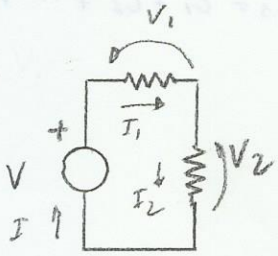
Estudo para P1 - Circuitos Elétricos II

Revisão circuitos I

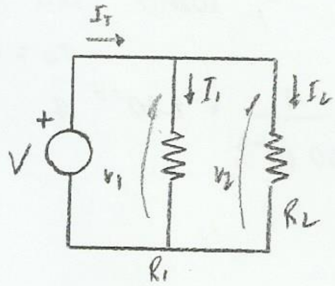
Divisor de Tensão:



Como $I = I_1 = I_2$, então

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V}{R_1+R_2} \therefore \boxed{V_2 = \frac{R_2}{R_1+R_2} . V}$$

Divisor de corrente



Como $V = V_1 = V_2$, então

$$I_1 R_1 = I_2 R_2$$

$$\Rightarrow \frac{I_1}{R_2} = \frac{I_2}{R_1} = \frac{I_1+I_2}{R_2+R_1} = \frac{I}{R_2+R_1} \therefore \boxed{I_2 = \frac{R_1}{R_1+R_2} . I}$$

Capítulo 6 - Capacitância e Indutância

Capacitância: é a propriedade de armazenar cargas elétricas

$$C = \frac{\varepsilon . A}{d} \quad \text{ou} \quad C = \frac{Q}{V} \qquad [C] = F \text{ (Farad)}$$

Como $C = \frac{dQ}{dv}$; $i = \frac{dQ}{dt}$ $\rightarrow i = \frac{C\,dv}{dt}$ e $v = \frac{1}{C}\int i\,dt + K$

$$\boxed{i = \frac{C\,dv}{dt} \text{ (A)} \qquad v = \frac{1}{C}\int i\,dt + v_0} \qquad \boxed{w = \frac{C . v^2}{2}}$$

Associação de capacitores

Quando esta em série: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$

Quando esta em paralelo: $C_s = C_1 + C_2 + \dots + C_n$

Problemas

6.1 $C = 100\,\mu F$ Qual a tensão após $4s$

$I = 1\,mA$

$$C = \frac{Q}{V} \therefore C = \frac{dQ}{dv} \; ; \; i = \frac{dQ}{dt} \text{ então: } i = \frac{C\,dv}{dt}$$

$\therefore v = \frac{1}{C}\int i\,dt + v_0$; como esta inicialmente descarregado $v_0 = 0$

$$v = \frac{1}{C} \times i(t - t_0) = \frac{1}{100\,10^{\cancel{-6}\,-3}} \times 1\,10^{-3}\,.\,4 \quad \therefore \boxed{v = 40V}$$

6.2 $C = 25\,\mu F$ Qual a tensão sobre o capacitor depois de 2,5 min

$v_0 = -10V$

$I = 2,5\,\mu A$

$$v = \frac{1}{C}\int i\,dt + v_0 \qquad 2{,}5\,min = 150\,s$$

$$v = \frac{1}{C}\,.\,i\,(t - t_0) + v_0$$

$$v = \frac{1}{25\,\cancel{10^{-6}}}\,.\,2{,}5\,\cancel{10^{-6}}\,.\,150 + (-10) \quad \therefore \boxed{v = 5V}$$

6.3 $C = 100\,\mu F$ Determine:

$v(t) = 120\,sen\,377t$

(a) corrente no capacitor

(b) expressão para energia

(a) $i = \frac{dQ}{dt} = \frac{C\,dv}{dt}$ $\qquad i = 100.10^{-6}\,.\,\frac{d(120\,sen\,377t)}{dt}$

$i = 100.10^{-6}\,.\,120\,.\,377\,.\,\cos(377t)$

$\boxed{\therefore i = 4{,}524\,\cos(377t)\,(A)}$

$w = \frac{Cv^2}{2} = \frac{100.10^{-6}}{2}\,.\,(120\,sen\,377t)^2$

$\boxed{w = 0{,}72\,.\,sen^2(377t)}$

6.4 $C = 25\mu F$ Determine I

$w(t) = 12 \sin^2 377t$ [J]

$$w = \frac{Cv^2}{2} \therefore v^2 = \frac{2w}{C} = \frac{2.12 \sin^2 377t}{25.10^{-6}}$$

$$\therefore v^2 = 960.10^3 \sin^2 377t$$

$$v = \frac{1}{C}\int i\,dt$$

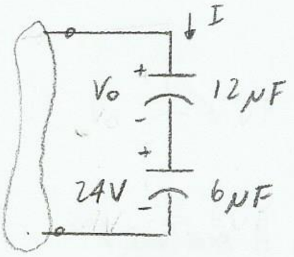
$$\frac{dv}{dt} = \frac{i}{C} \therefore i = C\frac{dv}{dt} = 25.10^{-6}.\frac{d[(960.10^3 \sin^2 377t)^{\frac{1}{2}}]}{dt}$$

$$i = 25.10^{-6}.\frac{d}{dt}[(960.10^3)^{\frac{1}{2}}.\sin 377t]$$

$$i = 25.10^{-6}(960.10^3)^{\frac{1}{2}}.377.\cos 377t$$

$$\therefore \boxed{i = 9{,}23 \cos 377t} \text{ (A)}$$

6.5



Associação em série

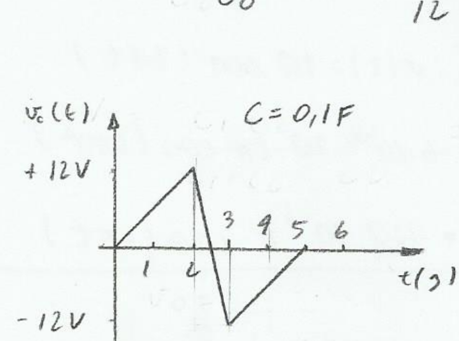
$V = V_0 + V_1$

$Q_1 = Q_2 = Q$; $C = \frac{Q}{V}$

$C_1V_1 = C_2V_2 = C_{eq}.V$

$$V_0 = \frac{C_2V_2}{C_0} = \frac{6.24}{12} \therefore \boxed{V_0 = 12V}$$

6.6



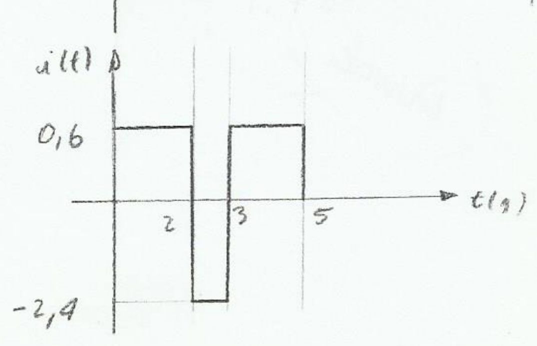
p/ $0<t<2 \Rightarrow v_c(t) = 6t$

$i = C\frac{dv}{dt} \therefore i(t) = 0{,}1.6 = 0{,}6$ (A)

p/ $2<t<3 \Rightarrow v_c(t) = -24t + 60$

$y - 12 = -24(x-2) \therefore i(t) = -24.0{,}1 = -2{,}4$ (A)

$y = -24x + 60$



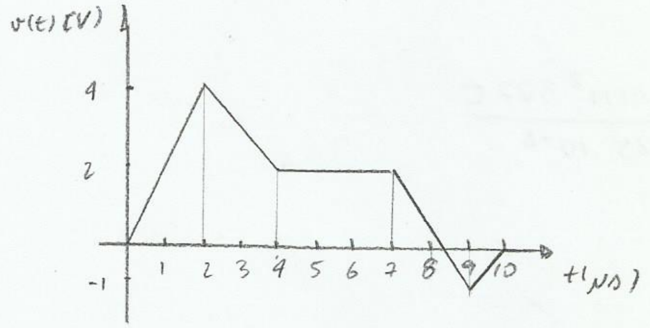
p/ $3<t<5 \Rightarrow v_c(t) = 6t - 30$

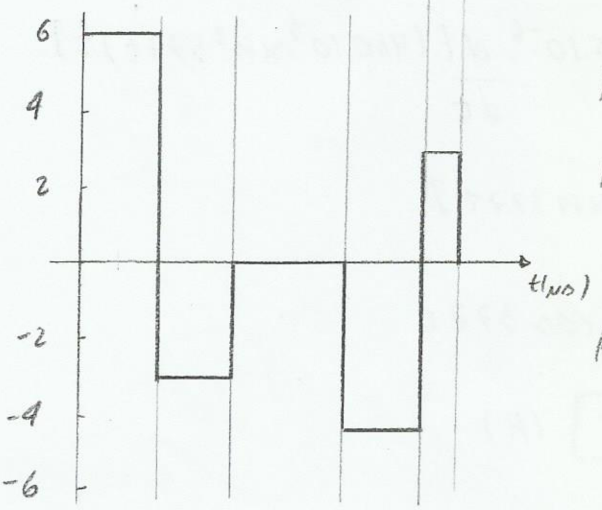
$y + 12 = 6(x-3) \therefore i(t) = 6.0{,}1 = 0{,}6$ (A)

$y = 6x - 30$

6.7

$C = 3\mu F$





p/ $0<t<2$

$v_c(t) = 2000\,t \quad i(t) = 3.10^{-3} \cdot 2000 = 6\,A$

p/ $2<t<4$

$i(t) = C.\frac{dv}{dt}$

$= 3.10^{-3}\left(-\frac{2}{2.10^{-3}}\right) = -3$

p/ $4<t<7 \quad i(t)=0$

p/ $7<t<9$

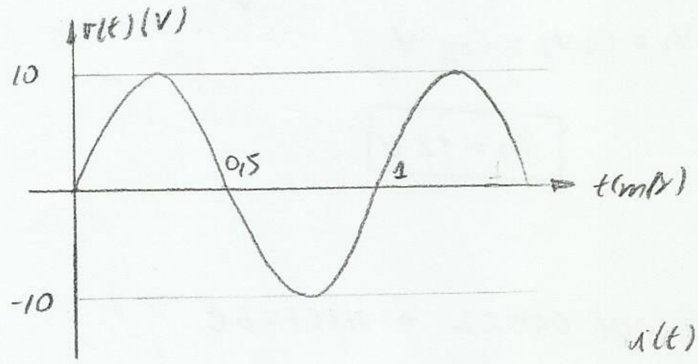
$i(t) = 3.10^{-3}.\left(\frac{-3}{2.10^{-3}}\right) = -4{,}5$

p/ $9<t<10$

$i(t) = 3.10^{-3}.\frac{1}{10^{-3}} = 3$

6.8 e 6.9 → São iguais ao 6.7

6.10 $\quad C = 6\mu F$



$v(t) = A\, sen(\omega t)$

$A = 10$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi$

$\therefore v(t) = 10\, sen\,(2\pi t)$

$i(t) = 6.10^{-6}.10.2\pi \cos(2\pi t)$

$\boxed{\therefore i(t) = 120.10^{-6}\pi \cos(2\pi t) \quad (A)}$

Dúvida

Indutores ou Bobinas

$$v = L \cdot \frac{di}{dt} \qquad [L] = \frac{V \cdot s}{A} = H \text{ (henry) (Auto indutância)}$$

Lei de Faraday

Associação de Indutores

- Associação série: $L = L_1 + L_2 + L_3 + \dots + L_n$
- Associação paralela: $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$

Energia armazenada no indutor

$$W = \frac{1}{2} L \cdot I^2$$

6.11 $L = 100\,mH$ Determine:

$i(t) = 2 \sin 377t$

(a) Tensão no indutor

(b) Energia armazenada

$$v = L \cdot \frac{di}{dt} \Rightarrow v = 100 \cdot 10^{-3} \frac{d(2 \sin 377t)}{dt}$$

$$v = 100 \cdot 10^{-3} \cdot 2 \cdot 377 \cdot \cos(377t) \Rightarrow \boxed{v = 75{,}4 \cos(377t) \text{ (V)}}$$

$$W = \frac{100 \cdot 10^{-3}}{\cancel{2}} \cdot (\cancel{2} \sin 377t)^2 \quad \therefore \quad \boxed{W = 0{,}1 \sin^2(377t) \text{ (J)}}$$

6.12 $L = 50\,mH$ $\quad w(t) = 1{,}2 \sin^2(377t)$ (J) $\quad$ Determine a tensão

$$W = \frac{L \cdot I^2}{2} \quad ; \quad v = L \frac{di}{dt} \Rightarrow v = 50 \cdot 10^{-3} \cdot \frac{d}{dt}\left[(48 \sin^2(377t))^{\frac{1}{2}} \right]$$

$$i^2 = \frac{2}{50 \cdot 10^{-3}} \cdot (1{,}2 \sin^2(377t)) \qquad v = 50 \cdot 10^{-3} \cdot \sqrt{48} \cdot 377 \cdot \cos(377t)$$

$$i = (48 \sin^2(377t))^{\frac{1}{2}} \qquad \boxed{v = \pm 130{,}60 \cdot \cos(377t) \text{ [V]}}$$

6.13

$i(t) = 0 \qquad t < 0$ (a) tensão no indutor

$i(t) = 10(1-e^{-t})$ (A) $\qquad t > 0$ (b) energia armazenada

$v = L\dfrac{di}{dt}$; $L = 25mH$

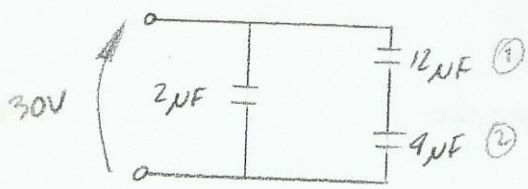
$$v = 25.10^{-3} . \frac{d}{dt}\left[10(1-e^{-t})\right] = 25.10^{-3}.10.e^{-t} \therefore \boxed{v = 0{,}25.e^{-t} \ (V)}$$

$$w = \frac{L I^2}{2} = \frac{25.10^{-3}}{2} \left[10(1-e^{-t})\right]^2 = \boxed{1{,}25(1-e^{-t})^2 \ [J]}$$

6.14 $\quad v(1) = 0{,}09V$ e $w = 0{,}5 J$

→ (Exercícios 6.15 ao 6.21 → fazer depois)

6.22



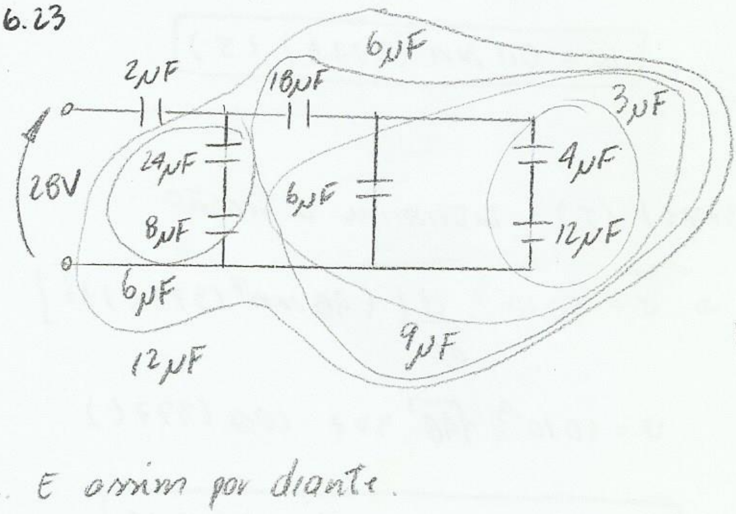
$Q_1 = Q_2 = Q$

$C_1 V_1 = C_2 V_2 = C_{eq} V$

$V_1 = \dfrac{\cancel{C_1} \times C_2}{C_1 + C_2} \dfrac{V}{\cancel{C_1}}$

$$V_1 = \frac{4}{4+12} \times 30 \therefore \boxed{V_1 = 7{,}5V} \qquad \therefore \boxed{V_2 = 22{,}5V}$$

6.23



$Q_1 = Q_2 = Q$

$C_1 V_1 = C_2 V_2 = C_{eq} V$

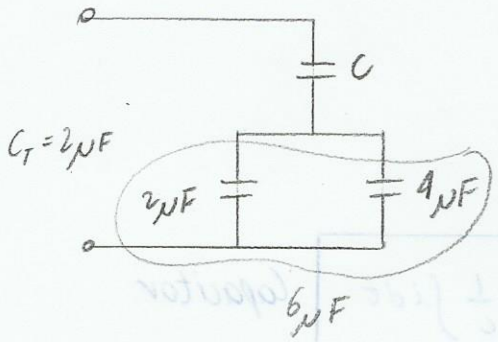
$V_2 = \dfrac{C_1 \cancel{C_2}}{C_1 + C_2} \dfrac{V}{\cancel{C_2}}$

$$V_2 = \frac{12}{12+2} . 28 \therefore \boxed{V_2 = 24V}$$

... E assim por diante.

