ME 7820 - Cinemática e Dinâmica

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Beer; Johnston. Mecanica Vetorial para Engenheiros: Cinemática e Dinamica

05-02-2015

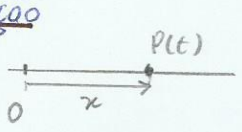
Cinemática do Ponto

Leitura Recomendada:

. BEER & JOHNSTON, 2006, Cap. 11, Pags 1-52

Movimento Retilíneo

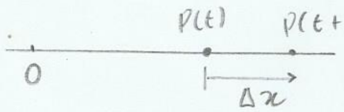
Posição



	posição	
derivação ↓	velocidade	↑ Integração
	aceleração	

"tranco"

Velocidade



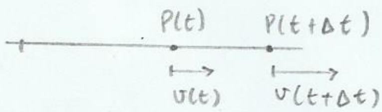
$\bar{v} = \frac{\Delta x}{\Delta t}$ (velocidade média)

$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ (velocidade instantânea)

$$\int_0^t dx = \int_0^t v(t)\,dt$$

$$x(t) - x_0 = \int_0^t v(t)\,dt \quad \therefore \quad x(t) = x_0 + \int_0^t v(t)\,dt$$

Aceleração



$\bar{a} = \frac{\Delta v}{\Delta t}$ (aceleração média)

$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$ (aceleração instantânea)

$$\int_0^t a(t)\,dt = \int_0^t dv$$

$$v(t) - v_0 = \int_0^t a(t)\,dt \qquad v(t) = v_0 + \int_0^t a(t)\,dt$$

Notas: Usamos apenas $v(t) = v_0 + at$ e suas derivações quando a aceleração é const.

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Exemplo: Um automóvel sofre desacelaração conforme:

$$a(t) = -2 + 0{,}04t \ [m/s^2]$$

$72 \frac{Km}{h} = 72 \times \frac{1000}{3600} \frac{m}{s}$

$\therefore 72\ km/h = 20\ m/s$

Sabendo que sua velocidade inicial é $v_0 = v(t=0) = 72$ km/h, Determinar a distância e o tempo de frenagem até o repouso.

$v(t) = v_0 + \int_0^t a(t)\,dt$

$v(t) = 20 + -2t + 0{,}02t^2$

Para o repouso temos: $0.02t^2 - 2t + 20 = 0$

$t = \frac{2 \pm \sqrt{4 - 1.6}}{2 \times 0.02}$ $\begin{cases} t_1 = 88{,}73\,s \\ t_2 = 11{,}27\,s \end{cases}$

Para distancia de frenagem temos: Δx

$$x(t) = x_0 + \int_0^t v(t)\,dt$$

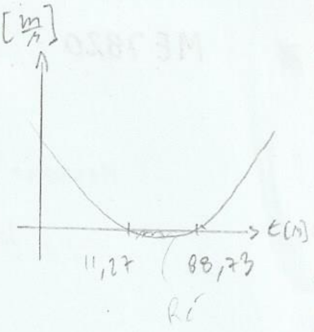
$$\Delta x = \int_0^t (0.02t^2 - 2t + 20)\,dt$$

$$\Delta x = \frac{0.02}{3}t^3 - t^2 + 20t$$

Portanto, temos.

$$\boxed{x(11,27) = 107,93\ m} \checkmark$$

$$x(88,73) = -1441.26 \ \times$$



R: O carro alcança o repouso depois de <u>11,27s</u> a <u>107,93 m</u>

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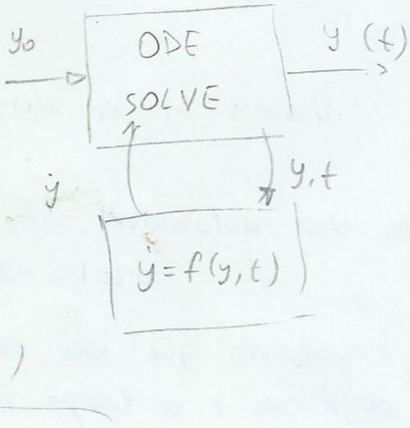
Transformação de 1 EDO de ordem n em n EDO's de ordem 1

$$x^{(n)} = f(x^{(n-1)}, x^{(n-2)}, \dots, x^{(1)}, x, t)$$

$$\begin{cases} x_1 = x \\ x_2 = x^{(1)} = \dot{x}_1 \\ x_3 = x^{(2)} = \dot{x}_2 \\ \vdots \\ x_n = x^{(n-1)} = \dot{x}_{n-1} \\ \dot{x}_n = x^{(n)} = f(x_n, x_{n-1}, \dots x_2, x_1, t) \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(x_n, x_{n-1}, \dots, x_2, x_1, t) \end{cases} \quad n \text{ EDO's de ordem 1}$$

Se $y = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $\boxed{\dot{y} = g(y,t)}$



Example: $\ddot{x} = -0.01\dot{x}^2 - 0.01x - 2$

duas variaveis

$$\begin{cases} x_1 = x \\ x_2 = \dot{x} = \dot{x}_1 \\ \ddot{x} = \dot{x}_2 = -0,01x_2^2 - 0,01x_1 - 2 \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.01x_2^2 - 0.01x_1 - 2 \end{cases}$$

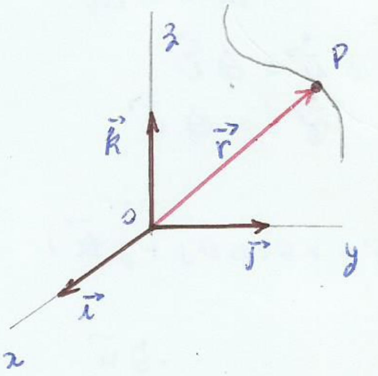
$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \dot{y} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overbrace{\begin{bmatrix} x_2 \\ -0.01x_2^2 - 0,01x_1 - 2 \end{bmatrix}}^{g(y,t)}$$

12-02-2015

Cinemática do ponto

Leitura Recomendada

- França & Matsumura, 2010, Cap 6. pg. 89-94
- Beer & Johnston, 2006, pag. 52-86



Sistema de coordenadas: Conjunto de coordenadas utilizadas para descrever a cinemática de um ponto ou um conjunto de pontos (Exs: Cartesianas, cilíndricas, esféricas, ...)

Referencial: É qualquer corpo em relação ao qual se estuda o problema e se determinam as características cinemáticas (Posição, velocidade, aceleração)

Base ou Sistema de referência: Base, geralmente ortonormal utilizada para escrever (exprimir) as quantidades vetoriais da cinemática em relação ao referencial. Ex: $(O\ \vec i\ \vec j\ \vec k)$. Pode ser fixa ao referencial ou móvel.

Base Fixa: $(O\ \vec i\ \vec j\ \vec k)$

Sistema de coordenadas: x, y, z

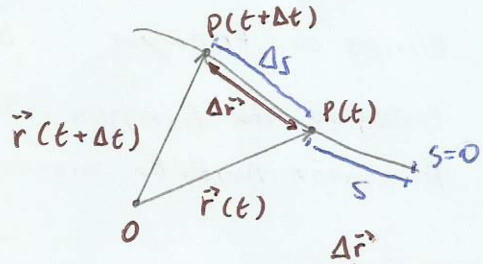
Posição $\vec r = x\vec i + y\vec j + z\vec k$

Velocidade $\vec v = \frac{d\vec r}{dt} = \dot x \vec i + \dot y \vec j$

Aceleração. $\vec a = \frac{d\vec v}{dt} = \ddot x \vec i + \ddot y \vec j + \ddot z \vec k$ ← Usando (I)

$\vec a = \dot v \vec t + v \dot{\vec t}$

aceleração tangencial — aceleração normal



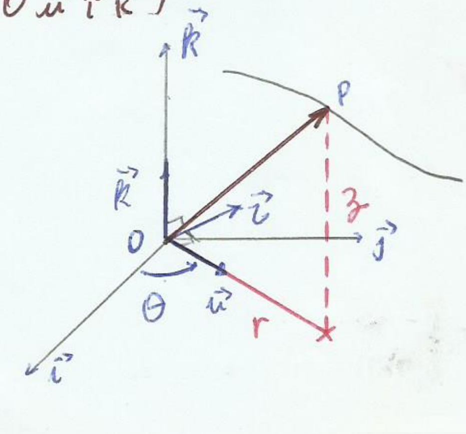
$$\frac{d\vec r}{dt} = \lim_{\Delta t \to 0} \frac{\overbrace{\vec r(t+\Delta t) - \vec r(t)}^{\Delta\vec r}}{\Delta t}$$

$$\frac{d\vec r}{dt} = \lim_{\Delta t \to 0} \underbrace{\left(\frac{\Delta s}{\Delta t}\right)}_{v} \vec t$$

$$\vec v = \frac{d\vec r}{dt} = \frac{ds}{dt}\vec t = v\vec t \quad (I)$$

Sistema de coordenadas cilíndricas: (r, θ, z)

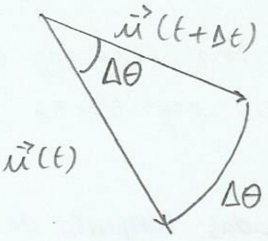
Base Móvel: $(O\ \vec u\ \vec\tau\ \vec K)$

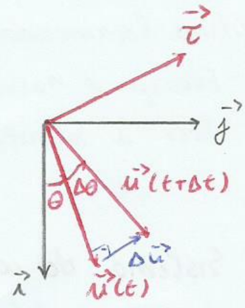


Posição: $\vec r = r\vec u + z\vec K$

$\vec r = r\cos\theta\,\vec i + r\sin\theta\,\vec j + z\vec K$

Velocidade: $\vec v = \frac{d\vec r}{dt} = \dot r \vec u + r\dot{\vec u} + \dot z \vec K + z\overset{0}{\dot{\vec K}}$





$$\dot{\vec{u}} = \lim_{\Delta t \to 0} \frac{\overbrace{\vec{u}(t+\Delta t) - \vec{u}(t)}^{\Delta \vec{u}}}{\Delta t}$$

$$\dot{\vec{u}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}\vec{\tau}$$

$$\dot{\vec{u}} = \theta\vec{\tau}$$

$$\dot{\vec{\tau}} = -\dot{\theta}\vec{u}$$

$$\therefore \quad \vec{v} = \dot{r}\vec{u} + r\theta\vec{\tau} + \dot{z}\vec{k}$$

$$(\vec{v} = \dot{r}\cos\theta\,\vec{\imath} - r\dot{\theta}\,\mathrm{sen}\,\theta\, i + r\,\mathrm{sen}\,\theta\,\vec{\jmath} + r\dot{\theta}\cos\theta\, j + \dot{z}\vec{k})$$

Aceleração

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{r}\vec{u} + \dot{r}\overset{\dot{\theta}\vec{\tau}}{\dot{\vec{u}}} + \dot{r}\dot{\theta}\vec{\tau} + r\ddot{\theta}\vec{\tau} + r\dot{\theta}\overset{-\dot{\theta}\vec{u}}{\dot{\vec{\tau}}} + \ddot{z}\vec{k} + \cancel{\dot{z}\dot{\vec{k}}}^{\,0}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{\tau} + \ddot{z}\vec{k}$$

