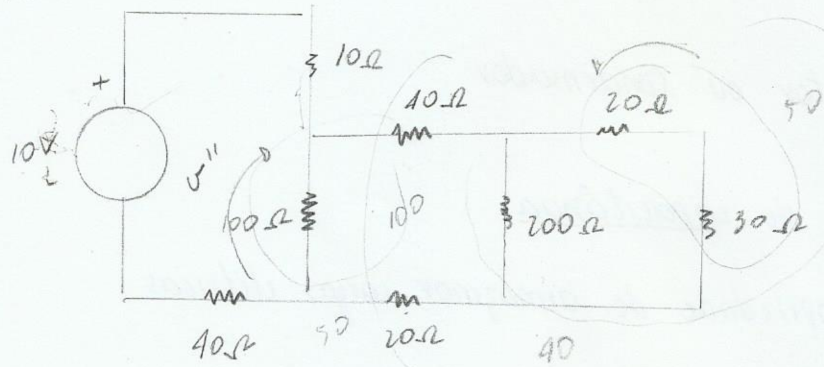
EL4220 - Circuitos Elétricos II

Livro texto: Análise de circuitos em Engenharia autor: Irwin

Capítulos: 6º ao 12º - $P_1 \to (6,7,8)$ $P_2 \to (9,10,11,12)$

Revisão de Circuitos I

Ex:



$V'' = \frac{50}{100} \times 10 = 5$

$V' = \frac{40}{100} \cdot 5 = \frac{200}{100} = 2$

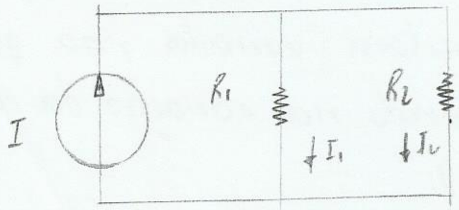
$V = \frac{20}{50} \cdot 2 = \frac{4}{5}$

Divisor de tensão

$$\boxed{V_2 = \frac{R_2}{R_1+R_2} \times V}$$

$\frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V}{R_1+R_2}$ ∴ $V_2 = \frac{R_2}{R_1+R_2} \cdot V$

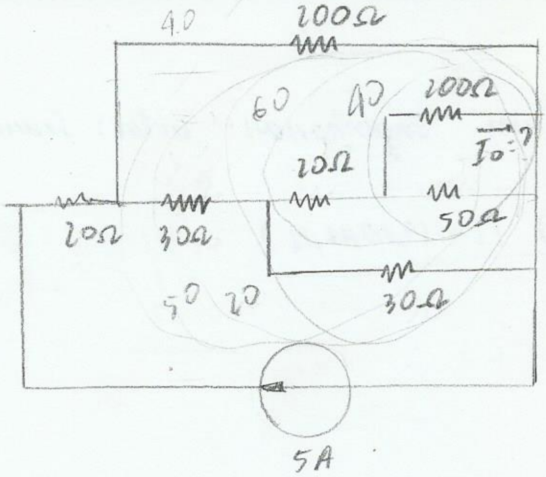
Divisor de corrente



$I_1 R_1 = I_2 R_2$

$\frac{I_1}{R_2} = \frac{I_2}{R_1} = \frac{I_1+I_2}{R_1+R_2} = \frac{I}{R_1+R_2}$

$$\boxed{I_2 = \frac{R_1}{R_1+R_2} \cdot I_2}$$



$$\frac{200}{200+50} \cdot 5 = 4$$

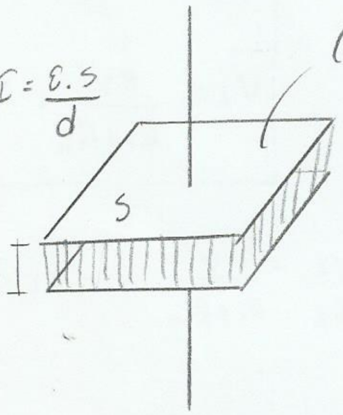
$$\frac{30}{30+60} \cdot 4 = \frac{120}{90} = \frac{4}{3}$$

$$\frac{50}{200+50} \times \frac{4}{3} = \frac{200}{750} = \boxed{\frac{4}{15}\,A}$$

Capítulo 6 - Capacitor ou Condensador

- apresenta o fenômeno de capacitância
- capacitância é a propriedade de armazenar cargas elétricas

$$C = \frac{\varepsilon . S}{d}$$



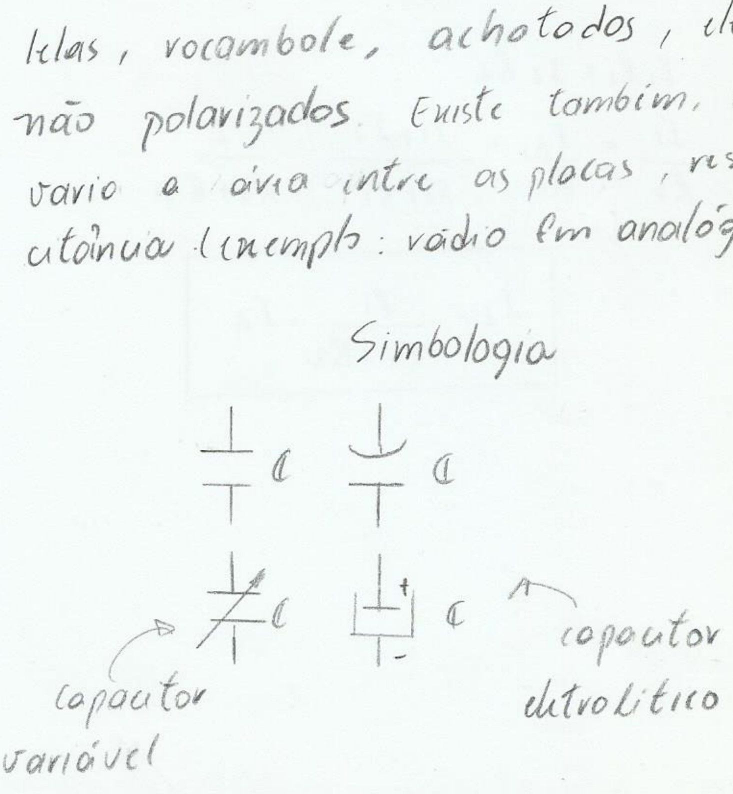
Capacitor: é formado por duas placas paralelas

$$\boxed{C = \frac{Q}{V}}$$

$[C] = F$ (Farad) $\begin{cases} mF \\ \mu F \\ nF \\ pF \end{cases}$

× Existe vários tipos de capacitores, como placas paralelas, rocambole, achatados, eletrolíticos polarizados e não polarizados. Existe também capacitores variados, no qual vario a área entre as placas, resultando na variação da capacitância (exemplo: rádio em analógico).

Simbologia



$$C = \frac{dq}{dv} \quad ; \quad i = \frac{dq}{dt} = \frac{C.dv}{dt} \quad \therefore \quad \boxed{u = C.\frac{dv}{dt}}$$

$$dv = \frac{i\,dt}{C} \Rightarrow v = \int \frac{i\,dt}{C} = \frac{1}{C}\int i\,dt + k \quad \therefore \quad \boxed{v = \frac{1}{C}\int i\,dt + v_0}$$

Energia armazenada no capacitor (w)

$$W = \int p.dt = \int v.i.dt$$

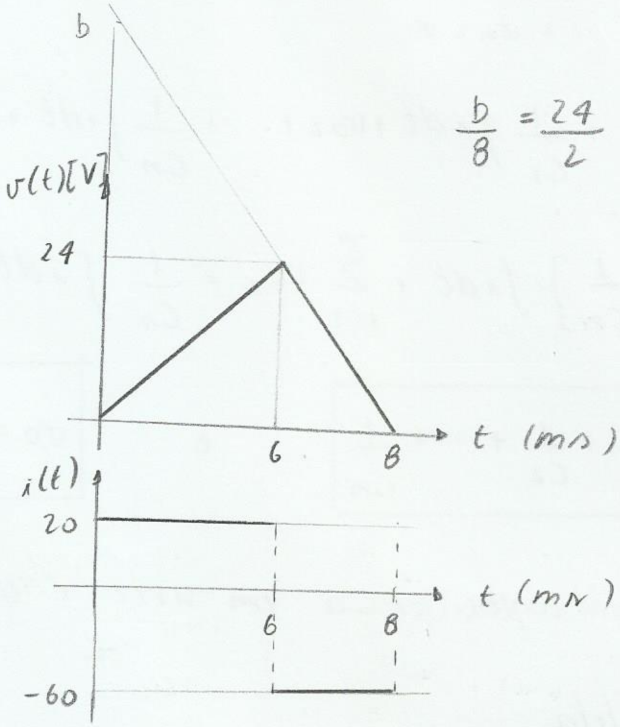
$$= \int v.\,C.\frac{dv}{\cancel{dt}}.\cancel{dt}$$

$$= C\int v\,dv \quad \therefore \quad \boxed{W = \frac{C.v^2}{2}}$$

Exemplo:

$C = 5\,\mu F$

$$\frac{b}{8} = \frac{24}{2} \quad \therefore \quad b = 96$$



Para $0 < t < 6 \Rightarrow v(t) = \frac{24}{6.10^{-3}}.t \quad \therefore \quad v(t) = 4.10^3 t$

$$i(t) = 5.10^{-6}.4.10^3 = 20.10^{-3} \quad \therefore \quad i(t) = 20\,mA$$

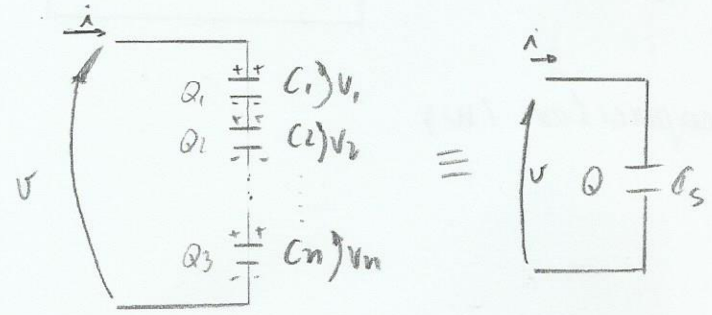
Para $6 < t < 8 \Rightarrow v(t) = -12.10^3 t + 96$

$$i(t) = 5.10^{-6}.(-12.10^3) \quad \therefore \quad i(t) = -60.\,mA$$

Para $t > 8 \Rightarrow v(t) = 0 \Rightarrow i(t) = 0$

Associação de capacitores

Associação série



Através da Lei de Kichorf dos tensões (LKT)

$$v_1 + v_2 + \dots + v_n = v$$

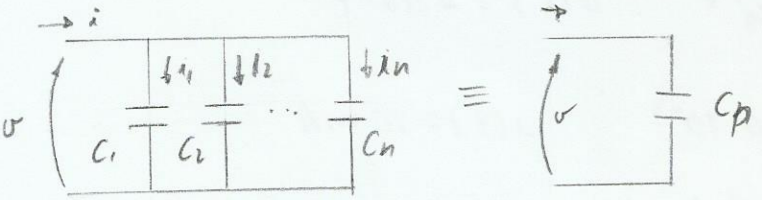
$$\frac{1}{C_1}\int i\,dt + v_{01} + \frac{1}{C_2}\int i\,dt + v_{02} + \dots + \frac{1}{C_n}\int i\,dt + v_{0n} = \frac{1}{C_s}\int i\,dt + v_0$$

$$\left[\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}\right]\cdot \int i\,dt + \sum_{i=1}^{n} v_{0i} = \frac{1}{C_s}\int i\,dt + v_0$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}} \quad \text{e} \quad \boxed{v_0 = \sum_{i=1}^{n} v_{0i}}$$

* Objetivo de capacitancia em série é diminuir ela

Associação paralela



Através de LKC (Lei de Kirchorf das correntes)

$$i_1 + i_2 + \dots + i_n = i$$

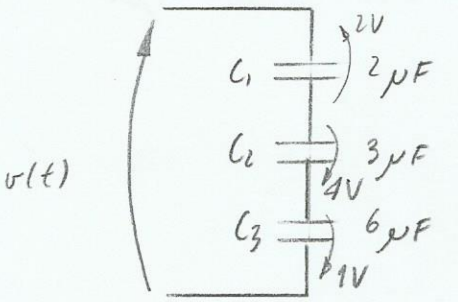
$$C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \dots + C_n\frac{dv}{dt} = C_p\cdot\frac{dv}{dt}$$

$$[C_1 + C_2 + \dots + C_n]\frac{dv}{dt} = C_p\cdot\frac{dv}{dt} \quad \therefore \quad \boxed{C_p = C_1 + C_2 + \dots + C_n}$$

① Determine

a) C_{eq}

b) W_{arm} na associação e no equivalente



a) $\frac{1}{C_{eq}} = \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{6}\right) \cdot \frac{1}{1 \cdot 10^{-6}}$

$\therefore \quad C_{eq} = 1 \cdot 10^{-6}\,F$ ou $1\,\mu F$

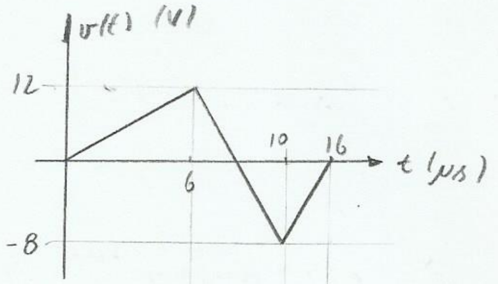
b) $W_1 = \frac{2 \cdot 10^{-6} \cdot 2^2}{2} = 4 \cdot 10^{-6}\,J$

$W_2 = \frac{3 \cdot 10^{-6} \cdot 4^2}{2} = 24 \cdot 10^{-6}\,J$ $\qquad W_3 = \frac{6 \cdot 10^{-6} \cdot 1^2}{2} = 3 \cdot 10^{-6}\,J$ $\qquad W_{arm\ ass.} = 31 \cdot 10^{-6}\,J$

$W_{eq} = \frac{1 \cdot 10^{-6} \cdot (-2+4+1)^2}{2} = 4{,}5 \cdot 10^{-6}\,J$

② Um $C = 12\,\mu F$

$i = C \cdot \frac{dv}{dt}$



Para $0 < t < 6 \rightarrow v(t) = 2t \cdot 10^{-6}$

$i(t) = 12 \cdot 10^{-6} \cdot 2 \cdot 10^{+6} = 24$

Para $6 < t < 10 \rightarrow v(t) =$

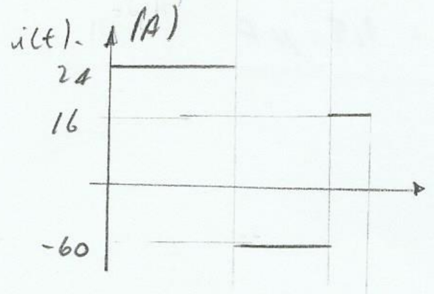
$v(t) = ax + b$

$\begin{cases} 12 = 6a + b \\ -8 = 10a + b \end{cases} \sim \begin{cases} 12 = 6a + b \\ 8 = -10a - b \end{cases}$

$20 = -4a \therefore a = -5$

$b = 42$

$\therefore v(t) = -5x + 18$



$i(t) = 12 \cdot 10^{-6} \cdot (-5) \cdot 10^{+6} = -60$

Para $10 < t < 16$

$\begin{cases} -8 = 10a + b \\ 0 = 16a + b \end{cases}$

$\begin{cases} 8 = -10a - b \\ 0 = 16a + b \end{cases}$

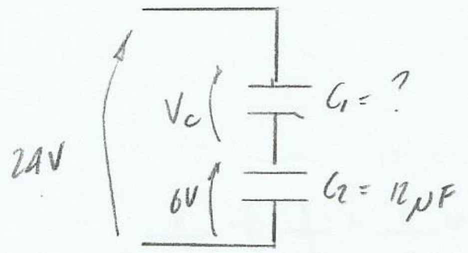
$6a = 8$

$a = \frac{8}{6} = \frac{4}{3}$

$i(t) = 12 \cdot 10^{-6} \cdot \frac{4}{3} \cdot 10^{+6}$

$i(t) = 16$

3)

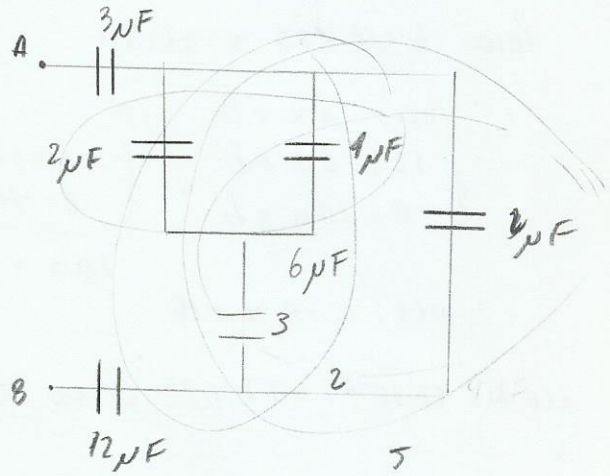


$$24 = V_c + 6 \quad \therefore \; V_c = 18V$$

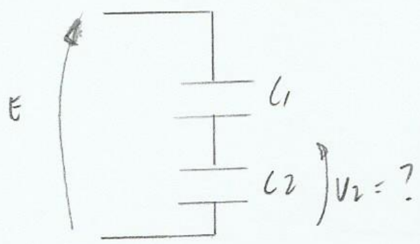
$$C = \frac{q}{V} \qquad Q_1 = Q_2 = 12.10^{-6} . 6 = 72.10^{-6}$$

$$C_1 = \frac{72.10^{-6}}{18} = 4.10^{-6} \quad \therefore \; C_1 = 4\mu F$$

4) $Ceq_{AB} = ?$



$$\boxed{Ceq = 1{,}5 \; \mu F}$$



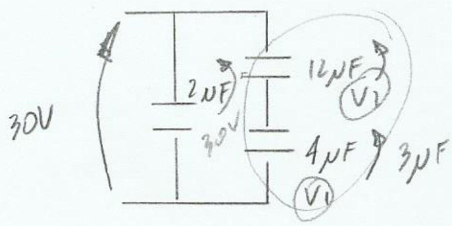
$$C_1V_1 = C_2V_2 = C_{eq}.E = Q$$

$$C_{eq} = \frac{C_1 . C_2}{C_1+C_2}$$

$$Q = \frac{C_1 . C_2}{C_1+C_2} . E = \cancel{C_2}V_2$$

$$\boxed{V_2 = \frac{C_1}{C_1+C_2} . E}$$

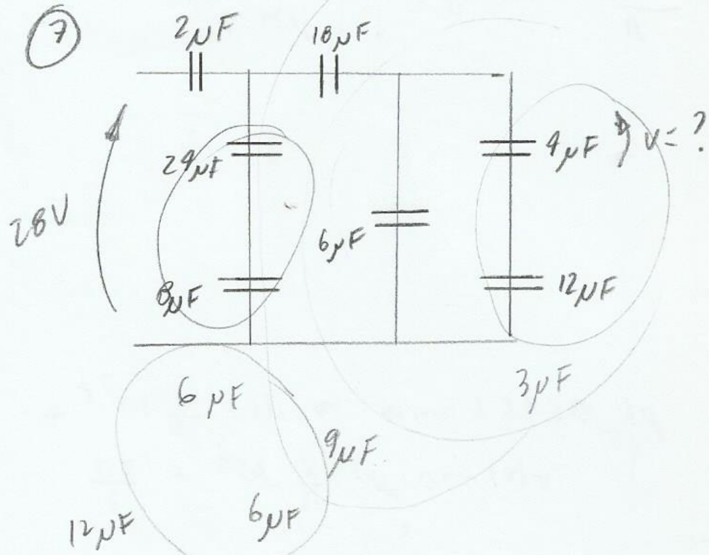
6)



$V_1 = \frac{12}{12+4} . 30 = \underline{22,5\,V}$ $\therefore V_2 = \underline{7,5\,V}$

$$V_2 = \frac{4}{12+4} . 30 = \frac{120}{16} = \frac{60}{8} = \frac{30}{4} = 7,5\,V$$

7

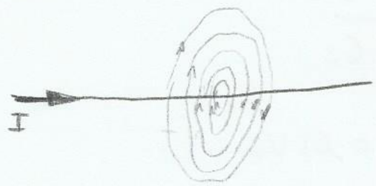


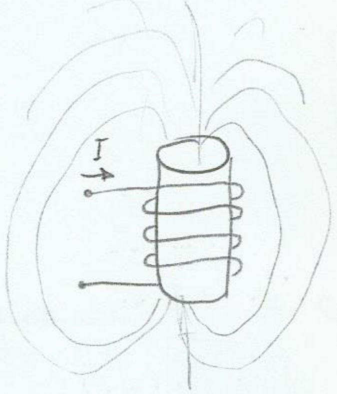
$$V_1 = \frac{2}{12+2} . 28 = \frac{2.\cancel{28}^{2}}{\cancel{14}} = 4$$

$$V_2 = \frac{18}{18+9} . 4 = \frac{\cancel{18}.4}{\cancel{27}} = \frac{\cancel{6}^{2}.4}{\cancel{9}\,3} = \frac{8}{3}$$

$$V = \frac{\cancel{12}^{1}}{(12+4)_{\cancel{16}\;2}} . \frac{\cancel{8}}{\cancel{3}} = \underline{2\,V}$$

Indutores ou Bobinas





L = autoindutância

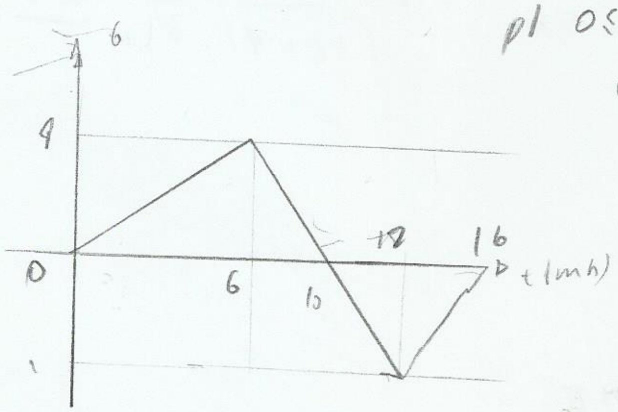
$$\boxed{v = L\frac{di}{dt}}$$

$$[L] = \frac{V.s}{A} = H \text{ (henry)} \quad \begin{cases} H \\ mH \\ \mu H \end{cases}$$

Lei de Faraday

$\Rightarrow \quad i = \cancel{L}\int v\,dt + i_0$

Exemplo: $L = 10\,mA$



p/ $0 \leq t \leq 6\,ms \Rightarrow i(t) = \frac{2}{3}10^3 t \Rightarrow$

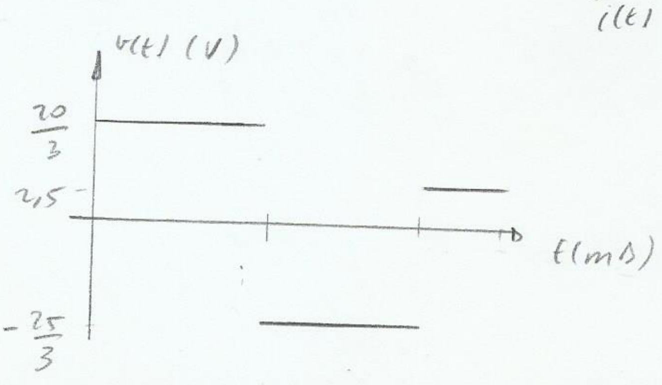
$v(t) = 10.\cancel{10^{-3}}.\frac{2}{3}.\cancel{10^3} = \frac{20}{3}$

p/ $6 \leq t \leq 15 \Rightarrow i(t) = -\frac{5}{6}.10^3 t + \cdots$

$v(t) = 10.\cancel{10^3}.\frac{5}{6}\cancel{10^3} = -\frac{25}{3}$

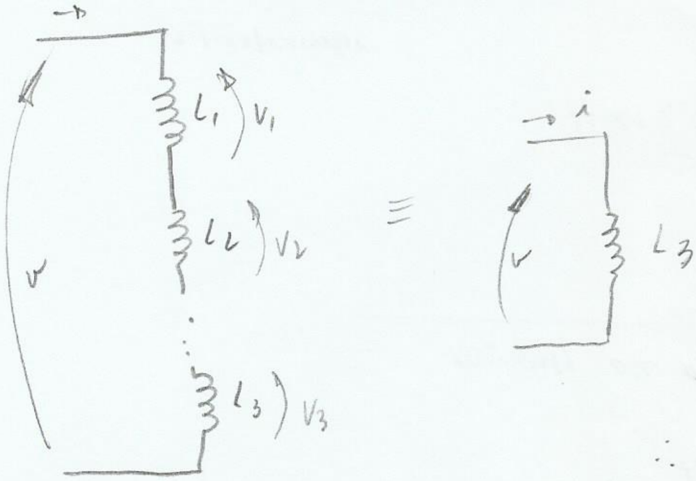
p/ $12 \leq t \leq 16\,ms$

$i(t) = \frac{1}{4}.10^3 t + \cdots \quad v(t) = 2{,}5\,V$



Associação de Indutores

Associação série



LKT

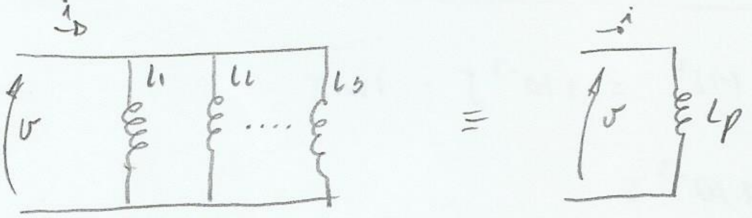
$$v_1+v_2+\cdots+v_n=v$$

$$L_1\frac{di}{dt}+L_2\frac{di}{dt}+\cdots L_n\frac{di}{dt}=L\frac{di}{dt}$$

$$\cancel{\frac{di}{dt}}\left(L_1+L_2+\cdots+L_n\right)=L\cancel{\frac{di}{dt}}$$

$$\therefore\ \boxed{L=L_1+L_2+\cdots+L_n}$$

Associação paralela



LKC

$$i_1+i_2+\cdots+i_n=i$$

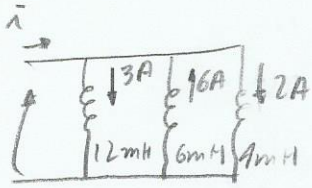
$$\frac{1}{L_1}\int v\,dt+i_{o_1}+\frac{1}{L_2}\int v\,dt+i_{o_2}+\cdots+\frac{1}{L_n}\int v\,dt+i_{o_n}=\frac{1}{L_p}\int v\,dt+i_o$$

$$\left[\frac{1}{L_1}+\frac{1}{L_2}+\cdots+\frac{1}{L_n}\right]\int v\,dt+\sum_{i=1}^{n}i_{o_i}=\frac{1}{L_p}\int v\,dt+i_o$$

$$\boxed{\frac{1}{L_1}+\frac{1}{L_2}+\cdots+\frac{1}{L_n}=\frac{1}{L_p}}\quad \text{ou}\quad \boxed{\sum_{i=1}^{n}i_{o_i}=i_o}$$

Exercícios

(1)



Pede-se:

a) $L_p = ?$

b) W_{arm} na associação e no equivalente

a) $L_p = 12 \parallel 6 \parallel 4 \therefore L_p = 2\,mH$

b) $i = 3 - 6 + 2 = -1$

Energia Armazenada no Indutor

$$W = \int p\,dt = \int v\,i\,dt$$
$$= \int L \frac{di}{dt} i\,dt = L \int i\,di$$
$$\boxed{W = \frac{1}{2} L I^2}$$

$W_{eq} = \frac{1}{2} \cdot 2 \cdot 10^{-3} \cdot (-1)^2 = 1 \cdot 10^{-3}\,J = 1\,mJ$

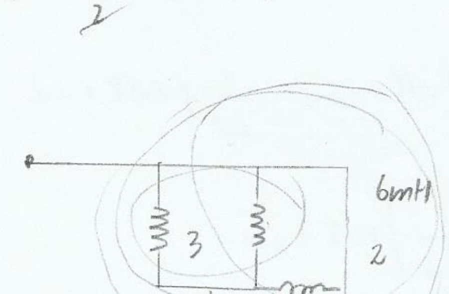
$W_1 = \frac{1}{2} \cdot 12 \cdot 10^{-3} \cdot 3^2 = 54 \cdot 10^{-3}\,J$

$W_2 = \frac{1}{2} \cdot 6 \cdot 10^{-3} \cdot 6^2 = 108 \cdot 10^{-3}\,J$ $\quad W_{asso} = 170 \cdot 10^{-3}\,J$

$W_3 = \frac{1}{2} \cdot 4 \cdot 10^{-3} \cdot 2^2 = 8 \cdot 10^{-3}\,J$

(2)

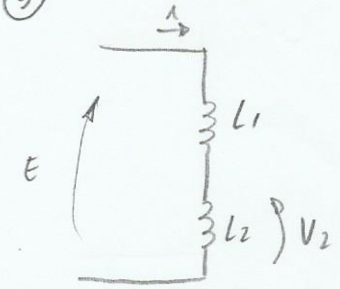
$L_{eq_{AB}}$



$\therefore L_{eq_{AB}} = \cancel{9H}$

$[(6 \parallel 6 \parallel 6) + 6] \parallel 6 + 6$

③

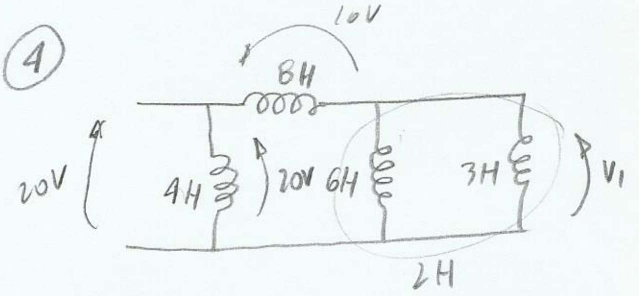


$$V = L_2 \cdot \frac{di}{dt} \qquad E = L_{eq} \cdot \frac{di}{dt}$$

$$\frac{V}{E} = L_2 \frac{di}{dt} \div L_{eq} \cdot \frac{di}{dt} \quad \therefore \quad \frac{V}{E} = \frac{L_2}{L_{eq}}$$

$$\therefore \quad V = \frac{L_2}{L_1 + L_2} \cdot E$$

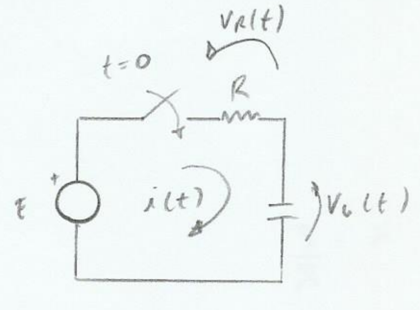
④



$$V_1 = \frac{3}{2+8} \cdot 20 = 6V$$

Capítulo 7 - Circuito RC e RL

Circuito RC em corrente contínua



obs.: Capacitor descarregado em t<0

LKT

$$v_R(t) + v_C(t) = E$$

$$\frac{d}{dt}\left(R \cdot i(t) + \frac{1}{C} \int i(t)dt \right) = \frac{d}{dt}(E)$$

$$R \cdot \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0$$

$$\therefore \quad \boxed{\frac{di(t)}{dt} + \frac{1}{RC} \cdot i(t) = 0}$$

a solução

$$\boxed{i(t) = K \cdot e^{-\frac{t}{T_c}}}$$

$$\therefore \frac{di(t)}{dt} = -K \cdot \frac{1}{T_c} \cdot e^{-\frac{t}{T_c}}$$

Substituindo

$$-K \cdot \frac{1}{T_c} \cdot e^{-\frac{t}{T_c}} + \frac{K}{RC} \cdot e^{-\frac{t}{T_c}} = 0$$

$$K \cdot e^{-\frac{t}{T_c}} \underbrace{\left(-\frac{1}{T_c} + \frac{1}{RC} \right)}_{=0} = 0$$

$$\therefore \boxed{T_c = R \times C}$$ cte de tempo

$$[T_c] = [R] \times [C]$$

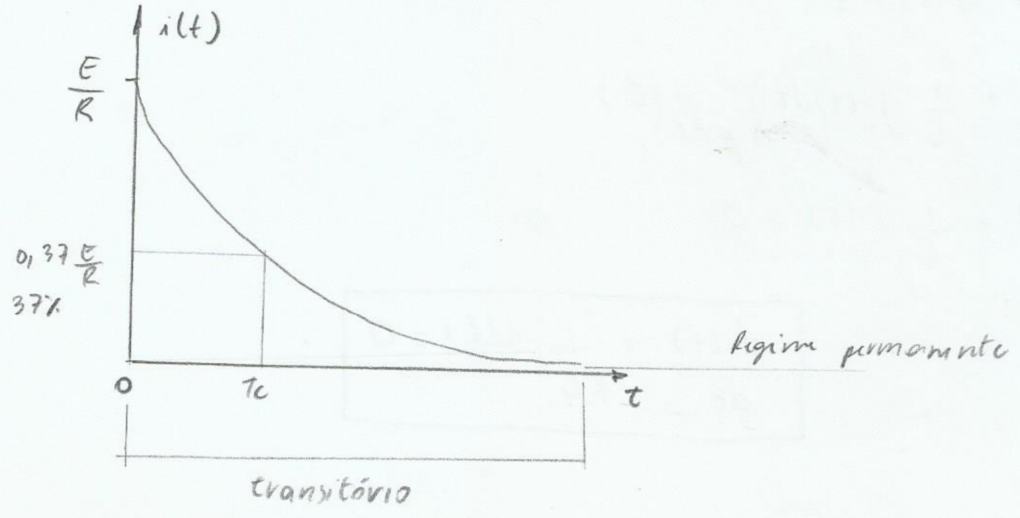
$$= \frac{\cancel{V}}{\cancel{A}} \cdot \frac{\cancel{A} s}{\cancel{V}} \therefore [T_c] = s$$

$$\therefore i(t) = K \cdot e^{-\frac{t}{T_c}}$$

$$i(t) = K \cdot e^{-\frac{t}{RC}}$$

$$p/\ t = 0 \quad i(0) = K = \frac{E}{R}$$

$$\therefore i(t) = \frac{E}{R} \cdot e^{-\frac{t}{RC}}$$



$$\therefore \; v_c(t) = \frac{1}{C} \int \frac{E}{R} e^{-\frac{t}{RC}} dt + K$$

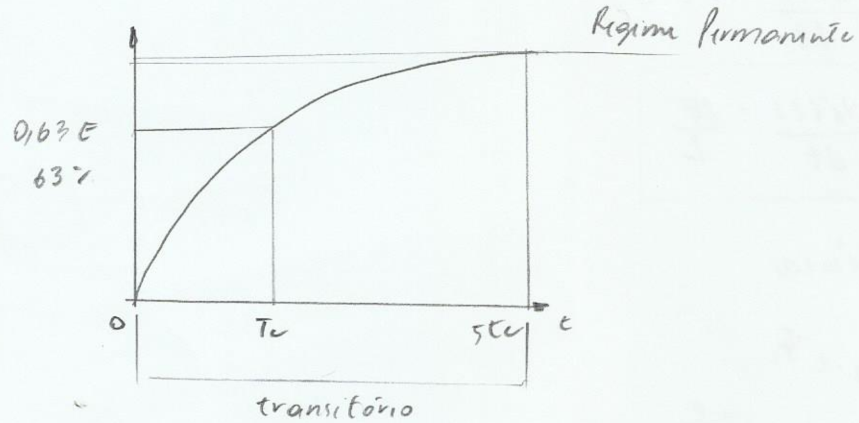
$$= \frac{1}{\cancel{C}} \cdot \frac{E}{\cancel{R}} \cdot \left(\frac{-1}{\frac{1}{\cancel{RC}}} \right) \cdot e^{\frac{-t}{RC}} + K$$

$$\therefore \; v_c(t) = -E \cdot e^{\frac{-t}{RC}} + K$$

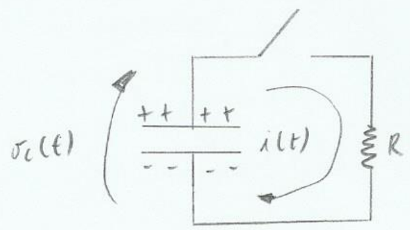
$$p/\; t = 0 \Rightarrow v_c(t) = -E + K = 0 \;\; \therefore \;\; E = K$$

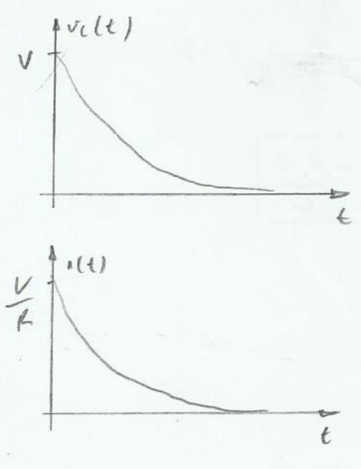
$$\therefore \; v_c(t) = E - E e^{\frac{-t}{RC}}$$

$$\therefore \; \boxed{v_c(t) = E\left(1 - e^{\frac{-t}{RC}}\right)}$$

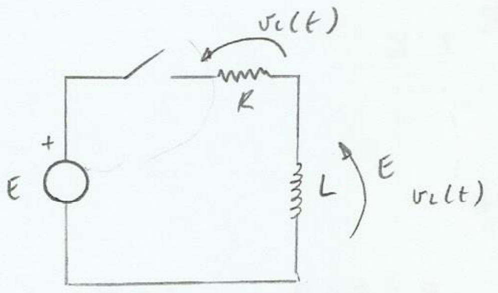


R.	C	Tc
100Ω	1μF	100μs
1KΩ	1μF	1ms
1MΩ	100mF	100Ks





Circuito RL em corrente contínua



Obs: Bobina sem campo inicial

LKT

$$v_R(t) + v_L(t) = E$$

$$R.\,i(t) + L\,\frac{di(t)}{dt} = E$$

$$\boxed{\frac{R.\,i(t)}{L} + \frac{di(t)}{dt} = \frac{E}{L}}$$

homogénea

a solução $\quad i(t) = K_2 . e^{-\frac{t}{T_C}}$

$$\frac{di(t)}{dt} = -K_2 . \frac{1}{T_C} . e^{-\frac{t}{T_C}}$$

$$\frac{R.\,K_2 . e^{-\frac{t}{T_C}}}{L} - \frac{K_2}{T_C} . e^{-\frac{t}{T_C}} = 0$$

$$\cancel{K_2 . e^{-\frac{t}{T_C}}}\underbrace{\left(-\frac{1}{T_C} + \frac{R}{L}\right)}_{=0} = 0 \qquad \therefore \boxed{T_C = \frac{L}{R}} \text{ cte de tempo}$$

$$[T_C] = \frac{[L]}{[R]} = \frac{\frac{\cancel{V}.s}{\cancel{A}}}{\frac{\cancel{V}}{\cancel{A}}} = s$$

$$\boxed{i_h(t) = K_2 . e^{-\frac{R.t}{L}}}$$

Particular

$i_p(t) = K_1$

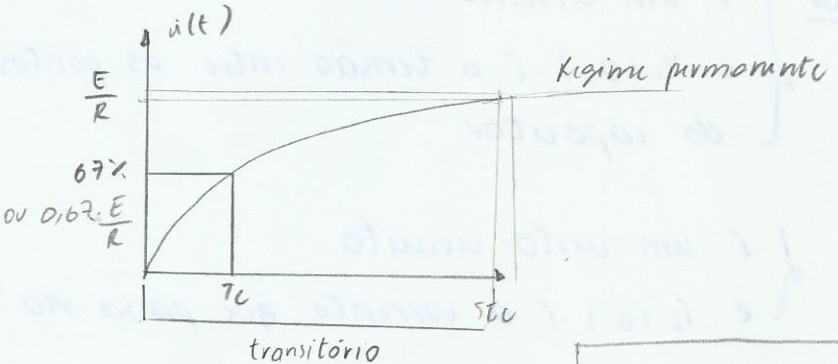
$$\frac{R \cdot K_1}{\cancel{L}} = \frac{E}{\cancel{L}} \quad \therefore \quad \boxed{K_1 = \frac{E}{R}}$$

$$\therefore \quad i(t) = \frac{E}{R} + K_2 \cdot e^{-\frac{R}{L} \cdot t}$$

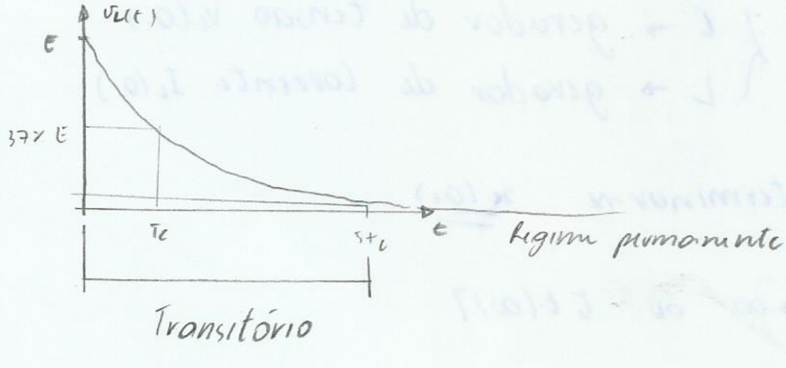
$p/\ t = 0 \rightarrow i(0) = 0$

$$0 = \frac{E}{R} + K_2 \quad \therefore \quad \boxed{K_2 = -\frac{E}{R}}$$

$$\therefore \quad i(t) = \frac{E}{R} - \frac{E}{R} \cdot e^{-\frac{R \cdot t}{L}} \quad \text{ou} \quad \boxed{i(t) = \frac{E}{R} \cdot \left(1 - e^{-\frac{R}{L} \cdot t}\right)}$$



$$v_L(t) = \cancel{L}\left(\frac{E}{\cancel{R}} \cdot \frac{\cancel{R}}{\cancel{L}} \cdot e^{-\frac{Rt}{L}}\right) \quad \therefore \quad \boxed{v_L(t) = E \cdot e^{-\frac{R \cdot t}{L}}}$$



Método dos 6 passos (circuitos com chave D.C)

- RL ou RC

um elemento armazenador de energia

A solução geral das soluções de 1ª ordem

$$\boxed{x(t) = k_1 + k_2 \cdot e^{-\frac{t}{\tau_c}}}$$

onde $x \begin{cases} i(t) \\ v(t) \end{cases}$

$\boxed{\text{1º Passo}}$ Adotar a solução geral

$\boxed{\text{2º Passo}}$ p/ $t<0$ determinar $x(0)$

- Capacitor $\begin{cases} \text{é um circuito aberto} \\ \text{e } V_c(0_-) \text{ é a tensão entre os pontos} \\ \text{do capacitor} \end{cases}$

- Bobina $\begin{cases} \text{é um curto-circuito} \\ \text{e } I_L(0_-) \text{ é a corrente que passa no "curto".} \end{cases}$

$\boxed{\text{3º Passo}}$ p/ $t>0$ ou $[t(0_+)]$

- substituir $\begin{cases} C \to \text{gerador de tensão } V_c(0_-) \\ L \to \text{gerador de corrente } I_L(0_-) \end{cases}$

e determinar $x(0_+)$

$\boxed{\text{4º passo}}$ p/ $t \to \infty$ ou $[t(\infty)]$

- substituir $\begin{cases} C \to \text{circuito aberto} \\ L \to \text{curto circuito} \end{cases}$

e determinar $x(\infty)$

5º passo Determinar T_c

- determinar a R_{eq} do circuito "vista" pelo elemento armazenador de energia

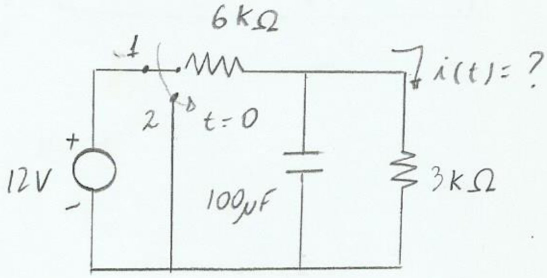
$$\text{Se for } \begin{cases} C \rightarrow T_c = R_{eq} \times C \\ L \rightarrow T_c = \dfrac{L}{R_{eq}} \end{cases}$$

6º passo Determinar K_1 e K_2

$$\begin{matrix} \text{em } t = 0_+ \rightarrow \\ \text{em } t = \infty \rightarrow \end{matrix} \begin{cases} x(0) = k_1 + k_2 \\ x(\infty) = k_1 \end{cases}$$

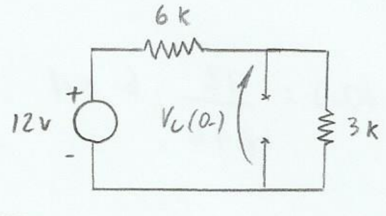
$$\therefore \boxed{x(t) = k_1 + k_2 \cdot e^{\frac{-t}{T_c}}}$$

Exemplo:



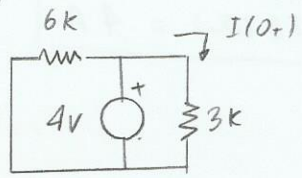
1º passo $i(t) = k_1 + k_2 \cdot e^{\frac{-t}{T_c}}$

2º passo p/ $t < 0$



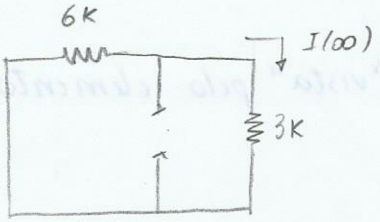
$\therefore \boxed{V_c(0) = 4V}$

3º passo



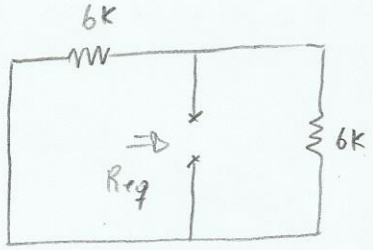
$\therefore \boxed{I(0_+) = \frac{4}{3} mA}$

4º Passo p/ $t \to \infty$



$\therefore \boxed{I(\infty) = 0}$

5º Passo



$\therefore R_{eq} = 6K // 3K = 2K\Omega$.