Basta aplicar $V_2 = \dfrac{C_1}{C_1 + C_2} V_1$

6.25 Determine C para produzir $C_T = 2\mu F$



$$C_T = \frac{C \cdot C_1}{C + C_1}$$

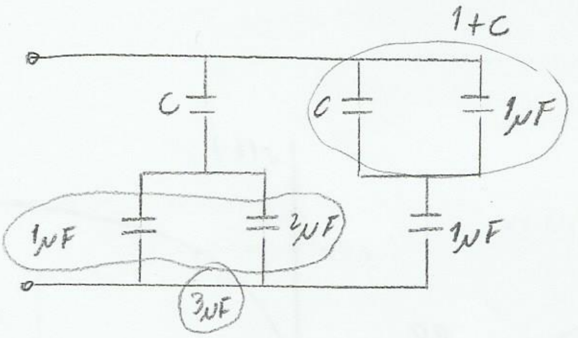
$$C_T (C + C_1) = C \cdot C_1$$

$$C_T \cdot C - C \cdot C_1 = -C_T C_1$$

$$C (C_T - C_1) = -C_T C_1$$

$$C = \frac{C_T C_1}{C_1 - C_T} = \frac{2 \cdot 6}{6 - 2} \therefore \boxed{C = 3\mu F}$$

6.26 Determine C para produzir $C_T = 1\mu F$



$$C_T = \frac{3C}{3+C} + \frac{(1+C) \cdot 1}{1+C+1} \; ; \; C_T = 1$$

$$1 = \frac{3C}{3+C} + \frac{1+C}{2+C}$$

$$1 = \frac{3C(2+C) + (1+C)(3+C)}{(3+C)(2+C)} \rightarrow 1 = \frac{6C + 3C^2 + 3 + 3C + C + C^2}{6 + 3C + 2C + C^2}$$

$$1 = \frac{4C^2 + 10C + 3}{C^2 + 5C + 6} \qquad C^2 + 5C + 6 = 4C^2 + 10C + 3$$

$$3C^2 + 5C - 3 = 0$$

$$C = \frac{-5 \pm \sqrt{25 + 4 \cdot 3 \cdot 3}}{6} \therefore C_1 = 0{,}47 \text{ ou } \cancel{C_2 = -2{,}13}$$

$$\therefore \boxed{C = 0{,}47\,\mu F}$$

Capítulo 7

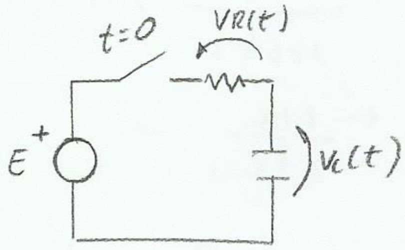
Resumo:

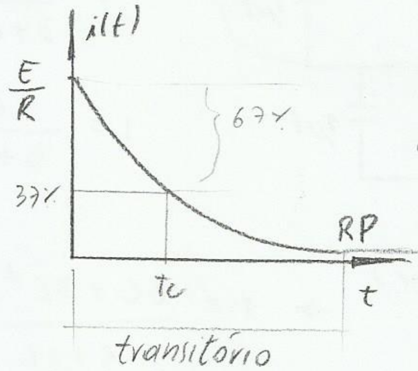
$$C = \frac{Q}{V} = \frac{dQ}{dv} \; ; \; \cancel{dQ = \frac{di}{dt} \rightarrow Cdv = \frac{di}{dt}}$$

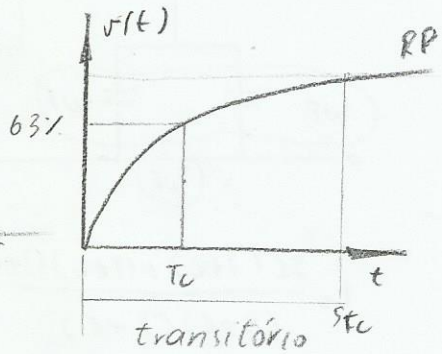
$$\therefore \; i = \frac{dQ}{dt} = \frac{Cdv}{dt} \; \therefore \; \boxed{i_c = C \cdot \frac{dv}{dt} \qquad v = \frac{1}{c}\int i\,dt} \text{ Capacitor}$$

$$\boxed{v_i = L\frac{di}{dt} \qquad i_i = \frac{1}{L}\int v\,dt} \text{ Indutor}$$

- Circuito RC em corrente contínua

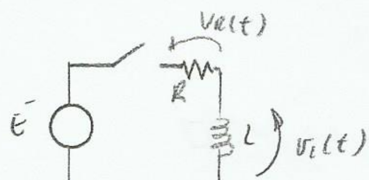


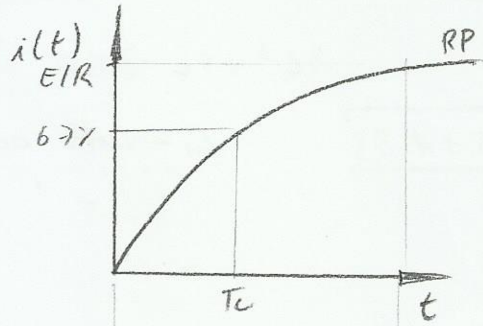


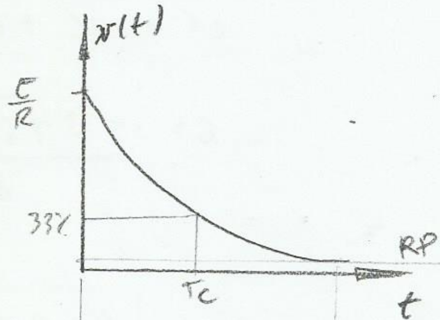


RP: Regime permanente

- Circuito RL em corrente contínua







Método dos 6 passos:

Solução geral para 1ª ordem $\boxed{x(t) = K_1 + K_2 \cdot e^{\frac{-t}{T_C}}}$

1º) Adotar solução geral

2º) p/ t<0 determinar $x(0)$

- Capacitor $\begin{cases} \text{é um circuito aberto} \\ \text{e } V_C(0-) \text{ é a tensão entre os pontos do capacitor} \end{cases}$

- Bobina $\begin{cases} \text{é um curto-circuito} \\ \text{e } I_L(0-) \text{ é a corrente que passa no "curto"} \end{cases}$

3º) p/ t>0

- Substituir $\begin{cases} C \to V_C(0-) \\ L \to I_L(0-) \end{cases}$ e Determinar $x(0+)$

4º) p/ $t \to \infty$

- Substituir $\begin{cases} C \to \text{circuito aberto} \\ L \to \text{curto circuito} \end{cases}$ e determinar $x(\infty)$

5º) Determinar T_C

$$\begin{cases} C \to & T_C = R_{eq} \times C \\ L \to & T_C = \dfrac{L}{R_{eq}} \end{cases}$$

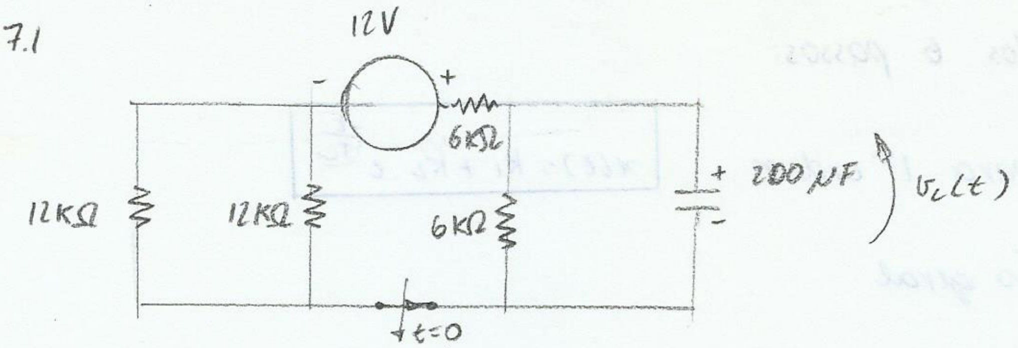
6º) Determinar K_1 e K_2

em $t = 0+$ $\quad x(0) = K_1 + K_2$

em $t = \infty$ $\quad x(\infty) = K_1$

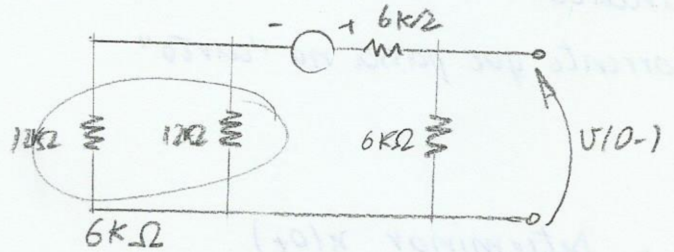
$$\therefore x(t) = K_1 + K_2\, e^{\frac{-t}{T_C}}$$

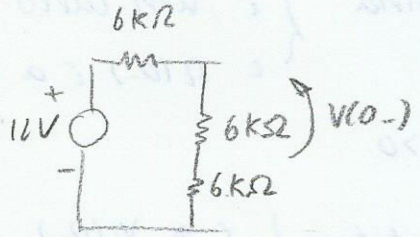
7.1



1º passo: $v_c(t) = K_1 + K_2 \cdot e^{\frac{-t}{T_c}}$

2º passo: p/ $t<0$



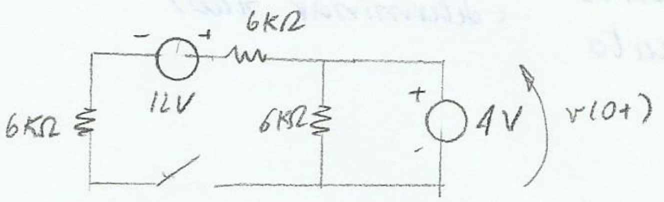


$\underline{V(0-) = 4V}$

C: Abre e V(0-)

B: curto e I(0-)

3º passo: p/ $t>0$

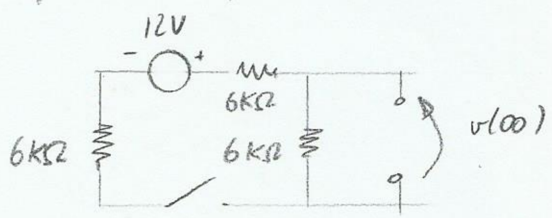


C: C → V(0-)   x(0+)

L: L → I(0-)

$\therefore \underline{V(0+) = 4V}$

4º passo: p/ $t \to \infty$

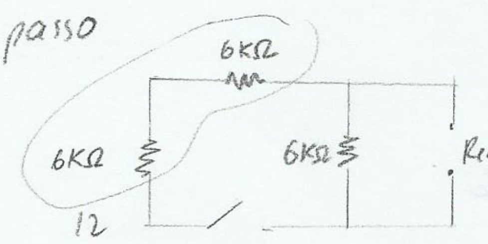


$\therefore \underline{v(\infty) = 0V}$

C: Abre e x(∞)

L: curto e x(∞)

5º passo



$\therefore R_{eq} = 6\Omega$

C: $T_c = C \cdot R_{eq}$

$T_c = \frac{L}{R_{eq}}$

$T_c = 6 \cdot K \cdot 200\mu \therefore T_c = 1{,}2s$

6º passo

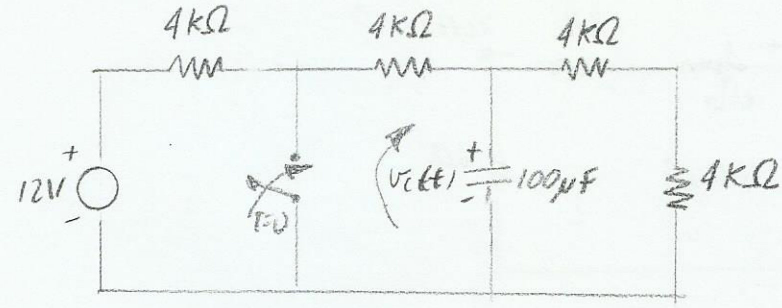
$v(0+) = K_1 + K_2 = 4$

$v(\infty) = K_1 = 0$

$\therefore K_1 = 0$ e $K_2 = 4$

$v_c(t) = 4e^{-0{,}83t}$

7.2

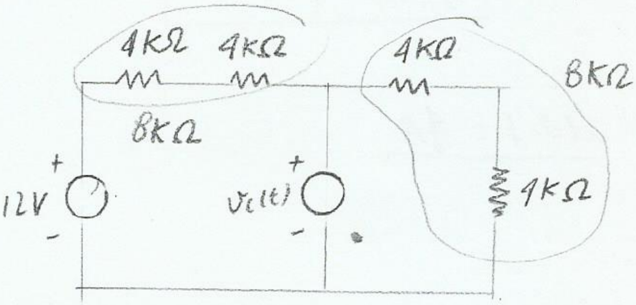


1º passo: $v_c(t) = K_1 + K_2 e^{\frac{-t}{T_c}}$

2º passo:

C: Abre e $v(0_-)$ $\quad$ p/ $t<0$.