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$$a = -2 - 0.01v^2 - 0.01x$$

a) tempo de frenagem 6 s

b) Distancia da frenagem 50

c) Aceleração Absoluta maxima

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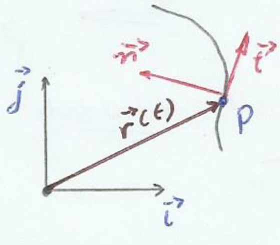
19-02-2015

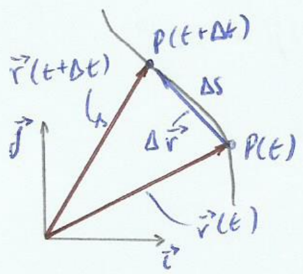
Componentes Intrinsecas (Triedro de Frenet)

- França & Matsumura, 2010, Pags 92-94
- Beer & Johnston, 2006, Pags. 82-86

Componentes Tangencial e Normal ($\vec{t}, \vec{n}$) (No plano)



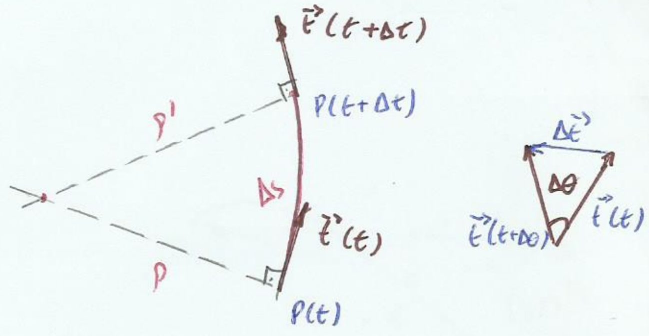
Versor tangente ($\vec{t}$)



$$\vec{t} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta s}$$

$$\vec{t} = \frac{\vec{v}}{v} = \frac{d\vec{r}/dt}{ds/dt} = \frac{d\vec{r}}{ds}$$

Versor Normal ($\vec{n}$)



$$\vec{n} = \lim_{\Delta t \to 0} \frac{\Delta \vec{t}}{\|\Delta \vec{t}\|} = \lim_{\Delta t \to 0} \frac{\Delta \vec{t}}{\Delta \theta} \Rightarrow \vec{n} = \frac{d\vec{t}}{d\theta}$$

$$\Delta s \to \rho \Delta \theta$$

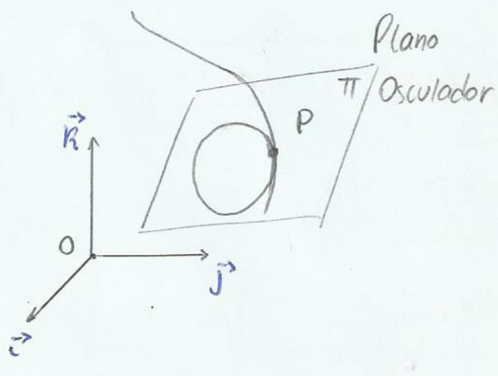
$$\rho = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta \theta} = \frac{ds}{d\theta}$$

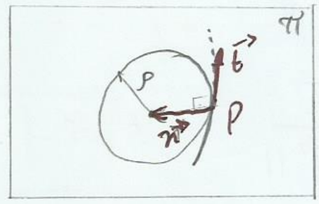
Velocidade $\quad \vec{v} = v\vec{t} = \frac{ds}{dt}\vec{t}$

Aceleração $\quad \vec{a} = \frac{d\vec{v}}{dt} = \dot{v}\vec{t} + v\frac{d\vec{t}}{dt}$

$$\frac{d\vec{t}}{dt} = \underbrace{\frac{d\vec{t}}{d\theta}}_{\vec{n}} \cdot \underbrace{\frac{d\theta}{ds}}_{1/\rho} \cdot \underbrace{\frac{ds}{dt}}_{v} \quad \therefore \quad \frac{d\vec{t}}{dt} = \frac{v}{\rho}\vec{n} \; //$$

$$\boxed{\vec{a} = \dot{v}\vec{t} + \frac{v^2}{\rho}\vec{n}}$$





$$\vec{t} = \frac{d\vec{r}}{ds}$$
$$\vec{n} = \frac{d\vec{r}}{d\theta}$$
$$\vec{b} = \vec{t} \wedge \vec{n}$$

$(P\,\vec{t}\,\vec{n}\,\vec{b}) \to$ triedro de Frenet

Para:
$\underbrace{(P-O)}_{\vec{r}} = (t\,\text{sen}\,t + \cos t)\vec{i} + (-t\cos t + \text{sen}\,t)\vec{j} + t^2\vec{k}$ com $(O\,\vec{i}\,\vec{j}\,\vec{k})$ Fixo ao referencial. Determinar:

a) $\vec{v}$ e v

b) $s = s(t)$ com $s(t=0) = 0$

c) $\vec{t}$, $\vec{n}$ e ρ em função de t

d) As acelerações tangenciais e normal

(a)

$\vec{r}\,(P-O) = (t\,\text{sen}\,t + \cos t)\,\vec{i} + (-t\cos t + \text{sen}\,t)\,\vec{j} + t^2\,\vec{k}$

$\dfrac{d\vec{r}}{dt} = (\text{sen}\,t + t\cos t - \text{sen}\,t)\,\vec{i} + (-\cos t + t\,\text{sen}\,t + \cos t)\,\vec{j} + 2t\,\vec{k}$

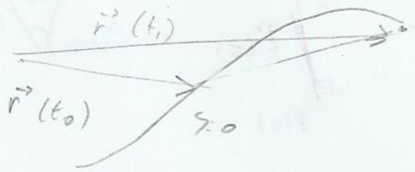
$\boxed{\vec{v} = \dfrac{d\vec{r}}{dt} = t\cos t\,\vec{i} + t\,\text{sen}\,t\,\vec{j} + 2t\,\vec{k}}$

$v = \|\vec{v}\| = \sqrt{t^2\cos^2 t + t^2\,\text{sen}^2 t + 4t^2}$

$= \sqrt{t^2(\cos^2 t + \text{sen}^2 t + 4)}$

$= \sqrt{5t^2} \quad \therefore \quad \boxed{v = \sqrt{5}\,t}$

(b) $v = \dfrac{ds}{dt}$



$s(t) - \overset{0}{s(t_0)} = \int_0^t v\,dt$

$s(t) = \int_0^t \sqrt{5}\,t\,dt$

$= \dfrac{\sqrt{5}}{2}\,t^2 \quad \therefore \quad \boxed{s(t) = \dfrac{\sqrt{5}}{2}\,t^2}$

(c) $\vec{t} = \dfrac{d\vec{r}}{ds} = \dfrac{\vec{v}}{v}$

$= \dfrac{t\cos t\,\vec{i} + t\,\text{sen}\,t\,\vec{j} + 2t\,\vec{k}}{\sqrt{5}\,t}$

$\boxed{\vec{t} = \dfrac{\sqrt{5}}{5}\left(\cos t\,\vec{i} + \text{sen}\,t\,\vec{j} + 2\,\vec{k}\right)}$

$\dfrac{d\vec{t}}{dt} = \dfrac{v}{\rho}\,\vec{n}$

$\dot{\vec{t}} = \dfrac{1}{\sqrt{5}}\left(-\text{sen}\,t\,\vec{i} + \cos t\,\vec{j}\right)$

$\|\dot{\vec{t}}\| = \sqrt{\dfrac{\text{sen}^2 t}{5}\,\vec{i} + \dfrac{\cos^2 t}{5}\,\vec{j}} = \dfrac{1}{\sqrt{5}}$

$\vec{n} = \dfrac{\dot{\vec{t}}}{\|\dot{\vec{t}}\|} \Rightarrow \boxed{\vec{n} = -\text{sen}\,t\,\vec{i} + \cos t\,\vec{j}}$

$\|\dot{\vec{t}}\| = \dfrac{v}{\rho}\|\vec{n}\| \Rightarrow \rho = \dfrac{v}{\|\dot{\vec{t}}\|} \Rightarrow \rho = \dfrac{\sqrt{5}\,t}{1/\sqrt{5}} \quad \therefore \quad \boxed{\rho = 5t}$

(d) $\vec{a} = \underbrace{\dot{v}\,\vec{t}}_{\vec{a}_t} + \underbrace{\dfrac{v^2}{\rho}\,\vec{n}}_{\vec{a}_n}$

$\vec{a}_t = \sqrt{5}\cdot\dfrac{1}{\sqrt{5}}\,(\cos t\,\vec{i} + \text{sen}\,t\,\vec{j} + 2\vec{k}) = \cos t\,\vec{i} + \text{sen}\,t\,\vec{j} + 2\vec{k}$ //

$\vec{a}_n = \dfrac{5t^2}{5t}\,(-\text{sen}\,t\,\vec{i} + \cos t\,\vec{j}) = -t\,\text{sen}\,t\,\vec{i} + t\cos t\,\vec{j}$ //

Cinemática do corpo Rígido

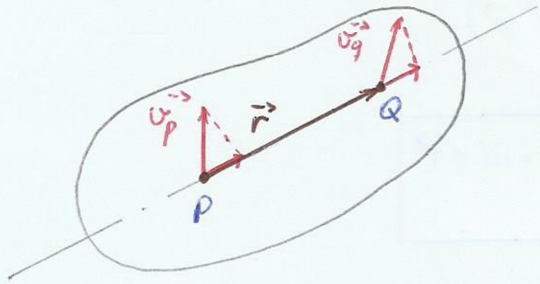
26-02-2015

- França & Matsumura, 2010, pags 94-115
- Beer & Johnston, 2006, pag. 397-531

Propriedade Fundamental

$\vec{r} = (Q-P) \quad \|\vec{r}\|$ - constante

↳ magnitude ou módulo constante



$\|\vec{r}\|^2 = \vec{r}\cdot\vec{r} = (Q-P)(Q-P) = \text{constante}$

$(\dot{Q}-\dot{P})(Q-P) + (Q-P)(\dot{Q}-\dot{P}) = 0$

$(\dot{Q}-\dot{P})(Q-P) + \cancel{(Q-P)(\dot{Q}-\dot{P})} = 0$

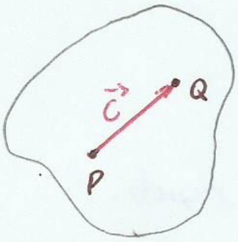
$(\vec{v_Q}-\vec{v_p})\cdot(Q-P) = 0$

$\boxed{\vec{v_Q}\cdot(Q-P) = \vec{v_p}(Q-P)}$

"A projeção das velocidades de dois pontos quaisquer P e Q sobre a reta que os une é a mesma."