$\therefore T_C = R_{eq} \times C = 2K \cdot 100\mu = 0{,}2\,s$

6º passo

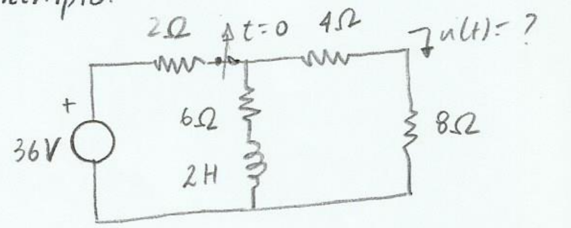
$I(0_+) = K_1 + K_2 = \frac{4}{3}\,mA$

$I(\infty) = K_1 = 0 \quad \therefore K_2 = \frac{4}{3}\,mA$

$\therefore i(t) = \frac{4}{3} \cdot e^{\frac{-t}{0,2}} \cdot (mA)$

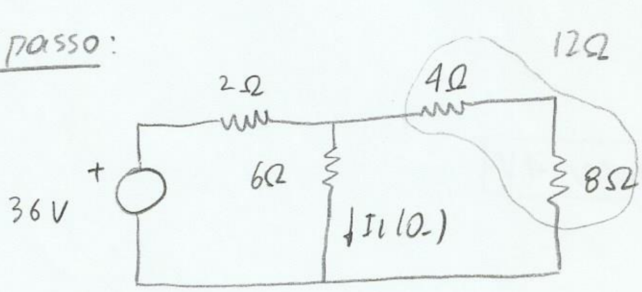
$\therefore \boxed{i(t) = \frac{4}{3} \cdot e^{-5t} \; (mA)}$

Exemplo:



1º passo: $i(t) = K_1 + K_2 \cdot e^{\frac{-t}{T_C}}$

2º passo:



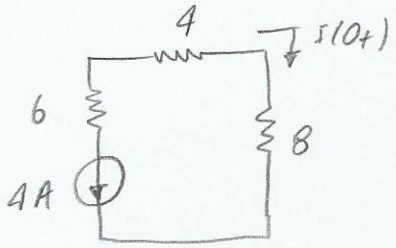
$\therefore IL(0-) = \frac{12}{12+6} \cdot 6 = 4$

$R_{eq} = 2 + 12 // 6 = 6$

$I_{eq} = \frac{36}{6} = 6$

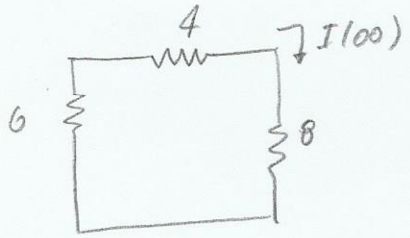
$\therefore \boxed{IL(0-) = 4\,A}$

3º passo $\quad p/\ t(0+)$



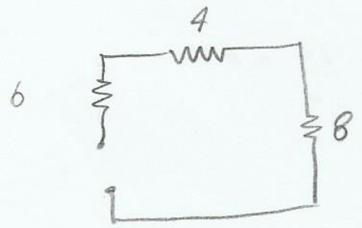
$\therefore \boxed{I(0+) = -4A}$

4º passo $\quad p/\ t \to \infty$



$\therefore \boxed{I(\infty) = 0}$

5º passo $\quad T_c = ?$



$R_{eq} = 18\,\Omega$

$$T_c = \frac{L}{R_{eq}} = \frac{2}{18} = \frac{1}{9}\ s$$

6º passo $\quad k_1$ e k_2

$I(0+) = k_1 + k_2 = -4 \quad \therefore\ k_2 = -4$

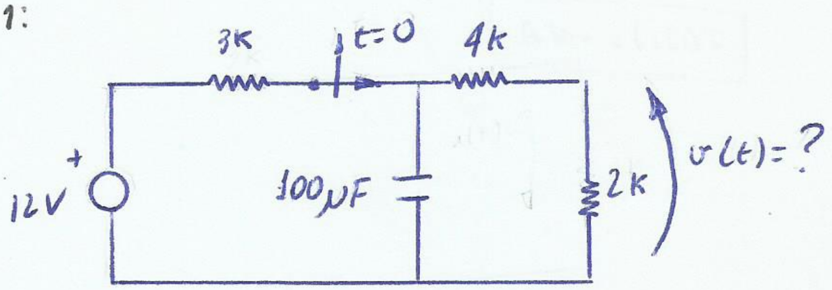
$I(\infty) = k_1 = 0$

$$\therefore\ i(t) = -4 \cdot e^{\frac{-t}{1/9}}\ (A)$$

$$\therefore\ \boxed{i(t) = -4 \cdot e^{-9t}}\ (A)$$

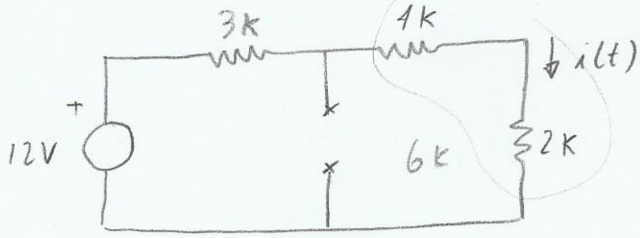
Exercícios

Ex 1:



1º passo: $i(t) = K_1 + K_2 . e^{\frac{-t}{T_C}}$

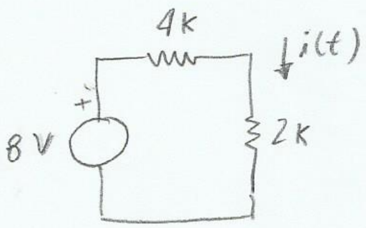
2º passo: $t<0$



$V(0-) = \frac{6}{6+3} . 12$

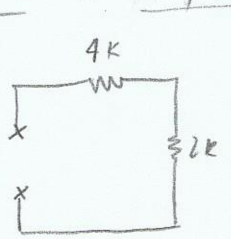
$\therefore V(0-) = \frac{24}{3} = 8V$

3º passo $t>0$



$i(0+) = \frac{24}{3} \div 6 = \frac{4}{3} A$

4º passo e 5º passo



$I(\infty) = 0$

$R_{eq} = 6k\Omega$

$T_C = R_{eq} \times C = 6.k.100\mu \quad \therefore \quad T_C = 0{,}6s$

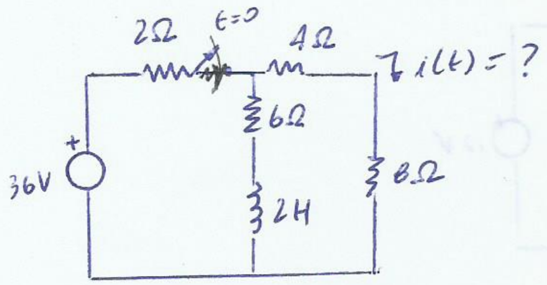
6º passo

$I(0_+) = K_1 + K_2 = 4/3$

$I(\infty) = K_1 = 0 \quad \therefore K_2 = 4/3$

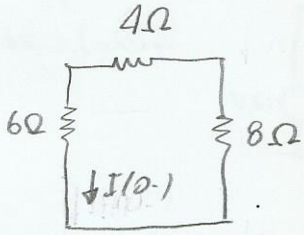
$i(t) = \frac{4}{3} . e^{\frac{-t}{0{,}6}} \quad \therefore \quad \boxed{v(t) = \frac{8}{3} . e^{\frac{-t}{0{,}6}} \ (V)}$

Ex 2:



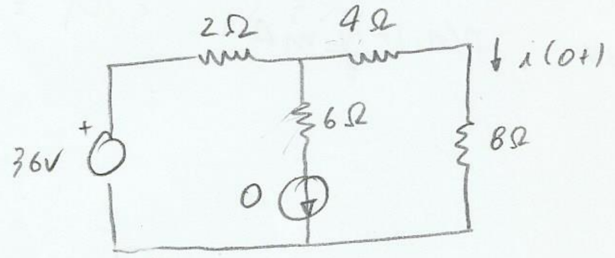
1º passo: $i(t) = K_1 + K_2 \cdot e^{\frac{-t}{\tau_c}}$

2º passo: p/ $t < 0$



$\therefore I(0-) = 0$

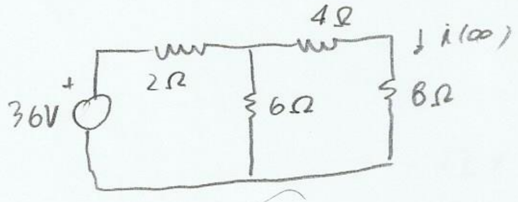
3º passo: p/ $t > 0$



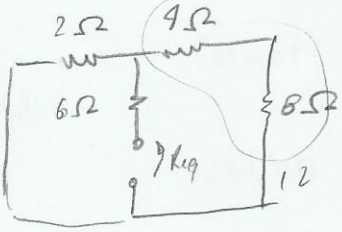
$i(0+) = \frac{36}{14} = \frac{18}{7}$ A

4º passo e 5º passo

p/ $t \to \infty$



$\therefore i(\infty) = 2$ A



$R_{eq} = 6 + 2 // 12 = \frac{54}{7}$

$\tau_c = 2 \div \frac{54}{7} = \frac{14}{54} = \frac{7}{27}$

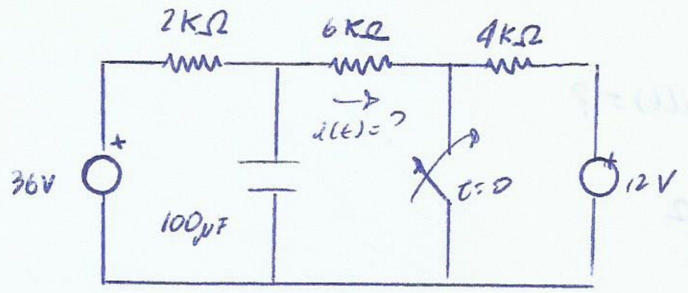
6º passo

$I(0+) = K_1 + K_2 = \frac{18}{7}$ $\quad \therefore K_2 = \frac{18}{7} - 2 = \frac{18 - 14}{7} = \frac{4}{7}$

$I(\infty) = K_1 = 2$

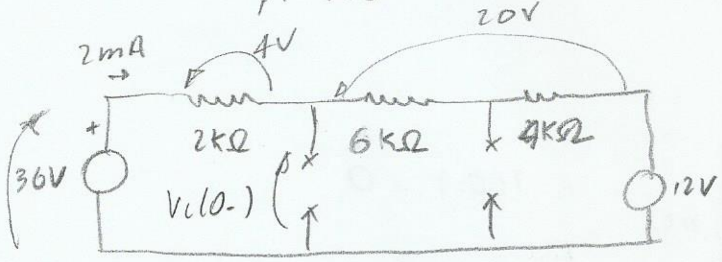
$\therefore i(t) = 2 + \frac{4}{7} \cdot e^{\frac{-54t}{14}}$

Ex.1



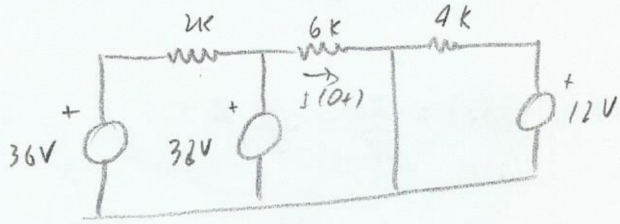
1º Passo $\quad i(t) = K_1 + K_2 . e^{-\frac{t}{T_C}}$

2º Passo $\quad$ p/ $t<0$



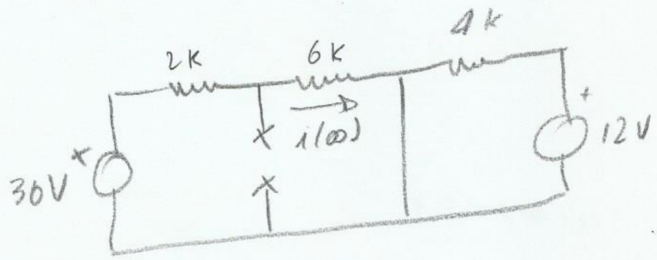
$V_c(0-) = 32V$

3º Passo $\quad$ p/ $t(0+)$



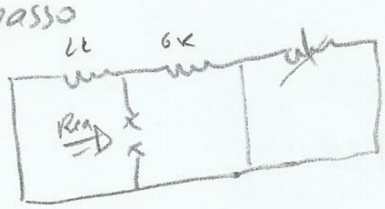
$I(0+) = \frac{16}{3}\, mA$

4º passo $\quad$ p/ $t \to \infty$



$\therefore I(\infty) = \frac{9}{2}\, mA$

5º passo



$\therefore R_{eq} = \frac{3}{2}\, k\Omega$

$T_C = \frac{3}{2} . 10^3 \times 100 . 10^{-6}$

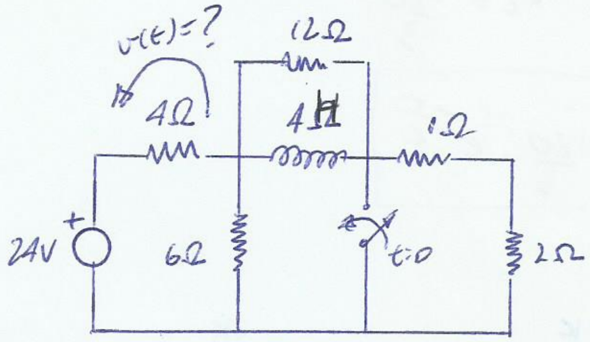
$\therefore T_C = 0{,}15 s$

6º passo $\quad I(0+) = K_1 + K_2 = \frac{16}{3}\, mA$

$I(\infty) = K_1 = \frac{9}{2}\, mA \quad \therefore K_2 = \frac{5}{6}\, mA$

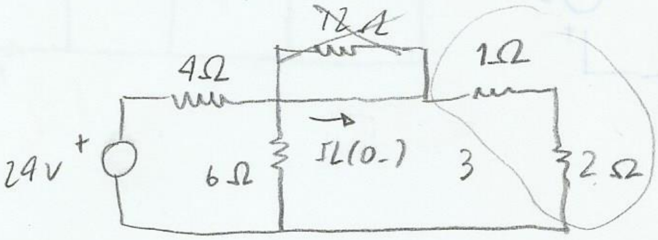
$$i(t) = \frac{9}{2} + \frac{5}{6} . e^{-\frac{t}{0{,}15}} \; (mA)$$

Ex:



1º passo: $v(t) = K_1 + K_2^{\frac{-t}{T_C}}$

2º passo: p/ $t<0$

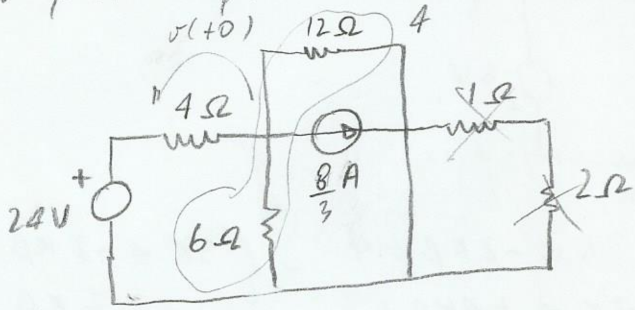


$R_{eq} = 4 + 6 \| 3 = 6$

$I_T = 4$

$I_L(0^-) = \frac{6}{3+6} \cdot 4 = \frac{8}{3}$

3º passo: p/ $t>0$

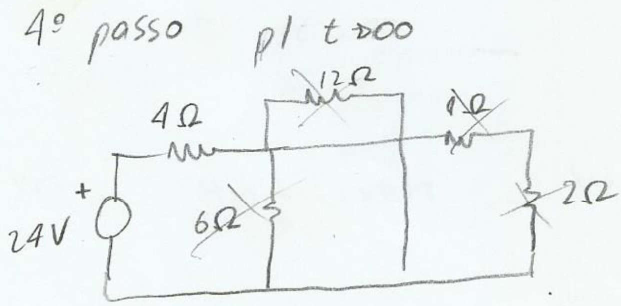


$V_1 = 12V$

$V_2 = \frac{4}{3} \cdot 4 = \frac{16}{3}$

$\therefore\ v(0^+) = 12 + \frac{16}{3} = \frac{52}{3}$

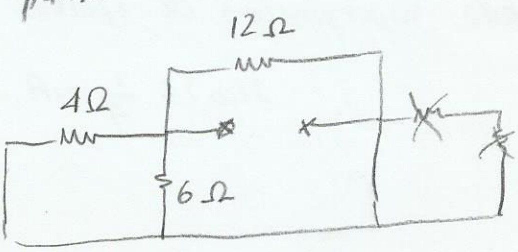
4º passo p/ $t \to \infty$



$v(\infty) = 24\ V$

5º passo

$R_{eq} = 4 \| 12 \| 6 \quad \therefore R_{eq} = 2$



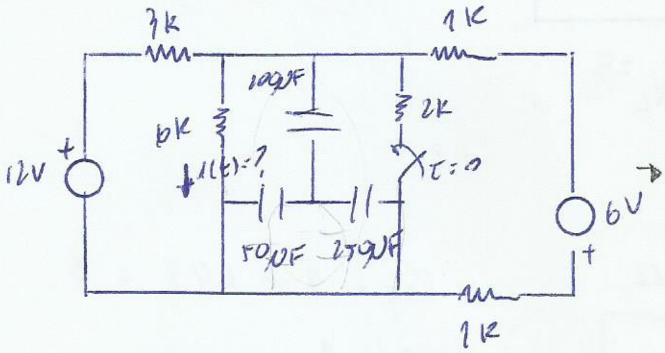
$T_C = \frac{4}{2} = 2$

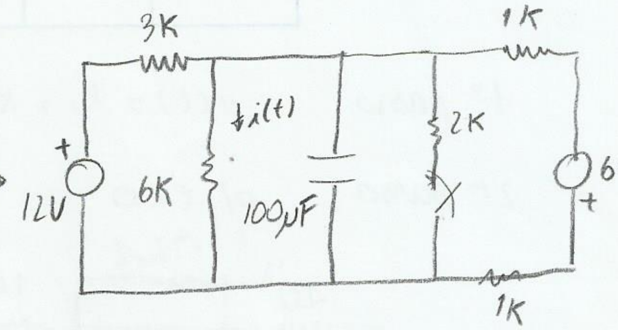
$v(0_+) = K_1 + K_2 = 52/3 \quad \therefore \quad K_2 = -\frac{20}{3}$

$v(\infty) = K_1 = 24$

$$\boxed{v(t) = 24 - \frac{20}{3} \cdot e^{-\frac{t}{2}}}$$

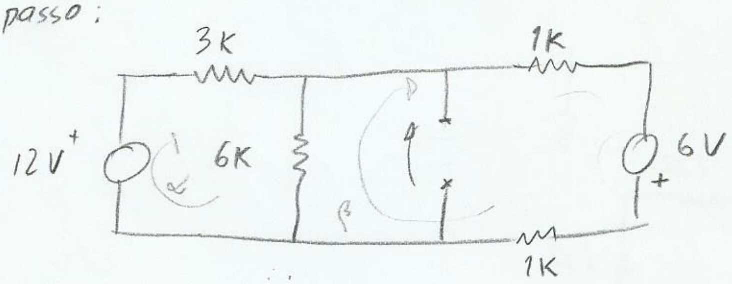
Ex:





1º passo: $i(t) = K_1 + K_2 e^{\frac{-t}{T_c}}$

2º passo:

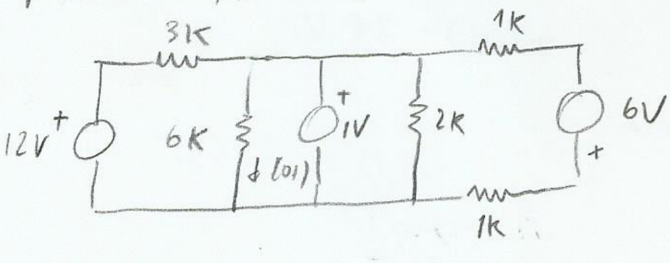


$$\begin{cases} 9K \cdot \alpha - 6K \cdot \beta = 12 \\ -6K\,\alpha + 8K\beta = 6 \end{cases} \sim \begin{cases} 3K\alpha - 2K\beta = 4 \\ -3K\alpha + 4K\beta = 3 \end{cases} \sim \begin{cases} 3K\alpha - 2K\beta = 4 \\ 2K\beta = 7 \end{cases}$$

$\therefore \beta = \frac{7}{2} mA \qquad \alpha = \frac{11}{3} mA$

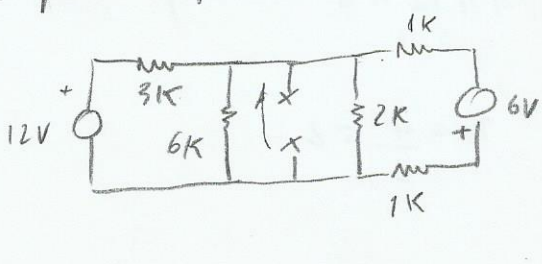
$I = \frac{11}{3} - \frac{7}{2} = \frac{1}{6} mA \quad \therefore \quad V_C(0_-) = 6K \cdot \frac{1}{6} m = 1V$

3º passo: p/ $t(0+)$



$\therefore I(0+) = \frac{1}{6} mA$

4º passo: p/ $t \to \infty$



Usando superposição de efeitos

$I(\infty) = \frac{1}{9} mA$

5° passo $\quad T_c = ?$

$$R_{eq} = \frac{2}{3}\ k\Omega$$

$$T_c = R_{eq} \times C = \frac{2}{3}\ k \times 100\mu = \frac{200}{3} \cdot 10^{-3} = \frac{0,2}{3}\ s$$

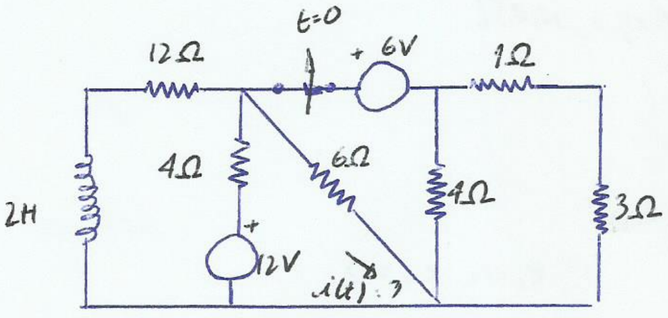
6° passo

$$I(0_+) = K_1 + K_2 = \frac{1}{6}\ mA$$

$$I(\infty) = K_1 = \frac{1}{9}\ mA \quad \therefore K_2 = \frac{1}{18}\ mA$$

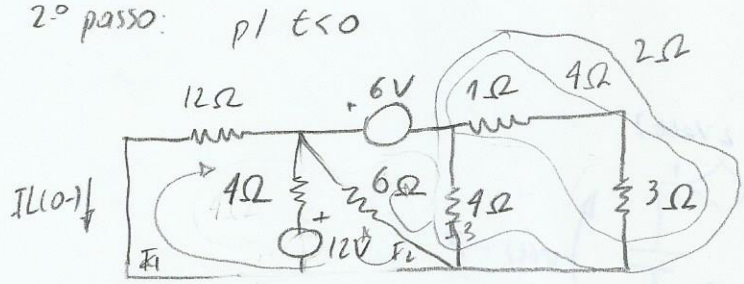
$$\therefore\ i(t) = \frac{7}{9} + \frac{1}{18} \cdot e^{-15t}\ (mA)$$

Ex:



1° passo: $\quad i(t) = K_1 + K_2 \cdot e^{\frac{-t}{T_c}}$

2° passo: $\quad p/\ t<0$



$$\begin{cases} 16 I_1 - 4 I_2 = -12 \\ 10 I_2 - 4 I_1 - 6 I_3 = 12 \\ 8 I_3 - 6 I_2 = -6 \end{cases} \sim \begin{cases} 4 I_1 - I_2 = -3 \\ 0 + 9 I_2 - 6 I_3 = 9 \\ -3 I_2 + 4 I_3 = -3 \end{cases} \sim \begin{cases} 4 I_1 - I_2 = -3 \\ 3 I_2 - 2 I_3 = 3 \\ 2 I_3 = 0 \end{cases}$$

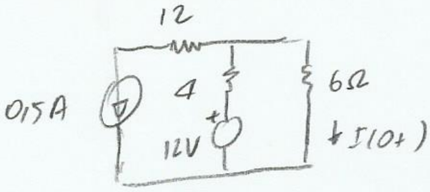
$$\therefore\ I_3 = 0 \quad I_2 = 1 \quad I_1 = \frac{1}{2} \quad \therefore\ I_L(0-) = \frac{1}{2}\ A$$

Por superposição

$$V_1 = 12 \times \frac{4/3}{4 + 4/3} = 3V \quad \therefore\ V = 3 + 3 = 6$$

$$V_2 = 3V \qquad \therefore\ I_L(0_-) = 0,5A$$

3º passo p/ $t(0+)$

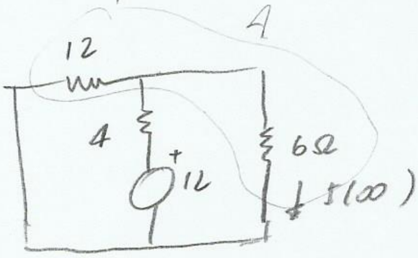


$$I_1 = \frac{12}{10} = 1,2\ A$$

$$I_2 = -\frac{4}{4+6} \therefore \frac{1}{2} = -\frac{1}{5}\ A = -0,2\ A$$

$I(0+) = 1,2 - 0,2 \therefore I(0+) = 1,0\ A$

4º passo p/ $t \to \infty$



$\therefore V = 6V$

$\therefore I(\infty) = 1A$

5º passo $R_{eq} = 12 + (6 \| 4) \therefore R_{eq} = 14,4\Omega$

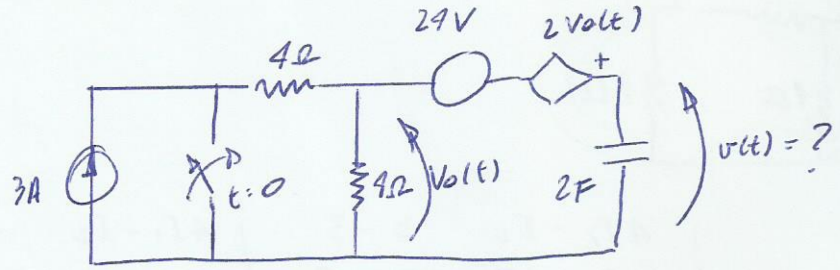
$$T_c = \frac{L}{R} = \frac{2H}{14,4\Omega} = \frac{1}{7,2}\ s$$

$\therefore$ $I(0+) = K_1 + K_2 = 1$ $\therefore K_1 = 1,\ K_2 = 0$

$I(\infty) = K_1 = 1$

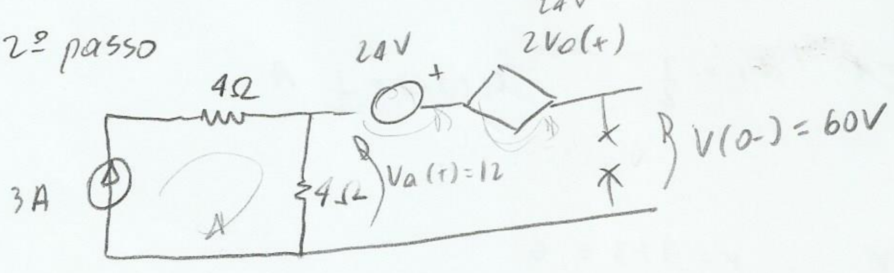
$\boxed{\therefore i(t) = 1A}$

Ex1

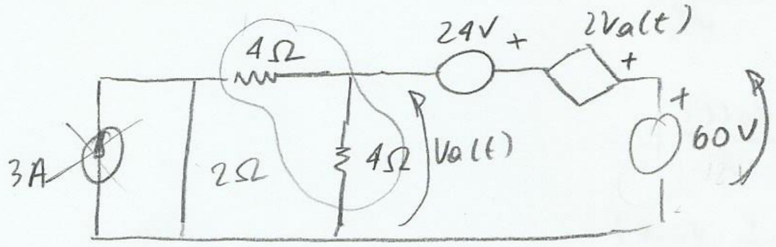


1º passo $v(t) = K_1 + K_2 e^{-\frac{t}{Tc}}$

2º passo

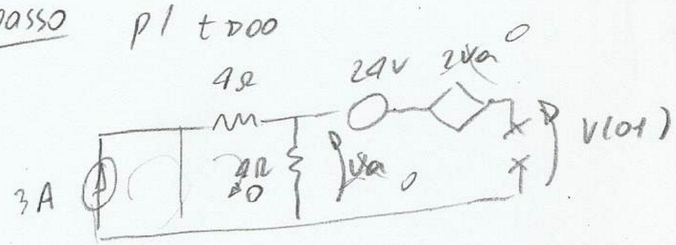


3º passo $\quad p/\ t>0$ ou $t(0+)$



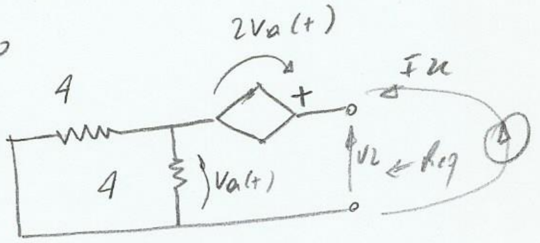
$V(0+) = 60V$

4º passo $\quad p/\ t \to \infty$



$\therefore \quad V(0+) = 24V$

5º passo



$Req = \dfrac{Vx}{Ix} \quad \therefore \quad Vx = 3Va(t) = 6Ix$

$Req = \dfrac{6Ix}{Ix} = 6\Omega$

$Va(t) = 2 . Ix$

$Tc = Req \times C = 6 \times 2 = 12s$

6º passo

$V(0) = K_1 + K_2 = 60$

$V(\infty) = K_1 = 24$

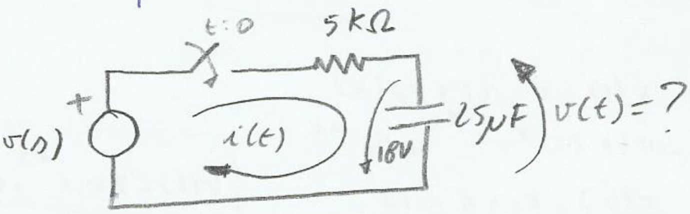
$K_1 = 24$

$K_2 = 36$

$$\boxed{v(t) = 24 + 36 . e^{\frac{-t}{12}} \ (V)}$$

Observações:

Se o gerador não for constante $\Rightarrow$ não vale o método dos seis passos



$v_s(t) = 6e^{-2t} \ (V)$

$$i(t) = \frac{v_s(t) - v(t)}{R} = C\frac{dv(t)}{dt}$$

$$C\frac{dv(t)}{dt} + \frac{1}{R}v(t) = \frac{v_s(t)}{R}$$

$$\frac{dv(t)}{dt} + \frac{1}{RC}v(t) = \frac{1}{RC}\cdot v_s(t)$$

$$\boxed{\frac{dv(t)}{dt} + 8v(t) = 48e^{-2t}}$$

Homogênea

$$\frac{dv(t)}{dt} + 8v(t) = 0$$

Solução: $v_H(t) = K_1 \cdot e^{\frac{-t}{\tau_C}}$

$$\frac{dv_H(t)}{dt} = -\frac{K_1}{\tau_C}\cdot e^{\frac{-t}{\tau_C}}$$

$$-\frac{K_1}{\tau_C}\cdot e^{\frac{-t}{\tau_C}} + 8 \cdot K_1 e^{\frac{-t}{\tau_C}} = 0$$

$$\left(-\frac{1}{\tau_C} + 8\right) K_1 e^{\frac{-t}{\tau_C}} = 0 \quad \therefore \boxed{\tau_C = \frac{1}{8}\, s}$$

$$\therefore \boxed{v_h(t) = K_1 \cdot e^{-8t}}$$

Particular

Solução $v_p(t) = K_2 \cdot e^{-2t}$

$$\frac{dv_p(t)}{dt} = -2K_2 \cdot e^{-2t}$$

Subst. $-2K_2 \cdot e^{-2t} + 8K_2 e^{-2t} = 48e^{-2t}$

$$6K_2 \cancel{e^{-2t}} = 48\cancel{e^{-2t}} \quad \therefore \boxed{K_2 = 8}$$

$$\boxed{v_p(t) = 8e^{-2t}}$$

$\therefore v(t) = v_h(t) + v_p(t)$

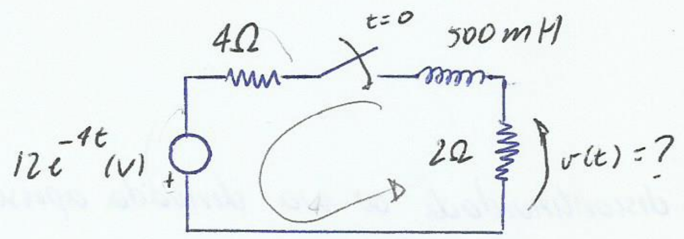
$\therefore v(t) = K_1 e^{-8t} + 8e^{-2t}$

$v(0) = K_1 + 8 = -18$

$\therefore K_1 = -26$

$$\therefore \boxed{v(t) = 26 \cdot e^{-8t} + 8e^{-2t}\ (V)}$$

Ex 7.55)



$v = L\dfrac{di}{dt}$

$di = \dfrac{1}{L} v\,dt$

$i = \dfrac{1}{L}\int v\,dt$

$$12e^{-4t} - 4i(t) - L\frac{di}{dt} - 2i(t) = 0$$

$$L\frac{di}{dt} + 6i(t) = 12e^{-4t}$$

$$\frac{di}{dt} + \frac{6}{0{,}5} i(t) = \frac{12e^{-4t}}{0{,}5} \Rightarrow \frac{di}{dt} + 12i(t) = 24e^{-4t}$$

homogênea: $\dfrac{di(t)}{dt} + 12i(t) = 0$

solução: $i_H(t) = K_1 \cdot e^{\frac{-t}{Tc}}$

$$\frac{di_H(t)}{dt} = \frac{-K_1}{Tc} \cdot e^{\frac{-t}{Tc}}$$

substituindo: $\dfrac{-K_1}{Tc} e^{\frac{-t}{Tc}} + 12K_1 e^{\frac{-t}{Tc}} = 0$

$$\left(\frac{-1}{Tc} + 12\right) e^{\frac{-t}{Tc}} = 0 \qquad \boxed{Tc = \frac{1}{12}\,s}$$

$$\therefore \boxed{i_h(t) = K_1 e^{-12t}}$$

Particular: $i_p(t) = K_2 e^{-4t}$

$$\frac{di_p(t)}{dt} = -4K_2 e^{-4t}$$

Substituindo: $-4K_2 e^{-4t} + 12K_2 e^{-4t} = 24e^{-4t}$

$$8K_2 \cancel{e^{-4t}} = 24\cancel{e^{-4t}} \quad \therefore \boxed{K_2 = 3}$$

$$i_p(t) = 3e^{-4t}$$

$i(t) = i_h(t) + i_p(t)$

$i(t) = K_1 e^{-12t} + 3e^{-4t}$

$i(0) = K_1 + 3 = 0 \quad \therefore K_1 = -3$

$i(t) = -3e^{-12t} + 3e^{-4t}$

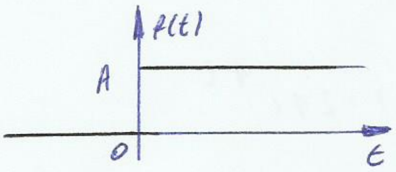
$$\boxed{\therefore v(t) = 6e^{-12t} - 6e^{-4t}}$$

Respostas ao pulso

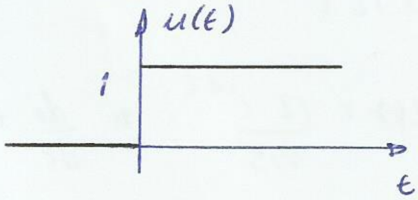
- Funções singulares -

são aquelas que apresentam discontinuidade ou sua derivada apresenta descontinuidade

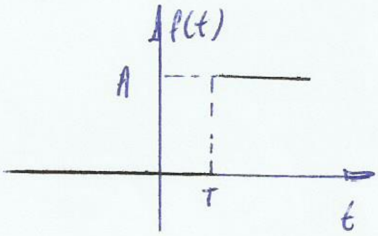
Função Degrau



Degrau unitário

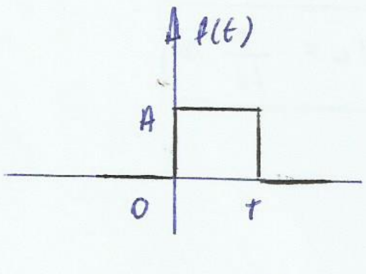


$$u(t) \begin{cases} 0 & p/\ t<0 \\ 1 & p/\ t>0 \end{cases}$$



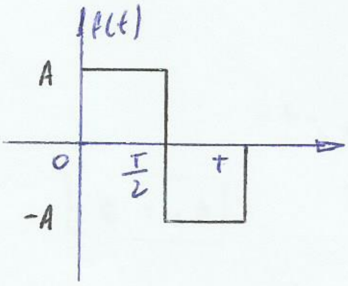
$$f(t) = Au(t-T) \begin{cases} 0 & p/\ t<T \\ A & p/\ t>T \end{cases}$$