I: Curto e $I(0_-)$

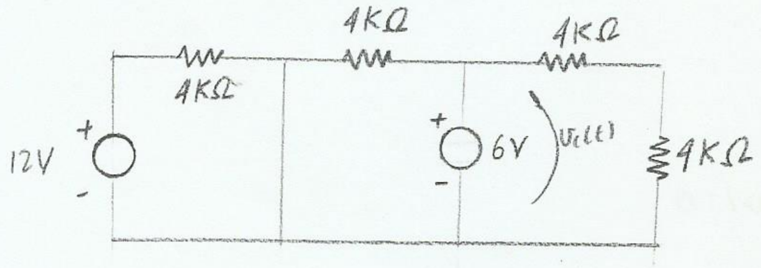


$\therefore \underline{v(0_-) = 6V}$

3º passo: p/ $t>0$

C: Substitui $v(0_-)$ $\quad$ e determina $x(0$

L: Substitui $I(0_-)$

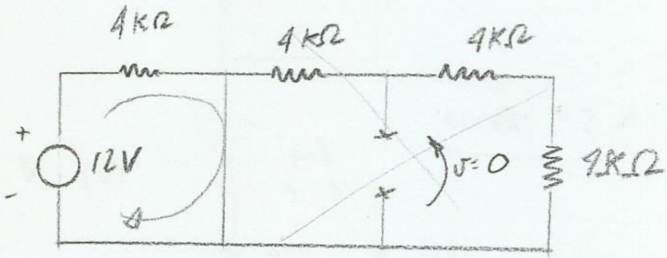


$\therefore \underline{v(0+) = 6V}$

4º passo: p/ $t \to \infty$

C: Abre e $x(\infty)$

I: Curto e $x(\infty)$

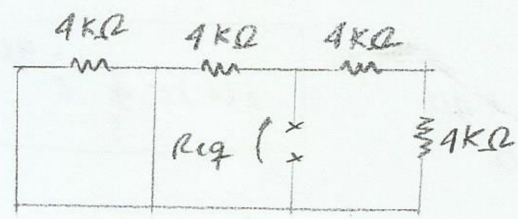


$v(\infty) = 0$

5º passo:

C: $T_c = R_{eq} \times C$

L: $T_c = \frac{L}{R_{eq}}$



$\therefore R_{eq} = (4+4) \parallel 4$

$\therefore R_{eq} = 2{,}67$

$T_c = 0{,}27$

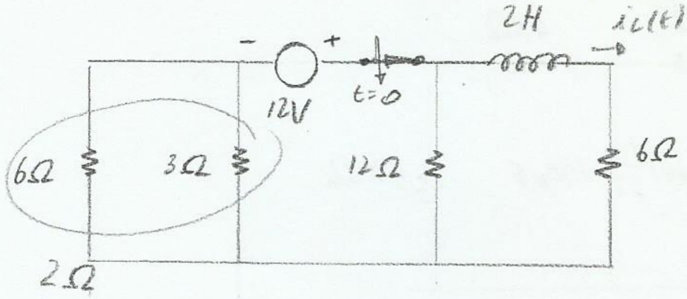
6º passo

$v(0) = K_1 + K_2 = 6 \quad \therefore K_2 = 6$

$v(\infty) = K_1 = 0$

$\therefore \boxed{v(t) = 6 \cdot e^{-3{,}75t}}$

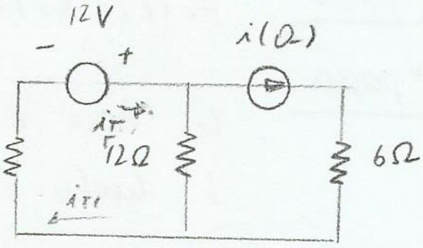
7.3



1º passo: $i(t) = K_1 + K_2 \cdot e^{\frac{-t}{\tau_c}}$

2º passo: B: Abre e $v(0_-)$

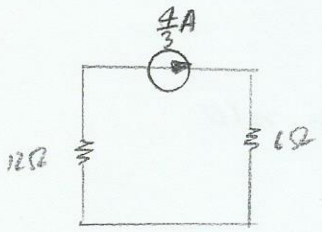
L: Curto e $I(0_-)$



$R_{eq} = 2 + (12 \| 6)$ ∴ $R_{eq} = 6\Omega$

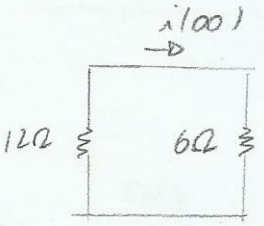
$I_T = 2\,A$ $\quad i(0_-) = \frac{12}{12+6} \cdot 2$ ∴ $i(0_-) = \frac{4}{3}A$

3º passo: p/ $t > 0$



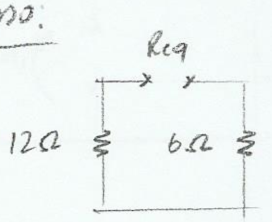
$i(0_+) = \frac{4}{3}A$

4º passo: p/ $t \to \infty$



$i(\infty) = 0$

5º passo:



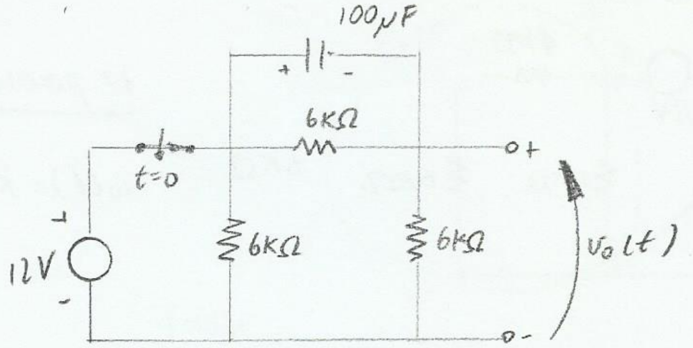
$R_{eq} = 18$ $\quad \tau_c = \frac{2}{18} = \frac{1}{9}$

6º passo

$i(0_+) = K_1 + K_2 = 4/3$ ∴ $\boxed{i(t) = \frac{4}{3} \cdot e^{-9t}}$

$i(\infty) = K_1 = 0$

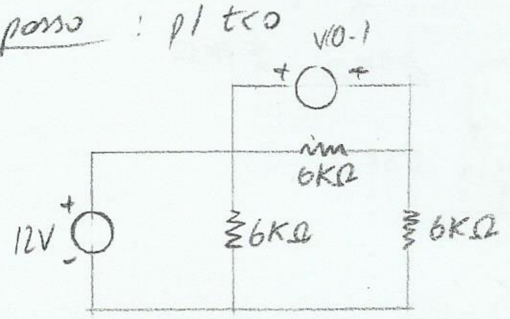
7.1)



1º passo

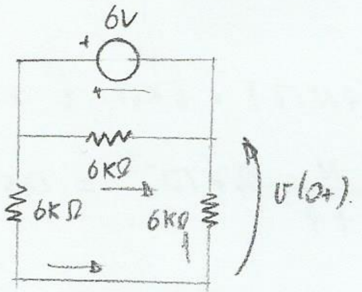
$v_o(t) = K_1 + K_2 e^{-\frac{t}{\tau_c}}$

2º passo : p/ t<0



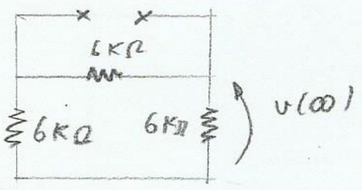
$V(0_-) = \frac{6}{6+6} \cdot 12 = 6V$

3º passo: p/ t>0



$v(0_+) = \frac{6}{6+6} \cdot 6 = -3V$

4º passo: p/ t→∞



$\therefore\ v(\infty) = 0$

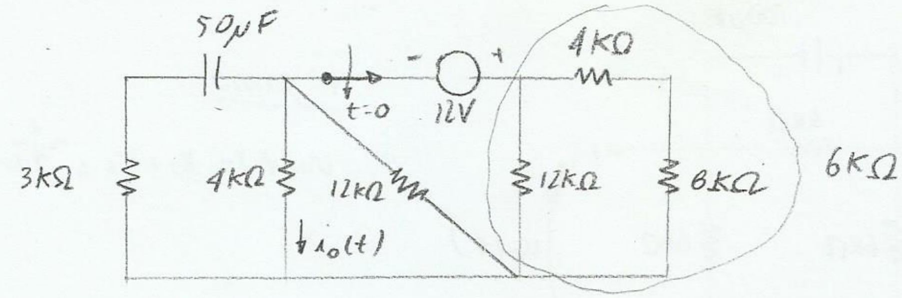
5º passo: $R_{eq} = 6 // (6+6) \ \therefore\ R_{eq} = 4k\Omega \ \therefore\ \tau_c = 4K \cdot 100\mu = 0{,}4\,s$

6º passo:

$v(0) = K_1 + K_2 = -3 \ \therefore\ K_2 = 3$

$v(\infty) = K_1 = 0 \ \therefore\ \boxed{v_o(t) = -3 \cdot e^{-2{,}5t}}$

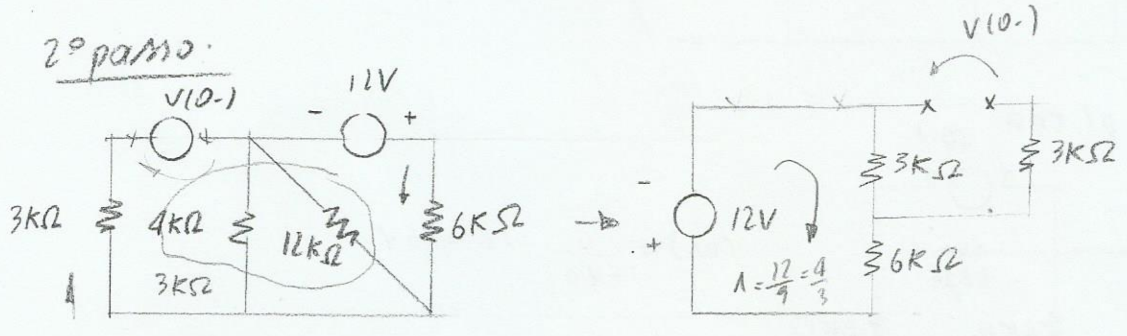
7.5



1° passo

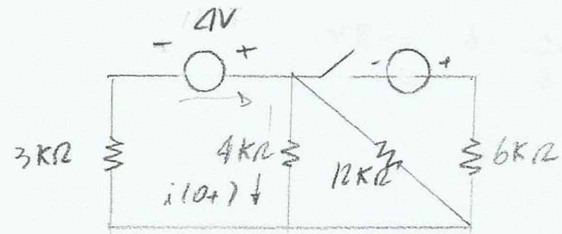
$$i_o(t) = K_1 + K_2 e^{-\frac{t}{T_c}}$$

2° passo:



$V(0-) = \frac{4}{3} \cdot 3 \quad \therefore \quad V(0-) = 4\,V$

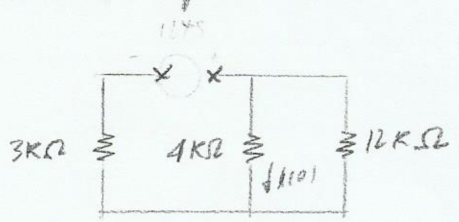
3° passo p/ t>0



$Req = 3 + (4 // 12) = 6\,K\Omega \quad e \quad i(t) = 0{,}67 . 10^{-3}$

$i(0+) = \frac{12}{12+4} \cdot 0{,}7\, 10^{-3} = 0{,}5 . 10^{-3}$

4° passo p/ t→∞



$i(\infty) = 0$

5° passo

$Req = 3 + (12 // 4) = 6\,\Omega \qquad T_c = Req \times C = 6 . 50 . 10^3 . 10^{-6} = 0{,}3$

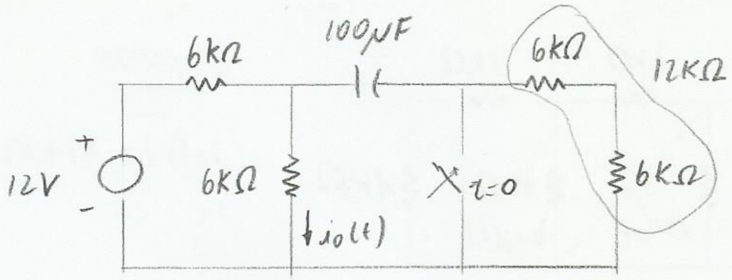
6° passo

$i(0+) = K_1 + K_2 = 0{,}5$

$i(\infty) = K_1 = 0$

$\therefore \boxed{i(t) = 0{,}5\, e^{\frac{-t}{0{,}3}} \ (A)}$

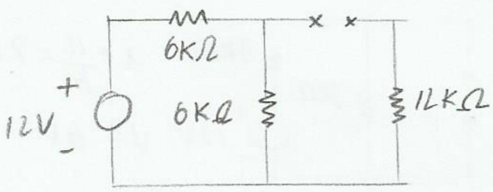
7.6



1° passo

$i_0(t) = K_1 + K_2 e^{-\frac{t}{T_C}}$

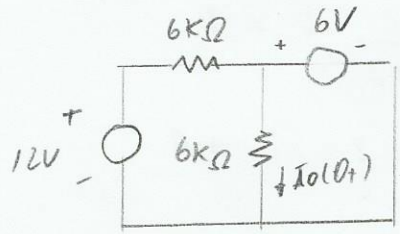
2° passo   p/ $t<0$



$i = \frac{12}{12} = 1$

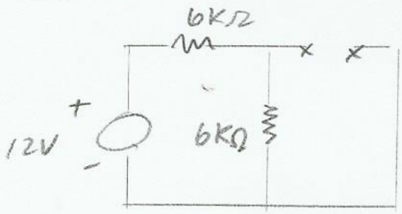
$v(0_-) = 6.1 = 6V$

3° passo   p/ $t>0$



$i_0(0_+) = \frac{6}{6} = 1.10^{-3}$ A

4° passo   p/ $t \to \infty$



$i(\infty) = 1.10^{-3}$ A

5° passo $R_{eq} = 3K\Omega$ $T_C = 3.10^3.100.10^{-6} = 0{,}3$

6° passo

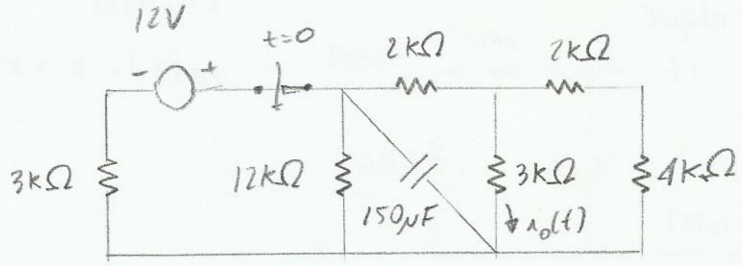
$i(0+) = K_1 + K_2 = 1.10^{-3}$

$i(\infty) = K_1 = 1.10^{-3}$

$K_2 = 0$

$\boxed{i_0(t) = 1.10^{-3}\ A}$

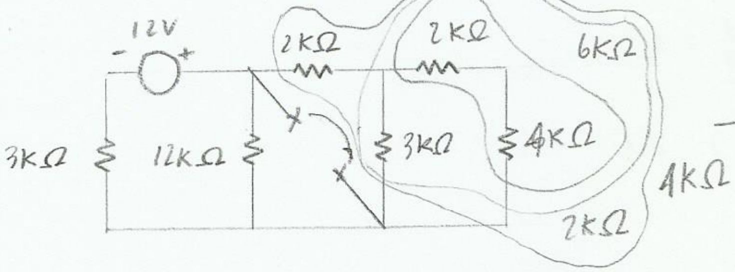
7.7

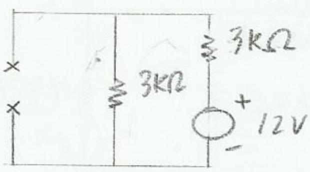


1º passo

$$i_o(t) = k_1 + k_2 e^{-\frac{t}{T_c}}$$

2º passo p/ t<0

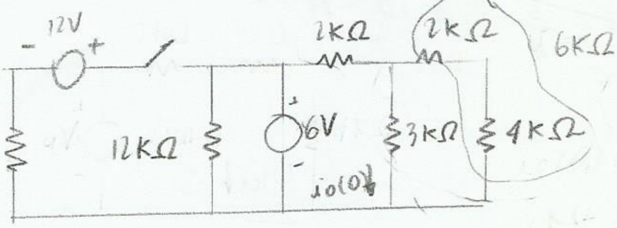




$i = \frac{12}{6} = 2mA$

$V = 6V$

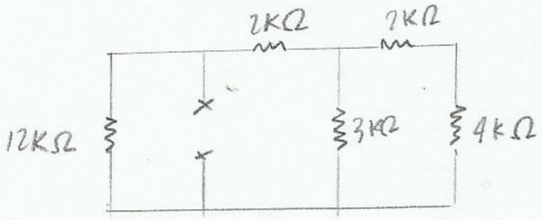
3º passo p/ t>0



$$V_o = \frac{2}{2+2} \cdot 6 = 3$$

$$i(0+) = 2 \cdot 3 = 6\ mA$$

4º passo p/ $t \to \infty$



$\therefore\ i(\infty) = 0$

5º passo

$$R_{eq} = 12 \parallel [2 + 3 \parallel (2+4)] = 3k\Omega$$

$$T_c = 3k\Omega \cdot 150\mu F = 0{,}45$$

6º passo

$$i(0+) = k_1 + k_2 = 6$$

$$i(\infty) = k_1 = 0$$

Revisão circuitos II

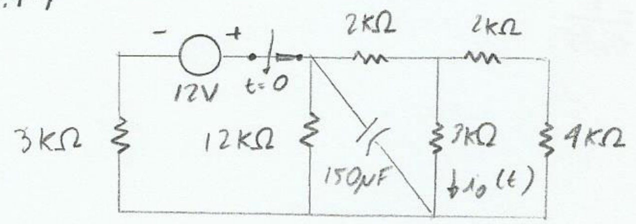
Divisor de tensão: $V_2 = \dfrac{R_2}{R_1+R_2} \cdot V$

