2 = i_2 \cdot 7$

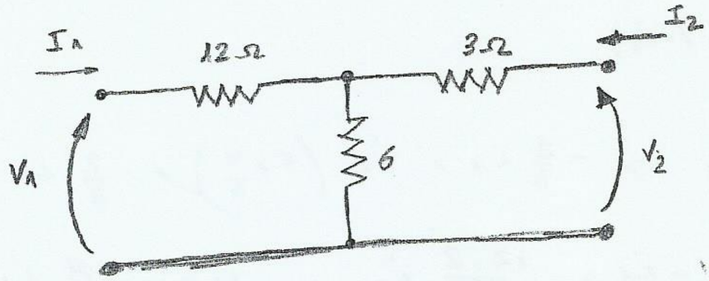
$\frac{i_2}{V_2} = 7^{-1}\,\Omega \Rightarrow Y_{22} = \frac{1}{7}\,\Omega^{-1}$

CIRCUITOS ELÉTRICOS III

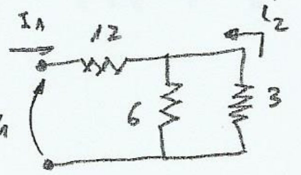
26/10/2012

• Parametros Hibridos (H):

$$H: \begin{cases} V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2 \\ I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2 \end{cases}$$



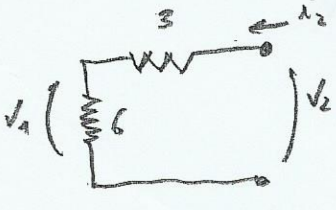
$h_{11} = \dfrac{V_1}{I_1}$ para $V_2 = 0$;



$R_{eq} = \dfrac{3 \cdot 6}{9} + 12 \Rightarrow R_{eq} = 14\Omega$

$\dfrac{V_1}{I_1} = 14\Omega \Rightarrow h_{11} = 14\Omega$

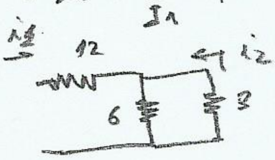
$h_{12} = \dfrac{V_1}{V_2}$ para $I_1 = 0$;



$V_1 = \dfrac{6}{9} \cdot V_2 \Rightarrow \dfrac{V_1}{V_2} = \dfrac{6}{9} \Rightarrow \dfrac{V_1}{V_2} = \dfrac{2}{3}\,\Omega$

$h_{12} = \dfrac{2}{3}$

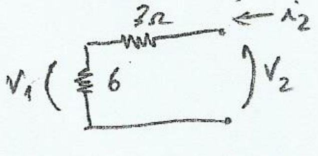
$h_{21} = \dfrac{I_2}{I_1}$ para $V_2 = 0$;



$-i_2 = \dfrac{6}{9} \cdot i_1 \Rightarrow \boxed{\dfrac{i_2}{i_1} = -\dfrac{2}{3}}$

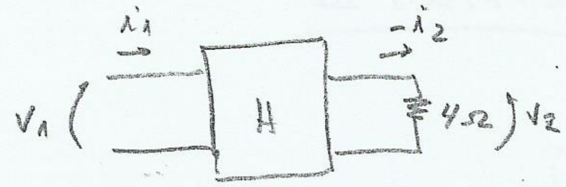
$h_{21} = -\dfrac{2}{3}$

$h_{22} = \dfrac{I_2}{V_2}$ para $I_1 = 0$;



$\dfrac{V_2}{I_2} = 9 \Rightarrow \dfrac{I_2}{V_2} = \dfrac{1}{9}\,\Omega^{-1}$

$h_{22} = \dfrac{1}{9}\,\Omega^{-1}$



$$V_1 = 14 \cdot i_1 + \frac{2}{3} \cdot V_2 \quad \text{(II)} \qquad V_2 = 4 \cdot (-i_2)$$

$$i_2 = -\frac{2}{3} \cdot i_1 + \frac{1}{9} \cdot V_2 \quad \text{(I)}$$

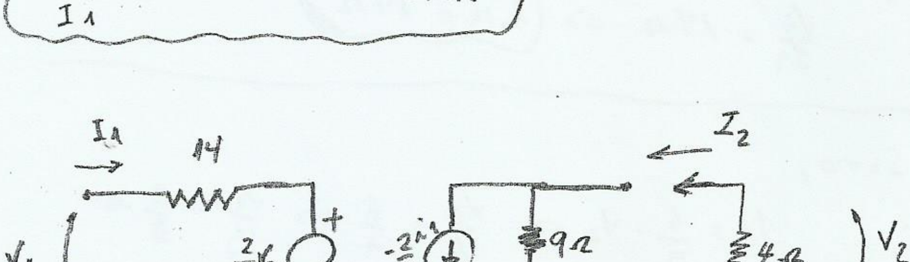
$$i_2 = -\frac{2}{3} \cdot i_1 + \frac{1}{9} \cdot 4 \cdot (-i_2)$$

$$i_2 + \frac{4}{9} i_2 = -\frac{2}{3} i_1 \Rightarrow -\frac{2}{3} i_1 = i_2 \left(1 + \frac{4}{9}\right) = -\frac{2}{3} i_1$$

$$i_2 \left(\frac{9+4}{9}\right) = -\frac{2}{3} i_1 \Rightarrow -i_1 = \frac{13 \cdot 3}{2 \cdot 9} i_2 \Rightarrow i_2 = -\frac{18}{39} i_1$$

$$V_1 = 14 I_1 + \frac{2}{3} \cdot 4 \cdot \left(-\frac{18}{39}\right) \cdot I_1 \Rightarrow \frac{V_1}{I_1} = 14 + \frac{8}{3} \cdot \left(-\frac{18}{39}\right) \cdot (-1)$$

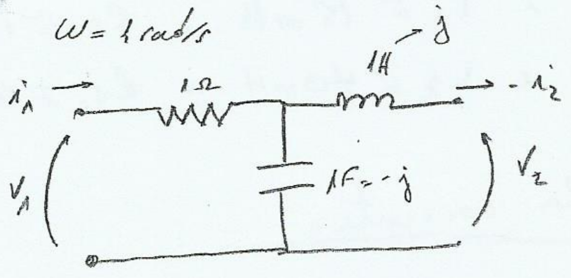
$$\boxed{\frac{V_1}{I_1} = 15{,}23\,\Omega = Z_{in}}$$



- Parâmetros de transmissão (t):

$$t: \begin{cases} V_1 = A \cdot V_2 - B \cdot I_2 \\ I_1 = C \cdot V_2 - D \cdot I_2 \end{cases} \qquad \begin{bmatrix} V_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

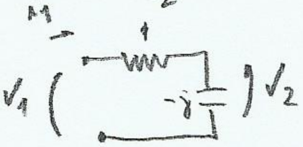
$\omega = 1\ rad/s$



Resp:

$$t = \begin{bmatrix} j+1 \,, & j\Omega \\ j\Omega^{-1} & 0 \end{bmatrix}$$

$A = \dfrac{V_1}{V_2}$ para $-I_2 = 0$:



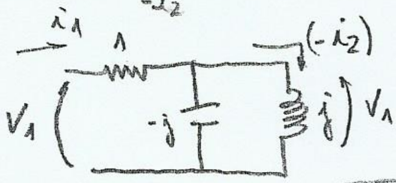
$V_2 = -j \cdot i_1$

$V_1 = (1-j) \cdot i_1 \Rightarrow i_1 = \dfrac{1}{1-j} \cdot V_1 \Rightarrow V_2 = -j \cdot \dfrac{1}{1-j} V_1$

$V_2 = \dfrac{-j}{1-j} \cdot V_1 \Rightarrow \dfrac{V_1}{V_2} = \dfrac{1-j}{-j} \Rightarrow \dfrac{V_1}{V_2} = \dfrac{1}{-j} + 1 \Rightarrow \dfrac{V_1}{V_2} = j+1$

$A = j + 1$

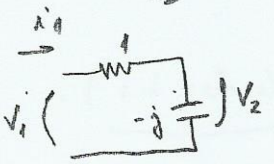
$B = \dfrac{V_1}{-I_2}$ para $V_2 = 0$:



$V_1' = \dfrac{j // (-j)}{1 + j//(-j)} \cdot V_1 \quad ; \quad V_1' = j \cdot (-i_2)$

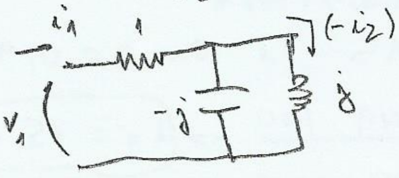
$j(-i_2) = \dfrac{\frac{j(-j)}{j-j}}{1 + \frac{j(-j)}{j-j}} \cdot V_1 \Rightarrow \dfrac{V_1}{-i_2} = j \Rightarrow B = j\Omega$

$C = \dfrac{i_1}{V_2}$ para $-i_2 = 0$:



$V_2 = i_1 \cdot (-j) \Rightarrow \dfrac{i_1}{V_2} = \dfrac{1}{-j} \Rightarrow \dfrac{i_1}{V_2} = j \Rightarrow C = j\ \Omega^{-1}$

$D = \dfrac{I_1}{-I_2}$ para $V_2 = 0$:



$-i_2 = \dfrac{-1}{j-j} \cdot i_1 \Rightarrow \dfrac{i_1}{-i_2} = \dfrac{0}{-j} = 0$

$D = 0$

Estudo: Laboratório

Medida dos Parâmetros L, M e C:

- Indutor de $N_1 = 800$ esp e $L_1 \cong 15\,mH$ e $R_{S_1} \cong 17,5\,\Omega$
  $N_2 = 1400$ esp e $L_2 \cong 40\,mH$ e $R_{S_2} \cong 40\,\Omega$

1) Método da tensão e da corrente:

$$f' = \frac{10 \cdot R_{S1}}{2 \cdot \pi \cdot L_1} (kHz) = 1,856\ kHz$$

$$f'' = \frac{10 \cdot R_{S_2}}{2 \cdot \pi \cdot L_2} \Rightarrow f'' = 1,592\ kHz$$

Indutor L_1:

$V_1 = 11,6\ V_{pp}$

$V_2 = 4,08\ V_{pp}$

$$L_1 = \frac{11,6 \cdot 100}{2 \cdot \pi \cdot 3 \cdot 10^3 \cdot 4,08} \Rightarrow L_1 \cong 15,08\ mH$$

Indutor L_2:

$V_1' = 13,2\ V_{pp}$

$V_2' = 1,52\ V_{pp}$

$$L_2 = \frac{13,2 \cdot 100}{2 \cdot \pi \cdot 3 \cdot 10^3 \cdot 1,52} \Rightarrow L_2 \cong 46,07\ mH$$

3) Método das 3 tensões (Apenas indutor L_1):

$V_2 = 10,6\ V_{pp}$

$V_1 = 6,4\ V_{pp}$

$$\cos\theta = \frac{14^2 - 6,4^2 - 10,6^2}{2 \cdot 6,4 \cdot 10,6} \Rightarrow$$

$\cos\theta = 0,315$ e $\sin\theta = 0,949$

$V_{L1} = 6,074\,V$

$V_{RS1} = 2,016\,V$

$$L_1 = \frac{6,4 \cdot 0,949 \cdot 100}{2 \cdot \pi \cdot 600 \cdot 10,6} \Rightarrow L_1 \cong 15,199\ mH$$

$$R_{S1} = \frac{6,4 \cdot 0,315 \cdot 100}{10,6} \Rightarrow R_{S1} \cong 19,02\,\Omega$$

4) Medida de L pelo método da defasagem:

$\Delta t = 160 \mu s$

$t = 1,640 ms$

$1,640 \cdot 10^{-3} - 360°$

$160 \cdot 10^{-6} - \phi$

$\phi = 35,12°$

$\cos\phi = 0,8179$

$\text{sen}\,\phi = 0,575$

$$L_1 = \frac{14 \cdot 0,575 \cdot 100}{2 \cdot \pi \cdot 600 \cdot 10,6} \Rightarrow L_1 \cong 20,14 \text{ mH}$$

$$Rs_1 = \frac{14}{10,6} \cdot 0,8179 \cdot 100 - 100 \Rightarrow$$

$Rs_1 = 8,02 \, \Omega$

6) Medida de C pelo método da tensão e da corrente:

$V_1 = 12,6 \text{ Vpp}$ $f = 6,1 \text{ kHz}$

$V_2 = 5,6 \text{ Vpp}$

$$C = \frac{5,6}{2 \cdot \pi \cdot 6 \cdot 10^3 \cdot 12,6 \cdot 100} \Rightarrow C = 0,117 \, \mu F$$

8) Medida de M usando F.e.m induzida:

$E_i = 3,76V$ $f = 2,55 KHz$

$V_2 = 4,72V$ $M = \dfrac{3,76 \cdot 100}{2 \cdot \pi \cdot 2,55 \cdot 4,72 \cdot 10^3} \Rightarrow \boxed{M = 4,972 mH}$

9) Medida de M pelas associações série aditiva e subtrativa:

$f = \dfrac{10(17,5 + 40)}{2 \cdot \pi \cdot (15m + 40m - 2M)} \Rightarrow f = 2,03 KHz$

QUADRIPOLOS:

1) Medida dos parametros Z:

Qa) $V_1 = 4,821V$; $V_2 = 3,665V$

$V_R = 7,260V$

$Z_{11a} = \dfrac{V_1}{i_1} = \dfrac{V_1}{V_R} \cdot 10k \Rightarrow Z_{11} = 6,64 k\Omega$

$Z_{21a} = \dfrac{V_2}{V_R} \cdot 10K \Rightarrow Z_{21a} = 5,048 K\Omega$

$V_1 = 3,502V$; $V_2 = 5,138V$; $V_R = 6,937V$

$Z_{12a} = \dfrac{V_1}{V_R} \cdot 10K \Rightarrow Z_{12a} = 5,048 K\Omega$

$Z_{22a} = \dfrac{V_2}{V_R} \cdot 10k \Rightarrow Z_{22} = 7,41 k\Omega$

$$[Z_a] = \begin{bmatrix} 6,64 & 5,048 \\ 5,048 & 7,41 \end{bmatrix} [k\Omega]$$

Qb)

$V_1 = 4,822V$ $V_2 = 2,397V$

$V_R = 7,253V$

$Z_{11b} = \frac{V_1}{V_R} \cdot 10k \Rightarrow Z_{11b} = 6,648\ k\Omega$

$Z_{21b} = \frac{V_2}{V_R} \cdot 10k \Rightarrow Z_{21b} = 3,3048\ k\Omega$

$V_1 = 2,391V$ $V_2 = 4,839V$

$V_R = 7,241V$

$Z_{12b} = \frac{V_1}{V_R} \cdot 10k \Rightarrow Z_{12b} = 3,30203\ k\Omega$

$Z_{22b} = \frac{V_2}{V_R} \cdot 10k \Rightarrow Z_{22b} = 6,68278\ k\Omega$

$$[Z_b] = \begin{bmatrix} 6,65 & 3,302 \\ 3,302 & 6,683 \end{bmatrix} [k\Omega]$$

Associação

$V_R = 5,185V$; $V_1 = 6,889V$; $V_2 = 4,328V$

$Z_{11} = \frac{V_1}{V_R} \cdot 10k \Rightarrow \underline{Z_{11} = 13,29\ k\Omega}$

$Z_{21} = \frac{V_2}{V_R} \cdot 10K \Rightarrow \underline{Z_{21} = 8,347\ k\Omega}$

$V_R = 5{,}014\,V \; ; \; V_1 = 4{,}185\,V$

$V_2 = 7{,}063\,V$

$$Z_{12} = \frac{V_1}{V_R} \cdot 10k \Rightarrow Z_{12} = 8{,}347\,k\Omega$$

$$Z_{22} = \frac{V_2}{V_R} \cdot 10k \Rightarrow Z_{22} = 14{,}087\,k\Omega$$

$$[Z] = \begin{bmatrix} 13{,}29 & 8{,}347 \\ 8{,}347 & 14{,}087 \end{bmatrix} [k\Omega]$$

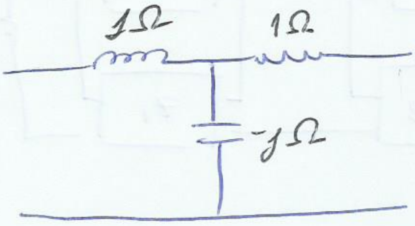
$$[Z_a] + [Z_b] = [Z]$$

$$[Z] = \begin{bmatrix} 6{,}64 & 5{,}048 \\ 5{,}048 & 7{,}41 \end{bmatrix} + \begin{bmatrix} 6{,}65 & 3{,}302 \\ 3{,}302 & 6{,}683 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} 13{,}29 & 8{,}35 \\ 8{,}35 & 14{,}093 \end{bmatrix} [k\Omega]$$

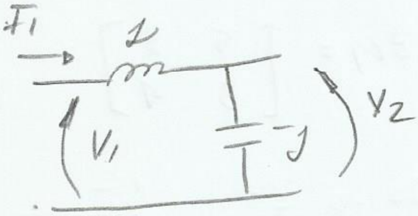
Continuação do final do caderno

3) Determine os parametros T do quadripolo



$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$



Para $I_2 = 0 \rightarrow A = \frac{V_1}{V_2}$ e $C = \frac{I_1}{V_2}$

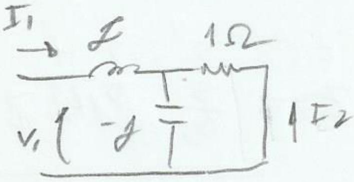
$V_1 = I_1 (j - j)$

$V_2 = \frac{-j}{j-j} I_1$

$A = \frac{\cancel{I_1}(j-j)}{\frac{-j}{(j-j)} \cdot \cancel{I_1}}$

$A = \frac{(j-j)^2}{-j} = 0$

$V_2 = -jI_1 \therefore C = \frac{I_1}{-j I_1} \therefore C = j$



Para $V_2 = 0 \rightarrow B = \frac{-V_1}{I_2}$ e $D = -\frac{I_1}{I_2}$

$V_1 = I_1 (j + (1 // -j))$

$= I_1 \left(j + \frac{(-j)}{1-j}\right) \rightarrow V_1 = I_1 \left(\frac{1 + \cancel{j} - \cancel{j}}{1-j}\right)$

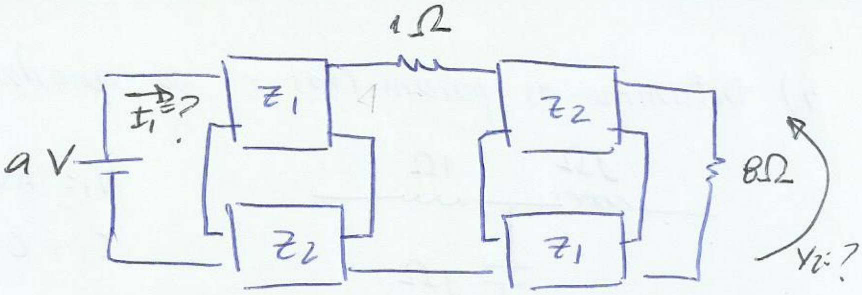
$-I_2 = \frac{-j}{1-j} I_1 \therefore B = \frac{-\cancel{I_1}\left(\frac{1}{\cancel{1-j}}\right)}{\frac{j}{\cancel{1-j}} \cancel{I_1}} \therefore B = j$

$D = \frac{(1-j)}{-j} = 1 + j$

$$T = \begin{bmatrix} 0 & j \\ j & 1+j \end{bmatrix}$$

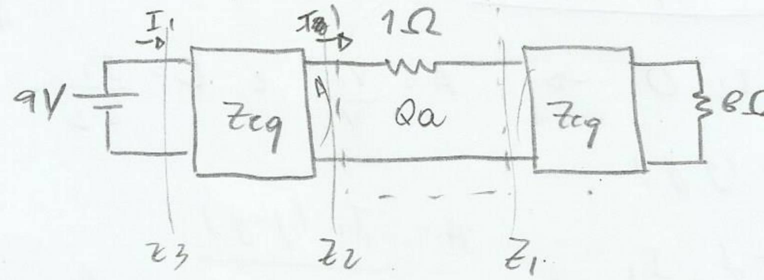
4) Determine I_1 e V_2

$Z_1 = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$ $Z_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$



Z_1 em série com Z_2

$Z_{eq} = \begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix}$



$T = \begin{bmatrix} \frac{5}{3} & \frac{11}{3} \\ \frac{1}{3} & \frac{4}{3} \end{bmatrix}$

$Z_1 = \dfrac{\frac{5}{3} \cdot 8 + \frac{11}{3}}{\frac{1}{3} \cdot 8 + \frac{4}{3}} = 4{,}25$ $\quad Z_2 = 1 + Z_1$

$Z_2 = 5{,}25$

$Z_3 = \dfrac{\frac{5}{3} \cdot 5{,}25 + \frac{11}{3}}{\frac{1}{3} \cdot 5{,}25 + \frac{4}{3}} \therefore Z_3 = 4{,}03$ $\quad \boxed{I_1 = 2{,}23\,A}$

$V_2' = 5{,}25\,I$

$V_1 = 5{,}25\,I - I$

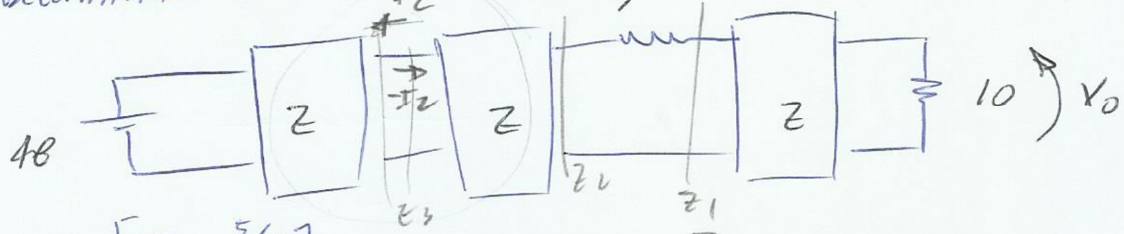
$V_1 = \frac{5}{3} V_2 - \frac{11}{3} \cdot I_2$;

$I_1 = \frac{1}{3} V_2 - \frac{4}{3} I_2$; $V_2 = -8 I_2$

$2{,}23 = -\frac{1}{2} \cdot 5{,}25\,I - \frac{4}{3}\,I \therefore I = 0{,}56$

$0{,}56 = \frac{1}{3} \cdot V_2 + \frac{4}{3} \cdot \frac{1}{8} V_2 \therefore V_2 = 1{,}13$

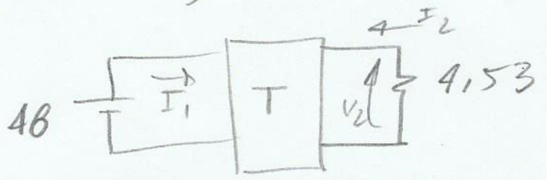
Determinar V_o



$$Z = \begin{bmatrix} 5 & 5/2 \\ \frac{5}{2} & \frac{15}{4} \end{bmatrix} \rightarrow T = \begin{bmatrix} 2 & 5 \\ \frac{2}{5} & \frac{3}{2} \end{bmatrix}$$

$$Z_1 = \frac{2\,10 + 5}{\frac{2}{5}\cdot 10 + \frac{3}{2}} = \frac{50}{11} = 4{,}54 \qquad Z_2 = 9{,}54\ \Omega$$

$$Z_3 = \frac{2 \cdot 9{,}54 + 5}{\frac{2}{5}\cdot 9{,}54 + \frac{3}{2}} \qquad Z_3 = 4{,}53\ \Omega$$



$$V_1 = 2V_2 - 5I_2$$

$$I_1 = \frac{2}{5}V_2 - \frac{3}{2}I_2$$

$$V_2 = -4{,}53\,I_2$$

$$48 = 2\cdot(-4{,}53 I_2) - 5I_2 \quad \therefore \quad I_2 = -3{,}41\ A$$

$$3{,}41 = \frac{2}{5}(-I_2 \cdot 9{,}54) - \frac{3}{2}I_2 \quad \therefore \quad I_2 = -0{,}64\ A$$

$$0{,}64 = \frac{2}{5}\cdot(-I_2 \cdot 10) - \frac{3}{2}\cdot I_2 \quad \therefore \quad I_2 = 0{,}12$$

$$V_o = 1{,}17$$

Exercícios de Aplicação das propriedades da Transformada de Laplace

1. $f(t) = 3t$

$\mathcal{L}[f(t)] = \mathcal{L}[3t] = 3\mathcal{L}[t] \quad \therefore \quad F(s) = \frac{3}{s^2}$

2. $f(t) = 5\cos 2t$

$\mathcal{L}[f(t)] = \mathcal{L}[5\cos 2t] = 5\mathcal{L}[\cos 2t] \quad \therefore \quad F(s) = \frac{5s}{s^2+4}$

3. $f(t) = t + e^{-2t}$

$\mathcal{L}[f(t)] = \mathcal{L}[t + e^{-2t}] = \mathcal{L}[t] + \mathcal{L}[e^{-2t}]$

$\therefore \quad F(s) = \frac{1}{s^2} + \frac{1}{s+2}$

4. $f(t) = 2 + 3\operatorname{sen} 10t - 5e^{-t}$

$\mathcal{L}[f(t)] = \mathcal{L}[2 + 3\operatorname{sen} 10t - 5e^{-t}]$

$= \mathcal{L}[2] + 3\mathcal{L}[\operatorname{sen} 10t] - 5\mathcal{L}[e^{-t}]$

$= \frac{2}{s} + 3 \cdot \frac{10}{s^2+100} - 5 \cdot \frac{1}{s+1}$

$\therefore \quad F(s) = \frac{2}{s} + \frac{30}{s^2+100} - \frac{5}{s+1}$

5. $f(t) = \cos 2(t-3) \cdot u(t-3)$

Observação: Existe dois tipos de deslocamento no tempo, Veja a seguir:

$f(t-t_0)\, u(t-t_0)$ ou $f(t)\, u(t-t_0)$

Transformada

$e^{-t_0 \cdot s} F(s)$ $\qquad$ $e^{-t_0 \cdot s} L[f(t+t_0)]$

$f(t) = \cos 2(t-3) \cdot u(t-3)$

$F(s) = e^{-3s} \cdot \mathcal{L}[\cos 2(t+3-3)]$

$= e^{-3s} \cdot \mathcal{L}[\cos 2t] \quad \therefore \quad F(s) = e^{-3s}\left(\frac{s}{s^2+4}\right)$

6-) $f(t) = t u(t-2)$

$f(t) = t \cdot u(t-2)$

$F(s) = e^{-2s} \mathcal{L}[t+2]$

$= e^{-2s} [\mathcal{L}[t] + \mathcal{L}[2]]$

$= e^{-2s} \left(\frac{1}{s^2} + \frac{2}{s} \right)$ $\therefore F(s) = e^{-2s} \left(\frac{1}{s^2} + \frac{2}{s} \right)$

7-) $f(t) = e^{-3t} \cdot u(t-1)$

$F(s) = e^{-s} \mathcal{L}[e^{-3(t+1)}]$

$= e^{-s} \mathcal{L}[e^{-3t} e^{-3}]$ $\therefore F(s) = \frac{e^{-(s+3)}}{s+3}$

$= e^{-(s+3)} \cdot \left(\frac{1}{s+3} \right)$

8-) $F(s) = e^{-3s} \left\{ \cos 6 \left(\frac{s}{s^2+4} \right) - \operatorname{sen} 6 \left(\frac{2}{s^2+4} \right) \right\}$ (Resposta)

$f(t) = \cos 2t \; u(t-3)$

$F(s) = e^{-3s} \mathcal{L}[\cos 2(t+3)]$

$= e^{-3s} \mathcal{L}[\cos(2t+6)]$

$= e^{-3s} \mathcal{L}[\cos 2t \cos 6 - \operatorname{sen} 2t \operatorname{sen} 6]$

$= e^{-3s} \left\{ \cos 6 \; \mathcal{L}[\cos 2t] - \operatorname{sen} 6 \; \mathcal{L}[\operatorname{sen} 2t] \right\}$

$\therefore F(s) = e^{-3s} \left\{ \cos 6 \cdot \left(\frac{s}{s^2+4} \right) - \operatorname{sen} 6 \left(\frac{2}{s^2+4} \right) \right\}$

9-) $f(t) = e^{-2t} \cdot t$

$F_1(s) = \frac{1}{s^2}$ $\therefore F(s) = \frac{1}{(s+2)^2}$

10-) $f(t) = e^{-2t} \cdot \underbrace{t \cdot u(t)}_{f_1(t)}$

Para $f_1(t) = t \cdot \underbrace{u(t)}_{\frac{1}{s}}$ $F_1(s) = \frac{-dF(s)}{ds} = \frac{1}{s^2}$

$F(s) = \frac{1}{(s+2)^2}$

11. $f(t) = e^{-t} \cdot \underbrace{\cos 2t}_{f_1(t)}$

$$F_1(s) = \frac{s}{s^2+4} \quad \therefore \quad F(s) = \frac{(s+1)}{(s+1)^2+4}$$

12. $f(t) = e^{-t} \cdot \underbrace{t \cdot \cos t}_{f_1(t)}$

$$F_1(s) = \frac{-d}{ds} \cdot \left(\frac{s}{s^2+1} \right)$$

$$= -\left[\frac{(s^2+1) - s \cdot 2s}{(s^2+1)^2} \right] = -\left[\frac{-s^2+1}{(s^2+1)^2} \right]$$

$$F(s) = \frac{(s+1)^2 - 1}{[(s+1)^2+1]^2}$$

13. $f(t) = e^{-t} \cdot \overbrace{t \cdot \cos t \cdot u(t)}^{F(s)}$

É a mesma coisa que $\mathcal{L}[e^{-t} \cdot t \cos t] = \dfrac{(s+1)^2 - 1}{[(s+1)^2+1]^2}$