Função pulso



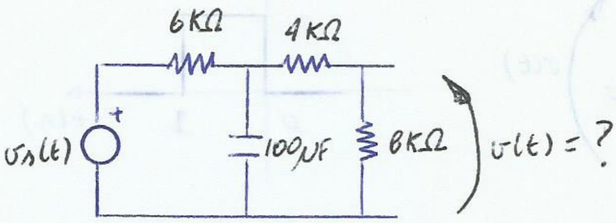
$$f(t) \begin{cases} 0 & p/\ t<0 \\ A & p/\ 0<t<T \\ 0 & p/\ t>T \end{cases}$$

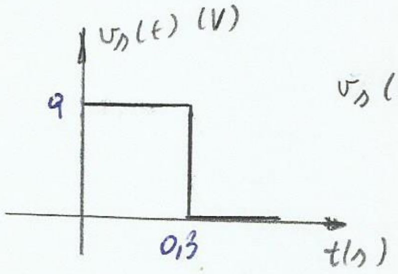
$$f(t) = Au(t) - Au(t-T)$$



$$f(t) = Au(t) - 2Au\left(t-\frac{T}{2}\right) + Au(t-T)$$

Exemplo:





$v_s(t) = 9u(t) - 9u(T - 0{,}3)$

p/ $t < 0 \rightarrow v(t) = 0$

p/ $0 < t < 0{,}3s$

1º passo: $v(t) = K_1 + K_2 e^{-t/T_C}$

2º passo: $V_C(0_-) = 0$

3º passo: $V(0+) = 0$

4º passo: $V(\infty) = \frac{8}{18} 9 = 4V$

5º passo: $T_C = 4K \times 100\mu = 0{,}4s$

$R_{eq} = 4K$

6º passo: $\left.\begin{array}{l} V(0+) = K_1 + K_2 = 0 \\ V(\infty) = K_1 = 4 \end{array}\right\} \begin{array}{l} K_1 = 4 \\ K_2 = -4 \end{array}$

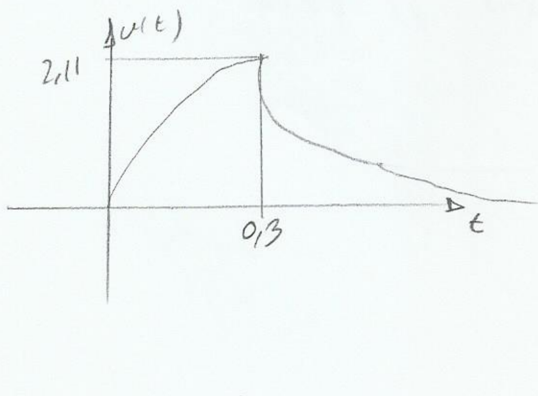
$v(t) = 4(1 - e^{-2{,}5t})$

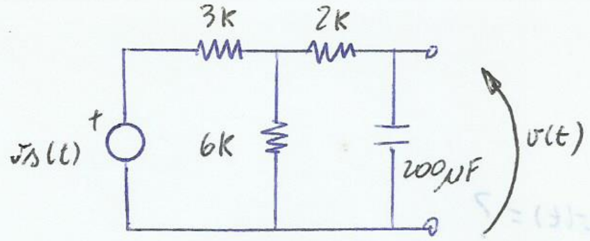
$v(0{,}3) \simeq 2{,}11 V$

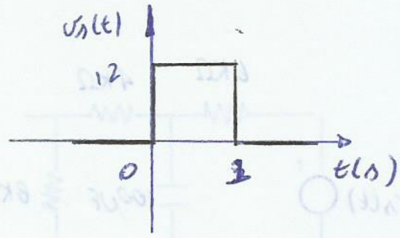
p/ $t > 0{,}3$

$v(t) = 2{,}11 e^{-2{,}5(t-0{,}3)}$

$\therefore v(t) \begin{cases} 0 & p/\ t < 0 \\ 4(1 - e^{-2{,}5t}) & p/\ 0 < t < 0{,}3s \\ 2{,}11 e^{-2{,}5(t-0{,}3)} & p/\ t > 0{,}3s \end{cases}$







p/ $t<0 \quad v(t)=0$

p/ $0<t<3$

1º passo $\quad v(t) = K_1 + K_2 e^{-\frac{t}{T_c}}$

2º passo $\quad V_c(0-) = 0$

3º passo $\quad V_c(0+) = 0$

4º passo $\quad v(\infty) = \dfrac{6 \cdot 12}{9} = 8\,V$

5º passo $\quad T_c = 4K \times 200\mu = 0{,}8s$

$R_{eq} = 4\,K\Omega$

6º passo

$$\left\{ \begin{array}{l} V(0+) = K_1 + K_2 = 0 \\ V(\infty) = K_1 = 8 \end{array} \right. \quad K_1 = 8 \ \text{e} \ K_2 = -8$$

$$\boxed{v(t) = 8(1 - e^{-1{,}25t})}$$

p/ $t > 1s$

$v(1s) \cong 5{,}71$

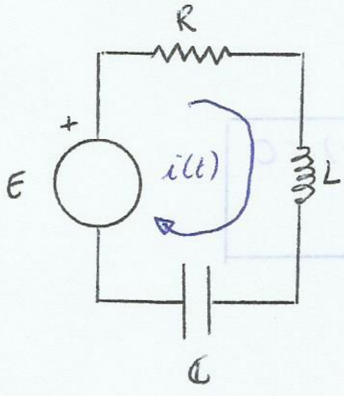
$v(t) = 5{,}71\, e^{-1{,}25(t-1)}$

$$\therefore \ v(t) = \left\{ \begin{array}{ll} 0 & \text{p/ } t<0 \\ 8(1-e^{-1{,}25t}) & \text{p/ } 0<t<1s \\ 5{,}71 \cdot e^{-1{,}25(t-1)} & \text{p/ } t>1s \end{array} \right.$$

Cap 8 Circuito RLC

- Em corrente contínua

<u>série</u>



<u>LKT</u>

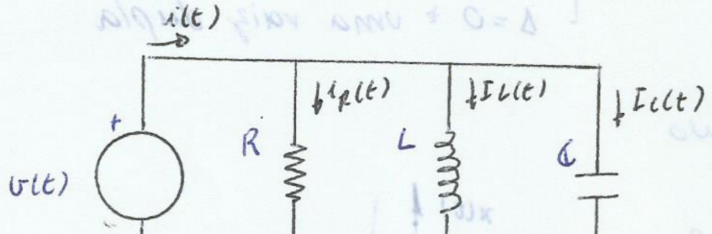
$$\frac{d}{dt}\left(R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt\right) = \frac{dE}{dt}$$

$$R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{i(t)}{C} = 0 \quad \div L$$

$$\therefore \boxed{\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = 0}$$

equação diferencial de 2ª ordem

<u>paralelo</u>



<u>LKC</u>

$$\frac{d}{dt}\left(\frac{v(t)}{R} + \frac{1}{L}\int v(t)dt + C\frac{dv(t)}{dt}\right) = \frac{d(i(t))}{dt}$$

$$\frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}.v(t) + C\frac{d^2v(t)}{dt^2} = \frac{di(t)}{dt} \quad (\div C)$$

$$\therefore \boxed{\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v(t) = \frac{di(t)}{dt}}$$

equação diferencial de 2ª ordem

Homogênea

Particular

Homogênea "genérica"

$$\boxed{\frac{d^2x(t)}{dt^2} + 2\alpha \frac{dx(t)}{dt} + w_0^2 x(t) = 0}$$

- equação característica

$$s^2 + 2\alpha s + w_0^2 = 0$$

$$s_{1,2} = \frac{-2\alpha \pm \sqrt{(2\alpha)^2 - 4.w_0^2}}{2}$$

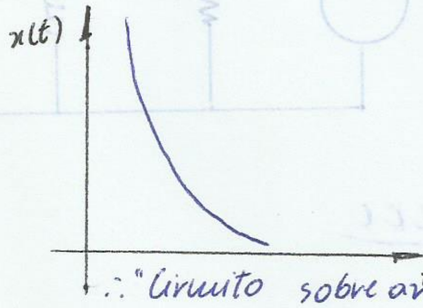
$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$$

$\Delta > 0 \rightarrow$ duas raizes distintas
$\Delta < 0 \rightarrow$ par de complexos conjugados
$\Delta = 0 \rightarrow$ uma raiz dupla

1º caso | $\Delta > 0 \quad \alpha > w_0$

a solução homogênea

$$\boxed{x(t) = K_1.e^{s_1 t} + K_2.e^{s_2 t}}$$



∴ "Circuito sobre amortecido"

2º caso | $\Delta < 0 \quad \alpha < w_0$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2} = -\alpha \pm \sqrt{(w_0^2 - \alpha^2).(-1)}$$

$$\therefore s_{1,2} = -\alpha \pm i\sqrt{w_0^2 - \alpha^2} \quad \therefore s_{1,2} = -\alpha \pm j\sqrt{w_0^2 - \alpha^2} = -\alpha \pm j w_d$$

$\sqrt{-1}$

$$\boxed{w_d = \sqrt{w_0^2 - \alpha^2}}$$

a solução

$$x(t) = K_1 e^{(-\alpha + jw_d)t} + K_2.e^{(-\alpha - jw_d)t}$$

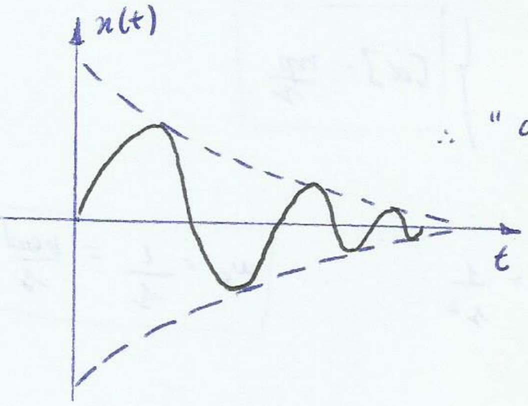
$$= K_1.e^{-\alpha t}e^{jw_d t} + K_2.e^{-\alpha t}.e^{-jw_d t}$$

$$\therefore x(t) = e^{-\alpha t}\left[K_1.e^{jw_d t} + K_2 e^{-jw_d t}\right]$$

$$x(t) = e^{-\alpha t}\left[K_1(\cos w_d t + j\sin w_d t) + K_2(\cos w_d t - j\sin w_d t)\right]$$

$$x(t) = e^{-\alpha t}\left[(K_1+K_2)\cos w_d t + j(K_1-K_2)\,\text{sen}\, w_d t \right]$$

$$\therefore \boxed{x(t) = e^{-\alpha t}\left(A_1 \cos w_d t + A_2 \,\text{sen}\, w_d t \right)}$$



∴ "circuito sub-amortecido"

3º caso | $\alpha = w_0 \;(\Delta = 0)$

$$\therefore \cancel{x(t) = K_1 e^{-\alpha t} + K_2 e^{-\alpha t}}$$

∴ a solução:

$$x(t) = K_1 e^{-\alpha t} \cdot y(t)$$

$$\frac{dx(t)}{dt} = -\alpha K_1 e^{-\alpha t} y(t) + K_1 e^{-\alpha t} \cdot y'(t)$$

$$\frac{d^2x(t)}{dt} = \alpha^2 K_1 e^{-\alpha t} \cdot y(t) - \alpha K_1 e^{-\alpha t} y'(t) - \alpha K_1 e^{-\alpha t} y'(t) + K_1 e^{-\alpha t} \cdot y''(t)$$

Substituindo:

$$\alpha^2 \cancel{K_1 e^{-\alpha t}} y(t) - \alpha \cancel{K_1 e^{-\alpha t}} y'(t) - \alpha \cancel{K_1 e^{-\alpha t}} y'(t) + K_1 e^{-\alpha t} y''(t) - 2\alpha^2 \cancel{K_1 e^{-\alpha t}} y(t) + 2\alpha \cancel{K_1 e^{-\alpha t}} y'(t) + \alpha^2 \cancel{K_1 e^{-\alpha t}} y(t) = 0$$

$$\therefore K_1 e^{-\alpha t} y''(t) = 0$$

$$\therefore y''(t) = 0$$

$$y'(t) = C$$

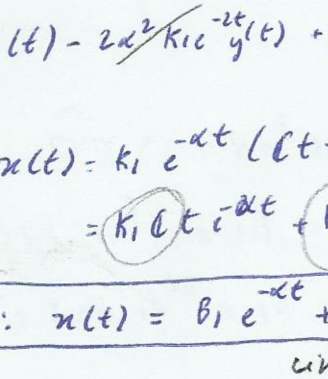
$$\therefore \boxed{y(t) = Ct + D}$$

$$\therefore x(t) = K_1 e^{-\alpha t}(Ct + D)$$

$$= K_1 C\, t\, e^{-\alpha t} + K_1 D\, e^{-\alpha t}$$

$$\boxed{\therefore x(t) = B_1 e^{-\alpha t} + B_2\, t\, e^{-\alpha t}}$$

circuito de amortecimento crítico



α e w_0 são características próprias e têm dimensão de frequência

∴ "frequências próprias" ou "frequências complexas próprias"

$$2\alpha = \frac{1}{RC} \qquad \alpha = \frac{1}{2}\frac{1}{RC}$$

$$2\alpha = \frac{R}{L} = \frac{1}{\frac{L}{R}} \rightarrow \alpha = \frac{1}{2}\frac{1}{\frac{L}{R}}$$

$$\boxed{[\alpha] = \frac{np}{s}}$$

$$w_0^2 = \frac{1}{LC} = \frac{1}{\left[\frac{V.s}{A}\right]} \cdot \frac{1}{\left[\frac{A.s}{V}\right]} = \frac{1}{s^2} \quad ∴ \boxed{w_0 = \frac{1}{s} = \frac{rad}{s}}$$

Exercício:

$$\frac{d^2 i(t)}{dt^2} + 10\frac{di(t)}{dt} + 25 i(t) = 0$$

Pede-se: a) frequências próprias ou frequências naturais do circuito

b) $i(t)$

(a)

homogênea genérica: $\frac{d^2 x(t)}{dt} + 2\alpha \frac{x(t)}{dt} + w_0^2 x(t) = 0$

equação característica: $s^2 + 2\alpha s + w_0^2 = 0$

$$s^2 + 10s + 25$$

$$10 = 2\alpha \quad ∴ \boxed{\alpha = 5 \frac{np}{s}} \qquad w_0^2 = 25 \quad ∴ \boxed{w_0 = 5\frac{rad}{s}}$$

(b) $\Delta = 10^2 - 4 \cdot 1 \cdot 25 = 0$ ∴ Amortecimento crítico

$$x(t) = B_1 e^{-\alpha t} + B_2 t e^{-\alpha t}$$

$$∴ i(t) = B_1 e^{-5t} + B_2 t e^{-5t}$$

(2) RLC paralelo

$R = 1\Omega$
$L = \frac{1}{5} H$
$C = \frac{1}{4} F$

Pede-se: Qual o tipo de amortecimento?

$$2\alpha = \frac{1}{RC} \therefore \alpha = \frac{1}{1 \cdot \frac{1}{4}} \cdot \frac{1}{2} = 2 \frac{np}{s}$$

$$w_0^2 = \frac{1}{LC} = \frac{1}{\frac{1}{5} \times \frac{1}{4}} = 20 \therefore w_0 = \sqrt{20} \frac{rad}{s}$$

Como $\alpha < w_0$ $\therefore$ Circuito sub amortecido

(3) RLC série

$R = 2\Omega$
$C = \frac{1}{8} F$
$L = ?$

p/ amortecimento crítico

$$2\alpha = \frac{R}{L} \quad w_0^2 = \frac{1}{LC} \quad ; \quad \alpha = w_0$$

$$\alpha = \frac{R}{2L} = \frac{1}{\sqrt{LC}} \rightarrow R\sqrt{LC} = 2L$$

$$R^2 \cdot \cancel{L}C = 4L^{\cancel{2}}$$

$$\therefore L = \frac{R^2 C}{4} = \frac{2^2 \cdot \frac{1}{8}}{4}$$

$$\therefore L = \frac{1}{8} H$$

Ex) $\frac{d^2 v(t)}{dt^2} + 4\frac{dv(t)}{dt} + 4v(t) = 0$ $\quad v(0) = 2V$ $\quad v'(0) = 4V$ $\quad v(t) = ?$

(Amortecimento crítico)

equação característica: $s^2 + 2\alpha s + w_0^2 = 0$

$$2\alpha = 4 \therefore \alpha = 2 \frac{np}{s} \text{ e } w_0^2 = 4 \therefore w_0 = 2 \frac{rad}{s}$$

$$\Delta = 4^2 - 4 \cdot 4 = 0 \therefore x(t) = B_1 e^{-\alpha t} + B_2 t e^{-\alpha t}$$

$$\therefore v(t) = B_1 \cdot e^{-\alpha t} + B_2 \cdot t e^{-\alpha t}$$

$$v'(t) = -\alpha B_1 e^{-\alpha t} + B_2 (e^{-\alpha t} - t \cdot \alpha \cdot e^{-\alpha t})$$

$$= -\alpha B_1 e^{-\alpha t} + B_2 \cdot e^{-\alpha t} - \alpha t \cdot B_2 \cdot e^{-\alpha t}$$

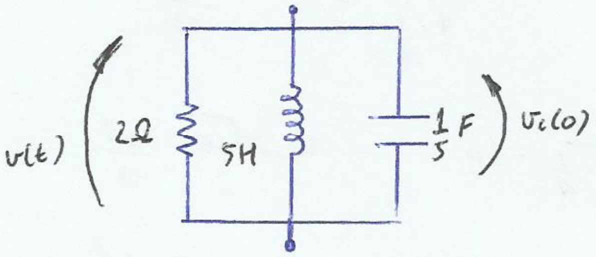
$v(0) = 2V \therefore B_1 = 2$

$v'(0) = 4V \therefore -\alpha B_1 + B_2 = 4 \therefore B_2 = 4 + 2 \cdot 2 = 8$

$$\therefore v(t) = 2e^{-\alpha t} + 8te^{-\alpha t}$$

Circuitos sem gerador

Exemplo 1:



$i_L(0) = -1A$

$v_c(0) = 4V$

$i_R(t) + i_C(t) + i_L(t) = 0$

$$\frac{d}{dt}\left(\frac{1}{R}v(t) + C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)dt \right) = \frac{d\,0}{dt} \quad \div C$$

$$\frac{d^2v(t)}{dt} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{RL}v(t) = 0$$

$$\frac{d^2v(t)}{\partial t} + 2{,}5\frac{dv(t)}{dt} + v(t) = 0$$

eq. característica

$$s^2 + 2{,}5s + 1 = 0$$

$$s = \frac{-2{,}5 \pm \sqrt{6{,}25-4}}{2} = \frac{-2{,}5 \pm 1{,}5}{2} \therefore \begin{cases} s_1 = -0{,}5 \\ s_2 = -2 \end{cases}$$

$\therefore$ "Circuito sobreamortecido"

$\therefore v(t) = k_1 . e^{-0{,}5t} + k_2 e^{-2t}$

$v(0) = k_1 + k_2 = 4$

$v'(0) = -0{,}5k_1 - 2k_2 = -5$

$\sim \begin{cases} k_1 + k_2 = 4 \\ -3k_2 = -6 \end{cases} \therefore k_2 = 2 \text{ e } k_1 = 2$

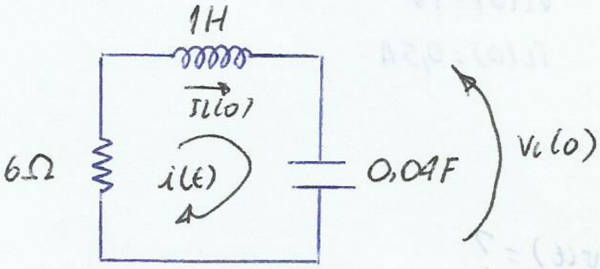
$$\frac{dv(t)}{dt} = \frac{-1}{RC}v(t) - \frac{1}{CL}\underbrace{\int v(t)dt}_{i(t)}$$

$\therefore \boxed{\begin{array}{l} v(t) = 2\,(e^{-0{,}5t} + e^{-2t})\,V \\ v(0) = 4\,V \end{array}}$

$$v'(t) = \frac{-1}{RC}v(t) - \frac{1}{RC} . i(t)$$

$v'(0) = -2{,}5 . 4 - 5(-1) \quad \therefore v'(0) = -5$

Exemplo 2:



$i_L(0) = 4A$

$V_c(0) = -4V$

LKT

$$\frac{d}{dt}\left(R\,i(t) + \frac{1}{C}\int i(t)\,dt + L\,\frac{di(t)}{dt} \right) = 0 \quad \div L$$

$$\frac{d^2 i(t)}{dt} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{RC}\, i(t) = 0$$

$$\frac{d^2 i(t)}{dt} + 6\,\frac{di(t)}{dt} + 25\, i(t) = 0$$

eq. característica : $s^2 + 6s + 25 = 0$

$$s = \frac{-6 \pm \sqrt{6^2 - 4\cdot 25}}{2} = -3 \pm j.4$$

∴ "sub-amortecido"

Solução : $i(t) = e^{-3t}(A_1 \cos 4t + A_2 \sin 4t)$

$i'(t) = -3e^{-3t}(A_1\cos 4t + A_2 \sin 4t) + e^{-3t}(-4A_1 \sin 4t + 4A_2 \cos 4t)$

∴ $i(0) = A_1 = 4$

$i'(0) = -3A_1 + 4A_2 = -20$

∴ $A_1 = 4$ e $A_2 = -2$

$$\boxed{\therefore\ i(t) = e^{-3t}(4\cos 4t - 2\sin 4t)}$$

$$R\,i(t) + \frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} = -\frac{R}{L}\, i(t) - \overbrace{\frac{1}{CL}\int i(t)\,dt}^{v_c}$$

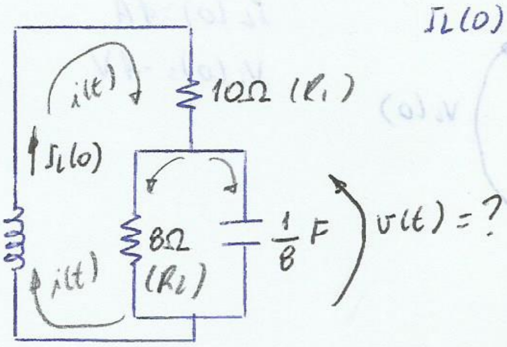
$$i'(t) = -\frac{R}{L}\, i(t) - \frac{v_c(t)}{L}$$

$$i'(0) = \frac{-6\cdot 4}{1} - \frac{(-4)}{1} \qquad \therefore\ i'(0) = -20\,A$$

Exemplo 3:

$V_c(0) = 1V$

$I_L(0) = 0{,}5A$



$$\boxed{\begin{aligned} -b &= x_1 + x_2 \\ c &= x_1 \cdot x_2 \end{aligned}}$$

Lembrete

LKT

$$\begin{cases} L\dfrac{di(t)}{dt} + R_1 i(t) + v(t) = 0 \\[2ex] i(t) = \dfrac{v(t)}{R_2} + C\dfrac{dv(t)}{dt} \end{cases}$$

$$\frac{L}{R_2}\frac{dv(t)}{dt} + LC\frac{d^2v(t)}{dt^2} + \frac{R_1}{R_2}v(t) + R_1 C\frac{dv(t)}{dt} + v(t) = 0$$

$$LC\frac{d^2v(t)}{dt^2} + \left(\frac{L}{R_2} + R_1 C\right)\frac{dv(t)}{dt} + \left(\frac{R_1}{R_2} + 1\right)v(t) = 0 \quad \div (LC)$$

$$\frac{d^2v(t)}{dt^2} + \left(\frac{1}{R_2 C} + \frac{R_1}{L}\right)\frac{dv(t)}{dt} + \left(\frac{R_1}{R_2 LC} + \frac{1}{LC}\right)v(t) = 0$$

$$\frac{d^2v(t)}{dt^2} + 6\frac{dv(t)}{dt} + 9v(t) = 0 \quad \therefore \quad \alpha = w_0 = -3$$

"Amortecimento crítico"

$v(t) = B_1 e^{-3t} + B_2 t e^{-3t}$

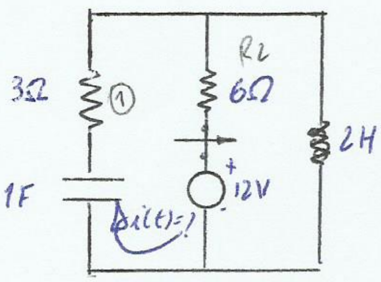
$v(0) = B_1 = 1 \quad \therefore \quad B_1 = 1$ e $B_2 = 6$

$v'(0) = -3B_1 + B_2 = 3$

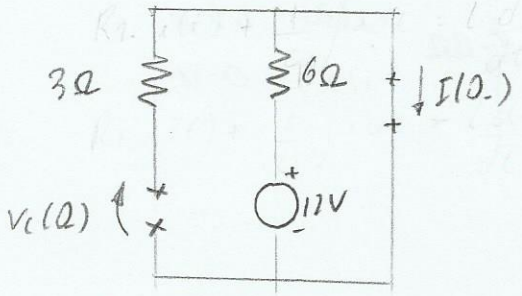
$$\frac{dv(t)}{dt} = \frac{i(t)}{C} - \frac{v(t)}{R_2 \cdot C} \Rightarrow \frac{dv(t)}{dt} = 8i(t) - v(t)$$

$v'(t) = 8i(t) - v(t) \quad \therefore v'(0) = 4 - 1 = 3$

$$\boxed{\therefore \; v(t) = e^{-3t} + 6te^{-3t} \; (V)}$$

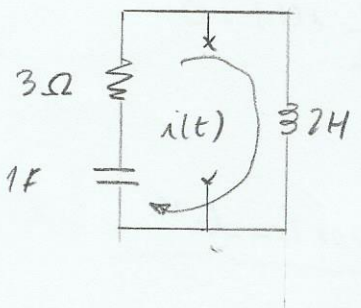


<u>p/ t<0</u>



$\therefore I(0_-) = 2A \quad V_c(0_-) = 0$

<u>p/ t>0</u>



<u>LKT</u>

$v_R(t) + v_c(t) + v_L(t) = 0$

$\left(R\,i(t) + \frac{1}{C}\int i\,dt + L\frac{di}{dt} = 0 \right) \quad \frac{d}{dt}$ e $\div L$

$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC} i(t) = 0$

$\frac{d^2 i(t)}{dt^2} + 1{,}5\frac{di(t)}{dt} + 0{,}5\,i(t) = 0$

equação característica $s^2 + 1{,}5 s + 0{,}5 = 0$ soma = $-1{,}5$ produto = $0{,}5$

$\therefore s_1 = -0{,}5 \quad s_2 = -1{,}0$ $\therefore$ "circuito sobreamortecido"

ou "superamortecido"

$\therefore i(t) = K_1 e^{-0{,}5t} + K_2 e^{-t}$

$$\boxed{\therefore i(t) = -2e^{-0{,}5t} + 4e^{-t}\ (A)}$$

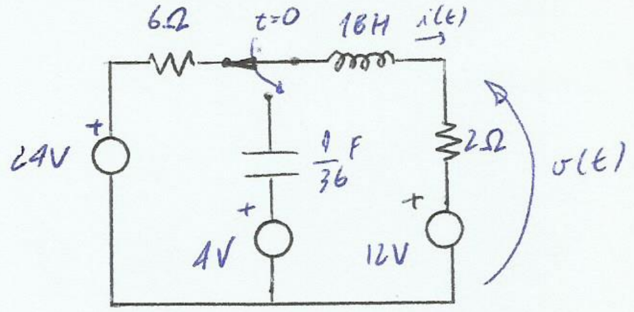
$i(0) = K_1 + K_2 = 2$ $\therefore K_1 = -2$, $K_2 = 4$

$i'(0) = -0{,}5K_1 - K_2 = ?\ -3$

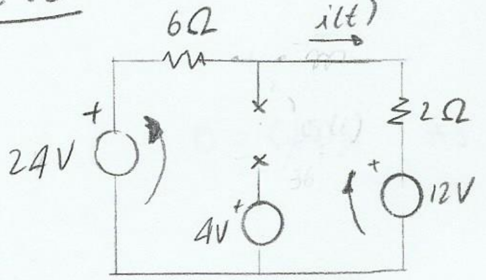
$\frac{di(t)}{dt} = I'(t) = \frac{-R}{L} i(t) - \frac{1}{LC}\int i\,dt$ $v_c(t)$

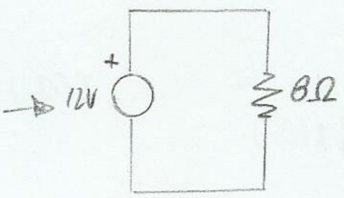
$= \frac{-R}{L} i(t) - \frac{1}{L} v_c(t) = \frac{-R}{L} I(0) - \frac{1}{L} V_c(0) = -3$

Ex



t<0

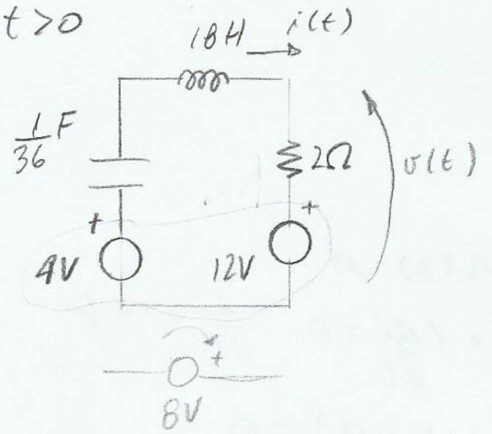




$\cdot\, i(0) = 1{,}5\ A$

$v(0_-) = 0\ V$

t>0



LKI

$$\left(R\,i(t) + \frac{1}{C}\int i\,dt + L\frac{di}{dt}\right) = (-8) \qquad \frac{d}{dt} : L$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{CL}\, i(t) = 0$$

$$\frac{\partial^2 i(t)}{\partial t^2} + \frac{1}{9}\frac{di(t)}{dt} + 2\, i(t) = 0 \quad \therefore \quad \text{"sub-amortecido"}$$

eq. caracteristica $s^2 + \frac{1}{9}s + 2 = 0$

$2\alpha = \frac{1}{9} \therefore \alpha = \frac{1}{18}$

$w_0^2 = 2 \therefore w_0 = \sqrt{2}$

$$s = \frac{-1/9 \pm \sqrt{1/81 - 8}}{2}$$

$$= \frac{-1}{18} \pm j\,1{,}41 \Rightarrow s_{1,2} \cong \underbrace{-0{,}06}_{\alpha} \pm j\cdot\underbrace{1{,}41}_{wd}$$

$$i(t) = e^{-0{,}06t}(A_1 \cos 1{,}41t + A_2 \sin 1{,}41t)$$

$I(0) = A_1 = 1{,}5 \quad ,\ I'(0) = 0{,}06\,A_1 + 1{,}41\,A_2 = -8{,}17$

$$-0{,}06\, e^{-0{,}06t}(A_1 \cos 1{,}41t + A_2 \sin 1{,}41t) + e^{-0{,}06}(-1{,}41 A \sin 1{,}41t + 1{,}41 A_2 \cos 1{,}41t)$$

$I'(t) = -8 - \frac{R}{L}\, i(t) - \frac{1}{LC}\int i(t)\,dt$

$I'(t) = -8 - \frac{1}{9}\, i(t) - \frac{1}{18}\, v_C(t)$

$I'(0) = -8 - \frac{1}{9}\cdot 1{,}5 = -\frac{49}{6} = -8{,}17$

$$\boxed{i(t) = e^{-0{,}06t}(1{,}5 \cos 1{,}41t + -5{,}73 \sin 1{,}41t)}$$

Ex) Um circuito apresenta:

$$\frac{d^2 i(t)}{dt^2} + 7\frac{di(t)}{dt} + 12 i(t) = 8e^{-2t} \qquad I(0) = I'(0) = 0$$

equação característica: $s^2 + 7s + 12 = 0$ ∴ "Circuito sobreamortecido"

$$s = \frac{-7 \pm \sqrt{49-48}}{2} \quad \therefore \quad s_1 = -3 \quad s_2 = -4$$

$$\therefore\ i_h(t) = k_1 e^{-3t} + k_2 e^{-4t} \quad \text{(homogênea)}$$

$i_p = Ae^{-2t}$ $\qquad 4Ae^{-2t} - 14Ae^{-2t} + 12Ae^{-2t} = 8e^{-2t}$

$i_p' = -2Ae^{-2t}$ $\qquad 2A = 8 \quad \therefore\ A = 4$

$i_p'' = 4Ae^{-2t}$

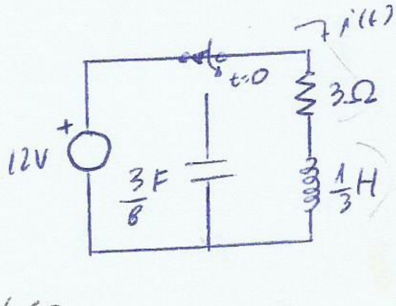
$$\begin{cases} i(t) = k_1 e^{-3t} + k_2 e^{-4t} + 4e^{-2t} \\ i'(t) = -3k_1 e^{-3t} - 4k_2 e^{-4t} - 8e^{-2t} \end{cases}$$

$i(0) = k_1 + k_2 = -4$ $\qquad \begin{cases} 3k_1 + 3k_2 = -12 \\ -k_2 = -4 \end{cases} \quad \therefore\ k_2 = 4 \text{ e } k_1 = -8$

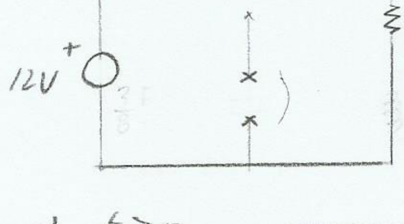
$i'(0) = -3k_1 - 4k_2 = 8$

$$\therefore\ i(t) = -8e^{-3t} + 4e^{-4t} + 4e^{-2t}$$

Ex:



p/ t<0

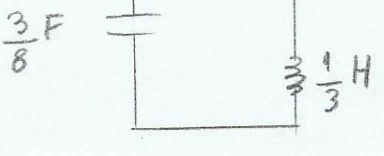


$i(0) = 4A$

$v(0_-) = 0V$

$$R i(t) + \frac{1}{C}\int i(t)dt + L\frac{di}{dt} = 0 \quad \Big| \ \frac{d}{dt} \div L$$

p/ t>0



$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{CL} i(t) = 0$$

$$\frac{d^2 i}{dt^2} + 9\frac{di(t)}{dt} + 8 i(t) = 0$$

equação característica:

$$s^2 + 9s + 8 = 0$$

$s_1 = -1 \quad s_2 = -8$

$\therefore$ "circuito sobrecrítico"

$$i(t) = K_1 e^{-t} + K_2 e^{-8t}$$

$$i'(t) = -K_1 e^{-t} - 8K_2 e^{-8t}$$

$$\frac{di(t)}{dt} = \frac{-Ri(t)}{L} - \frac{1}{LC}\int i(t)\,dt$$

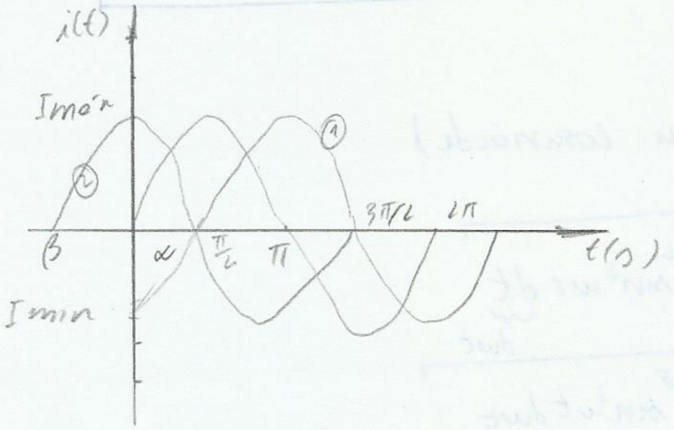
$$i'(0) = \frac{-Ri(t)}{L} - \frac{1}{L}v_c(t) = -\frac{3.4}{\frac{1}{3}} - 3.0 = -36$$

$$\begin{cases} K_1 + K_2 = 4 \\ -K_1 - 8K_2 = -36 \end{cases} \quad \therefore K_2 = \frac{32}{7} \text{ e } K_1 = -\frac{4}{7}$$

$$-7K_2 = -32$$

$$\therefore \boxed{i(t) = -\frac{4}{7}e^{-t} + \frac{32}{7}e^{-8t}}$$

Cap. Regime Permanente Sinoidal



$i(t) = I_{máx} \operatorname{sen} wt \quad (A)$

$i_1(t) = I_{máx} \operatorname{sen}(wt - \alpha) \quad (A)$

$i_2(t) = I_{máx} \operatorname{sen}(wt + \beta) \quad (A)$

$I_{máx}$; I_p, I_c

Valor Máximo | Valor de Crista

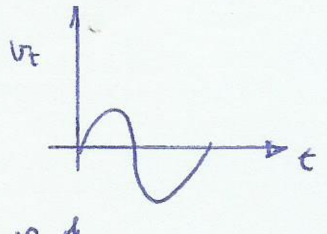
Valor de Pico

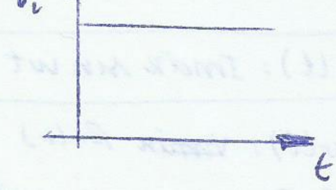
w: Velocidade angular

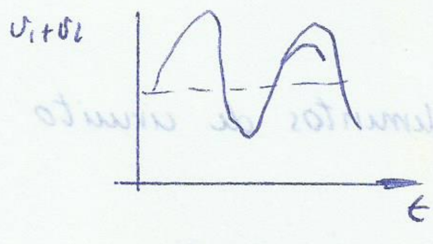
T: período

$$\boxed{w = 2\pi f = \frac{2\pi}{T} \; ; \; f = \frac{1}{T} \text{ (frequência)}}$$

$$\boxed{\text{Valor Médio} = \frac{1}{T}\int_0^T f(t)\,dt}$$

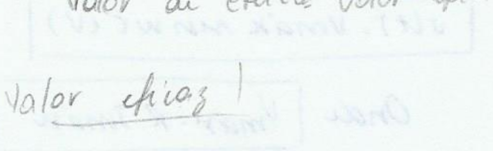






Valor de Efeito Valor efetivo = Valor eficaz

Valor eficaz !