Divisor de corrente: $I_2 = \dfrac{R_1}{R_1+R_2} I$

Capacitor: $i = C\dfrac{dv}{dt}$ $\quad v = \dfrac{1}{C}\int i\,dt$ $\quad w = \dfrac{1}{2}Cv^2$

Indutor: $v = L\dfrac{di}{dt}$ $\quad i = \dfrac{1}{L}\int v\,dt$ $\quad w = \dfrac{1}{2}L \cdot I^2$

7.7-)

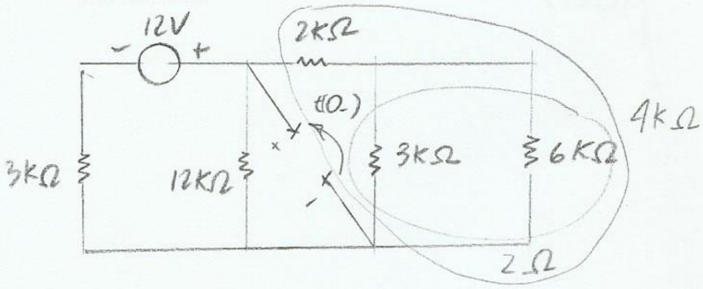


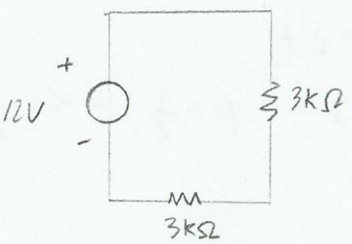
1º passo: $i(t) = K_1 + K_2 \cdot e^{-\frac{t}{\tau c}}$

2º passo: p/ $t<0$

C: Abre → Tensão

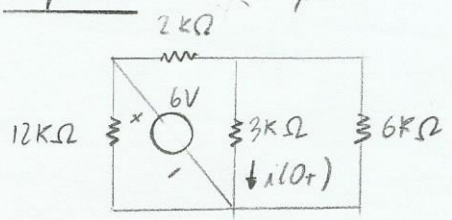
L: curto → Corrente

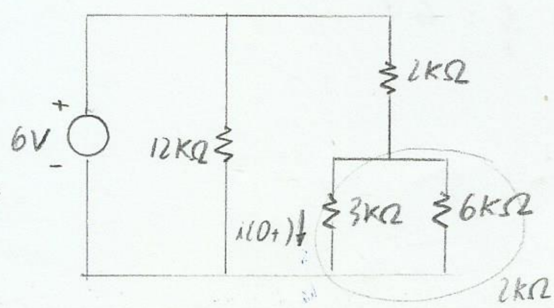




$v(0_-) = 6V$

3º passo p/ $t>0$



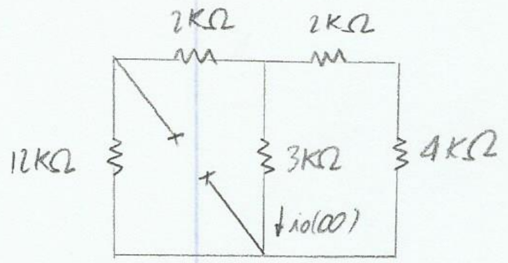


$i' = \dfrac{6}{4} = \dfrac{3}{2}$

$i(0_+) = \dfrac{6}{3+6} \cdot \dfrac{3}{2}$

$\therefore i(0_+) = 1$

4º passo   p/ $t \to \infty$



$i(\infty) = 0$

5º passo

$R_{eq} = 12 // (2 + 3 // (2+4)) \therefore R_{eq} = 3\Omega$

$T_c = R_{eq} \cdot C = 3 \cdot 10^3 \cdot 150 \cdot 10^{-6} = 0{,}45$

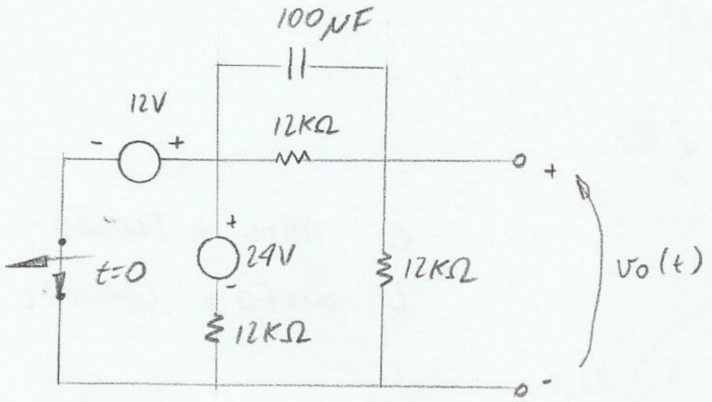
6º passo

$i(0+) = K_1 + K_2 = 1 \quad \therefore K_2 = 1$

$i(\infty) = K_1 = 0$

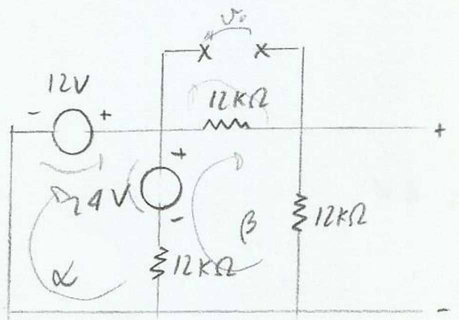
$\therefore \; i(t) = e^{\frac{-t}{0{,}45}}$

7.B)



1º passo: $v(t) = K_1 + K_2 \cdot e^{\frac{-t}{\tau c}}$

2º passo:

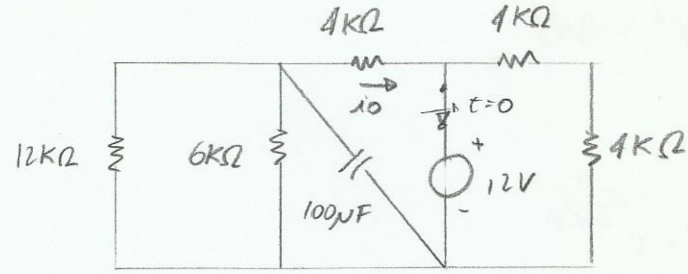


$$\begin{cases} 12\alpha - 12\beta = 12 - 24 \\ -12\alpha + 36\beta = 24 \end{cases}$$

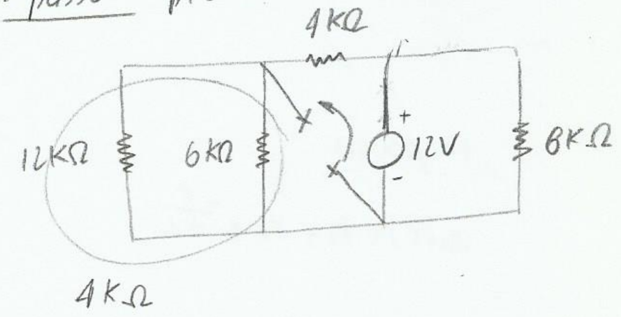
$$\begin{cases} \alpha - \beta = -1 \\ 2\beta = 1 \end{cases} \quad \therefore \beta = \frac{1}{2} \text{ e } \alpha = \frac{-1}{2}$$

$v(0_-) = 12 \cdot \frac{1}{2} = 6V$

7.9

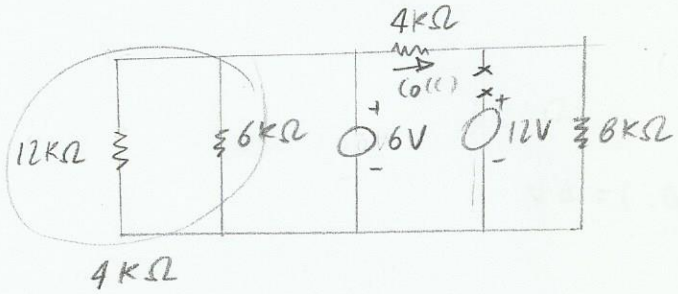


2º passo p/ t<0



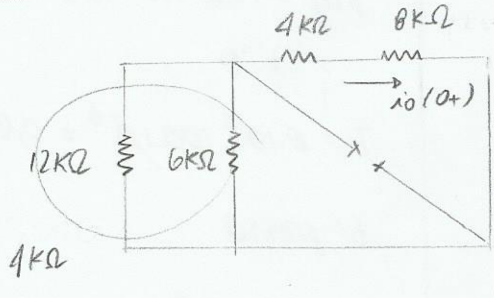
$V(0_-) = 6V$

3º passo p/ t>0



$i(0+) = \frac{6}{12} = 0{,}5\ A$

4º passo p/ t→∞



$i(\infty) = 0$

5º passo

$R_{eq} = 4 // (4+8) = 3Ω$

$T_C = 3.10^3 . 100.10^{-6} = 0{,}3$

6º passo

$K_1 + K_2 = 0{,}5$

$K_1 = 0$

$i(t) = 0{,}5\, e^{\frac{t}{-0{,}3}}$

5.º passo:

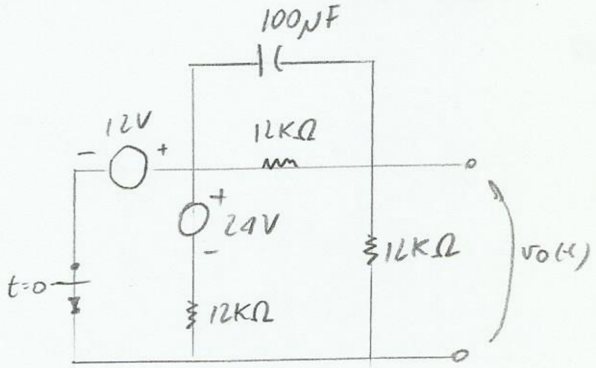
$R_{eq} = 3\Omega$ $\qquad T_C = 3.10^3 . 150.10^{-6} = 0{,}45$

6.º passo:

$K_1 + K_2 = 1$ $\qquad i(t) = e^{-\frac{t}{0{,}45}}$

$K_1 = 0$

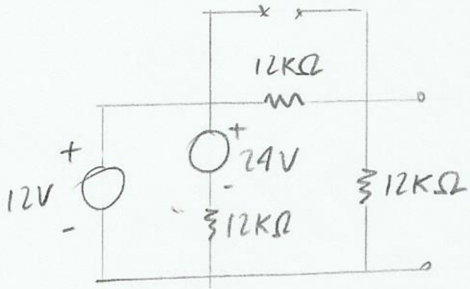
7.8



1.º passo

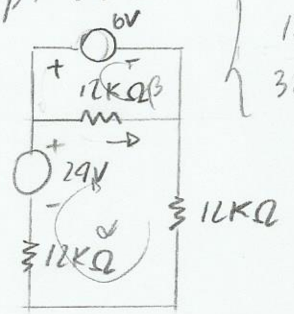
$v_0(t) = K_1 + K_2 e^{-\frac{t}{T_C}}$

2.º passo p/ t<0



$v(0_-) = 6V$

3.º passo p/ t>0



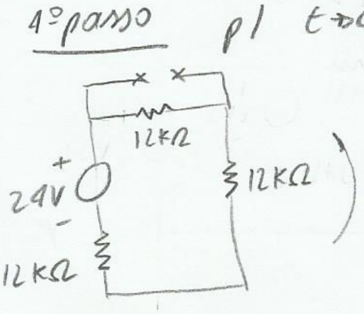
$\begin{cases} 12\beta - 12\alpha = -6 \\ 36\alpha - 12\beta = 24 \end{cases}$

$\begin{cases} -2\alpha + 2\beta = -1 \\ 6\alpha - 2\beta = 4 \end{cases}$

$4\alpha = 3$

$\alpha = \frac{3}{4} \therefore V = \frac{3.12}{4} = 9V$

4.º passo p/ $t \to \infty$



$V(\infty) = \frac{24}{3} = 8V$

5.º passo

$R_{eq} = 12 // (12+12)$

$= 8$

$T_C = 8.10^3 . 100.10^{-6} = 0{,}8$

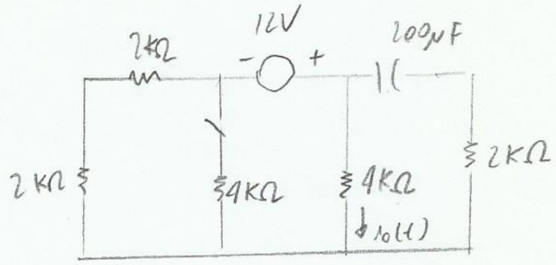
6.º passo

$K_1 + K_2 = 9$

$K_1 = 8 \therefore K_2 = 1$

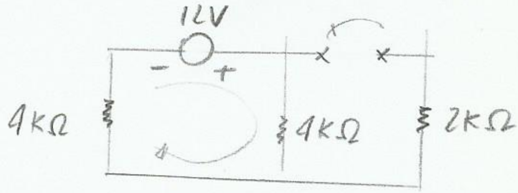
$v(t) = 8 + e^{-\frac{t}{0{,}8}} (V)$

7.12



1º passo: $i(t) = K_1 + K_2 \cdot e^{-\frac{t}{\tau_C}}$

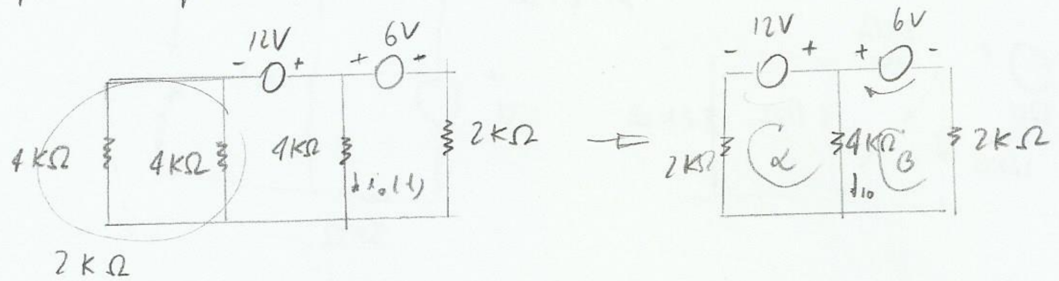
2º passo: p/ $t<0$



$v(0_-) = \frac{3}{2} \cdot 4 = \frac{12}{2} = 6$

$i = \frac{12}{8} = \frac{6}{4} = \frac{3}{2}$

3º passo: p/ $t>0$



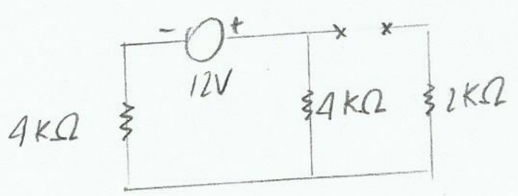
$$\begin{cases} 6\alpha - 4\beta = 12 \\ 6\beta - 4\alpha = -6 \end{cases} \sim \begin{cases} 6\alpha - 4\beta = 12 \\ -4\alpha + 6\beta = -6 \end{cases} \sim \begin{cases} 24\alpha - 16\beta = 48 \\ -24\alpha + 36\beta = -36 \end{cases}$$