Momento de Translação

$\vec{c} = (Q-P) = \text{constante}$



(Mesma direção
Mesmo módulo)

$\dot{\vec{c}} \xrightarrow{} 0 = \vec{v_Q} - \vec{v_p}$

$\therefore \boxed{\vec{v_Q} = \vec{v_p}}$

Quando é translação pura, todos os pontos do corpo rígido tem a mesma velocidade.

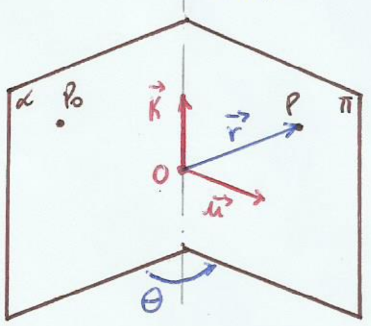
Rotação (Eixo Fixo)

α : Plano Fixo

π : Plano Solidário ao corpo rígido

$(O\,\vec{u}\,\vec{t}\,\vec{k})$ - Base solidária ao corpo rígido

Eixo Fixo $(O\vec{k})$



$\omega = \dot{\theta}$: Velocidade angular

$\boxed{\vec{\omega} = \omega\vec{k}}$ Vetor Rotação

$\dfrac{d\vec{r}}{dt} = \vec{v_p} - \cancel{\vec{v_0}}^{\,0} \quad (I)$

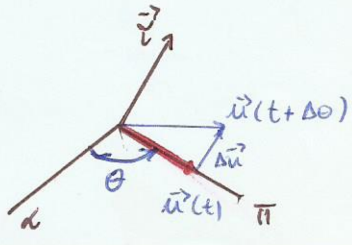
$\vec{r}$ na Base $(O\,\vec{u}\,\vec{t}\,\vec{k})$:

$\vec{r} = (\vec{r}\cdot\vec{u})\vec{u} + (\vec{r}\cdot\vec{k})\vec{k}$

Usando (I):

$\dfrac{d\vec{r}}{dt} = \cancel{(\dot{\vec{r}}\cdot\vec{u})}^{\,0}\,\vec{u} + (\vec{r}\cdot\vec{u})\dot{\vec{u}} + \cancel{(\dot{\vec{r}}\cdot\vec{k})}^{\,0}\,\vec{k} + \cancel{(\vec{r}\cdot\vec{k})\,\dot{\vec{k}}}^{\,0}$ const.

$\dfrac{d\vec{r}}{dt} = (\vec{r}\cdot\vec{u})\,\dot{\vec{u}}$



$\dfrac{d\vec{u}}{dt} = \lim_{\Delta t \to 0} \dfrac{\overbrace{\vec{u}(t+\Delta t) - \vec{u}(t)}^{\Delta\vec{u}}}{\Delta t}$

$\dfrac{d\vec{u}}{dt} = \lim_{\Delta t \to 0} \dfrac{\|\Delta\vec{u}\|}{\Delta t}\cdot\vec{t} \quad \xrightarrow{} \Delta\theta$

$\therefore \boxed{\dot{\vec{u}} = \omega\,\vec{t}}$

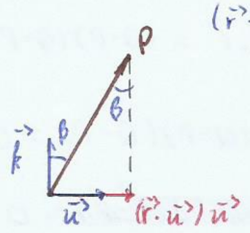
A partir de $\frac{d\vec{r}}{dt} = (\vec{r}\cdot\vec{u})\dot{\vec{u}}$ $\therefore$ $\dot{\vec{r}} = (\vec{r}\cdot\vec{u})\,\omega\,\vec{\tau}$ mas $\vec{\tau} = \vec{k}\wedge\vec{u}$

$\dot{\vec{r}} = \omega\vec{k}\wedge(\vec{r}\cdot\vec{u})\,\vec{u}$

$\therefore \quad \dot{\vec{r}} = \underbrace{\omega\vec{k}}_{\vec{\omega}}\wedge(\vec{r}\cdot\vec{u})\,\vec{u} = \vec{\omega}\wedge\vec{r}$

$\|\vec{\omega}\wedge(\vec{r}\cdot\vec{u})\vec{u}\| = \|\vec{\omega}\|\,(\vec{r}\cdot\vec{u})\cdot \cancel{\operatorname{sen} 90°}^{\,1}$

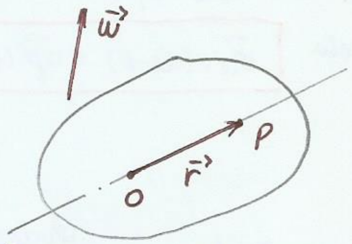
$\|\vec{\omega}\wedge\vec{r}\| = \|\vec{\omega}\|\,\underbrace{\|\vec{r}\|\cdot\operatorname{sen}\beta}_{(\vec{r}\cdot\vec{u})} = \|\vec{\omega}\|\cdot(\vec{r}\cdot\vec{u})$



$$\boxed{\frac{d\vec{r}}{dt} = \vec{\omega}\wedge\vec{r}}$$

Movimento Geral

$$\boxed{\frac{d\vec{r}}{dt} = \vec{\omega}\wedge\vec{r}}$$



$\dot{\vec{r}} = \vec{v_P} - \vec{v_O} = \vec{\omega}\wedge\vec{r}$

$\vec{v_P} = \vec{v_O} + \vec{\omega}\wedge\vec{r}$

$\boxed{\vec{v_P} = \vec{v_O} + \vec{\omega}\wedge(P-O)}$ (Fórmula de Poisson)

Relaciona a velocidade de dois pontos quaisquer do corpo rígido.

$\therefore$ Derivando em relação ao tempo:

$\vec{a_P} = \dot{\vec{v_P}} = \vec{a_O} + \dot{\vec{\omega}}\wedge(P-O) + \vec{\omega}\wedge\underbrace{\dot{\vec{r}}}_{\vec{\omega}\wedge\vec{r}}$

$\boxed{\vec{a_P} = \vec{a_O} + \dot{\vec{\omega}}\wedge(P-O) + \vec{\omega}\wedge[\vec{\omega}\wedge(P-O)]}$ (Fórmula de Poison para aceleração)

Exemplo 2, Seção 5, Kaminski (2000)

(Base ijk no O

(a) Velocidade angular Ω do eixo AB

$\vec{v}_D = -v\,\vec{i}$

Poisson: $\vec{v}_P = \vec{v}_O + \vec{\omega} \wedge (P-O)$

Aplicando poisson entre C e D

$\vec{v}_D = \cancel{\vec{v}_C}^{\,\vec{0}} + \Omega \wedge (D-C)$ $\quad$ $C \in$ eixo fixo OC

$-v\,\vec{i} = \Omega \vec{k} \wedge R\vec{j} = -R\Omega\,\vec{i}$

$\Omega = \dfrac{v}{R}$; $\vec{\Omega} = \dfrac{v}{R} \cdot \vec{k}$

(b) Velocidade angular w do disco A

$i\ j\ k\ i\ j$ (+ / −)

Olhando para corpo AB

$\vec{v}_A = \cancel{\vec{v}_O}^{\,0} + \vec{\Omega} \wedge (A-O)$

$\vec{v}_A = \dfrac{v}{R} \cdot \vec{k} \wedge 2R\vec{j} = 2v\,\vec{i}$ //

Olhando para o disco de centro A:

$\vec{w} = w_x \vec{i} + w_y \vec{j} + w_z \vec{k}$

$\vec{v}_A = \cancel{\vec{v}_E}^{\,0} + \vec{w} \wedge (A-E)$

↳ $\vec{0}$ (Contato sem escorregamento)

$-2v\vec{i} = (w_x \vec{i} + w_y \vec{j} + w_z \vec{k}) \wedge r(\vec{k})$

$-2v\vec{i} = -r w_x \vec{j} + r w_y \vec{i}$

Em $\vec{i}$: $-2v = r w_y \Rightarrow w_y = \dfrac{-2v}{r}$

Em $\vec{j}$: $0 = -r w_x \Rightarrow w_x = 0$

$w_z = ?$

"Cone Imaginário"
$O \in$ Disco

Aplicando Poisson entre A e O:
↳ p/ o disco

$\vec{v}_A = \cancel{\vec{v}_O}^{\,0} + \vec{w} \wedge (A-O)$

$-2v\vec{i} = (w_x \vec{i} + w_y \vec{j} + w_z \vec{k}) \wedge 2R\vec{j}$

$-2v\vec{i} = 2R w_x \vec{k} - 2R w_z \vec{i}$

Em $\vec{i}$: $-2\vec{v} = -2R w_z \quad \therefore \quad w_z = \dfrac{v}{R}$

Em $\vec{k}$: $0 = 2R w_x \quad\quad w_x = 0$

$\vec{w} = -2\dfrac{v}{R}\vec{j} + \dfrac{v}{R}\vec{k}$ //

c) Velocidade do ponto P do disco A, quando $AP // x$

Aplicando a fórmula de poisson para os pontos P e A do disco

$$\vec{v_P} = \vec{v_A} + \vec{\omega} \wedge (P-A)$$

$$\vec{v_P} = -2v\vec{i} + \left(-2\frac{v}{r}\vec{j} + \frac{v}{R}\vec{k}\right) \wedge r\vec{i}$$

$$\vec{v_P} = -2vi + 2v\vec{k} + \frac{v}{R} r\vec{j}$$

$$\vec{v_P} = -2v\vec{i} + \frac{r}{R} v\vec{j} + 2v\vec{k}$$

Movimento Plano

Todas as partículas do corpo movimentam-se em planos paralelos a um plano α

Adotando-se $\vec{k}$ normal ao plano α:

$$\vec{\omega} = \omega\vec{k}$$

- Fórmula de Poisson em velocidade

$$\vec{v_P} = \vec{v_O} + \omega\vec{k} \wedge (P-O)$$

- Fórmula de Poisson em aceleração $\quad \dot{\vec{\omega}} = \frac{d\vec{\omega}}{dt} = \frac{d}{dt}\omega\vec{k} = \dot{\omega}\vec{k}$

$$\vec{a_P} = \vec{a_O} + \dot{\omega}\vec{k} \wedge (P-O) + \omega\vec{k} \wedge [\omega\vec{k} \wedge (P-O)]$$

— 11 —

Problema Resolvido 11.1 (Beer & Johnston)

Ponto se desloca em linha reta: $x = t^3 - 6t^2 - 15t + 40 \quad x\,[m] \quad t\,[s]$

(a) instante para velocidade nula

(b) distancia percorrida ate $v=0$

(c) aceleração neste ponto

(d) distancia percorrida de $t=4s$ a $t=6s$

x Checar esboço do gráfico de uma curva de terceiro grau

$v(t) = \frac{dx}{dt} = 3t^2 - 12t - 15$; Para $v=0 \Rightarrow \frac{dx}{dt} = 0$