14 $f(t) = 8 e^{-2t} \cdot t \cdot \cos 3t \cdot \cos t$

$$F(s) = 8 \cdot \mathcal{L}[e^{-2t} \cdot t \cdot \cos 3t \cdot \cos t]$$

$$= 8 \cdot \mathcal{L}[e^{-2t} \cdot t \cdot \underbrace{\frac{1}{2}(\cos 2t + \cos 4t)}_{f_1(t)}]$$

$$F_1(s) = \frac{1}{2} \cdot \left(\frac{s}{s^2+4} + \frac{s}{s^2+16} \right)$$

$$F_2(s) = -\frac{d}{ds} \left[\frac{1}{2} \left(\frac{s}{s^2+4} + \frac{s}{s^2+16} \right) \right]$$

$$= -\frac{1}{2} \cdot \left(\frac{s^2+4 - s \cdot 2s}{(s^2+4)^2} + \frac{s^2+16 - s \cdot s \cdot 2}{(s^2+16)^2} \right)$$

$$= -\frac{1}{2} \left[\frac{4 - s^2}{(s^2+4)^2} + \frac{16 - s^2}{(s^2+16)^2} \right]$$

$$\therefore F(s) = -4 \cdot \left[\frac{4 - (s+2)^2}{((s+2)^2+4)^2} + \frac{16 - (s+2)^2}{[(s+2)^2+16]^2} \right]$$

15-) $f(t) = 4 \operatorname{sen}^2(2t-4) \cdot u(t-2)$

$$F(s) = 4 \cdot \mathcal{L}[\operatorname{sen}^2(2(t+2)-4)] \cdot e^{-2s}$$

$$= 4 \cdot \mathcal{L}[\operatorname{sen}^2(2t)] \cdot e^{-2s} \quad , \quad \operatorname{sen}^2(2t) = \operatorname{sen}(2t) \operatorname{sen}(2t)$$

$$= 4 \cdot \mathcal{L}\left[\frac{1}{2}\left(\cos(0) - \cos(4t)\right)\right] \cdot e^{-2s}$$

$$= \frac{4}{2} \cdot \mathcal{L}[1 - \cos(4t)] \cdot e^{-2s}$$

$$= 2 \cdot \left(\frac{1}{s} - \frac{s}{s^2+16}\right) \cdot e^{-2s}$$

Exercício do livro texto

16.2 $f(t) = \dfrac{e^{-(a+t)}}{4x} = \dfrac{e^{-a} \cdot e^{-t}}{4x}$

$$F(s) = \frac{e^{-a}}{4x} \cdot \left(\frac{1}{s+1}\right)$$

16.3 $f(t) = a + a^2 t + a e^{-at} + a^2 \cos at$

$$\mathcal{L}[f(t)] = \frac{a}{s} + \frac{a^2}{s^2} + a \cdot \left(\frac{1}{s+a}\right) + a^2 \cdot \left(\frac{s}{s^2+a^2}\right)$$

16.4 $f(t) = e^{-at} \operatorname{sen} wt \cdot u(t-1)$

$$\mathcal{L}[f(t)] = e^{-s} \mathcal{L}[e^{-a(t+1)} \operatorname{sen} w(t+1)]$$

$$= e^{-s} \mathcal{L}[e^{-at} e^{-a} \operatorname{sen}(wt+w)]$$

$$= e^{-s} \mathcal{L}[e^{-at} e^{-a} (\operatorname{sen} wt \cos w + \operatorname{sen} w \cos wt)]$$

$$= e^{-(s+a)} \mathcal{L}[e^{-at} \operatorname{sen} wt \cos w + e^{-at} \operatorname{sen} w \cos wt]$$

$$= e^{-(s+a)} \cdot \left[\cos w \cdot \frac{w}{(s+a)^2+w^2} + \operatorname{sen} w \cdot \frac{s+a}{(s+a)^2+w^2}\right]$$

$$\therefore \quad F(s) = \frac{e^{-(s+a)}}{(s+a)^2+w^2} \left(w \cos w + (s+a) \operatorname{sen} w\right)$$

16.5 $f(t) = t e^{-at} u(t-4)$

$$F(s) = e^{-4s} \mathcal{L}[(t+4) e^{-a(t+4)}]$$

$$= e^{-4s} \mathcal{L}[t e^{-at} e^{-4a} + 4 e^{-at} e^{-4a}]$$

$$= e^{-4s} \times \left[e^{-4a} \mathcal{L}[t e^{-at}] + 4 e^{-4a} \mathcal{L}[e^{-at}] \right]$$

$$= e^{-4s} \times \left[e^{-4a} \cdot \frac{1}{(s+a)^2} + 4 e^{-4a} \frac{1}{(s+a)} \right]$$

$$= e^{-4(s+a)} \left[\frac{1}{(s+a)^2} + \frac{4}{(s+a)} \right]$$

16.6 $f(t) = e^{-at} u(t-1)$

$$\mathcal{L}[f(t)] = F(s) = e^{-s} \mathcal{L}[e^{-a(t+1)}]$$

$$= e^{-s} \mathcal{L}[e^{-at} e^{-a}]$$

$$= e^{-(s+a)} \mathcal{L}[e^{-at}]$$

$$\therefore F(s) = \frac{e^{-(s+a)}}{s+a}$$

16.7 Igual ao 16.6, ou seja, usar o mesmo teorema

$f(t) = t e^{-at} u(t-1)$

$$F(s) = \mathcal{L}[f(t)] = e^{-s} \mathcal{L}[(t+1) e^{-a(t+1)}]$$

$$= e^{-s} \mathcal{L}[t e^{-at} e^{-a} + e^{-at} e^{-a}]$$

$$= e^{-(s+a)} \mathcal{L}[t e^{-at} + e^{-at}]$$

$$F(s) = e^{-(s+a)} \left(\frac{1}{(s+a)^2} + \frac{1}{(s+a)} \right)$$

16.8 $f(t) = [t-1+e^{-(t-1)}]u(t-1)$

$$F(s) = \mathcal{L}[f(t)] = e^{-s}\mathcal{L}[(t+1)-1+e^{-(t+1-1)}]$$

$$= e^{-s}\mathcal{L}[t+e^{-t}]$$

$$\therefore F(s) = e^{-s}\left[\frac{1}{s^2}+\frac{1}{s+1}\right]$$

16.9 $f(t) = [e^{-(t-2)} - e^{-2(t-2)}]u(t-2)$

$$F(s) = \mathcal{L}[f(t)] = e^{-2s}\mathcal{L}[e^{-(t+2-2)} - e^{-2(t+2-2)}]$$

$$= e^{-2s}\mathcal{L}[e^{-t} - e^{-2t}]$$

$$\therefore F(s) = e^{-2s}\left[\frac{1}{s+1} - \frac{1}{s+2}\right]$$

16.10 $f(t) = t\cos wt \cdot u(t-1)$

$$F(s) = \mathcal{L}[f(t)] = e^{-s}\mathcal{L}[(t+1)\cos w(t+1)]$$

$$= e^{-s}\mathcal{L}[t(\cos wt\cos w - \sin wt\sin w) + \cos wt\cos w - \sin wt\sin w]$$

$$= e^{-s}\{\cos w\,\mathcal{L}[t\cos wt] - \sin w\,\mathcal{L}[t\sin wt] + \cos w\,\mathcal{L}[\cos wt] - \sin w\,\mathcal{L}[t\sin wt]$$

$$\mathcal{L}[t\cos wt] = -\frac{d}{ds}\mathcal{L}[\cos wt] = -\frac{d}{ds}\left(\frac{s}{s^2+w^2}\right)$$

$$= -\left(\frac{s^2+w^2 - s.2s}{(s^2+w^2)^2}\right) = \frac{s^2-w^2}{(s^2+w^2)^2}$$

$$\mathcal{L}[t\sin wt] = -\frac{d}{ds}\mathcal{L}[\sin wt] = -\frac{d}{ds}\left(\frac{w}{s^2+w^2}\right)$$

$$= +w.(s^2+w^2)^{-2}.2s = \frac{2ws}{(s^2+w^2)^2}$$

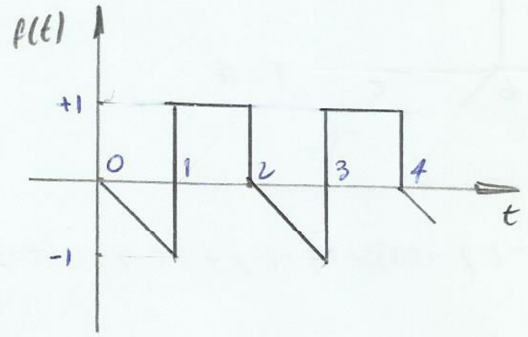
$$F(s) = e^{-s}\left[\cos w.\frac{(s^2-w^2)}{(s^2+w^2)^2} - \sin w.\frac{2ws}{(s^2+w^2)^2} + \cos w.\frac{s}{s^2+w^2} - \sin w.\frac{w}{s^2+w^2}\right]$$

$$\therefore F(s) = \frac{e^{-s}}{(s^2+w^2)}\left\{\left(\frac{s^2-w^2}{s^2+w^2}+s\right)\cos w - \left(\frac{2ws}{s^2+w^2}+w\right)\sin w\right\}$$

Exercícios 16.21 até 16.25

$$\boxed{F(s) = \frac{F_1(s)}{1 - e^{-sT}}}$$

16.21



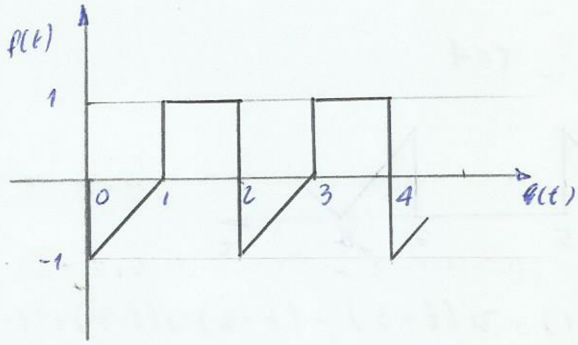
T = 2

$$f_1(t) = -t\,u(t) + (t-1)u(t-1) + 2u(t-1) - u(t-2)$$

$$F_1(s) = \frac{-1}{s^2} + \frac{e^{-s}}{s^2} + 2 \cdot \frac{1}{s} \cdot e^{-s} - \frac{1}{s} \cdot e^{-2s}$$

$$F(s) = \frac{1}{s^2}\left(\frac{-1 + e^{-s} + 2se^{-s} + s \cdot e^{-2s}}{1 - e^{-2s}}\right)$$

16.22



T = 2

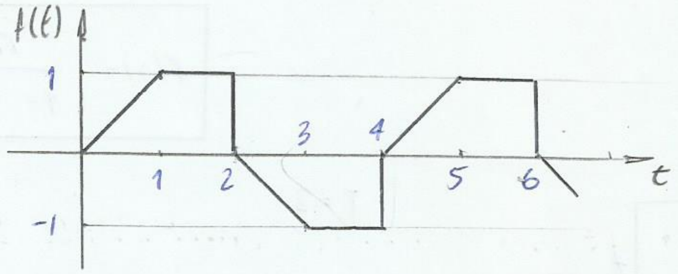
$$f_1(t) = -u(t) + t\,u(t) - (t-1)u(t-1) + u(t-1) - u(t-2)$$

$$F_1(s) = \frac{-1}{s} + \frac{1}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s} - \frac{e^{-2s}}{s}$$

$$F(s) = \left(-\frac{1}{s} + \frac{1}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s} - \frac{e^{-2s}}{s}\right) \cdot \frac{1}{1 - e^{-st}}$$

$$F(s) = \frac{1}{s^2}\left(\frac{-s + 1 - e^{-s} + se^{-s} - se^{-2s}}{1 - e^{-2s}}\right)$$

16.23



$T=4$

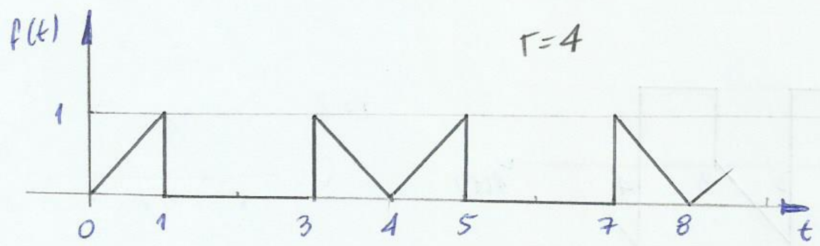
$$f_1(t) = t\,u(t) - (t-1)\,u(t-1) - u(t-2) - (t-2)\,u(t-2) + (t-3)\,u(t-3) + u(t-4)$$

$$F_1(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s} - \frac{e^{-2s}}{s^2} + \frac{e^{-3s}}{s^2} + \frac{e^{-4s}}{s} .$$

$$F_1(s) = \frac{1}{s^2}\left(1 - e^{-s} - s e^{-2s} - e^{-2s} + e^{-3s} + s e^{-4s}\right)$$

$$\therefore\ F(s) = \frac{1}{s^2}\left(\frac{1 - e^{-s} - s e^{-2s} - e^{-2s} + e^{-3s} + s e^{-4s}}{1 - e^{-4s}}\right)$$

16.24



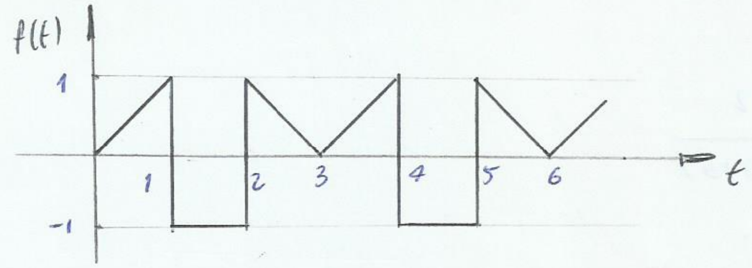
$$f_1(t) = t\,u(t) - (t-1)\,u(t-1) - u(t-1) + u(t-3) - (t-3)\,u(t-3) + (t-4)\,u(t-4)$$

$$F_1(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} + \frac{e^{-3s}}{s} - \frac{e^{-3s}}{s^2} + \frac{e^{-4s}}{s^2}$$

$$\therefore\ F(s) = \frac{F_1(s)}{1 - e^{-4s}}$$

$$F(s) = \frac{1}{s^2}\left(\frac{1 - e^{-s} - s e^{-s} + s e^{-3s} + e^{-3s} + e^{-4s}}{s^2}\right)$$

16.25



$F(s) = \frac{F_1(s)}{1 - e^{-sT}}$

$T = 3$

$f_1(t) = t\,u(t) - (t-1)u(t-1) - 2u(t-1) + 2u(t-2) - (t-2)u(t-2) + (t-3)u(t-3)$

$$F_1(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{2e^{-s}}{s} + \frac{2e^{-2s}}{s} - \frac{e^{-2s}}{s^2} + \frac{e^{-3s}}{s^2}$$

$$F(s) = \frac{F_1(s)}{1 - e^{-3s}}$$

$$F(s) = \frac{1}{s^2}\left(\frac{1 - e^{-s} - 2se^{-s} + 2se^{-2s} - e^{-2s} + e^{-3s}}{1 - e^{-3s}}\right)$$

Transformada Inversa de Laplace

Transformada inversa de laplace: $\mathcal{L}^{-1}[F(s)] = f(t)$

- Para: $F(s) = \frac{P(s)}{Q(s)} = \frac{a_m \cdot s^m + a_{m-1} s^{m-1} + \dots a_1 s + a_0}{b_n \cdot s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \dots b_1 s + b_0}$

- As raízes do polinômio $P(s)$ são chamados de "Zeros".
- As raízes do polinômio $Q(s)$ são chamados de "pólos"

-1- Polos simples

$$F(s) = \frac{K_1}{s+p_1} + \frac{K_2}{s+p_2} \dots \frac{K_n}{s+p_n}$$

$$K_i = (s + p_i) \cdot F(s) \Big|_{s=-p_i}$$

$$K_i \, \mathcal{L}^{-1}\left(\frac{1}{s+p_i}\right) = K_i \cdot e^{-p_i t}$$

Veja os exercícios a seguir para melhor compreensão

16.30

(a) $F(s) = \dfrac{s^2+7s+12}{(s+2)(s+4)(s+6)}$

$$F(s) = \frac{s^2+7s+12}{(s+2)(s+4)(s+6)} = \frac{k_1}{(s+2)} + \frac{k_2}{(s+4)} + \frac{k_3}{(s+6)}$$

$$k_1 = (s+2)\,F(s)\Big|_{s=-2} = \frac{s^2+7s+12}{(s+4)(s+6)}\Big|_{-2} = \frac{2}{8} = \frac{1}{4}$$

$$k_2 = (s+4)\,F(s)\Big|_{s=-4} = \frac{s^2+7s+12}{(s+2)(s+6)} = \frac{0}{-4} = 0$$

$$k_3 = (s+6)\,F(s)\Big|_{s=-6} = \frac{s^2+7s+12}{(s+2)(s+4)} = \frac{6}{8} = \frac{3}{4}$$

$$F(s) = \frac{1/4}{s+2} + \frac{3/4}{s+6} \quad ; \text{ logo } \quad f(t) = \frac{e^{-2t}+3e^{-6t}}{4}\,.u(t)$$

(b) $F(s) = \dfrac{(s+3)(s+6)}{s(s^2+10s+24)}$

$$s^2+10s+24=0$$

$$s = \frac{-10 \pm \sqrt{100-96}}{2} \quad \begin{cases} s_1 = -4 \\ s_2 = -6 \end{cases}$$

$$F(s) = \frac{(s+3)(s+6)}{s(s^2+10s+24)} = \frac{(s+3)\cancel{(s+6)}}{s(s+4)\cancel{(s+6)}} = \frac{k_1}{s} + \frac{k_2}{s+4}$$

$$k_1 = s\,F(s)\Big|_{s=0} = \frac{s+3}{s+4} = \frac{3}{4}$$

$$k_2 = (s+4)\,F(s)\Big|_{s=-4} = \frac{s+3}{s} = \frac{-1}{-4} = \frac{1}{4}$$

$$\therefore F(s) = \frac{3/4}{s} + \frac{1/4}{s+4}$$

$$\therefore f(t) = \left(\frac{3}{4} + \frac{1}{4}e^{-4t}\right)u(t)$$

16.31 a) $F(s) = \dfrac{s^3+2s^2+s}{s^2(s^2+5s+4)}$

$s^2+5s+4=0$

$s = \dfrac{-5 \pm \sqrt{25-16}}{2}$ ⟨ $s_1=-1$, $s_2=-4$

$F(s) = \dfrac{\cancel{s}(s^2+2s+1)}{s^{\cancel{2}}(s+1)(s+4)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+1} + \dfrac{K_3}{s+4}$

$K_1 = sF(s)\Big|_{s=0} = \dfrac{s^2+2s+1}{(s+1)(s+4)} = \dfrac{1}{4}$

$K_2 = (s+1)F(s)\Big|_{s=-1} = \dfrac{s^2+2s+1}{s(s+4)} = 0$

$K_3 = (s+4)F(s)\Big|_{s=-4} = \dfrac{s^2+2s+1}{s(s+1)} = \dfrac{3}{4}$

$F(s) = \dfrac{1/4}{s} + \dfrac{3/4}{s+4}$; logo $f(t) = \left(\dfrac{1}{4} + \dfrac{3}{4}e^{-4t}\right)u(t)$

b) ~~$F(s) = \dfrac{\quad}{s(s^2+10s+24)}$~~ $F(s) = \dfrac{s^2+4s+3}{(s^2+2s+1)(s^2+7s+12)}$

$s^2+2s+1=0$

$s = \dfrac{-2 \pm \sqrt{4-4}}{2}$ / $s_1=s_2=-1$

$s^2+7s+12=0$

$s = \dfrac{-7 \pm \sqrt{49-48}}{2}$ ⟨ $s_1=-3$, $s_2=-4$

$F(s) = \dfrac{\cancel{(s+1)(s+3)}}{(s+1)\cancel{(s+1)(s+3)}(s+4)}$

$s^2+4s+3=0$

$s = \dfrac{-4 \pm \sqrt{16-12}}{2}$ ⟨ $s_1=-3$, $s_2=-1$

$F(s) = \dfrac{1}{(s+1)(s+4)} = \dfrac{K_1}{s+1} + \dfrac{K_2}{s+4}$

$K_1 = (s+1)F(s)\Big|_{s=-1} = \dfrac{1}{s+4} = \dfrac{1}{3}$

$K_2 = (s+4)F(s)\Big|_{s=-4} = \dfrac{1}{s+1} = -\dfrac{1}{3}$

$\therefore F(s) = \dfrac{1}{3}\left(\dfrac{1}{s+1} - \dfrac{1}{s+4}\right)$

$\therefore f(t) = \dfrac{1}{3}\cdot\left(e^{-t} - e^{-4t}\right)u(t)$

16.33

(a) $F(s) = \dfrac{s+8}{s^2(s+6)}$

Observação: $K_{ij} = \dfrac{1}{(r-j)!} \dfrac{d^{r-j}}{ds^{r-j}} \left[(s+p_1)^r \cdot F(s) \right] \Big|_{s=-p_1}$

$$F(s) = \frac{s+8}{s^2(s+6)} = \frac{K_1}{s} + \frac{K_2}{s^2} + \frac{K_3}{s+6}$$

$$K_3 = (s+6)F(s) \Big|_{s=-6} = \frac{s+8}{s^2} = \frac{2}{36} = \frac{1}{18}$$

$$K_2 = s^2 F(s) \Big|_{s=0} = \frac{s+8}{s+6} = \frac{8}{6} = \frac{4}{3}$$

$$K_1 = \frac{1}{1!} \cdot \frac{d}{ds} \left[\frac{s+8}{s+6} \right] \Big|_{s=0}$$

$$K_1 = \frac{(s+6)-(s+8)}{(s+6)^2} \Big|_{s=0} \quad \therefore \quad K_1 = \frac{-1}{18}$$

$$F(s) = \frac{-1/18}{s} + \frac{4/3}{s^2} + \frac{1/18}{(s+6)}$$

$$\therefore \quad f(t) = \left(\frac{-1}{18} + \frac{4}{3} t + \frac{1}{8} e^{-6t} \right) u(t)$$

(b) $F(s) = \dfrac{1}{s^2(s+1)^2} = \dfrac{K_1}{s} + \dfrac{K_2}{s^2} + \dfrac{K_3}{(s+1)} + \dfrac{K_4}{(s+1)^2}$

$$K_2 = s^2 F(s) \Big|_{s=0} = \frac{1}{(s+1)^2} = 1$$

$$K_1 = \frac{d}{ds} \left[s^2 F(s) \right] \Big|_{s=0} = \frac{d}{ds} \left(\frac{1}{(s+1)^2} \right) = -2(s+1)^{-3} \Big|_{s=0} = -2$$

$$K_4 = (s+1)^2 F(s) \Big|_{s=-1} = \frac{1}{s^2} \Big|_{-1} = 1$$

$$K_3 = \frac{d}{ds} \left[(s+1)^2 F(s) \right] \Big|_{s=-1} = \frac{d}{ds} \left(\frac{1}{s^2} \right) \Big|_{s=-1} = \frac{-2}{s^3} \Big|_{s=-1} = 2$$

$$F(s) = \frac{1}{s^2(s+1)^2} = \frac{-2}{s} + \frac{1}{s^2} + \frac{2}{s+1} + \frac{1}{(s+1)^2}$$

$$\therefore f(t) = (-2 + t + 2e^{-t} + te^{-t})u(t)$$

16.34) a) $F(s) = \frac{s+4}{(s+2)^2} = \frac{K_1}{s+2} + \frac{K_2}{(s+2)^2}$

$$K_2 = (s+2)^2 \cdot F(s)\Big|_{s=-2} = s+4\Big|_{-2} = 2$$

$$K_1 = \frac{d}{ds}\left((s+2)^2 F(s)\right)\Big|_{-2} = \frac{d}{ds}(s+4)\Big|_{-2} = 1$$

$F(s) = \frac{2}{s+2} + \frac{1}{(s+2)^2}$ $\quad\therefore\quad f(t) = (2e^{-2t} + te^{-2t})u(t)$

b) $F(s) = \frac{s+6}{s(s+1)^2} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{(s+1)^2}$

$$K_1 = s\,F(s)\Big|_{s=0} = \frac{s+6}{(s+1)^2} = 6$$

$$K_3 = (s+1)^2 F(s)\Big|_{s=-1} = \frac{s+6}{s} = -5$$

$$K_2 = \frac{d}{ds}\left((s+1)^2 F(s)\right)\Big|_{s=-1} = \frac{d}{ds}\left(\frac{s+6}{s}\right)\Big|_{-1} = \frac{s-(s+6)}{s^2} = -6$$

$$F(s) = \frac{6}{s} - \frac{6}{s+1} - \frac{5}{(s+1)^2}$$

$$\therefore f(t) = (6 - 6e^{-t} - 5te^{-t})u(t)$$

16.35

a) $F(s) = \dfrac{s^2}{(s+1)^2(s+2)} = \dfrac{K_1}{(s+1)} + \dfrac{K_2}{(s+1)^2} + \dfrac{K_3}{(s+2)}$

$K_3 = (s+2)F(s)\Big|_{s=-2} = \dfrac{s^2}{(s+1)^2} = 4$

$K_2 = (s+1)^2 F(s)\Big|_{s=-1} = \dfrac{s^2}{s+2} = 1$

$K_1 = \dfrac{1}{(2-1)!}\dfrac{d}{ds}\left((s+1)^2F(s)\right)\Big|_{s=-1} = \dfrac{d}{ds}\left(\dfrac{s^2}{s+2}\right)_{s=-1}$

$K_1 = \dfrac{2s(s+2)-s^2}{(s+2)^2} = \dfrac{\cancel{2}s^2+4s-\cancel{s^2}}{(s+2)^2} = \dfrac{1-4}{1} = -3$

$F(s) = \dfrac{-3}{s+1} + \dfrac{1}{(s+1)^2} + \dfrac{4}{s+2}$ $\therefore$ $f(t) = (-3e^{-t} + te^{-2t} + 4e^{-2t})u(t)$

b) $F(s) = \dfrac{s^2+9s+20}{s(s+4)^3(s+5)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+5} + \dfrac{K_3}{(s+4)^3} + \dfrac{K_4}{(s+4)^2} + \dfrac{K_5}{(s+4)}$

$K_1 = sF(s)\Big|_{s=0} = \dfrac{s^2+9s+20}{(s+4)^3(s+5)}\Big|_0 = \dfrac{1}{16}$

$K_2 = (s+5)F(s)\Big|_{s=-5} = \dfrac{s^2+9s+20}{s(s+4)^3}\Big|_{-5} = 0$

$K_3 = (s+4)^3F(s)\Big|_{s=-4} = \dfrac{s^2+9s+20}{s(s+5)} = 0$

$K_4 = \dfrac{d}{ds}\left((s+4)^3F(s)\right)\Big|_{-4} = \dfrac{d}{ds}\left(\dfrac{s^2+9s+20}{s(s+5)}\right)$

$= \dfrac{(2s+9)s(s+5) - (s^2+9s+20)\cdot(2s+5)}{s^2(s+5)^2}\Big|_{-4}$

$= \dfrac{s(2s^2+10s+9s+45) - (2s^3+18s^2+40s+5s^2+45s+100)}{s^2(s^2+10s+25)}$

$$= \frac{\cancel{2s^3}+10s^2+9s^2+\cancel{45s}-\cancel{2s^3}-18s^2-40s-5s^2-\cancel{45s}-100}{s^4+10s^3+25s^2}$$

$$= \frac{-4s^2-40s-100}{s^4+10s^3+25s^2}\Big|_{-4} = \frac{-1}{4}$$

$$K_5 = \frac{d^2}{ds^2}\left((s+4)F(s)\right)\Big|_{-4} = \frac{d}{ds}\left(\frac{-4s^2-40s-100}{s^4+10s^3+25s^2}\right)\Big|_{-4}$$

$$= \frac{(-8s-40)(s^4+10s^3+25s^2)-(-4s^2-40s-100).(4s^3+30s^2+50s)}{(s^4+10s^3+25s^2)^2}$$

$$\therefore K_5 = \frac{-1}{8}$$

$$F(s) = \frac{\frac{1}{16}}{s} \quad \frac{-\frac{1}{4}}{(s+4)^2} \quad \frac{-\frac{1}{8}}{(s+4)} \qquad \therefore F(t) = \frac{1}{16} - \frac{1}{4}te^{-4t} - \frac{1}{8}e^{-4t}$$

16.36 (a) $F(s) = \dfrac{s^2+6s+8}{s^2(s+2)(s^2+10s+16)}$

$$s^2+10s+16=0 \qquad s = \frac{-10 \pm \sqrt{100-64}}{2} \quad s_1 = -2,\ s_2 = -8$$

$$s^2+6s+8=0 \qquad s = \frac{-6 \pm \sqrt{36-32}}{2} \quad s_1 = -2,\ s_2 = -4$$

$$F(s) = \frac{\cancel{(s+2)}(s+4)}{s^2\cancel{(s+2)}(s+2)(s+8)} = \frac{K_1}{s} + \frac{K_2}{s^2} + \frac{K_3}{s+2} + \frac{K_4}{s+8}$$

$$K_4 = (s+8)F(s)\Big|_{s=-8} = \frac{s+4}{s^2(s+2)}\Big| = \frac{1}{96}$$

$$K_3 = (s+2)F(s)\Big|_{s=-2} = \frac{s+4}{s^2(s+8)}\Big|_{-2} = \frac{1}{12}$$

$$K_2 = s^2F(s)\Big|_{s=0} = \frac{s+4}{(s+2)(s+8)} = \frac{4}{16} = \frac{1}{4}$$

$$K_1 = \frac{d}{ds}\left(s^2F(s)\right)\Big|_{s=0} = \frac{d}{ds}\left(\frac{s+4}{s^2+10s+16}\right)\Big|_{s=0}$$

$$= \frac{s^2+10s+16-(s+4)(2s+10)}{(s^2+10s+16)^2}\Big|_{s=0} \qquad \therefore K_1 = \frac{-3}{32}$$

$$F(s) = \frac{-3/32}{s} + \frac{1/4}{s^2} + \frac{1/12}{s+2} + \frac{1/96}{s+8}$$

$$\therefore f(t) = \left(\frac{-3}{32} + \frac{1}{4}t + \frac{1}{2}e^{-2t} + \frac{1}{96}e^{-8t}\right)u(t)$$

b) $F(s) = \dfrac{s^2+9s+18}{s(s+4)(s^2+10s+24)}$

$s^2+9s+18=0$ $\qquad s^2+9s+18=(s+3)(s+6)$

$s = \dfrac{-9 \pm \sqrt{81-72}}{2} < \begin{matrix} s_1=-3 \\ s_2=-6 \end{matrix}$