$20\beta = 12 \quad \therefore \beta = \frac{12}{20} = \frac{3}{5}$

$\alpha = \frac{12+4\beta}{6} = \frac{12 + 4 \cdot \frac{3}{5}}{6} = \frac{60+12}{30} = \frac{12}{5}$

$I(0_+) = \frac{12}{5} - \frac{3}{5} = \frac{9}{5}$

4º passo: p/ $t \to \infty$



$I(\infty) = \frac{12}{8} = \frac{3}{2}$

5º passo:

$R_{eq} = (4 // 4) + 2 = 4\,k\Omega \qquad \tau_C = 4 \cdot 10^3 \cdot 200 \cdot 10^{-6} = 0{,}8$

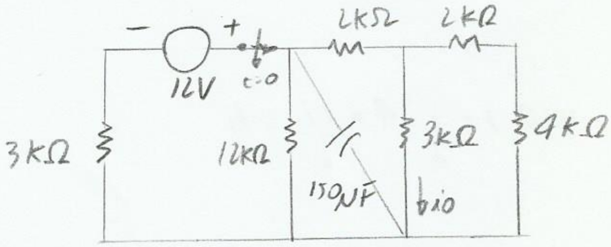
6º passo

$$K_1 + K_2 = \frac{9}{5} \qquad \therefore K_2 = \frac{3}{10}$$

$$K_1 = \frac{3}{2}$$

$$i(t) = \frac{3}{2} + \frac{3}{10} \cdot e^{-1,25t}$$

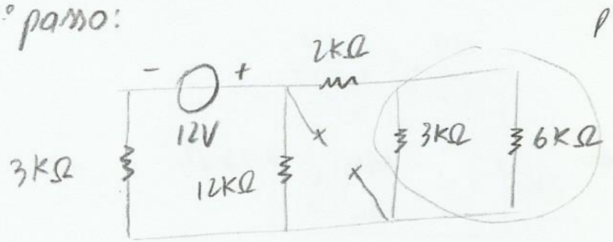
7.7)

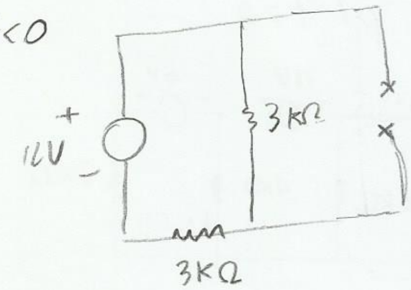


1º passo: $i_o(t) = K_1 + K_2 \cdot e^{-\frac{t}{T_C}}$

2º passo:

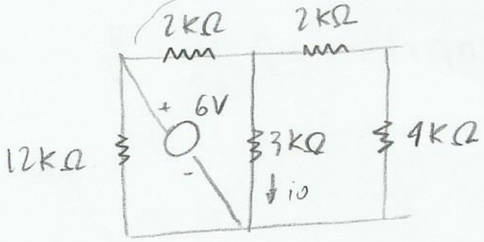
p/ t<0

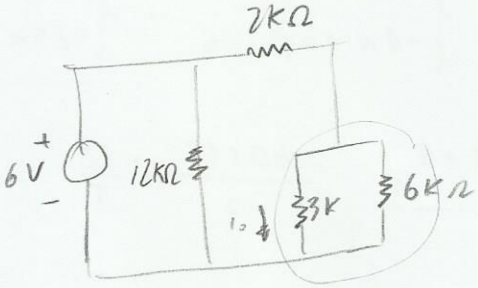




$v(0_-) = 6V$

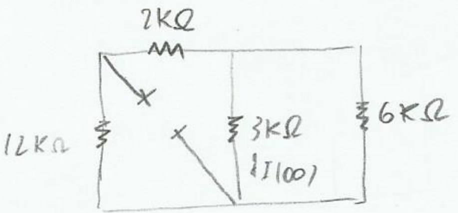
3º passo: p/ t>0





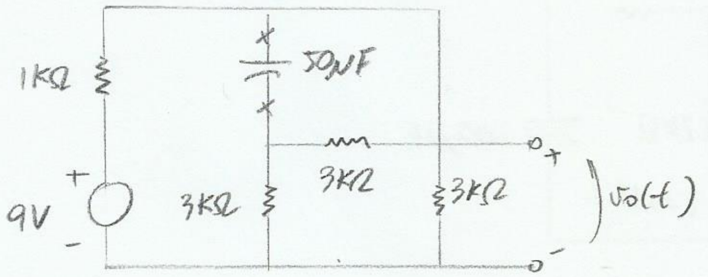
$$i(0+) = \frac{3}{3} = 1A$$

4º passo: p/ t→∞



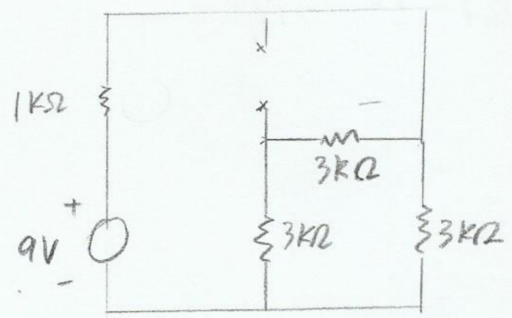
$i(\infty) = 0$

7.10



1º passo : $v(t) = K_1 + K_2 \cdot e^{-\frac{t}{\tau_c}}$

2º passo $t<0$



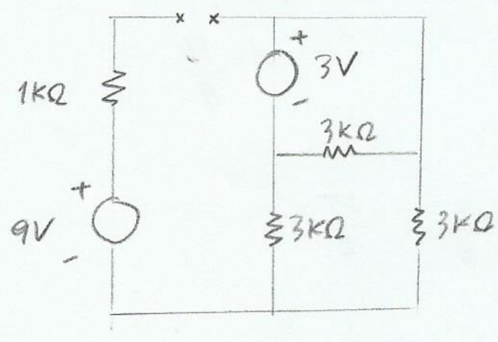
$R_{eq} = 1 + 3 // 6 = 3\,K\Omega$

$I_T = 3$

$I_T' = \frac{3}{9} \cdot 3 = 1$

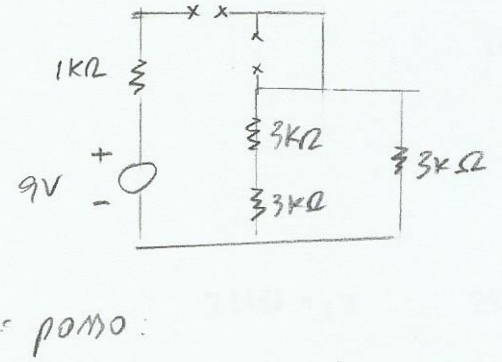
$\therefore\ V(0-) = 3V$

3º passo: $t>0$



$V(0+) = 1{,}5V$

4º passo: $t \to \infty$



$V(\infty) = 0$

5º passo:

$R_{eq} = 2$ $\quad \tau_c = 2 \cdot 10^3 \cdot 50 \cdot 10^{-6}$

$\tau_c = 0{,}1$

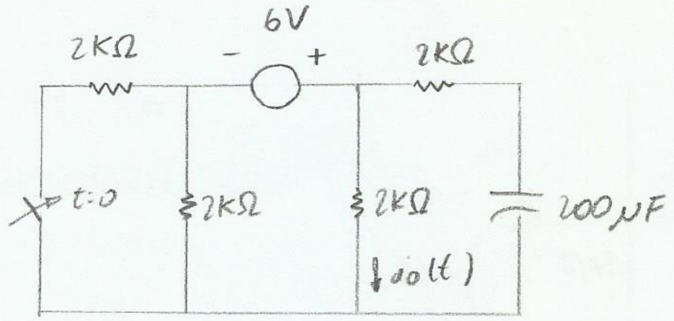
6º passo:

$K_1 + K_2 = 1{,}5$

$K_1 = 0$

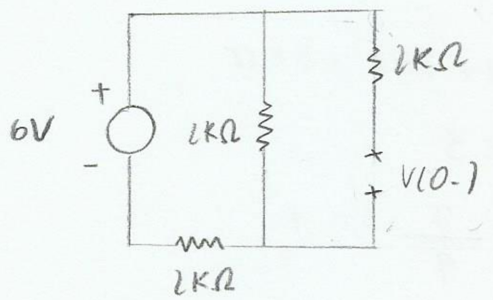
$\therefore\ v_o = 1{,}5\, e^{\frac{-t}{0{,}1}}$

7,11)



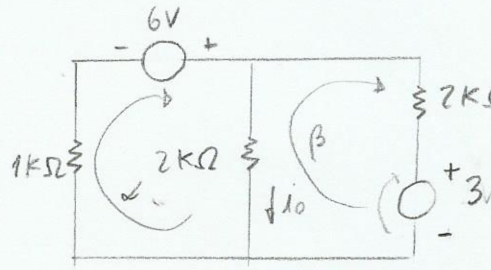
1º passo: $i(t) = K_1 + K_2 e^{\frac{-t}{\tau c}}$

2º passo: p/ $t<0$



$V(0_-) = 3V$

3º passo: p/ $t>0$



$$\begin{cases} 3\alpha - 2\beta = 6 \\ -2\alpha + 4\beta = -3 \end{cases} \sim \begin{cases} 6\alpha - 4\beta = 12 \\ -2\alpha + 4\beta = -3 \end{cases}$$

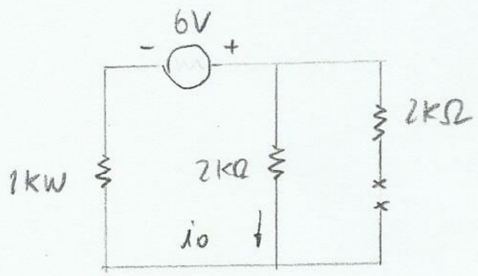
$4\alpha = 9$

$\alpha = \frac{9}{4}$

$\beta = \left(3 \cdot \frac{9}{4} - 6\right) \div 2 = \frac{3}{8}$

$i(0+) = \frac{9}{4} - \frac{3}{8} = \frac{15}{8}$

4º passo: p/ $t \to \infty$



$i_o(\infty) = \frac{6}{3} = 2\ A$

5º passo:

$R_{eq} = 2 + 1 \| 2 = \frac{8}{3}$

$\tau c = \frac{8}{3} \cdot 10^3 \cdot 200 \cdot 10^{-6} = 0,533$

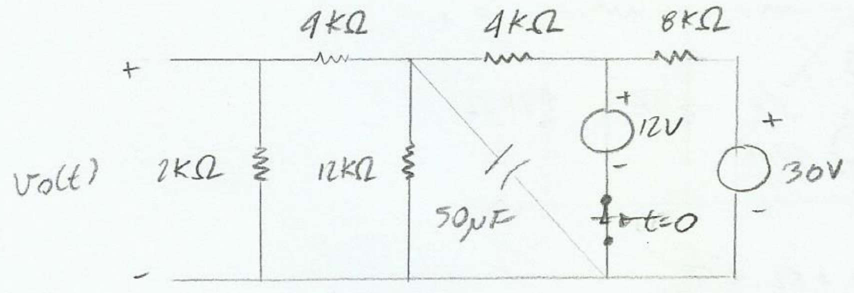
6º passo

$K_1 + K_2 = 1,875 \therefore K_2 = 0,125$

$K_1 = 2$

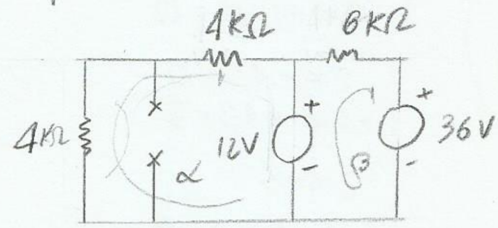
$\therefore i(t) = 2 + 0,125 \cdot e^{\frac{-t}{0,533}}$

7.13)



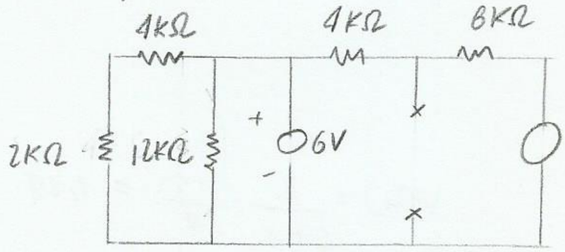
1º passo: $v_o(t) = K_1 + K_2 \cdot e^{\frac{-t}{T_c}}$

2º passo: p/ $t<0$



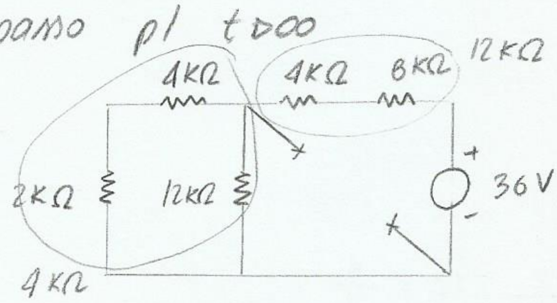
$V(0_-) = 6V$

3º passo p/ $t>0$



$v(0_+) = \frac{2}{2+4} \cdot 6 = 2V$

4º passo p/ $t \to \infty$



$V' = \frac{4}{4+12} \cdot 36 = 9$

$V(\infty) = \frac{2}{2+4} \cdot 9 = 3V$

5º passo

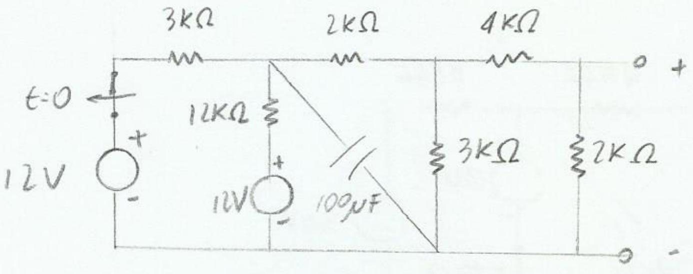
$R_{eq} = (4+2) \| 12 \| 12 = 3 \quad \therefore \quad T_c = 3 \cdot 10^3 \cdot 50 \cdot 10^{-6} = 0{,}15$

6º passo

$K_1 + K_2 = 2 \quad \therefore \quad K_2 = -1 \quad \therefore \quad v_o(t) = 3 - e^{\frac{-t}{0{,}3}} \ (V)$

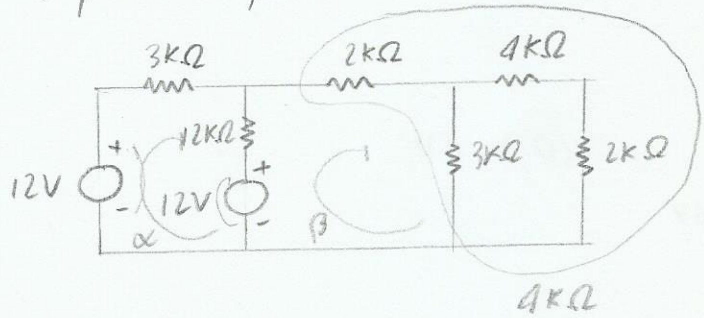
$K_1 = 3$

7.14)