$3t^2 - 12t - 15 = 0$

$$t = \frac{12 \pm \sqrt{12^2 + 4.3.15}}{2.3}$$

$\boxed{t_1 = 5s}$

$\cancel{t_2 = -1}$

$a(t) = \ddot{x} = 6t - 12$

Para $t = 5s$; $\boxed{a(5) = 18\ m/s^2}$

Para $t=0 \Rightarrow x_0 = 40$

$t=5 \Rightarrow x_5 = -60$

$x_5 - x_0 = -60 - 40 \therefore \boxed{\Delta x = -100\,m}$

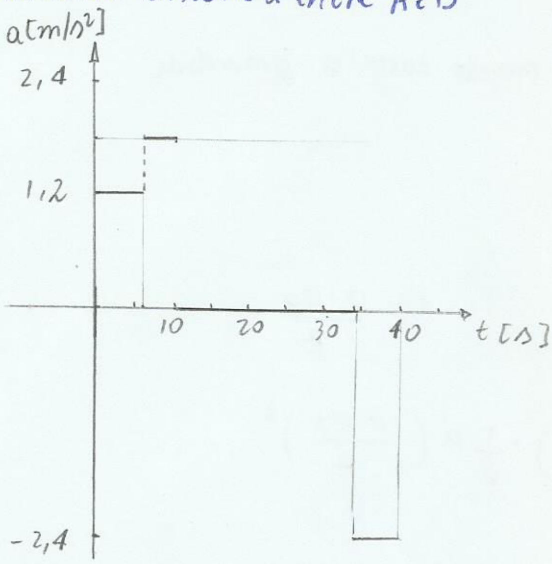
Problema Resolvido 11.6

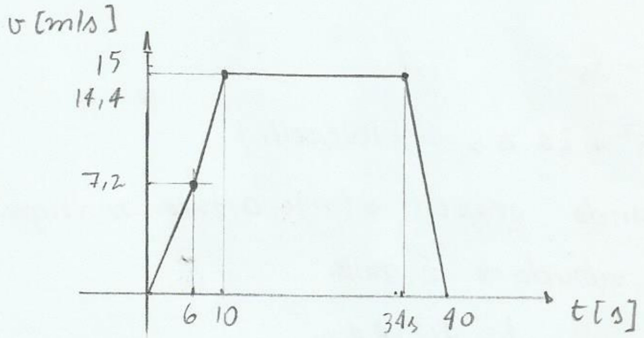
+ Comportamento em A: - acelera à razão $1,2\ m/s^2$ em $6s$
- " " " $1,8\ m/s^2$ até alcançar $v = 14,4\ m/s$ e mantém velocidade até estação B

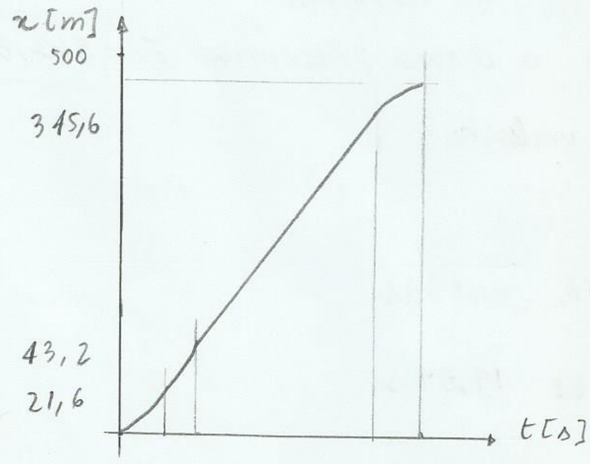
(a) Trace diagramas a-t
v-t
x-t

+ Tempo de frenagem: 6s
+ Tempo total da viagem AB: 40s

(b) Determine distancia entre A e B







$$a = \frac{\Delta v}{\Delta t} \ ; \ v = \frac{\Delta x}{dt}$$

Para $0 < t < 6s$:

$a = 1,2\ m/s^2$

$\Delta v = 7,2\ m/s$

$\Delta x = 43,2\ m$

Para $6 < t < t_2$

$v = 14,4\ m/s$

$$a = \frac{14,4 - 7,2}{t_2 - 6} = 1,8 \quad \therefore t_2 = 10s$$

Distancia Percorrida:

$$0 \to 6 = \frac{7,2 \times 6}{2} = 21,6\ m$$

$$6 \to 10 = \frac{(14,4 + 7,2) \cdot 4}{2} = 43,2\ m$$

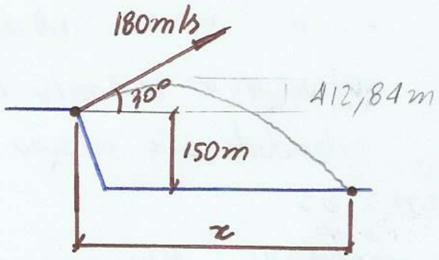
$$10 \to 34 = 24 \times 14,4 = 345,6\ m$$

$$34 \to 40 = \frac{14,4 \times 6}{2} = 43,2\ m$$ distancia de Frenagem

$$\therefore \boxed{0 \to 40 = 453,6\ m}$$

$$a_{34 \to 40} = \frac{-14,4}{6} = -2,4$$

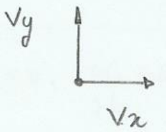
Problema Resolvido 11.7



- Desprezar a resistência do ar

(a) x

(b) altura máxima do projétil



I Know that: $V_x = const$

V_y = Varia de acordo com a gravidade

$V_x = V_{0x} = 180 . \cos 30 \therefore V_x = 155{,}88 \ m/s$

$V_y = V_{0y} = 180 \sin 30 \therefore V_y = 90 \ m/s$

$\therefore$ Sabendo que: $v = v_0 + at$ $\qquad t = \dfrac{v - v_0}{a}$

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

$$s = s_0 + v_0 \left(\frac{v - v_0}{a} \right) + \frac{1}{2} a \left(\frac{v - v_0}{a} \right)^2$$

$$s = s_0 + \frac{v_0 v}{a} - \frac{v_0^2}{a} + \frac{1}{2}\frac{v^2}{a} - \frac{1}{2}\frac{v_0^2}{a}$$

$$2a(s - s_0) = \cancel{2 v_0 v} - 2v_0^2 + v^2 + v_0^2 - \cancel{2 v v_0}$$

$$2a(s - s_0) = -v_0^2 + v^2 \quad \therefore \quad v^2 = v_0^2 + 2a\Delta s \quad \text{(Torricelli)}$$

$v_y^2 = 90^2 + 2 \times (-9{,}81) . h$ $\qquad$ Quando projétil atinge altura máxima a velocidade é nula.

$v_y^2 = 8100 - 19{,}62 h$

$\therefore h = 412{,}84 m$

Portanto a distância entre o solo e a altura maxima é $\boxed{562{,}84 \ m}$

Usando equação do movimento retilínco uniforme:

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

$$0 = 150 + 90t + \frac{1}{2}(-9{,}81) t^2$$

$$-4{,}905 t^2 + 90t + 150 = 0 \quad \begin{cases} t_1 = \cancel{-1{,}538 s} \\ t_2 = 19{,}89 s \end{cases}$$

Para V_x = const

$v = \dfrac{\Delta s}{\Delta t} \therefore$ $\qquad x = \Delta s = 19{,}89 \times 155{,}88$

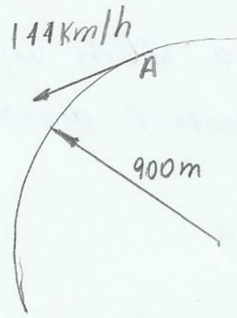
$$\therefore \boxed{x = 3{,}1 \ Km = 3{,}1 . 10^3 m}$$

Problema Resolvido 11.10

Trem descacelera com desaceleracao constante

Após 6s → $v = 96$ Km/h

x Determinar desaceleração



$$\frac{Km}{h} = \frac{1000\ m}{3600\ s} \therefore 3{,}6\ \frac{Km}{h} = 1\ \frac{m}{s}$$

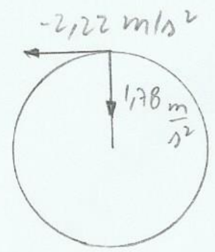
144 Km/h → 40 m/s

96 Km/h → 26,67 m/s

Lembrando que: $\vec{a} = \dot{v}\vec{t} + \frac{v^2}{\rho}\vec{n}$

$$a_t = \frac{\Delta v}{\Delta s} = \frac{26{,}67 - 40}{6} \qquad a_t = -2{,}22\ m/s^2$$

(Aceleração tangencial)



Determinando aceleração normal

$$a_n = \frac{40^2}{900} = 1{,}78\ \frac{m}{s^2}$$

$$\therefore \|a\| = \sqrt{(-2{,}22)^2 + 1{,}78^2}$$

$$\boxed{\|a\| = 2{,}85\ m/s^2}$$

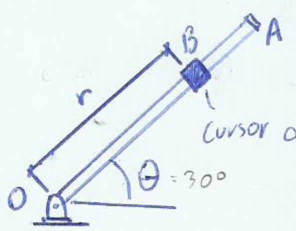
Problema Resolvido 11.12

OA = 0,9 m

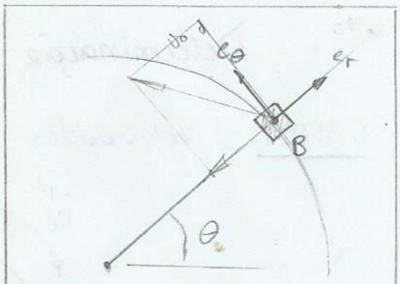
$\theta = 0{,}15t^2$ ~~rad~~

$r = 0{,}9 - 0{,}12t^2$

- Determinar a velocidade e aceleração total do cursor B após o braço OA ter girado 30°



Para $\theta = 30°$ → $\frac{30\pi}{180} = 0{,}15t^2 \therefore t = 1{,}87s$



$v = v_r . e_r + v_\theta e_\theta$

$a = a_r e_r + a_\theta e_\theta$

$$a_c = \frac{v^2}{R} \Rightarrow \frac{(\omega R)^2}{R}$$

$$a_c = R.\omega^2$$

$r = 0{,}9 - 0{,}12t^2$

$\dot{r} = -0{,}24t$

$\ddot{r} = -0{,}24\ \frac{m}{s^2}$

$\therefore$

$r(1{,}87) = 0{,}481\ m$

$\dot{r}(1{,}87) = -0{,}448\ m/s$

$\ddot{r}(1{,}87) = -0{,}24\ m/s^2$

— " —

$\theta = 0{,}15t^2$

$\dot{\theta} = 0{,}30t$

$\ddot{\theta} = 0{,}30$

$\therefore$

$\theta(1{,}87) = 0{,}52$ rad

$\dot{\theta}(1{,}87) = 0{,}56$ rad/s → 0,27 m/s

$\ddot{\theta} = 0{,}30$ rad/s²

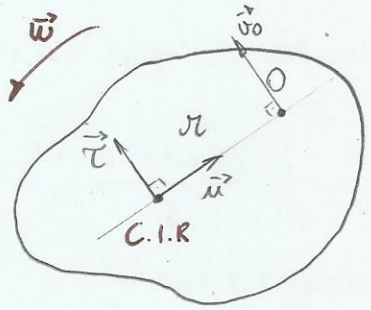
$$\vec{v} = -0{,}448\vec{e_r} + 0{,}27\vec{e_\theta} \rightarrow \|\vec{v}\| = 0{,}52\ m/s$$

$$\vec{a} = (-0{,}24 + 0{,}481 \times 0{,}56^2)\vec{e_r} + (0{,}481 \times 0{,}30 + 2 \times (-0{,}448) . 0{,}56)\vec{e_\theta}$$

$$\vec{a} = -0{,}391\vec{e_r} + 0{,}531\vec{e_\theta}$$

Centro Instantâneo de Rotação (C.I.R.) 5-3-2015

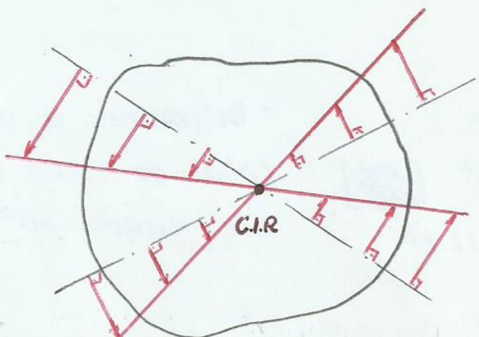
Para movimento plano, existe um ponto solidário ao sólido cuja velocidade é nula em um determinado instante. Este ponto é denominado Centro Instantâneo de Rotação (C.I.R.)