$s^2+10s+24=0$ $\qquad s^2+10s+24=(s+4)(s+6)$

$s = \dfrac{-10 \pm \sqrt{100-96}}{2} < \begin{matrix} s_1=-4 \\ s_2=-6 \end{matrix}$

$F(s) = \dfrac{(s+3)\cancel{(s+6)}}{s(s+4)(s+4)\cancel{(s+6)}} = \dfrac{K_1}{s} + \dfrac{K_2}{s+4} + \dfrac{K_3}{(s+4)^2}$

$K_1 = sF(s)\Big|_{s=0} = \dfrac{(s+3)}{(s+4)^2} = \dfrac{3}{16}$

$K_3 = (s+4)^2F(s)\Big|_{s=-4} = \dfrac{s+3}{s} = \dfrac{1}{4}$

$K_2 = \dfrac{d}{ds}\left(\dfrac{s+3}{s}\right)\Big|_{-4} = \dfrac{s-(s+3)}{s^2}\Big|_{-4} = \dfrac{-3}{16}$

$\therefore F(s) = \dfrac{3/16}{s} + \dfrac{3/16}{s+4} + \dfrac{1/4}{(s+4)^2}$

$\therefore f(t) = \left(\dfrac{3}{16} - \dfrac{3}{16}e^{-4t} + \dfrac{1}{4}t.e^{-4t}\right)u(t)$

16.37) (a) $F(s) = \dfrac{s(s+6)}{(s+3)(s^2+6s+18)}$

$s^2+6s+18=0$

$s = \dfrac{-6 \pm \sqrt{36-72}}{2} \quad \therefore s = -3 \pm j3$

$F(s) = \dfrac{K_1}{s+3} + \dfrac{K_2}{s-(-3+j3)} + \dfrac{K_2^*}{s-(-3-j3)}$

$= \dfrac{K_1}{s+3} + \dfrac{K_2}{s+3-j3} + \dfrac{K_2^*}{s+3+j3}$

$K_1 = (s+3)F(s)\Big|_{s=-3} = \dfrac{s(s+6)}{s^2+6s+18}\Big|_{-3} = -1$

$$K_2 = (s+3-j3)\cdot F(s)\Big|_{s=-3+j3} = \frac{s(s+6)}{(s+3)(s+3+j3)}$$

$$= \frac{(-3+j3)(-3+j3+6)}{(\cancel{-3}+j3+\cancel{3})(\cancel{-3}+j3+\cancel{3}+3+j3)} = \frac{9-9j-18-9j-9+18j}{-9+j6-9} = \frac{-18}{-18+j6}$$

$$= \frac{-18\angle 0}{18{,}97\angle 161{,}6} = -0{,}94\angle -161{,}6$$

$$\therefore f(t) = [-e^{-3t} + 2.\,0{,}94\,e^{-3t}\cos(3t-161{,}6)]\,u(t)$$

16.38) (a) $F(s) = \dfrac{s+2}{(s^2+4s+5)(s^2+4s+8)}$

$s^2+4s+5=0$ $\qquad\qquad$ $s^2+4s+8=0$

$s = \dfrac{-4 \pm \sqrt{16-20}}{2} = -2 \pm j1$ $\qquad\qquad$ $s = \dfrac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$

$$F(s) = \frac{s+2}{(s+2-j1)(s+2+j1)(s+2-j2)(s+2+j2)}$$

$$= \frac{K_1}{s+2-j1} + \frac{K_1^*}{s+2+j1} + \frac{K_2}{s+2-j2} + \frac{K_2^*}{s+2+j2}$$

$$K_1 = (s+2-j)\,F(s)\Big|_{s=-2+j} = \frac{\cancel{-2}+j+\cancel{2}}{(\cancel{-2}+j+\cancel{2}+j)(\cancel{-2}+j+\cancel{2}-j2)(\cancel{-2}+j+\cancel{2}+j2)}$$

$$= \frac{\cancel{j}}{(j2)(-j)(j3)} = \frac{1}{6} = \frac{1}{6}\angle 0$$

$$K_2 = (s+2-j2)F(s)\Big|_{s=-2+2j} = \frac{\cancel{-2}+2j+\cancel{2}}{(\cancel{-2}+2j+\cancel{2}-j)(\cancel{-2}+2j+\cancel{2}+j)(\cancel{-2}+2j+\cancel{2}+j2)}$$

$$= \frac{2j}{j\times(3j)\overset{\times}{\underset{2}{}}(4j)} = \frac{-1}{6} = \frac{1}{6}\angle 0$$

$$F(t) = \frac{1}{3}e^{2t}\cos(-t) + \frac{1}{3}e^{2t}\cos(2t+180)$$

b) $F(s) = \dfrac{s(s+2)}{s^2+2s+2}$

$s^2+2s+2=0$

$s = \dfrac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$

$F(s) = \dfrac{s(s+2)}{s^2+2s+2} = \dfrac{K_1}{s+1-j} + \dfrac{{}^*K_1}{s+1+j}$

$K_1 = (s+1-j)F(s)\Big|_{s=-1+j} = \dfrac{s(s+2)}{s+1+j} = \dfrac{(-1+j)(-1+j+2)}{\cancel{-1}+j+\cancel{1}+j}$

$K_1 = \dfrac{(-1+j)(1+j)}{j2} = \dfrac{j^2-1}{j2} = \dfrac{-1-1}{j2} = \dfrac{-2}{j2} = \dfrac{-1}{j} \dfrac{\times j}{\times j}$

$K_1 = \dfrac{-j}{j^2} = \dfrac{-j}{-1} = j = 1\angle 90$

$\boxed{F(t) = (2e^{-t}\cos(t+90))u(t)}$

16.39

(a) $F(s) = \dfrac{(s+1)(s+3)}{(s+2)(s^2+4s+8)}$

$s^2+4s+8=0$

$s = \dfrac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$

$F(s) = \dfrac{(s+1)(s+3)}{(s+2)(s^2+4s+8)} = \dfrac{(s+1)(s+3)}{(s+2)(s+2-j2)(s+2+j2)}$

$= \dfrac{K_1}{(s+2)} + \dfrac{K_2}{(s+2-j2)} + \dfrac{{}^*K_2}{(s+2+j2)}$

$K_1 = (s+2)F(s)\Big|_{s=-2} = \dfrac{(-2+1)(-2+3)}{(-2)^2+4(-2)+8} = \dfrac{-1}{4}$

$K_2 = (s+2-j2)F(s)\Big|_{s=-2+j2} = \dfrac{(-2+j2+1)(-2+j2+3)}{(-2+j2+2)(-2+j2+2+j2)}$

$= \dfrac{-5}{-8} = \dfrac{5}{8} = \dfrac{5}{8}\angle 0$ $\quad f(t) = -\dfrac{1}{4}e^{-2t} + \dfrac{5}{4}e^{-2t}\cos(2t)$

b) $F(s) = \dfrac{(s+2)^2}{s^2+4s+8}$

$s^2+4s+8=0$

$s = \dfrac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$: $\dfrac{(s+2)^2}{s^2+4s+8} = \dfrac{K_1}{s+2-j2} + \dfrac{{}^*K_1}{s+2+j2}$

$K_1 = (s+2-j2)\Big|_{s=-2+j2} = \dfrac{(\cancel{-2}+j2+\cancel{2})^2}{\cancel{-2}+j2+\cancel{2}+j2} = \dfrac{-4}{j4} = j = 1\angle 90$

$f(t) = (\ 2e^{-2t}\cos(2t+90))\,u(t)$

16.42) a)

$F(s) = \dfrac{12(s+2)}{s^2(s+1)(s^2+4s+8)}$

$s^2+4s+8=0$ $\qquad s^2+4s+8 = (s+2-j2)(s+2+j2)$

$s = \dfrac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$

$\dfrac{12(s+2)}{s^2(s+1)(s^2+4s+8)} = \dfrac{K_1}{s} + \dfrac{K_2}{s^2} + \dfrac{K3}{(s+1)} + \dfrac{K4}{s+2-j2} + \dfrac{{}^*K4}{s+2+j2}$

$K_2 = \dfrac{1}{(2-2)!} \cdot \dfrac{d^{2-2}}{ds^{2-2}} \left[s^2F(s) \right]\Big|_{s=0} = \dfrac{12(0+2)}{(0+1)(0^2+4.0+8)} = 3$

$K_1 = \underset{1}{\dfrac{1}{(2-1)!}} \cdot \dfrac{d^{2-1}}{ds^{2-1}} \left[s^2F(s) \right]\Big|_{s=0} = \dfrac{d}{ds}\left(\dfrac{12(s+2)}{(s+1)(s^2+4s+8)} \right)$

$= 12 \times \left(\dfrac{(s+1)(s^2+4s+8) - (s+2).[(s^2+4s+8) - (s+1).(2s+4)]}{((s+1)(s^2+4s+8))^2} \right)$

$= 12 \times \left(\dfrac{8 - 2\times(8-4)}{8^2} \right) = 0$

$K_3 = (s+1)F(s)\Big|_{s=-1} = \dfrac{12(s+2)}{s^2(s^2+4s+8)} = \dfrac{12((-1)+2)}{(-1)^2((-1)^2+4(-1)+8)} = \dfrac{12}{5}$

$$K_4 = (s+2-j2)F(s)\Big|_{s=-2+j2} = \frac{12(s+2)}{s^2(s+1)(s+2+j2)}$$

$$= \frac{12(\cancel{-2}+j2\cancel{+2})}{(-2+j2)^2(-2+j2+1)(\cancel{-2}+j2+\cancel{2}+j2)} = \frac{24j}{-32+j64}$$

$$= 0{,}3 - j0{,}15 = \frac{3}{10}\angle{-26{,}6}$$

$$f(t) = \left(3t + \frac{12}{5}e^{-t} + \frac{3}{5}e^{-2t}\cos(2t-26{,}6)\right)u(t)$$

16.45)

$$F(s) = \frac{(s+1)e^{-s}}{s(s+2)(s^2+2s+2)}$$

$$s^2+2s+2=0$$

$$s = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$$

$$F(s) = \frac{(s+1)e^{-s}}{s(s+2)(s^2+2s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+1+j} + \frac{K_3^*}{s+1-j}$$

$$K_1 = sF(s)\Big|_{s=0} = \frac{(0+1)\cdot e^{-0}}{(0+2)(0+0+2)} = \frac{1}{4}$$

$$K_2 = (s+2)F(s)\Big|_{s=-2} = \frac{[(-2)+1]e^{-(-2)}}{(-2)[\cancel{(-2)^2}+\cancel{2(-2)}+2]} = \frac{-e^2}{-4} = \frac{e^2}{4}$$

$$K_3 = (s+1+j)F(s)\Big|_{s=-1-j} = \frac{(\cancel{-1}-j\cancel{+1})e^{-(-1-j)}}{(-1-j)(-1-j+2)(-1-j+1-j)}$$

$$= \frac{\cancel{j}e^1e^j}{j4} = -0{,}271 - j0{,}815 = 0{,}86\angle{-108{,}4}$$

$$f(t) = \left(\frac{1}{4} + \frac{e^2}{4} \times e^{-2t} + 1{,}72\,e^{-t}\cos(t-108{,}4)\right)u(t)$$

X

16.47) $F(s) = \frac{1-e^{-s}}{s^2(s+1)}$

$$F(s) = \frac{1-e^{-s}}{s^2(s+1)} = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+1}$$

$$K_1 = s^2 F(s)\Big|_{s=0} = \frac{1-e^{-s}}{s+1}\Big|_{s=0} = 0$$

$$K_2 = \frac{d}{ds}\left(s^2F(s)\right)\Big|_{s=0} = \frac{d}{ds}\left(\frac{1-e^{-s}}{s+1}\right) = \frac{e^{s}(s+1)+(1-e^{-s})}{(s+1)^2}$$

$$K_2 = 1$$

$$K_3 = (s+1)F(s)\Big|_{s=-1} = \frac{1-e^{-(-1)}}{(-1)^2} = \frac{1-e}{1}$$

Dúvida: ?

16.51 Determine o valor inicial e final de $f(t)$

teorema do valor Inicial

$$VI = \lim_{t\to 0} f(t) = \lim_{s\to\infty} s\,F(s)$$

teorema do valor final

$$V.F = \lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F(s)$$

(a) $F(s) = \frac{2(s+2)}{(s+1)(s)}$

$$\frac{K_1}{s+1} + \frac{K_2}{s}$$

$$K_1 = (s+1)F(s)\Big|_{s=-1} = \frac{2(s+2)}{s} = -2$$

$$K_2 = s\,F(s)\Big|_{s=0} = \frac{2(s+2)}{s+1} = 4$$

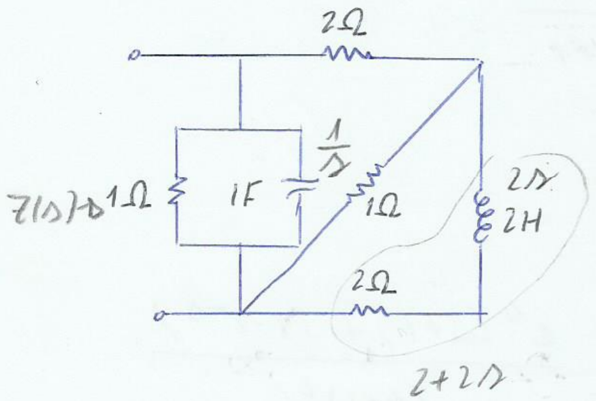
$$f(t) = (4-2e^{-t})\,u(t)$$

$$VI = \lim_{t\to 0}(4-2e^{-t}) = 4-2 = 2 \quad ; \lim_{s\to\infty} = \frac{2(s+2)}{s+1} = 2$$

$$VF = \lim_{t\to\infty}(4-2e^{-t}) = \lim_{s\to 0}\left(\frac{2(s+2)}{s+1}\right) = 4$$

Capítulo 17

Exercício 17.1 Determine a impedância $Z(s)$



$$1 \parallel \frac{1}{s} = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{\frac{1}{s}}{\frac{s+1}{s}}$$

$$1 \parallel \frac{1}{s} = \frac{1}{s+1}$$

$$1 \parallel 2+2s = \frac{2+2s}{3+2s} \to 2+\frac{2+2s}{3+2s} = \frac{6+4s+2+2s}{3+2s}$$

$$= \frac{8+6s}{3+2s} \quad ; \quad \frac{1}{s+1} \parallel \frac{8+6s}{3+2s}$$

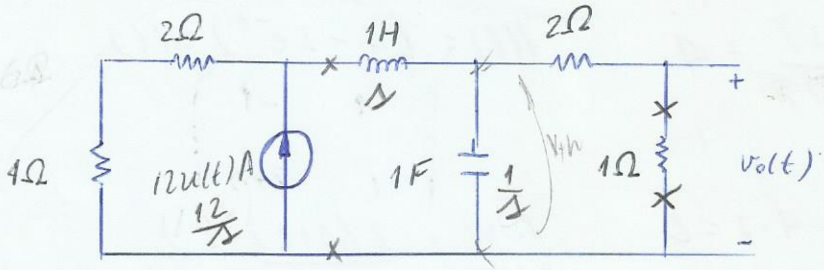
$$= \frac{8+6s}{(s+1)(3+2s)} \div \left(\frac{1}{s+1} + \frac{8+6s}{3+2s} \right)$$

$$= \frac{8+6s}{(s+1)\cancel{(3+2s)}} \div \frac{3+2s+(8+6s)(s+1)}{(s+1)\cancel{(3+2s)}}$$

$$= \frac{8+6s}{3+2s+(8s+8+6s^2+6s)}$$

$$Z(s) = \frac{8+6s}{6s^2+16s+11}$$

Exercício 17.4

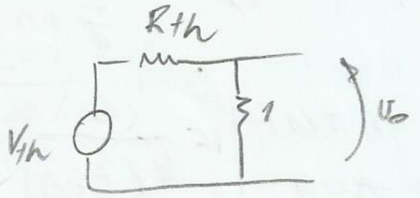


$$R_{th} = [(6+s) \| \tfrac{1}{s}] + 2 = \frac{\frac{6+s}{s}}{\frac{s^2+6s+1}{s}} + 2$$

$$= \frac{6+s}{s^2+6s+1} + 2 = \frac{6+s+2s^2+12s+2}{s^2+6s+1}$$

$$= \frac{2s^2+13s+8}{s^2+6s+1}$$

$$I = \frac{6}{6+s+\frac{1}{s}} \cdot \frac{12}{s} = \frac{72}{s^2+6s+1}$$



$$V_{th} = \frac{72}{(s^2+6s+1)s}$$

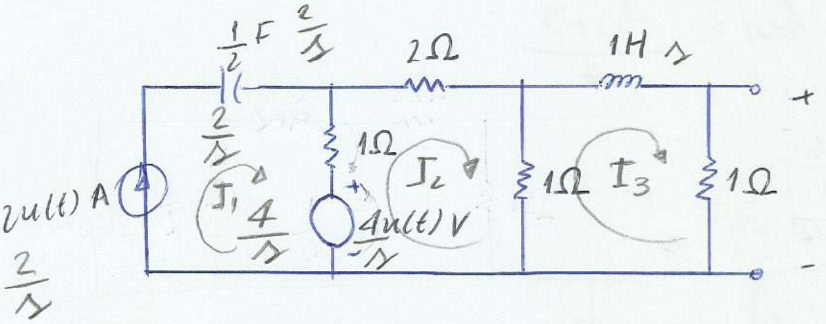
$$v_o = \frac{1}{1+R_{th}} \cdot V_{th}$$

$$= \frac{1}{1+\frac{2s^2+13s+8}{s^2+6s+1}} \times \frac{72}{(s^2+6s+1)s}$$

$$= \frac{1}{\frac{s^2+6s+1+2s^2+13s+8}{\cancel{s^2+6s+1}}} \times \frac{72}{(\cancel{s^2+6s+1})s}$$

$$\lim_{t\to\infty} v_o = \lim_{s\to 0} s\, v_o = \frac{72}{3s^2+21s+9} = \boxed{8\,V}$$

Exercício 17.12



$$\begin{cases} 4I_2 - I_1 - I_3 = \frac{4}{s} \\ (2+s)I_3 - I_2 = 0 \end{cases} ; \quad I_1 = \frac{2}{s}$$

$$\begin{vmatrix} 4 & -1 \\ -1 & 2+s \end{vmatrix} = $$

$$\begin{cases} 4I_2 - I_3 = \frac{6}{s} \\ -I_2 + (2+s)I_3 = 0 \end{cases}$$

$$= 8+4s-1 = 7+4s$$

$$\begin{vmatrix} 4 & \frac{6}{s} \\ -1 & 0 \end{vmatrix} = \frac{6}{s}$$

$$I_3 = \frac{\frac{6}{s}}{7+4s} = \frac{6}{(7+4s)s} = \frac{6}{4s\left(\frac{7}{4}+s\right)}$$

$$= \frac{6}{4}\left(\frac{K_1}{s} + \frac{K_2}{\frac{7}{4}+s}\right)$$

$$K_1 = s.F(s)\Big|_{s=0} = \frac{6}{\cancel{4}\left(\frac{7}{\cancel{4}}+s\right)} = \frac{6}{7}$$

$$K_2 = \left(\frac{7}{4}+s\right)F(s)\Big|_{s=-\frac{7}{4}} = \frac{6}{4s} = \frac{6}{\cancel{4}\left(\frac{-7}{\cancel{4}}\right)} = \frac{-6}{7}$$

$$v_0(t) = \frac{6}{7}\left(1 - e^{-\frac{7}{4}t}\right)u(t)$$

E17.13 Utilize o teorema de thévenin para o problema anterior

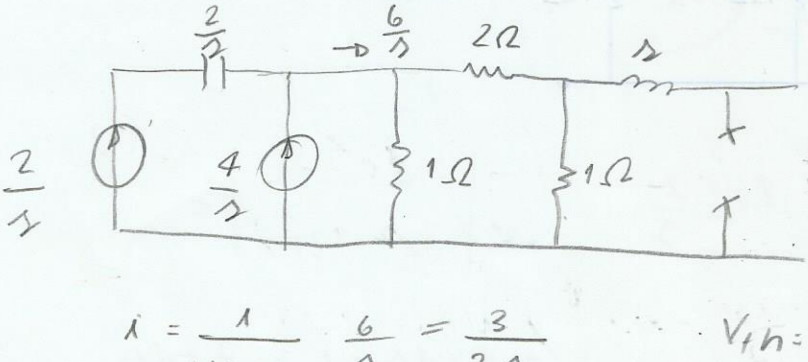
Determinando R_{th}

$(1+2)||1 + s$

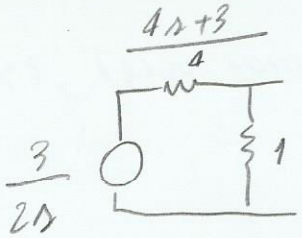
$3||1 + s$

$\frac{3}{4} + s \quad \therefore \quad R_{th} = \frac{4s+3}{4}$

Determinando V_{th}



$i = \frac{1}{1+2+1}\cdot\frac{6}{s} = \frac{3}{2s} \qquad V_{th} = \frac{3}{2s} \times 1 = \frac{3}{2s}$

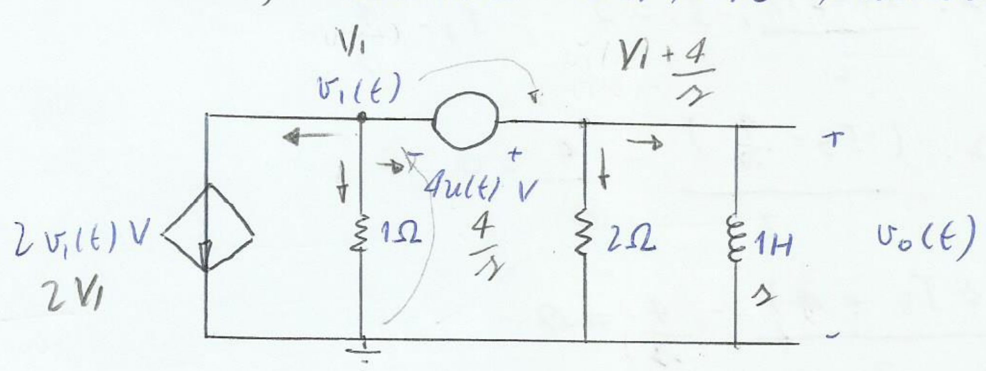


$$\therefore\ V_o(s) = \frac{1}{\frac{4s+3}{4}+1} \cdot \frac{3}{2s}$$

$$= \frac{4\,2}{4s+7}\,\frac{3}{\cancel{2}s} = \frac{6}{4s+7} = \frac{6}{4\left(s+\frac{7}{4}\right)}$$

$$\therefore\ v_o(t) = \frac{6}{7}\left(1 - e^{-\frac{7}{4}t}\right)u(t)$$

Ex: 17.16 (livro) Determine $v_o(t)$, $t>0$, usando as equações nodais



$$\begin{cases} 2V_1 + V_1 + I_g = 0 \\ \frac{V_1}{2} + \frac{2}{s} + \frac{V_1}{s} + \frac{4}{s^2} - I_g = 0 \end{cases} \quad +$$

$$V_o = V_1 + \frac{4}{s}$$

$$2V_1 + V_1 + \frac{V_1}{2} + \frac{2}{s} + \frac{V_1}{s} + \frac{4}{s^2} = 0 \qquad 2s^2$$

$$4s^2V_1 + 2s^2V_1 + s^2V_1 + 4s + 2sV_1 + 8 = 0$$

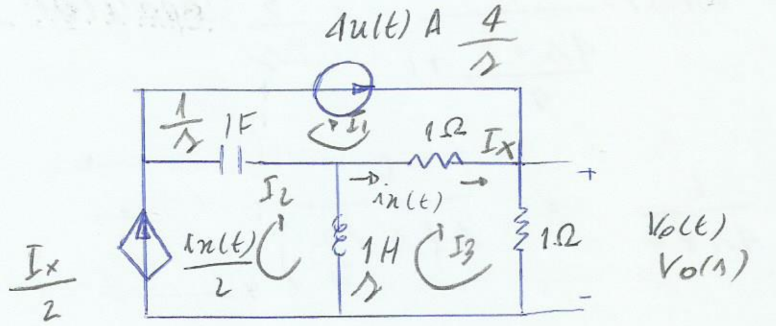
$$V_1(4s^2 + 2s^2 + s^2 + 2s) = -8 - 4s$$

$$V_1 = \frac{-(8+4s)}{7s^2+2s} \qquad V_o = \frac{-(8+4s)}{7s^2+2s} + \frac{4}{s}$$

$$V_o = \frac{-4s^2 - 8s + 28s^2 + 8s}{(7s^2+2s)s} = \frac{-24\cancel{s^2}}{\cancel{s^2}(7s+2)}$$

$$= \frac{-24}{7\left(s+\frac{2}{7}\right)} = \left(\frac{-24}{7}\cdot e^{-\frac{2}{7}t}\right)u(t)\ [V]$$

Ex: 17.21 Utilize análise de malha para determinar $v_o(t)$, $t>0$



$(2+s)I_3 - sI_2 - I_1 = 0 \quad ; \quad I_2 = \frac{I_x}{2} \quad ; \quad I_x = I_3 - I_1$

$(2+s).I_3 - s.\frac{(I_3 - I_1)}{2} - I_1 = 0 \quad ; \quad I_1 = \frac{4}{s}$

$(2+s)I_3 - s.\frac{(I_3 - \frac{4}{s})}{2} - \frac{4}{s} = 0$

$2I_3 + sI_3 - \frac{(sI_3 - 4)}{2} - \frac{4}{s} = 0$

$4sI_3 + 2s^2I_3 - (s^2I_3 - 4s) - 8 = 0$

$4sI_3 + 2s^2I_3 - s^2I_3 + 4s - 8 = 0$

$s^2I_3 + 4sI_3 + 4s - 8 = 0$

$I_3(s^2 + 4s) = 8 - 4s$

$I_3 = \frac{8-4s}{s^2+4s}$ logo $v_o = I_3 .. 1$

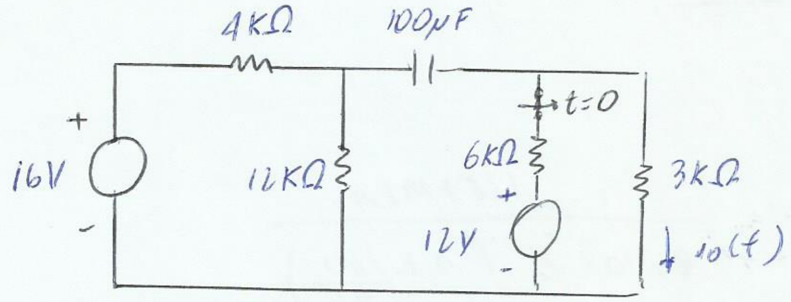
$V_o = \frac{8-4s}{s^2+4s} = \frac{8-4s}{s(s+4)} = \frac{K_1}{s} + \frac{K_2}{s+4}$

$K_1 = sF(s)\Big|_{s=0} = \frac{8-4s}{s+4} = \frac{8}{4} = 2$

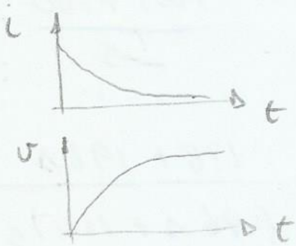
$K_2 = (s+4)F(s)\Big|_{s=-4} = \frac{8-4s}{s} = \frac{8-4(-4)}{-4} = -6$

$v_o(t) = (2 + 2e^{-4t})u(t)$ (V)

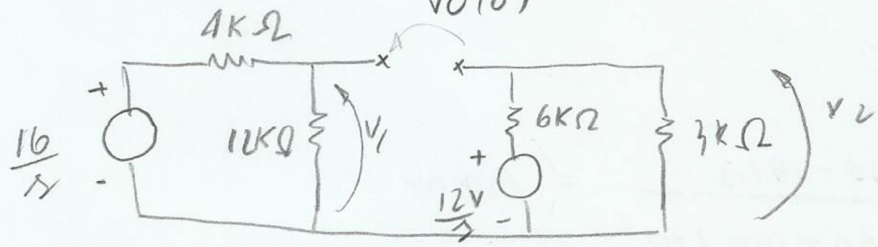
Ex. 17.27 Determine $i_o(t)$, $t>0$



Capacitor



P/ $t<0$

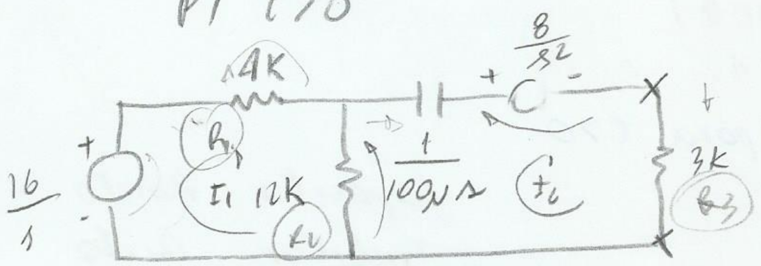


$$V_1 = \frac{12}{12+4} \cdot \frac{16}{s} = \frac{12}{s}$$

$$V_2 = \frac{3}{3+6} \cdot \frac{12}{s} = \frac{4}{s}$$

$$V_c(0) = V_1 - V_2 = \frac{12}{s} - \frac{4}{s} = \frac{8}{s}$$

P/ $t>0$



$$\begin{cases} 16K\, I_1 - 12K\, I_2 = \frac{16}{s} \\ \left(15K + \frac{10K}{s}\right) \cdot I_2 - 12K\, I_1 = \frac{8}{s^2} \end{cases}$$

$$\begin{vmatrix} 16K & -12K \\ -12K & 15K + \frac{10K}{s} \end{vmatrix} = 240 \cdot 10^6 + \frac{160}{s} 10^6 - 144 \cdot 10^6$$

$$= \left(96 + \frac{160}{s}\right) \cdot 10^6$$

$$\begin{vmatrix} 16K & \frac{16}{s} \\ -12K & \frac{8}{s^2} \end{vmatrix} = \frac{128K}{s^2} + \frac{192K}{s} = \frac{128K + 192K \cdot s}{s^2}$$

$$I_2 = \frac{128K + 192Ks}{s^2} \div \left(96 + \frac{160}{s}\right) \cdot 10^6$$

$$I_2 = \frac{\dfrac{(128 + 192s) \cdot \cancel{10^3}}{s^2}}{\dfrac{96s + 160}{\cancel{s}} \cdot 10^{\cancel{6}3}}$$

$$= \frac{128 + 192s}{(96s + 160)s \cdot 10^3} = \frac{128 + 192s}{96 \cdot 10^3 \, s \left(s + \frac{160}{96}\right)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + \frac{160}{96}}$$