1º passo: $v_o(t) = K_1 + K_2 \cdot e^{-\frac{t}{T_c}}$

2º passo: p/ $t<0$



$$\begin{cases} 15\alpha - 12\beta = 0 \\ -12\alpha + 16\beta = 12 \end{cases}$$

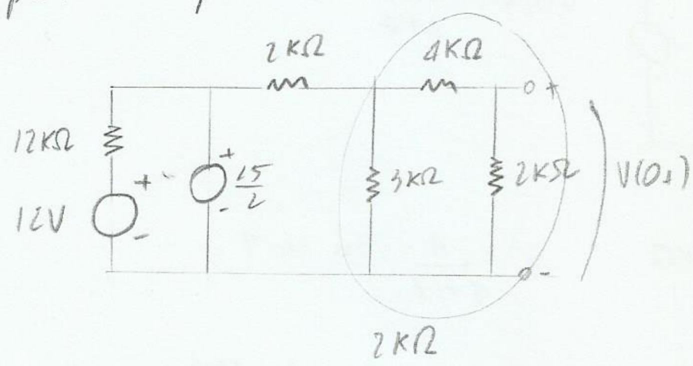
$$\begin{cases} 5\alpha - 4\beta = 0 \\ -3\alpha + 4\beta = 3 \end{cases}$$

$2\alpha = 3$ ∴

$\alpha = \frac{3}{2}$ e $\beta = \frac{15}{8}$

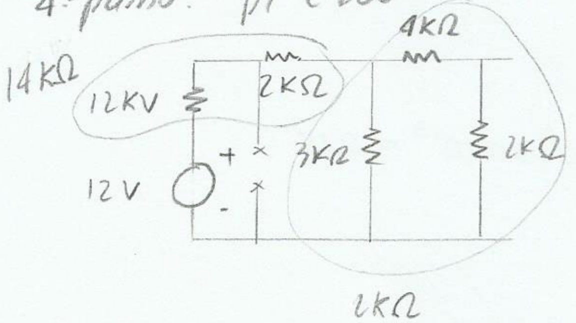
$v(0-) = 4 \cdot \frac{15}{8} = \frac{15}{2}$

3º passo: p/ $t>0$



$V(0+) = \frac{2}{2+4} \cdot \frac{15}{4} = 1{,}25$

4º passo: p/ $t \to \infty$



$V' = \frac{2}{14+2} \cdot 12 = \frac{3}{2}$

$V(\infty) = \frac{2}{2+4} \cdot \frac{3}{2} = 0{,}5$

5º passo:

$R_{eq} = 12 \,//\, (2+2) = 3\Omega$ $\quad T_c = 3 \cdot 10^3 \cdot 100 \cdot 10^{-6} = 0{,}3$

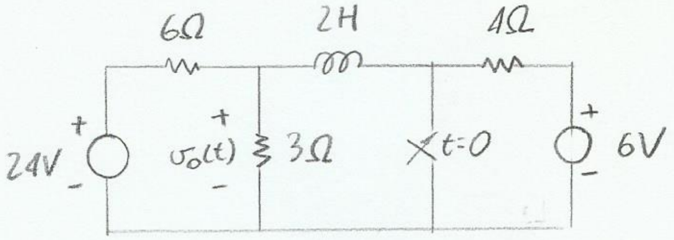
6º passo:

$K_1 + K_2 = 1{,}25$

$K_1 = 0{,}5$ ∴ $K_2 = 0{,}75$

∴ $v(t) = \frac{1}{2} + \frac{3}{4} \cdot e^{-\frac{t}{0,3}}$ (V)

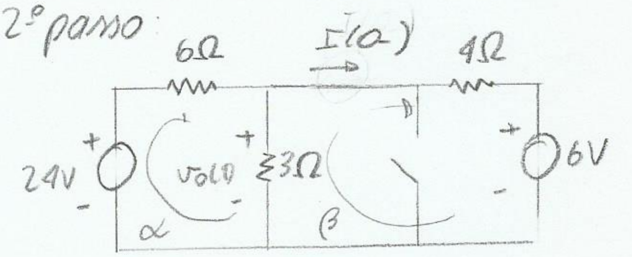
7.31)



1º passo:

$v_o(t) = K_1 + K_2 e^{-\frac{t}{\tau_c}}$

2º passo

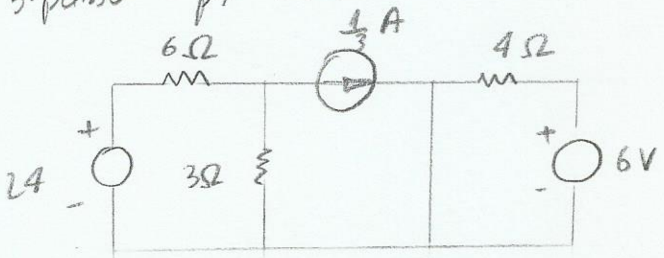


$\begin{cases} 9\alpha - 3\beta = 24 \\ -3\alpha + 7\beta = -6 \end{cases} \rightarrow \begin{cases} 9\alpha - 3\beta = 24 \\ -9\alpha + 21\beta = -18 \end{cases}$

$9\alpha - 3\beta = 24$

$18\beta = 6 \therefore \beta = \frac{1}{3} \therefore I(0_-) = \frac{1}{3}$ A

3º passo p/ $t>0$



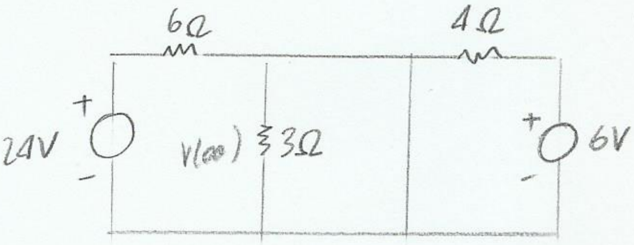
$V_{01} = \frac{3}{3+6} \cdot 24 = 8$

$V_{02} = 0$

$V_{03} = \frac{6}{3+6} \cdot \frac{1}{3} = \frac{2}{9}$

$\therefore V(0+) = \frac{74}{9}$

4 passo p/ $t \to \infty$



$V(\infty) = \frac{3}{3+6} \cdot 24 = 8$

$V_2(\infty) = 0$

$\therefore v(\infty) = 8$

5º passo

$R_{eq} = (6 \| 3) = 2 \therefore \tau_c = \frac{2}{2} = 1$

6º passo

$K_1 + K_2 = \frac{74}{9}$

$K_1 = 8$

?

E14

Ex: 7.40

1º passo: $i_o(t) = K_1 + K_2 \cdot e^{\frac{-t}{T_c}}$

2º passo: p/ $t<0$

$$16\alpha - 4\beta = -12$$
$$-4\alpha + 10\beta - 6\gamma = 12$$
$$-6\beta + 8\gamma = -6$$

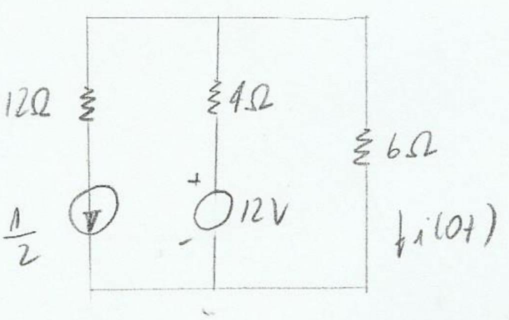
$$\begin{bmatrix} 16 & -4 & 0 & -12 \\ & 36 & -24 & 36 \\ & & 24 & 0 \end{bmatrix}$$

$$16\alpha - 4\beta = -12$$
$$36\beta - 24\gamma = 36$$
$$24\gamma = 0$$

$\therefore \gamma = 0 \quad \beta = 1 \quad \alpha =$

$\therefore I(0^-) = \frac{-1}{2}$

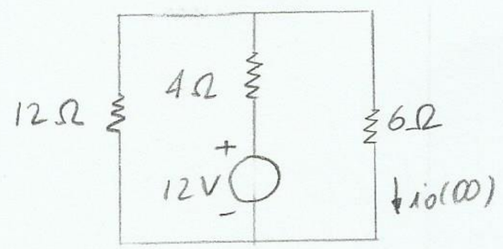
3º passo: p/ $t>0$



$V = \frac{12}{2} = 6$

$\therefore i(0^+) = \frac{6}{6} = 1A$

4º passo: p/ $t \to \infty$



$V_o(\infty) = 6V$

$\therefore I(\infty) = 1A$

5º passo:

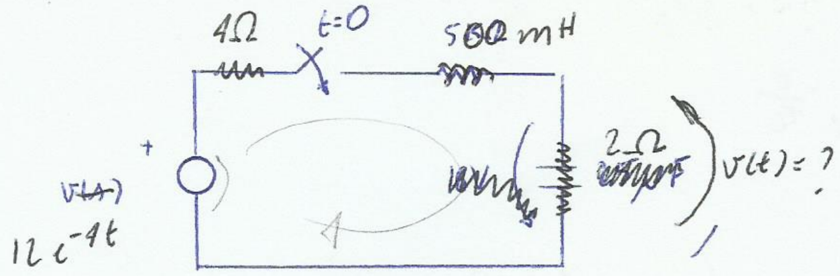
$R_{eq} = 12 + 4//6 = 14,4 \qquad T_c = \frac{14,4}{2} = 7,2$

6º passo:

$K_1 + K_2 = 1$

$K_1 = 1$

$\therefore \boxed{i(t) = 1A}$



$$L \quad i = \frac{1}{L}\int v\,dt$$

$$v = L\frac{di}{dt}$$

$$-12e^{-4t} + Ri(t) + L\frac{di}{dt} = 0$$

$$12e^{-4t} + 6i(t) + 0{,}5\frac{di}{dt} = 0$$

$$\frac{di(t)}{dt} + 12i(t) = 24e^{-4t}$$

homogênea: $\frac{di(t)}{dt} + 12i(t) = 0$

solução

$$i_H = K_1 \cdot e^{\frac{-t}{\tau c}}$$

$$\frac{di_H}{dt} = -\frac{1}{\tau c}K_1 \cdot e^{\frac{-t}{\tau c}}$$

Substituindo

$$-\frac{1}{\tau c}K_1 e^{\frac{-t}{\tau c}} + 12K_1 e^{\frac{-t}{\tau c}} = 0$$

$$K_1 e^{\frac{-t}{\tau c}}\left(-\frac{1}{\tau c} + 12\right) = 0 \qquad \therefore \tau c = \frac{1}{12}$$

$$i(h) = K_1 \cdot e^{-12t}$$

Particular

$$i_p(t) = K_2 e^{-4t}$$

$$\frac{di_p(t)}{dt} = -4K_2 e^{-4t}$$

$$-4K_2 e^{-4t} + 12K_2 e^{-4t} = 24e^{-4t}$$

$$8K_2 = 24 \quad \therefore K_2 = 3 \qquad \therefore i_p = 3e^{-4t}$$

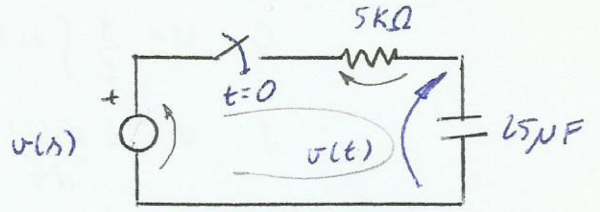
$$i(t) = K_1 e^{-12t} + 3e^{-4t}$$

$$i(0) = K_1 + 3 = 0$$

$$\therefore K_1 = -3$$

$$\therefore i(t) = -3e^{-12t} + 3 \cdot e^{-4t}$$

$$v(t) = -6e^{-12t} + 6e^{-4t}$$



$v(s): 6\,e^{-2t}$

$v(s) = v_R(t) + v(t)$

Capacitor $v = \frac{1}{C}\int i\,dt \quad i = C\frac{dv}{dt}$

Indutor $i = \frac{1}{L}\int v\,dt \quad v = L\frac{di}{dt}$

$$i(t) = \frac{v(s) - v(t)}{R} = C\frac{dv}{dt}$$

$$\frac{v_s}{RC} - \frac{v(t)}{RC} = \frac{dv}{dt} \rightarrow \frac{dv(t)}{dt} + \frac{v(t)}{RC} = \frac{v_s}{RC}$$

$$\frac{dv(t)}{dt} + 8\,v(t) = 48\,e^{-2t}$$

homogênea: $v_h(t) = K_1 e^{\frac{-t}{\tau c}}$ $\qquad \frac{dv(t)}{dt} + 8v(t) = 0$

$$\frac{dv_h(t)}{dt} = -\frac{K_1}{\tau c}\,e^{\frac{-t}{\tau c}}$$

Substituindo: $-\frac{K_1}{\tau c} \cdot e^{\frac{-t}{\tau c}} + 8K_1 e^{\frac{-t}{\tau c}} = 0$

$$K_1 e^{\frac{-t}{\tau c}}\left(-\frac{1}{\tau c} + 8\right) = 0 \quad \therefore \quad \tau_c = \frac{1}{8}$$

particular: $v_p = K_2 e^{-2t}$

$$\frac{dv_p}{dt} = -2K_2 e^{-2t}$$

Substituindo:

$$-2K_2 \cancel{e^{-2t}} + 8K_2 \cancel{e^{-2t}} = 48 \cancel{e^{-2t}}$$

$$6K_2 = 48 \quad \therefore \quad K_2 = 8$$

$$v(t) = K_1 e^{-8t} + 8\,e^{-2t}$$

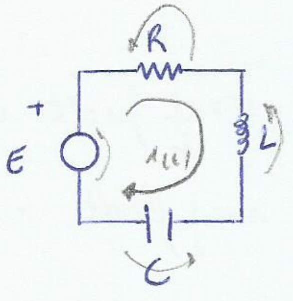
$$v(0) = K_1 + 8 = 0 \quad \therefore K_1 = -8$$

$$\therefore \quad v(t) = -8e^{-8t} + 8\,e^{-2t}$$

Circuito RLC

C: $v = \frac{1}{C}\int i(t)\,dt$

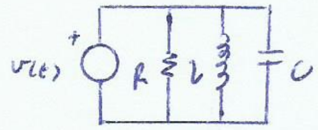
I: $v = L\frac{di(t)}{dt}$



LKT

$$\left(Ri(t) + \frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt} = E\right) \frac{d}{dt} \div L$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{RL}\, i(t) = 0$$



LKT

$$\frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt + C\frac{dv(t)}{dt} = v(t)$$

$$\frac{d^2 v(t)}{dt} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{RL}\, v(t) = 0$$

Homogénea particular $\frac{d^2 x(t)}{dt} + 2\alpha\frac{dx(t)}{dt} + w_0 = 0$

equação característica

$$s^2 + 2\alpha s + w_0 = 0$$

$$s_{1,2} = \frac{-2 \pm \sqrt{4\alpha^2 - 4w_0}}{2} = -1 \pm \sqrt{\alpha^2 - w_0^2}$$

Para $\Delta > 0$ ou $\alpha > w$, temos 2 raízes

$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$ "circuito sobreamortecido"

Para $\Delta < 0$ ou $\alpha < w$, temos 0 raízes

$x(t) = e^{-\alpha t}(A_1 \cos w_d t + A_2 \sin w_d t)$ "amortecimento sub-amortecido"

Para $\Delta = 0$ ou $\alpha = w$, temos duas raízes iguais

$x(t) = B_1 e^{-\alpha t} + B_2 t e^{-\alpha t}$ "amortecimento crítico"

$$\frac{d^2 i(t)}{dt^2} + 10\frac{di(t)}{dt} + 25i(t) = 0$$

$2\alpha = 10 \therefore \alpha = 5\frac{np}{s}$ $w_0^2 = 25 \therefore w_0 = 5\frac{rad}{s}$