$P = R.I^2 \qquad p(t) = R.i^2(t)$

$$P_{med} = \frac{1}{T}\int_0^T R\,i(t)^2\,dt$$

$R \cdot I^2 = \frac{1}{T}\int_0^T R\, i(t)^2\, dt$ ∴ $\boxed{I_{ef} = \sqrt{\frac{1}{T}\int_0^T i(t)^2\, dt}}$ idem V_{ef} (rms)

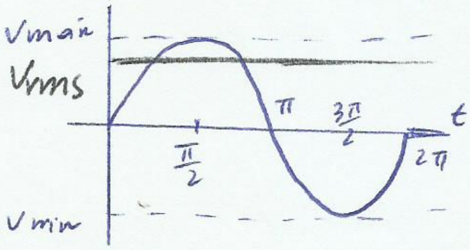
Para a senóide (ou cossenóide)

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_{máx}^2 \operatorname{sen}^2 \omega t\, \underbrace{dt}_{d\omega t}}$$

$$= I_{máx}\sqrt{\frac{1}{2\pi}\int_0^{2\pi} \operatorname{sen}^2 \omega t\, d\omega t}$$

$$= I_{máx}\sqrt{\frac{1}{4\pi}\left[\int_0^{2\pi} d\omega t - \cancelto{0}{\int_0^T \cos 2\omega t\, d\omega t}\right]}$$

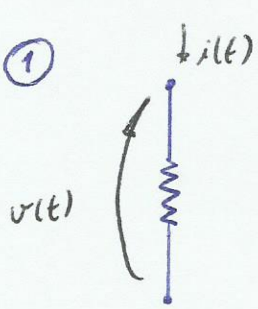
$$= I_{máx}\sqrt{\frac{1}{4\pi}\cdot 2\pi} \quad \therefore I_{rms} = \frac{I_{máx}}{\sqrt{2}} \cong 0{,}707\, I_{máx}$$



$V_{rms} \cong 0{,}7\, V_{máx}$

Elementos de circuito

1



Lei de Ohm

$v(t) = R \cdot i(t)$

$i(t) = \frac{v(t)}{R}$

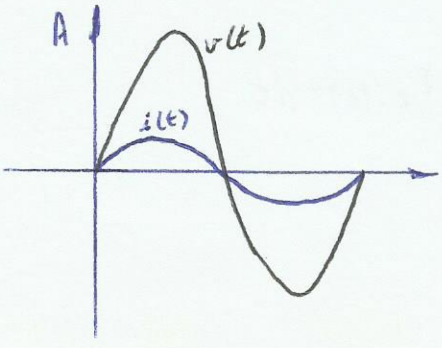
Se $\boxed{i(t) = I_{máx} \operatorname{sen} \omega t \text{ (A)}}$

então $v(t) = \cancel{V_{máx}}\, R \cdot i(t)$

$= R \cdot I_{máx} \operatorname{sen} \omega t$ (V)

∴ $\boxed{v(t) = V_{máx} \operatorname{sen} \omega t \text{ (V)}}$

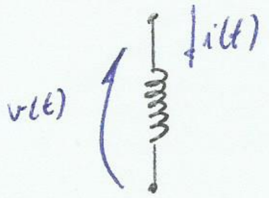
Onde $\boxed{V_{máx} = R \cdot I_{máx}}$



∴ na resistência

v(t) e i(t) estão em fase

Indutor



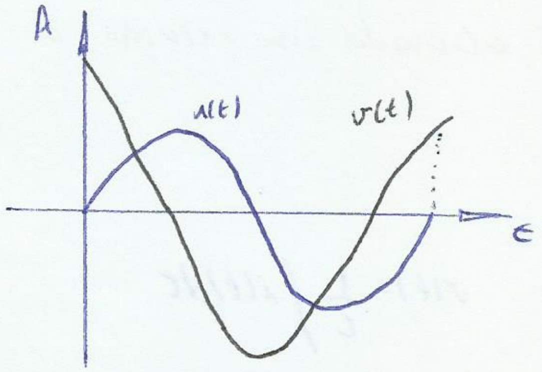
Lei de Faraday

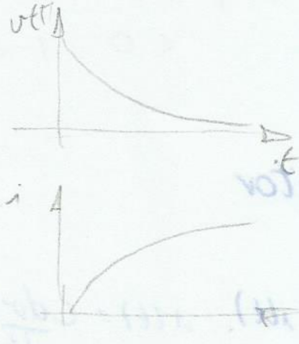
$$v(t) = L \frac{di(t)}{dt} \quad \therefore \quad i(t) = \frac{1}{L}\int v(t)\,dt$$

Se $\boxed{i(t) = I_{máx} \operatorname{sen} wt \ (A)}$

então $v(t) = \underbrace{wL \cdot I_{máx}}_{V_{máx} = wL \cdot I_{máx}} \cos wt$

$$\therefore \boxed{v(t) = V_{máx} \cdot \cos(wt) \ (V)}$$



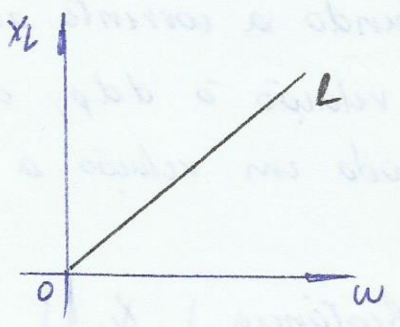


- apareceu uma defasagem entre i(t) e v(t) de 90° no tempo
- Sendo a v(t) adiantada em relação à corrente ou a corrente atrasada em relação à d.d.p.

$$\frac{V_{máx}}{I_{máx}} = \frac{V_{rms}}{I_{rms}} = wL \ (\Omega)$$

Reatância Indutiva

$$[X_L] = [w] \cdot [L] = \frac{rad}{s} \cdot \frac{V \cdot s}{A} = \frac{V}{A} = \Omega$$



Defasagem

$i(t) = Imáx \cdot sen(wt - \alpha)$
$i(t) = Imáx\, sen(wt + 0)$
$i(t) = Imáx\, sen(wt + \alpha)$

$\measuredangle i(t) - \measuredangle_1 i(t) =$ defasagem

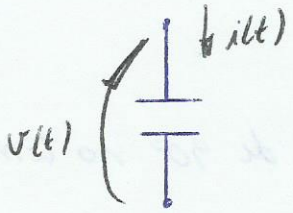
$0° - \beta = -\beta$

$0° - (-\alpha) = \alpha$

1 - 2 > 0 1 está adiantado em relação a 2

< 0 1 está atrasada em relação 2

Capacitor

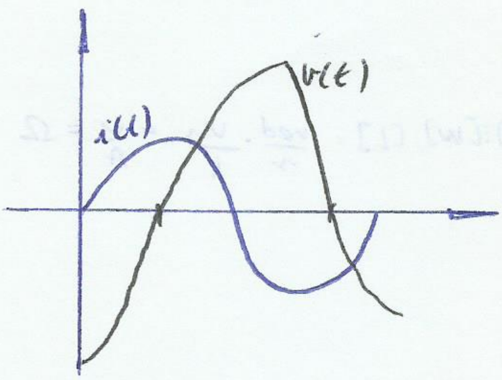


$i(t) = C\dfrac{dv(t)}{dt}$ $v(t) = \dfrac{1}{C}\int i(t)dt$

Se $\boxed{i(t) = Imáx\, sen wt (A)}$

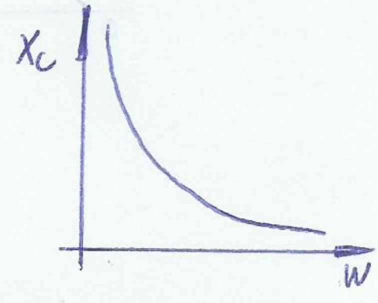
então $v(t) = \dfrac{-1}{wC} Imáx \cos wt$ $\therefore$ $\boxed{v(t) = -Vmáx \cos wt\ (V)}$

$Vmáx = \dfrac{1}{wC} Imáx$

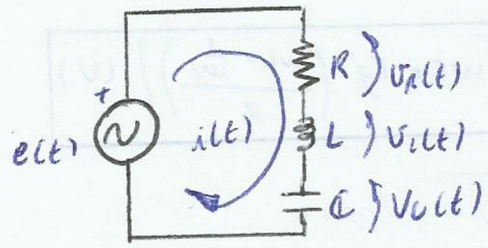


- Existe defasagem entre a ddp e a corrente de 90° no tempo
- Sendo a corrente avançada em relação à d.d.p; ou a ddp atrasada em relação a corrente.

$\dfrac{Vmáx}{Imáx} = \dfrac{1}{wC} = \dfrac{Vrms}{Irms}\ (\Omega): X_C$ (Reatância capacitiva)



Circuito RLC série



<u>LKT</u>

$e(t) = v_R(t) + v_L(t) + v_C(t)$

<u>Se</u> $i(t) = I_{máx} . sen(wt)$ (A)

$v_R(t) = R . I_{máx} \, sen(wt)$ (V)

$v_L(t) = wL \, I_{máx} \cos(wt)$ (V)

$v_C(t) = \frac{1}{wC} I_{máx} \cos(wt)$ (V)

$\therefore e(t) = R . I_{máx} . \underline{sen(wt)} + \left(wL - \frac{1}{wC}\right) I_{máx} . \underline{\cos(wt)}$ (V)

$e(t) = A \, sen(wt + \varphi)$

$= A \, \underline{sen \, wt} . \cos\varphi + A \, sen \, \varphi \, \underline{\cos wt}$

$\therefore \begin{cases} A \, sen \, \varphi = \left(wL - \frac{1}{wC}\right) I_{máx} \\ A \cos\varphi = R . I_{máx} \end{cases} \div$

$\therefore tg \, \varphi = \dfrac{wL - \frac{1}{wC}}{R} = \dfrac{X_L - X_C}{R}$

$\boxed{\varphi = tg^{-1} \dfrac{wL - \frac{1}{wC}}{R}} = \boxed{tg^{-1} \dfrac{X_L - X_C}{R}}$

$A^2 sen^2 \varphi = \left(wL - \frac{1}{wC}\right)^2 I^2_{máx}$

$A^2 \cos^2\varphi = R^2 . I^2_{máx}$

$\sqrt{A^2 (sen^2\varphi + \cos^2\varphi)} = \sqrt{I^2_{máx} \left[R^2 + \left(wL - \frac{1}{wC}\right)^2\right]}$

$$\therefore \boxed{A = I_{máx}\sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2} = I_{máx}\sqrt{R^2 + (x_L - x_C)^2}}$$

$$\therefore \boxed{e(t) = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2} \cdot I_{máx} \sin\left[wt + tg^{-1}\left(\frac{wL - \frac{1}{wC}}{R}\right)\right] (V)}$$

$$V_{máx} = \sqrt{R^2 + (x_L - x_C)^2} \, I_{máx}$$

$$\frac{V_{máx}}{I_{máx}} = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2} \; (\Omega) = Z_{eq} \text{ (Impedância)}$$

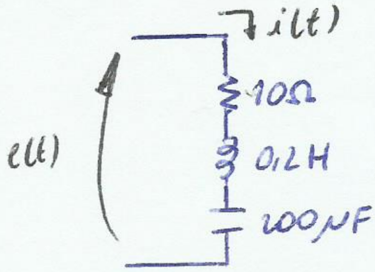
$$tg^{-1}\frac{x_L - x_C}{R} = \varphi$$

$90° > \varphi > 0$ -> circuito é indutivo

$-90° < \varphi < 0$ -> circuito é capacitivo

$\varphi = 0$ -> circuito é resistivo

Exercício



$i(t) = 2 \sin(100t + 30°)$ (A)

Pede-se:
a) $V_R(t), v_L(t), v_C(t)$
b) $e(t)$;
c) Z_{eq};
d) tipo de circuito.

a) $\boxed{V_R(t) = 20 \sin(100t + 30°) \; (V)}$

$V_L(t) = 100 \cdot 2 \cos(100t + 30°)$ (V) $\therefore \boxed{V_L(t) = 200 \cos(100t + 30) \; (V)}$

$V_C(t) = -\frac{1}{100 \cdot 200 \, 10^{-6}} \, 2 \cos(100t + 30°)$ $\boxed{V_C(t) = -100 \cos(100t + 30°) \, V}$

b) $v_R(t) + v_L(t) + v_C(t) = v(t)$

$\therefore v(t) = 20 \sin(100t + 30) - 60 \cos(100t + 30°)$

$v(t) = A \sin[(100t + 30) + \varphi]$

$= A \sin(100t + 30) \cos\varphi + A \sin\varphi \cos(100t + 30)$

$$\begin{cases} A \sin\varphi = -60 \\ A \cos\varphi = 20 \end{cases}$$

$\therefore \ tg\,\varphi = -3 \rightarrow \boxed{\varphi \cong -71{,}6^\circ}$

$A = \sqrt{(-60)^2 + 20^2} \cong 63{,}2$

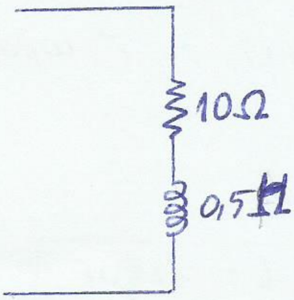
$\therefore \ v(t) = 63{,}2 \ sen\,[100t + 30^\circ - 71{,}6]$

$\therefore \ v(t) = 63{,}2 \ sen\,[100t - 41{,}6^\circ] \ (V)$

c) $Z_{eq} = \dfrac{V_{máx}}{I_{máx}} = \dfrac{63{,}2}{2} = 31{,}6\,\Omega$

d) "Circuito é capacitivo"

Ex:)



$i(t) = 2\cos(200t - 30^\circ) \ (A)$

Det: a) $v_r(t)$ e $v_L(t)$;

b) $v(t)$;

c) Z_{eq}

a) $\boxed{v_r(t) = 20\cos(200t - 30^\circ) \ (V)}$

$v_L(t) = 200 \cdot 0{,}5 \cdot 2\cos(200t - 30) \ \therefore \ \boxed{v_L(t) = -200\,sen(200t - 30)}$

b) $v(t) = v_R(t) + v_L(t)$

$v(t) = 20\cos(200t - 30) - 200\,sen(200t - 30)$

$v(t) = A\cos[200t - 30 + \varphi]$

$= A\cos(200t - 30)\cos\varphi - A\,sen(200t - 30)\,sen\,\varphi$

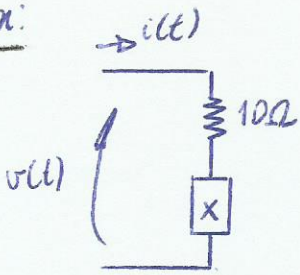
$\begin{cases} A\cos\varphi = 20 \\ A\,sen\,\varphi = 200 \end{cases}$ $\quad tg\,\varphi = \dfrac{200}{20} \ \therefore \ \varphi = 84{,}29$

$\therefore \ v(t) = \sqrt{20^2 + 200^2}\ \cos[200t - 30 + 84{,}29]$

$\boxed{v(t) = 201 \ \cos[200t + 54{,}29]}$

c) $Z_{eq} = \dfrac{201}{2}$ $\quad \boxed{Z_{eq} = 100{,}5\,\Omega}$

Ex:



$v(t) = 100 \sin(100t - 10°)$ (V)

$i(t) = I \cos(100t - 30°)$ (A)

Dt: a) elemento x;

b) I.

(a)

$\sin x = \cos(x - 90)$

$\cos x = \sin(x + 90)$

$v(t) = 100 \sin(100t - 10)$

$= 100 \cos(100t - 10 - 90) \therefore v(t) = 100 \cos(100t - 100)$

corrente − tensão $= -30 - (-100) = 70 \therefore \varphi = 70°$ $\varphi = -70$

$\therefore$ Como a corrente está adiantada a tensão $\therefore$ é capacitivo

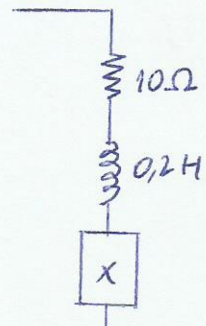
Com isso $\boxed{x: \text{é capacitor}}$

$tg^{-1}\left[-\frac{x_c}{10}\right] = 70 \therefore x_c = 27{,}47 = \frac{1}{wC} \therefore \boxed{C = 365\,\mu F}$

(b) $Z_{eq} = \sqrt{R^2 + x_c^2} = \sqrt{10^2 + 27{,}47^2} = 29{,}23\,\Omega$

$I = \frac{V_{máx}}{Z_{eq}} = \frac{100}{29{,}16} \therefore \boxed{I = 3{,}42\text{ A}}$

Ex:



$v(t) = 100 \sin(100t + 60°)$ (V)

$i(t) = I \cos(100t - 40°)$ (A)

$\sin(wt) = \cos(wt - 90)$

$\cos(wt) = \sin(wt + 90)$

Dt. a) Elemento x;

b). I

$$\therefore\ i(t) = \frac{V_{máx}}{R} \underline{\sin wt} + \left(wC - \frac{1}{wL}\right) V_{máx} \underline{\cos wt}$$

$$\downarrow$$

$$i(t) = B \sin(wt + \varphi')$$

$$= B \underline{\sin wt} \cdot \cos\varphi' + B \sin\varphi' \underline{\cos wt}$$

$$B \cos\varphi' = \left(wC - \frac{1}{wL}\right) V_{máx}$$

$$B \sin\varphi' = \frac{V_{máx}}{R}$$

$$\therefore\ tg\,\varphi' = R\left(wC - \frac{1}{wL}\right)$$

$$\therefore\ \boxed{\varphi' = tg^{-1}\left[R\left(wC - \frac{1}{wL}\right)\right]} \quad \text{ou} \quad \varphi' = tg^{-1}\left[\frac{\frac{1}{x_C} - \frac{1}{x_L}}{\frac{1}{R}}\right]$$

$$\boxed{B = \sqrt{\left(\frac{1}{R}\right)^2 + \left(wC - \frac{1}{wL}\right)^2} \cdot V_{máx}}$$

$$\therefore\ \boxed{i(t) = \underbrace{V_{máx} \sqrt{\left(\frac{1}{R}\right)^2 + \left(wC - \frac{1}{wL}\right)^2}}_{I_{máx}} \sin\left[wt + tg^{-1} R\left(wC - \frac{1}{wL}\right)\right] \ (A)}$$

$$I_{máx} = V_{máx} \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{x_C} - \frac{1}{x_L}\right)^2}$$

$$\frac{I_{máx}}{V_{máx}} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{x_C} - \frac{1}{x_L}\right)^2} \ (S) = Y_{eq}$$

$$Y_{eq} = \frac{1}{Z_{eq}} \ (\text{Admitância})$$

Quando:

$\varphi' > 0 \rightarrow$ circuito capacitivo

$\varphi' = 0 \rightarrow$ circuito resistivo

$\varphi < 0 \rightarrow$ circuito indutivo

$v(t) = 100 \,\text{sen}\, (100t + 60)$

$i(t) = I \cos(100t - 40°)$ (A) $\rightarrow$ $i(t) = I \,\text{sen}\, (100t + 50°)$ (A)

Caso Resistivo

$$tg\,\theta = \frac{x_c}{x + R} \rightarrow tg\,10 = \frac{20}{10 + x} \therefore x = 103{,}43\,\Omega$$

$$Zeq = \sqrt{x_L^2 + R^2} \rightarrow Zeq = 115{,}18\,\Omega \;;\; Im = \frac{Vm}{Zeq} \therefore \boxed{Im \simeq 0{,}87\,A}$$

Como capacitor

$$tg\,\varphi = \frac{x_L - x_c}{R} \rightarrow (tg10).10 - 20 = -x_c \therefore x_C = 18{,}24\,\Omega$$

$$x_c = \frac{1}{wc} \quad \boxed{C = 548\,\mu F}$$

$$Zeq = \sqrt{(x_L - x_c)^2 + R^2} \rightarrow Zeq = 10{,}15\,\Omega \;;\; Im = \frac{Vm}{Zeq} \therefore \boxed{Im = 9{,}85\,A}$$

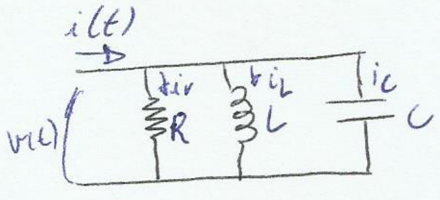
Como bobina

$$tg\,\varphi = \frac{x_L - x_L'}{R} \quad \therefore x_L' = -18{,}24\,\Omega$$

$$(tg10).10 - 20 = x_L$$

$\llcorner$ Impossível

Circuito RLC paralelo



Se $\boxed{v(t) = Vmáx \,\text{sen}\, wt \text{ (V)}}$

então $\boxed{i_R(t) = \frac{Vmáx}{R} \,\text{sen}\, wt \text{ (A)}}$

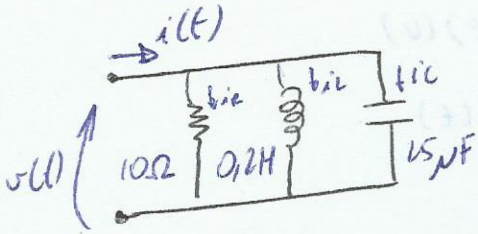
$\boxed{i_C(t) = w.C.Vmáx \cos wt}$

$\boxed{i_L(t) = \frac{1}{wL} Vmáx \cos wt \text{ (A)}}$

L.K.C

$$i_R(t) + i_L(t) + i_C(t) = i(t)$$

Exercício



$v(t) = 100 \operatorname{sen}(200t - 10°)$ (V)

Det: a) $i_R(t)$; $i_L(t)$; $i_C(t)$;

b) $i(t)$

c) Y_{eq}

d) tipo de circuito

a) $\boxed{i_R(t) = 10 \operatorname{sen}(200t - 10) \ (A)}$

$i_L(t) = -\dfrac{1}{200 \cdot 0{,}2} \cdot 100 \operatorname{sen}(200t - 10°)$

$\therefore \boxed{i_L(t) = -2{,}5 \cos(200t - 10°) \ (A)}$

$i_C(t) = 200 \cdot 25 \cdot 10^{-6} \cdot 100 \cdot \cos(200t - 10°)$

$\therefore \boxed{i_C(t) = 0{,}5 \cos(200t - 10°) \ (A)}$

b) $i(t) = 10 \operatorname{sen}(200t - 10°) - 2 \cos(200t - 10°)$

$i(t) = B \operatorname{sen}[(200t - 10) - \varphi']$

$i(t) = B \operatorname{sen}(200t - 10°) \cdot \cos\varphi' - B \operatorname{sen}\varphi' \cos(200t - 10°)$

$\begin{cases} -B \operatorname{sen}\varphi' = -2 \\ B \cos \varphi = 10 \end{cases}$

$tg\,\varphi' = 0{,}2 \rightarrow \varphi' = 11{,}3$

$B = \sqrt{(-2)^2 + 10^2} \cong 10{,}2$

$\therefore i(t) \cong 10{,}2 \operatorname{sen}(200t - 10° - 11{,}3°) \quad \therefore \boxed{i(t) = 10{,}2 \operatorname{sen}(200t - 21{,}3°) \ (A)}$

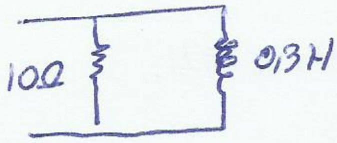
c) $Y_{eq} = \dfrac{I_{máx}}{V_{máx}} = \dfrac{10{,}2}{100} \cong 0{,}1$ (S)

d) $\varphi' > 0 \therefore$ ~~capacitivo~~ $\therefore$ indutivo

$\varphi = \measuredangle V - \measuredangle I = (-10°) - (-21{,}3°) = 11{,}3°$

$\varphi' = \measuredangle I - \measuredangle V = -21{,}3° - (-10) = -11{,}3°$

Ex 1



$v(t) = 2\cos(100t + 10°)\,(V)$

Def: a) $i_R(t)$ e $i_L(t)$

b) $i(t)$;

c) Y_{eq}

a) $\boxed{i_R(t) = 0{,}2\cos(100t + 10°)}$

$i_L(t) = \frac{1}{0{,}3}\int 2\cos(100t + 10°) \quad \therefore \quad \boxed{i_L = 0{,}067\,\text{sen}(100t + 10°)}$

b) $i(t) = i_R(t) + i_L(t)$

$i(t) = 0{,}2\cos(100t + 10) + 6{,}67\,\text{sen}(100t + 10)$

$i(t) = B\cos(100t + 10 - \varphi')$

$= B\,\underline{\cos(100t + 10)}\cos\varphi' + B\,\underline{\text{sen}(100t + 10)}\,\text{sen}\,\varphi'$

$\begin{cases} B\cos\varphi' = 0{,}2 \\ B\,\text{sen}\,\varphi' = 0{,}067 \end{cases} \Rightarrow \quad tg\,\varphi = 0{,}333 \quad \therefore \quad \varphi' = 18{,}43$

$B = \frac{0{,}2}{\cos 18{,}43} \quad \therefore \quad B = 0{,}21$

$\therefore \quad \boxed{i(t) = 0{,}21\cos(100t - 08{,}43)}\ (A)$

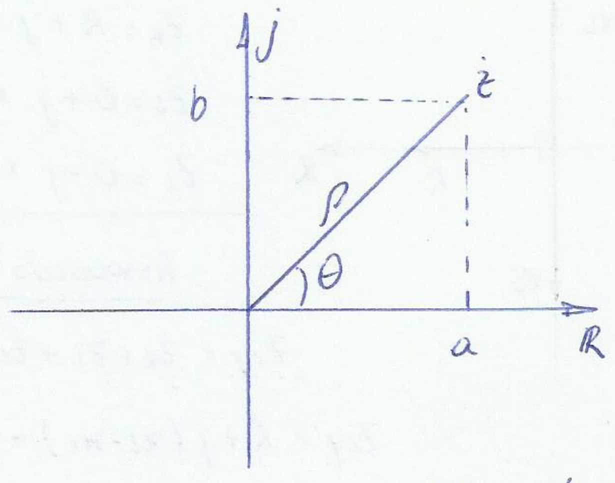
c) $Y_{eq} = \frac{0{,}21}{2} \quad \therefore \quad \boxed{Y_{eq} = 0{,}105\,\Omega}$

- Resistência, Reatância, Impedância e seus inversos Condutância, Susceptância e Admitância "<u>são números complexos</u>"

- d.d.p. e corrente são "fasores" e ficam bem determinados por número complexo na <u>forma</u> <u>polar</u>.

fasor: "vetor girante"

Números Complexo



$\dot{z} = a + jb$ (Cartesiana ou Retangular)

$z = \rho \angle \theta$ (Polar)

$$\begin{cases} \rho = \sqrt{a^2+b^2} \\ \theta = tg^{-1}\frac{b}{a} \end{cases} \qquad \begin{cases} a = \rho\cos\theta \\ b = \rho\sin\theta \end{cases}$$

Operações

- Soma e subtração
  (usa-se forma retangular)

$\dot{z}_1 = a + jb$

$\dot{z}_2 = c + jd$

$\dot{z}_1 + \dot{z}_2 = (a+c) + j(b-d)$

$\dot{z}_1 - \dot{z}_2 = (a-c) + j(b+d)$

- Multiplicação, Divisão, exponenciação e Radiação
  (usa-se na forma polar)

$\dot{z}_1 = \rho_1 \angle \theta_1$

$\dot{z}_2 = \rho_2 \angle \theta_1$

$\dot{z}_1 \times \dot{z}_2 = \rho_1 \cdot \rho_2 \angle \theta_1 + \theta_2$

$\frac{\dot{z}_1}{\dot{z}_2} = \frac{\rho_1}{\rho_2} \cdot \angle \theta_1 - \theta_2$

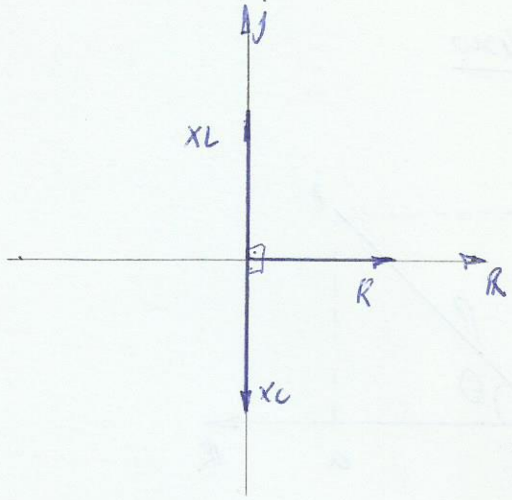
$\dot{z}_1^2 = \rho_1^2 \angle 2\theta_1$

$\sqrt{\dot{z}_1} = \sqrt{\rho_1} \cdot \angle \frac{\theta_1}{2}$

Complexo conjugados

$z_1 = a + jb \quad z_1 = \rho \angle \theta$

$z_1^* = a - jb \quad z_1^* = \rho \angle -\theta$



$$\dot{Z}_R = R + j\cdot 0 = R\angle 0^\circ\,\Omega$$

$$\dot{Z}_L = 0 + j\cdot X_L = X_L\angle 90^\circ\,\Omega$$

$$\dot{Z}_C = 0 - j\,X_C = X_C\angle -90^\circ\,\Omega$$

Associação série

$$\dot{Z}_{eq} = \dot{Z}_R + \dot{Z}_L + \dot{Z}_C$$

$$\therefore\ \dot{Z}_{eq} = R + j(X_L - X_C) = \sqrt{R^2 + (X_L - X_C)^2}\,\angle tg^{-1}\frac{X_L - X_C}{R}$$

Associação Paralelo

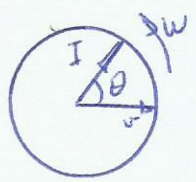
$$\frac{1}{\dot{Z}_{eq}} = \frac{1}{\dot{Z}_R} + \frac{1}{\dot{Z}_L} + \frac{1}{\dot{Z}_C}$$

$$Y_{eq} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$Y_{eq} = \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$ ou $$Y_{eq} = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right) = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}\,\angle tg^{-1}\left[R\left(\frac{1}{X_C} - \frac{1}{X_L}\right)\right.$$

na

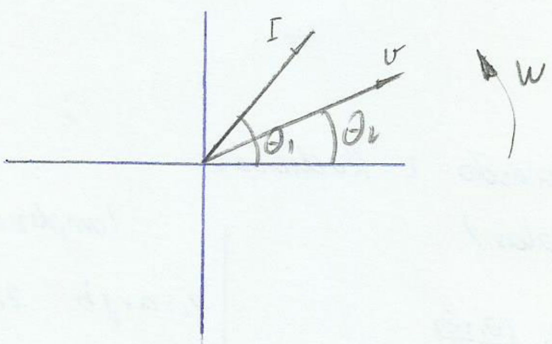
Valor Eficaz



$$\dot{I} = I\angle\theta$$

$$\dot{V} = V\angle 0^\circ$$

Diagrama Fasorial



$\cos(\theta) = \sin(\theta + 90)$

$Z = 3 + j4 = 5 \angle 53{,}13$
$Pol(3,4) = 5$
$Rec(5;53{,}13) = 3$
Obs: Alfa F

Ex:

$v_1(t) = 100 \sin(100t - 30^\circ)$ (V)

$v_2(t) = 200 \cos(100t + 10)$ (V) $= 200 \sin(100t + 100)$

$v_3(t) = 50 \sin(100t + 30^\circ)$ (V)

$v_4(t) = 100 \cos(100t - 10^\circ)$ (V) $= 100 \sin(100t + 80)$

$v_5(t) = 50 \sin(100t + 50^\circ)$ (V)

$v_1(t) = 100 \sin(100t - 30) = \frac{100}{\sqrt{2}} \angle -30^\circ = 61{,}24 - 35{,}35j$

$v_2(t) = 200 \sin(100t + 100) = \frac{200}{\sqrt{2}} \angle 100 = -24{,}56 + 139{,}27j$

$v_3(t) = 50 \sin(100t + 30) = \frac{50}{\sqrt{2}} \angle 30^\circ = 30{,}62 + 17{,}68j$

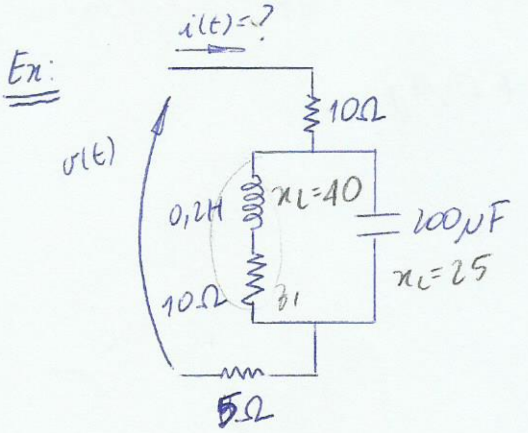
$v_4(t) = 100 \sin(100t + 80) = \frac{100}{\sqrt{2}} \angle 80^\circ = 12{,}28 + 69{,}64j$

$v_5(t) = 50 \sin(100t + 50) = \frac{50}{\sqrt{2}} \angle 50^\circ = 22{,}73 + 27{,}08j$

$v(t) = v_1(t) + v_2(t) + v_3(t) + v_4(t) + v_5(t)$

$v(t) = 102{,}31 + 218{,}32j = 241{,}104 \angle 64{,}89$

Ex:



$v(t) = 100 \sin(200t - 40^\circ)$ (V)

$Z_1 = 0 + 40j + 10 + 0j = 10 + 40j = 41{,}23 \angle 75{,}96$

$$Z_2 = \frac{41{,}23 \angle 75{,}9 \times 25 \angle -90}{10 + 40j + 0 - 25j}$$

$$Z_2 = \frac{1030{,}75 \angle -14{,}1}{18{,}03 \angle 56{,}3} = 57{,}17 \angle -70{,}4 = 19{,}18 - 53{,}86j$$

$\theta = \angle V - \angle I$
$= -40 - (17{,}6) = \boxed{-57{,}6^\circ}$

$Z_{eq} = 34{,}18 - 53{,}86j = 63{,}79 \angle -57{,}6$

$v(t) = \frac{100}{\sqrt{2}} \angle -40 \quad \therefore \quad i(t) = \frac{v(t)}{Z_{eq}} = 1{,}11 \angle +17{,}6 = 1{,}06 + 0{,}34j$

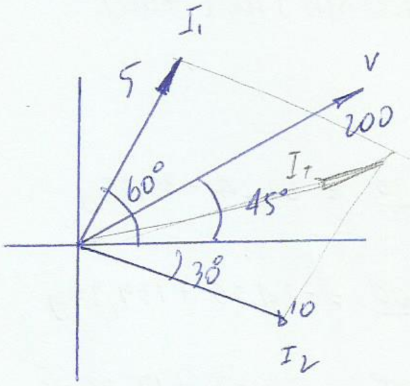
$\boxed{\therefore i(t) = 1{,}11\sqrt{2} \sin(200t + 17{,}6^\circ) \text{ A}}$

Ex 1 Dado o diagrama fasorial de um circuito paralelo, determinar:

a) os possíveis elementos do circuito;

b) $\dot{Z}_{eq}$

c) tipo de circuito



$\omega = 100 \ \frac{rad}{s}$

a) RC → $R = 38{,}64\Omega$ e $X_C = -10{,}35$ ∴ $C = 0{,}96mF$

RL → $R = 5{,}18\Omega$ e $X_L = 19{,}32$ ∴ $F = 0{,}19H$

b) $\dot{V} = 200 \angle 45$ $\quad X_1 = 40 \angle -15°$

$\dot{I}_1 = 5 \angle 60$ $\quad X_2 = 20 \angle 75$

$\dot{I}_2 = 10 \angle -30°$

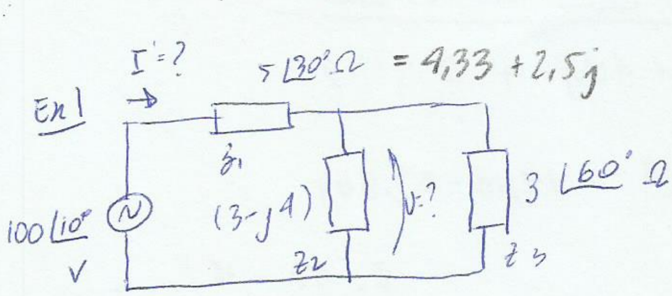
$$Z_{eq} = \frac{40 \angle -15 \times 20 \angle 75}{38{,}64 - 10{,}4j + 5{,}18 + 19{,}32j}$$

$$Z_{eq} = \frac{800 \angle 60}{44{,}72 \angle 11{,}5} = 17{,}89 \angle 48{,}5 = 11{,}85 + 13{,}4j$$

c) $\varphi = 48{,}5$ ∴ é Indutivo

Ex 1

$I = ?$ $\quad 5 \angle 30° \Omega = 4{,}33 + 2{,}5j$



$\dot{V} = 100 \angle 10°$

$$z_2 // z_3 = \frac{5 \angle -53{,}1 \times 3 \angle 60°}{3 - j4 + 1{,}5 + 2{,}6j}$$

$$z_2 // z_3 = \frac{15 \angle 6{,}9}{4{,}5 - 1{,}4j} = \frac{15 \angle 6{,}9}{4{,}71 \angle -17{,}3} = 3{,}18 \angle 24{,}2 = 2{,}9 + 1{,}3j$$

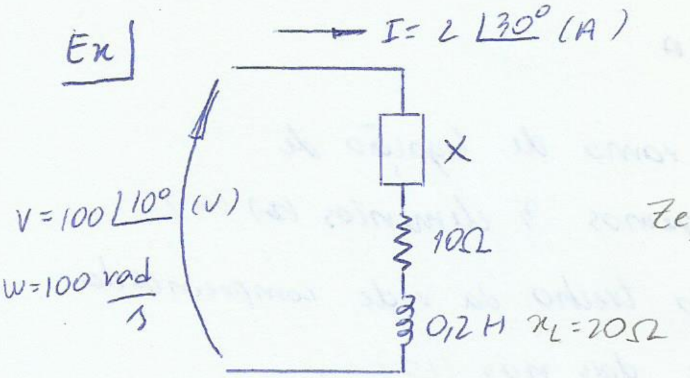
$z_T = 7{,}23 + 3{,}8j$

$$\dot{I} = \frac{100 \angle 10}{8{,}17 \angle 27{,}73} = 12{,}24 \angle -17{,}7$$

$\dot{V} = 12{,}24 \angle -17{,}73 \times 3{,}18 \angle 24{,}2$

∴ $\dot{V} = 38{,}92 \angle 6{,}5$

Ex



$V = 100 \angle 10°$

$I = 2 \angle 30°$

$Zeq = \frac{V}{I} = \frac{100 \angle 10°}{2 \angle 30°} = 50 \angle -20°$

$Zeq = 46,98 - 17,10j$

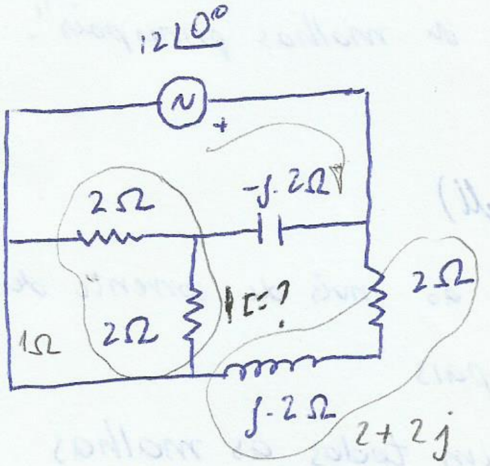
$Zeq = 10 + 20j + X = 46,98 - 17,10j$

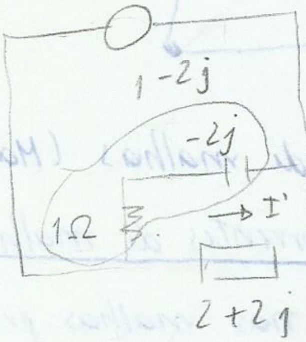
$\therefore \quad X = 36,98 - 37,1j = 52,38 \angle -45,1$

$\phi = -45,1 < 0 \therefore$ Capacitor

$\frac{1}{wC} = x_C \quad \therefore \quad \frac{1}{100 . C} = 52,38 \quad \therefore C = 190,9 \mu F$

Ex 2





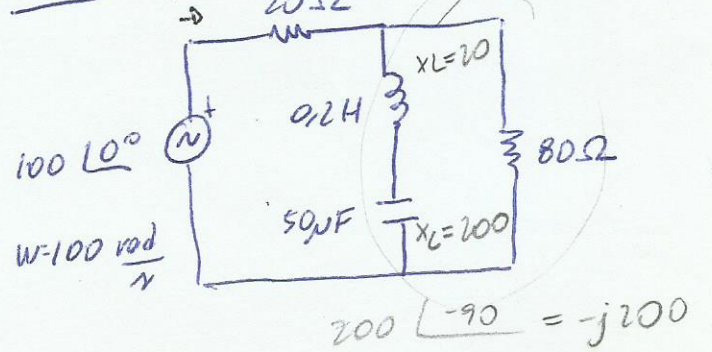
(Importante)

$I' = \frac{12 \angle 0°}{2,24 \angle -63,4} = 5,36 \angle 63,4°$

$V'' = -5,36 \angle 63,4 \quad \therefore \quad I = \frac{-5,36 \angle 63,4}{2} = -2,68 \angle 63,4 \ (A)$

$= 2,68 \angle -116,6° \ (A)$

Ex 3



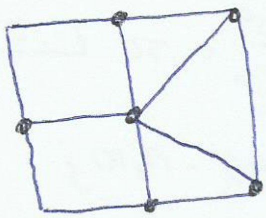
$10 \angle 90 = j20$

$200 \angle -90 = -j200$

$Zeq = 20 + (80 // -j180) = 91,73 \angle -18,9°$

$I = \frac{100 \angle 0°}{91,73 \angle -18,9°} = 1,09 \angle 18,9° (A)$

Cap 10 - Análise de Redes em CA



nó: é o ramo de ligação de pelo menos 3 elementos (6)

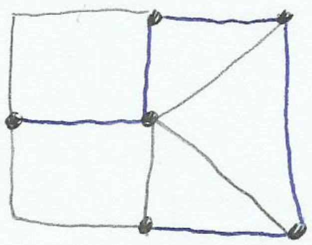
ramo: é o trecho da rede compreendida entre dois nós (10)

Obs.: Cada ramo passa uma corrente diferente

malha: é qualquer circuito fechado de uma rede (n)

rede - é um conjunto de malhas

árvores - é um conjunto de ramos que envolvem todos os nós uma única vez.