$\vec{v}_{C.I.R} = \vec{O}$

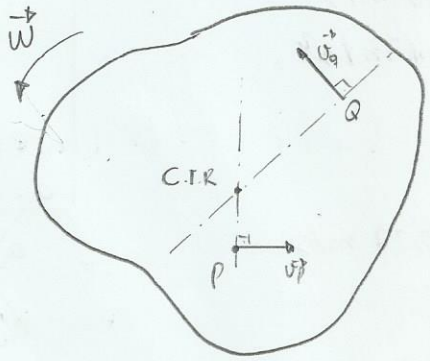
$\vec{v}_O = \cancel{\vec{v}_{CIR}} + \vec{w} \wedge (O - CIR)$

$\vec{v}_O = \vec{w}\vec{k} \wedge r\,\vec{u} = w r \vec{\tau}$

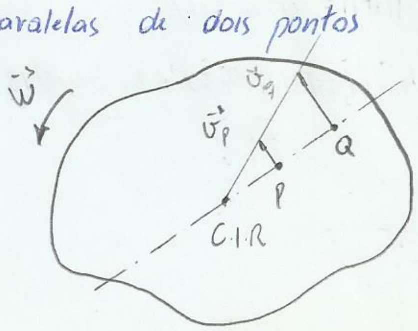


Determinação Gráfica do C.I.R.

CASO 1: Velocidades de dois pontos não paralelos



CASO 2: Velocidades paralelas de dois pontos

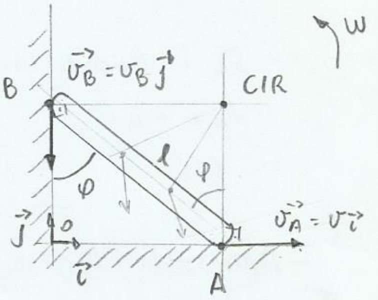


Exemplo 6.2 (Matsumura)

a) Determine CIR

b) $\omega(\varphi) = ?$

v_b e $a_b = ?$



Fórmula de Poison em velocidade:

$$\boxed{\vec{v_p} = \vec{v_o} + \omega_{\vec{k}} \wedge (P-O)}$$

$+$

$\vec{i} \quad \vec{j} \quad \vec{k} \quad \vec{i} \quad \vec{j}$

$-$

Aplicando Poison nos pontos CIR e A:

$$\vec{v_A} = \vec{v_{CIR}} + \vec{\omega} \wedge (A - CIR)$$

$$v\vec{i} = 0 + \omega\vec{k} \times l\cos\varphi(-\vec{j})$$

$$v\vec{i} = \omega l \cos\varphi \vec{i}$$

$$\boxed{\omega = \frac{v}{l\cos\varphi} \; ; \; \vec{\omega} = \frac{v}{l\cos\varphi}\vec{k}}$$

Aplicando Poison entre os pontos CIR e B:

$$\vec{v_B} = \vec{v_{CIR}} + \vec{\omega} \wedge (B - CIR)$$

$$v_b\vec{j} = 0 + \frac{v}{l\cos\varphi}\vec{k} \wedge (-l\,sen\varphi\,\vec{i})$$

$$v_b\vec{j} = -\frac{l\,sen\varphi}{l\cos\varphi} v\vec{j}$$

$$\therefore \boxed{\begin{array}{l} v_b = -v\,tg\varphi \\ \vec{v_b} = -v\,tg\varphi\,\vec{j} \end{array}}$$

Fórmula de Poison em aceleração

$$\boxed{\vec{a_p} = \vec{a_o} + \dot{\omega}\vec{k} \wedge (P-O) + \omega\vec{k} \wedge [\omega\vec{k} \wedge (P-O)]}$$

Aplicando Poison em aceleração (A e B)

$$a_B\vec{j} = \vec{0} + \dot{\omega}\vec{k} \wedge (-l\,sen\varphi\,\vec{i} + l\cos\varphi\,\vec{j}) + \omega\vec{k} \wedge [\omega\vec{k} \wedge (-l\,sen\varphi\,\vec{i} + l\cos\varphi\,\vec{j})]$$

→ Movimento retilíneo uniforme

→ Movimento retilíneo

$$a_B\vec{j} = -\dot{\omega} l\,sen\varphi\,\vec{j} - \dot{\omega} l\cos\varphi\,\vec{i} + \omega^2 l\,sen\varphi\,\vec{i} - \omega^2 l\cos\varphi\,\vec{j}$$

$$a_B\vec{j} = (-\dot{\omega} l\cos\varphi + \omega^2 l\,sen\varphi)\vec{i} - (\dot{\omega} l\,sen\varphi + \omega^2 l\cos\varphi)\vec{j}$$

Em $\vec{i}$: $\quad 0 = -\dot{\omega} l\cos\varphi + \omega^2 l\,sen\varphi \quad \therefore \quad \dot{\omega} = \omega^2 tg\varphi$

$$\dot{\omega} \cancel{l}\cos\varphi = \omega^2 \cancel{l}\,sen\varphi$$

Em $\vec{j}$: $\quad a_B = -(\dot{\omega} l\,sen\varphi + \omega^2 l\cos\varphi)$

$$a_B = -\omega^2 l\,sen\varphi . tg\varphi - \omega^2 l\cos\varphi$$

$$a_B = -\omega^2 l\left(\frac{sen^2\varphi + \cos^2\varphi}{\cos\varphi}\right)$$

$$a_B = \frac{-\omega^2 l}{\cos\varphi}$$

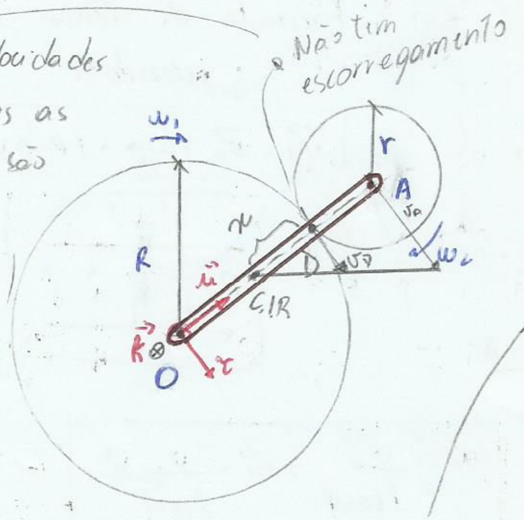
$$\boxed{a_B = \frac{-v^2}{l\cos^3\varphi}\vec{j}}$$

Exemplo 6.3, (França e Matsumura)

Dados: ω_1, ω_2

- Ω (Velocidade angular da Barra)?
- V_A e a_A
- CIR

Implica velocidades iguais, mas as acelerações são diferentes
(Tang = igual
Normal = dif)



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- Aplicando Poison para o disco 2 nos pontos D e A

$\vec{v_A} = \vec{v_D} + \vec{\omega_2} \wedge (A - D)$

$\vec{v_A} = \omega_1 . R \vec{\tau} + \vec{\omega_2} K \wedge r \vec{u}$

$\boxed{\vec{v_A} = (\omega_1 R + \omega_2 . r) \vec{\tau}}$

- Aplicando Poison para o disco 1 nos pontos O e D

$\vec{v_D} = \vec{v_O} + \vec{\omega_1} \wedge (K - O)$

$\vec{v_D} = O + \omega_1 \vec{K} \wedge R \vec{u}$

$\Rightarrow \vec{v_D} = \omega_1 . R \vec{\tau}$

- Aplicando Poison para a barra nos pontos O e A

$\vec{v_A} = \vec{v_O} + \vec{\Omega} \wedge (A - O)$

$(\omega_1 R + \omega_2 . r) . \vec{\tau} = O + \Omega \vec{K} \wedge (R + r) \vec{u}$

$(\omega_1 R + \omega_2 r) . \vec{\tau} = \Omega (R + r) \vec{\tau}$

$$\boxed{\Omega = \frac{\omega_1 R + \omega_2 r}{R + r} \; ; \; \vec{\Omega} = \frac{\omega_1 R + \omega_2 r}{R + r} \vec{\tau}}$$

- Aplicando Poison em aceleração nos pontos O e A

$\vec{a_A} = \vec{a_O} + \dot{\Omega} \vec{K} \wedge (A - O) + \Omega \vec{K} \wedge [\Omega \vec{K} \wedge (A - O)]$

$\vec{a_A} = O + \dot{\Omega} \vec{K} \wedge (R + r) \vec{u} + \Omega \vec{K} \wedge [\Omega \vec{K} \wedge (R + r) \vec{u}]$

$\vec{a_A} = \dot{\Omega} (R + r) \vec{\tau} - \Omega^2 (R + r) \vec{u}$

$$\vec{a_A} = \frac{\dot{\omega}_1 R + \dot{\omega}_2 r}{\cancel{(R + r)}} \cancel{(R + r)} \vec{\tau} - \frac{(\omega_1 R + \omega_2 r)^2}{(R + r)^{\cancel{2}}} \cancel{(R + r)} \vec{u}$$

$$\boxed{\vec{a_A} = (\dot{\omega}_1 R + \dot{\omega}_2 r) \vec{\tau} - \frac{(\omega_1 R + \omega_2 r)^2}{(R + r)} \vec{u}}$$

- Para determinar o CIR graficamente, temos as velocidade $\vec{v_A}$ e $\vec{v_D}$ que são paralelas.