$s^2 + 10s + 25 = 0$

$s = \frac{-10 \pm \sqrt{100-100}}{2} \Rightarrow s_{1,2} = -5$

"Circuito de amortecimento crítico"

Solução particular: $x(t) = K_1 e^{-\alpha t} + K_2 . t e^{-\alpha t}$

$i(t) = K_1 e^{-5t} + K_2 . e^{-5t}$

$$\frac{d^2 v(t)}{dt^2} + \frac{4dv(t)}{dt} + 4v(t) = 0$$

$v(0) = 2V$

$v'(0) = 4V$

$\therefore v(t) = ?$

equação característico: $s^2 + 4s + 4 = 0$

$\therefore \alpha = 2\frac{np}{s}$ $w_0 = 2\frac{rad}{s}$

$s_{1,2} = \frac{-4 \pm \sqrt{16-16}}{2} = -2 \therefore$ "Amortecimento crítico"

Equação homogênea $v_h(t) = K_1 e^{-\alpha t} + K_2 . t e^{-\alpha t}$

$v_h = K_1 . e^{-2t} + K_2 . t e^{-2t}$

$\frac{dv_h}{dt} = -2K_1 e^{-2t} + K_2(e^{-2t} - 2te^{-2t})$

$= -2K_1 e^{-2t} + K_2 e^{-2t} + 2te^{-2t}$

$\frac{d^2 v(t)}{dt^2} = 4K_1 e^{-2t} + 2K_2 e^{-2t} + 2(e^{-2t} - 2te^{-2t})$

$= 4K_1 e^{-2t} + 2K_2 e^{-2t} + 2e^{-2t} - 4te^{-2t}$

$v(0) = K_1 = 2$

$v'(0) = -2K_1 + K_2 = 4 \quad \therefore K_2 = 8$

$\therefore v(t) = 2e^{-2t} + 8te^{-2t}$

B.1 $\dfrac{d^2v(t)}{dt^2} + 2.\dfrac{dv(t)}{dt} + 5v(t) = 0$

(a) equação característica: $s^2+2s+5=0$

(b) $2\alpha = 2 \therefore \alpha = 1 \dfrac{np}{s}$ $\quad w_0^2 = 5 \therefore w_0 = \sqrt{5} \dfrac{rad}{s}$

(c) $s^2+2s+5=0$

$s = \dfrac{-2 \pm \sqrt{4-4.5}}{2}$; $\Delta < 0 \therefore$ Amortecimento sub-crítico

$v(t) = e^{-\alpha t}\left(A_1 \cos(w_d t) + A_2 \sin(w_d t)\right)$

$v(t) = e^{-t}\left(A_1 \cos(\sqrt{5}t) + A_2 \sin(\sqrt{5}t)\right)$

B.2) $\dfrac{d^2vo(t)}{dt^2} + 6\left(\dfrac{dvo(t)}{dt}\right) + 10vo(t) = 0$

eq. característica: $s^2+6s+10=0$

$s_{1,2} = \dfrac{-6 \pm \sqrt{36-40}}{2} = -3 \pm 2j$; $\Delta<0 \therefore$ Amortecimento sub-crítico

$\alpha = 3 \dfrac{np}{s}$ $\quad w_0 = \sqrt{10} \dfrac{rad}{s}$

$v(t) = e^{-3t}\left(A_1 \cos\sqrt{10}t + A_2 \sin\sqrt{10}t\right)$

B.3 $\dfrac{d^2io(t)}{dt^2} + 10\dfrac{dio(t)}{dt} + 25io(t) = 0$

$s^2+10s+25=0$

$\alpha = 5 \dfrac{np}{s}$ $\quad w_0 = 5$ rad $\quad \Delta = 100-25.4 = 0 \therefore$ Amortecimento crítico

$\therefore io(t) = K_1 e^{-5t} + K_2 . e^{-5t}$

8.9) $\dfrac{d^2 i(t)}{dt^2} + 6\cdot\left[\dfrac{di(t)}{dt}\right] + 10\, i(t) = 0$ $\quad i(0) = 1A$, $i'(0) = 0$

eq. característica: $s^2 + 6s + 10 = 0$

$$s = \frac{-6 \pm \sqrt{6^2 - 4.10}}{2} \; ; \; \Delta < 0 \therefore \text{ Sub crítico}$$

$$s_{1,2} = -3 \pm j$$

$$\therefore i(t) = e^{-3t}(A_1 \cos t + A_2 \operatorname{sen} t)$$

$$i'(t) = -3e^{-3t}(A_1 \cos t + A_2 \operatorname{sen} t) + e^{-3t}(-A_1 \operatorname{sen} t + A_2 \cos t)$$

$$i(0) = A_1 = 1$$

$$i'(0) = -3A_1 + A_2 = 0 \quad \therefore A_2 = 3$$

$$\therefore i(t) = e^{-3t}(\cos t + 3 \operatorname{sen} t)$$

8.10) $\dfrac{d^2 i(t)}{dt^2} + 4\left[\dfrac{di(t)}{dt}\right] + \dfrac{17}{4} i(t) = 0$ $\quad i(0) = 1$ $\quad i(\pi) = 0$ $\quad \cancel{i'(0) = 0}$

eq. característica: $s^2 + 4s + \frac{17}{4} = 0$ $\quad e^{-2t}(\cos(\frac{t}{2}))$

$$s_{1,2} = \frac{-4 \pm \sqrt{4^2 - 17}}{2} = -2 \pm \frac{j}{2}$$

solução homogênea:

$$i_h = e^{-2t}\left(A\cos\left(\tfrac{t}{2}\right) + B \operatorname{sen}\left(\tfrac{t}{2}\right)\right)$$

$$i_h' = -2e^{-2t}\left(A\cos\left(\tfrac{t}{2}\right) + B\operatorname{sen}\left(\tfrac{t}{2}\right)\right) + e^{-2t}\left(-\frac{A}{2}\operatorname{sen}\left(\tfrac{t}{2}\right) + \frac{B}{2}\cos\left(\tfrac{t}{2}\right)\right)$$

$$i(0) = A = 1$$

$$i(\pi) = \cancel{e^{-2\pi}}\left(A \underbrace{\cos\left(\tfrac{\pi}{2}\right)}_{0} + B\operatorname{sen}\left(\tfrac{\pi}{2}\right)\right) = 0 \quad \therefore B = 0$$

$$i(t) = e^{-2t} \cdot \cos\left(\frac{t}{2}\right)$$

8.15.)

$$\frac{d^2 io(t)}{dt^2} + 4\left[\frac{d io(t)}{dt}\right] + 3io(t) = 6 \qquad \begin{matrix} i(0) = 1 \\ i'(0) = 0 \end{matrix}$$

$s^2 + 4s + 3 = 0$

$s_{1,2} = \dfrac{-4 \pm \sqrt{16 - 4.3}}{2} \quad \therefore s_1 = -1 \quad s_2 = -3$

$ih = A e^{-t} + B e^{-3t} \qquad i(0) = A + B = 1 \qquad A + B = 1 \qquad \therefore B = -0{,}5$

$i'h = -Ae^{-t} - 3Be^{-3t} \qquad i'(0) = -A - 3B = 0 \qquad -2B = 1 \qquad A = 1{,}5$

$\therefore ih = 1{,}5 e^{-t} - 0{,}5 e^{-3t}$

Solução particular

$ip = A \qquad \therefore \qquad 0 + 4.0 + 3.A = 6 \quad \therefore A = 2$

$i'p = 0$

$i''p = 0$

$\therefore io(t) = 1{,}5 e^{-t} - 0{,}5 e^{-1{,}5t} + 2$

8.16

$$\frac{d^2 vo(t)}{dt^2} + 7\left[\frac{d vo(t)}{dt}\right] + 10 vo(t) = 20 e^{-3t} \qquad \begin{matrix} vo(0) = 4V \\ v'o(0) = 0V \end{matrix}$$

$s^2 + 7s + 10 = 0 \qquad s_{1,2} = \dfrac{-7 \pm \sqrt{49 - 4.10}}{2} \quad \therefore s_1 = -2 \quad s_2 = -5$

solução homogênea : $v_h(t) = A e^{-2t} + B e^{-5t}$

$v'h(t) = -2Ae^{-2t} - 5Be^{-5t}$

$v(0) = A + B = 4$

$v'(0) = -2A - 5B = 0$

$\rightarrow \begin{bmatrix} 1 & 1 & 4 \\ -2 & -5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 8 \\ 0 & -3 & 8 \end{bmatrix} \therefore \begin{matrix} A + B = 4 \\ -3B = 8 \end{matrix} \quad \therefore B = -\frac{8}{3} \quad A = \frac{20}{3}$

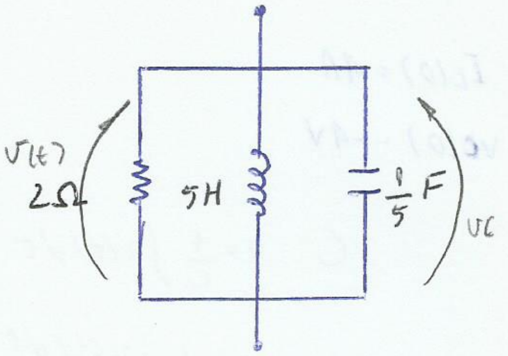
$vh(t) = \frac{20}{3} e^{-2t} - \frac{8}{3} e^{-5t}$

Solução particular : $9A\cancel{e^{-3t}} - 21A\cancel{e^{-3t}} + 10A\cancel{e^{-3t}} = 20\cancel{e^{-3t}}$

$vp = Ae^{-3t} \qquad -2A = 20$

$v'p = -3Ae^{-3t} \qquad \therefore A = -10$

$v''p = 9Ae^{-3t} \qquad \therefore vh(t) = \frac{20}{3} e^{-2t} - \frac{8}{3} e^{-5t} - 10 e^{-3t}$



$i_L(0) = -1A$
$v_C(0) = 4V$

C: $v = \frac{1}{C}\int i(t)\,dt$

$i = C\frac{di(t)}{dt}$

I: $i = \frac{1}{L}\int v(t)\,dt$

$v = L\frac{dv(t)}{dt}$

$i_R + i_L + i_C = 0$

(I) $\left(\frac{v(t)}{R} + C\frac{dv}{dt} + \frac{1}{L}\int v(t)dt = 0\right)\ \frac{d}{dt}\ e \div C$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v(t) = 0$$

$$\frac{d^2v}{dt^2} + 2{,}5\frac{dv(t)}{dt} + v(t) = 0$$

$s^2 + 2{,}5s + 1 = 0 \quad s_{1,2} = \frac{-2{,}5 \pm 1{,}5}{2} \quad \therefore s_1 = -0{,}5 \quad s_2 = -2$

$v_h(t) = Ae^{-0{,}5t} + Be^{-2t}$

$\frac{dv_h(t)}{dt} = -0{,}5Ae^{-0{,}5t} - 2Be^{-2t}$

$v(0) = A + B = 4$

$v'(0) = -0{,}5A - 2B = ?$

(I) $\frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)dt = 0$

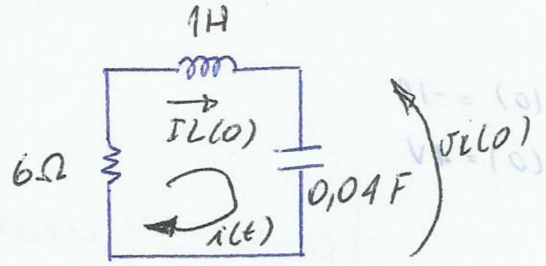
$$\frac{v(t)}{RC} + \frac{dv(t)}{dt} + \underbrace{\frac{1}{LC}\int v(t)\,dt}_{i(t)} = 0$$

$\frac{dv(t)}{dt} = v'(t) = -2{,}5\,v(t) - 5\,i(t)$

$v'(0) = -2{,}5 \cdot v(0) - 5 \cdot i(0) = -2{,}5 \cdot 4 - 5(-1) = -5$

$\therefore \begin{cases} A + B = 4 \\ -0{,}5A - 2B = -5 \end{cases} \rightarrow \begin{cases} A + B = 4 \\ -3B = -6 \end{cases} \quad \therefore B = 2 \quad A = 2$

$\therefore\ v(t) = 2e^{-0{,}5t} + 2e^{-2t} \quad e \quad v_C(0) = 4V$



$I_L(0) = 4A$

$v_C(0) = -4V$

$C: \; v = \frac{1}{C}\int i(t)\,dt$

$L: \; i = \frac{1}{L}\int v(t)\,dt$

$$R\,i(t) + \frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt} = 0$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}\,i(t) = 0$$

$$\frac{d^2 i(t)}{dt^2} + 6\frac{di(t)}{dt} + 25\,i(t) = 0 \qquad s^2 + 6s + 25 = 0$$

$$s_{1,2} = \frac{-6 \pm \sqrt{36 - 4\cdot 25}}{2} = -3 \pm 4j \qquad \text{Circuito sub-critico}$$

$$\begin{cases} i_n(t) = e^{-3t}(A\cos 4t + B\sin 4t) \\ i'(t) = -3e^{-3t}(A\cos 4t + B\sin 4t) + e^{-3t}(-4A\sin 4t + 4B\cos 4t) \end{cases}$$

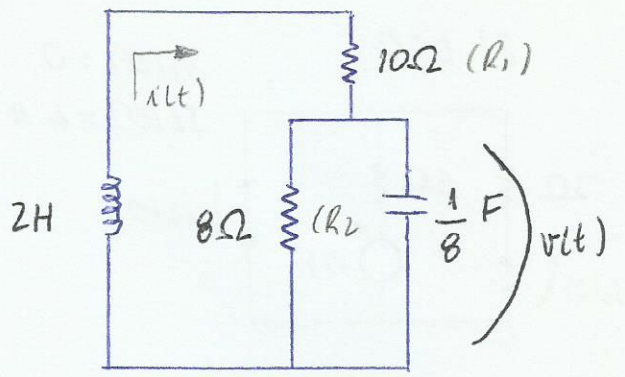
$$\begin{cases} i(0) = A = 4 \\ i'(0) = -3A + 4B = ? \end{cases}$$

$$\frac{R}{L}\,i(t) + \frac{1}{LC}\int i(t)\,dt + \frac{di(t)}{dt} = 0 \qquad \therefore A = 4 \;\text{ e }\; B = -2$$

$$\frac{di(t)}{dt} = i'(t) = -6\,i(t) - \frac{v_c(t)}{1}$$

$$i'(0) = -6\cdot 4 - \frac{(-4)}{1} = -20\,A$$

$$i(t) = e^{-3t}(4\cos 4t - 2\sin 4t)$$



$V_C(0) = 1V$

$I_L(0) = 0{,}5\,A$

$C: \; v = \frac{1}{C}\int i(t)dt$

$I: \; i = \frac{1}{L}\int v(t)dt$

$$\begin{cases} L\frac{di(t)}{dt} + R_1 i(t) + v(t) = 0 \\ i(t) = \frac{v(t)}{R_2} + C\frac{dv(t)}{dt} \end{cases}$$

$$\frac{L}{R_2}\frac{dv(t)}{dt} + LC\frac{d^2v(t)}{dt^2} + \frac{R_1 v(t)}{R_2} + R_1 C\frac{dv(t)}{dt} + v(t) = 0$$

$$\frac{d^2v(t)}{dt^2} + \left(\frac{1}{R_2C} + \frac{R_1}{L}\right)\frac{dv(t)}{dt} + \left(\frac{R_1}{LCR_2} + \frac{1}{LC}\right)v(t) = 0$$

$$\frac{d^2v(t)}{dt^2} + 6\frac{dv(t)}{dt} + 9v(t) = 0$$

$\therefore$ "Amortecimento crítico"

$$s^2 + 6s + 9 = 0 \qquad s_{1,2} = \frac{-6 \pm \sqrt{36-36}}{2} = -3$$

$\alpha = w_0 = -3$

$$v(t) = Ae^{-3t} + B.te^{-3t}$$

$$v'(t) = -3Ae^{-3t} + B.(e^{-3t} + t.(-3)e^{-3t})$$

$v(0) = A = 1$

$v'(0) = -3A + B = ?$

$$i(t) = \frac{v(t)}{R_2} + C\frac{dv(t)}{dt} \quad \therefore \quad \frac{dv(t)}{dt} = \left(i(t) - \frac{v(t)}{R_2}\right) \div C$$