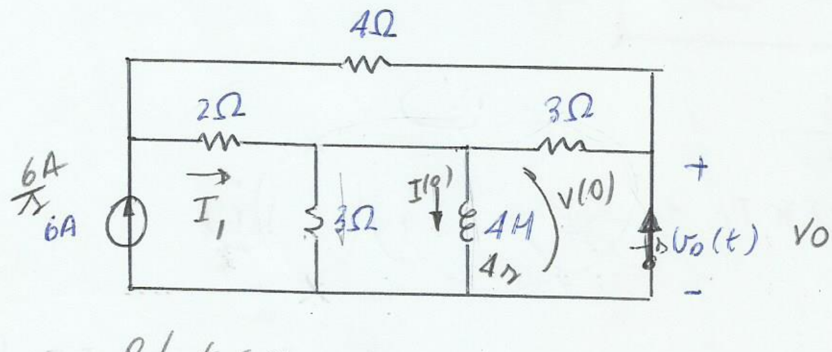
$$K_1 = sF(s)\Big|_{s=0} = \frac{128 + 192s}{(96s + 160) \cdot 10^3} = 0{,}8m$$

$$K_2 = \left(s + \frac{160}{96}\right) \cdot F(s)\Big|_{s = -\frac{160}{96}} = \frac{128 + 192 \cdot \left(-\frac{160}{96}\right)}{96 \cdot 10^3 \cdot \left(-\frac{160}{96}\right)} = \frac{-192}{-160\,000} = 1{,}2m$$

(wrong)

17.30 Determine $v_0(t)$ para $t > 0$

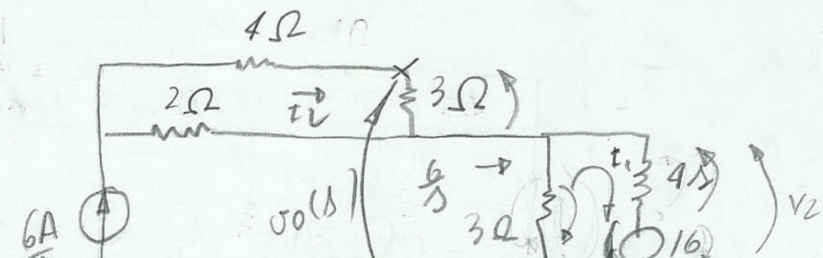
Capacitor Aberto
Indutor Curto



P/ $t < 0$

$$I = \frac{4}{\cancel{2+4}} \cdot 6 = 4$$

P/ $t > 0$



$V_2 = 4sI_1 - 16$

$4sI + 3(I - \frac{6}{s}) = 16$ $\qquad I = \dfrac{\frac{16s+18}{s}}{4s+3} = \dfrac{16s+18}{(4s+3)s}$

$4sI + 3I - \frac{18}{s} = 16$

$(4s+3)I = 16 + 18$

$V = V_1 + V_2$

$I_2 = \frac{2}{4+3+2} \cdot \frac{6}{s} = \frac{4}{3s}$ $\qquad V_2 = 3I_2 = \frac{4}{s}$

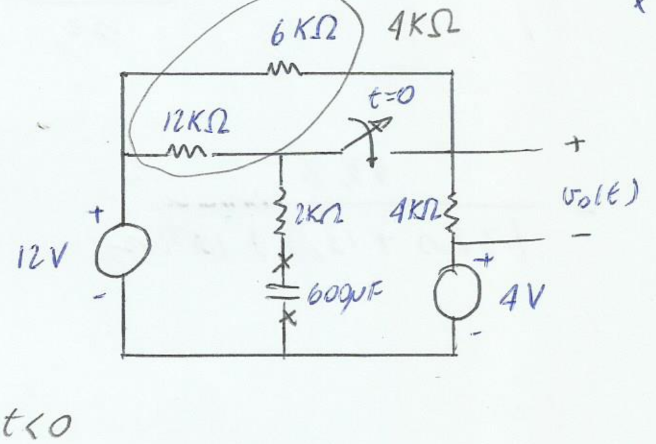
$V_1 = 4\cancel{s}\left(\frac{16s+18}{(4s+3)\cancel{s}}\right) - 16$

$= \frac{\cancel{64s} + 72 - \cancel{64s} - 48}{4s+3} = \frac{24}{4s+3} = \frac{24}{4(s+\frac{3}{4})} = \frac{6}{s+\frac{3}{4}}$

$V = \_\_ + \frac{6}{s+\frac{3}{4}}$

$V(t) = (4 + 6e^{-0{,}75t})u(t)$

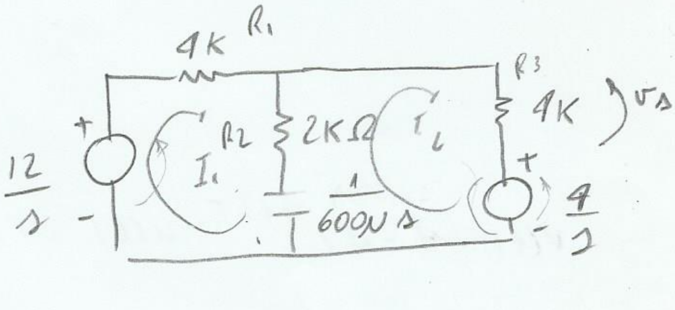
Ex: 17.31 $\qquad$ × Determine $v_o(t)$



P/ $t<0$

$V_c(0) = 12$

P/ $t>0$



$(R_1+R_2+\frac{1}{Cs})I_1 - (R_2+\frac{1}{Cs})I_2 = \frac{12}{s}$

$(R_2+R_3+\frac{1}{Cs})I_2 - (R_2+\frac{1}{Cs})I_1 = -\frac{4}{s}$

$$\begin{vmatrix} R_1+R_2+\frac{1}{Cs} & -(R_2+\frac{1}{Cs}) \\ -(R_2+\frac{1}{Cs}) & R_2+R_3+\frac{1}{Cs} \end{vmatrix} =$$

$= (R_1+R_2+\frac{1}{Cs})(R_2+R_3+\frac{1}{Cs}) - (R_2+\frac{1}{Cs})^2$

$= (4K+2K+\frac{1}{600\mu s})(2K+4K+\frac{1}{600\mu s}) - (2K+\frac{1}{600\mu s})^2$

$$= \left(6K + \frac{1}{600\mu . s}\right)^2 - \left(2K + \frac{1}{600\mu s}\right)^2$$

$$= 36.10^6 + \frac{20.10^6}{s} + \cancel{\frac{2,8.10^6}{s^2}} - \left(4.10^6 + \frac{6,7.10^6}{s} + \cancel{\frac{2,8.10^6}{s^2}}\right)$$

$$= 32.10^6 + \frac{13,3\ 10^6}{s} = \frac{32s + 13,3}{s} . 10^6$$

$$\begin{vmatrix} R_1 + R_2 + \frac{1}{Cs} & \frac{12}{s} \\ -\left(R_2 + \frac{1}{Cs}\right) & -\frac{4}{s} \end{vmatrix} = \left(4K + 2K + \frac{1}{600\mu s}\right) \cdot \left(\frac{-4}{s}\right) + \frac{12}{s}\left(2K + \frac{1}{600\mu s}\right)$$

$$= \frac{-24K}{s} - \frac{1}{150\mu s^2} + \frac{24K}{s} + \frac{1}{50\mu s^2}$$

$$= \frac{-24.10^3}{s} - \frac{6,7.10^3}{s^2} + \frac{24.10^3}{s} + \frac{20.10^3}{s^2}$$

$$= \frac{\cancel{-24.10^3 s} - 6,7.10^3 + \cancel{24.10^3 s} + 20.10^3}{s^2} = \frac{13,3.10^3}{s^2}$$

$$I_2 = \frac{\frac{13,3.\cancel{10^3}}{s^{\cancel{2}}}}{\frac{32s + 13,3}{\cancel{s}} 10^{\cancel{6}\,3}} = \frac{13,3}{(32s + 13,3).10^3 s}$$

$V_0 = 4K . I_2$

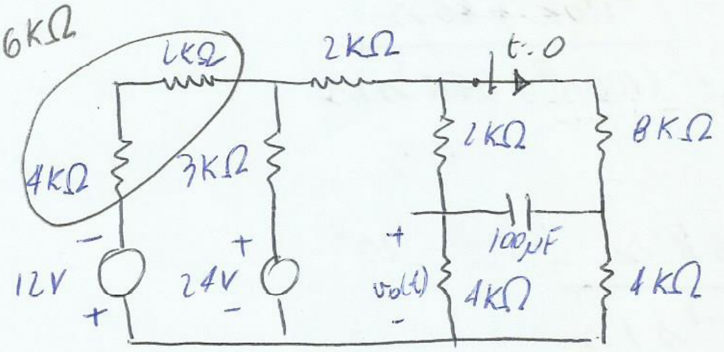
$$= \frac{53,3}{(32s + 13,3)s} = \frac{53,3}{32s\left(s + \frac{13,3}{32}\right)} = \frac{1,67}{s(s + 0,42)}$$

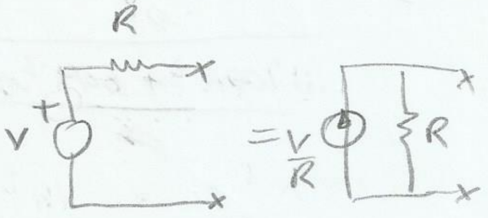
$$= \frac{K_1}{s} + \frac{K_2}{s + 0,42}$$

$$K_1 = sF(s)\Big|_{s=0} = \frac{53,3}{13,3} = 4 \qquad \therefore V(t) = (4 - 4e^{-0,42t})u(t)\ (V)$$

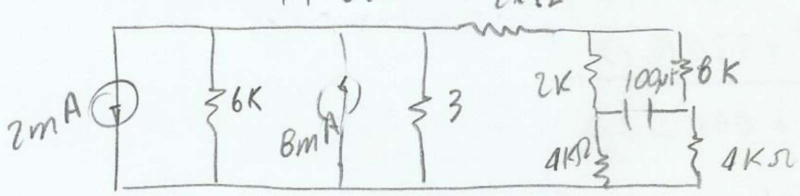
$$K_2 = (s + 0,42)\Big|_{s=-0,42} = \frac{53,3}{-32.0,42} = -4$$

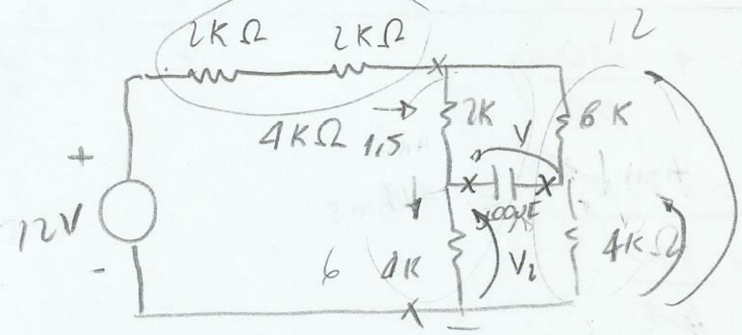
17.32 Determine $v_o(t)$ para $t>0$





P/ t<0





6

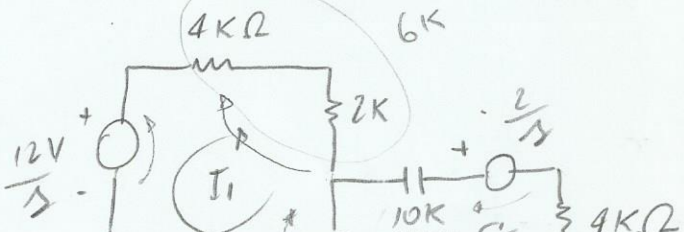
$V = \frac{R}{I}$

$\frac{4}{6} - \frac{4}{12}$

$V_1 = \frac{4}{8} \cdot 12 = 6V$

$V = 4.K.1m - 4K.0,5m = 2V$

P/ t>0



$$\begin{cases} 10K\,I_1 - 4K\,I_2 = \frac{12}{s} \\ \left(8K + \frac{10K}{s}\right) I_2 - 4K\,I_1 = -\frac{2}{s} \end{cases}$$

$$\begin{vmatrix} 10K & -4K \\ -4K & 8K + \frac{10K}{s} \end{vmatrix} = 80K^2 + \frac{100K^2}{s} - 16K^2 = \frac{100K^2 + 64K^2 s}{s}$$

$$\begin{vmatrix} \frac{12}{s} & -4K \\ -\frac{2}{s} & 8K + \frac{10K}{s} \end{vmatrix} = \frac{96K}{s} + \frac{120K}{s^2} - \frac{8K}{s} = \frac{120K + 88s}{s^2}$$

$$I = \frac{\frac{120K + 88s}{\cancel{s}}}{\frac{100K^2 + 64K^2 s}{\cancel{s}}} = \frac{120K + 88s}{(100K^2 + 64K^2 s)s}$$

$$V = 6K \cdot I = \frac{720K^2 + 528Ks}{(100K^2 + 64K^2 s)s}$$

$$v_o(s) = \frac{12}{s} - \frac{720K^2 + 528Ks}{(100K^2 + 64K^2 s)s}$$

$$= \frac{1200K^2 + 768K^2 s - 720K^2 - 528Ks}{64K^2 s\left(s + \frac{100}{64}\right)} = \frac{K_1}{s} + \frac{K_2}{s + \frac{100}{64}}$$

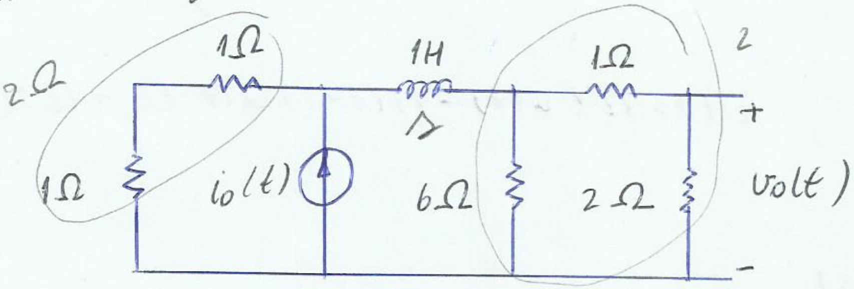
$$K_1 = sF(s)\Big|_{s=0} = \frac{1200\cancel{K^2} - 720\cancel{K^2}}{\cancel{64K^2} \cdot \frac{100}{\cancel{64}}} = 4{,}8$$

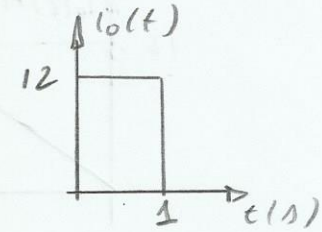
$$K_2 = \left(s + \frac{100}{64}\right)F(s)\Big|_{s = -\frac{100}{64}} = \frac{480K^2 + 766K^2 \cdot \left(-\frac{100}{64}\right) - 528K\left(\frac{100}{64}\right)}{\cancel{64}K^2 \cdot \left(\frac{-100}{\cancel{64}}\right)}$$

4

Resposta: $v_o(t) = (4{,}8 - 1{,}05\, e^{-1{,}56t})\, u(t)$

Exercício 17.46





$i_0(t) = 12u(t) - 12u(t-1)$

$I_0 = \frac{12}{s} - 12 \cdot e^{-s} = \frac{12 - 12e^{-s} \cdot s}{s}$

$i_1 = \frac{2}{2+2+s} \cdot \frac{12 - 12e^{-s} \cdot s}{s} = \frac{24 - 24e^{-s} \cdot s}{(4+s)s}$

$i_2 = \frac{\cancel{6}\ 2}{\cancel{6+3}\ 3} \cdot \frac{24 - 24e^{-s} \cdot s}{(4+s)s} = \frac{16 - 16e^{-s} \cdot s}{(4+s)s}$

logo $v_0(s) = 2i_2 = 32 \times \frac{1 - e^{-s} \cdot s}{(4+s)s}$

$V_0 = \frac{1}{(4+s)s} - \frac{1}{(4+s)s} \cdot e^{-s}$

V_{01} V_{02}

$V_{01} = \frac{K_1}{s} + \frac{K_2}{4+s}$

$K_1 = sF(s)\Big|_{s=0} = \frac{32}{4+s} = 8$

$K_2 = (4+s)F(s)\Big|_{s=-4} = \frac{32}{s} = -8$

$v_{01}(t) = (8 - 8e^{-4t})u(t)$

~~Exercício~~ $V_{02} = \left[\frac{K_1}{s} + \frac{K_2}{4+s}\right] e^{-s}$

$K_1 = sF(s)\Big|_{s=0} = \frac{32}{4+s} = 8$

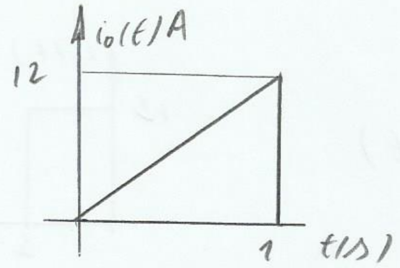
$K_2 = (s+4)F(s)\Big|_{s=-4} = \frac{32}{s} = -8$

$V_0(s) = \left[\frac{8}{s} - \frac{8}{4+s}\right] e^{-s}$

$v_{02}(t) = (8 - 8e^{-4(t-1)})u(t-1)$

$v_0(t) = (8 - 8e^{-4t})u(t) - (8 - 8e^{-4(t-1)})u(t$

17.46 Resolva o problema anterior caso a entrada seja:



$$i_o(t) = 12\,t\,u(t) - \underline{12}(t-1)\,u(t-1) - 12\,u(t-1)$$

$$I_o(s) = \frac{12}{s^2} - \frac{12e^{-s}}{s^2} - \frac{12}{s}e^{-s}$$

$$i = \frac{2}{4+s} \times \left(\frac{12}{s^2} - \frac{12e^{-s}}{s^2} - \frac{12}{s}e^{-s} \right)$$

$$= \frac{24}{4+s}\left(\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} \right)$$

$$V = 2i = \frac{48}{4+s}\left(\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} \right)$$

$$\therefore v_o(s) = \frac{2}{3} \cdot V = \frac{32}{4+s}\left(\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} \right)$$

$$= \frac{32}{(4+s)s^2} - \frac{32e^{-s}}{(4+s)s^2} - \frac{32e^{-s}}{(4+s)s}$$

$$= \frac{32 - 32e^{-s} - 32e^{-s}s}{(4+s)s^2}$$

$$= \underbrace{\frac{32}{(4+s)s^2}}_{V_1} - \underbrace{\frac{32e^{-s} + 32e^{-s}\cdot s}{(4+s)s^2}}_{V_2}$$

$$(4+s)^{-1}$$
$$-(4+s)^{-2}$$

$$V_1 = \frac{K_{11}}{s} + \frac{K_{12}}{s^2} + \frac{K_2}{4+s}$$

$$K_{11} = \frac{d}{ds}\left(s^2 F(s) \right)_{s=0} = \frac{d}{ds}\left(\frac{32}{4+s} \right)\Bigg|_{s=0}$$

$$= \frac{32}{(4+s)^2}\Bigg|_0 = 2$$

$$K_{12} = s^2F(s)\Big|_{s=0} = \frac{32}{4+s} = 8$$

$$K_2 = (4+s).F(s)\Big|_{s=-4} = \frac{32}{s^2} = 2$$

logo $V_1 = \frac{2}{s} + \frac{8}{s^2} + \frac{2}{4+s}$

$$V_1(t) = (2 + 8t + 2.e^{-4t})\,u(t)$$

$$V_2(s) = \frac{32\,(1+s)\,e^{-s}}{(4+s)\,s^2} = \frac{K_{11}}{s} + \frac{K_{12}}{s^2} + \frac{K_2}{4+s}$$

$$K_{11} = \frac{d}{ds}\left(\frac{32(1+s)}{4+s}\right)\Big|_{s=0} = 32\times\frac{(4+s)-(1+s)}{(4+s)^2}$$

$$= 32\times\frac{3}{(4+s)^2} = \frac{96}{(4+s)^2}\Big|_{s=0} = 6$$

$$K_{12} = s^2F(s)\Big|_{s=0} = \frac{32(1+s)}{(4+s)} = 8$$

$$K_2 = (s+4)F(s)\Big|_{s=-4} = \frac{32(1+s)}{s^2} = \frac{32(1-4)}{(-4)^2} = -6$$

$$V_2 = \left(\frac{24}{s} + \frac{8}{s^2} + \frac{(-6)}{4+s}\right)e^{-s}$$

$$V_2(t) = (6 + 8(t-1) - 6.e^{-4(t-1)})\,e^{-s}$$

$$\therefore V(t) = (2 + 8t + 2e^{-4t})\,u(t) - (6 + 8(t-1) - 6e^{-4(t-1)})\,u(t-1)\ (V)$$

Ex: 17.58

$$x(t) = [e^{-2t} - e^{-6t}]\,u(t) \;\langle\!-\!\rangle\; y(t) = [t - 1 + e^{-t}]\,u(t)$$

Determine a resposta $y_2(t)$, caso $x_2(t) = [e^{-t} - e^{-2t}]\,u(t)$

$$X(s) = \frac{1}{s+2} - \frac{1}{s+6} = \frac{s+6-(s+2)}{(s+2)(s+6)} = \frac{4}{(s+2)(s+6)}$$

$$Y(s) = \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} = \frac{s+1-s(s+1)+s^2}{s^2(s+1)}$$

$$Y(s) = \frac{\cancel{s^2} - \cancel{s^2} \cancel{- s} \cancel{+ s} + 1}{s^2(s+1)} = \frac{1}{s^2(s+1)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2(s+1)} \div \frac{4}{(s+2)(s+6)}$$

$$= \frac{(s+2)(s+6)}{4s^2(s+1)}$$

Para $x_2(t) = [e^{-t} - e^{-2t}]u(t)$

$$X_2(s) = \frac{1}{s+1} - \frac{1}{s+2} = \frac{s+2-(s+1)}{(s+1)(s+2)} = \frac{1}{(s+1)(s+2)}$$

$$Y(s) = \frac{1}{(s+1)\cancel{(s+2)}} \times \frac{\cancel{(s+2)}(s+6)}{4s^2(s+1)} = \frac{s+6}{4s^2(s+1)^2}$$

$$Y(s) = \frac{K_{11}}{s} + \frac{K_{12}}{s^2} + \frac{K_{21}}{(s+1)} + \frac{K_{22}}{(s+1)^2}$$

$$K_{11} = \frac{d}{ds}\left(\frac{s+6}{4(s+1)^2}\right)\Bigg|_{s=0}$$

$$= \frac{1}{4} \times \left(\frac{(s+1)^2 - (s+6).2(s+1)}{(s+1)^4}\right)\Bigg|_{s=0}$$

$$= \frac{1}{4} \times \left(\frac{1-6+2}{1}\right) = \frac{-11}{4}$$

$$K_{12} = \frac{s+6}{4(s+1)^2}\Bigg|_{s=0} = \frac{6}{4} = \frac{3}{2}$$

$$K_{21} = \frac{d}{ds}\left(\frac{s+6}{4s^2}\right) = \frac{1}{4}\left(\frac{s^2 - (s+6).2s}{s^4}\right)\Bigg|_{s=-1} = \frac{11}{4}$$

$$K_{22} = \frac{s+6}{4s^2}\Bigg|_{s=-1} = \frac{5}{4}$$

$$y(t) = \left(\frac{-11}{4} + \frac{3}{2}t + \frac{11}{4}.e^{-t} + \frac{5}{4}t.e^{-t}\right)u(t)$$

Ex: 17.60 Resposta da tensão da rede a uma entrada em degrau unit. e

$$V_o(s) = \frac{2(s+1)}{s(s^2+12s+27)}$$

x ~~Qual~~ a resposta é sobreamortecida

$X(t) = 1u(t)$ $\quad X(s) = \frac{1}{s}$

$$H(s) = \frac{V_o(s)}{X(s)} = \frac{2(s+1)}{s^2+12s+27}$$

$2\xi w_0 = 12$ e $27 = w^2$ $\quad w = \sqrt{27}$

$$\xi = \frac{12}{2\cdot\sqrt{27}} = 1{,}155$$

$s^2 + 2\xi w_0 \cdot s + w_0^2 = 0$

ξ: coef. de amortecimento

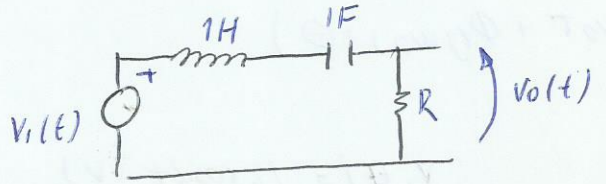
w_0: frequencia natura

$$s_1, s_2 = \frac{-2\xi w_0 \pm \sqrt{4\xi^2 w_0^2 - 4w_0^2}}{2}$$

$$= \frac{-2\xi w_0 \pm 2w_0\sqrt{\xi^2 - 1}}{2}$$

∴ Circuito sobreamortecido

Exercício) Determinar R p/ que a rede tenha $\xi = \frac{\sqrt{2}}{2}$ e calcular $v_o(t)$ p/ $v(t) = u(t)$



$$H(s) = \frac{V_o(s)}{V_1(s)}$$

$$V_o(s) = \frac{R}{R+s+\frac{1}{s}} \times V_1(s) \quad \therefore \quad \frac{V_o(s)}{V_1(s)} = H(s) = \frac{R}{R+s+\frac{1}{s}}$$

$$H(s) = \frac{Rs}{s^2+Rs+1} \quad ; \quad s^2 + 2\xi w_0 s + w_0^2 = 0$$

$R = 2\xi w_0$ e $w_0^2 = 1$

$$\therefore R = 2\cdot\frac{\sqrt{2}}{2}\cdot 1 = \sqrt{2} \quad \therefore \boxed{R = \sqrt{2}\,\Omega}$$

Para $V(t) = u(t)$ $\quad V_1(t) = \frac{1}{s}$

$$v_o = \frac{\sqrt{2}\cancel{s}}{s^2+\sqrt{2}s+1} \times \frac{1}{\cancel{s}}$$

$$s^2 + \sqrt{2}s + 1 = 0$$

$$s = \frac{-\sqrt{2} \pm \sqrt{2-4}}{2}$$

$$s = \frac{-\sqrt{2} \pm \sqrt{2}j}{2}$$

$$v_o(s) = \frac{K_1}{s+\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}} + \frac{K_1^*}{s-\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}}$$

$$K_1 = \left(s+\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}\right)F(s)\Bigg|_{s=-\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{\left(s+\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)}$$

$$= \frac{\sqrt{2}}{\left(-\cancel{\frac{\sqrt{2}}{2}}+j\frac{\sqrt{2}}{2}+\cancel{\frac{\sqrt{2}}{2}}+j\frac{\sqrt{2}}{2}\right)}$$

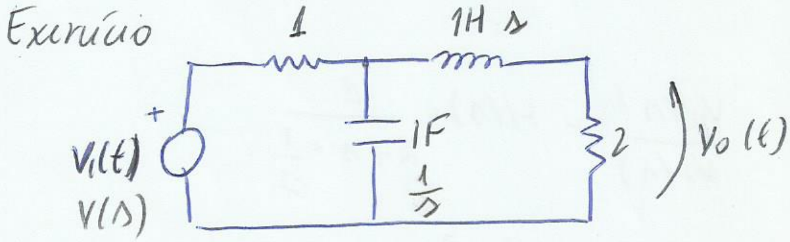
$$= \frac{\sqrt{2}}{j\sqrt{2}} = \frac{1}{j} = -j = 1\angle{-90}$$

$$f(t) = 2e^{-\frac{\sqrt{2}}{2}t}\cos\left(\frac{\sqrt{2}}{2} - 90\right)u(t)$$

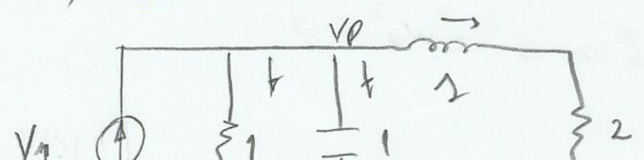
Resposta de redes em estado estacionário

$$y_{ss}(t) = X_{max}\,|H(jw_0)|\cos(w_0 t + \phi(jw_0) + \theta)$$

Exercício



$V_1(t) = 12\cos 2t$ (V)



$$V_o = \frac{2}{s+2}\cdot V_p$$

$$V_p = \frac{(s+2)}{2}V_o$$

$$V_p + V_p\cdot s + \frac{V_p}{2+s} = V_1$$

$$V_p(s+2) + V_p(s+2)s + V_p = V(2+s)$$

$$V_p(s^2+3s+2+1) = V(2+s)$$

$$\frac{(\cancel{s+2})}{2}V_o(s^2+3s+3) = V(\cancel{2+s}) \quad \therefore \quad \frac{V_o}{V_1} = \frac{2}{s^2+3s+3} = H(s)$$

$$\frac{V_o}{V_1} = H \quad ; \quad V_o = H \cdot V_1$$

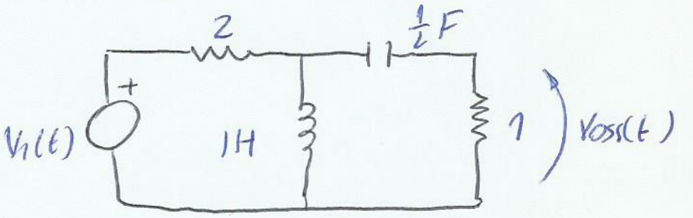
$$H(2j) = \frac{2}{(2j)^2 + 3 \cdot 2j + 3} = \frac{2}{6j - 1} = \frac{2 \angle 0}{6{,}08 \angle 99{,}46}$$

$$H(2j) = 0{,}316 \angle -99{,}5$$

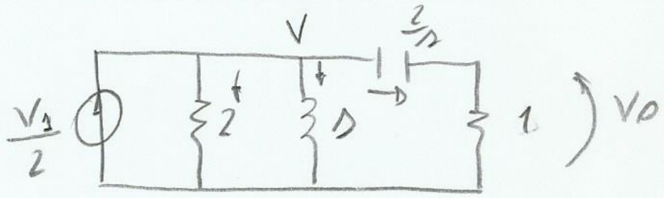
$$\therefore V_o(t) = 12 \cdot 0{,}316 \cdot \cos(2t + (-99{,}5))$$

$$\therefore V_o(t) = 3{,}8 \cos(2t - 99{,}5)$$

Determinar $V_{oss}(t)$ p/ $t > 0$



$V_1(t) = 10 \cos 2t$



$$V_o = \frac{1}{\frac{2}{s} + 1} \cdot V$$

$$V = \frac{2 + s}{s} V_o$$

$$\frac{V}{2} + \frac{V}{s} + \frac{V}{\frac{2}{s} + 1} = \frac{V_1}{2}$$

$$\frac{V(2+s)s + V(2+s) \cdot 2 + 2Vs^2}{(2+s)2s} = \frac{V_1}{2}$$

$$V(2s + s^2 + 4 + 2s + 2s^2) = V_1(2+s)s$$

$$V(3s^2 + 4s + 4) = V_1(2+s)s$$

$$\frac{\cancel{(2+s)}}{s} \cdot V_o(3s^2 + 4s + 4) = V_1 \cancel{(2+s)} s$$

$$\frac{V_o}{V_1} = \frac{s^2}{3s^2 + 4s + 4} = H \quad \therefore V_o = H\, V_1$$

$$H(2j) = \frac{(2j)^2}{3(2j)^2 + 4 \cdot 2j + 4} = \frac{-4}{-8 + 8j} = \frac{4 \angle 180}{11{,}31 \angle 135}$$

$$H(2j) = 0{,}35 \angle 45$$

$$\therefore V_o(t) = 3{,}5 \cos(2t + 45) \ (V)$$

Frequências Complexas próprias

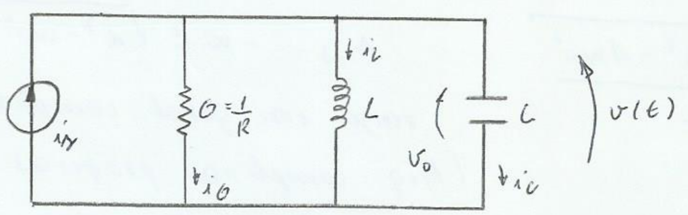
1- Objetivos

- Execução de medidas em transitorios repetivos.
- Determinação experimental de F.C.P de redes simples
- Medida do indice de mérito do circuito
- Modelamento de indutor real