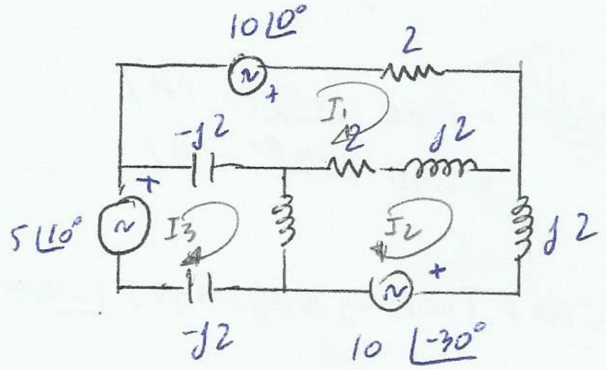
"O número de ramos que faltou para completar a árvore é igual ao número de malhas principais". (5)

Análise de malhas (Maxwell)

- Considerar "correntes de malhas" ao invés de corrente de ramo
- Aplicar LKT nas malhas principais

Obs.: Adotar o mesmo sentido em todas as malhas

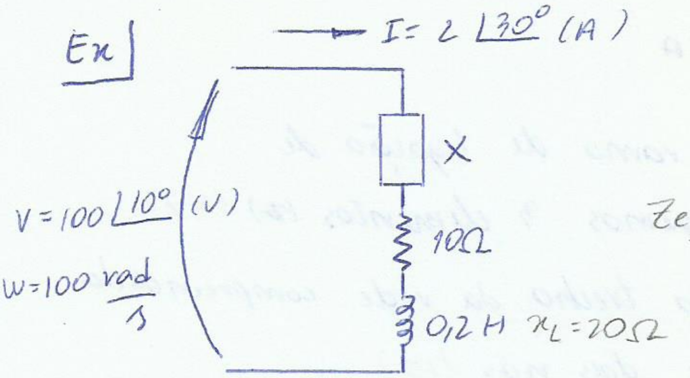
Gerador: (+)
Receptor: (−)



Malha I:

$(2+j2)(I_1 - I_2)$

Ex



$V = 100 \angle 10°$

$I = 2 \angle 30°$

$$Z_{eq} = \frac{V}{I} = \frac{100 \angle 10°}{2 \angle 30°} = 50 \angle -20°$$

$$Z_{eq} = 46{,}98 - 17{,}10j$$

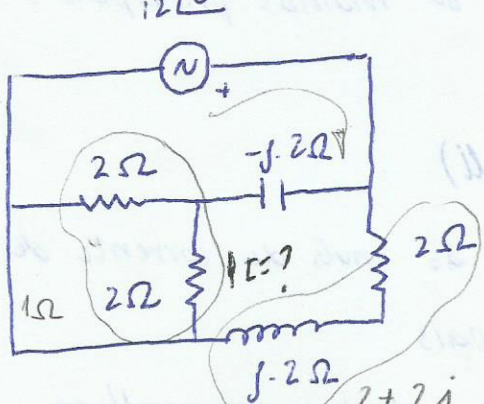
$$Z_{eq} = 10 + 20j + X = 46{,}98 - 17{,}10j$$

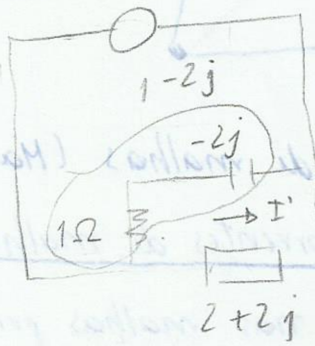
$$\therefore X = 36{,}98 - 37{,}1j = 52{,}38 \angle -45{,}1$$

$$\phi = -45{,}1 < 0 \therefore \text{Capacitor}$$

$$\frac{1}{wC} = x_C \therefore \frac{1}{100 \cdot C} = 52{,}38 \therefore C = 190{,}9\,\mu F$$

Ex 2



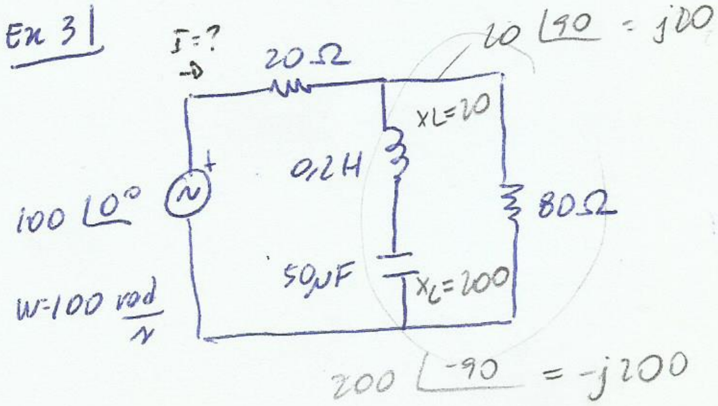


(Importante)

$$I' = \frac{12 \angle 0°}{2{,}24 \angle -63{,}4} = 5{,}36 \angle 63{,}4°$$

$$V'' = -5{,}36 \angle 63{,}4 \therefore I = \frac{-5{,}36 \angle 63{,}4}{2} = -2{,}68 \angle 63{,}4 \text{ (A)} = 2{,}68 \angle -116{,}6° \text{ (A)}$$

Ex 3

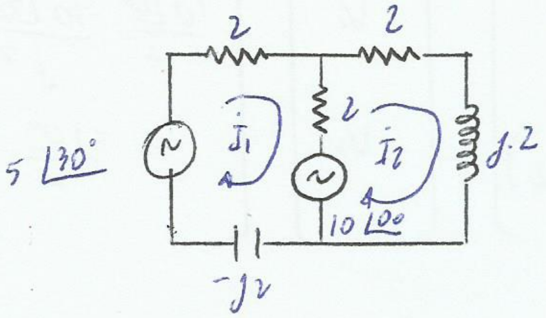


$$Z_{eq} = 20 + (80{,}11 - j\,180) = 91{,}73 \angle -18{,}9°$$

$$I = \frac{100 \angle 0°}{91{,}73 \angle -18{,}9°} = 1{,}09 \angle 18{,}9° \text{ (A)}$$

Gerador : +

Receptor : −

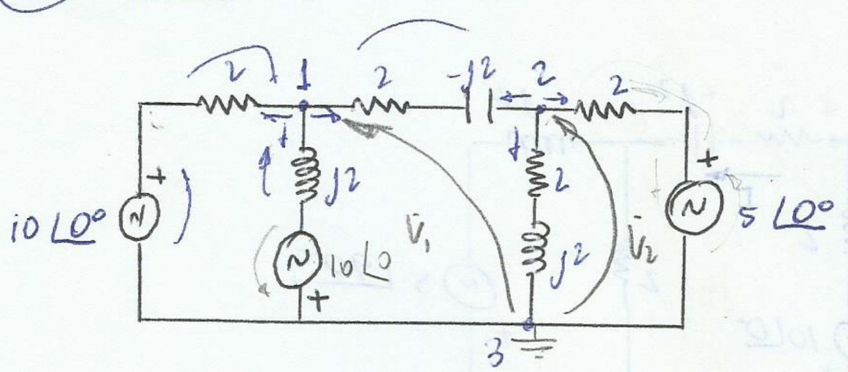


$$\begin{bmatrix} (4-j2) & 2 \\ 2 & (4+j2) \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° + 5\angle 30° \\ -10\angle 0° \end{bmatrix}$$

Análise Nodal

- É feita à partir das tensões entre os nós
- nomear os nós
- escrever as tensões dos nós em relação aos nós de referência (incógnitas)
- Aplicar LKC nos nós

Obs. Adotar o mesmo sentido em todos os nós



nó 1: $$\frac{V_1 - 10\angle 0°}{2} + \frac{V_1 + 10\angle 0°}{j2} + \frac{V_1 - V_2}{2-j2} = 0$$

nó 2: $$\frac{V_2 - 5\angle 0°}{2} + \frac{V_2}{2+j2} + \frac{V_2 - V_1}{2-j2} = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{j2} + \frac{1}{2-j2}\right) - V_2 \cdot \left(\frac{1}{2-j2}\right) = \frac{10\angle 0°}{2} - \frac{10\angle 90°}{j2}$$

$$-V_1\left(\frac{1}{2-j2}\right) + V_2\left(\frac{1}{2} + \frac{1}{2+j2} + \frac{1}{2-j2}\right) = \frac{5\angle 0°}{2}$$

$$\begin{bmatrix} \left(\frac{1}{2}+\frac{1}{j2}+\frac{1}{2-j2}\right) & -\left(\frac{1}{2-j2}\right) \\ -\left(\frac{1}{2-j2}\right) & \left(\frac{1}{2}+\frac{1}{2+j2}+\frac{1}{2-j2}\right)\end{bmatrix}\begin{bmatrix}\dot V_1\\ \dot V_2\end{bmatrix} = \begin{bmatrix}\frac{10\angle 0^\circ}{2}-\frac{10\angle 90^\circ}{j2} \\ \frac{5\angle 0^\circ}{2}\end{bmatrix}$$

$$[Y][V]=[I]$$

$[Y]$:

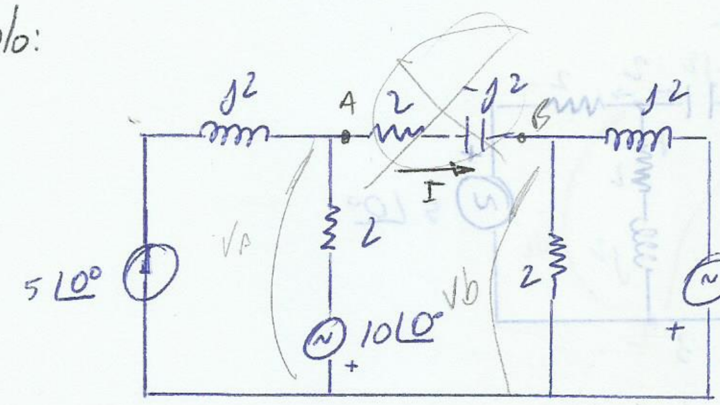
Diagonal Principal (+) = Y_{11}, Y_{22} ... (Soma das admitâncias ligadas no nó)

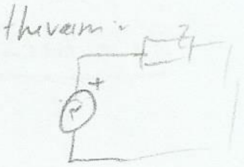
Fora da Diagonal Principal (-) = $Y_{12}=Y_{21}$; $Y_{31}=Y_{13}$ ....
(a admitância entre os nós)

2º membro

gerador de corrente = entrando no nó (+)
receptor de corrente = saindo do nó (-)

Exemplo:





$\dot V_{AB} = \dot E_{th} = V_A - V_b$

$R_{th} = 2 + (2 \,//\, j2)$

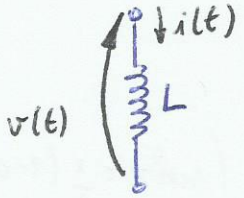
Cap. 11 | Potência em corrente alternada

$$P = V.I$$

$$p(t) = v(t).\,i(t)$$

Elementos de circuito

a) Indutor



Se $\boxed{v(t) = V_{máx} \operatorname{sen} wt \quad (V)}$

então $i(t) = \dfrac{1}{L}\int v(t)\,dt$

$= \dfrac{1}{L}\int V_{máx} \operatorname{sen} wt\, dt$

$= \underbrace{\dfrac{-V_{máx}}{wL}}_{I_{máx}} \cos wt$

$$\boxed{\therefore\ i(t) = -I_{máx} \cos wt}$$

$\therefore\ p(t) = v(t).\,i(t)$

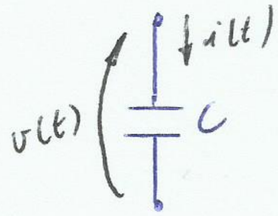
$\therefore\ p(t) = -V_{máx}\, I_{máx} \operatorname{sen} wt \cos wt \quad (W)$

$\therefore\ p(t) = -\dfrac{V_{máx}}{\sqrt{2}}\,\dfrac{I_{máx}}{\sqrt{2}} \operatorname{sen} 2wt$

$$\boxed{\therefore\ p(t) = -VI \operatorname{sen} 2wt \quad (W)}$$

$$\boxed{P_{média} = 0}$$

b) Capacitor



Se $v(t) = V_{máx} \operatorname{sen} wt$ (V)

então $i(t) = \underbrace{wC V_{máx}}_{I_{máx}} \cos wt$

$$\boxed{i(t) = I_{máx} \cos wt \text{ (A)}}$$

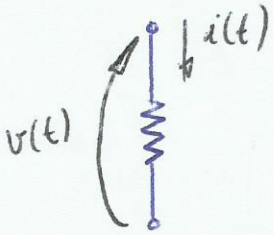
$\therefore\ p(t) = V_{máx} . I_{máx} \operatorname{sen} wt \cos wt$ (w)

$p(t) = \dfrac{V_{máx} I_{máx}}{2} \operatorname{sen} 2wt$

$\operatorname{sen}^2 x = \frac{1}{2}(1 - \cos 2x)$

$$\boxed{\therefore\ p(t) = VI \operatorname{sen} 2wt \text{ (W)}} \qquad \boxed{P_{méd} = 0}$$

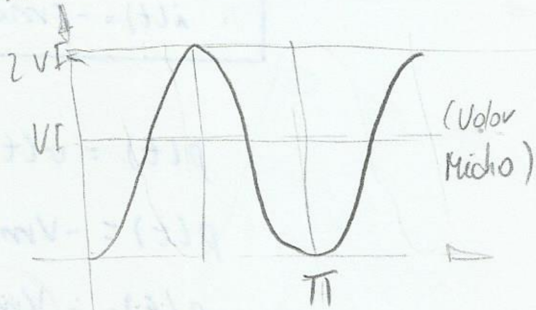
c) Resistor



Se $v(t) = V_{máx} . \operatorname{sen} wt$ (V)

então $i(t) = I_{máx}\ \cancel{\operatorname{sen}} wt$ (V)

$\dfrac{V_{máx}}{I_{máx}}$

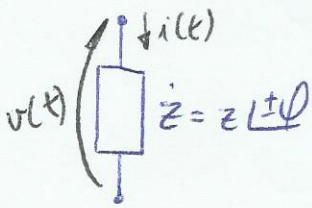


$\therefore\ p(t) = V_{máx} . I_{máx} . \operatorname{sen}^2 wt$ (w)

$= \dfrac{V_{máx} I_{máx}}{2} (1 - \cos 2wt)$

$$\therefore\ \boxed{p(t) = V.I - V.I \cos 2wt \text{ (w)}} \qquad \therefore\ \boxed{P_{média} = V.I}$$

d) Impedância



Se $v(t) = V_{máx} \operatorname{sen} wt$ (V)

então $i(t) = \dfrac{V_{máx}}{z} \operatorname{sen}(wt \mp \varphi) = I_{máx} \operatorname{sen}(wt \mp \varphi)$

$\therefore\ p(t) = V_{máx} I_{máx} \operatorname{sen} wt \operatorname{sen}(wt \mp \varphi)$

$= \dfrac{V_{máx} I_{máx}}{2} [\cos(\pm\phi) - \cos(2wt \mp \varphi)]$

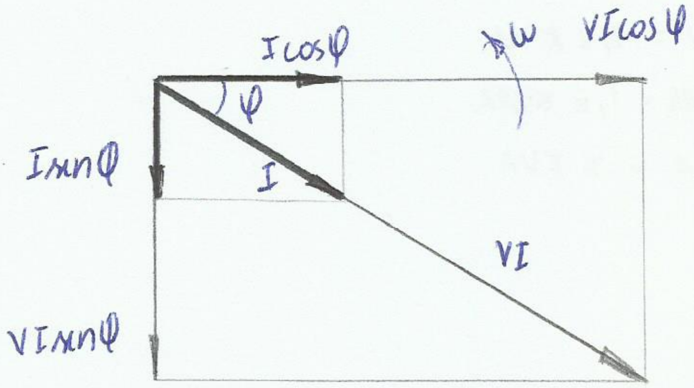
$\cos(a - b) = \cos a \cos b + \operatorname{sen} a \operatorname{sen} b$ ⊖
$\cos(a + b) = \cos a \cos b - \operatorname{sen} a \operatorname{sen} b$

$\operatorname{sen} a \operatorname{sen} b = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$

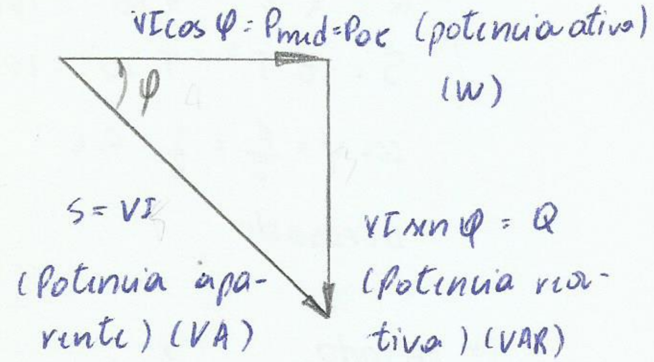
$$\boxed{\therefore\ p(t) = VI \cos\phi - VI \cos(2wt \mp \phi)}$$

$$\boxed{\therefore\ P_{média} = VI \cos\phi}$$

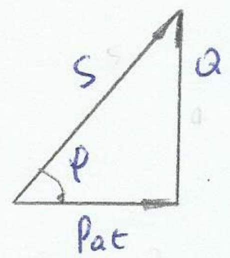
Indutivo



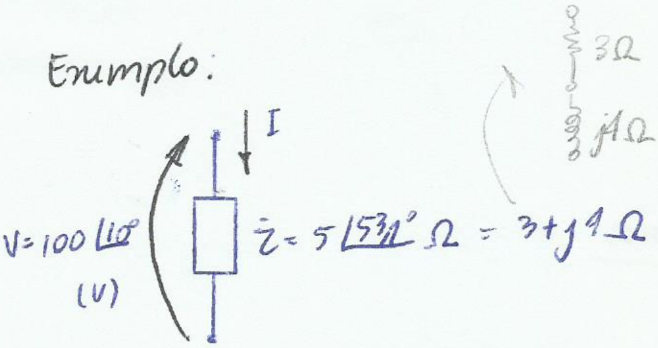
Triangulo das Potências



Capacitivo



Exemplo:



$$I = \frac{100 \angle 10^\circ}{5 \angle 53,1^\circ} = 20 \angle -43,1^\circ \text{ A}$$

x Calcular o triângulo das potências

1º Método

$S = V \cdot I = 100 \cdot 20 = 2000 \text{ VA} = 2 \text{ KVA}$

$P_{at} = VI \cos\varphi = S \cos\varphi = 1200 \text{ W} = 1{,}2 \text{ KW}$

$Q = VI \operatorname{sen}\varphi = S \operatorname{sen}\varphi = 1600 \text{ VAR} = 1{,}6 \text{ KVAR}$

$\cos\varphi = 0{,}6$

Indutivo ou atrasado

2º Método

$$Pat = R.I^2 = 3.20^2 = 1200\,W = 1{,}2\,KW$$
$$Q = X.I^2 = 4.20^2 = 1600\,VAR = 1{,}6\,KVAR$$
$$S = Z.I^2 = 5.20^2 = 2000\,VA = 2\,KVA$$
$$\cos\varphi = \frac{R}{Z} = \frac{3}{5} = 0{,}6$$

atrasado

3º Método

$$Pat = \frac{V_R^2}{R} = \frac{60^2}{3} = 12000\,W = 1{,}2\,KW$$
$$Q = \frac{V_X^2}{X} = \frac{80^2}{4} = 1600\,VAR = 1{,}6\,KVAR$$
$$S = \frac{V^2}{Z} = \frac{100^2}{5} = 2000\,VA = 2\,KVA$$
$$\cos\varphi = \frac{R}{Z} = \frac{3}{5} = 0{,}6$$

atrasado

4º Método Potência Complexa

$$\dot{S} = \dot{V}\dot{I}^* = S\angle\pm\varphi = Pat \pm j\,\varphi$$
$$\dot{S} = 100\angle 10^\circ \cdot 20\angle 43{,}1 = \underbrace{2000}_{S}\,\angle\underbrace{53{,}1^\circ}_{\varphi} = \underbrace{1200}_{Pat} + j\underbrace{1600}_{Q}$$

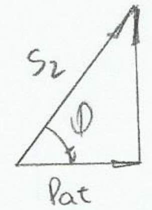
Ex

× Determinar:

a) O triângulo das potências da carga total

b) A leitura do amperímetro

Ramo 2:



$S = Z . I^2 = 30$

$I = \sqrt{\frac{30}{5}} = 2,45 \text{ A}$

$Z_2 = 3 - j4$

$Z_1 = 2 + j2$

$\cos \varphi = \frac{R}{Z} = \frac{3}{5} = 0,6$

$Pat = VI \cos \varphi$

$Pat = S_2 \cos \varphi$

$VI \cancel{\cos\varphi} = S_2 \cos \varphi$

$V = \frac{30}{2,45} = 12,25$

$$Zeq = \frac{(3-j4)(2+j2)}{(3-j4)+(2+j2)} = \frac{6+j6-j8+8}{5-j2} = \frac{14-j2}{5-j2}$$

$$Zeq = \frac{14,14 \angle -8,1}{5,39 \angle -21,8} = 2,62 \angle 13,7$$

(b) $I = \frac{12,25}{2,62} = \boxed{4,68 \text{ A}}$

$S = V . I = 12,25 . 4,68 = 57,33 \text{ W}$

$Pat = 12,25 . 4,68 . \cos 13,7 = 55,7 \text{ VA}$

$Q = 12,25 . 4,68 . \operatorname{sen} 13,7 = 13,58 \text{ VAR}$

Como $\cos \varphi = 0,97$ ∴ atrasado

Ex: Determinar o triangulo das potências da associação das seguintes cargas.

Carga 1 { 100 W Pat_1; f.p. 0,6 $\cos\varphi_1$; atrasada

Carga 2 { 150 VA S_2; f.p. 0,8 $\cos\Phi_2$; adiantado

Carga 3 { 100 VAR Q_3; 200 VA $- S_3$; atrasada

Ex: Um transformador de 200 kVA alimenta uma carga de 100 kW, fator de potência 0,6 atrasado

Determine:

a) Qual a porcentagem de plena carga que o transformador alimenta

b) Quantos kW de carga de fator de potência unitário podem ser acrescentados no transformador?

Resolução

① Lembrete: atrasado - indutivo
adiantado - capacitivo

Carga 1

$P_{1at} = 100\ W$

$S_1 = \frac{100}{0,6} = 166,67\ VA$

$Q_1 = 166,67 \cdot 0,8 \cong 133,33\ VAR$

Carga 2

$S_2 = 150\ VA$

$Pat_2 = 120\ W$

$Q_2 = 90\ VAR$

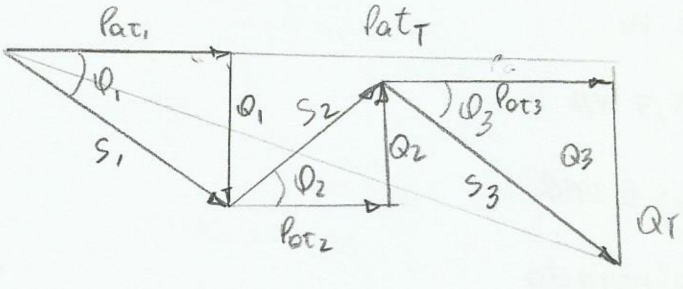
Carga 3

$Q_3 = 100\ VAR$

$S_3 = 200\ VA$

$Pat_3 \cong 173,2\ W$

e $\cos\varphi_3 \cong 0,866$



$Pat_T = Pat_1 + Pat_2 + Pat_3$

$Q_t = Q_1 - Q_2 + Q_3$

$S_T = \sqrt{Pat_T^2 + Q_T^2}$

$\cos\varphi_T = \frac{Pat_T}{S_T}$

atrasada

$Pat_T = 393,2\ W$

$Q_T = 143,3\ VAR$

$S_T = 418,5\ VA$

$\cos\varphi = 0,93$

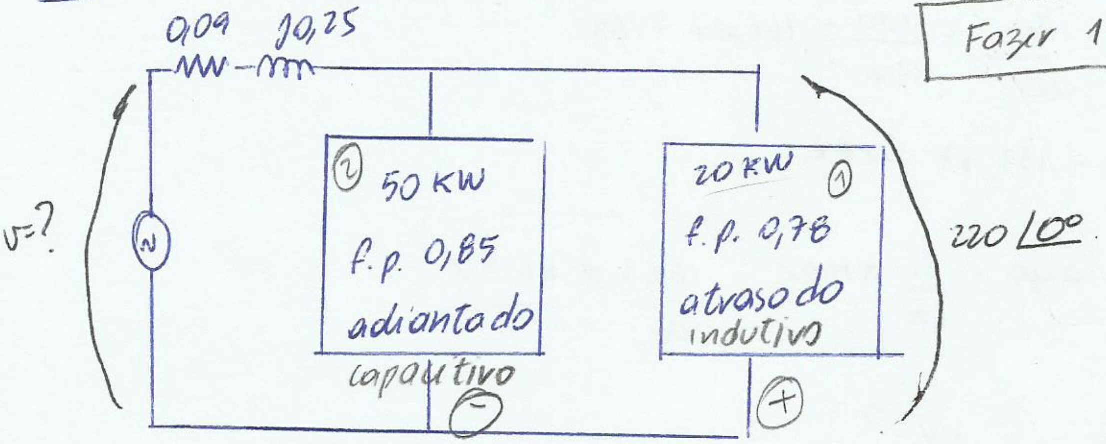
$$\frac{sen\,\beta}{S_N} = \frac{sen\,\alpha}{S_1}$$

$$\frac{sen\,\gamma}{S_2} = \frac{sen\,\beta}{S_N}$$

$S_2 = 111$ KVA

Ex|

Fazer 11.57



① Pat = 20 KW

$$S = \frac{20\,000}{0{,}78} = 25{,}6.10^3 \therefore \dot{S} = 25{,}6.10^3 \angle{-38{,}7°}$$

$$\overset{*}{I} = \frac{25{,}6.10^3 \angle 38{,}7}{220 \angle 0°} = 116{,}55 \angle{+38{,}7°}$$

② Pat = 50 KW

$$S = \frac{50\,000}{0{,}85} = 58823{,}53 \therefore \dot{S} = 58823{,}53 \angle 31{,}8°$$

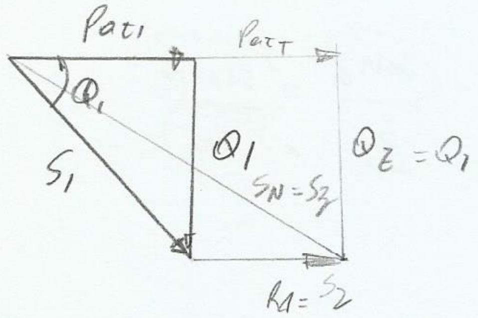
$$\overset{*}{I} = \frac{58\,823{,}53 \angle 31{,}8°}{220 \angle 0°} \therefore \dot{I} = 267{,}38 \angle 31{,}8°$$

$$I = 90{,}96 - j\,72{,}87 + 227{,}24 + j\,140{,}9 = 318{,}2 + j\,68{,}03$$

Ex2:

$S_N = 200$ KVA

Carga 1 { 100 KW (Pat_1)
f.p. 0,6 ∴ $\cos\varphi_1$
atrasado



$$S_1 = \frac{Pot_1}{\cos\theta_1} = \frac{100}{0,6} = 166,67 \text{ KVAR}$$

$$Q_1 = 133,33 \text{ KVAR}$$

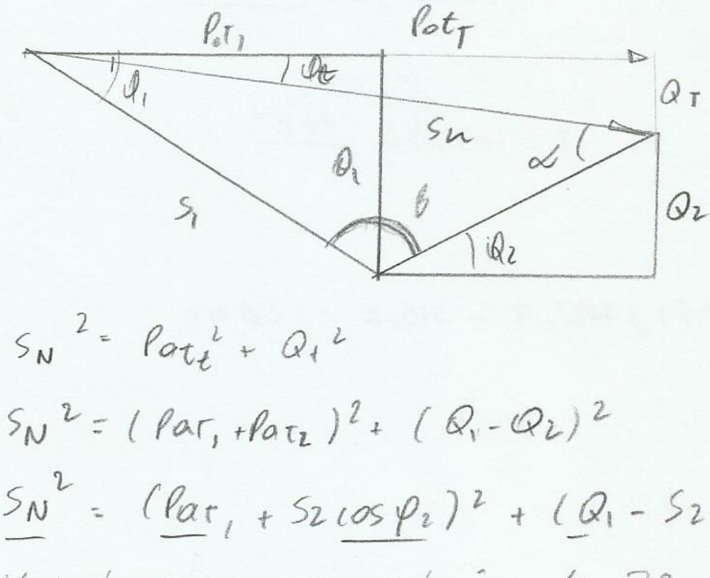
a) % plena carga = $\frac{S_1}{S_N} \times 100\% = \frac{166,7}{200} \simeq \underline{83,3\%}$

b) Carga 2 { KW = ?
f.p. = 1
∴ Carga resistiva, pois $\varphi = 0$

$$Pat_2 = \sqrt{200^2 - 133,33^2} = 149,1 \text{ KW}$$

$\boxed{\text{Resposta: } 49,1 KW}$

Ex1 Se no exercício anterior, as cargas adicionais forem de fator de potência 0,8 adiantado. Quantos KVA delas podemos acrescentar?



$$\frac{\operatorname{sen}\beta}{S_N} = \frac{\operatorname{sen}\alpha}{S_1}$$

Lei dos senos

$$S_N^2 = Pot_t^2 + Q_t^2$$

$$S_N^2 = (Pat_1 + Pat_2)^2 + (Q_1 - Q_2)^2$$

$$S_N^2 = (Pat_1 + S_2\cos\varphi_2)^2 + (Q_1 - S_2\operatorname{sen}\varphi_2)^2$$

↳ Vai chegar numa solução do 2° grau
- se < 0 Não existe, complexo: Não é solução

Correção do fator de potência

Carga $\begin{cases} 5\,KW = P_{at} \\ f.p.\ 0{,}6 = \cos\varphi \\ \text{atrasado} \\ V = 220\,V \end{cases}$ $\longrightarrow$ $\{ \cos\varphi = 0{,}9$

$\therefore I = \dfrac{5K}{220 \times \cos\varphi} \cong \underline{37{,}9\ A}$ $\qquad I = \dfrac{5K}{220 \cdot 0{,}9} \cong \underline{25{,}3\ A}$

Ex.) Dada uma carga

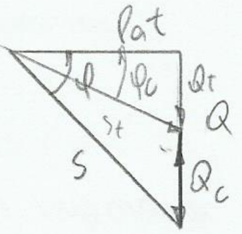
$\begin{cases} 10\,KW = P_{at} \\ f.p.\ 0{,}6 = \cos\varphi \\ \text{atrasada} \\ 110V \\ 60Hz \end{cases}$

Pede-se:

a) Qtos KVAR de capacitor são necessários para corrigir o fator de potência à 0,9 atrasada

b) Qual o valor da capacitância?

a)



$Q = 13{,}3\,KAR$
$S = 16{,}6\,KA$

$tg\,\varphi_T = \dfrac{Q_T}{P_{at}}$

$Q_T = 10K \times 0{,}48 = 4{,}8\,KVAR$

$Q_C = Q - Q_T = 13{,}3\,KVAR - 4{,}8\,KVAR$

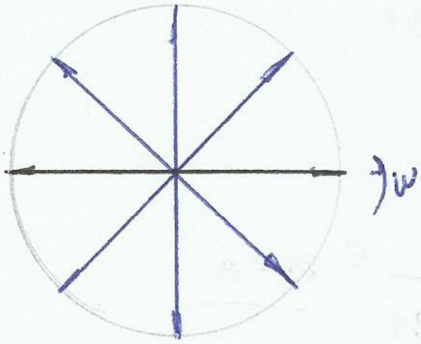
$\boxed{\therefore Q_C \doteq 8{,}5\,KVAR}$

b)

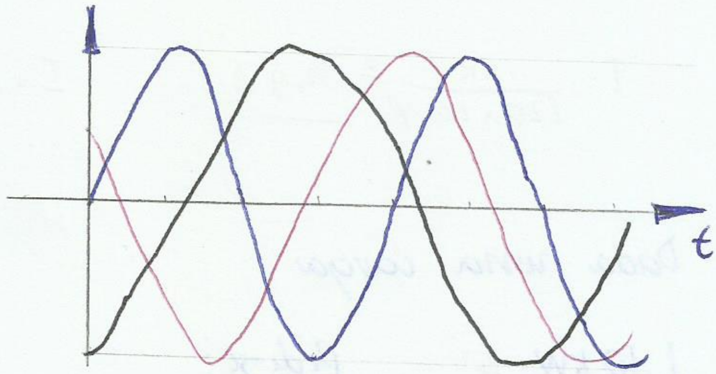
$Q_C = \dfrac{V_X^2}{X_C} = \dfrac{V^2}{X_C} \rightarrow X_C = \dfrac{110^2}{8{,}5K} = 1{,}42\,\Omega$

$1{,}42 = \dfrac{1}{2\pi f C} \quad \therefore \boxed{C \doteq 1{,}9\,mF}$

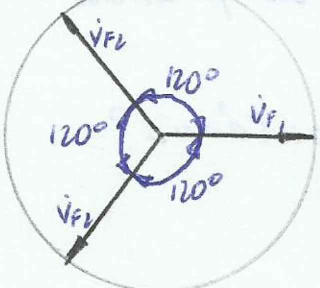
Capítulo 12 - Circuitos polifásicos



A Usina gera o sinal trifásico que é transmitido pelas linhas



Circuitos trifásicos



$\dot{V}_{F1} = V_F \angle 0° \; V$

$\dot{V}_{F2} = V_F \angle 120° \; V$

$\dot{V}_{F3} = V_F \angle 240° \; V$

$V_F = \angle -120° \; V$

$\boxed{\dot{V}_{F1} + \dot{V}_{F2} + \dot{V}_{F3} = 0}$ As somas de três grandezas quaisquer é 0, se a fase é 120°

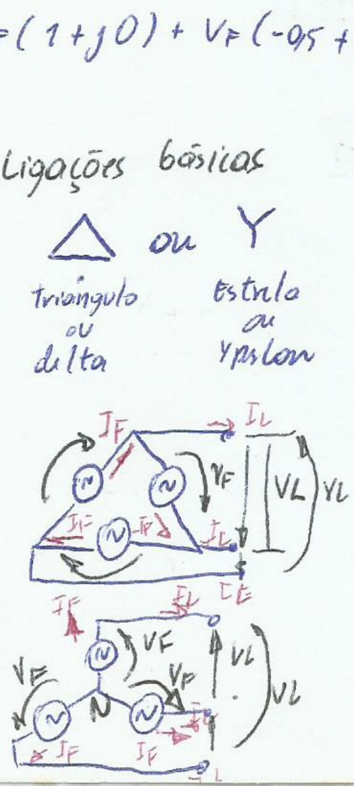
$V_F \angle 0° + V_F \angle 120° + V_F \angle -120° =$

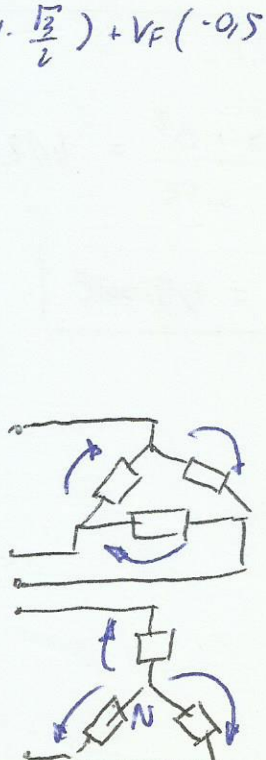
$V_F(1 + j0) + V_F(-0{,}5 + j \cdot \frac{\sqrt{3}}{2}) + V_F(-0{,}5 - j\frac{\sqrt{3}}{2}) = 0$

Ligações básicas

△ ou Y

triângulo ou delta — estrela ou ypsilon





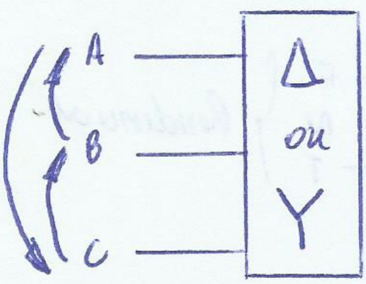
Delta | **Estrela**

$\boxed{V_L = V_F}$ | ?

$\times$ | $\boxed{I_L = I_F}$

$$\dot{V}_L = \dot{V}_{F1} - \dot{V}_{F3}$$
$$= V_F \angle 0^\circ - V_F \angle -120^\circ$$
$$= V_F (1 + j0) - V_F \left(-0{,}5 - j\frac{\sqrt{3}}{2}\right) =$$
$$= V_F \left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = V_F \cdot \sqrt{3} \left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)$$
$$\therefore \boxed{\dot{V}_L = \dot{V}_F \sqrt{3} \angle 30^\circ}$$

Referência (Tensão de linha)

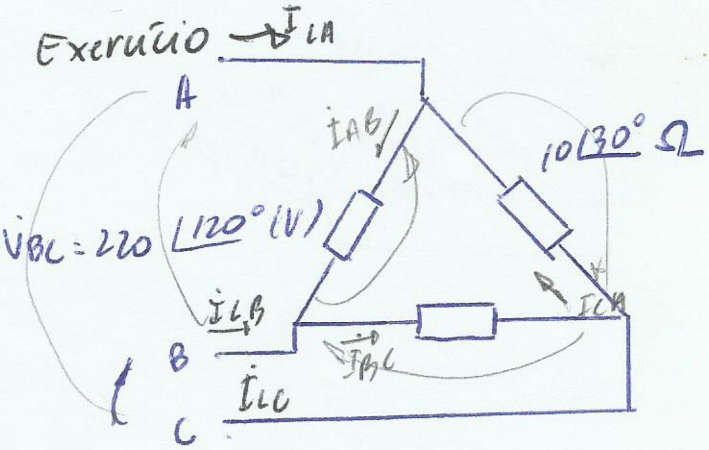


$$\dot{V}_{AB} = V \angle 0^\circ$$
$$\dot{V}_{BC} = V \angle 120^\circ$$
$$\dot{V}_{CA} = V \angle -120^\circ$$

$$V_{AN} = \frac{V_{AB}}{\sqrt{3} \angle 30}$$
$$V_{BN} = \frac{V}{\sqrt{3}} \angle 90^\circ$$
$$V_{CN} = \frac{V}{\sqrt{3}} \angle -150$$

Sequência ABC

Exercício



Carga equilibrada
- quando as impedâncias de fase são iguais

$$\dot{I}_{AB} = \frac{\dot{V}_{AB}}{\dot{Z}_{AB}} = \frac{220 \angle 0^\circ}{10 \angle 30^\circ} = 22 \angle -30^\circ \text{ A}$$

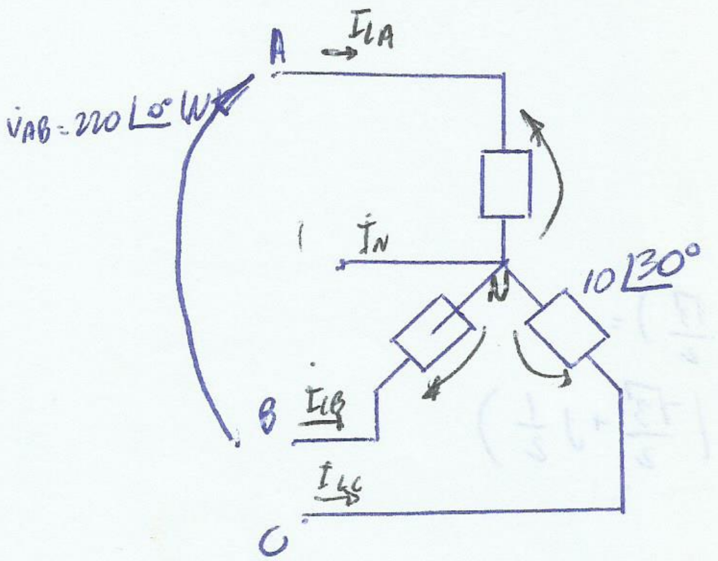
$$\dot{I}_{BC} = \frac{\dot{V}_{BC}}{\dot{Z}_{BC}} = \frac{220 \angle +120^\circ}{10 \angle 30^\circ} = 22 \angle 90^\circ \text{ A}$$

$$\dot{I}_{CA} = \frac{\dot{V}_{CA}}{\dot{Z}_{CA}} = \frac{220 \angle -120^\circ}{10 \angle 30^\circ} = 22 \angle -150^\circ \text{ A}$$

$\dot{I}_{LA} = \dot{I}_{AB} - \dot{I}_{CA} = 22\angle{-30°} - 22\angle{-150°} =$

$\dot{I}_{LB} = \dot{I}_{BC} - \dot{I}_{AB} = 22\angle{90°} - 22\angle{-30°} =$

$\dot{I}_{LC} = \dot{I}_{CA} - \dot{I}_{BC} = 22\angle{-150} - 22\angle{90°} =$



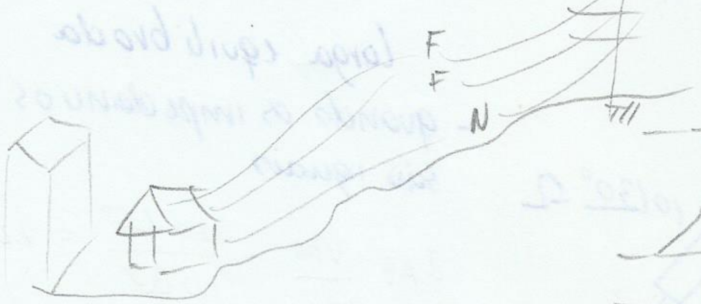
$$\dot{I}_{LA} = \frac{\dot{V}_{AN}}{\dot{Z}_{AN}} = \frac{220\angle{0°}}{10\angle{30°}\ \angle{30°}\sqrt{3}} = 12{,}7\angle{-60}\ (A)$$

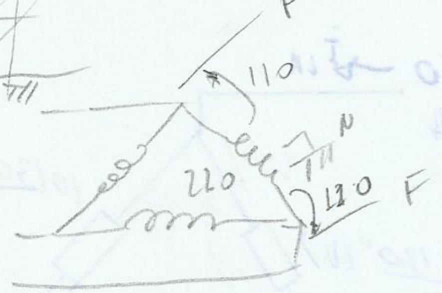
$$\dot{I}_{LB} = \frac{\dot{V}_{BN}}{\dot{Z}_{BN}} = \frac{220°\angle{120°}}{10\angle{30°}\ \angle{30°}\sqrt{3}} = 12{,}7\angle{60}\ (A)$$

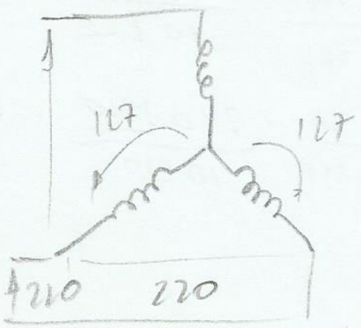
$$\dot{I}_{LC} = \frac{\dot{V}_{CN}}{\dot{Z}_{CN}} = \frac{220°\angle{-120°}}{10\angle{30°}\angle{30°}\sqrt{3}} = 12{,}7\angle{-160°}\ (A)$$