$$\frac{v_A}{x + r} = \frac{v_D}{x}$$

$v_A . x = v_D x + v_D r$

$(v_A - v_D) . x = v_D . r$

$$x = \frac{v_D}{v_A - v_D} r$$

$$x = \frac{\omega_1 . R}{\cancel{\omega_1 R} + \omega_2 R - \cancel{\omega_1 R}} r$$

$$\boxed{x = \frac{\omega_1 . r}{\omega_2}}$$

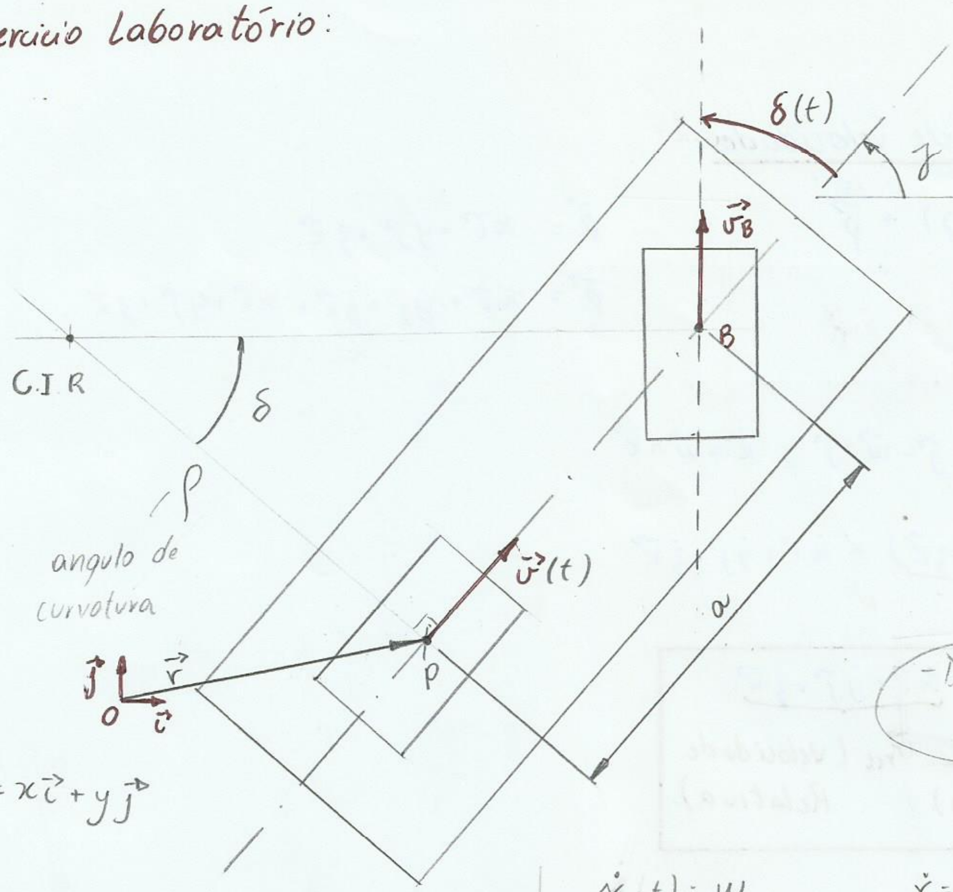
Aplicando Poison para Disco 2

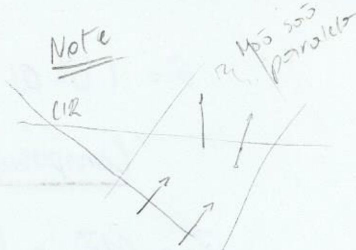
$\vec{v_D} = \vec{v}_{CIR}^{\,\to 0} + \omega_2 \vec{K} \wedge \underbrace{(D - CIR)}_{x}$

$\omega_1 R \vec{\tau} = \omega_2 \, x \, \vec{\tau}$

$$\boxed{x = \frac{\omega_1 r}{\omega_2}}$$

Exercício Laboratório:





$$tg\,\delta = \frac{a}{\rho}$$

$$\therefore \rho = \frac{a}{tg\,\delta}$$

- Determinar as relações de γ e $\vec{r}$ com v e δ

$\vec{r} = x\vec{I} + y\vec{J}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}\vec{I} + \dot{y}\vec{J}$$

$\downarrow$

$$v\cos\gamma\,\vec{I} + v\,sen\,\gamma\,\vec{J} = \dot{x}\vec{I} + \dot{y}\vec{J}$$

$$\dot{x} = v(t)\cos\gamma$$

$$\dot{y} = v(t)\,sen\,\gamma$$

$$\dot{\gamma}(t) = \omega$$

$$\omega = \frac{v}{\rho}$$

$$\Rightarrow \dot{\gamma} = \frac{v}{\rho}$$

$$= \dot{\gamma} = \frac{v}{\frac{a}{tg\,\delta}}$$

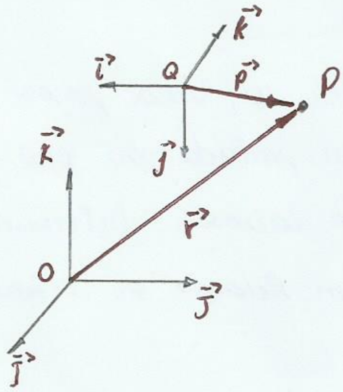
$$\dot{\gamma} = \frac{v(t)\cdot tg\,\delta}{a}$$

$$\boxed{\begin{aligned} \dot{x} &= v(t)\cos\gamma \\ \dot{y} &= v(t)\,sen\,\gamma \\ \dot{\gamma} &= \frac{v(t)}{a}\,tg(\delta(t)) \end{aligned}}$$

Composição de Movimentos (Pontos)

12-03-2015

- Franco & Matsumura, 2010, Pg 117-121
- Beer & Johnston, 2006, Pg. 502-513



$(O\vec{I}\vec{J}\vec{K})$ - Base Fixa

$(Q\vec{i}\vec{j}\vec{k})$ - Base Móvel

<u>Movimento Relativo de P</u>: Seu movimento em relação ao referencial móvel

<u>Movimento absoluto de P</u>: Seu movimento em relação ao referencial fixo

<u>Movimento de arrastamento de P</u>: Movimento de P (em relação ao referencial fixo) se P estivesse ligado (rigidamente) ao referencial móvel no instante considerado.

$$\vec{r} = (Q - O) + \vec{\rho}$$

Composição de velocidades

$$\vec{v}_P = \frac{d\vec{r}}{dt} = (\dot{Q} - O) + \dot{\vec{\rho}} \qquad \vec{\rho} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{v}_P = \vec{v}_Q + \dot{\vec{\rho}} \qquad \dot{\vec{\rho}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} + x\dot{\vec{i}} + y\dot{\vec{j}} + z\dot{\vec{k}}$$

$$\dot{\vec{i}} = \vec{\omega} \wedge \vec{i} \ ; \quad \dot{\vec{j}} = \vec{\omega} \wedge \vec{j} \ ; \quad \dot{\vec{k}} = \vec{\omega} \wedge \vec{k}$$

$$\dot{\vec{\rho}} = \vec{\omega} \wedge \underbrace{(x\vec{i} + y\vec{j} + z\vec{k})}_{\vec{\rho}} + \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$

$$\Rightarrow \boxed{\vec{v}_P = \underbrace{\vec{v}_Q + \vec{\omega} \wedge \vec{\rho}}_{\vec{v}_{arr}} + \underbrace{\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}}_{\vec{v}_{rel}}}$$

Poisson

$\vec{v}_{arr}$ (Velocidade de arrastamento) $\vec{v}_{rel}$ (Velocidade Relativa)

Composições de acelerações

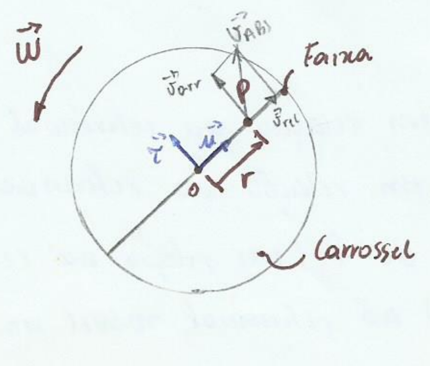
$$\vec{a}_P = \frac{d\vec{v}_P}{dt} = \vec{a}_Q + \dot{\vec{\omega}} \wedge \vec{\rho} + \vec{\omega} \wedge \dot{\vec{\rho}} + \dot{x}\dot{\vec{i}} + \dot{y}\dot{\vec{j}} + \dot{z}\dot{\vec{k}} + \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$

$$\vec{a}_P = \vec{a}_Q + \dot{\vec{\omega}} \wedge \vec{\rho} + \vec{\omega} \wedge [\vec{\omega} \wedge \vec{\rho} + \vec{v}_{rel}] + \vec{\omega} \wedge \underbrace{(\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k})}_{\vec{v}_{rel}} + \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$

$$\boxed{\vec{a}_P = \underbrace{\vec{a}_Q + \dot{\vec{\omega}} \wedge \vec{\rho} + \vec{\omega} \wedge (\vec{\omega} \wedge \vec{\rho})}_{\text{Ac. de arrastamento}} + \underbrace{\ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}}_{\text{Ac. Relativa}} + \underbrace{2\vec{\omega} \wedge \vec{v}_{rel}}_{\text{Ac. de Coriolis}}}$$

Exemplo:

Um carrosel gira com velocidade angular constante ω. Uma pessoa no carrosel (Ponto P) locomove-se sobre uma faixa radial pintada no piso do carrosel com velocidade constante v em relação ao carrosel. Determinar as velocidades e aceleração absolutas da pessoa (em Relação ao referencial fixo).



$$\vec{v}_{P,ABS} = \vec{v}_{ARR} + \vec{v}_{REL}$$

$$\vec{v}_{ARR} = \cancel{\vec{v}_O}^{\to 0} + \vec{\omega} \wedge (P-O) = \omega \vec{k} \wedge r \vec{u}$$

$$\vec{v}_{REL} = v \vec{u}$$

$$\vec{v}_{ARR} = \omega r . \vec{\tau}$$

$$\therefore \quad \vec{v}_{P,ABS} = v \vec{u} + \omega . r . \vec{\tau}$$

$$\vec{a}_{P,ABS} = \vec{a}_{ARR} + \vec{a}_{REL} + \vec{a}_c$$

$$\vec{a}_{ARR} = \cancel{\vec{a}_O}^{\vec{0}} + \cancel{\dot{\omega}}^{\to 0} \vec{k} \wedge (P-O) + \omega \vec{k} \wedge [\omega \vec{k} \wedge (P-O)]$$

$$\vec{a}_{ARR} = \omega \vec{k} \wedge [\omega \vec{k} \wedge r \vec{u}] = -\omega^2 r \vec{u}$$