2- Introdução teórica

- Resposta em comportamento livre da rede
- Critério de estabilidade de redes

2.1 Aplicação - Rede RLC paralela



$i_s = i_C + i_G + i_L = C\frac{dv}{dt} + G.v + \frac{1}{L}\int v\,dt \quad (1)$

- Resposta em comportamento livre

$i_s = 0$

i_0 e ou $v_0 \neq 0$

$C\frac{d^2v}{dt^2} + G\frac{dv}{dt} + \frac{v}{L} = 0$

$\frac{d^2v}{dt^2} + \frac{G}{C}\frac{dv}{dt} + \frac{v}{LC} = 0$

$\alpha = \frac{G}{2C} = \frac{1}{2RC}$ (cte de amortecimento)

$\frac{G}{C} = 2\alpha$

$w_0 = \frac{1}{\sqrt{LC}}$ (freq. de ressonancia não amortecida)

$LC = \frac{1}{w_0^2}$

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + w_0^2 v = 0 \quad (2)$$

Solução $v = Ve^{st}$ (3)

V = cte real

s = cte complexa

(3) em (2)

$$\frac{d^2}{dt^2} Ve^{st} + 2\alpha\frac{d}{dt} Ve^{st} + w_0^2 Ve^{st} = 0 \ \therefore$$

$$\therefore \ s^2 Ve^{st} + 2\alpha s Ve^{st} + w_0^2 Ve^{st} = 0 \ \therefore$$

$$Ve^{st}(s^2 + 2\alpha s + w_0^2) = 0 \ \therefore$$

logo $s^2 + 2\alpha s + w_0^2 = 0$ (4) (equação característica da rede)

Raízes de (4)

$$s_{1,2} = \frac{-2\alpha \pm \sqrt{4\alpha^2 - 4w_0^2}}{2} \ \therefore \ s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$$

raízes em geral complexas (freq. complexas próprias)

1º caso: $\alpha > w_0 \rightarrow$ 2 raízes reais distintas

resposta sobre amortecida

$$v(t) = V_1 e^{s_1 t} + V_2 e^{s_2 t}$$

2º caso: $\alpha = w_0 \ \therefore \ s_1 = s_2 = -\alpha$ (real dupla)

amortecimento crítico

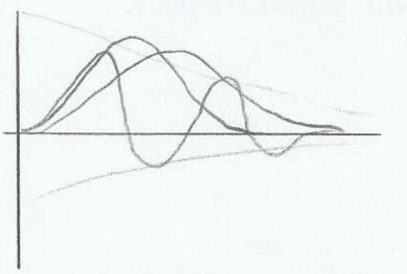
$$v(t) = V_1 e^{-\alpha t} + V_2 e^{-\alpha t}$$

3º caso: $\alpha < w_0$

$$s_{1,2} = -\alpha \pm j\underbrace{\sqrt{w_0^2 - \alpha^2}}_{w_d} = -\alpha \pm jw_d$$

resposta oscilatória

$$v(t) = V_0 e^{-st} \operatorname{sen} w_d t$$



$\alpha > w_0$

$\alpha = w_0$

$\alpha < w_0$

Circuitos Ressonantes

1- Objetivos

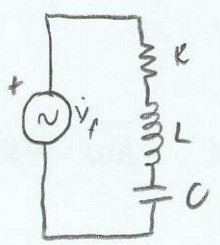
- Estudo de CR em regime permanente senoidal
- Medida da banda passante e de ~~modo~~ índice de mérito

1- $w_0 = \frac{1}{\sqrt{LC}} \rightarrow X_L = X_C$

2- $w_{r_1} \begin{cases} \dot{Z} \rightarrow \text{puramente resistiva (serie)} \\ \dot{Y} \rightarrow \text{" " (paralelo)} \end{cases}$

corrente e tensão em fase

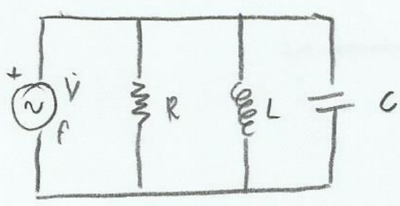
3- $w_{r_2} \begin{cases} Z \rightarrow \text{mínimo (série)} \\ Y \rightarrow \cancel{\text{máximo}} \text{ (paralelo)} \end{cases}$



$$\dot{Z} = R + j\left(wL - \frac{1}{wC}\right)$$

$$\dot{Z} = \sqrt{R^2 + \left(wL - \frac{1}{w}\right)^2}$$

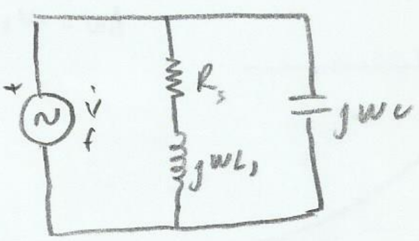
$$w_0 = w_{r_1} = w_{r_2}$$



$$w_0 = w_{r_1} = w_{r_2}$$

$$\dot{Y} = G + j\left(wC - \frac{1}{wL}\right)$$

$$Y = \sqrt{G^2 + \left(wC - \frac{1}{wL}\right)^2}$$



$$\dot{Y} = jwC + \frac{1}{R_s + jwL_s}$$

$$w_0 \neq w_{r_1} \neq w_{r_2}$$

$\dot{Y}$ puramente resistivo

$$w_{r_1}^2 = w_0^2 - \left[\frac{R_s}{L_s}\right]^2$$

Y mínimo

$$w_{r_2}^2 = w_0^2 \sqrt{1 + \frac{2}{w_0^2}\left(\frac{R_s}{L_s}\right)^2} - \left(\frac{R}{L_s}\right)^2$$

Indice de mérito do indutor na frequência própria não amortecida

$$Q_{b0} = \frac{w_0 L_s}{R_s}$$

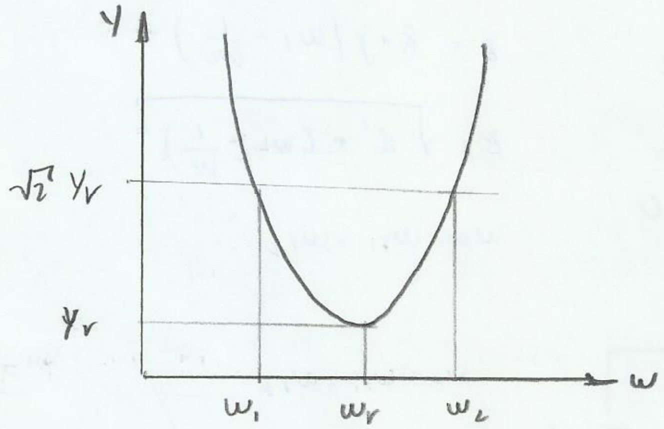
$$w_{r1} = w_0 \sqrt{1 - \frac{1}{Q_{b0}^2}}$$

$$w_{r2} = w_0 \sqrt{\sqrt{1 + \frac{2}{Q_{b0}^2}} - \frac{1}{Q_{b0}^2}}$$

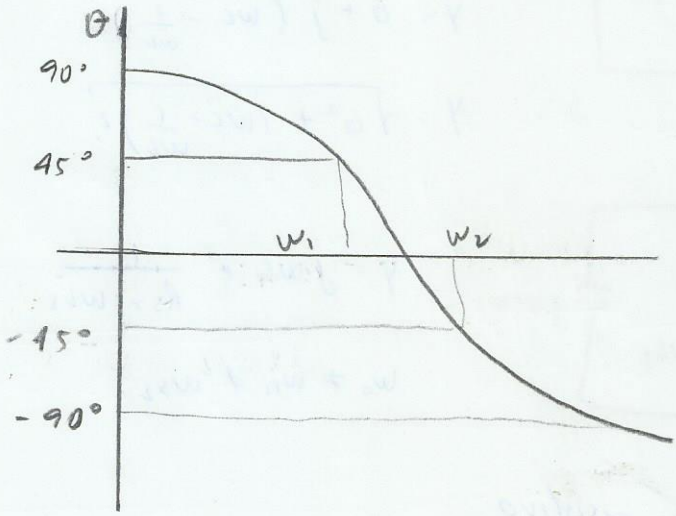
$$w_{r1} \neq w_{r2} \neq w_0$$

Se $Q_{b0} \geq 10 \rightarrow w_{r1}, w_{r2}, w_{r3}$ irão diferir de menos de 1%

Banda passante



$\Delta w = w_2 - w_1$



$\Delta w = w_2 - w_1$

$$Q = \frac{w_r}{\Delta w}$$

Exercícios P2

Redes magnéticas acopladas

Para uma associação de indutores, quando não há interação magnética entre as indutâncias, podemos determinar essas associações por semelhança com associações série e paralelo de resistores:

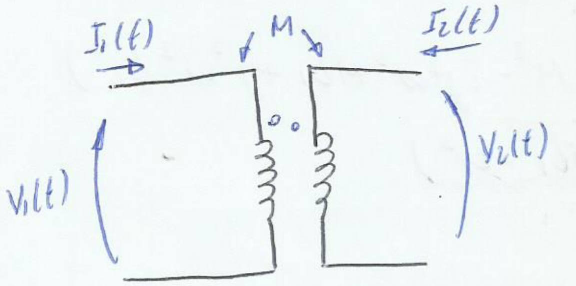
Série: $L_{eq} = L_1 + L_2 + \cdots + L_n$

Paralelo: $\dfrac{1}{L_{eq}} = \dfrac{1}{L_1} + \dfrac{1}{L_2} + \cdots + \dfrac{1}{L_n}$

Caso exista interação de fluxo uma com as outras devem ser contabilizados por análise de redes com indutância acoplada.

$V(t) = N\dfrac{d\Phi(t)}{dt}$ ou $V(t) = L\dfrac{dI(t)}{dt}$

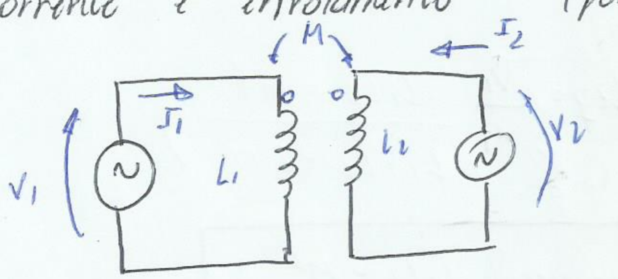
Para um transformador:



$V_1(t) = L_1\dfrac{dI_1(t)}{dt} + M\dfrac{dI_2(t)}{dt}$

$V_2(t) = L_2\dfrac{dI_2(t)}{dt} + M\dfrac{dI_1(t)}{dt}$

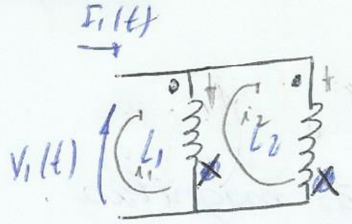
Para o regime permanente senoidal, de acordo com o sentido da corrente e enrolamento "o" (ponto) temos:



$V_1 = j\omega L_1 I_1 + j\omega M I_2$

$V_2 = j\omega L_2 I_2 + j\omega M I_1$

Determine L_{eq}:



$$\begin{cases} j\omega L_1(i_1-i_2) + j\omega M i_2 - V_1 = 0 \\ j\omega L_2 i_2 - j\omega M(i_2-i_1) + j\omega L_1(i_2-i_1) - j\omega M i_2 = 0 \end{cases}$$

sendo $I_1 = i_1$

$$\begin{cases} j\omega L_1 i_1 - j\omega L_1 i_2 + j\omega M i_2 - V_1 = 0 \\ j\omega L_2 i_2 - j\omega M i_2 + j\omega M i_1 + j\omega L_1 i_2 - j\omega L_1 i_1 - j\omega M i_2 = 0 \end{cases}$$

$$\begin{bmatrix} (j\omega L_1) & (j\omega M - j\omega L_1) \\ (j\omega M - j\omega L_1) & (j\omega L_2 - j\omega M + j\omega L_1 - j\omega M) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} j\omega L_1 & j\omega M - j\omega L_1 \\ j\omega M - j\omega L_1 & j\omega L_2 - 2j\omega M + j\omega L_1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

$$\Delta = j^2\omega^2(L_1L_2 - 2L_1M + L_1^2) - (j\omega M - j\omega L_1)^2$$

$$= j^2\omega^2(L_1L_2 - 2L_1M + L_1^2) - (j^2\omega^2M^2 - 2j^2\omega^2 ML_1 + j^2\omega^2L_1^2)$$

$$= j^2\omega^2(L_1L_2 - \cancel{2L_1M} + \cancel{L_1^2} - M^2 + \cancel{2ML_1} - \cancel{L_1^2})$$

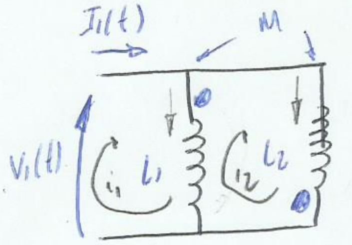
$$\therefore \quad \Delta = j^2\omega^2(L_1L_2 - M^2)$$

$$\Delta i_1 = \begin{vmatrix} V_1 & j\omega M - j\omega L_1 \\ 0 & j\omega L_2 - 2j\omega M + j\omega L_1 \end{vmatrix} = j\omega V_1(L_2 - 2M + L_1)$$

$$i_1 = \frac{\Delta i}{\Delta} = \frac{\cancel{j\omega} V_1(L_1+L_2-2M)}{j^{\cancel{2}}\omega^{\cancel{2}}(L_1L_2-M^2)} \quad ; \quad L_{eq} = \frac{V_1}{i_1}$$

$$L_{eq} = \frac{V_1}{\frac{V_1}{j\omega}\left(\frac{L_1+L_2-2M}{L_1L_2-M^2}\right)} \quad \therefore \quad \boxed{L_{eq} = j\omega\left(\frac{L_1L_2-M^2}{L_1+L_2-2M}\right)}$$

Determine L_{eq}



$$j\omega L_1(i_1-i_2) - j\omega M i_2 - V = 0$$

$$j\omega L_2 i_2 + j\omega M(i_2 - i_1) + j\omega L_1(i_2 - i_1) + j\omega M i_2 = 0$$

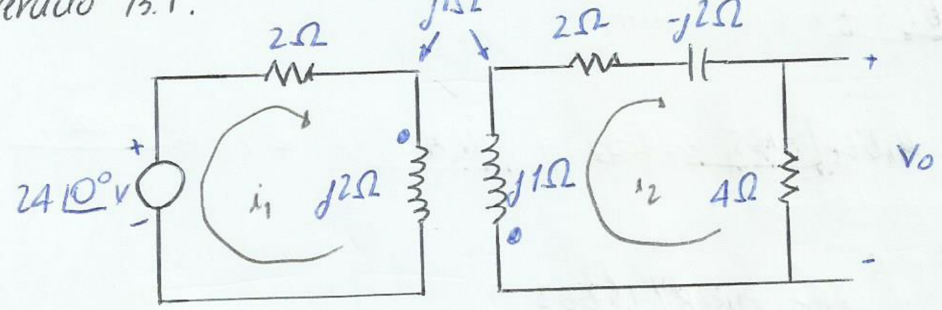
$$\begin{cases} i_1(j\omega L_1) - i_2(j\omega L_1 + j\omega M) = V \\ -i_1(j\omega M + j\omega L_1) + i_2(j\omega L_2 + 2j\omega M + j\omega L_1) = 0 \end{cases}$$

$\vdots$

$$\boxed{\text{logo} \quad L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

Exercício 13.1:

× Determine V_o



$$\begin{cases} i_1(2+j2) + i_2 \cdot j = 24 \\ i_1 \cdot j + i_2(2 - j2 + j + 4) = 0 \end{cases} \quad ; \text{ sendo } V_o = 4 i_2$$

$$\begin{bmatrix} 2+j2 & j \\ j & 6-j \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \end{bmatrix}$$

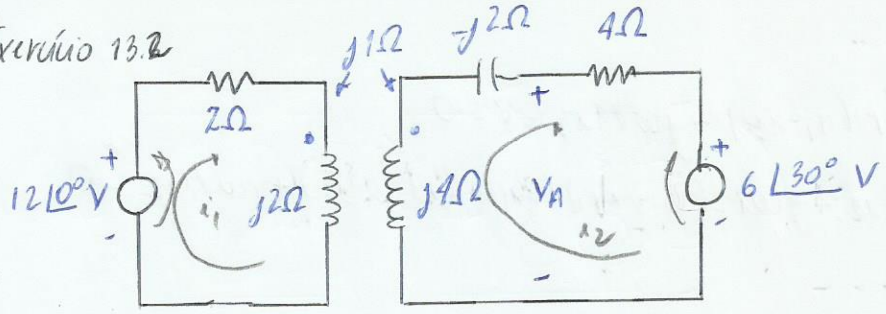
$$\Delta = (2+2j)(6-j) - j^2$$
$$= 12 - 2j + 12j - 2j^2 - j^2$$
$$= 12 + 10j + 3 = 15 + j10 = 18{,}03\angle 33{,}69$$

$$\Delta i_2 = \begin{vmatrix} 2+j2 & 24 \\ j & 0 \end{vmatrix} = -24j = 24\angle -90$$

$$i_2 = \frac{24\angle -90}{18{,}03\angle 33{,}69} = 1{,}33\angle -123{,}69$$

$$V_o = 5{,}32\angle -123{,}69 \quad (V)$$

Exercício 13.2



$$V_A = 6\angle 30^\circ + 4 i_2$$

$$\begin{cases} i_1(2+j2) - j \cdot i_2 = 12 \\ -j \cdot i_1 + i_2(4+j2) = -(5{,}2+j3) \end{cases}$$

$$\begin{bmatrix} 2+j2 & -j \\ -j & 4+j2 \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -5{,}2-j3 \end{bmatrix} \qquad \Delta = \begin{vmatrix} 2+j2 & -j \\ -j & 4+j2 \end{vmatrix} = 5+j12$$

$$\Delta = 5+j12 = 13\angle 67{,}38$$

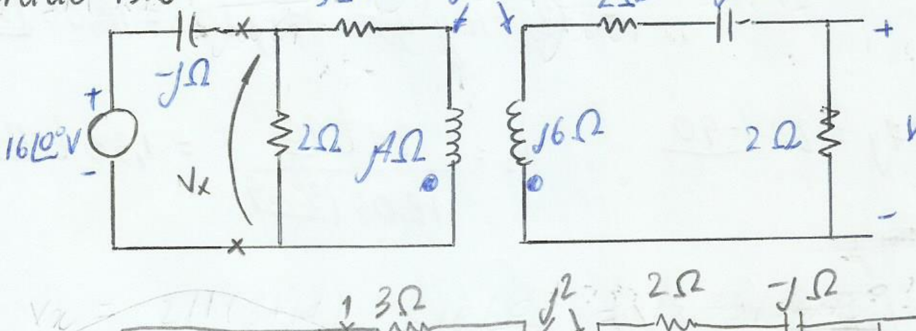
$$\Delta i_2 = \begin{vmatrix} 2+j2 & 12 \\ -j & -5{,}2-j3 \end{vmatrix} = 6{,}22\angle -135$$

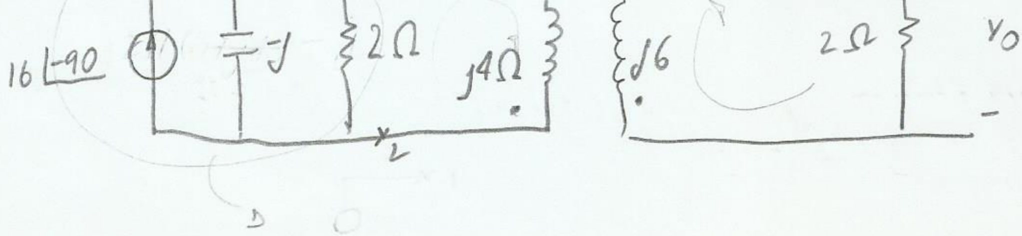
$$i_2 = \frac{6{,}22\angle -135}{13\angle 67{,}38} \quad \therefore \quad i_2 = 0{,}48\angle 157{,}62$$

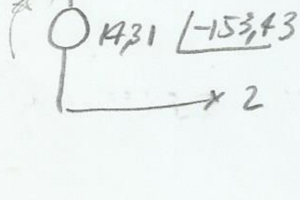
$$V_A = 6\angle 30 + 4 \times 0{,}48\angle 157{,}62 \quad \therefore \quad \boxed{V_A = 5{,}06\angle 47{,}48^\circ}$$

Exercício 13.4

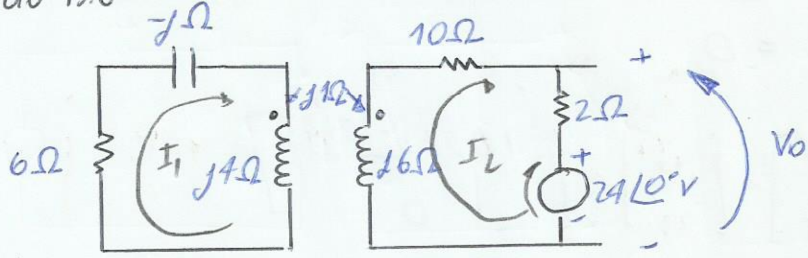
Exercício 13.6







Exercício 13.8



$$I_1(6+j3) - I_2 \cdot j = 0$$

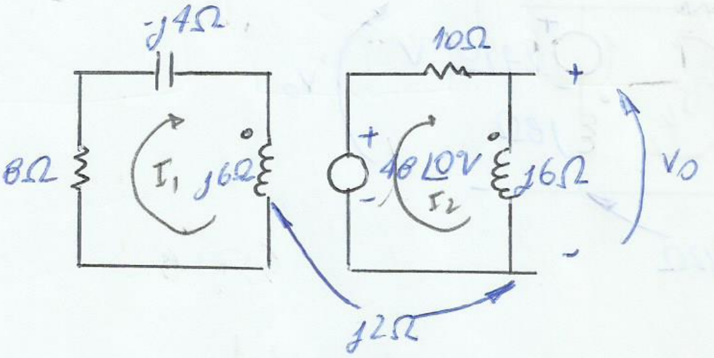
$$-I_1 \times j + I_2(12+j6) = -(24\angle 0)$$

$$\begin{bmatrix} 6+j3 & -j \\ -j & 12+j6 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -24\angle 0 \end{bmatrix} \quad \therefore I_2 = -1{,}596 + j\,0{,}781$$

$$V_0 = 24\angle 0 + 2 \times (-1{,}596 + j\,0{,}781)$$

$$V_0 = 20{,}81 + j\,1{,}56 \quad \therefore \quad V_0 = 20{,}87\angle 4{,}29$$

Exercício 13.9



$$I_1(8+j2) + j \cdot 2 \cdot I_2 = 0$$

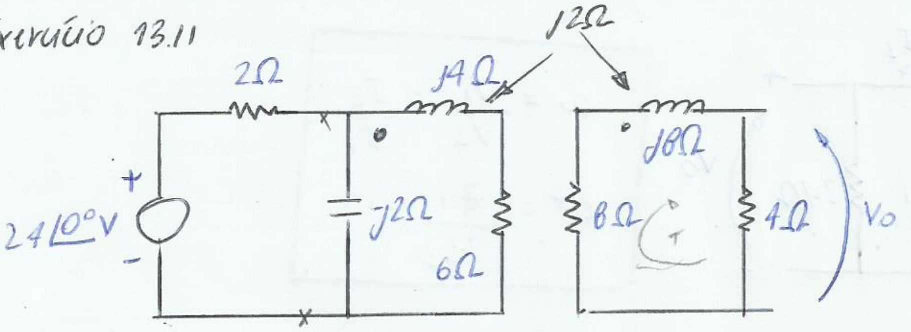
$$I_1 \cdot j \cdot 2 + I_2(10+j6) = 48\angle 0$$

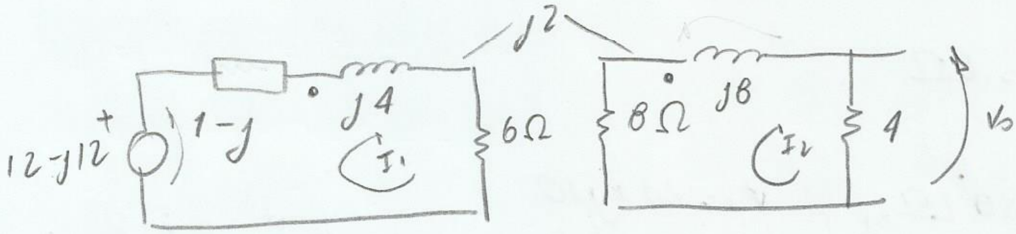
$$\therefore I_2 = 3{,}4845 - j\,1{,}9576 \quad e \quad I_1 = -0{,}6656 - j0{,}7047$$

$$V_0 = j6 \times (3{,}4845 - j\,1{,}9576) + j2 \times (-0{,}6656 - j\,0{,}7047)$$

$$\therefore V_0 = 13{,}155 + j\,19{,}5759 \quad \therefore \quad \boxed{V_0 = 23{,}59\angle 56{,}1°}$$

Exercício 13.11 $\quad 1{,}21\ \angle 190{,}18$





$$I_1(1-j+4j) + I_2 \cdot j2 = 12 - j12$$

$$I_1(j2) + I_2(12+j8) = 0$$

(Dados errados)

$$\therefore \quad I_1 = -2{,}016 - j\,5{,}088$$

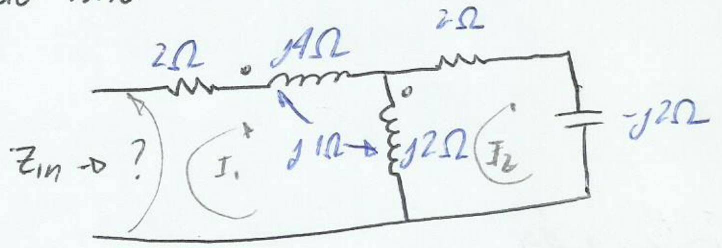
$$I_2 = -0{,}4320 + j\,0{,}624$$

?

$$V_o = 8 \cdot I_2 + j8 \cdot I_2 + j2 \cdot I_1 \quad \therefore \quad V_o = 1{,}728 - j\,2{,}496$$

$$\therefore \boxed{V_o = 3{,}04\ \angle -55{,}3^\circ \ \ V}$$

Exercício 13.18 $\quad 6{,}5 + j8$



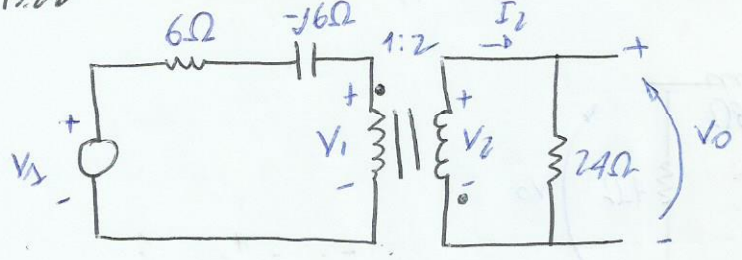
$$\begin{cases} I_1(2+j4) + j2(I_1 - I_2) - I_2 j = V \\ I_2(2-j2) + j2(I_2 - I_1) - I_1 \cdot j = 0 \end{cases}$$

$$\begin{cases} I_1(2+j6) + I_2(-j3) = V \\ I_1(-j3) + I_2(2) = 0 \end{cases} \qquad \Delta = \begin{vmatrix} 2+j6 & -j3 \\ -j3 & 2 \end{vmatrix} = 13 + 12i$$

$$\Delta i = \begin{vmatrix} V & -j3 \\ 0 & 2 \end{vmatrix} = 2V \qquad i = \frac{2V}{13+12i} \qquad Z = \frac{V}{i} = \frac{V}{\frac{2V}{13+12i}}$$

$$\therefore \quad Z = 6{,}5 + j6$$

13.29



$$\boxed{a = \frac{V_1}{V_2} = \frac{I_2}{I_1} \qquad a^2 = \frac{Z_1}{Z_2}}$$

$V_s = 50\angle 0$

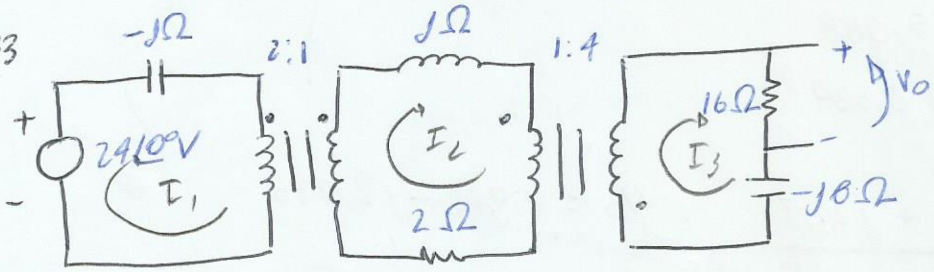
$$Z_1 = 24 \cdot \left(\frac{1}{2}\right)^2 = 6\Omega$$

$$V_1 = \frac{6}{12 - j6} \cdot 50\angle 0 \quad \therefore V_1 = 20 + j10$$

$$V_0 = \frac{20 + j10}{\frac{1}{2}} \quad \therefore \quad V_0 = 40 + j20 \quad \therefore \quad V_0 = 44{,}72\angle 26{,}57^\circ \text{ V}$$