$\dot{I}_N = -(\dot{I}_{LA} + \dot{I}_{LB} + \dot{I}_{LC}) = 0$

F, F, F, N, P		Comercial / Industrial
F, N, T		Residencial







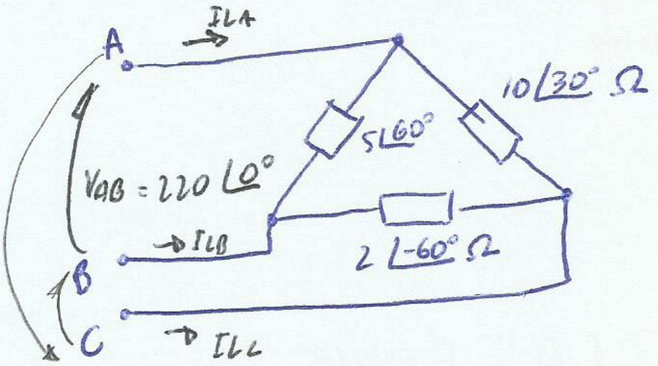
Cargas desiquilibradas

- quando as cargas por fase forem diferentes

$$\boxed{I_{L\Delta} = 3\, I_{L\lambda}}$$

Para um sistema equilibrado

Em triângulo



$$\dot I_{AB} = \frac{220\angle 0°}{5\angle 60°} = 44\angle -60° \qquad I_{BC} = \frac{220\angle 120}{2\angle -60} = 110\angle 180°$$

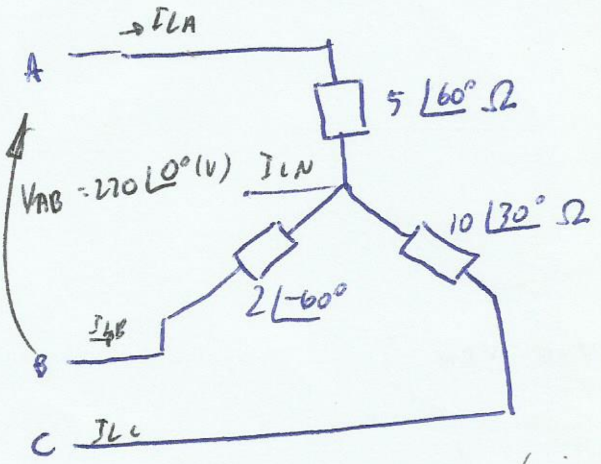
$$\dot I_{CA} = \frac{220\angle -120}{10\angle 30°} = 22\angle -150°$$

$\dot I_{LA} = \dot I_{AB} - \dot I_{LA} = 49{,}2\angle -33{,}4°$ A

$\dot I_{LB} = \dot I_{BC} - \dot I_{AB} = 137{,}4\angle 163{,}9$ A

$\dot I_{LC} = \dot I_{CA} - \dot I_{BC}$

Em Estrela a 4 fios



$$\dot I_{LA} = \frac{\dot V_{AN}}{\dot Z_{AN}} = \frac{\frac{220\angle 0°}{\sqrt3\angle 30°}}{5\angle 60} = 25{,}4\angle -90°$$

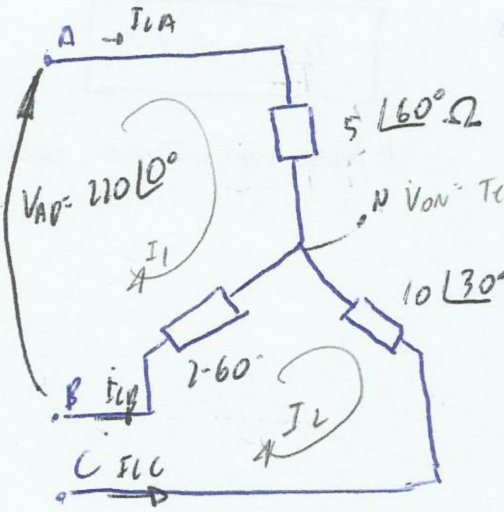
$$I_{LB} = \frac{V_{BN}}{\dot Z_{BN}} = \frac{220\angle 120°}{\sqrt3\angle 30 \cdot 2\angle -60} = 63{,}4\angle 150°$$

$$\dot I_{LC} = \frac{\dot V_{CN}}{\dot Z_{CN}} = \frac{220\angle -120}{\sqrt3\angle 30 \cdot 10\angle 30} = 12{,}7\angle -180°$$

$$\dot I_{LN} = -(\dot I_{LA} + \dot I_{LB} + \dot I_{LC})$$

$$I_{LN} = -(25{,}4\angle -90 + 63{,}4\angle 150 + 12{,}7\angle -180) = 67{,}60\angle 0{,}3$$

Estrela à 3 Fios



V_{ON}: Tensão de desequilíbrio no neutro

$$\begin{bmatrix} 5\angle 60° + 2\angle -60 & -2\angle -60° \\ -2\angle -60° & 2\angle -60° + 10\angle 30° \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} 220\angle 0° \\ 220\angle 120° \end{bmatrix}$$

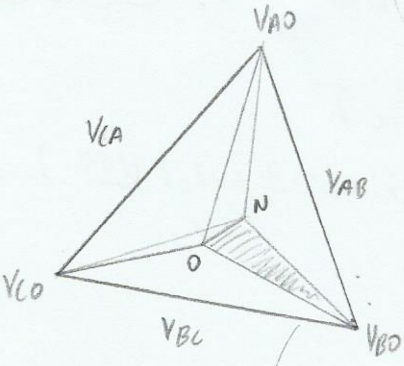
$$\left.\begin{array}{l} \dot{I}_1 = \\ \\ \dot{I}_2 = \end{array}\right\} \begin{array}{l} \dot{I}_{LA} = \dot{I}_1 \\ \dot{I}_{LB} = \dot{I}_2 - \dot{I}_1 \\ \dot{I}_{LC} = -\dot{I}_2 \end{array}$$

$\dot{V}_{AO} = \dot{I}_{LA} \cdot 5\angle 60° =$

$\dot{V}_{BO} = \dot{I}_{LB} \cdot 2\angle -60° =$

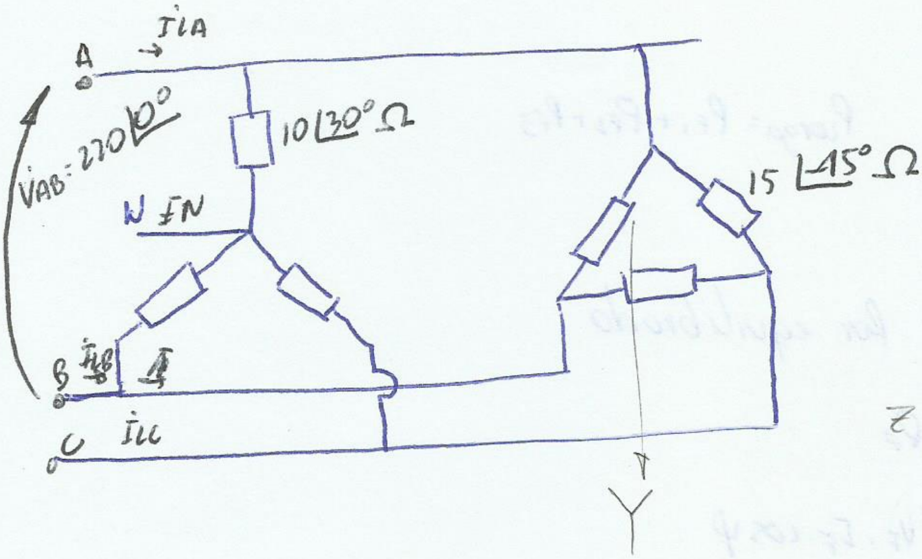
$\dot{V}_{CO} = \dot{I}_{LC} \cdot 10\angle 30° =$

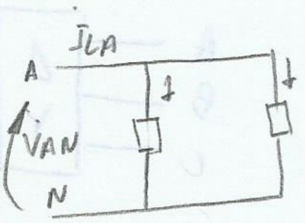
$\dot{V}_{ON} = \dot{V}_{BO} - \dot{V}_{BN}$



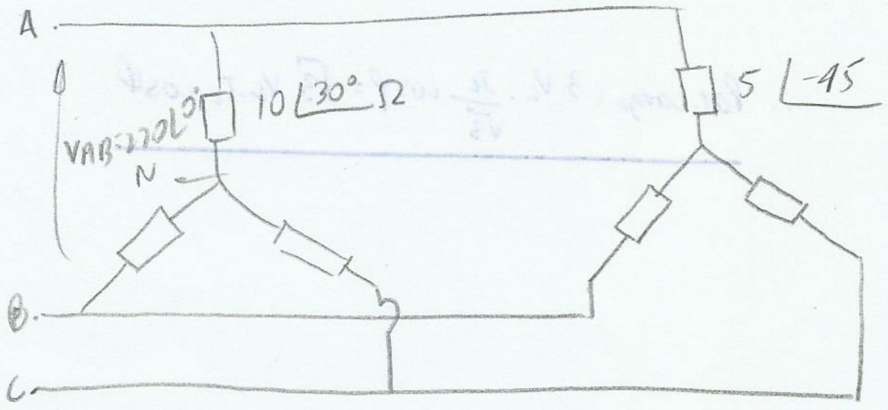
Triângulo equilátero

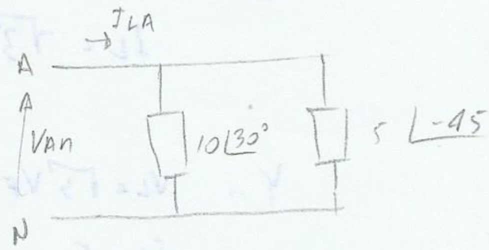
Se o sistema é equilibrado





$$Z = \frac{(15\angle -45°)^2}{3(15\angle 45)} = \frac{225\angle -90}{45\angle -45}$$





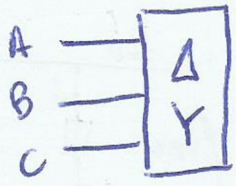
$$V_{AN} = \frac{220\angle 0°}{\sqrt{3}\angle 30} = 127\angle -30°$$

$$\dot{I}_1 = \frac{127\angle -30°}{10\angle 30} = 12{,}7\angle -60°\ A$$

$$\dot{I}_2 = \frac{127\angle -30}{5\angle -45} = 25{,}4\angle 15°\ A$$

$$\dot{I}_1 + \dot{I}_2 = \boxed{\begin{array}{l} \dot{I}_{LA} \cong 31{,}20\angle -8{,}2° \\ \dot{I}_{LB} \cong 31{,}2\angle 118{,}8° \\ \dot{I}_{LC} \cong 31{,}2\angle -128{,}2° \end{array}}$$

Potência em sistema trifásico



$P_{carga} = P_{F1} + P_{F2} + P_{F3}$

Se o sistema for equilibrado

$P_{carga} = 3\,P_F$

$P_{at\ carga} = 3 \cdot V_F \cdot I_F \cdot \cos\varphi$

$\Delta \rightarrow \quad V_L = V_F$

$I_L = \sqrt{3}\, I_F$

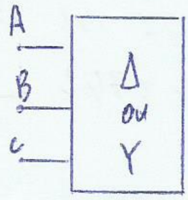
$\therefore \; P_{at.\,carga} = 3 \cdot V_L \cdot \frac{I_L}{\sqrt{3}} \cos\varphi = \sqrt{3}\, V_L \cdot I_L \cdot \cos\varphi$

$Y \rightarrow \quad V_L = \sqrt{3}\, V_F$

$I_L = I_F$

$P_{at\ carga} = 3 \cdot \frac{V_L \cdot I_L}{\sqrt{3}} \cos\varphi = \sqrt{3}\, V_L \cdot I_L \cdot \cos\varphi$

Sistema Equilibrado



$$\left(\begin{array}{l} P_{at_{carga}} = \sqrt{3}\, V_L \cdot I_L \cos\varphi \\ \therefore S_{carga} = \sqrt{3}\, V_L\, I_L \\ Q_{carga} = S_{carga} \operatorname{sen}\varphi \end{array} \right)$$

Medida da Potência em sistemas polifásicos

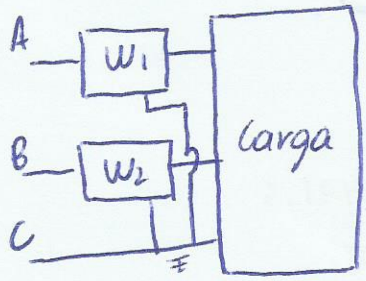
Teorema de Blundall

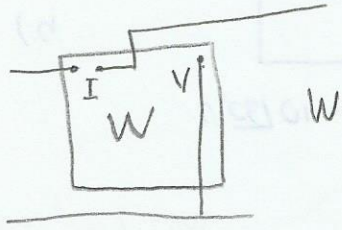
"Nos sistemas polifásicos são necessários $n-1$ (n: nº de fases) wattímetros"

$$\boxed{P_{at} = \Sigma W}$$

∴ Sistemas Trifásico

são necessários 2 wattímetros (2 leituras)





$W = V_L I_L \cos \angle^{V_L}_{I_L}$

$$P_{at} = W_1 + W_2$$

$$W_1 = V_L I_L \cos(30° - \varphi) = \frac{\sqrt{3}}{2} V_L I_L \cos\varphi + \frac{1}{2} V_L I_L \operatorname{sen}\varphi$$

$$W_2 = V_L I_L \cos(30° + \varphi) = \frac{\sqrt{3}}{2} V_L I_L \cos\varphi - \frac{1}{2} V_L I_L \operatorname{sen}\varphi$$

$\pm$

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\varphi = P_{at\,carga}$$

$$W_1 - W_2 = V_L I_L \operatorname{sen}\varphi$$

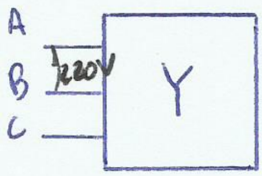
$\div$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\cancel{V_L I_L} \operatorname{sen}\varphi}{\sqrt{3}\,\cancel{V_L I_L} \cos\varphi} = \frac{tg\,\varphi}{\sqrt{3}} \quad \therefore \quad tg\,\varphi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

Sabendo a sequência (ABC)

$$tg\,\varphi = \sqrt{3}\,\frac{W_A - W_B}{W_A + W_B} = \sqrt{3}\,\frac{W_B - W_C}{W_B + W_C}$$

Exercício



$W_B = -600\,W$

$W_C = 1000\,W$

$Z_F = ?$

$$tg\,\varphi = \sqrt{3} \cdot \left[\frac{(-600) - 1000}{(-600) + 1000}\right]$$

$\therefore \varphi = -81{,}79$

$P_{at} = 400\,W$

$\therefore$ ~~$\dot{V} = 220\,\angle 0°$~~

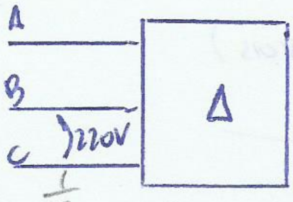
$$400 = \sqrt{3} \cdot \frac{220^2}{\left(\frac{Z}{\sqrt{3}}\right)} \cos(-81{,}79) \quad \therefore \quad Z = \frac{29{,}93\,\Omega}{\sqrt{3}} \quad \therefore \quad Z = 17{,}28\,\angle{-81{,}8}$$

---

ou $Z_F = \frac{V_F}{I_F}$; $I_L = I_F$, $V_F = \frac{V_L}{\sqrt{3}}$ $\quad Z_F = \frac{V_L}{\sqrt{3}\, I_L}$; $P_{at} = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos\varphi$

$$= \sqrt{3} \cdot \frac{V_L \cdot V_L}{Z_F \cdot \sqrt{3}} \cos\varphi \quad \therefore \quad Z = 27{,}22°$$

Ex:



a) $W_A = ?$

$W_B = ?$

b) $Pot_{carga} = ?$

$Z_F = 10\angle 30^\circ\ \Omega$

Δ: $V_L = V_F$

$I_L = \sqrt{3}\, I_F$

~~$W_A = \frac{220^2}{10} \cdot \cos 30 = 4191{,}6$~~

$\dot{I}_{AB} = \frac{V_F}{Z_{AB}} = \frac{220\angle 0^\circ}{10\angle 30^\circ} = 22\angle -30^\circ$

$\dot{I}_{BC} = 22\angle 90$

$\dot{I}_{CA} = 22\angle -150^\circ$

$I_{LA} = I_{AB} - I_{LD} = 38{,}1\angle -150$

$I_{LB} = 38{,}1\angle 120^\circ$

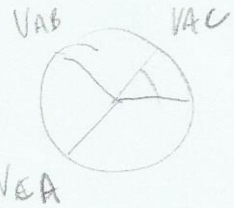
$I_{LC} = 38{,}1\angle -120^\circ$

$Pot_{carga} = \sqrt{3} \cdot 220 \times 38{,}1 \times \cos 30^\circ = \underline{12.573\,W}$

$W_A = V_{AC} \times I_{LA} \cdot \cos \angle^{AC}_{A} = 220 \times 38{,}1 \cdot \cos(60^\circ - 0^\circ) = 4191\,W$

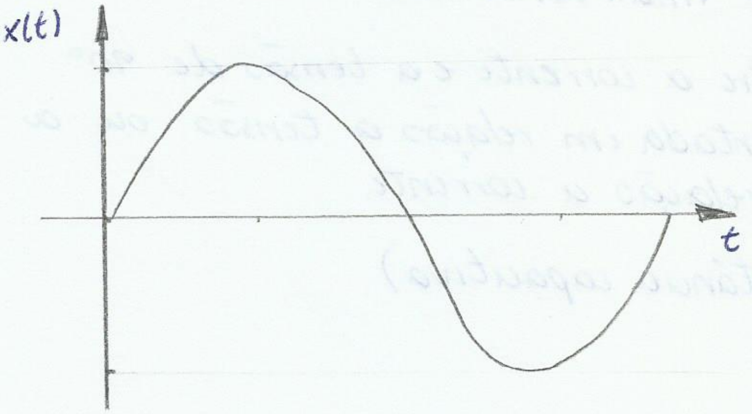
$W_B = V_{BC} \times I_{LB} \cdot \cos \angle^{AB}_{B} = 220 \times 38{,}1 \cdot \cos(120^\circ - 120) = \underline{8382\,W}$

12573 W



Circuitos II - Estudo Para P2

Capítulo 9 - Senóides e Fasores



$x(t) = X_m \operatorname{sen} \omega t$

$\omega = 2\pi f = \frac{2\pi}{T}$

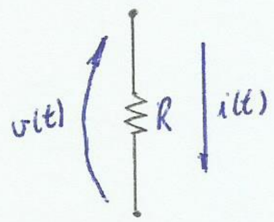
- Em corrente alternada a corrente e a tensão é uma onda senoidal:
  - O valor médio é:

$I_{rms} = I_{medio} = I_{eficaz} = \frac{I_{máx}}{\sqrt{2}}$

$V_{rms} = V_{medio} = V_{eficaz} = \frac{V_{máx}}{\sqrt{2}}$

Elementos de circuito

Resistor:



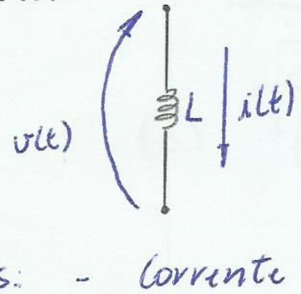
$i(t) = I_{máx} \operatorname{sen} \omega t \quad (A)$

$v(t) = \underbrace{R \cdot I_{máx}}_{V_{máx}} \operatorname{sen} \omega t$

$v(t) = V_{máx} \operatorname{sen} \omega t \quad (V)$

* Obs.: Na resistência, a tensão e a corrente estão em fase

Indutor:



$i(t) = I_{máx} \operatorname{sen} \omega t \quad (A)$

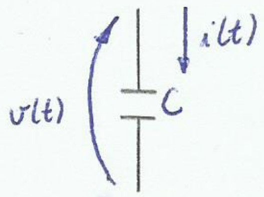
$v(t) = \underbrace{\omega L I_{máx}}_{V_{máx}} \cos \omega t$

$v(t) = V_{máx} \cos \omega t \quad (V)$

* Obs.: - Corrente e tensão estão defasado em 90°
- v(t) adiantado em relação a corrente i(t) ou a corrente atrasada em relação a tensão.

$X_L = \omega L = \frac{V}{I} \quad (\Omega)$

Capacitor:



$i(t) = I_{máx} \, \text{sen} \, wt \; (A)$

$v(t) = \underbrace{\frac{-1}{wL} I_{máx}}_{V_{máx}} \cos wt$

$v(t) = V_{máx} \cos wt \; (V)$

- Existe defasagem entre a corrente e a tensão de 90°
- A corrente está adiantada em relação a tensão ou a tensão atrasada em relação a corrente.

$\frac{V}{I} = \frac{1}{wC} = x_C$ (Reatância capacitiva)

Circuito RLC série

Lembretes: $90° > \varphi > 0 \rightarrow$ Circuito é indutivo

$-90° < \varphi < 0 \rightarrow$ Circuito é capacitivo

$\varphi = 0 \rightarrow$ Circuito é resistivo

Dicas: $e(t) = v_R(t) + v_L(t) + v_C(t)$

$$\varphi = tg^{-1} \frac{wL - \frac{1}{wC}}{R} = tg^{-1} \frac{x_L - x_C}{R}$$

$$Z_{eq} = \frac{V}{I} = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2} \quad \text{(Impedância)}$$

Circuito RLC paralelo

Lembrete: $\varphi' > 0$: circuito capacitivo

$\varphi' < 0$: circuito indutivo

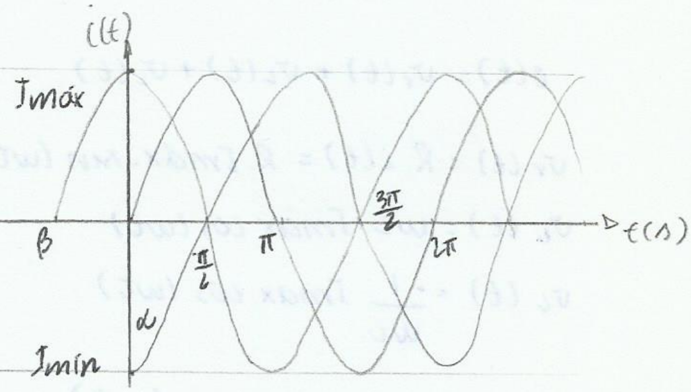
$\varphi = 0$: circuito resistivo

Dicas: $i(t) = i_R(t) + i_L(t) + i_C(t)$

$$\varphi' = tg^{-1} R\left(wC - \frac{1}{wL}\right) \quad \text{ou} \quad tg^{-1} \frac{\frac{1}{x_C} - \frac{1}{x_L}}{\frac{1}{R}}$$

$$Y_{eq} = \frac{1}{Z_{eq}} = \frac{I}{V} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{x_C} - \frac{1}{x_L}\right)^2} \quad (S)$$

Capítulo 9 - Senóides e fasores (Resumo)



$i(t) = I_{máx} \cdot sen\, wt$ (A)

$i(t) = I_{máx}\, sen(wt - \alpha)$ (A)

$i_1(t) = I_{máx}\, sen(wt + \beta)$ (A)

Corrente e tensão eficaz

$$\boxed{I_{rms} = I_{ef} = \frac{I_{máx}}{\sqrt{2}} \qquad V_{rms} = V_{ef} = \frac{V_{máx}}{\sqrt{2}}}$$

Elementos de Circuito

Resistor: Para $i(t) = I_{máx} \cdot sen\, wt$ (A), temos

$v(t) = V_{máx} \cdot sen\, wt$ (V) ; $V_{máx} = R \cdot I_{máx}$

× Obs.: A corrente e a resistência estão em fase

Indutor: Para $i(t) = I_{máx} \cdot sen\, wt$ (A), temos

$v(t) = I_{máx} \cdot L \cdot w \cdot cos\, wt$ (V)

$\therefore v(t) = V_{máx}\, cos\, wt$ (V) ; $V_{máx} = I_{máx} \cdot L \cdot w$

lembrete

$v(t) = L \dfrac{di(t)}{dt}$

$i(t) = \dfrac{1}{L} \int v(t)\, dt$

× Obs.: - A corrente e a tensão estão defasada em 90° no tempo

- $v(t)$ adiantada em relação a $i(t)$ ou $i(t)$ atrasada em relação a $v(t)$.

- $\dfrac{V_{máx}}{I_{máx}} = \dfrac{V_{rms}}{I_{rms}} = wL$ (Ω) $\quad x_L = wL$ (Reatância indutiva)

Capacitor: Para $i(t) = I_{máx} \cdot sen\, wt$ (A), temos

$v(t) = -\dfrac{I_{máx}}{wC} cos\, wt \therefore v(t) = -V_{máx}\, cos\, wt$

$V_{máx} = \dfrac{I_{máx}}{wC}$

lembrete:

$i(t) = C \dfrac{dv(t)}{dt}$

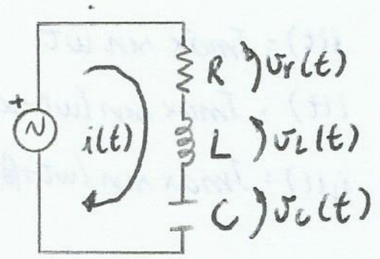
$v(t) = \dfrac{1}{C} \int i(t)\, dt$

× Obs.: - A corrente e a tensão estão defasada em 90°

- Tensão é atrasada em relação a corrente, ou corrente adiantada em relação a tensão

$\dfrac{V_{máx}}{I_{máx}} = \dfrac{V_{rms}}{I_{rms}} = \dfrac{1}{wC} = x_C$ (Reatância capacitiva)

Circuito RLC série



$$e(t) = v_r(t) + v_L(t) + v_c(t)$$

$$v_r(t) = R.\,i(t) = R.\,I_{máx}.\,sen\,(wt)$$

$$v_L(t) = wL.\,I_{max}\cos(wt)$$

$$v_c(t) = \frac{-1}{wC} I_{max}\cos(wt)$$

$$e(t) = R\,I_{máx}\,\underline{sen(wt)} + wL\,I_{max}\,\underline{\cos(wt)} - \frac{1}{wC}\,I_{max}\,\underline{\cos(wt)}$$

$$e(t) = A.\,sen\,(wt + \varphi)$$

$$= A\,\underline{sen\,wt}.\cos\varphi + A\,sen\,\varphi\,\underline{\cos wt}$$

$$\begin{cases} A\,sen\,\varphi = \left(wL + \dfrac{1}{wC}\right) \cdot I_{max} & (I) \\ A\cos\varphi = R.\,I_{máx} & (II) \end{cases}$$

Dividindo (I) por (II)

$$tg\,\varphi = \frac{wL - \frac{1}{wC}}{R} = \frac{X_L - X_C}{R}$$

$$\boxed{\varphi = tg^{-1}\frac{wL - \frac{1}{wC}}{R} = tg^{-1}\frac{X_L - X_C}{R}}$$

$$\begin{cases} A^2 sen^2\varphi = \left(wL + \dfrac{1}{wC}\right)^2 I^2_{max} \\ A^2\cos^2\varphi = R^2\,I^2_{máx} \end{cases}$$

Somando, racionalizando e simplificando

$$\boxed{A = I_{máx}\sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2} = I_{máx}\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\boxed{e(t) = A\,sen\left[wt + tg^{-1}\left(\frac{wL - \frac{1}{wC}}{R}\right)\right]}$$

Impedância: $$Z_{eq} = \frac{V_{máx}}{I_{máx}} = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}$$

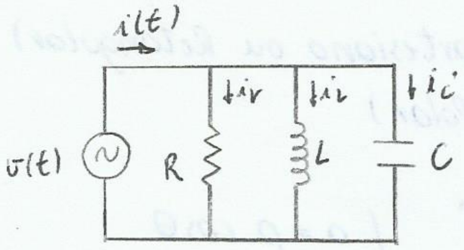
$$\varphi = tg^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$90° > \varphi > 0$ ∴ Circuito indutivo

$-90° < \varphi < 0$ ∴ Circuito capacitivo

$\varphi = 0$ ∴ Circuito resistivo

Circuito RLC Paralelo



Se $v(t) = V_{máx} \cdot sen\, wt$, temos:

$$i_r(t) = \frac{V_{máx}}{R} sen\, wt$$

$$i_L(t) = -\frac{1}{wL} V_{máx} \cos wt$$

$$i_C(t) = wC\, V_{máx} \cos wt$$

L.K.C

$$i(t) = i_r(t) + i_L(t) + i_C(t)$$

$$= \frac{V_{máx}}{R} sen\, wt + \left(wC - \frac{1}{wL} \right) V_{máx} \cos wt$$

$$i(t) = B\, sen\,(wt + \varphi')$$

$$= B\, sen\, wt \cos\varphi' + B\, sen\,\varphi' \cos wt$$

$$\begin{cases} B\, sen\,\varphi' = \left(wC - \frac{1}{wL} \right) V_{máx} & (I) \\ B\cos\varphi' = \frac{V_{máx}}{R} & (II) \end{cases}$$

Dividindo (I) por (II)

$$tg\,\varphi' = \frac{wC - \frac{1}{wL}}{R}$$

$$\boxed{\varphi' = tg^{-1}\left[R \left(wC - \frac{1}{wL} \right) \right] = tg^{-1}\left[\frac{\frac{1}{x_C} - \frac{1}{x_L}}{\frac{1}{R}} \right]}$$

$$B = \sqrt{\left(\frac{1}{R}\right)^2 + \left(wC - \frac{1}{wL} \right)^2}\; V_{máx}$$

$$\boxed{i(t) = \underbrace{V_{máx} \sqrt{\left(\frac{1}{R}\right)^2 + \left(wC - \frac{1}{wL} \right)^2}}_{B}\; sen\left[wt + tg^{-1} R \left(wC - \frac{1}{wL} \right) \right] \text{(A)}}$$

$$I_{máx} = V_{máx} \sqrt{\left(\frac{1}{R}\right)^2 + \left(wC - \frac{1}{wL} \right)^2}$$

(Admitância)

$$\frac{I_{máx}}{V_{máx}} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{x_C} - \frac{1}{x_L} \right)^2} \;(S) = Y_{eq} = \frac{1}{Z_{eq}}$$

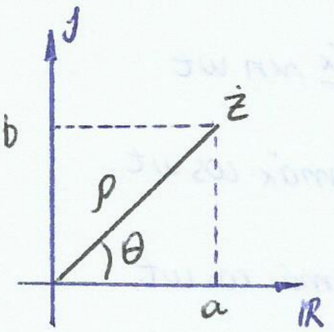
Quando:

$\varphi' > 0$: circuito capacitivo

$\varphi' = 0$: circuito resistivo

$\varphi' < 0$: circuito indutivo

Números complexo



$\dot{Z} = a + jb$ (Cartesiano ou Retangular)

$\dot{Z} = \rho \angle \theta$ (Polar)

$$\begin{cases} \rho = \sqrt{a^2+b^2} \\ \theta = tg^{-1} \dfrac{b}{a} \end{cases} \qquad \begin{cases} a = \rho . \cos\theta \\ b = \rho . \operatorname{sen}\theta \end{cases}$$

Operações

- Soma e subtração (usa-se a forma

$\dot{Z}_1 = a + jb \qquad \dot{Z}_2 = c + jd$

$\dot{Z}_1 \pm \dot{Z}_2 = (a \pm c) + j(b \pm d)$

- Multiplição, Divisão, exponenciação e Radiação (usa-se na forma polar)

$\dot{Z}_1 = \rho_1 \underline{|\theta_1} \qquad \dot{Z}_2 = \rho_2 \underline{|\theta_2}$

$\dot{Z}_1 \times \dot{Z}_2 = \rho_1 \times \rho_2 \underline{|\theta_1 + \theta_2}$

$\dfrac{\dot{Z}_1}{\dot{Z}_2} = \dfrac{\rho_1}{\rho_2} \underline{|\theta_1 - \theta_2}$

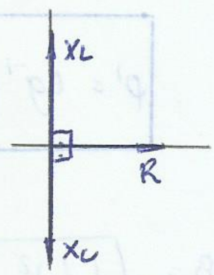
$\dot{Z}_1^2 = \rho_1^2 \underline{|2\theta_1}$

$\sqrt{\dot{Z}_1} = \sqrt{\rho_1} \underline{|\theta_1/2}$

Complexo Conjudos

$Z_1 = a + jb \qquad \dot{Z}_1 = \rho \underline{|\theta}$

$Z_1^* = a - jb \qquad \dot{Z}_1^* = \rho \underline{|-\theta}$

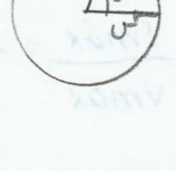


$\dot{Z}_R = R + j0 = R \underline{|0^\circ} \ \Omega$

$\dot{Z}_L = 0 + jXL = XL \underline{|90^\circ} \ \Omega$

$\dot{Z}_C = 0 - jX_C = X_C \underline{|-90^\circ} \ \Omega$

$\dot{Z}eq = \dot{Z}_R + \dot{Z}_L + \dot{Z}_C$

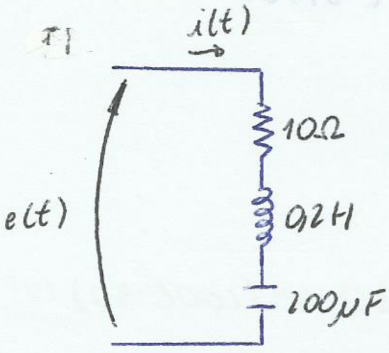


Valor Eficaz

$\dot{I} = I \underline{|\theta}$

$\dot{V} = V \underline{|0^\circ}$

Problemas propostos do livro e sala de aula



$i(t) = 2\,\text{sen}\,(100t + 30°)\ (A)$

Pede-se: a) $v_R(t),\ v_L(t),\ v_C(t)$

b) $e(t)$

c) Z_{eq}

d) tipo de circuito

a) $v_R(t) = 20\,\text{sen}\,(100t + 30°)$

$v_L(t) = \dfrac{L\,di(t)}{dt} = 0{,}2 \cdot 2 \cdot 100 \cdot \cos(100t + 30) \therefore v_L(t) = 40\cos(100t + 30°)$

$v_C(t) = -\dfrac{1}{200 \cdot 10^{-6} \cdot 100} \cdot 2\cos(100t + 30) \therefore v_C(t) = -100\cos(100t + 30°)$

b) $e(t) = v_R(t) + v_L(t) + v_C(t)$

$= 20\,\text{sen}\,(100t + 30) + 40\cos(100t + 30) - 100\cos(100t + 30)$

$e(t) = A\,\text{sen}\,(100t + 30 + \varphi)$

$= A\,\text{sen}\,(100t + 30)\cos\varphi + A\,\text{sen}\,\varphi\cos(100t + 30)$

$\begin{cases} A\,\text{sen}\,\varphi = 40 - 100 \\ A\cos\varphi = 20 \end{cases} \qquad tg^{-1}\left(\dfrac{-60}{20}\right) = \varphi \quad \therefore \varphi = -71{,}6°$

$A^2 = (-60)^2 + 20^2 \therefore A = 63{,}25$

logo; $e(t) = 63{,}25\,[100t - 41{,}6]\ (V)$

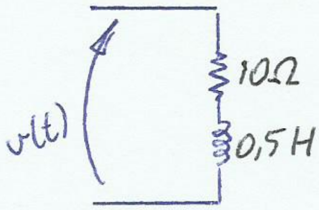
c) $Z_{eq} = \dfrac{V_{máx}}{I_{máx}} = \dfrac{63{,}25}{2} = 31{,}63\,\Omega$

ou

$Z_{eq}^2 = R^2 + (X_L - X_C)^2$

$Z_{eq}^2 = 10^2 + \left(100 \cdot 0{,}2 - \dfrac{1}{100 \cdot 200\,10^{-6}}\right)^2 \quad \therefore Z_{eq} = 31{,}6\,\Omega$

e) Neste circuito em série, o $\varphi < 0$, logo é capacitivo. Pois a corrente esta adiantado em relação a tensão.