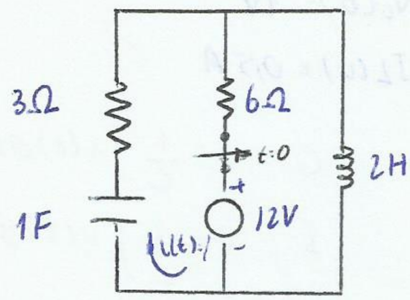
$$\frac{dv(0)}{dt} = v'(0) = \left(0{,}5 - \frac{1}{8}\right) \div \frac{1}{8} = 3$$

$-3A + B = 3 \; ; \; A = 1$

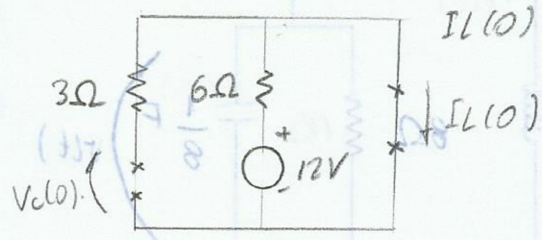
$\therefore B = 6$

$$v(t) = e^{-3t} + 6te^{-3t}$$

E20



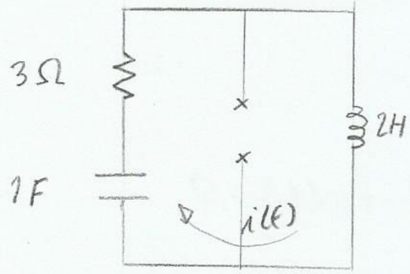
p/ $t<0$



$V_c(0)=0$

$I_L(0)=2A$

p/ $t>0$



LKT

$$Ri(t)+\frac{1}{C}\int i(t)\,dt+L\frac{di(t)}{dt}=0$$

$$\frac{d^2i(t)}{dt^2}+\frac{R}{L}\frac{di(t)}{dt}+\frac{1}{LC}i(t)=0$$

$$\frac{d^2i(t)}{dt^2}+1{,}5\frac{di(t)}{dt}+0{,}5\,i(t)=0 \qquad s^2+1{,}5s+0{,}5$$

(Circuito Amortecido)

$$s_{1,2}=\frac{-1{,}5\pm\sqrt{1{,}5^2-4\times0{,}5}}{2}=\frac{-1{,}5\pm0{,}5}{2} \quad \therefore s_1=-0{,}5 \quad s_2=-1$$

$$i_h(t)=Ae^{-0{,}5t}+Be^{-t} \qquad i(0)=A+B=2$$

$$i'(t)=-0{,}5Ae^{-0{,}5t}-2Be^{-t} \qquad i'(0)=-0{,}5A-2B=\ ?$$

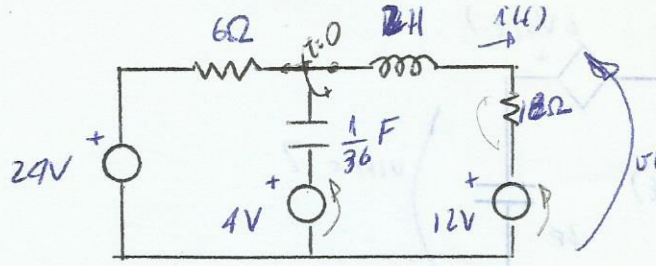
$$Ri(t)+\frac{1}{C}\int i(t)\,dt+L\frac{di(t)}{dt}=0$$

$$\frac{di(t)}{dt}=i'(t)=\frac{-Ri(t)}{L}-\frac{1}{LC}\int i(t)\,dt$$

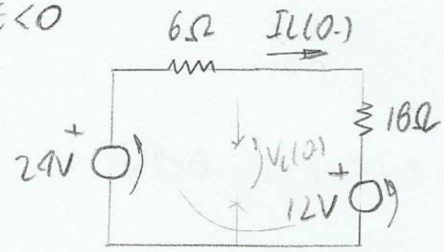
$$i'(0)=\frac{-3}{2}\cdot2-\frac{0}{1}=-3$$

$$\begin{cases}A+B=2\\-0{,}5A-B=-3\end{cases}\sim\begin{cases}A+B=2\\0{,}5A=-1\end{cases}\quad\therefore A=-2\quad\therefore B=4$$

$$i(t)=-2\cdot e^{-0{,}5t}+4e^{-t}$$



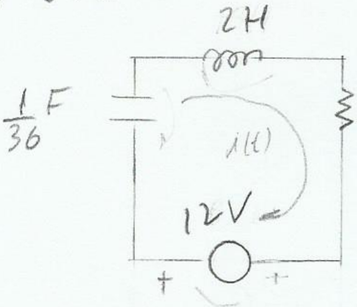
p/ $t<0$



$V_c(0)=0$

$I_L(0)=0,5$

p/ $t>0$



$$R\,i(t)+\frac{1}{C}\int i(t)\,dt+L\frac{di(t)}{dt}=12$$

$$\frac{d^2 i(t)}{dt^2}+\frac{R}{L}\frac{di(t)}{dt}+\frac{1}{LC}i(t)=0$$

$$\frac{d^2 i(t)}{dt^2}+9\frac{di(t)}{dt}+18\,i(t)=0$$

$$s^2+9s+18=0$$

$$s_{1,2}=\frac{-9\pm\sqrt{81-72}}{2} \quad \therefore s_1=-3 \quad s_2=-6$$

$$i(t)=K_1 e^{-3t}+K_2 e^{-6t}$$

$$i'(t)=-3K_1e^{-3t}-6K_2e^{-6t}$$

$i(0)=K_1+K_2=0,5$

$i'(0)=-3K_1-6K_2=\ ?$

$$\frac{di(t)}{dt}=\frac{12}{L}-\frac{R\,i(t)}{L}-\frac{1}{LC}\int i(t)\,dt$$

$$i'(0)=\frac{12}{2}-\frac{18\cdot 0,5}{2}-\frac{0}{2}=1,5$$

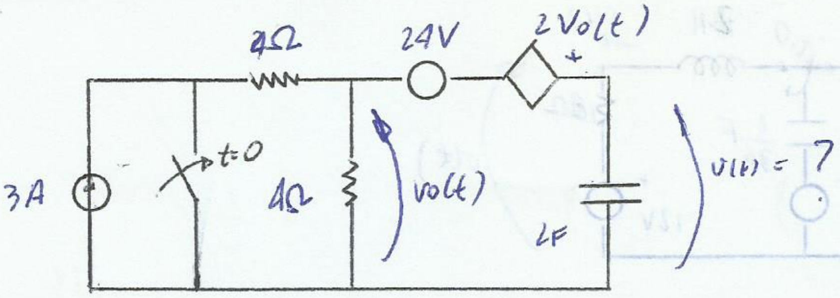
$$\begin{cases}K_1+K_2=0,5\\-3K_1-6K_2=1,5\end{cases}$$

$$\begin{cases}3K_1+3K_2=1,5\\-3K_1-6K_2=1,5\end{cases}$$

$$\begin{cases}K_1+K_2=0,5\\-3K_2=3\end{cases} \quad \therefore K_2=-1 \quad K_1=0,5$$

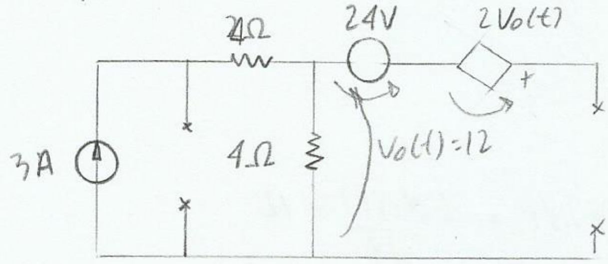
$$\therefore i(t)=\frac{1}{2}e^{-3t}-e^{-6t}$$

$$v(2)=\left(\frac{1}{2}e^{-3.2}-e^{-6.2}\right)\cdot 18+12=12,022$$



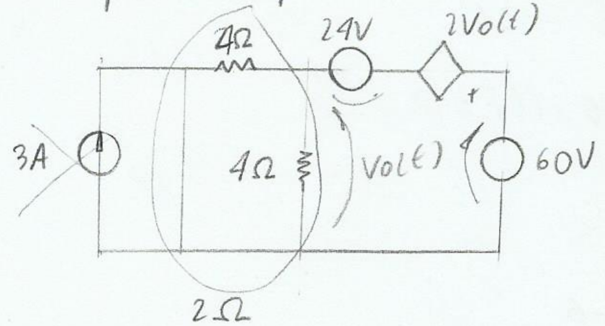
1º passo: $v(t) = K_1 + K_2 e^{-\frac{t}{\tau_c}}$

2º passo: p/ $t<0$



$V(0_-) = 12 + 24 + 2.12 = 60V$

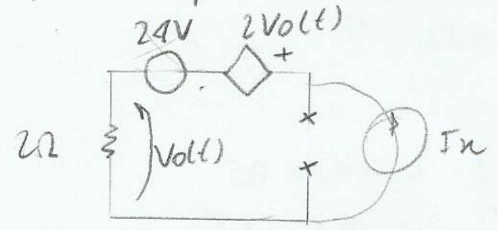
3º passo: p/ $t>0$



$v(0_+) = 60V$

$V(\infty) = 24V$

4º passo p/ $t \to \infty$



$R_{eq} = \frac{V_x}{I_x}$ $V_x = 3V_o$; $V_o = 2I_x$

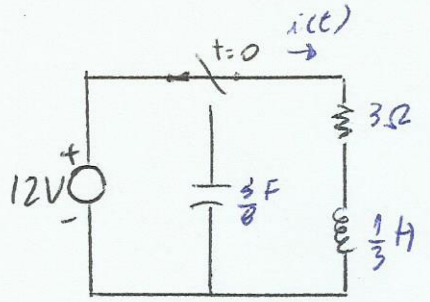
$V_x = 6I_x$

$\therefore R_{eq} = \frac{6I_x}{I_x} = 6$

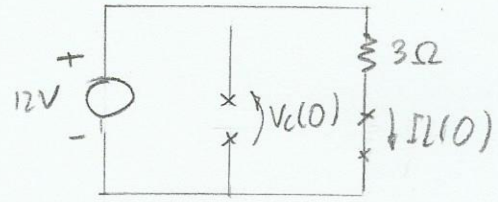
$K_1 + K_2 = 60$

$K_1 = 24$ $\therefore$ $K_2 = 36$

$v(t) = 24 + 36 \cdot e^{-\frac{t}{12}}$



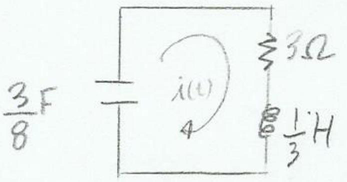
p/ t<0



$V_c(0)=0$

$I_L(0)=4A$

p/ t>0



$$Ri(t)+\frac{1}{C}\int i(t)dt+L\frac{di(t)}{dt}=0$$

$$\frac{d^2(t)}{dt^2}+\frac{R}{L}\frac{di(t)}{dt}+\frac{1}{LC}i(t)=0$$

$$\frac{d^2(t)}{dt^2}+9\frac{di(t)}{dt}+8=0 \qquad s^2+9s+8$$

"Circuito sobre amortecido"

$$s_{1,2}=\frac{-9\pm\sqrt{9^2-4\cdot8}}{2} \quad \therefore \quad s_1=-1 \quad s_2=-8$$

$i_h(t)=Ae^{-t}+Be^{-8t}$

$i'(t)=-Ae^{-t}-8Be^{-8t}$

$i(0)=A+B=4$

$i'(0)=-A-8B=?$

$$\frac{di(t)}{dt}=i'(t)=-\frac{Ri(t)}{L}-\frac{1}{LC}\int i(t)dt \quad v_c(t)$$

$$i'(0)=\frac{-3\cdot4}{\frac{1}{3}}-\frac{0}{\frac{1}{3}}=-36$$

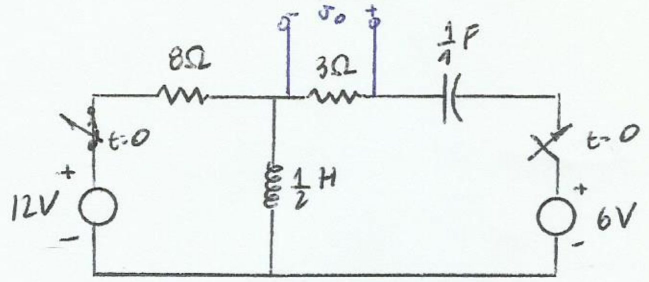
$$\begin{cases} A+B=4 \\ -A-8B=-36 \end{cases}$$

$$i(t)=\frac{-4}{7}e^{-t}+\frac{32}{7}e^{-8t}$$

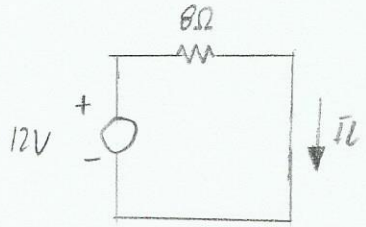
$A+B=4$

$-7B=-32$

$B=\frac{32}{7} \qquad A=\frac{-4}{7}$

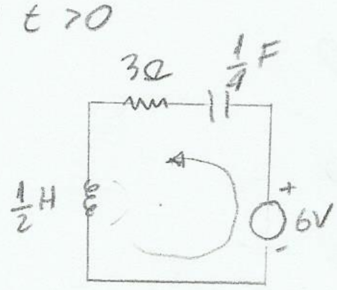


p/ $t<0$



$v_c = 0 \therefore I_L = \frac{12}{8} = 1{,}5A$

p/ $t>0$



$$Ri(t) + \frac{1}{C}\int i(t)\,dt + L\frac{di}{dt} = 6$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{CL}i(t) = 0$$

$$\frac{d^2 i}{dt^2} + 6\frac{di(t)}{dt} + 8 = 0 \qquad s^2 + 6s + 8 = 0$$

$$s_{1,2} = \frac{-6 \pm \sqrt{6^2 - 4.8}}{2} \qquad s_1 = -2 \quad s_2 = -4$$

$$i(t) = Ae^{-2t} + Be^{-4t} \qquad i(0) = A + B = 1{,}5$$

$$i'(t) = -2Ae^{-2t} - 4Be^{-4t} \qquad i'(0) = -2A - 4B = ?$$

$$\frac{di}{dt} = \frac{6}{L} - \frac{Ri(t)}{L} - \frac{1}{LC}\int i(t)\,dt \quad v(0)$$

$$\frac{di}{dt} = \frac{6}{\frac{1}{2}} - \frac{3.1{,}5}{\frac{1}{2}} - \frac{0}{\frac{1}{2}} \quad \therefore i'(0) = 3$$

$$\begin{cases} A + B = 1{,}5 \\ -A - 2B = 1{,}5 \end{cases}$$

$$-B = 3$$

$$B = 3 \quad \therefore A = -1{,}5$$

$$\therefore i(t) = -1{,}5e^{-2t} + 3e^{-4t}$$