$15{,}94\angle 175{,}2$

13.33



$$Z_3 = 16 - j8$$

$$Z_3' = (16 - j8)\left(\frac{1}{4}\right)^2 = 1 - 0{,}5j$$

$$Z_2 = 2 + j + Z_3' = 3 + 0{,}5j$$

$$Z_2' = (3 + 0{,}5j) \cdot 2^2 = 12 + j2$$

$$V_2' = \frac{12 + j2}{12 + j} \cdot 24\angle 0 = 24{,}17 + j1{,}99$$

$$V_2 = \frac{V_2'}{2} = 12{,}08 + j0{,}99$$

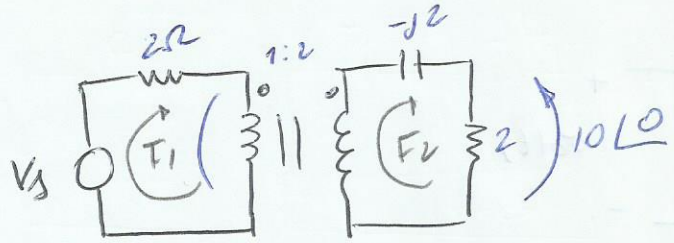
$$V_3' = \frac{1 - 0{,}5j}{3 + 0{,}5j} \cdot V_2 = 3{,}81 - j2{,}32$$

$$V_3 = \frac{V_3'}{\frac{1}{4}} = 15{,}23 - j9{,}27$$

$$V_0 = \frac{16}{16 - j8} \cdot V_3 \quad \therefore V_0 = 15{,}89 - j1{,}32 \quad \therefore \boxed{V_0 = 15{,}94\angle 175{,}2^\circ \text{ V}}$$

~~Lista~~ Exercícios do Caderno

x Determinar V_s

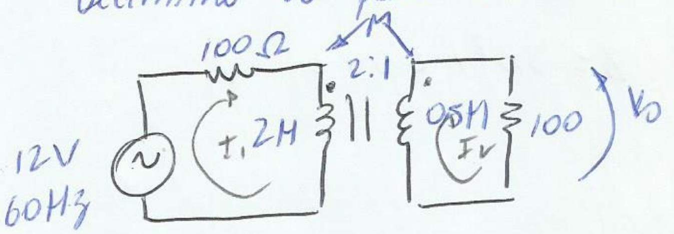


$Z_2 = 2 - j2 \quad Z_2' = 0{,}5 - j0{,}5$

$I_2 = \dfrac{10\angle 0}{2} = 5\angle 0 \quad \therefore \quad I_2' = \dfrac{5\angle 0}{\frac{1}{2}} = 10\angle 0$

$V_s = 10\angle 0 \times (2 + 0{,}5 - j0{,}5) \quad \therefore \quad \boxed{V_s = 25{,}5 - j11{,}31 \text{ V}}$

Determine V_o para trafo real e ideal ; $K = 1$



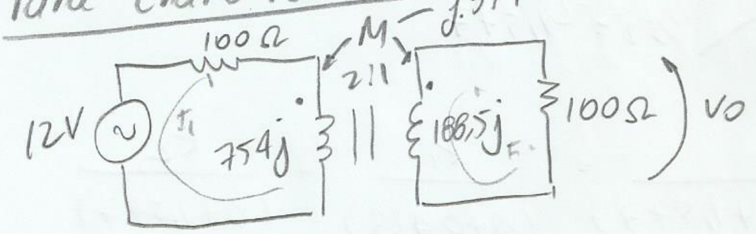
Para trafo ideal

$Z' = 100 \cdot 2^2 = 400$

$V' = \dfrac{400}{500} \cdot 12\angle 0 = 9{,}6\angle 0 \quad \therefore \quad \boxed{V_o = 4{,}8\angle 0^\circ \text{ V}}$

Para trafo real

$M = K\sqrt{L_1 L_2} = 1\sqrt{2 \cdot 0{,}5} = 1$

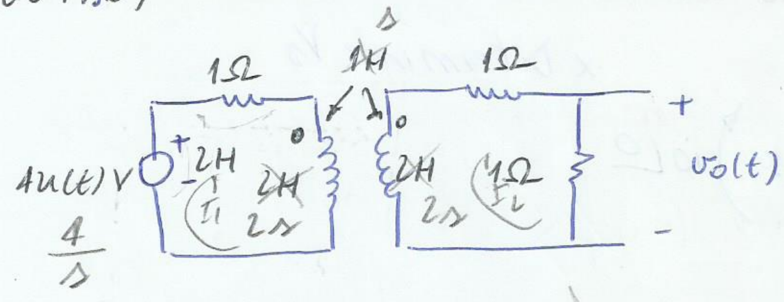


$$\begin{cases} I_1(100 + j754) - j377 I_2 = 12 \\ -j377 I_1 + I_2(100 + 188{,}5j) = 0 \end{cases}$$

$I_2 = 0{,}0475 + j0{,}005$

$\therefore V_o = 100 I_2 \quad \therefore \quad V_o = 4{,}78\angle 6{,}01^\circ \text{ V}$

Exercício 17.3B)



x Condições iniciais é nula

$$\begin{cases} I_1(1+2s) - I_2 s = \frac{4}{s} \\ -I_1 s + I_2(2+2s) = 0 \end{cases} \qquad \begin{bmatrix} 1+2s & -s \\ -s & 2+2s \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{4}{s} \\ 0 \end{bmatrix}$$

$$\Delta = (1+2s)(2+2s) - (-s)^2$$

$$= 2 + 2s + 4s + 4s^2 - s^2 \quad \therefore \quad \Delta = 3s^2 + 6s + 2$$

$$\Delta i_2 = \begin{vmatrix} 1+2s & \frac{4}{s} \\ -s & 0 \end{vmatrix} = 4$$

$$\therefore \; i_2 = \frac{\Delta i_2}{\Delta} = \frac{4}{3s^2+6s+2} = \frac{4}{3} \cdot \left(\frac{1}{s^2 + 2s + \frac{2}{3}} \right)$$

$$s^2 + 2s + \frac{2}{3} = 0$$

$$s = \frac{-2 \pm \sqrt{4 - 4 \cdot \frac{2}{3}}}{2} \quad \begin{cases} s_1 = -0{,}423 \\ s_2 = -1{,}577 \end{cases}$$

$$V_o(s) = \frac{4}{3} \cdot \frac{1}{(s+0{,}423)(s+1{,}577)} = \frac{k_1}{(s+0{,}423)} + \frac{k_2}{(s+1{,}577)}$$

$$k_1 = (s+0{,}423)\,V_o(s) \Big|_{s=-0{,}423} = \frac{4}{3 \cdot (-0{,}423 + 1{,}577)} \quad \therefore \; k_1 = 1{,}155$$

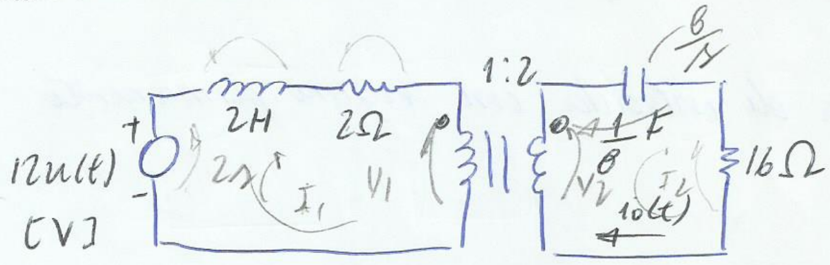
$$k_2 = (s+1{,}577)\,V_o(s) \Big|_{s=-1{,}577} = \frac{4}{3(-1{,}577 + 0{,}423)} \quad \therefore \; k_2 = -1{,}155$$

$$\therefore \; V_o(s) = \frac{1{,}155}{(s+0{,}423)} - \frac{1{,}155}{(s+1{,}577)} \quad \therefore \; V_o(t) = 1{,}155\left(e^{-0{,}423t} - e^{-1{,}577t}\right)u(t)$$

[V]

Exercício 17.40

* Determine $i_o(t)$



$\frac{12}{s}$

Lembrete

$V_1 = aV_2$

$I_2 = aI_1$

$Z_1 = a^2 Z_2$

$a = \frac{1}{2}$

$$I_1(2s+2) + V_1 = \frac{12}{s} \Rightarrow 2I_2(2s+2) + \frac{V_2}{2} = \frac{12}{s}$$

$$I_2\left(\frac{8}{s}+16\right) - V_2 = 0$$

$$\begin{cases} I_2(4s+4) + \frac{V_2}{2} = \frac{12}{s} \\ I_2\left(\frac{8}{s}+16\right) - V_2 = 0 \end{cases} \qquad \Delta = \begin{vmatrix} 4s+4 & \frac{1}{2} \\ \frac{8}{s}+16 & -1 \end{vmatrix} =$$

$$\Delta = -4s - 4 - \frac{4}{s} - 8 = \frac{-4s^2 - 12s - 4}{s}$$

$$\Delta i_2 = \begin{vmatrix} \frac{12}{s} & \frac{1}{2} \\ 0 & -1 \end{vmatrix} = -\frac{12}{s} \qquad i_2 = \frac{\Delta i_2}{\Delta} = \frac{\frac{-12}{s}}{\frac{-4s^2-12s-4}{s}}$$

$$i_2(s) = \frac{3}{s^2+3s+1} \qquad s^2+3s+1=0 \quad s = \frac{-3 \pm \sqrt{9-4}}{2} \begin{cases} s_1 = -0,382 \\ s_2 = -2,618 \end{cases}$$

$$i_2(s) = \frac{3}{(s+0,382)(s+2,618)} = \frac{K_1}{(s+0,382)} + \frac{K_2}{(s+2,618)}$$

$$K_1 = (s+0,382)\, i_2(s) \Big|_{s=-0,382} = \frac{3}{(-0,382+2,618)} \therefore K_1 = 1,34$$

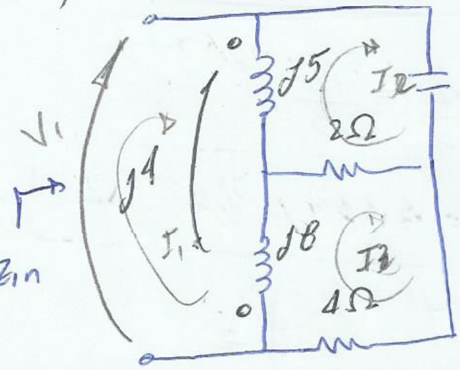
$$K_2 = (s+2,618)\, i_2(s) \Big|_{s=-2,618} = \frac{3}{(-2,618+0,382)} \therefore K_2 = -1,34$$

$$i_2(s) = \frac{1,34}{s+0,382} + \frac{(-1,34)}{s+2,618} \therefore \boxed{i_2(t) = 1,34\cdot\left(e^{-0,382t} - e^{-2,618t}\right) (A)}$$

Lista de Exercícios (Redes magnéticas) Prof. Benko

1) Determinar a impedância de entrada em regime permanente senoidal

a)



$$Z_{in} = \frac{V_1}{I_1}$$

$I_1(j13) - I_2(j5) - I_3(j8) - 2I_1 j4 + j4 \cdot I_2 + j4\, I_3 = V_1$

ou

$\to j5 \cdot (I_1 - I_2) + j8(I_1 - I_3) - j4(I_1 - I_2) - j4(I_1 - I_3) = V_1$

$I_1(j5) + I_2(-j) + I_3(-4j) = 0$ (Malha 1)

$I_2(2 + j2) - I_1 j5 - 2I_3 + j4(I_1 - I_3) = 0$

$I_1(-j) + I_2(2 + j2) + I_3(-2 - j4) = 0$ (Malha 2)

$I_3(6 + j8) - I_1 \cdot j8 - 2I_2 + j4(I_1 - I_2) = 0$

$I_1(-j4) + I_2(-2 - j4) + I_3(6 + j8) = 0$ (Malha 3)

$$\begin{bmatrix} j5 & -j & -4j \\ -j & 2+j2 & -2-j4 \\ -j4 & -2-j4 & 6+j8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \left| \begin{matrix} j5 & -j & -4j \\ -j & 2+j2 & -2-j4 \\ -j4 & -2-j4 & 6+j8 \end{matrix} \right| \begin{matrix} j5 & -j \\ -j & 2+j2 \\ -j4 & -2-j4 \end{matrix} = 132 + j122$$

$\Delta = -6 + j112$

$\Delta = 112{,}16 \angle 93{,}07$

$$\Delta I_1 = \begin{vmatrix} V_1 & -j & -j4 \\ 0 & 2+2j & -2-j4 \\ 0 & -2-j4 & 6+j6 \end{vmatrix}\begin{matrix} V_1 & -j \\ 0 & 2+2j \\ 0 & -2-j4 \end{matrix} = V_1.(2+2j)(6+j6) - (V_1.(-2-j4)^2)$$

$$\therefore \Delta I_1 = V_1(-4+j28) - V_1(-12+j16)$$

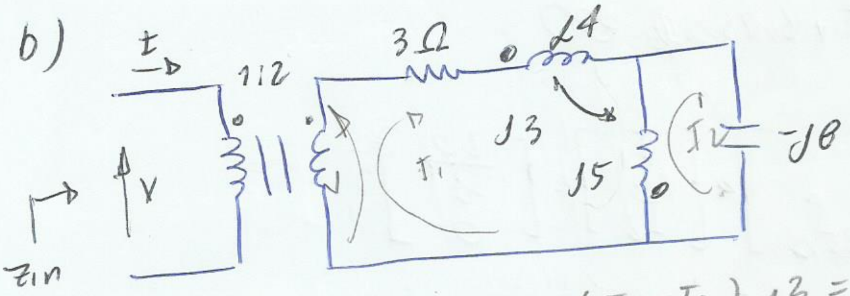
$$= V_1(-4+j28+12-j16) \quad \therefore \Delta I_1 = V_1(8+j12)$$

$$\therefore I_1 = \frac{V_1(8+j12)}{-6+j112} \quad \therefore I_1 = V_1.(0{,}103 - j0{,}077)$$

$$\therefore Z_{in} = \frac{V_1}{I_1} = \frac{V_1}{V_1(0{,}103-j0{,}077)}$$

$$\boxed{Z_{in} = 6{,}23 + j.\,4{,}65\ \Omega}$$

b)



$$\begin{cases} I_1(3+j9) - I_2.j5 - (I_1-I_2).j3 = V_1 \\ I_2(-3j) - j5.I_1 + I_1.j3 = 0 \end{cases}$$

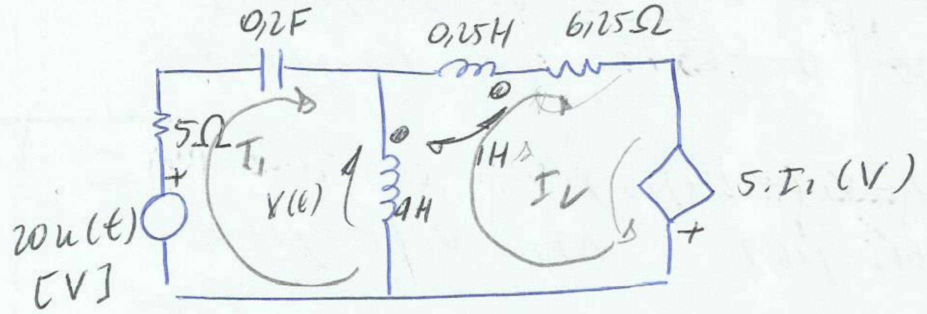
$$\begin{bmatrix} 3+j6 & -j2 \\ -j2 & -j3 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix} \quad \Delta = \begin{vmatrix} 3+j6 & -j2 \\ -j2 & -j3 \end{vmatrix} = 22-j9$$

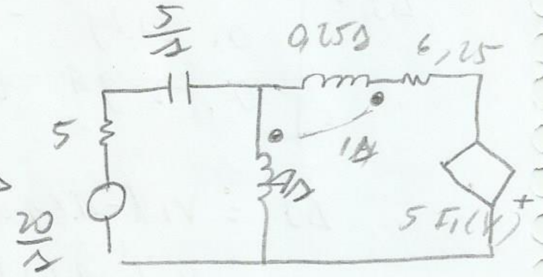
$$\Delta I_1 = \begin{vmatrix} V_1 & -j2 \\ 0 & -j3 \end{vmatrix} = -j3.V_1 \quad \therefore I_1 = \frac{-j3V_1}{22-j9} = (0{,}046-j0{,}12)V_1$$

$$Z_1 = \frac{V_1}{I_1} = \frac{V_1}{(0{,}046-j0{,}12)V_1} \quad \therefore Z_1 = 3 + \frac{22}{3}j$$

logo $Z_{in} = \left(\frac{1}{2}\right)^2 . Z_1 \quad \therefore Z_{in} = 0{,}75 + j\,1{,}83\ \Omega$

2) Calcular: a) equações de malha b) $v(t)$





$I_1(5+\frac{5}{s}+4s) - 4s\, I_2 - s I_2 = \frac{20}{s}$

$I_2(6,25+1,25s) - 4s\, I_1 + (I_2 - I_1)s + I_2 s = 5I_1$

$$\begin{cases} I_1(4s+\frac{5}{s}+5) - I_2(5s) = \frac{20}{s} \\ I_1(-5s-5) + I_2(6,25+6,25s) = 0 \end{cases}$$

$$\begin{bmatrix} 4s+\frac{5}{s}+5 & -5s \\ -5s-5 & 6,25+6,25s \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{20}{s} \\ 0 \end{bmatrix}$$

$\Delta = (4s+\frac{5}{s}+5)(6,25+6,25s) + 5s(-5s-5)$

$= \cancel{25s} + \cancel{25s^2} + \frac{31,25}{s} + 31,25 + 31,25 + 31,25s - \cancel{25s^2} - \cancel{25s}$

$= \frac{31,25s^2 + 62,5s + 31,25}{s}$

$\Delta I_1 = \begin{vmatrix} \frac{20}{s} & -5 \\ 0 & 6,25+6,25s \end{vmatrix} = \frac{125}{s} + 125$

$\therefore I_1 = \dfrac{\frac{125s+125}{\cancel{s}}}{\frac{31,25s^2+62,5s+31,25}{\cancel{s}}} = \frac{4s+4}{s^2+2s+1} = \frac{4\cancel{(s+1)}}{(s+1)^2} = \frac{4}{(s+1)}$

$\Delta I_2 = (4s+\frac{5}{s}+5) \cdot 0 - (-5s-5) \cdot \frac{20}{s} = \frac{100s+100}{s} = \frac{100(s+1)}{s}$

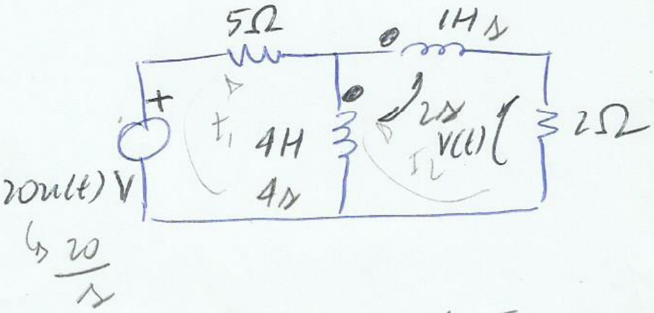
$I_2 = \dfrac{\frac{100(s+1)}{\cancel{s}}}{31,25s^2+62,5s+31,25/\cancel{s}} = \frac{100(s+1)}{31,25(s^2+2s+1)} = \frac{3,2}{(s+1)}$

$$V(s) = (I_1 - I_2) \cdot 4s - I_2 \cdot s$$

$$= \left(\frac{4}{s+1} - \frac{3{,}2}{s+1}\right) \cdot 4s - \left(\frac{3{,}2 \cdot s}{s+1}\right)$$

$$= \frac{3{,}2 s}{s+1} - \frac{3{,}2 s}{s+1} \quad \therefore \quad V(s) = 0 \text{ logo } \boxed{v(t) = 0V}$$

3) Calcular: a) equações de malha b) $v(t)$ sabendo $K = 1$



Lembrete:

$$M = K\sqrt{L_1 L_2}$$

logo $M = 1 \cdot \sqrt{4 \cdot 1} \therefore M = 2H$

$$\begin{cases} I_1(5+4s) - 4s\, I_2 + 2s\, I_2 = \frac{20}{s} \\ I_2(5s+2) - 4s\, I_1 - 2s(I_2 - I_1) - 2s\, I_2 = 0 \end{cases}$$

$$\begin{cases} I_1(5+4s) + I_2(-2s) = \frac{20}{s} \\ I_1(-2s) + I_2(s+2) = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} 5+4s & -2s \\ -2s & s+2 \end{vmatrix}$$

$$\begin{bmatrix} 5+4s & -2s \\ -2s & s+2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{20}{s} \\ 0 \end{bmatrix}$$

$$\Delta = (5+4s)(s+2) - (-2s)^2$$

$$= 5s + 10 + \cancel{4s^2} + 8s - \cancel{4s^2}$$

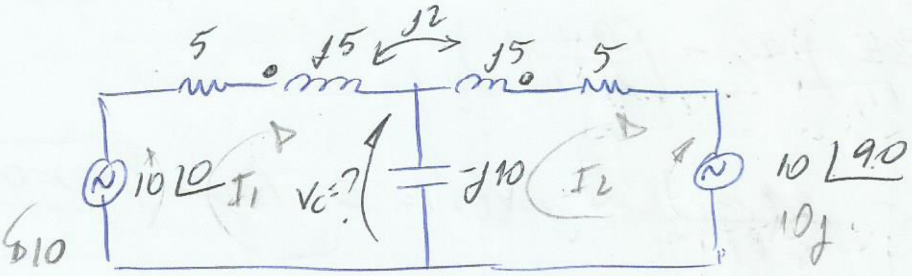
$$\therefore \Delta = 13s + 10$$

$$\Delta_2 = \begin{vmatrix} 5+4s & \frac{20}{s} \\ -2s & 0 \end{vmatrix} = 40$$

$$I_2 = \frac{40}{13s+10} = \frac{40}{13\left(s + \frac{10}{13}\right)} = 3{,}08\, e^{-0{,}77t}\, u(t)$$

$$V(t) = 2 I_2 \rightarrow \boxed{v(t) = 6{,}15\, e^{-0{,}77t}\, u(t)}$$

4) Calcular a tensão no capacitor V_C?



$$I_1(5-j5)+j10\,I_2-j2\cdot I_2=10$$

$$I_2(5-j5)+j10\,I_1-j2\,I_1=10j$$

$$\begin{bmatrix} 5-j5 & j8 \\ +j8 & 5-j5 \end{bmatrix}\cdot\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} 10 \\ j10 \end{bmatrix}$$

$$\Delta=(5-j5)^2-(j8)^2=64-j50$$

$$\Delta_{I_1}=10(5-j5)-j10\cdot j8=130-j50$$

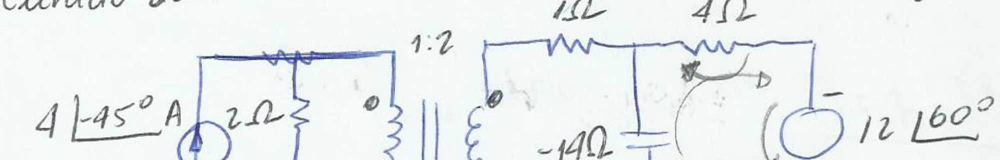
$$\Delta_{I_2}=(5-j5)\cdot j10-j8\cdot 10=50-j30$$

$$I_1=\frac{\Delta_{I_1}}{\Delta}=1{,}71\angle 16{,}96 \qquad I_2=\frac{\Delta I_2}{\Delta}=0{,}72\angle 7{,}03^\circ$$

$$V_C=(I_1-I_2)\cdot -j10$$

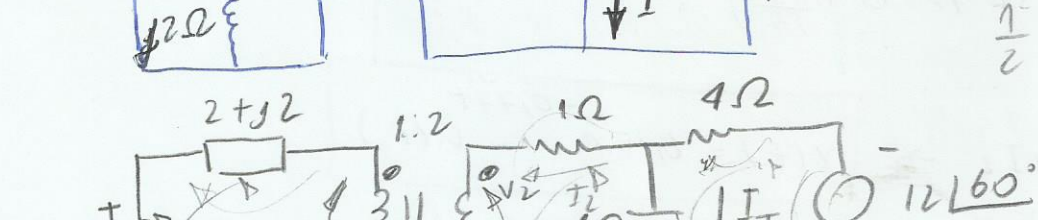
$$\boxed{V_C=10{,}15\angle -66{,}04^\circ}$$

Exercício do livro) (13.36) Determine I?



$$\frac{1}{2}=\frac{V_1}{V_2}$$

$$\frac{1}{2}=\frac{I_2}{I_1}$$



$$I_1(2+j2)+V_1=11{,}31 \rightarrow \quad I_2(4+j4)+\frac{V_2}{2}=11{,}31$$

$$I_2(1-j4)+j4\,I_3-V_2=0$$

$$I_3(4-j4)+j4\,I_2=+12\angle 60$$

$$\begin{bmatrix} 4+j4 & 0 & 0{,}5 \\ 1-j4 & j4 & -1 \\ j4 & 4-j4 & 0 \end{bmatrix} \times \begin{bmatrix} I_2 \\ I_3 \\ V_2 \end{bmatrix} = \begin{bmatrix} 11{,}31 \\ 0 \\ +12\angle 60 \end{bmatrix}$$

$2{,}61 \angle -33{,}4^\circ$

$$\Delta = \left| \begin{matrix} 4+j4 & 0 & 0{,}5 \\ 1-j4 & j4 & -1 \\ j4 & 4-j4 & 0 \end{matrix} \right| \begin{matrix} 4+j4 & 0 \\ 1-j4 & j4 \\ j4 & 4-j4 \end{matrix}$$

$\Delta = 0{,}5(1-j4)(4-j4) - 0{,}5(j4)^2 - (-1)\cdot(4+j4)(4-j4)$

$\Delta = 34 - j10$

$$\Delta_{I_2} = \left| \begin{matrix} 11{,}31 & 0 & 0{,}5 \\ 0 & j4 & -1 \\ -12\angle 60 & 4-j4 & 0 \end{matrix} \right| \begin{matrix} 11{,}31 & 0 \\ 0 & j4 \\ 12\angle 0 & 4-j4 \end{matrix} =$$

$\Delta I_2 = -(-12\angle 60)(j4)\cdot 0{,}5 - (-1)\cdot 11{,}31\cdot(4-j4) = \cancel{24{,}47} - j\,\cancel{33}{,}25$

$$\Delta I_3 = \left| \begin{matrix} 4+j4 & 11{,}31 & 0{,}5 \\ 1-j4 & 0 & -1 \\ j4 & -12\angle 60 & 0 \end{matrix} \right| \begin{matrix} 4+j4 & 11{,}31 \\ 1-j4 & 0 \\ j4 & -12\angle 60 \end{matrix}$$

$= 11{,}31\cdot(-1)\cdot(j4) + 0{,}5(1-j4)(+12\angle 60) + (+12\angle 60)(4+j4)$

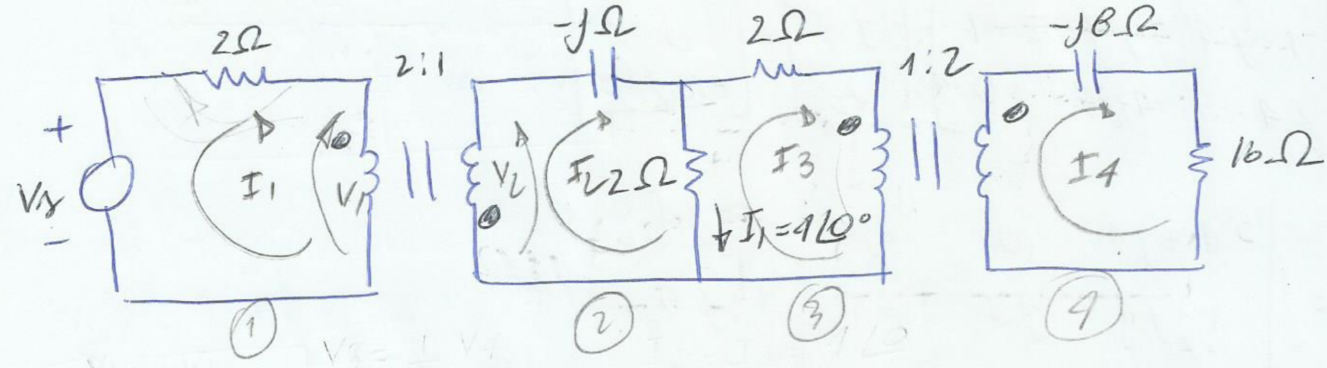
$\therefore \Delta I_3 = -6{,}\cancel{22} - j\,1\cancel{04}{,}02$

Através do MatLab

$I = \dfrac{\Delta I_2}{\Delta} - \dfrac{\Delta I_3}{\Delta} \quad \therefore \quad \boxed{I = 2{,}61 \angle 146{,}6}$