$i(t) = 2\cos(200t - 30)$ (A)

Determinar: a) $v_r(t)$ e $v_L(t)$

b) $v(t)$

c) Z_{eq}

a) $v_r(t) = 20\cos(200t - 30)$ (V)

$v_L(t) = 0{,}5 \cdot 200 \cdot 2\,\text{sen}(200t - 30) \therefore v_L(t) = 200\,\text{sen}(200t - 30)$ (V)

b) $v(t) = v_r(t) + v_L(t)$

$= 20\cos(200t - 30) + 200\,\text{sen}(200t - 30)$

$v(t) = A\cos(200t - 30 + \varphi)$

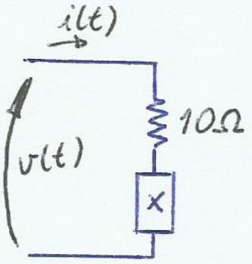
$= A\cos(200t - 30)\cos\varphi - A\,\text{sen}(200t - 30)\,\text{sen}\varphi$

$\begin{cases} A\,\text{sen}\varphi = 200 \\ A\cos\varphi = 20 \end{cases}$ $\quad \varphi = tg^{-1}\left(\frac{200}{20}\right) \therefore \varphi = 84{,}3^{o}$

$A^2 = 200^2 + 20^2 \therefore A = 201$

$\therefore v(t) = 201\cos(200t + 54{,}3^{o})$ (V)

c) $Z_{eq} = \frac{201}{2} = 100{,}5\,\Omega$



$v(t) = 100\,\text{sen}[100t - 10^{o}]$ (V) $\qquad \text{sen}\,x = \cos(x - 90)$

$i(t) = I\cos[100t - 30^{o}]$ (A)

Determine: a) o elemento x

b) I

$v(t) = 100\cos[100t - 100^{o}]$

$i(t) = I\cos[100t - 30^{o}]$

$\varphi = \measuredangle T - \measuredangle I$

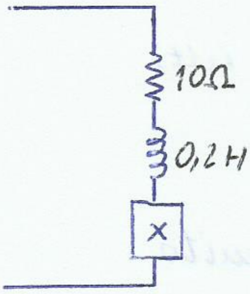
$= -100 - (-30) \therefore \varphi = -70$

$\therefore$ Circuito Capacitivo

$\varphi = tg^{-1}\left(\frac{\overset{\to 0}{x_L} - x_C}{R}\right) \quad -70 = tg^{-1}\left(\frac{-x_C}{10}\right)$

logo $x_C = \frac{1}{wC} = 27{,}47 \therefore C = \frac{1}{100 \cdot 27{,}47} \therefore C = 364\,\mu F$

$Z_{eq}^2 = 10^2 + 27{,}47^2 \therefore Z_{eq} = 29{,}23 \therefore Z_{eq} = \frac{V_{max}}{I_{max}} = \frac{100}{I} = 29{,}23$ $\boxed{I = 3{,}42A}$



$v(t) = 100 \sin(100t + 60°)$ (V)

$i(t) = I \cos(100t - 40°)$ (A)

Determinar a) Elemento x

b) I

$\sin x = \cos(x - 90)$ ou $\cos x = \sin(x + 90)$

$v(t) = 100 \sin(100t + 60°)$ (V)

$i(t) = I \sin(100t + 50°)$ (A)

$\varphi = \measuredangle T - \measuredangle I$

$= 60 - 50 \therefore \varphi = 10$

"Circuito Indutivo"

$$tg\,\theta = \frac{x_L - x_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

- Caso seja um resistor

$$tg\,10 = \frac{20}{10 + x} \therefore x = 103{,}43\,\Omega$$

$$Zeq^2 = 20^2 + (103{,}43 + 10)^2 \therefore Zeq = 115{,}18\,\Omega$$

$$I = \frac{100}{115{,}18} \therefore I = 0{,}87\,A$$

- Caso seja um capacitor

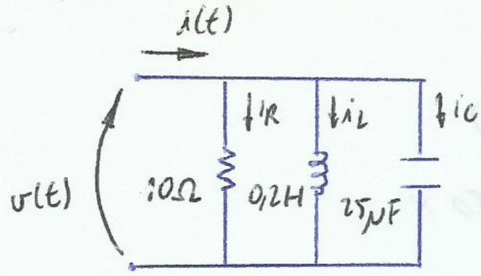
$$tg\,10 = \frac{0{,}2.100 - \frac{1}{100x}}{10} \therefore x = 548\,\mu F$$

$$Zeq^2 = 10^2 + \left(0{,}2.100 - \frac{1}{100.548.10^{-6}}\right)^2 \therefore Zeq = 9{,}84\,\Omega$$

$$I = \frac{100}{9{,}84} \therefore I = 10{,}16\,A$$

- Caso seja uma bobina

$$tg\,10 = \frac{20 + x_L'}{10} \therefore x_L' = -18{,}24 \therefore \text{Impossível}$$



$v(t) = 100 \operatorname{sen}(200t - 10°)$ (V)

Determinar: a) $i_R(t)$; $i_L(t)$; $i_C(t)$

b) $i(t)$;

c) Y_{eq};

d) tipo de circuito

a) $i_R(t) = 10 \operatorname{sen}(200t - 10°)$ //

$i_L(t) = -\dfrac{100}{200 \cdot 0{,}2} \cos(200t - 10°) \therefore i_L(t) = -2{,}5 \cos(200t - 10°)$ //

$i_C(t) = 25 \cdot 10^{-6} \cdot 100 \cdot 200 \cos(200t - 10°) \therefore i_C(t) = 0{,}5 \cos(200t - 10°)$ //

b) $i(t) = i_R(t) + i_L(t) + i_C(t)$

$= 10 \operatorname{sen}(200t - 10) - 2{,}5 \cos(200t - 10°) + 0{,}5 \cos(200t - 10°)$

$= \underline{10 \operatorname{sen}(200t - 10)} + (0{,}5 - 2{,}5) \underline{\cos(200t - 10)}$

$i(t) = B \operatorname{sen}(200t - 10° - \varphi')$

$= \underline{B \operatorname{sen}(200t - 10) \cos\varphi'} - B \operatorname{sen}\varphi' \underline{\cos(200t - 10)}$

d

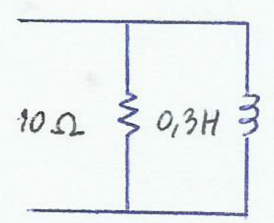
$\begin{cases} -B \operatorname{sen}\varphi' = -2 \\ B \cos\varphi' = 10 \end{cases} \therefore \varphi' = -tg^{-1}\left(\dfrac{-2}{10}\right) \therefore \varphi' = 11{,}3°$ logo $\varphi = -11{,}3°$

"Circuito Capacitivo" //

$B^2 = (-2)^2 + 10^2 \therefore B = 10{,}20$

$\therefore i(t) = 10{,}20 \operatorname{sen}(200t - 21{,}3°)$ (A) //

c) $Y_{eq} = \dfrac{I_{máx}}{V_{máx}} = \dfrac{10{,}20}{100} \therefore Y_{eq} = 0{,}102\ \Omega$ ///



$v(t) = 2 \cos(100t + 10°)$

Determinar: a) $i_R(t)$ e $i_L(t)$

b) $i(t)$

c) Y_{eq}

a) $i_R(t) = 0{,}2 \cos(100t + 10°)$ //

$i_L(t) = -\dfrac{2}{0{,}3 \cdot 100} \operatorname{sen}(100t + 10°) \therefore i_L(t) = 0{,}07 \operatorname{sen}(100t + 10°)$ ///

b) $i(t) = i_R(t) + i_L(t)$

$= \underline{0{,}2 \cos(100t + 10)} + 0{,}07 \underline{\operatorname{sen}(100t + 10°)}$

$$i(t) = B\cos(100t + 10° - \varphi')$$
$$= \underline{B\cos(100t+10)\cos\varphi'} + \underline{B\,sen(100t+10)\,sen\,\varphi'}$$

$$\begin{cases} B\,sen\,\varphi' = 0{,}067 \\ B\cos\varphi' = 0{,}2 \end{cases} \quad \varphi' = tg^{-1}\left(\frac{0{,}07}{0{,}2}\right) \therefore \varphi' = 18{,}43 \therefore \varphi = -18{,}43$$

∴ "Circuito Capacitivo"

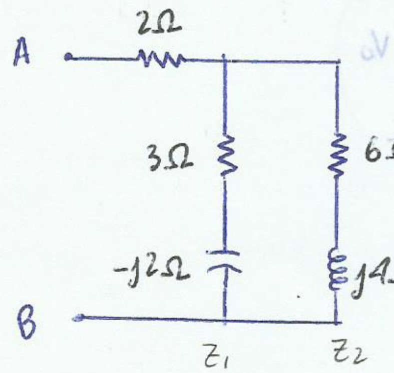
$$B^2 = 0{,}067^2 + 0{,}2^2 \therefore B = 0{,}21$$

$$i(t) = 0{,}21\cos(100t + 28{,}43°) \;//$$

c) $Yeq = \dfrac{0{,}21}{2} \therefore Yeq = 0{,}105\,s \;//$

Exercícios do Livro

9.10



× Calcule a impedância equivalente

$$\dot{z}_1 = 3 - j2 \qquad \dot{z}_2 = 6 + j4$$

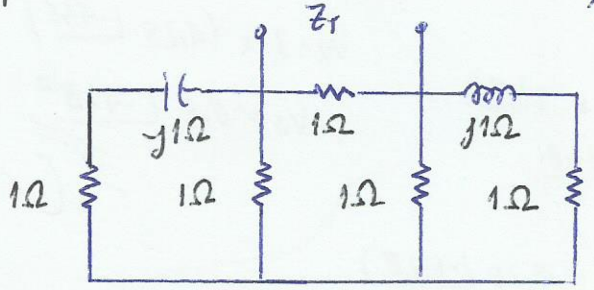
$$\dot{z}_1 // \dot{z}_2 = \frac{(3-j2)(6+j4)}{(3-j2)+(6+j4)}$$

$$(3-j2)(6+j4) = 18 + j12 - j12 + 8 = 26 + j\cdot 0 = 26\underline{/0}$$
$$(3-j2)+(6+j4) = 9 - j2 = 9{,}22\underline{/-12{,}5°}$$

$$\dot{z}_1 // \dot{z}_2 = \frac{26\underline{/0°}}{9{,}22\underline{/-12{,}5}} = 2{,}82\underline{/12{,}5°} = 2{,}75 + j0{,}61$$

$$\therefore Zeq = 4{,}75 + j0{,}61 \text{ ou } Zeq = 4{,}79\underline{/7{,}3°}$$

9.11



× Calcule Z_T

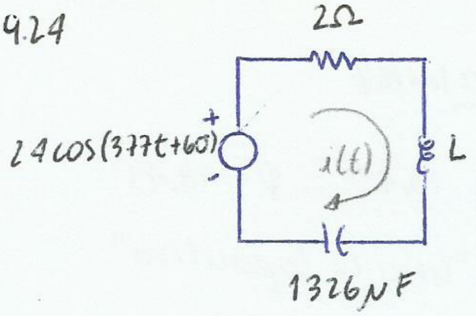
$$Z_T = [(1-j1)//(1+j0) + (1+j0)//(1+j1\Omega)]//(1+j0)$$

$$(1-j1)//(1+j0) = \frac{(1-j1)(1+j0)}{(1-j1)+(1+j0)} = \frac{1-j}{2-j}$$

$$(1-j1)//(1+j0) = 0{,}6 - j0{,}2 \quad | \quad (1+j0)//(1+j1) = 0{,}6 + j0{,}2$$

$$Z_T = (1{,}2 + j0)//(1+j0) \therefore Z_T = \frac{6}{11}\Omega \;//$$

9.24



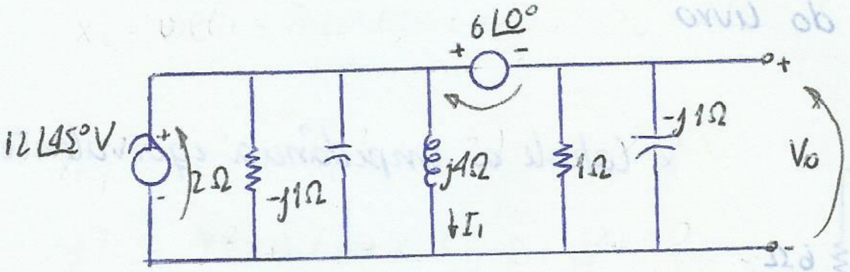
× Determine o valor da indutancia para que a corrente esteja em fase com a fonte de tensão.

$v(t) = 24\cos(377t + 60°) \therefore \dot{V} = 16{,}97\angle 60°$

$$X_C = \frac{1}{1326 \cdot 10^{-6} \cdot 377} = 2\,\Omega$$

$$\phi = tg^{-1}\left(\frac{x_L - x_C}{R}\right) \therefore \quad 60 = tg^{-1}\left(\frac{377L - 2}{2}\right) \therefore L = 14{,}5\,mH$$

9.25



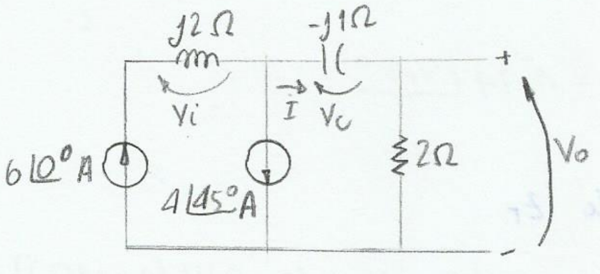
× Determine I_1 e Vo:

$$\dot{I}_1 = \frac{12\angle 45°}{4\angle 90} = 3\angle -45°$$

$12\angle 45° = 6\angle 0° + \dot{V}o$

$\dot{V}o = 8{,}49 + j8{,}49 - 6 + j0 = 2{,}49 + j8{,}49 \therefore Vo = 8{,}85\angle 73{,}7°\ V$

9.26 Determine Vo no circuito.



$\dot{I} = 6\angle 0° - 4\angle 45°$

$= (6 + j0) - (2{,}83 + j2{,}83)$

$= 3{,}17 - j2{,}83 = 4{,}25\angle -41{,}8°$

$\dot{V}o = 2 \times (4{,}25\angle -41{,}8)$

$\therefore \dot{V}o = 8{,}5\angle -41{,}8°$

$\dot{V}_i = 6\angle 0° \times j2$

$= 6\angle 0° \times 2\angle 90$

$= 12\angle 90°$

$\dot{V}c = 4{,}25\angle -41{,}8 \times 1\angle -90$

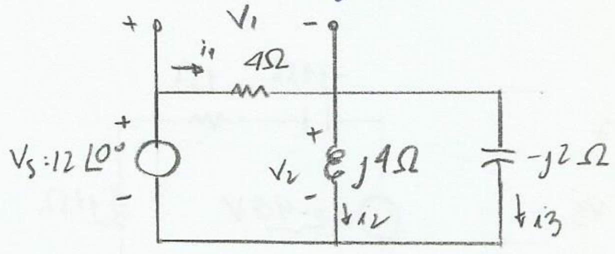
$\dot{V}c = 4{,}25\angle -131{,}8$

$\dot{V}o = -(12\angle 90 + 4{,}25\angle -131{,}8)$

$= -[(0 + j12) + (-2{,}83 - j3{,}17)]$

$\therefore \dot{V}o = 2{,}83 + j8{,}83 \therefore Vo = 9{,}27\angle 72{,}20°\ (V)$

4.27 Determine o diagrama fasorial, usando vs como referência:

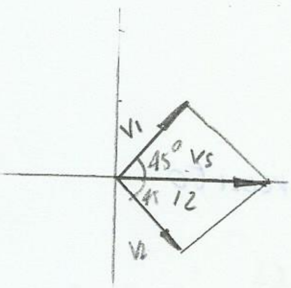


$$Z_{23} = \frac{4\angle 90 \times 2\angle -90}{j4 + (-j2)} = \frac{8\angle 0}{j2} = 4\angle -90$$

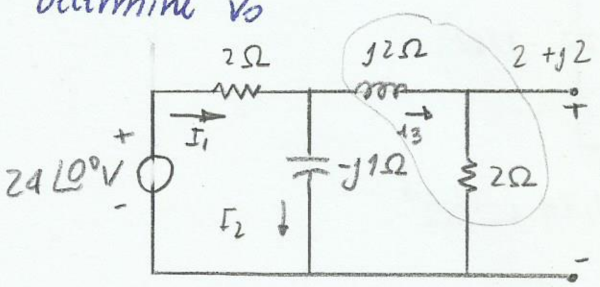
$$V_2 = \frac{4\angle -90}{4\angle -90 + 4} \times 12\angle 0^\circ = 8{,}49\angle -45 = 6 - 6i$$

$$\dot{V}_S = \dot{V}_1 + V_2$$

$$12\angle 0^\circ = \dot{V}_1 + 8{,}49\angle -45 \quad \therefore \quad \dot{V}_1 = 6 + 6i = 8{,}49\angle 45$$



4.28 Determine Vo

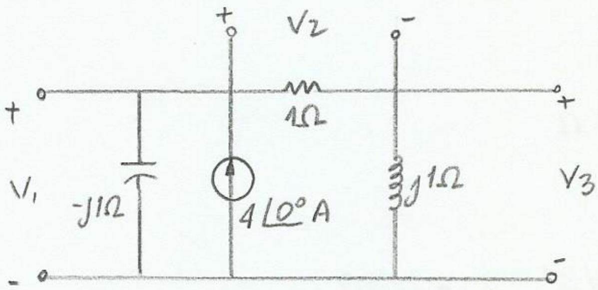


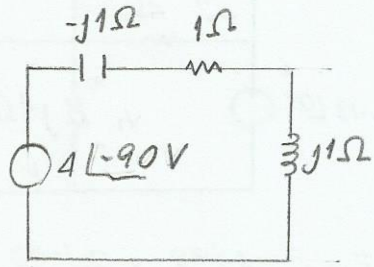
$$Z_1 = (-j1) \parallel (2 + j2) = \frac{(-j)(2 + j2)}{-j + 2 + j2} = 0{,}4 - 1{,}2i = 1{,}26\angle -71{,}6^\circ$$

$$V_2 = \frac{+0{,}4 - 1{,}2i}{2 + 0{,}4 - 1{,}2i} \, 24\angle 0^\circ = 8 - j8 = 11{,}31\angle -45$$

$$V_o = \frac{2}{2 + j2} \cdot (8 - j8) \therefore \boxed{V_o = 8\angle -90 \, (V)}$$

4.29 Determine V_1, V_2 e V_3



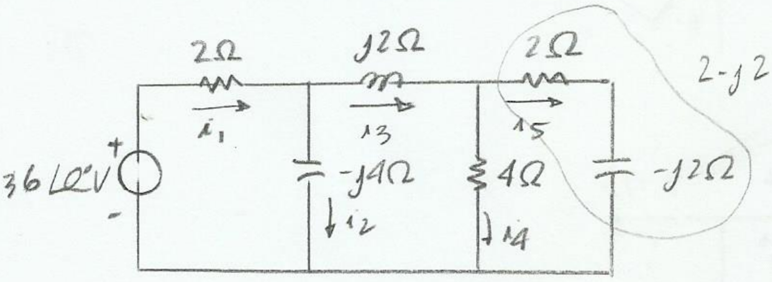


$$V_1 = \frac{-j1}{\cancel{-j1}+1+\cancel{j1}} \cdot 4\angle{-90} = 4\angle{180}$$

$$V_2 = \frac{1}{-j1+j1+1} \cdot 4\angle{-90} = 4\angle{-90}$$

$$V_3 = \frac{j1}{\cancel{-j1}+\cancel{j1}+1} \cdot 4\angle{-90} = 4\angle{0}$$

4.31 Calcule as correntes do circuito



$$Z_1 = 4 \parallel (2-j2) = 1{,}6 - j0{,}8 = 1{,}79\angle{-26{,}6^\circ}$$

$$Z_2 = j2 + 1{,}6 - j0{,}8 = 1{,}6 + j1{,}2$$

$$Z_3 = -j4 \parallel Z_2 = 2{,}46 + j0{,}31 = 2{,}48\angle{7{,}1^\circ}$$

$$Z_{eq} = 4{,}46 + j0{,}31 = 4{,}47\angle{3{,}9^\circ}$$

$$i_1 = \frac{36\angle{0^\circ}}{4{,}47\angle{3{,}9}} = 8{,}05\angle{-3{,}95^\circ} \text{ A}$$

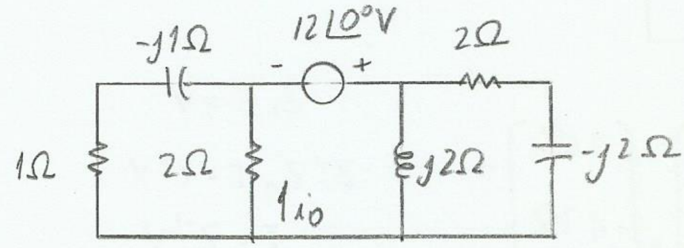
$$i_2 = \frac{1{,}6+j1{,}2}{1{,}6+j1{,}2-j4} \cdot 8{,}05\angle{-3{,}95} = 4{,}99\angle{93{,}17^\circ} \text{ A}$$

$$i_3 = 8{,}05\angle{-3{,}95} - 4{,}99\angle{93{,}17} = 9{,}98\angle{-33{,}7} \text{ A}$$

$$i_4 = \frac{2-j2}{2-j2+4} \cdot 9{,}98\angle{-33{,}7} = 4{,}47\angle{-60{,}3^\circ}$$

$$i_5 = 9{,}98\angle{-33{,}7} - 4{,}47\angle{-60{,}3} = 6{,}31\angle{-15{,}2^\circ}$$

9.32 Determine Io na rede da Figura a seguir

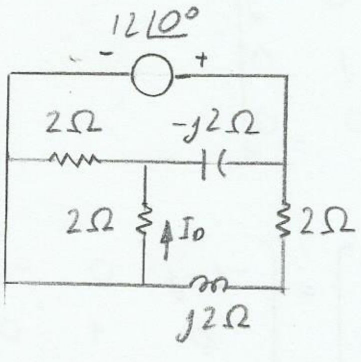


$$Zeq = (2-j2) \parallel j2 + (1-j1) \parallel 2 = 2{,}8 + 1{,}6j = 3{,}22\angle{29{,}7^\circ}$$

$$I = \frac{12\angle{0^\circ}}{3{,}22\angle{29{,}7^\circ}} = 3{,}72\angle{-29{,}74}$$

$$i_o = \frac{(1-j1)}{1-j1+2} \cdot 3{,}72\angle{-29{,}74} \quad \therefore \quad \boxed{i_o = 1{,}66\angle{-56{,}3^\circ}}$$

9.33 Determine Io na rede

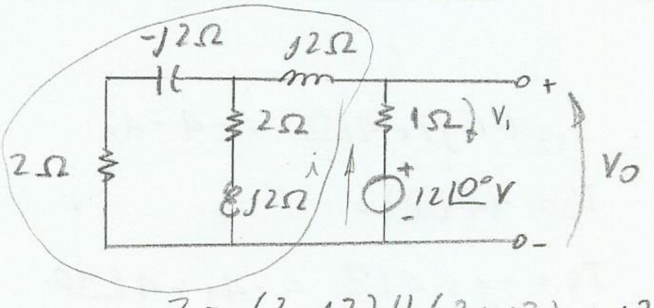


$$V_o = \frac{1}{1-j2} \, 12\angle{0^\circ} = 2{,}4 + 4{,}8j$$

$$I_o = \frac{2{,}4 + j4{,}8}{2} = 1{,}2 + 2{,}4j$$

$$\therefore \quad \boxed{I_o = 2{,}68\angle{63{,}4^\circ} \text{ A}}$$

9.34 Determine Vo na rede

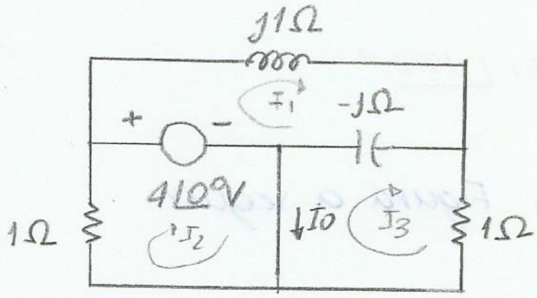


$$Z = (2-j2) \parallel (2+j2) + j2 = 2+2j$$

$$V_1 = \frac{1}{3+2j} \, 12\angle{0^\circ} = 3{,}33\angle{-33{,}69}$$

$$V_o = 12\angle{0^\circ} - 3{,}33\angle{-33{,}69} \quad \therefore \quad V_o = 9{,}41\angle{11{,}31^\circ} \text{ V}$$

9.35) Determine I_o



$0I_1 + j\,I_3 = 4\angle 0$

$I_2 = -4\angle 0° \,V$

$(1-j)\,I_3 + j\,I_1 = 0$

$$\begin{bmatrix} 0 & -0 & -(-j) \\ -0 & 1 & -0 \\ -(-1) & -0 & 1-j \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4\angle 0 \\ -4\angle 0 \\ 0 \end{bmatrix}$$

$Z \times I = V$

$Z \times I = V$

$Z^{-1} Z \times I = Z^{-1} V$

$I = Z^{-1} V$

Onde $Z^{-1} = \dfrac{\text{Adj}\,A}{|A|}$

Adj A: é a transposta da cofatora.

$$Z = \begin{bmatrix} 0 & 0 & j \\ 0 & 1 & 0 \\ 1 & 0 & (1-j) \end{bmatrix}$$

$|Z| = (1-j).0 - 0 + 1.j = j$

$$\text{cof}\,Z = \begin{bmatrix} (1-j) & 0 & 1 \\ 0 & j & 0 \\ -j & 0 & 0 \end{bmatrix} \qquad \text{Adj}\,Z = \begin{bmatrix} (1-j) & 0 & -j \\ 0 & j & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$Z^{-1} = \frac{1}{j} \begin{bmatrix} (1-j) & 0 & -j \\ 0 & j & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{(1-j)}{j} & 0 & -1 \\ 0 & 1 & 0 \\ \underbrace{\frac{1}{j}}_{-j} & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1-j & 0 & -1 \\ 0 & 1 & 0 \\ -j & 0 & 0 \end{bmatrix}$$

$I = Z^{-1} V$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -1-j & 0 & -1 \\ 0 & 1 & 0 \\ -j & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 4\angle 0 \\ -4\angle 0 \\ 0 \end{bmatrix}$$

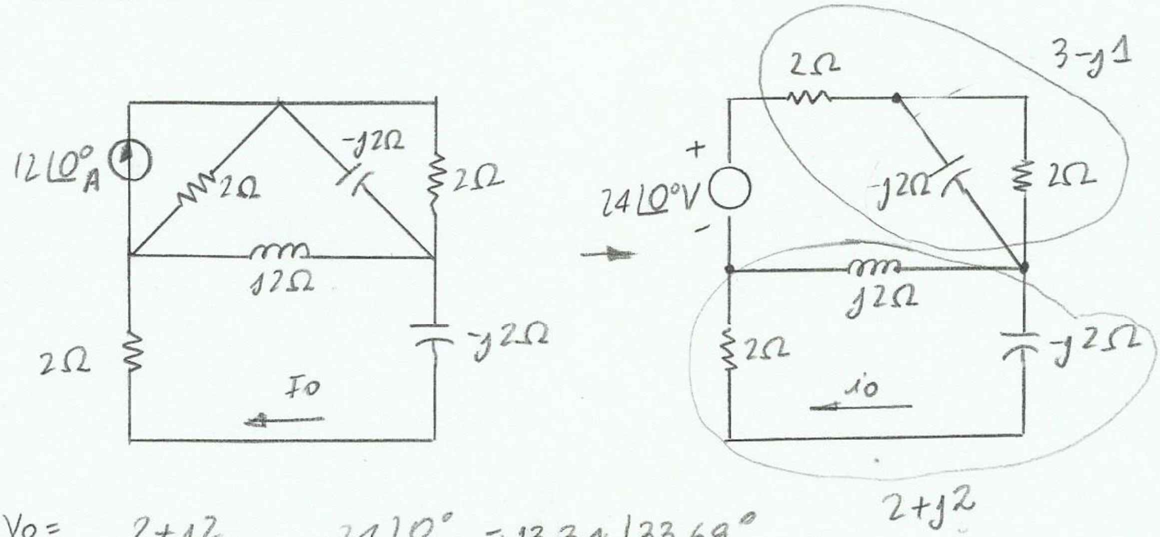
$I_1 = (-1-j) \times 4\angle 0 = -4 - 4i$

$I_2 = -(4\angle 0) = -4$

$I_3 = -j \times 4\angle 0 = -4i = 4\angle -90$

$I_o = I_2 - I_3 = -(4\angle 0) - (4\angle -90) = \underline{-4 + j4\ A}$

9.36 Determine I_o na rede



$$V_o = \frac{2+j2}{2+j2+3-j} \cdot 24\angle 0° = 13{,}31\angle 33{,}69°$$

$$i_o = \frac{13{,}31\angle 33{,}69}{2-j2} = 4{,}71\angle 78{,}7° \ (A)$$

Capítulo 10 - Análise Senoidal em Regime Permanente

Lembrete:

× Análise de Malhas

- Considerar "correntes de malhas" ao invés de corrente de ramo
- Aplicar LKT nas malhas principais
- Adotar o mesmo sentido em todas as malhas

Obs.: Gerador (+) Receptor (-)

+ Método para resolver o sistema de equações

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{33} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \end{bmatrix} = \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \end{bmatrix}$$

$$\dot{Z} \cdot I = \dot{V}$$

- Na diagonal principal (x +1)

Z_{11}, Z_{22}, Z_{33} - É a soma das impedâncias da malha

- Fora da diagonal principal (x -1)

$Z_{12} = Z_{21}$ $Z_{13} = Z_{31}$ $Z_{23} = Z_{32}$

é a impedância comuns as malhas

$$Z^{-1} \times Z \times I = Z^{-1} V$$

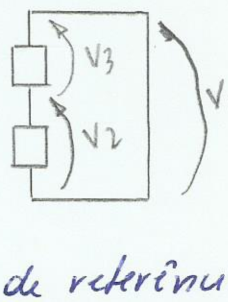
$$I = Z^{-1} V$$

$$Z^{-1} = \frac{Adj\, Z}{|Z|}$$

adj Z = É a transposta da co-fatora de Z

× Análise Nodal

$V_1 = V_2 + V_3$



- É feita à partir das tensões entre os nós
- nomear os nós
- escrever as tensões dos nós em relação aos nós de referência (incógnito)
- Aplicar LKC nos nós

Obs.: Adotar o mesmo sentido em todos os nós

Método para calcular

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \qquad [Y][V]=[I]$$

$[Y]$: Diagonal Principal (+): Y_{11}, Y_{22}, Y_{33} (Soma das alimentações no nó).

Fora da Diagonal Principal (-): $Y_{12} = Y_{21}$; $Y_{31} = Y_{13}$; $Y_{32} = Y_{23}$

(Admitancia entre os nós)

$[I]$: Gerador de corrente: entrando no nó (+)

Receptor de corrente: saindo do nó (-)

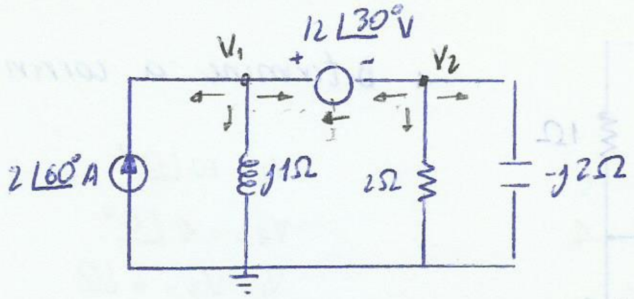
× Superposição

Substituir: fonte de tensão por curto-circuito

fonte de corrente por circuito aberto

Para determinar x, temos $x = x' + x'' + \dots x^n$

Onde n é o números de fontes.



Determinar a tensão V_1 e V_2

$$V_1 = V_2 + 12\angle 30^\circ$$

Nó 1: $$\frac{V_1}{j1} + \frac{V_1 - 12\angle 30^\circ}{1-j} - 2\angle 60 = 0$$

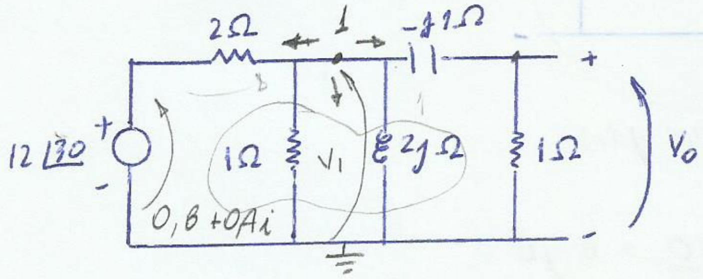
$$-V_1 \cdot j + \frac{(V_1 - 12\angle 30^\circ)(1+j)}{2} - 2\angle 60^\circ = 0$$

$$(-2V_1 \cdot j) + V_1 - 12\angle 30^\circ + (V_1 \cdot j) - (12\angle 30^\circ \times j) - 4\angle 60 = 0$$

$$V_1(-2j + 1 + j) = 12\angle 30^\circ + 4\angle 60 + 12\angle 30^\circ j$$

$$V_1 = \frac{12\angle 30^\circ + 4\angle 60^\circ + 12\angle 30^\circ j}{1-j} = 14{,}75\angle 117{,}16^\circ$$

$$\therefore V_2 = 18{,}55\angle 157{,}40^\circ$$



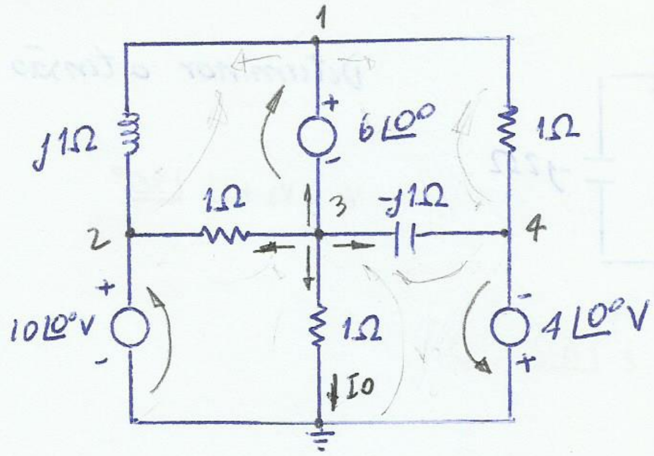
Determinar V_o

$$V_o = \frac{1}{1-j} \cdot V_1$$

nó 1: $$\frac{V_1 - 12\angle 30}{2} + \frac{V_1}{0{,}8 + j0{,}4} + \frac{V_1}{1-j1} = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{0{,}8 + j0{,}4} + \frac{1}{1-j1}\right) = \frac{12\angle 30}{2}$$

$$\therefore V_1 = 3\angle 30 \qquad V_o = \frac{3\angle 30}{1-j} = 2{,}12\angle 75^\circ \text{ (V)}$$



× Determine a corrente I_o

$V_2 = 10\angle 0°$

$V_4 = -4\angle 0°$

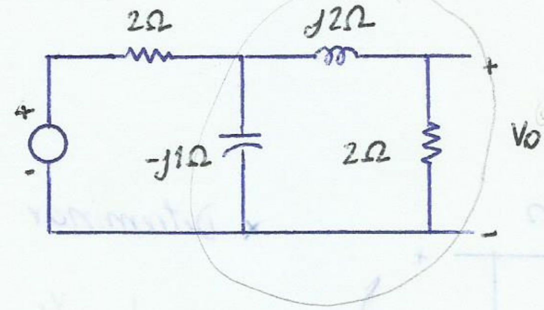
$V_1 = V_3 + 6\angle 0$

Nó: $V_3 + \frac{V_3 - V_2}{1} + \frac{V_4 + V_3}{-j1} + \frac{V_1 - V_2}{j1} + \frac{V_1 + V_4}{1} = 0$

$(1-j)V_1 + (-1+j)V_2 + (1+j)V_3 + (1+j)V_4 = 0$

(Depois eu faço :D)

10.1



$V_s = 24\angle 0°$

Qual o valor de V_o

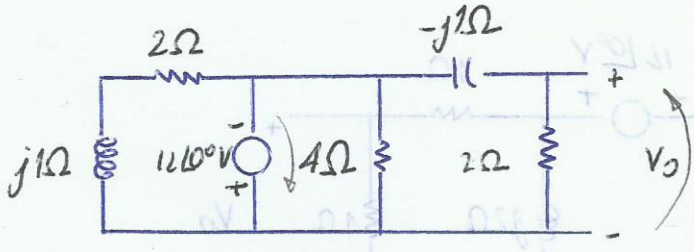
$0{,}4 - j1{,}2$

$V = \frac{0{,}4 - j1{,}2}{2{,}4 - j1{,}2} \cdot 24\angle 0 = 8 - j8$

$V_o = \frac{2}{2+j2} \cdot (8 - j8) = 8\angle -90 \; V$

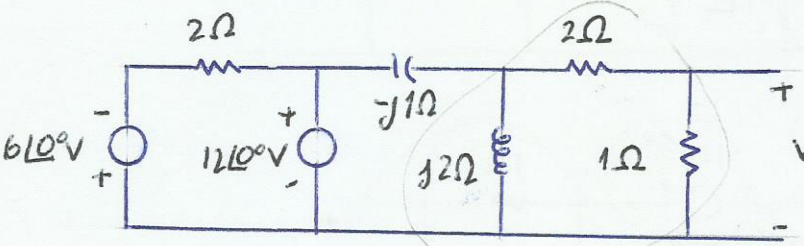
Fazer pelo método de linearidade

10.4 Determine Vo na rede



$$V_o = -\left(\frac{2}{2-j} \times 12\angle 0^\circ\right) \therefore V_o = 10{,}73\angle 26{,}57 \ (V)$$

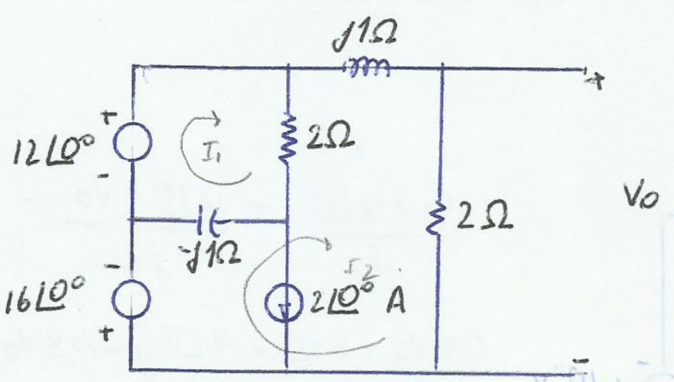
10.5 Determine Vo no circuito



$$V = \frac{3+j2}{3-j1} \times 12\angle 0^\circ \therefore V = 8{,}4 + j10{,}8$$

$$V_o = \frac{1}{1+2} \times (8{,}4 + j10{,}8) \therefore V_o = 2{,}8 + j3{,}6 = 4{,}56\angle 52{,}13^\circ$$

10.6 Determine Vo na rede



$$\begin{bmatrix} 2-j & -2+j \\ -2+j & 4 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12\angle 0 \\ -16\angle 0^\circ \end{bmatrix}$$

$$Z^{-1} = \begin{bmatrix} 4 & 2-j \\ 2-j & 2-j \end{bmatrix} \qquad |Z| = 4(2-j) - (-2+j)(-2+j) = 5$$

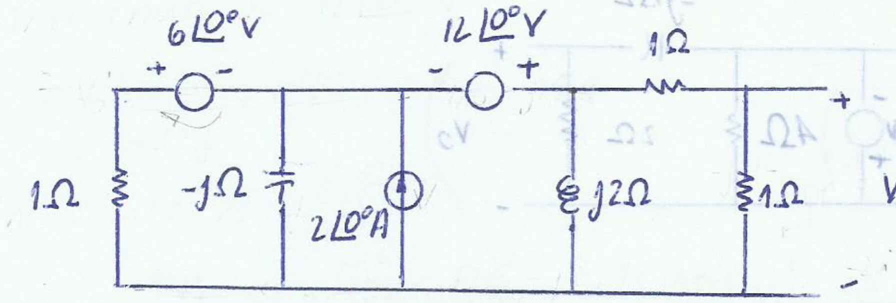
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4 & 2-j \\ 2-j & 2-j \end{bmatrix} \times \begin{bmatrix} 12\angle 0 \\ -16\angle 0 \end{bmatrix}$$

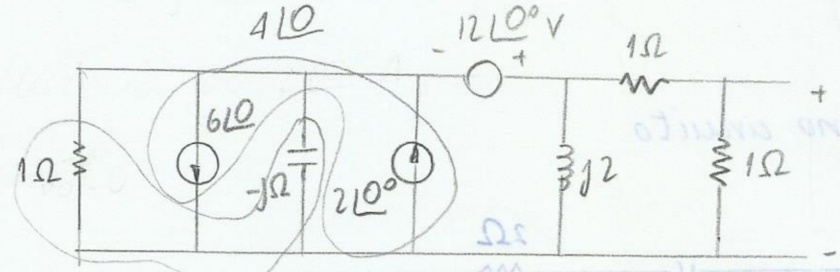
$I_1 = 3{,}2 + j3{,}2 = 4{,}53\angle 45^\circ$ A

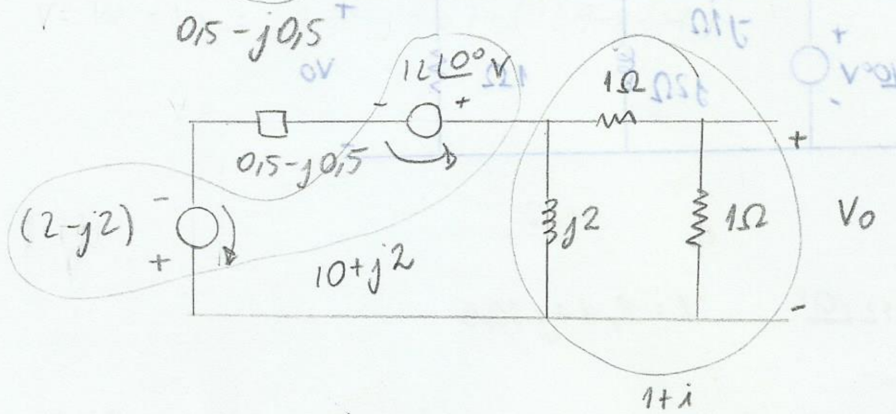
$I_2 = -1{,}6 + j0{,}8 = 1{,}79\angle 153{,}4$ A

$V_o = 2I_2 = 3{,}58\angle 153{,}4$ A

10.7 Determinar V_o



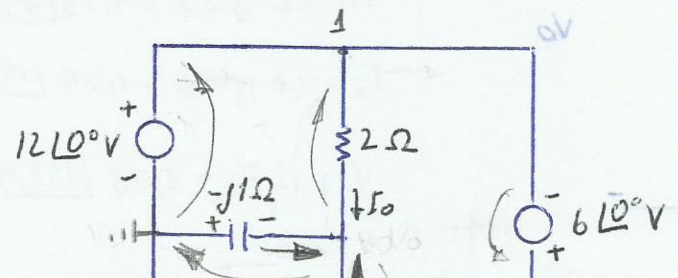




$$V = \frac{1+i}{(1+j)+(0{,}5-j0{,}5)} \times (10+j2) = 7{,}2 + j5{,}6$$

$$V_o = \frac{1}{2}(7{,}2+j5{,}6) \quad \therefore \quad V_o = 4{,}56\angle 37{,}67^\circ$$

10.8 Determine I_o



$$\frac{V_o + 12\angle 0}{2} + \frac{18\angle 0 + V_o}{j2} - \frac{V_o}{j} = 0$$

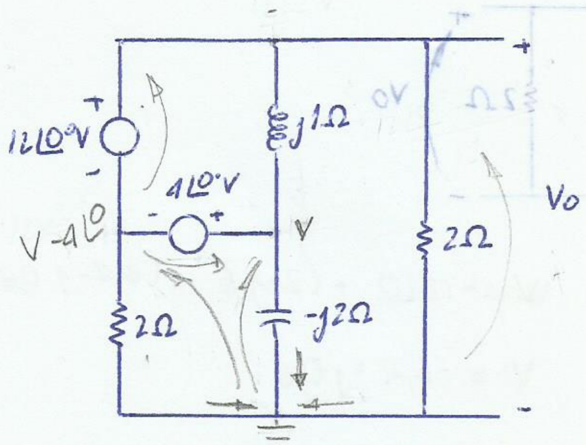
$$0{,}5V_o + 6\angle 0 - 9\angle 0\, j - 0{,}5V_o j + V_o j = 0$$

$$V_o(0{,}5 - 0{,}5j + j) = -6\angle 0 + 9\angle 0\, j$$

$$V_o = 3 + j15$$

$$I_o = \frac{V_o + 12\angle 0}{2} = \underline{10{,}61\angle 45^\circ \text{ A}}$$

10.9 Determine Vo no circuito



$V_o = V - 4\angle 0 + 12\angle 0$

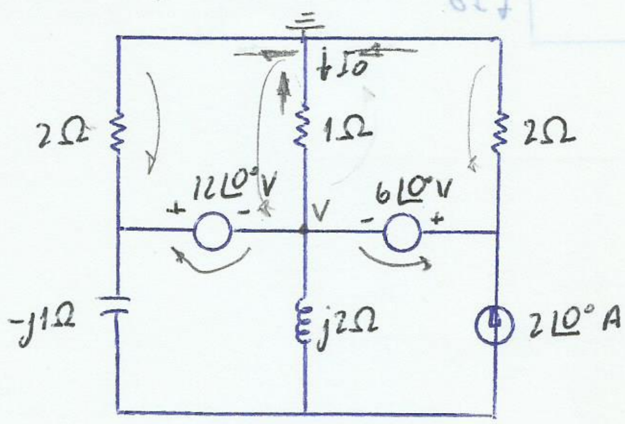
$V_o = V + 8\angle 0$

$$\frac{V}{-j2} + \frac{V - 4\angle 0}{2} + \frac{12\angle 0 + V - 4\angle 0}{2} = 0$$

$$V\left(-\frac{1}{2j} + \frac{1}{2} + \frac{1}{2}\right) = \frac{4\angle 0}{2} + \frac{4}{2}\angle 0 - \frac{12\angle 0}{2} \quad \therefore V = -1{,}6 + 0{,}8i$$

$$V_o = V + 8\angle 0 \quad \therefore \quad V_o = 6{,}45\angle 7{,}13^\circ \ (V)$$

10.10 Determine Io na rede



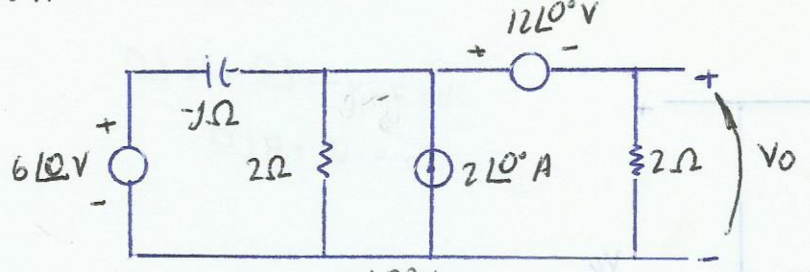
$$V + \frac{V + 12\angle 0}{2} + \frac{6\angle 0 + V}{2} = 0$$