$\vec{a}_{REL} = 0$

Movimento Uniforme Retilíneo

$$\vec{a}_c = 2 \vec{\omega} \wedge \vec{v}_{rel}$$

$$\vec{a}_c = 2 \omega \vec{k} \wedge v \vec{u}$$

$$\vec{a}_c = 2 \omega v \vec{\tau}$$

$$\vec{a}_{P,ABS} = -\omega^2 r \vec{u} + 2 \omega v \vec{\tau}$$

— // —

Continuação do Exercício (laboratório) $(x_0 = 0,\ y_0 = 0,\ \gamma = 0)$

Equações:

P/ $a = 2m$

$$\begin{cases} \dot{x} = v(t) \cos \gamma \\ \dot{y} = v(t) \sin \gamma \\ \dot{\gamma} = \dfrac{v}{a} \operatorname{tg} \delta(t) \end{cases}$$

+ Determinar $x(t)$, $y(t)$, $\gamma(t)$

(a) Para $v = 7{,}2$ Km/h e $\delta = 10°$:

(a.1) Plotar a trajetória do veículo

(a.2) Determinar o raio de curvatura

(b) Para $v = 7{,}2$ Km/h (constante), Plotar a trajetória do veículo e a posição do C.I.R. em resposta a um esterçamento das rodas dianteiras $\delta(t)$ dado por:

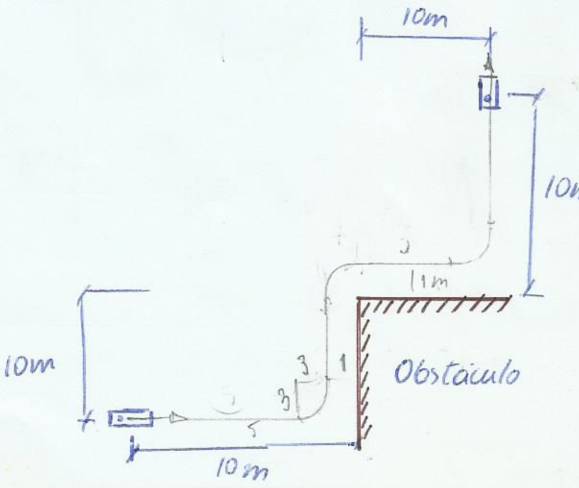
$$\delta(t) = \begin{cases} 0{,}2\pi \operatorname{sen}(0{,}5\pi t) & p/ \quad 0 \le t < 4s \\ 0 & p/ \quad 4s \le t \end{cases}$$

$\delta = \frac{s}{t}$

(c) Para a trajetória obtida em (b), determinar e plotar a trajetória do CG do veículo, localizado no ponto médio entre os eixos dianteiros e traseiro

(d) Encontre uma trajetória possível e as funções correspondentes $v(t)$ e $\delta(t)$ para que o veículo saia da posição inicial ilustrada no instante $t = 0s$ e chegue na condição final no instante $t = 60s$, s/ que o veículo colida com obstáculo

$6 \to 3 \circlearrowleft 5 \to 3 \circlearrowright \to 6 \circlearrowright 3 \to 7$



Exercício 11.1) (Beer & Johnston)

$x = 2t^3 - 15t^2 + 36t - 10$ [ ]

- Determinar a posição, velocidade e aceleração, quando $t = 4s$

$x = 2t^3 - 15t^2 + 36t - 10$ $\qquad x(4) = 22m$

$\dot{x} = 6t^2 - 30t + 36$ $\qquad \dot{x}(4) = 12 m/s$

$\ddot{x} = 12t - 30$ $\qquad \ddot{x}(4) = 18 m/s^2$

Exercício 11.2) (Beer & Johnston)

$x = t^4 - 3t^3 + t$ [m] t [s]

- Determinar $\dot{x}(3)$; $\ddot{x}(3)$

$x = t^4 - 3t^3 + t$ $\qquad x(3) = 3m$

$\dot{x} = 4t^3 - 9t^2 + 1$ $\quad \therefore \quad \dot{x}(3) = 28 m/s$

$\ddot{x} = 12t^2 - 18t$ $\qquad \ddot{x}(3) = 54 m/s^2$

Exercício 11.6) (Beer & Johnston)

$x = 2t^3 - 15t^2 + 24t + 4$ $\quad$ (a) Instante para velocidade nula

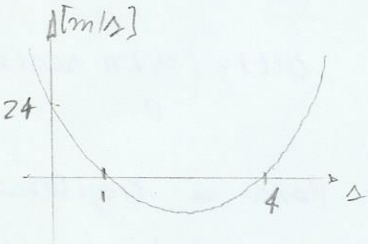
(b) Posição e distancia até aceleração nula

(a)

$\dot{x} = 6t^2 - 30t + 24$

Para $v = 0$: $\quad 6t^2 - 30t + 24 = 0 \qquad t = \dfrac{30 \pm \sqrt{30^2 - 4 \cdot 6 \cdot 24}}{2 \times 6}$

$\boxed{t_1 = 4s \quad \text{ou} \quad t_2 = 1s}$

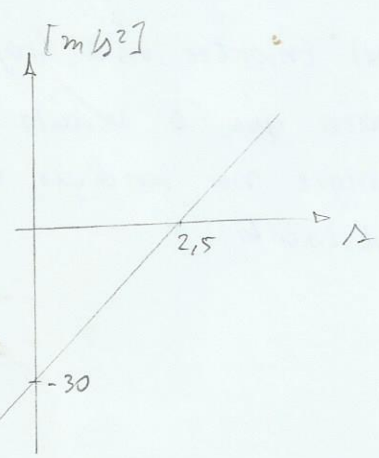


(b) $\ddot{x} = 12t - 30$

Para $a = 0$: $\quad 12t - 30 = 0 \quad \therefore t = 2{,}5s$

$x(0) = 4 m$

$\boxed{x(2{,}5) = 1{,}5m}$



Exercício 11.10) (Beer & Johnston)

$a = kt^2$

(a) Para $v = -24\ m/s$ em $t = 0$; Determinar K:
$v = +40\ m/s$ em $t = 4s$

(b) Equação do movimento, quando $x = 6m$ em $t = 2s$.

(a) $v = at^3 + bt^2 + ct + d$

$\dot{v} = 3at^2 + 2bt + c$; Como "a" apenas tem a parcela t^2

$\therefore v = at^3 + d$ $\begin{cases} -24 = a.0 + d \\ 40 = a.4^3 + d \end{cases}$ $\therefore$ $d = -24$, $a = 1$ $\therefore \boxed{v = t^3 - 24}$

$a = \dot{v} = 3t^2$ $\therefore$ $\boxed{K = 3\ m/s^4}$

(b) $x = at^4 + bt^3 + ct^2 + dt + e$

$\dot{x} = 4at^3 + 3bt^2 + 2ct + d$; Entretanto, $v(t) = t^3 - 24$

$\dot{x} = 4at^3 + d$; $a = \frac{1}{4}$ e $d = -24$

Logo $x = \frac{1}{4}t^4 - 24t + e$ $\begin{cases} \text{Para } t = 2s \Rightarrow x = 6 \\ 6 = \frac{1}{4}.2^4 - 24 \times 2 + e \quad \therefore e = 50 \end{cases}$

$\therefore \boxed{x(t) = \frac{1}{4}t^4 - 24t + 50}$

Exercício 11.14) (Beer & Johnston)

$a = 90 - 6x^2$; $a\ [m/s^2]$ $x\ [m/s]$

$v_0 = 0$ em $x = 0$

Determine:
(a) $v(5m) = ?$ 20 m/s
(b) $x(v=0) = ?$ 6,71 m
(c) $x(v_{max}) = ?$ 3,87 m

$a = \frac{dv}{dt} \cdot \frac{dx}{dx} = \frac{dx}{dt} \cdot \frac{dv}{dx}$ $\therefore a = v\frac{dv}{dx} \Rightarrow a\,dx = v\,dv$

$\int_{x_0}^{x_1} a\,dx = \int_{v_0}^{v_1} v\,dv$

$\int_{v_0}^{v_1} v\,dv = \int_{x_0}^{x_1} (90 - 6x^2)\,dx$

$\frac{v_1^2}{2} = 90x - 2x^3 \Big|_0^5$

$\boxed{v(5m) = 20\frac{m}{s}}$

$\frac{v^2}{2} = 90x - 2x^3 \Big|_{x_0=0}^{x}$

Para $v = 0$

$x(-2x^2 + 90) = 0$

$x_1 = 0$

$\boxed{x_2 = 6,71m}$

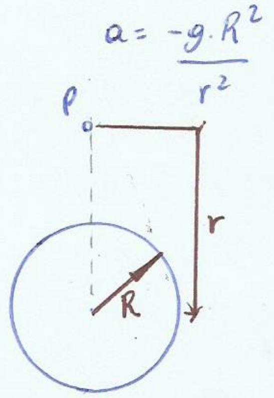
$v = (180x - 4x^3)^{\frac{1}{2}}$

$\frac{dv}{dt} = \frac{1}{2}(180x - 4x^3)^{-1/2} \cdot (180 - 12x^2)$

$\frac{dv}{dt} = \frac{180 - 12x^2}{2(180x - 4x^3)} = 0$

$\boxed{x = 3,87m}$

Exercício 11.26) (Beer & Johnston)



Deduza uma expressão para a velocidade de escape; isto é, a mínima velocidade com a qual um ponto material deve ser lançado verticalmente para uma, a partir da terra, para que não haja retorno à superfície

(Sugestão: $v=0$ para $r \to \infty$)

$$a = \frac{dv}{dt} \cdot \frac{dx}{dx} = \frac{dx}{dt}\frac{dv}{dx} = v\frac{dv}{dx}$$

$$a\,dx = v\,dv$$

$$\int_{r_0}^{r_1} -\frac{g \cdot R^2}{r^2}\,dr = \int_{v_0}^{v_1} v\,dv$$

$$\frac{v^2 - v_0^2}{2} = \frac{gR^2}{r}\Big|_{r_0}^{r_1}$$

p/ $r = \infty \to v = 0$

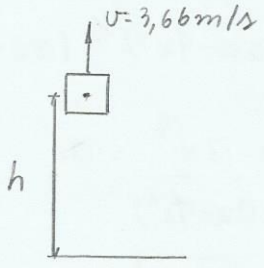
$$-\frac{v_0^2}{2} = \frac{gR^2}{R}$$

$$\therefore v_0^2 = 2gR$$

— // —

Exercício 11.32) (Beer & Johnston)

- elevador em movimento ascendente, com $v = 3{,}66$ m/s
- Pedra abandonada e cai em 2,5
  (a) altura do elevador
  (b) v da pedra ao chegar no solo



(a)

$$S = S_0 + v_0 t + \frac{at^2}{2}$$

$$h = S - S_0 = v_0 t + \frac{at^2}{2}$$

$$h = 3{,}66 \cdot 2{,}5 + \frac{(-9{,}8) \cdot 2{,}5^2}{2} \quad \therefore h = 21{,}48\,m$$