15,66 $\angle -153,43$

Exercício 13.40 $I_1 = 4\angle 0$ Determine Vs



$Z4 = 16 - j8$

$Z_3 = \left(\frac{1}{2}\right)^2 \cdot Z4 = 4 - j2$

$4 = \frac{6 - j2}{6 + j2 + 2} \cdot I_2 \quad \therefore \quad I_2 = 5,2 + j0,4$

$I_1 = \frac{-I_2}{a} = -2,6 - 0,2j$

$Z_2 = -j + 2 \,||\, 6 - j2 = 1,53 - j1,12$

$V_2 = I_2 \cdot Z_2 = 8,4 - j5,2$

$V_1 = -2V_2 = -16,8 + j10,4$

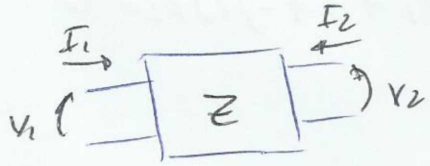
$V(s) = 2I_1 + V_1 \quad \therefore \quad V(s) = 24,17 \angle 155,56° \text{ V}$

Quadripolos

<u>Quadripolo</u> é uma estrutura de rede que relaciona tensão e corrente entre 2 pares de terminais (entrada e saída)

Parâmetros de Impedância - Matriz (Z)

- Se o quadripolo é uma rede linear que não contém fontes independentes, por meio da superposição podemos definir:



$$V_1 = Z_{11} \cdot I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} \cdot I_1 + Z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

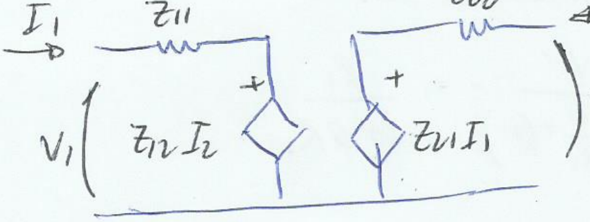
Onde: $Z_{11} = \dfrac{V_1}{I_1}\Big|_{I_2=0}$ -> Impedância de entrada com a saída aberta

$Z_{12} = \dfrac{V_1}{I_2}\Big|_{I_1=0}$ -> Impedância de transferência

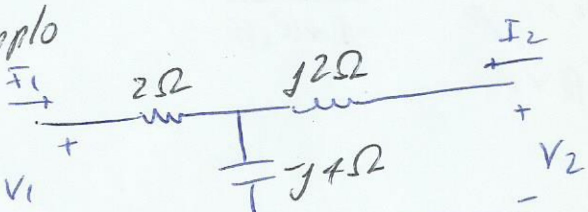
$Z_{21} = \dfrac{V_2}{I_1}\Big|_{I_2=0}$ -> " " "

$Z_{22} = \dfrac{V_2}{I_2}\Big|_{I_1=0}$ -> Impedância de saída com a entrada aberta

Circuito equivalente:



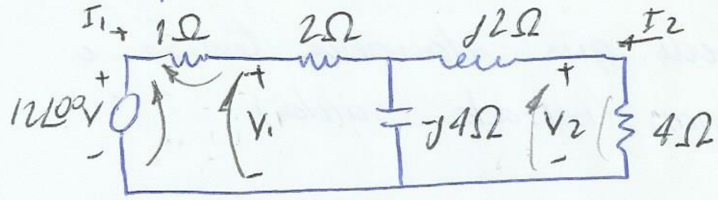
Exemplo



$Z_{11} = 2 - j4\ \Omega$
$Z_{12} = -j4\ \Omega$
$Z_{21} = -j4\ \Omega$
$Z_{22} = -j2\ \Omega$

$V_1 = I_1(2 - j4) + I_2(-j4)$
$V_2 = I_1(-j4) + I_2(-j2)$

Sabendo que o circuito foi modificado. Determine a corrente no resistor de $4\,\Omega$



$V_1 = 12\angle 0 - I_1 \qquad V_2 = -4I_2$

$(2-j4)I_1 - j4(I_2) = 12\angle 0 - I_1 \rightarrow (3-j4)I_1 - j4(I_2) = 12\angle 0$

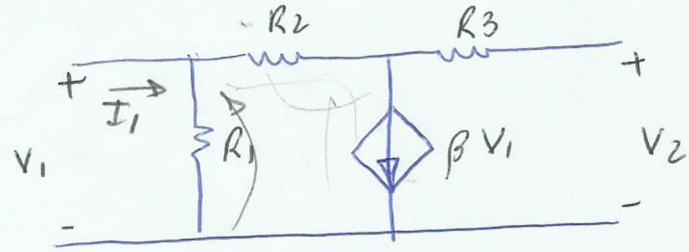
$(-j4)I_1 - j2(I_2) = -4I_2 \qquad (-j4)I_1 + (4-j2)I_2 = 0$

$$\begin{bmatrix} 3-j4 & -j4 \\ -j4 & 4-j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12\angle 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3-j4 & -j4 \\ -j4 & 4-j2 \end{vmatrix} = (3-j4)(4-j2) - (-j4)^2 = 20 - j22$$

$$\Delta i_2 = \begin{vmatrix} 3-j4 & 12\angle 0 \\ -j4 & 0 \end{vmatrix} = 48j \qquad i_2 = \frac{48j}{20-j22} \quad \therefore \underline{i_2 = 1{,}61\angle 137{,}73°}$$

Exemplo Determine o parâmetro z



Determinando z_{11}

$I_1 = \frac{V_1}{R_1} + \beta V_1 \quad \therefore \quad z_{11} = \frac{V_1}{I_1} = \frac{V_1}{V_1\left(\frac{1}{R_1} + \beta\right)} = \frac{R_1}{1+\beta R_1}$

Determinando z_{12}

$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \; ; \quad V_1 = R_1(I_2 - \beta V_1) \nearrow \quad V_1 = \frac{R_1 . I_2}{1 + R_1\beta}$

$V_1 = R_1 I_2 - R_1 \beta V_1$

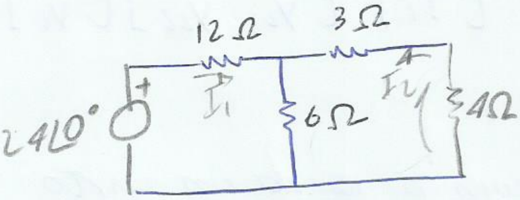
$\therefore z_{12} = \frac{R_1}{1 + R_1 . \beta}$

Determinando z_{21}

$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad I_1 = \frac{V_1}{R_1} + \beta V_1 \qquad V_2 = V_1 - R_2 . \beta V_1$

$z_{21} = \frac{V_1(1 - R_2\beta)}{V_1\left(\frac{1}{R_1} + \beta\right)} \quad \therefore z_{21} = \frac{R_1(1 - R_2\beta)}{1 + R_1\beta}$

Exercício: Determine o parametro z. Então calcule a corrente de carga de 4Ω, caso uma fonte de $24\angle 0°$ V seja conectada na porta de entrada.



$z_{11} = 18\Omega$

$z_{12} = 6\Omega$

$z_{21} = 6\Omega$

$z_{22} = 9\Omega$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V_1 = 24\angle 0°$ $\quad V_2 = -4I_2$

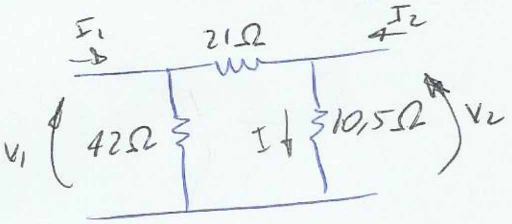
$\Delta = 198$

$$\begin{bmatrix} 18 & 6 \\ 6 & 9+4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24\angle 0 \\ 0 \end{bmatrix}$$

$$I_2 = \begin{vmatrix} 18 & 24\angle 0 \\ 6 & 0 \end{vmatrix} = -144$$

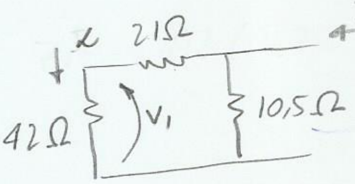
$$I_2 = \frac{-144}{198} = -0{,}73\angle 0° \text{ A}$$

Exercício: Determine os parametros z:



$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \therefore z_{11} = 42 \,||\, (21+10{,}5) \therefore z_{11} = 18\Omega$

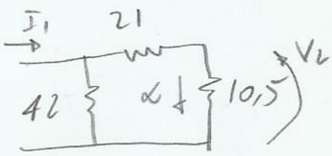
$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \therefore$



$\alpha = \frac{10{,}5}{10{,}5+42+21} \cdot I_2 = 0{,}143\, I_2 \quad \therefore V_1 = 42\alpha = 6\, I_2$

$z_{12} = \frac{6I_2}{I_2} \therefore z_{12} = 6\Omega$

$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$



$\alpha = \frac{42}{42+21+10{,}5} I_2 = 0{,}571\, I_2$

$V_2 = 10{,}5\,\alpha = 6\, I_2$

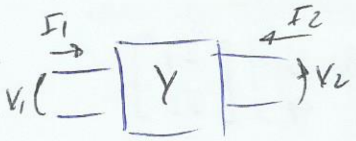
$\therefore z_{21} = \frac{6I_2}{I_2} = 6\Omega$

$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \quad z_{22} = (21+42) \,||\, 10{,}5 \therefore z_{22} = 9\Omega$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$[V] = [Z] \quad [I]$

Parâmetros de Admitancia - Matriz (Y)



$I_1 = Y_{11} V_1 + Y_{12} V_2$
$I_2 = Y_{21} V_1 + Y_{22} V_2$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Onde:

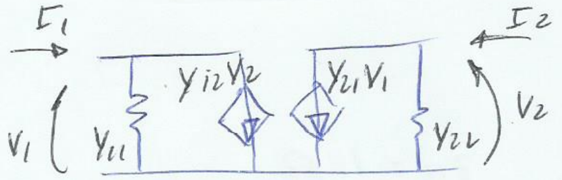
$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2 = 0}$ -> Admitancia de entrada com a saída em curto

$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1 = 0}$ -> Admitancia de transferência

$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2 = 0}$ -> " " "

$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1 = 0}$ -> Admitancia de transferencia com a entrada em curto

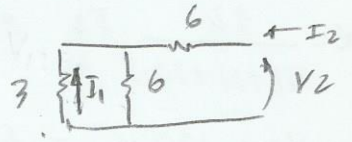
Circuito equivalente:



Exemplo: Determine o parametro Y

$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2 = 0}$ $\quad Y_{11} = \frac{1}{Z_{11}} = \frac{1}{3 + 6//6} = \frac{1}{6}$

$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1 = 0}$



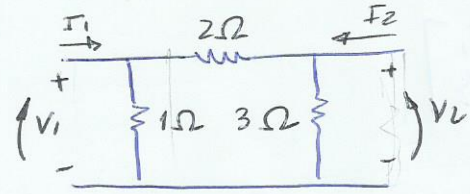
$I_1 = \frac{-6}{6+3} \cdot I_2 \qquad V_2 = -I_2 \cdot (6 + 6//3)$

$\therefore \quad Y_{12} = \frac{-\frac{2}{3} \cancel{I_2}}{\cancel{I_2}(6 + 6//3)} \quad \therefore \quad Y_{12} = \frac{-1}{12}$

$$Y = \begin{bmatrix} \frac{1}{6} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{8} \end{bmatrix}$$

$Y_{21} = \frac{-1}{12} \qquad Y_{22} = \frac{I_2}{V_2} = \frac{1}{Z_{22}} = \frac{1}{6 + 6//3} = \frac{1}{8}$

Exemplo:



a) Determinar o parametro Y.

b) Determinar a corrente em uma carga de 4Ω (na saída) para uma fonte de corrente de 2A (na entrada)

a) $Y_{11} = \frac{I_1}{V_1}\Big| V_2 = 0 \qquad Y_{11} = \frac{1}{Z_{11}}$

$Z_{11} = 1 // 2 = \frac{2}{3} \qquad \therefore \; Y_{11} = \frac{3}{2} S$

$Y_{12} = \frac{I_1}{V_2}\Big| V_1 = 0 \qquad I_1 = -\frac{3}{2+3} \cdot I_2$

$V_2 = I_2 (2 // 3)$

$$\therefore Y = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{6} \end{bmatrix}$$

$Y_{12} = \dfrac{-\frac{3}{5} \cancel{I_2}}{\cancel{I_2} \, \frac{6}{5}} \qquad Y_{12} = -\frac{1}{2} S \qquad \therefore Y_{21} = -\frac{1}{2} S$

$Y_{22} = \frac{1}{Z_{22}} \; ; \; Z_{22} = 2 // 3 \qquad \therefore Y_{22} = \frac{5}{6} S$

---------- // ----------

b) $I_1 = 2 A \qquad I_2 = \frac{-V_2}{4}$

$$\begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{13}{12} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\Delta = \frac{11}{6} \qquad \Delta V_2 = 1 \qquad \therefore \; V_2 = \frac{1}{\frac{11}{6}} = \frac{6}{11}$

logo $\quad I_2 = \frac{V_2}{4} = \boxed{\frac{-2}{11} A}$

Parâmetros híbridos - Matriz (H)

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

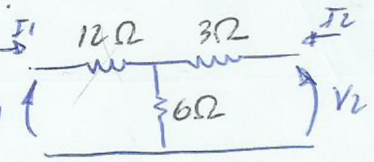
$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ Impedância de entrada com a saída em curto

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ Ganho reverso de tensão com entrada aberta

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ Ganho direto de corrente com saída em curto

$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$ Impedância de saída com a saída aberta

Exemplo:



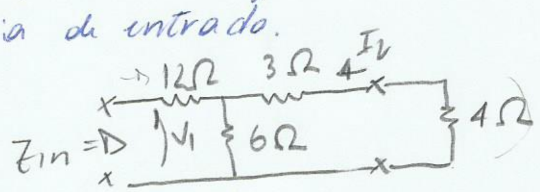
$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = Z_{11} = 12 + 3//6 \quad \therefore h_{11} = 14\,\Omega$

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} \qquad V_1 = 6 I_2 \qquad V_2 = +9 I_2 \qquad \therefore h_{12} = +\frac{2}{3}\ \frac{V}{V}$

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0} \qquad -I_2 = \frac{6}{6+3} \cdot I_1 \quad \therefore I_2 = -\frac{2}{3} I_1 \quad \therefore h_{21} = -\frac{2}{3}\ \frac{A}{A}$

$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = \frac{I_2}{I_2(3+6)} = \frac{1}{9}\,S \qquad h = \begin{bmatrix} 14 & +\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{9} \end{bmatrix}$

- Se uma carga de 4Ω for conectada na saída. Determine a impedância de entrada.



$$V_1 = 14 I_1 + \frac{2}{3} V_2$$
$$I_2 = -\frac{2}{3} I_1 + \frac{1}{9} V_2$$

$V_2 = -4 I_2 \qquad -I_2 = \frac{-2I_1}{3} - \frac{4I_2}{9} \ ; \quad I_1 = \left(1 + \frac{4}{9}\right) I_2 \cdot \frac{3}{-2} = -\frac{13}{6} I_2$

$\therefore I_1 = -\frac{13}{6} I_2 \qquad V_1 = 14 I_1 - \frac{8}{3} I_2 \qquad Z_{in} = \frac{V_1}{I_1}$

$I_2 = -\frac{6}{13} I_1 \qquad V_1 = 14 I_1 + \frac{16}{13} I_1$

$$\boxed{Z_{in} = 15{,}23\,\Omega}$$

Parametros de transmissão

$V_1 = A V_2 - B I_2$

$I_1 = C V_2 - D I_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

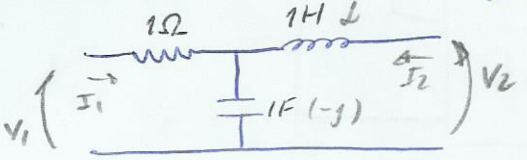
$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0}$: ganho reverso de tensão com saída aberta

$B = \left.-\frac{V_1}{I_2}\right|_{V_2=0}$: Impedância de transferência com saída em curto

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$: Admitância de transferência com saída aberta

$D = \left.-\frac{I_1}{I_2}\right|_{V_2=0}$: ganho reverso de corrente com saída em curto

Exercício: $\omega = 1 rad/s$



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ $\quad V_2 = \frac{-j\,V_1}{1-j}$ $\quad A = \frac{V_1}{\frac{-j}{1-j} V_1}$ $\quad \therefore A = 1+j \; \frac{V}{V}$

$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$ $\quad V_1 = I_1(1 + (-j)\,||\,(j))$

$= I_1\left(1 + \frac{1}{-j+j}\right) = I_1 \frac{1}{-j+j}$

Resposta:

$$\begin{bmatrix} 1+j & j \\ j & 0 \end{bmatrix}$$

$-I_2 = \frac{-j}{-j+j} \cdot I_1$ $\quad \therefore B = -\frac{\cancel{I_1} \frac{1}{-j+j}}{\frac{+j}{-j+j} \cdot \cancel{I_1}}$ $\quad \therefore B = j\,\Omega$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ $\quad I_1 = \frac{V_1}{1-j}$ $\quad V_2 = \frac{-j}{1-j} V_1$ $\quad C = \frac{\frac{\cancel{V_1}}{1-j}}{\frac{-j}{1-j} \cancel{V_1}} = j\,S$

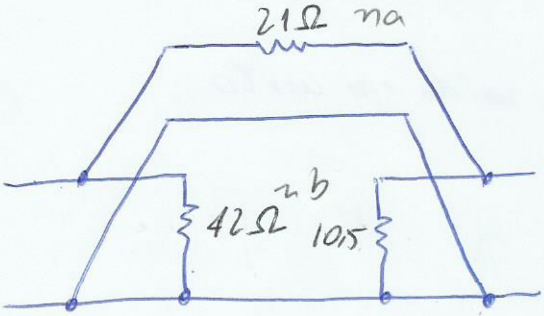
$D = \frac{-I_1}{I_2} \Big| V_2 = 0$ $\quad I_1 = \frac{V_1}{1+(-j)\,||\,(j)} = \frac{V_1}{1 + \frac{1}{j-j}}$

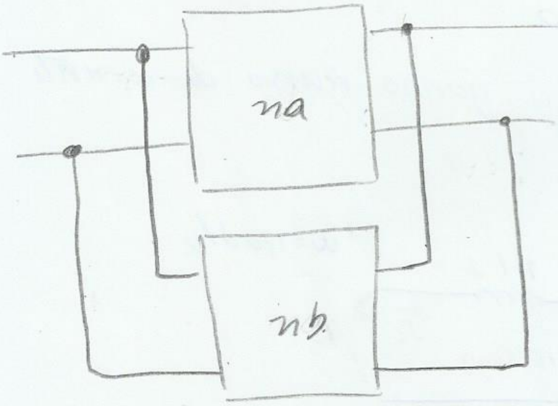
$-I_2 = \frac{-j}{-j+j} \cdot I_1$ $\quad \therefore D = \frac{\frac{V_1}{j-j+1}(j-j)}{\frac{-j}{-j+j}} = \frac{V_1 \cancel{(j-j)}^{2=0}}{(\cancel{j-j}^{0}+1)\cdot(-j)} = 0$

Associação de quadripolos

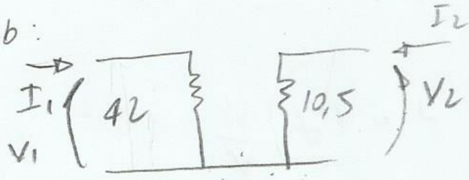
Matriz (Z) → Serie - Serie ∴ Basta somar os parametros
Matriz (Y) → Paralelo - Paralelo: " " " "
Matriz (H) → Serie - Paralelo: " " "
Matriz (T) → Cascata : Basta multiplicar os parametros

Exercícios Determine os parâmetros Y da rede





nb:



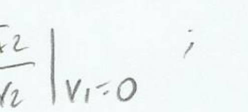
$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$; $I_1 = \frac{V_1}{42}$ logo $Y_{11} = \frac{\frac{V_1}{42}}{V_1} = \frac{1}{42}$

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$; $= 0$

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = 0$

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$; $I_2 = \frac{V_2}{10,5}$ ∴ $Y_{22} = \frac{\frac{V_2}{10,5}}{V_2} = \frac{2}{21}$

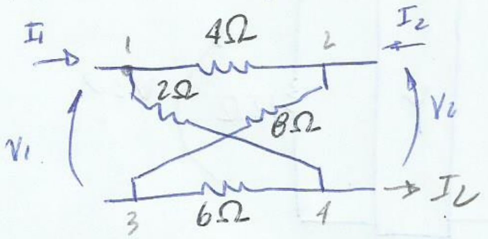
na :



$$Y = \begin{bmatrix} \frac{1}{42} + \frac{1}{21} & \frac{1}{21} \\ \frac{1}{21} & \frac{1}{21} + \frac{2}{21} \end{bmatrix} \therefore Y = \begin{bmatrix} \frac{1}{14} & \frac{1}{21} \\ \frac{1}{21} & \frac{1}{7} \end{bmatrix} S$$

Lista de Quadripolos I

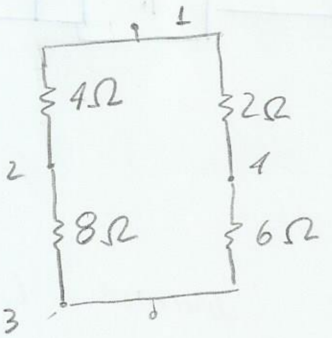
1) Determinar parâmetro Z



$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

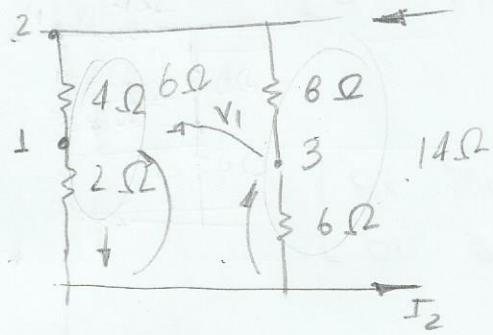
$Z_{11} = \dfrac{V_1}{I_1}$ para $I_2 = 0$



$\therefore Z_{11} = (8+4) \parallel (2+6)$

$Z_{11} = 4{,}8\,\Omega$

$Z_{12} = \dfrac{V_1}{I_2}$ para $I_1 = 0$



$6I' = 14I''$

$I' = \dfrac{7}{3} I''$

$I' + I'' = I_2$

$\dfrac{7}{3} I'' + I'' = I_2$

$I''\left(\dfrac{7}{3} + 1\right) = I_2$

$\therefore I'' = \dfrac{3}{10} I_2$

$I' = \dfrac{7}{10} I_2$

$V_1 = 2I' - 6I''$

$V_1 = 2 \cdot \dfrac{7 I_2}{10} - 6 \cdot \dfrac{3 I_2}{10} = -0{,}4\, I_2$ $\therefore$ $Z_{12} = \dfrac{-0{,}4 I_2}{I_2}$ $\therefore Z_{12} = -0{,}4$

$Z_{21} = Z_{12} = -0{,}4$

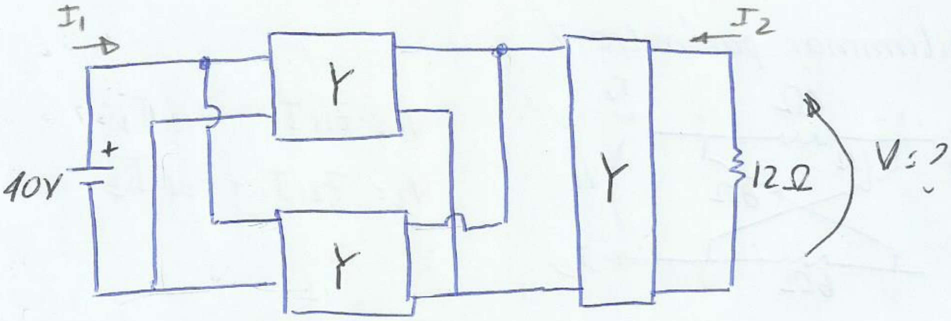
$Z_{22} = \dfrac{V_2}{I_2}$ para $I_1 = 0$

Z_{22}: $(4+2) \parallel (8+6)$ $\therefore Z_{22} = 4{,}2$

$$Z = \begin{bmatrix} 4{,}8 & -0{,}4 \\ -0{,}4 & 4{,}2 \end{bmatrix} \Omega$$

2) Determinar a tensão na carga de 12Ω, sabendo que o quadripolo

$$Y: \begin{bmatrix} \frac{1}{5} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix}$$



$$Y \parallel Y = \begin{bmatrix} \frac{2}{5} & -\frac{2}{10} \\ \frac{-2}{10} & \frac{2}{5} \end{bmatrix}$$

$(Y \parallel Y)$ em cascata com Y, devemos transformar para matriz (T)

$(Y \parallel Y)$ em T : $\begin{bmatrix} 2 & 5 \\ 0{,}6 & 2 \end{bmatrix}$ Y em T = $\begin{bmatrix} 2 & 10 \\ 0{,}3 & 2 \end{bmatrix}$

$$T_{eq} = \begin{bmatrix} 2 & 5 \\ 0{,}6 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 10 \\ 0{,}3 & 2 \end{bmatrix} = \begin{bmatrix} 5{,}5 & 30 \\ 1{,}8 & 10 \end{bmatrix}$$

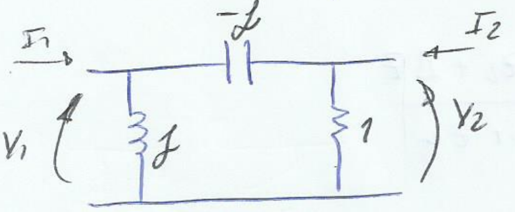
$V_1 = 5{,}5\, V_2 - 30\, I_2$ $V_1 = 40$

$I_1 = 1{,}8\, V_2 - 10\, I_2$ $V_2 = -12 I_2$

$40 = 5{,}5(-12 I_2) - 30 I_2 \quad \therefore \quad I_2 = \frac{-5}{12}$

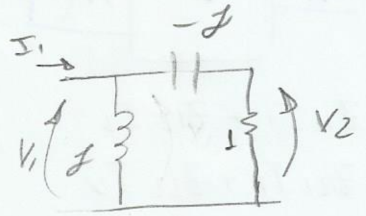
$V = -12 I_2 \quad \therefore \quad \boxed{V = 5V}$

3) Determine os parametros z do quadripolo abaixo.