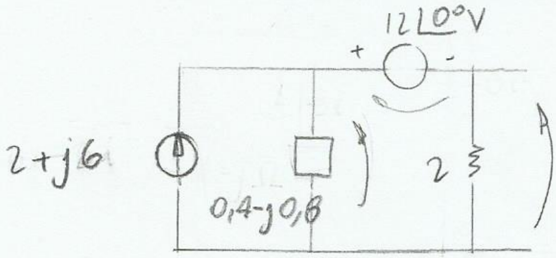
$$2V + V + 12\angle 0 + 6\angle 0 + V = 0$$

$$V(2+1+1) = -(12\angle 0 + 6\angle 0) \quad \therefore V = -4{,}5 + 0j$$

$$I_o = -\frac{V}{1} = -\frac{(-4{,}5+0)}{1} = 4{,}5 = \underline{4{,}5\angle 0^\circ \ V}$$

10.11 Determine V_o



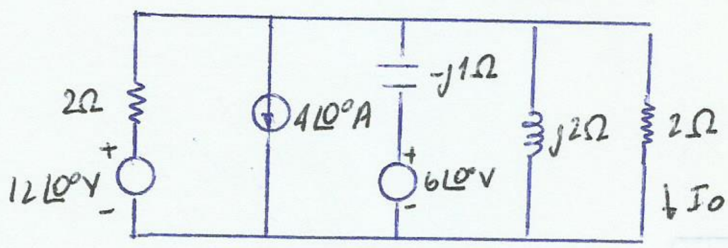


$V_o = -12\angle 0 + (2+j6) \times (0{,}4 - j0{,}8)$

$V = -6{,}4 + j0{,}8$

$V_o = \dfrac{2}{(2{,}4 - j0{,}8)} \times (-6{,}4 + j0{,}8) \quad \therefore \quad V_o = 5{,}1\angle -169 \ (V)$

10.12 Determinar I_o



Capítulo 11 - Análise de potência em regime permanente

$P = V \cdot I$

$\operatorname{sen}(a+b) = \operatorname{sen} a \cos b + \operatorname{sen} b \cos a$

$\operatorname{sen}(2a) = \operatorname{sen} a \cos a + \operatorname{sen} a \cos a$

$\operatorname{sen}(2a) = 2 \operatorname{sen} a \cos a$

$\operatorname{sen} a \cos a = \frac{1}{2} \operatorname{sen} 2a$

- Elementos de circuito

Indutor:

$v(t) = V_{máx} \operatorname{sen} \omega t \quad (V)$

$i(t) = -\underbrace{\frac{V_{máx}}{\omega L}}_{I_{máx}} \cos \omega t \quad (A)$

$p(t) = -VI \operatorname{sen} 2\omega t \quad (W)$ $\qquad P_{media} = 0$

Capacitor:

$v(t) = V_{máx} \operatorname{sen} \omega t \quad (V)$

$i(t) = \underbrace{\omega C\, V_{máx}}_{I_{máx}} \cos \omega t \quad (A)$

$p(t) = VI \operatorname{sen} 2\omega t \quad (W)$ $\qquad P_{media} = 0$

Resistor

$v(t) = V_{máx} \operatorname{sen} \omega t \quad (V)$

$i(t) = I_{máx} \operatorname{sen} \omega t \quad (A)$

$p(t) = VI - VI \cos 2\omega t \quad (W)$ $\qquad P_{media} = VI$

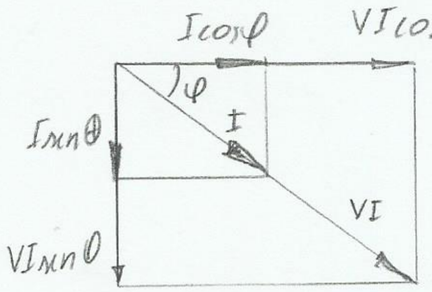
Impedância

$v(t) = V_{máx} \operatorname{sen} \omega t$

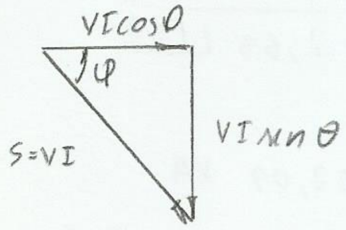
$i(t) = \underbrace{\frac{V_{máx}}{Z}}_{I_{máx}} \operatorname{sen}(\omega t \pm \varphi)$

$p(t) = VI \cos \varphi - VI \cos(2\omega t \mp \varphi)$ $\qquad P_{media} = VI \cos \varphi$

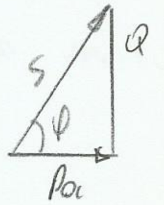
- Triângulo das Potências



Indutivo



Capacitivo



$\varphi = \measuredangle T - \measuredangle I$

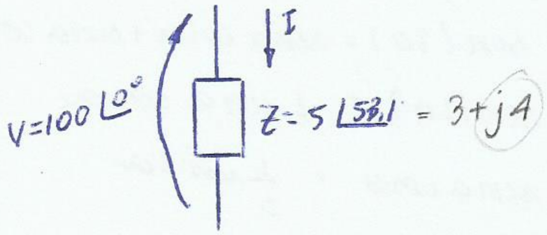
Indutivo: $Q > 0$

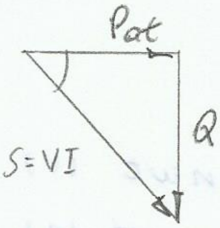
Capacitivo $Q < 0$

Exemplo de fixação

$\phi = \measuredangle V - \measuredangle I$

$\phi > 0 \therefore$ Indutivo ou atrasado



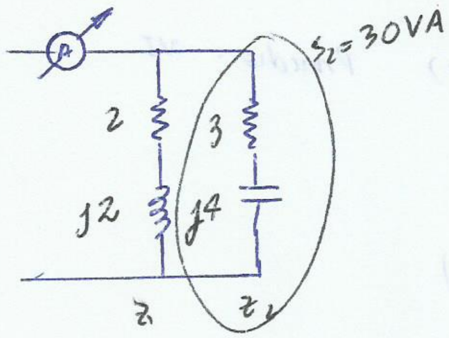


$$I = \frac{100\angle 0}{3+j4} = 12 - j16 = 20\angle{-53,1}$$

$S = VI = 100.20 = 2000VA = 2KVA$

$Pat = 2000 \cos 53,1 = 1200\,W = 1,2W$

$Q = 2000\, \text{sen}\, 53,1 = 1600\,VAR = 1,6KVAR$



$Z_2 = 3+j4 = 5\angle 53,1°$

$P = VI = \frac{V^2}{R}$

$V = \sqrt{30.5} = 12,25$

$\dot{V} = 12,25\angle 53,1$

$Z = (2+j2) \,//\, (3-j4) \therefore Z_{eq} = 2,63\angle 13,7$ ϕ

$$I = \frac{12,25\angle 53,1}{2,63\angle 13,7} \therefore \dot{I} = 4,66\angle 39,4°\ (A)$$

$S = 57,09$ VA

$Pat = 57,09.\cos 13,7 = 55,7$ W

$Q = 57,09\, \text{sen}\, 13,7 = 13,52$ VAR

Como $\phi > 0 \therefore$ Indutiva atrasada

Determinar o triangulo das potências da associação das seguintes cargas

Carga 1 { 100W Pot; f.p. 0,6 $\cos\varphi$; atrasada

Carga 2 { 150 VA; f.p. 0,8; adiantado

Carga 3 { 100 VAR; 200 VA; atrasada

Carga 1 : $P_{at} = 100$ W

$S = 166,7$ VA (atrasado)

$Q = 133,3$ VAR

$P_{at} = S \cdot \cos\varphi$

atrasado: indutivo

adiantado: capacitivo

Carga 2: $S = 150$ VA

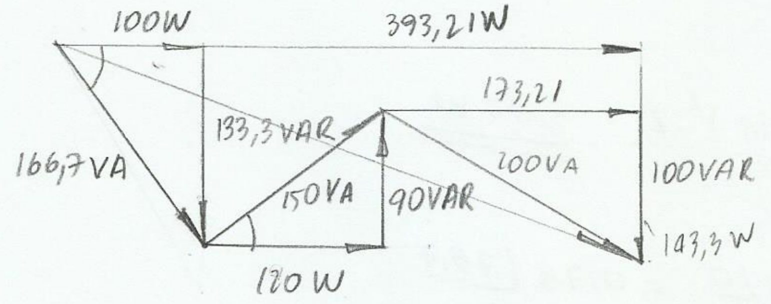
$P_{at} = 120$ W (adiantado)

$Q = 90$ VAR

Carga 3: $Q = 100$ VAR (atrasado)

$S = 200$ VA

$P = 173,21$ W



$P_{at} = 393,21$ W

$Q = 143,3$ W

$S = 418,5$ VA

$\cos\theta = 0,94$

Ex: 200 KVA ; 100 KW ; f.p = 0,6 atrasado

a) $S_n = 200\,000$ VA

$S_c = \frac{100\,000}{0,6} = 166\,667$ VA

% pleno $= \frac{166\,667}{200000} \cdot 100 = 83,3\%$

b) $166\,667^2 = 100\,000^2 + Q^2 \therefore Q = 133\,333$ VAR

$200\,000^2 = P^2 + 133\,333^2 \therefore P = 149\,071$ W

Problemas do Livro

11.1

$$I = \frac{-j2}{4-j2} \times (2\angle 30) = 0{,}89 \angle -33{,}43$$

$$P = \frac{0{,}89^2 \cdot 4}{2} = 1{,}58W$$

11.2

$$I_1 = \frac{2-j}{4+j}(6\angle 0) = 2{,}47 - 2{,}12j = 3{,}25 \angle -40{,}6$$

$$I_2 = 4{,}12 \angle 31°$$

$$P = \frac{3{,}25^2 \cdot 2 + 4{,}12^2 \cdot 2}{2} = 27{,}54 \text{ W}$$

11.3

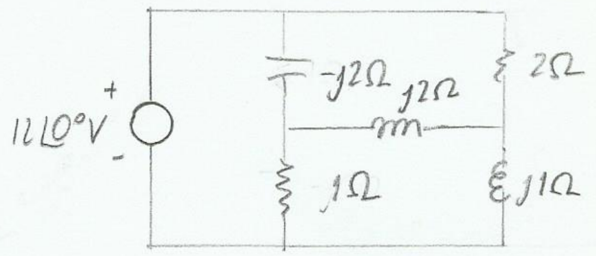
$$I = \frac{0{,}24 + j1{,}06}{2{,}24 + j1{,}06} \times (4\angle 0) = 1{,}07 + j1{,}38$$

$$(R_2): \quad P = \frac{(1{,}07 + j1{,}38)^2 \cdot 2}{2} = \underline{\underline{3{,}07 \text{ W}}}$$

$$I_{(R_4)} = \frac{2 \| j1}{0{,}4 + 0{,}8j + 4 - j2} \times 1\angle 0 = 0{,}78 \angle 78{,}7$$

$$(R_4): \quad P = \frac{0{,}78^2 \cdot 4}{2} = \underline{\underline{1{,}22 \text{ W}}}$$

11.5)



Depois eu faço

11.6)

$$\begin{bmatrix} 1-2j & j2 \\ j2 & 2-j \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12\angle 0° \\ -(6\angle 0°) \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{(2-j)(1-2j)-(j2)(j2)} \begin{bmatrix} 2-j & -j2 \\ -j2 & 1-2j \end{bmatrix} \times \begin{bmatrix} 12\angle 0 \\ -6\angle 0 \end{bmatrix}$$

$$\dot{I}_1 = \frac{1}{4-j5} \cdot (2-j)(12\angle 0) + (-2j)(-6\angle 0) = 3{,}75\angle 51{,}34$$

$$\dot{I}_2 = \frac{1}{4-j5} (-j2)(12\angle 0) + (1-2j) \times -(6\angle 0) = 2{,}1\angle -65{,}2°$$

$$P_{Gerador\,(12\angle 0)} = \frac{12}{2} \times 3{,}75 \times \cos(0-51{,}34) = 14{,}06\ W$$

$$P_{Receptor\,(6\angle 0)} = \frac{6 \times 2{,}1}{2} \cos(0+65{,}2) = 2{,}64\ W$$

$$R_{1\Omega} = \frac{3{,}75^2 \times 1}{2} = 7{,}03\ W$$

$$R_{2\Omega} = \frac{2{,}1^2 \times 2}{2} = 4{,}41\ W$$

11.7)

$$\frac{V_1 - 12\angle 0}{2} + \frac{V_1}{-j} + \frac{V_1}{4+j2} = 6\angle 0$$

$$V_1 \left(\frac{1}{2} + j + \frac{1}{4+j2}\right) = 6\angle 0 + 6\angle 0 \quad \therefore V_1 = 10{,}52\angle -52{,}1$$

$$I_1 = \frac{10{,}52\angle -52{,}1 - 12\angle 0}{2} = 4{,}99\angle -123{,}7$$

$$I_2 = \frac{10{,}52\angle -52{,}1}{-j} = 10{,}52\angle 37{,}9 \qquad I_3 = \frac{10{,}52\angle -52{,}1}{4+j2} = 2{,}35\angle -78{,}7$$

$P_{medio} = \frac{1}{2}\left(4{,}99^2 \cdot 2 + 2{,}35^2 \times 4\right) = 35{,}95\ W$

11.8

$$\frac{V}{1-j2} + \frac{V-4\angle 0}{2+j} = 12\angle 0$$

$$V\left(\frac{1}{1-j2} + \frac{1}{2+j}\right) = 12\angle 0 + \frac{4\angle 0}{2+j} \quad \therefore V = 21{,}54\angle -21{,}8$$

$$I_1 = \frac{21{,}54\angle -21{,}8}{1-j2} = 9{,}63\angle 41{,}6$$

$$I_2 = \frac{21{,}54\angle -21{,}8 - 4\angle 0}{2+j} = 8\angle -53{,}1$$

?

$$P_{fornecido} = \frac{1}{2}\left(9{,}63 \times 1^2 + 8 \cdot 2^2\right)$$

11.9 (Depois eu faço)

11.10

$$\frac{V}{4} + \frac{V}{j4} + \frac{V-12\angle 90}{2} = 4\angle 0$$

$$V\left(\frac{1}{4} + \frac{1}{j4} + \frac{1}{2}\right) = 4\angle 0 + 6\angle 90 \quad \therefore V = 7{,}2 + j5{,}6$$

$$P = \frac{9{,}12^2}{8} = 10{,}4\ W$$

11.11

$$\frac{V}{2} + \left(\frac{V-12\angle 0}{j2}\right) = 2I_x \quad ; \quad I_x = \frac{12\angle 0 - V}{j2}$$

$$\frac{V}{2} + \left(\frac{V-12\angle 0}{j2}\right) - \left(\frac{12\angle 0 - V}{j}\right) = 0$$

$$V\left(\frac{1}{2} + \frac{1}{j2} + \frac{1}{j}\right) = +\frac{12\angle 0}{j2} + \frac{12\angle 0}{j} \quad \therefore V = 11{,}38\angle -18{,}03$$

$$P = \frac{11{,}38^2}{4} = 32{,}46\ W$$

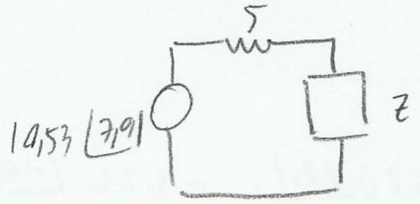
11.12 (Fazer Depois)

$P = V.I = \frac{V^2}{R}$

11.15

$V_{th} = 12\angle 30 + 4\angle 0°(1-j) = 14,53\angle 7,91$

$R_{th} = 1 + (-j1) + 4 + j1 = 5\,\Omega \therefore R_L = 5\Omega$

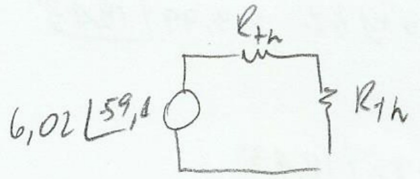


$P_L = \frac{1}{2} \cdot \frac{(14,53/2)^2}{5} = \underline{\underline{5,28\,W}}$

11.16

$V = \frac{-j4}{4+j2} \times 6,79\angle 57,12 = 6,02\angle -59,4$

$R_{th} = 4+j2 \,||\, -j4 = 3,2 - j2,4$



$\therefore P = \frac{1}{2}\,\frac{(6,02/2)^2}{3,2} = 1,42\,W$

11.18

$I_1 = \frac{V}{1-j2}$ $\quad I_2 = \frac{V-4\angle 0}{2+j}$

$\frac{V}{1-j2} + \frac{V-4\angle 0}{2+j} = 12\angle 0$

$V\left(\frac{1}{1-j2} + \frac{1}{2+j}\right) = 12\angle 0 + \frac{4\angle 0}{2+j}$ $\quad \therefore V = 20 - j8 = 21,54\angle -21,8°$

$I_1 = \frac{20-j8}{1-j2} = 7,2 + j6,4$ $\quad 9,63\angle 41,63$ $\quad I_2 = \frac{20-j8-4\angle 0}{2+j} = 4,8 - j6,4 = 8\angle -53,1$

$P_{medio} = \frac{1}{2}\left(8^2 \cdot 2 + 9,63^2 \times 1 + 8.4\cos(53,13)\right) = \underline{119,97\,W}$

11.17

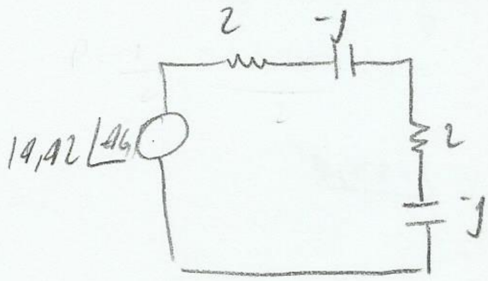
$12\angle 30 - I_1(1-j) + (I_1 - I_2)j = 0$; $I_2 = 2\angle 0$

$12\angle 30 - I_1(1-j) - (I_1 - 2\angle 0)j = 0$

$I_1(-(1-j) - 1j) = -12\angle 30 - 2\angle 0 \times j \therefore I_1 = 13{,}11\angle 37{,}6$

$V_{oc} = 1 \times 2\angle 0 + 13{,}11\angle 37{,}6 \times (-j) = 14{,}42\angle -46{,}1$

$Z_{th} = (1+j1) \,//\, (-j) + 1 = 2-j$



$I = \dfrac{14{,}42\angle -46{,}1}{4-2j} = 3{,}22\angle -19{,}5$

$P = \frac{1}{2}\left(3{,}22^2 \cdot 4\right) = 20{,}73 \text{ W}$

(Dúvida)

11.18

$V_{oc} = \dfrac{12\angle 0}{2+j} \times 2 - \dfrac{12\angle 0}{1-j} \times (-j) = 3{,}6 + j1{,}2 = 3{,}79\angle 18{,}43°$

$Z_{th} = (2 \,//\, j) + (1 \,//\, -j) = 0{,}9 + j0{,}3 = 0{,}95\angle 18{,}43°$

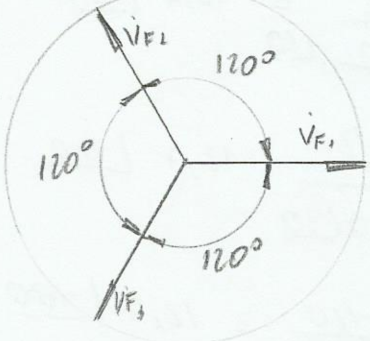
$Z_L = 0{,}9 - j0{,}3$

$P_m = \frac{1}{2}\left(\dfrac{(3{,}79/2)^2}{1{,}8}\right) = \underline{\underline{2\text{W}}}$

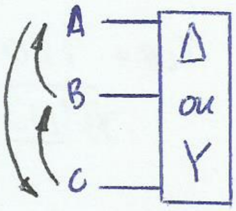
Fazer 11.57

Capítulo 12 - Circuitos Polifásicos

Circuitos trifásicos



Referência (tensão de linha)



$\dot{V}_{AB} = V\angle 0°$

$\dot{V}_{BC} = V\angle 120°$

$\dot{V}_{CA} = V\angle -120°$

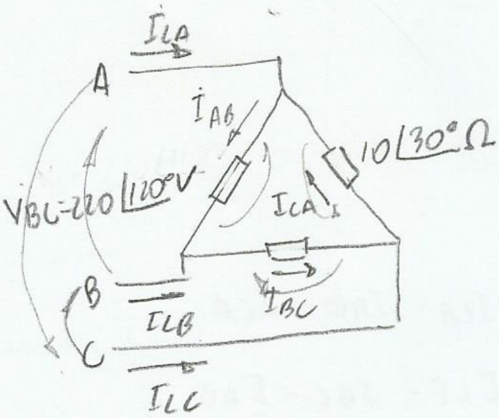
Sequência ABC

$$\boxed{I_{L\Delta} = 3\, I_{LY}}$$

Delta (Δ)	Estrela
$V_L = V_F$	$\dot{V}_L = V_F\sqrt{3}\angle 30$
$I_L = \sqrt{3}\, I_F$	$I_L = I_F$

$$\dot{V}_L = V_F\sqrt{3}\angle 30$$

Exercício de fixação



ligação triângulo (Δ)

$$\dot{I}_{AB} = \frac{220\angle 0}{10\angle 30} = 22\angle -30 \text{ A}$$

$$\dot{I}_{BC} = \frac{220\angle 120}{10\angle 30} = 22\angle 90 \text{ A}$$

$$\dot{I}_{CA} = \frac{220\angle -120}{10\angle 30} = 22\angle -150 \text{ A}$$

$V_{AB} = 220\angle 0 \text{ V}$

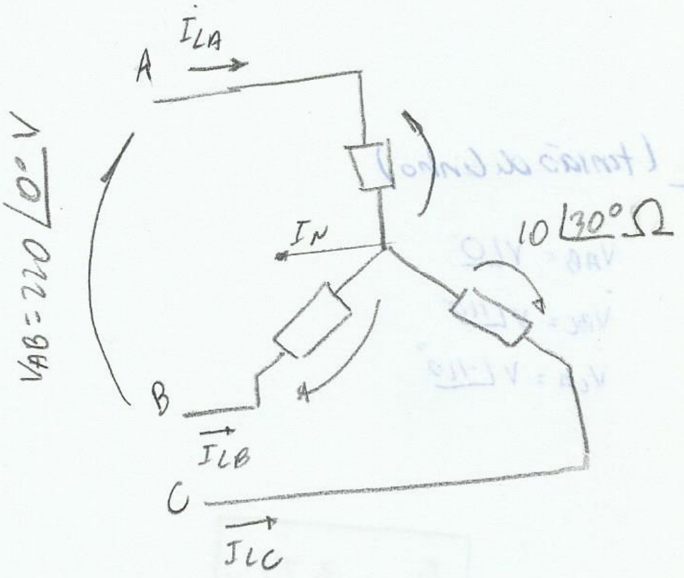
$\dot{V}_{BC} = 220\angle 120° \text{ V}$

$\dot{V}_{CA} = 220\angle -120° \text{ V}$

$\dot{I}_{LA} = I_{AB} - I_{CA} = \ldots$

Exemplo 27

$V_L = V_F \sqrt{3} \angle 30$



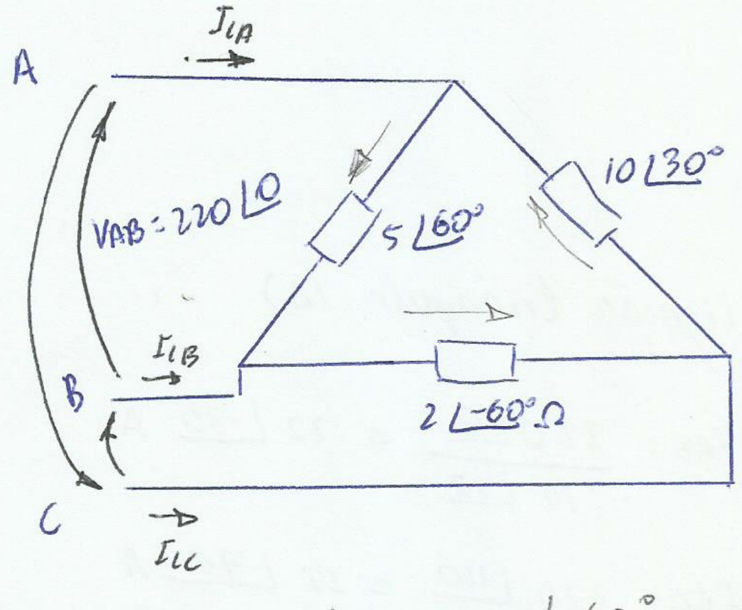
$$\dot{I}_{LA} = \frac{220 \angle 0}{\sqrt{3} \times 10 \angle 30 \cdot \angle 30} = 12{,}7 \angle -60$$

$$\dot{I}_{LB} = \frac{220 \angle 120}{\sqrt{3} \angle 30 \times 10 \angle 30} = 12{,}7 \angle 60$$

$$I_{LC} = \frac{220 \angle -120}{\sqrt{3} \angle 30 \times 10 \angle 30} = 12{,}7 \angle -180$$

$$I_N = -(I_{LA} + I_{LB} + I_C) = 0$$

Cargas distribuidas



$$I_{AB} = \frac{220 \angle 0}{5 \angle 60} = 44 \angle -60° \text{ A}$$

$$I_{BC} = \frac{220 \angle 120}{2 \angle -60} = 110 \angle 180° \text{ A}$$

$$I_{CA} = \frac{220 \angle -120}{10 \angle 30} = 22 \angle -150° \text{ A}$$

$I_{LA} = I_{AB} - I_{CA}$

$I_{LB} = I_{BC} - I_{AB}$

$I_{LC} = I_{CA} - I_{BC}$

Medida da Potência em sistemas polifásicos

$P_{at} = \Sigma W$

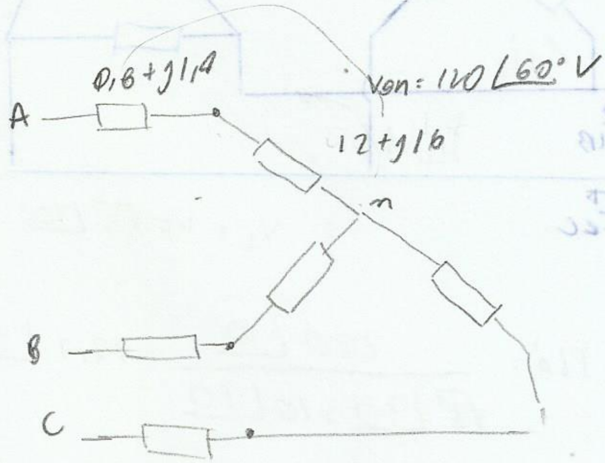
$$tg\varphi = \sqrt{3}\,\frac{W_A - W_B}{W_A + W_B} = \sqrt{3}\,\frac{W_B - W_C}{W_B + W_C}$$

Exercícios do Livro

12.1

$V_{an} = 120\angle 60°\,V$ (rms)

$Z = 12 + j16\ \Omega$



$$I_A = \frac{120\angle 60}{0{,}8 + j1{,}4 + 12 + j16} = 5{,}56\angle 6{,}34°$$

$I_b = 5{,}56\angle -113{,}7°$

$I_C = 5{,}56\angle -233{,}67°$

$$V_{C_a} = \frac{12 + j16}{0{,}8 + 12 + j1{,}4 + j16} \times 120\angle 60 = 111{,}1\angle 59{,}5°\,V$$

$V_{BN} = 111{,}1\angle -60{,}5°\,V$

$V_{CN} = 111{,}1\angle -180{,}5°\,V$

12.4

$$I_{LA} = \frac{116{,}63\angle 10°}{8 + j12} = 8{,}09\angle -46{,}3°\,A$$

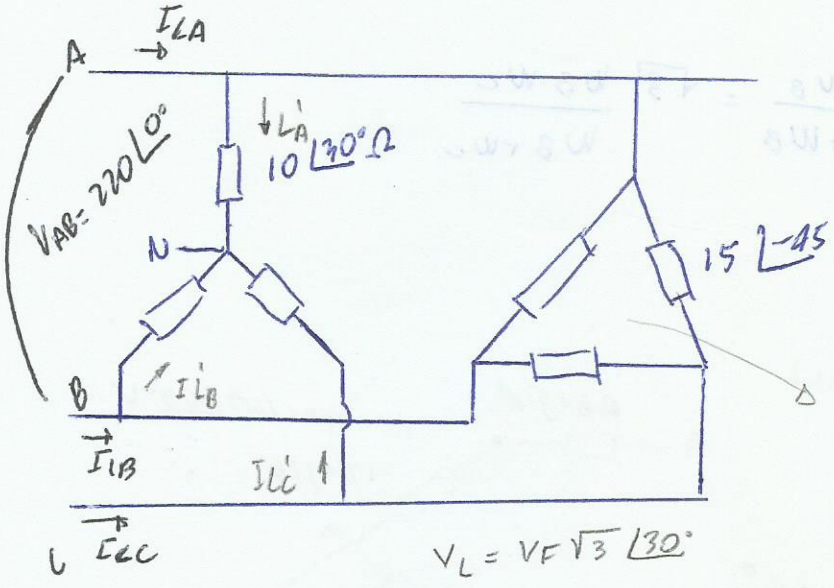
$I_{LB} = 8{,}09\angle -166{,}3°\,A$ $\quad I_{LC} = 8{,}09\angle -286{,}3°\,A$

$V_{An} = 8{,}09\angle -46{,}3 \times (0{,}6 + j0{,}9 + 8 + j12) = 125{,}42\angle 10°$

$V_L = V_F \cdot \sqrt{3}\angle 30$

$V_{ab} = 125{,}42\angle 10° \times \sqrt{3}\angle 30$ $\quad V_{ab} = 125{,}92\sqrt{3}\angle 40\ V$

Ex) Sistema equilibrado



$\dot{V}_{AB} = 220 \angle 0°$

$\dot{V}_{BC} = 220 \angle 120°$

$\dot{V}_{CA} = 220 \angle -120°$

Δ transformando para Y

temos $Z = 5 \angle -45$

$V_L = V_F \sqrt{3} \angle 30°$

$$I'_{LA} = \frac{220 \angle 0}{\sqrt{3} \angle 30 \times 10 \angle 30} = 12{,}7 \angle -60°$$

$$I''_{LA} = \frac{220 \angle 0}{\sqrt{3} \angle 30 \times 5 \angle -45} = 25{,}4 \angle 15$$

$I_{LA} = 31{,}2 \angle -8{,}2°$ (A)

$I_{LB} = 31{,}2 \angle 111{,}8°$ (A)

$I_{LC} = 31{,}2 \angle -128{,}2°$ (A)

Potência em sistema trifásico

$P_{carga} = P_{F1} + P_{F2} + P_{F3}$

$P_{carga} = 3 P_F$

$P_{at\ carga} = 3 V_F I_F . \cos\varphi$

$P_{at\ carga} = \sqrt{3}\, V_L I_L . \cos\varphi$

12.11

$V_{an} = 440 \angle 10^\circ \ V$

$V_{AN} = 398{,}32 \angle 8{,}72^\circ \ V$

$Z_{carga} = 20 + j24 \ \Omega$

$$I_{LA} = \frac{398{,}32 \angle 8{,}72}{20 + j24} = 12{,}75 \angle -41{,}5$$

$$Z_L = \frac{440 \angle 10}{12{,}75 \angle -41{,}5} - (20 + j24) \quad \therefore \ Z_L = 1{,}5 + j3 \ \Omega$$

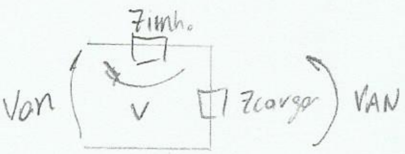
12.15

$V_{an} = 440 \angle 30^\circ$

$V_{AN} = 413{,}28 \angle 29{,}78^\circ$

$Z_{linha} = 1{,}2 + j1{,}5 \ \Omega$

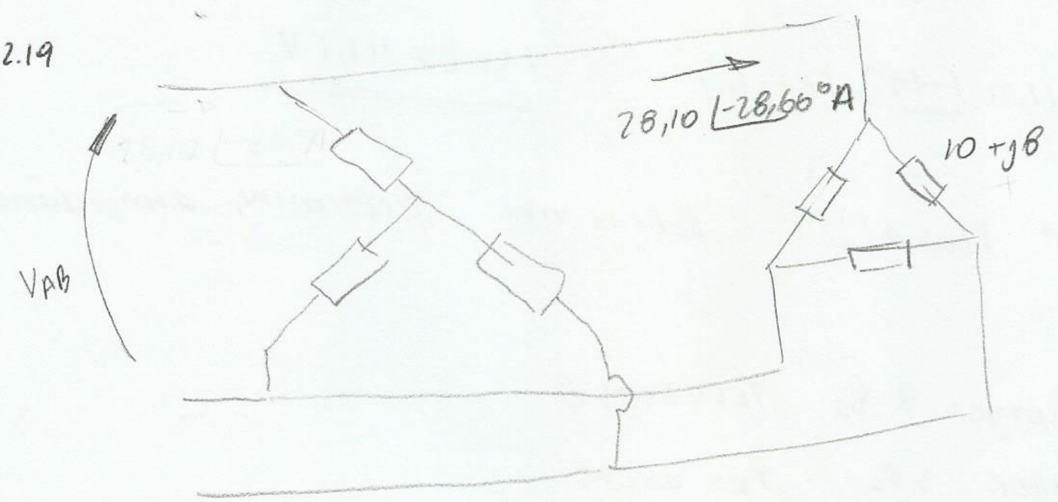
Determine impedância da carga



$$I = \frac{440 \angle 30 - 413{,}28 \angle 29{,}78}{1{,}2 + j1{,}5} = 13{,}94 \angle -17{,}9^\circ \ A$$

$$Z_{carga} = \frac{413{,}28 \angle 29{,}78}{13{,}94 \angle -17{,}9^\circ} = 19{,}96 + j21{,}92 \ \Omega$$

12.19



Δ

$V_F = V_L$

$I_L = \sqrt{3} \angle 30 \, I_F$

Y

$I_L = I_F$

$V_L = \sqrt{3} \angle 30 \, V_F$

$I_L = \sqrt{3} \angle 30 \, I_F$

$$I_F = \frac{28{,}10 \angle -28{,}66}{\sqrt{3} \cdot \angle 30} = 16{,}22 \angle -58{,}66$$

$$\therefore \ V = 16{,}22 \angle -58{,}66 \times (10 + j8) = 207{,}76 \angle -20^\circ \ V$$

12.23

$$\begin{bmatrix} 16,8+j13,6 & -8-j6 \\ -8-j6 & 16,8+j13,6 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 208\angle 20 \\ 208\angle -100 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{69,28+j360,96} \begin{bmatrix} 16,8+j13,6 & 8+j6 \\ 8+j6 & 16,8+j13,6 \end{bmatrix} \times \begin{bmatrix} 208\angle 20 \\ 208\angle -100 \end{bmatrix}$$

$I_1 = 7 - j7,68 = 10,39 \angle -47,6° A$

$I_2 = 10,81 \angle -112,6° A$

ou

$V_L = \sqrt{3} \cdot \angle 30 \; V_F$

$$V_F = \frac{208\angle 20}{\sqrt{3}\angle 30} = 120,09 \angle -10° V$$

$$I_{LA} = \frac{120,09\angle -10}{8,4+j6,8} = 11,11 \angle -49° A$$

$I_{LB} = 11,11 \angle -168,99 \; A \qquad I_{LC} = 11,11 \angle 71,01 \; A$

$|V_F| = 11,11 \angle -49 \times (8+j6) \quad \therefore \quad |V_F| = 111,1 V$

12.27 $\theta_{Vab} = 40°$ $I_{ba} = 4\angle 15°$ $P_{at} = 1400W$ Determine a impedância da carga.

$P_{carga} = 3 \cdot P_F \; ; P_F = VI\cos\theta$

$1400 = 3P_F \quad \therefore P_F = 467W$

$467 = |V| \cdot 4 \cdot \cos 25 \quad \therefore V = 128,73V$

$$Z = \frac{128,73\angle 40}{4\angle 15} = 32,18 \angle 25 \; \Omega$$

12.30

Y

$V_{ab} = 208\angle 60°$

$Z = 8 + j5\ \Omega$

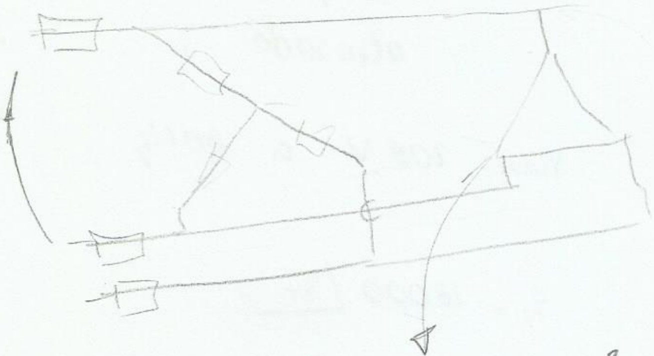
$Z_L = 1{,}2 + j$

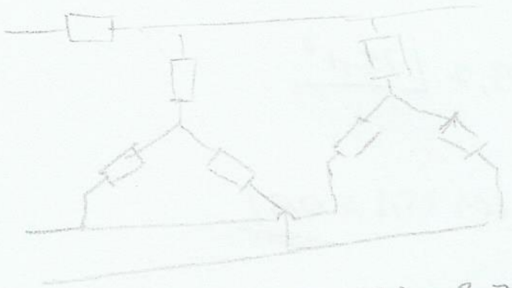
$\|$

Δ

$Z = 21 + j12\ \Omega$

$208\angle 60$





$$Z_Y' = \frac{(21+j12)^2}{3(21+j12)} = 7 + 4j$$

$(8+j5) \parallel (7+j4) = 3{,}74 + j\,2{,}23 = 4{,}35\angle 30{,}8$

$V_L = \sqrt{3}\angle 30\ V_F$

$V_F = 120{,}09\angle 30$

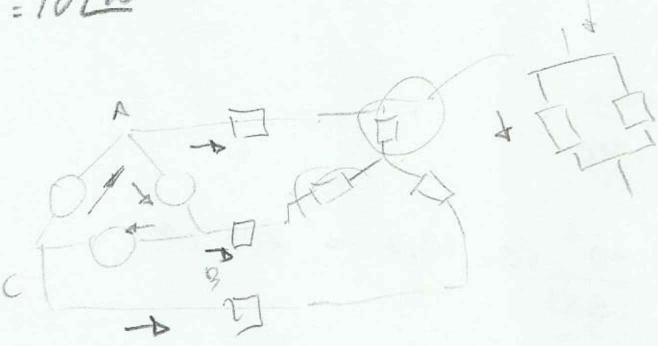
$$I_L' = \frac{120{,}09\angle 30}{4{,}94 + j3{,}2} = 20{,}36\angle -3{,}2$$

$$I_{LA} = \frac{7+4j}{15+j9} \times 20{,}36\angle -3{,}2 \quad \therefore \quad I_{LA} = 9{,}38\angle -4{,}4\ \text{A}$$

12.34

$Z_{F_1} = 4 + j4 \quad Z_A = 10 + j4 \quad Z_L = 0{,}3 + j0{,}2$

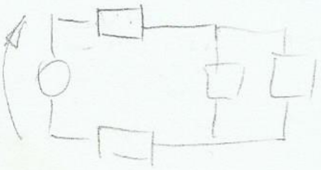
$I_{AN_1} = 10\angle 20°$



$10\angle 20 \times (4+j4) = (10+j4) \cdot I \quad \therefore I = 3{,}83 + j\,3{,}60$

$I_{LA} = 10\angle 20 + 3{,}83 + j3{,}6 = 14{,}97\angle 27{,}94\ \text{A}$

$I_{LB} = 14{,}97\angle 147{,}94 \quad I_{LC} = 14{,}97\angle -92{,}06$



12.46-

Carga 1 18KVA 0,8 fp atrasado

Carga 2 8KVA 0,8 fp avanço

Carga 3: 12 KVA 0,75 atrasado

$V_{carg} = 208\ V$ a $60 Hz$

$S_1 = 18000 \angle 37$

$S_2 = 8000 \angle -37$

$S_3 = 12000 \angle 41{,}41$

$\therefore S_T = 32873{,}7 \angle 25{,}1°$

$\therefore fp = \cos 25{,}1 = \underline{0{,}91}$

12.47

$V_L = 13{,}8\ KV$

$S_1 = 500 \angle 37$

$S_2 = 400 \angle 31{,}8$

$S_3 = 300 \angle 25{,}8$

$S_T = 1196{,}4 \angle 32{,}5$

$S_T = V I \cos \varphi$

$1196{,}4 = 13{,}8 . I \sqrt{3}$

$\therefore \underline{I = 50{,}05\ A}$

12.59

$V_{an} = 90 \angle 30°\ V$

$Z = 24 + j16\ \Omega$

$tg\varphi = \sqrt{3}\dfrac{W_A - W_B}{W_A + W_B} = \sqrt{3}\dfrac{W_B - W_C}{W_B + W_C}$

$Pat = W_A + W_B = \sqrt{3} . \dfrac{90^2}{28{,}84} \cos 33{,}7 = 405{,}14$

$0{,}67 = \sqrt{3} . \dfrac{(W_A - W_B)}{405{,}14}$

$\therefore \begin{cases} W_A - W_B = 156 \\ W_A + W_B = 405{,}14 \end{cases}$

$2W_A = 561{,}14$

$W_A = 280{,}6\ w$

$W_B = 124{,}6\ w$