$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

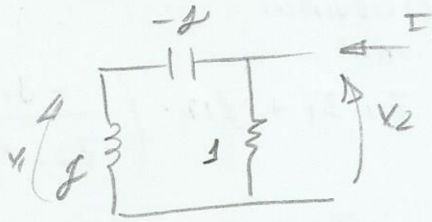


Para $I_2 = 0$ $\quad z_{11} = \dfrac{V_1}{I_1}$ e $z_{21} = \dfrac{V_2}{I_1}$

$z_{11} = j \,||\, (-j+1) = 1+j$

$z_{21} = \dfrac{V_2}{I_1}$; $I' = \dfrac{j}{1 \cancel{-j} \cancel{+j}} \cdot I_1 \quad \therefore V_2 = 1 I' = j I_1$

logo $z_{21} = \dfrac{j I_1}{I_1} \quad \therefore z_{21} = j$



Para $I_1 = 0$ $\quad z_{12} = \dfrac{V_1}{I_2}$ e $z_{22} = \dfrac{V_2}{I_2}$

$z_{22} = (-j+j) \,||\, 1 = 0$

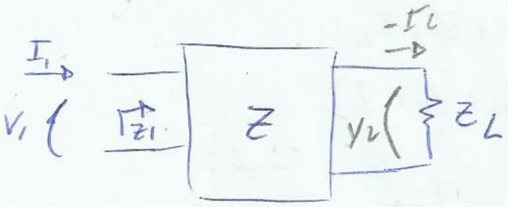
$I' = \dfrac{1}{\cancel{j} \cancel{-j} + 1} \cdot I_2 = I_2 \quad \therefore V_1 = I' \cdot j$

$V_1 = j I_2$

$z_{12} = \dfrac{j \cdot I_2}{I_2} \quad \therefore z_{12} = j$

$$Z = \begin{bmatrix} 1+j & j \\ j & 0 \end{bmatrix} (\Omega)$$

4) Demonstrar que:



$$Z_1 = \frac{Z_{22} + Z_L + \Delta Z}{Z_{22} + Z_L}$$

$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases} \quad ; \quad V_2 = -I_2 Z_L$$

$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ 0 = Z_{21} I_1 + (Z_{22} + Z_L) I_2 \end{cases}$$

$$\hookrightarrow I_2 = \frac{-Z_{21} I_1}{Z_{22} + Z_L}$$

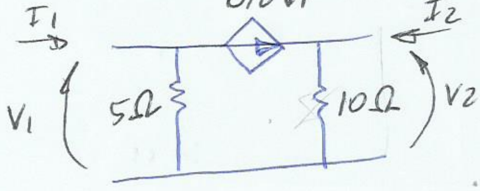
Substituindo

$$V_1 = Z_{11} I_1 + Z_{12} \cdot \left(\frac{-Z_{21} I_1}{Z_{22} + Z_L} \right)$$

$$\frac{V_1}{I_1} = Z_{11} + \frac{Z_{12}(-Z_{21})}{Z_{22} + Z_L} = \frac{Z_{11} Z_{22} + Z_{11} Z_L - Z_{12} Z_{21}}{Z_{22} + Z_L}$$

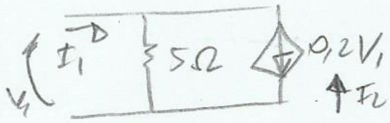
$$\therefore Z_{in} = \frac{\Delta Z + Z_{11} Z_{LL}}{Z_{22} + Z_L}$$

5) Determine os parâmetros Y do quadripolo



$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Para $V_2 = 0$ temos $Y_{11} = \frac{I_1}{V_1} = \frac{1}{Z_1}$ e $Y_{21} = \frac{I_2}{V_1}$



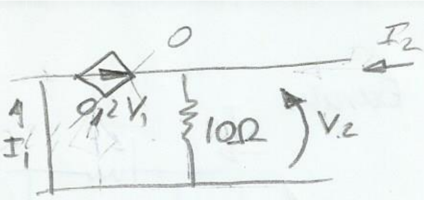
$$I_1 = \frac{V_1}{5} + 0{,}2 V_1 = \frac{V_1 + V_1}{5} \therefore I_1 = \frac{2V_1}{5}$$

$$\therefore \underline{Y_{11}} = \frac{2}{5} = 0{,}4\,S$$

$$I_2 = -0{,}2 V_1$$

$$Y_{21} = \frac{-0{,}2 V_1}{V_1} \therefore Y_{21} = -0{,}2\,S$$

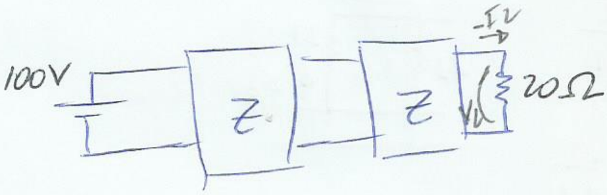
Para $V_1=0$ temos $Y_{12}=\frac{I_1}{V_2}$ e $Y_{22}=\frac{I_2}{V_2}$



$$Y_{12}=\frac{0}{V_2}=0$$

$$Y_{22}=\frac{\frac{V_2}{10}}{V_2}=\frac{1}{10}=0{,}1 \qquad Y=\begin{bmatrix}0{,}4 & 0\\ -0{,}2 & 0{,}1\end{bmatrix}$$

6) Calcular tensão V_o



$$Z=\begin{bmatrix}15 & 10\\ 10 & 20\end{bmatrix}$$

$$Z\to T=\begin{bmatrix}1{,}5 & 20\\ 0{,}1 & 2\end{bmatrix} \qquad \begin{array}{l}V_1=AV_2-BI_2\\ I_1=CV_2-DI_2\end{array}$$

$$\begin{bmatrix}V_1\\ I_1\end{bmatrix}=\begin{bmatrix}1{,}5 & 20\\ 0{,}1 & 2\end{bmatrix}\begin{bmatrix}1{,}5 & 20\\ 0{,}1 & 2\end{bmatrix}\begin{bmatrix}V_2\\ -I_2\end{bmatrix} \therefore \begin{array}{l}V_1=4{,}25V_2-70I_2\\ V_2=0{,}35V_2-6I_2\end{array}$$

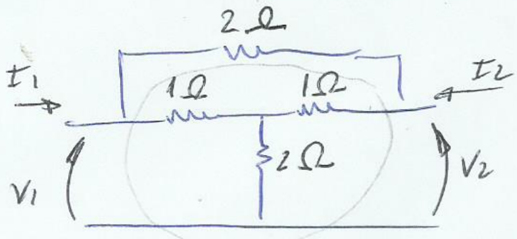
$V_2=-20I_2$ $\quad V_1=100$

$$100=4{,}25.(-20I_2)-70.I_2$$

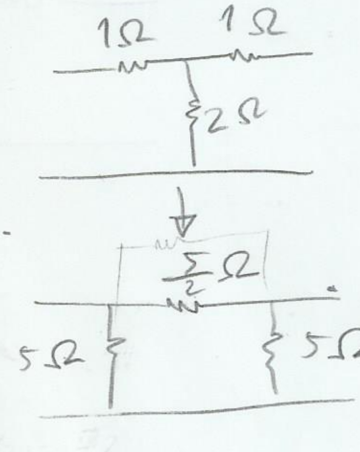
$$100=-85I_2-70I_2 \quad \therefore I_2=-0{,}65 \quad \therefore \boxed{V_2=12{,}90V}$$

Redes equivalentes $T\Rightarrow\pi\Rightarrow T$

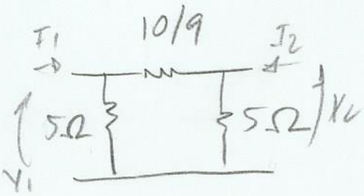
Determinar equivalentes π e T da rede



$$Z=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$$



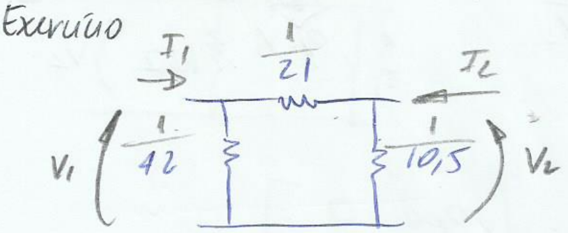
$|Z|=5$ $\quad Y_A=\frac{1}{5}$ $\quad Y_B=\frac{2}{5}$ $\quad Y_C=\frac{1}{5}$

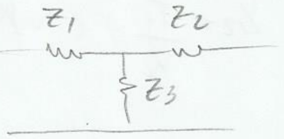


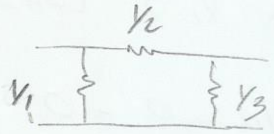
Determinando o parametro Y, obtemos: $Y=\begin{bmatrix}\frac{11}{10} & \frac{-9}{10}\\ \frac{-9}{10} & \frac{11}{10}\end{bmatrix}$ $\quad |Y|=\frac{2}{5}$

$$Z_1=\frac{\frac{1}{5}}{\frac{2}{5}}=\frac{1}{2}\Omega \qquad Z_2=\frac{\frac{1}{5}}{\frac{2}{5}}=\frac{1}{2}\Omega \qquad Z_3=\frac{\frac{9}{10}}{\frac{2}{5}}=\frac{9}{4}\Omega$$

Exercício





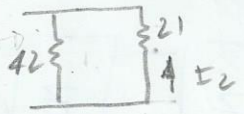


$$Y_1 = \frac{Z_2}{|Z|} \; ; \; Y_2 = \frac{Z_3}{|Z|} \; ; \; Y_3 = \frac{Z_1}{|Z|}$$

$$Z_1 = \frac{Y_3}{|Y|} \; ; \; Z_2 = \frac{Y_3}{|Y|} \; ; \; Z_3 = \frac{Y_1}{|Y|}$$

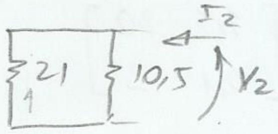
$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Para $V_2 = 0 \rightarrow \boxed{Y_{11} = \frac{I_1}{V_1}}$ e $Y_{21} = \frac{I_2}{V_1}$



$\therefore Y_{11} = \frac{1}{14}$ $\quad Y_{21} = \dfrac{\frac{-42}{42+21} \cdot \cancel{I_1}}{(42 \| 21)\cancel{I_1}} = \frac{-1}{21}$

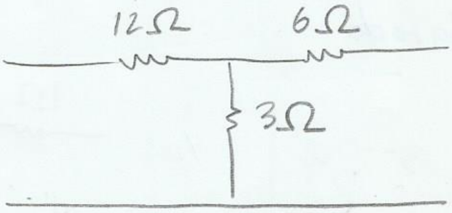
Para $V_1 = 0 \rightarrow Y_{22} = \frac{I_2}{V_2}$ e $Y_{12} = \frac{I_1}{V_2}$



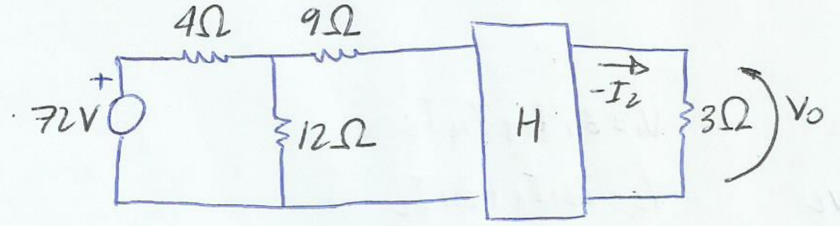
$Y_{22} = \frac{1}{21 \| 10,5} = \frac{1}{7}$ $\quad Y_{12} = \dfrac{-\frac{10,5}{10,5+21} \cancel{I_2}}{(21 \| 10,5)\cancel{I_2}}$ $\quad Y_{12} = -\frac{1}{21}$

$$Y = \begin{bmatrix} \frac{1}{14} & -\frac{1}{21} \\ -\frac{1}{21} & \frac{1}{7} \end{bmatrix} \qquad |Y| = \frac{1}{126}$$

$$Z_1 = \frac{\frac{1}{10,5}}{\frac{1}{126}} = 12\Omega \quad Z_2 = \frac{\frac{1}{21}}{\frac{1}{126}} = 6\Omega \quad Z_3 = \frac{\frac{1}{42}}{\frac{1}{126}} = 3\Omega$$



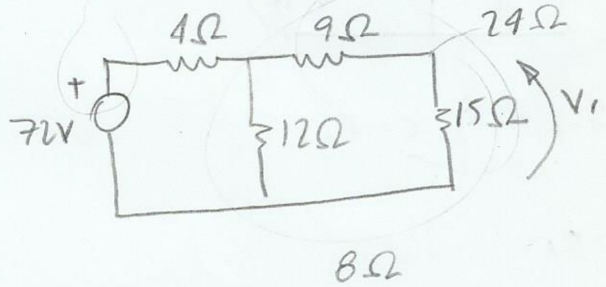
Quadripolos inserido em uma rede



$$H = \begin{bmatrix} 14 & \frac{2}{3} \\ \frac{-2}{3} & \frac{1}{9} \end{bmatrix}$$

$$\begin{array}{ccc} & A & B \\ H \rightarrow T = & \left[\begin{array}{cc} 3 & 21 \\ \frac{1}{6} & \frac{3}{2} \end{array}\right] \\ & C & D \end{array}$$

$$Z_{in} = \frac{3.3 + 21}{\frac{1}{6}.3 + \frac{3}{2}} \quad \therefore \quad Z_{in} = 15\Omega$$



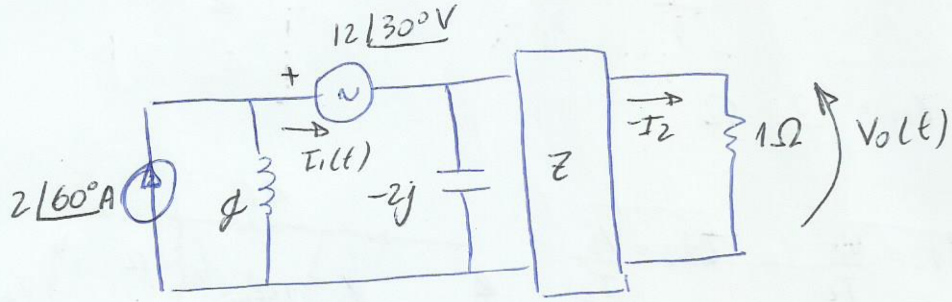
$$V' = \frac{8}{8+4} \cdot 72 = 48$$

$$V_1 = \frac{15}{15+9} \cdot 48 \quad \therefore \quad \boxed{V_1 = 30\,V}$$

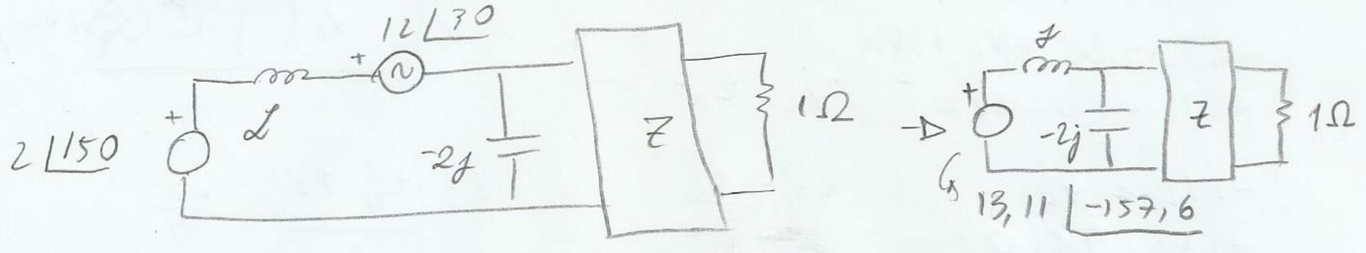
$$I_1 = CV_2 - DI_2 \quad ; \quad \frac{V_2}{I_2} = -3$$

$$2 = \frac{1}{6} \cdot V_2 + \frac{\cancel{3}}{2} \cdot \frac{V_2}{\cancel{3}}$$

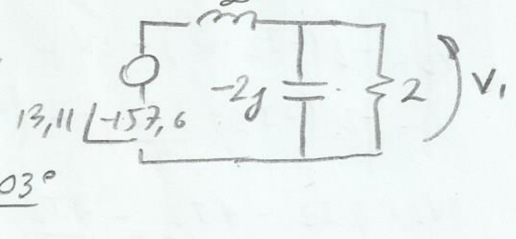
$$2 = \left(\frac{1}{6} - \frac{1}{2}\right) V_2 \quad \therefore \quad \boxed{V_2 = 3\,V}$$



$$Z = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$



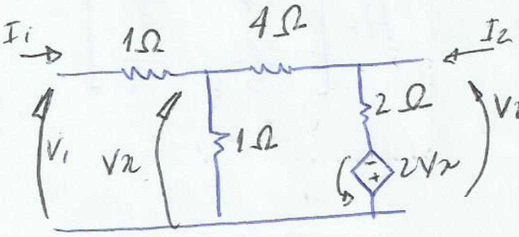
$$Z \rightarrow T = \begin{bmatrix} \frac{3}{2} & \frac{5}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix} \qquad Z_{in} = \frac{\frac{3}{2} \cdot 1 + \frac{5}{2}}{\frac{1}{2} \cdot 1 + \frac{3}{2}} = 2$$



$$V_1 = \frac{2-2j}{2-j} \cdot 13{,}11\angle{-157{,}6} \quad \therefore \quad V_1 = 16{,}58\angle{-176{,}03°}$$

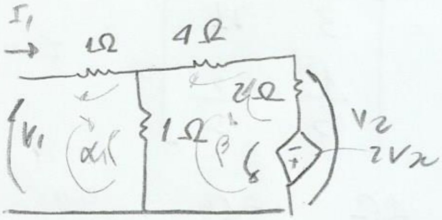
Lista de Exercícios · Quadripolos II

1) Determinar parâmetros Z



$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$



Para $I_2 = 0$ temos $Z_{11} = \frac{V_1}{I_1}$ e $Z_{21} = \frac{V_2}{I_1}$

$\begin{cases} \alpha(1+1) - \beta = V_1 \\ -\alpha + \beta . 7 = 2V_x \end{cases}$; $V_x = \alpha - \beta$

$\begin{cases} 2\alpha - \beta = V_1 \\ -3\alpha + 9\beta = 0 \end{cases}$ $\quad -\alpha + 7\beta = 2\alpha - 2\beta$; $\alpha = I_1$

$\beta = \frac{3\alpha}{9} = \frac{\alpha}{3}$ $\rightarrow$ $2\alpha - \frac{\alpha}{3} = V_1$; $\alpha = I_1$

$\frac{5 I_1}{3} = V_1 \quad \therefore \quad \boxed{Z_{11} = \frac{V_1}{I_1} = \frac{5}{3}\ \Omega}$

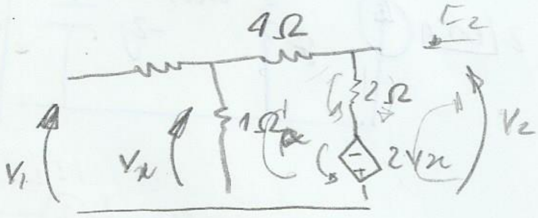
$V_x = \alpha - \beta$; $\beta = \frac{\alpha}{3}$ $\quad \therefore \quad V_x = \frac{2}{3} I_1$

$= \alpha - \frac{\alpha}{3} = \frac{2}{3}\alpha$

$V_2 = 2\beta - 2V_x$

$= \frac{2 I_1}{3} - 2 . \frac{2}{3} I_1 \Rightarrow \therefore V_2 = \frac{-2}{3} I_1$

$\therefore Z_{21} = \frac{\frac{-2}{3} I_1}{I_1}$ $\quad \boxed{Z_{21} = \frac{-2}{3}\ \Omega}$



Para $I_1 = 0$ temos $Z_{12} = \frac{V_1}{I_2}$ e $Z_{22} = \frac{V_2}{I_2}$

$\begin{cases} -2 I_2 - 2\alpha = V_2 + 2V_x \\ +2 I_2 + 7\alpha = 2V_x \end{cases}$; $V_x = -\alpha$

$\begin{cases} -2 I_2 = V_2 \\ +2 I_2 + 9\alpha = 0 \end{cases}$ $\quad \alpha = \frac{-2 I_2}{9} \quad \therefore \quad Z_{12} = \frac{+\frac{2}{9} I_2}{I_2} = +\frac{2}{9}$

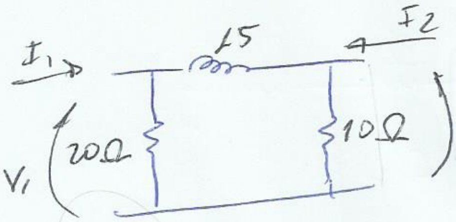
$V_2 = -2(-I_2 - \alpha) - 2V_x$

$= -2\left(-I_2 + \frac{2 I_2}{9}\right) - 2 . \left(\frac{2 I_2}{9}\right)$

$V_2 = 2 I_2 - \frac{4 I_2}{9} - \frac{4 I_2}{9}$ $\rightarrow V_2 = \frac{10}{9} I_2$ $\quad \therefore Z_{22} = \frac{\frac{10}{9} I_2}{I_2}$

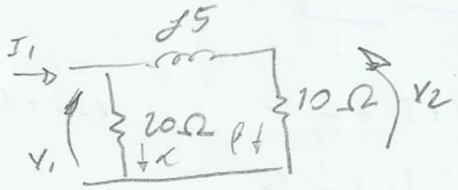
$\therefore Z_{22} = \frac{10}{9}\ \Omega$

2) Determinar parâmetros T do quadripolo.



$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$



Para $I_2 = 0 \rightarrow$ $A = \dfrac{V_1}{V_2}$ $\quad C = \dfrac{I_1}{V_2}$

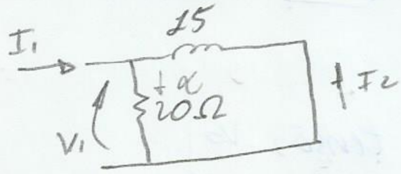
$\beta = \dfrac{20}{20+10+j5} \cdot I_1 = (0{,}65 - j\,0{,}11)\,I_1$

$V_2 = 10\beta = (6{,}49 - j1{,}08)\,I_1$

$\alpha = \dfrac{10+j5}{20+10+j5} \cdot I_1 = (0{,}35 + j0{,}108)\,I_1 \quad \therefore \quad V_1 = 20\alpha$

$A = \dfrac{20\alpha}{10\beta} \quad \therefore \quad \boxed{A = 1 + j\frac{1}{2}\ \frac{V}{V}}$

$I_1 = \alpha + \beta \quad \therefore \quad C = \dfrac{\alpha+\beta}{10\beta} \quad \therefore \quad \boxed{C = \frac{3}{20} + j\frac{1}{40}}$



Para $V_2 = 0 \rightarrow$ $B = \dfrac{-V_1}{I_2}$ e $D = -\dfrac{I_1}{I_2}$

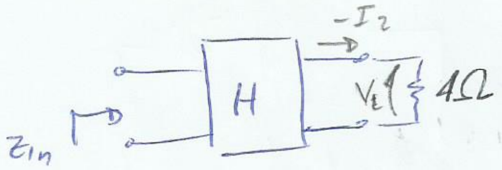
$I_2 = \dfrac{-20}{20+j5} I_1 = (-0{,}94 + j0{,}24)\,I_1$

$\alpha = \dfrac{j5}{20+j5} I_1 = (0{,}06 + j0{,}24)\,I_1$

$V_1 = 20\alpha$ $\qquad B = \dfrac{-V_1}{I_2} = 5j$ e $D = \dfrac{-I_1}{I_2} = 1 + j\frac{1}{4}$

$$T = \begin{bmatrix} 1 + \frac{j}{2} & 5j \\ \frac{6+j}{40} & 1 + \frac{j}{4} \end{bmatrix}$$

4) Determine a impedancia de entrada



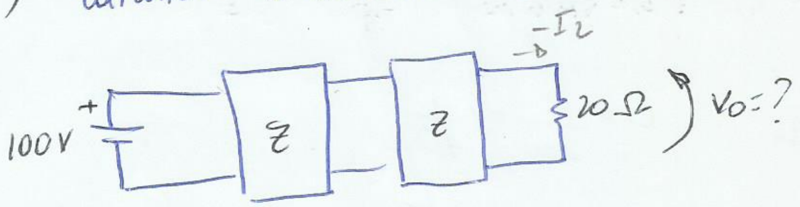
$$H = \begin{bmatrix} 14 & \frac{2}{3} \\ \frac{-2}{3} & \frac{1}{9} \end{bmatrix}$$

$$\begin{cases} V_1 = 14 I_1 + \frac{2}{3} V_2 \\ I_2 = -\frac{2}{3} I_1 + \frac{1}{9} V_2 \end{cases} \qquad V_2 = -4 I_2$$

$$\begin{cases} V_1 = 14 I_1 - \frac{8}{3} I_2 \\ 0 = -\frac{2}{3} I_1 - \frac{13}{9} I_2 \end{cases} \qquad I_2 = \frac{-6}{13} I_1$$

$$V_1 = 14 I_1 - \frac{8}{3}\left(-\frac{6}{13} I_1\right) \qquad Z_{in} = \frac{V_1}{I_1} = \underline{15{,}23\,\Omega}$$

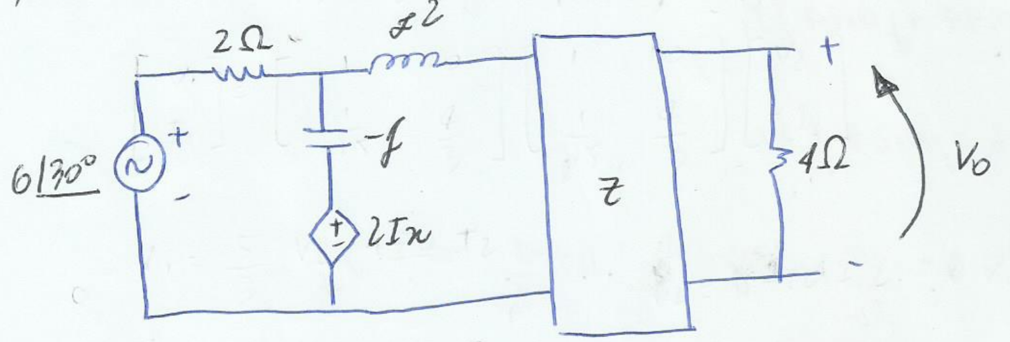
5) Calcular a tensão V_o



$$Z = \begin{bmatrix} 15 & 10 \\ 10 & 20 \end{bmatrix}$$

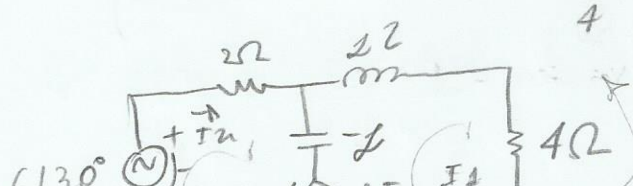
$$\boxed{V_o = 12{,}90\,V}$$

6) Calcular a tensão V_o



$$Z = \begin{bmatrix} 5 & 4 \\ 4 & 12 \end{bmatrix}$$

$$Z \to T = \begin{bmatrix} \frac{5}{4} & 11 \\ \frac{1}{4} & 3 \end{bmatrix} \qquad Z_{in} = \frac{\frac{5}{4} \cdot 4 + 11}{\frac{1}{4} \cdot 4 + 3} = 4\Omega$$



$$\begin{cases} \alpha(2-j) + j I_1 = 6\angle 30 - 2I_x \\ +j\alpha + I_1(4+j) = 2I_x \end{cases}$$

Sabendo que $\alpha = I_x$

$$\begin{cases} \alpha(4-j) + j \cdot I_1 = 6\angle 30 \\ \alpha(-2+j) + (4+j) I_1 = 0 \end{cases}$$

$$\alpha = \frac{-(4+j)\, I_1}{(-2+j)} \qquad I_1 = 0{,}74\angle -29$$

$$\frac{-(4-j) \times (4+j)\, I_1}{(-2+j)} + j I_1 = 6\angle 30$$

$$\begin{cases} V_1 = 5I_1 + 4I_2 \\ V_2 = 4I_1 + 12I_2 \end{cases} \qquad V_1 = 4I_1 \quad V_2 = -4I_2$$

$$\begin{cases} 4I_1 = 5I_1 + 4I_2 \\ -4I_2 = 4I_1 + 12I_2 \end{cases} \rightarrow \begin{cases} 0 = I_1 + 4I_2 \\ 0 = 4I_1 + 16I_2 \end{cases}$$

$\therefore I_1 = -4I_2$; $I_1 = 0{,}74 \angle -29$ $\therefore I_2 = -0{,}18 + 0{,}0094j$

$\therefore V_2 = -4I_2 = 0{,}74 \angle -4{,}9$ V

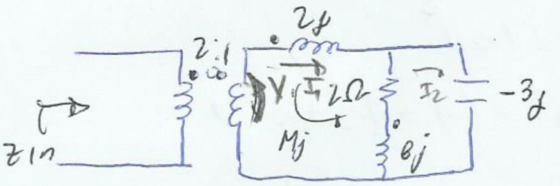
ou

$$\begin{cases} V_1 = \frac{5}{4} V_2 - 11 I_2 \\ I_1 = \frac{1}{4} V_2 - 3 I_2 \end{cases} \qquad V_1 = 4I_1 \quad V_2 = -4I_2$$

$\{ 4I_1 = -5I_2 - 11I_2$ $\qquad I_2 = \frac{4I_1}{-16} = \frac{I_1}{-4}$ ✓ A mesma coisa

Provas antigas

1-) Determine a impedância de entrada da rede magnética acoplada $K = 1$



$M = K\sqrt{L_1 L_2}$ $\qquad M = 1\sqrt{2 \cdot 8} = 4$

$(2+10j)I_1 - (2+8j)I_2 + 4j(I_1 - I_2) + 4jI_1 = V$

$\hookrightarrow (2+10j + 4j + 4j)I_1 + (-2-8j-4j)I_2 = V$

$(2+18j)I_1 - (2+12j)I_2 = V$ (Malha 1)

$(2+5j)I_2 - (2+8j)I_1 - 4jI_1 = 0$

$(-2-8j-4j)I_1 + (2+5j)I_2 = 0$

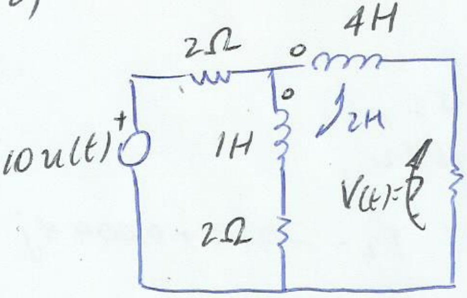
$-(2+12j)I_1 + (2+5j)I_2 = 0$ (Malha 2)

$$\begin{cases} (2+18j)I_1 - (2+12j)I_2 = V \\ -(2+12j)I_1 + (2+5j)I_2 = 0 \end{cases} \qquad \begin{aligned} \Delta &= (2+18j)(2+5j) - (2+12j)^2 \\ \Delta &= 54 - j2 \end{aligned}$$

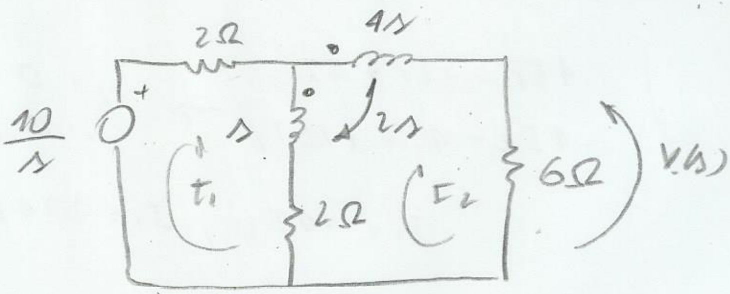
$\Delta I_1 = V \times (2+5j) I_2$

$I_1 = \dfrac{V \times (2+5j)}{54 - j2}$ $\quad \therefore Z = \dfrac{V}{I} = \dfrac{54 - j2}{2+5j}$ $\qquad Z_{in} = 2^2 \cdot Z$ $\therefore$ $\boxed{Z_{in} = 40{,}1 \angle -70{,}3° \, \Omega}$

2)



Domínio S ⇒



$$I_1(4+s) - (2+s)I_2 + 2s \cdot I_2 = \frac{10}{s}$$

$$I_1(4+s) + I_2(-2-s+2s) = \frac{10}{s}$$

$$I_1(4+s) + I_2(-2+s) = \frac{10}{s} \quad \text{(Malha 1)}$$

$$I_2(6+2+4s+s) - I_1(s+2) + 2s(I_1 - I_2) - 2sI_2 = 0$$

$$I_1(-s-2+2s) + I_2(8+5s-2s-2s) = 0$$

$$I_1(-2+s) + I_2(8+s) = 0 \quad \text{(Malha 2)}$$

$$\begin{cases} I_1(4+s) + I_2(-2+s) = \frac{10}{s} \\ I_1(-2+s) + I_2(8+s) = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} 4+s & -2+s \\ -2+s & 8+s \end{vmatrix} = (4+s)(8+s) - (-2+s)^2$$

$$= 32 + \cancel{4s} + 8s + \cancel{s^2} - (4 - \cancel{4s} + \cancel{s^2})$$

$$\Delta = 16s + 28$$

$$\Delta_{I_2} = \begin{vmatrix} 4+s & \frac{10}{s} \\ -2+s & 0 \end{vmatrix} = \frac{-10}{s}(-2+s) = \frac{20}{s} - 10$$

$$I_2 = \frac{\frac{20-10s}{s}}{16s+28} \quad \therefore \quad I_2 = \frac{20-10s}{16s^2+28s} = \frac{20-10s}{16(s^2+\frac{7}{4}s)} = \frac{K_1}{s} + \frac{K_2}{s+\frac{7}{4}}$$

$$K_1 = sI_2 \Big|_{s=0} = \frac{20-10s}{16(s+\frac{7}{4})} = 0{,}71 \qquad K_2 = (s+\tfrac{7}{4})I_2 \Big|_{s=-\frac{7}{4}} = \frac{20-10s}{16s} = -1{,}33$$

$$I_2 = 0{,}71 - 1{,}33\, e^{-\frac{7}{4}t} \quad \therefore \quad V(t) = 6I_2$$

$$\boxed{V(t) = 4{,}29 - 8{,}03\, e^{-1{,}75t} \ [V]}$$