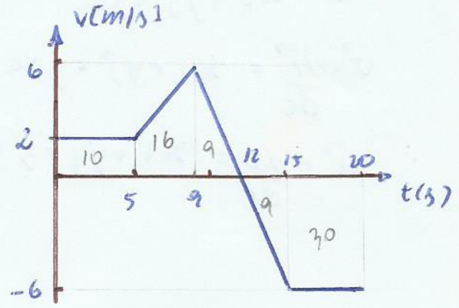
(b)

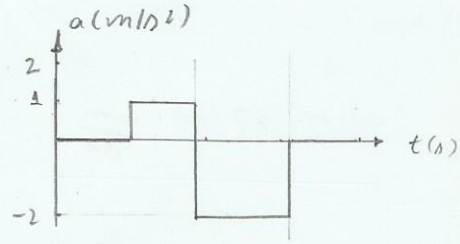
$$v = v_0 + at$$

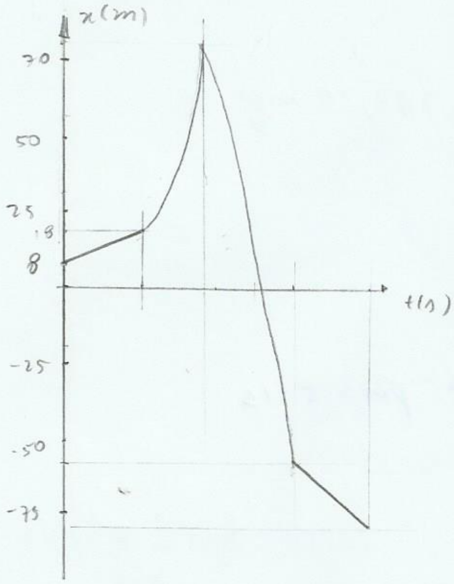
$$v = 3{,}66 + (-9{,}8) \cdot 2{,}5 \qquad v = -20{,}84\,\frac{m}{s}$$

Exercício 11.53) (Beer & Johnston)

Para $t_0 = 0$; $x = -8m$







Esboce os diagramas a-t, x-t, para $0<t<20s$

(a) máximo valor da coordenada de posição do ponto material.

(b) Os valores de t para os quais o ponto dista 18 m da origem

$a = \frac{dv}{dt}$ $\quad S = S_0 + v_0 t$

$S = S_0 + v_0 t + \frac{1}{2} a t^2$

$t = 0 \Rightarrow 8m$

$t = 5 \Rightarrow S = 8 + 2 \times 5 \therefore S = 18m$

$t = 9 \Rightarrow S = 18 + 2 \times 9 + \frac{1}{2} \cdot 1 \cdot 9^2 \Rightarrow S = 76,5$

$t = 15 \Rightarrow S = 76,5 + 6 \times 15 - \frac{1}{2} 2 \times 15^2 \therefore S = -58,5$

$t = 20 \Rightarrow S = -58,5 + (-6) \times 20 \therefore S = -78,5$

Exercício 11.54) (Beer & Johnston)

a) $x(0 \to 15) = \Delta_{0 \to 15} = 44s$

11,6 s

$v = v_0 + at$

$v = 1,8 \cdot 11,6 = 21,24 \frac{m}{s} = 76,46 \frac{Km}{h}$

$S = S_0 + v_0 t + \frac{1}{2} a t^2$

$S = \frac{1}{2} \cdot 1,8 \cdot 11,6^2 = 125,316$

$S = S_0 + v_0 t$

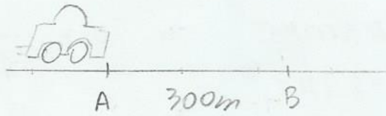
$S = 174,166$

Exercício 11.56) (Beer & Johnston)

$v = 54 \frac{Km}{h}$ $\quad a_m = 1,8 m/s^2$

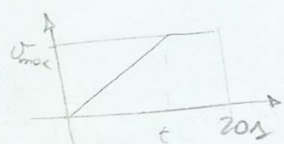
$\to 15 \frac{m}{s}$

$\frac{Km}{h} = \frac{1000m}{3600s}$



t_{AB} deveria ser: $S = S_0 + v_0 t$

$t = \frac{300}{15} \therefore t = 20s$



MRUA

$S = S_0 + v_0 t + \frac{at^2}{2}$ | $v_1 = v_0 + at$

$S_1 = \frac{at^2}{2}$ | $v_1 = at$

MRU

$S = S_0 + v_0 t$

$S_2 = v_1 (20 - t)$

$300 = \frac{1,8}{2} \times t^2 + 1,8 \cdot t (20 - t)$

$300 = 0,9t^2 + 36t - 1,8t^2$

$-0,9t^2 + 36t - 300 = 0$ $\quad \boxed{11,84 s}$ ~~28 s~~ e $\boxed{v = 21,3 \frac{m}{s} = 76,7 \frac{Km}{h}}$

Exercício 11.82) (Beer & Johnson)

$x = 120\,sen\left(\frac{1}{2}\pi t\right)$ $\quad \frac{dx}{dt} = 120 \cdot \frac{1}{2}\pi \cos\left(\frac{1}{2}\pi t\right)$

$y = 40t^2$ $\quad \frac{dy}{dt} = 80t$

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}i + \dot{y}\vec{j} + \dot{z}k$

$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}i + \ddot{y}j + \ddot{z}k$

$\frac{dx}{dt} = 0$ $\quad \frac{dy}{dt}(1) = 80mm$

$\ddot{x} = \frac{120}{2}\pi \cdot \frac{1}{2}\pi\,sen\left(\frac{1}{2}\pi t\right) \rightarrow \ddot{x}(1) = 296{,}09\ mm$

$\ddot{y} = 80$ $\quad a^2 = \ddot{x}^2 + \ddot{y}^2$ $\quad \therefore a(1) = 306{,}71\ \frac{mm}{s^2}$

—//—

Para $t = 2s$:

$\dot{x}(2) = -188{,}5$ $\quad \dot{y}(2) = 160$ $\quad v = 247{,}25\ \frac{mm}{s}$

$\ddot{x}(2) = 0$ $\quad \ddot{y}(2) = 80\ \frac{mm}{s^2}$

Exercício 11.84) (Beer & Johnston)

$r = (4\,sen\,\pi t)i - (\cos 2\pi t)j$ $\quad$ (a) v e a para $t = 1s$

$v = \dot{r} = (4\pi\cos\pi t)i + (2\pi\,sen\,2\pi t)j$

$v(1) = -4\pi\,i$

$y = \frac{1}{2}x^2 - 1$

$a = \ddot{r} = (-4\pi^2 sen\,\pi t)i + (4\pi^2\cos 2\pi t)j$

$a(1) = 4\pi^2\vec{j}$

$x = 4\pi\,sen\,\pi t \rightarrow t = \frac{x}{4\pi\,sen\,\pi t}$

$y = \cos 2\pi t \rightarrow t = \frac{y}{\cos 2\pi t}$

$\frac{x}{4\pi\,sen\,\pi t} = \frac{y}{\cos 2\pi t}$

$y = \frac{\cos 2\pi t}{4\pi\,sen\,\pi t} \cdot x = \frac{\cos^2\pi t - sen^2\pi t}{4\pi\,sen\,\pi t}\,x$

Recapitulação

Movimento Retilíneo de um Ponto Material

$\boxed{v = \frac{dx}{dt}}$ $\boxed{a = \frac{dv}{dt} = \frac{d^2x}{dt^2}}$

$$a = \frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{dx}{dx} = \overset{v}{\frac{dx}{dt}} \cdot \frac{dv}{dx} \qquad \boxed{a = v\frac{dv}{dx}}$$

Movimento Retilíneo Uniforme (MRU) $v = cte$

$$\frac{dx}{dt} = v$$

$$\boxed{x = x_0 + vt} \qquad \int_{x_0}^{x} dx = v\int_{t_0}^{t} dt \quad \therefore (x - x_0) = v(t - t_0)$$

Movimento Retilíneo Uniformemente Variado (MRUV) $a = cte$

$$\boxed{v = v_0 + at} \qquad \frac{dv}{dt} = a = cte \qquad \frac{dx}{dt} = v_0 + at$$

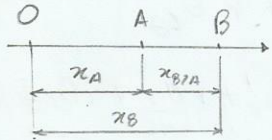
$$\int_{v_0}^{v} dv = a\int_{t_0}^{t} dt \qquad \int_{x_0}^{x} dx = \int_{t_0}^{t} v_0 + at\, dt$$

$$\boxed{x - x_0 = v_0 t + \frac{1}{2}at^2}$$

$$v - v_0 = at \qquad x - x_0 = v_0 t + \frac{a}{2}t^2$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

Movimento Curvilíneo de um Ponto Material



$$x_{B/A} = x_B - x_A$$

$$\therefore x_B = x_A + x_{B/A}$$

$$v_B = v_A + v_{B/A}$$

$$a_B = a_A + a_{B/A}$$

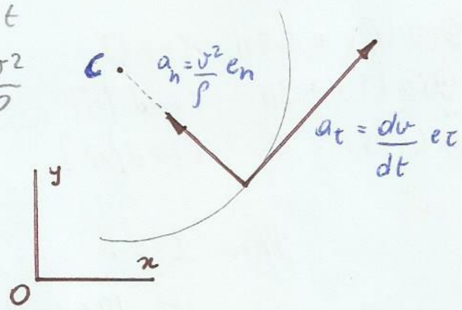
$$r = xi + yj + zk$$

$$v = \frac{dr}{dt} = \dot{x}i + \dot{y}j + \dot{z}k$$

$$a = \frac{dv}{dt} = \ddot{x}i + \ddot{y}j + \ddot{z}k$$

Componentes Tangencial e Normal

$$\boxed{a = \frac{dv}{dt}e_t + \frac{v^2}{\rho}e_n} \qquad a_T = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$$



$$v = \dot{r}e_r + r\dot{\theta}e_\theta \qquad v = \dot{R}e_R + R\dot{\theta}e_\theta + \dot{z}K$$

$$a = (\ddot{r} - r\dot{\theta}^2)e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})e_\theta \qquad a = \frac{dv}{dt} = (\ddot{R} - R\dot{\theta}^2)e_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})e_\theta + \ddot{z}K$$

11.112-) (Beer)

11.140-) (Beer)

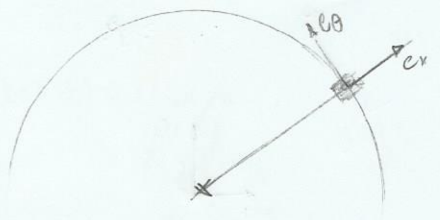
$\theta = \frac{1}{2}\pi(4t - 3t^2)$

$r = 1{,}25t^2 - 0{,}9t^3$

$\dot{r} = 2{,}5t - 2{,}7t^2 \qquad \dot{\theta} = 2\pi - 3\pi t$

$\ddot{r} = 2{,}5 - 5{,}4t \qquad \ddot{\theta} = -3\pi$

$v = \omega R$



$v = v_r e_r + v_\theta e_\theta$

$v = (2{,}5t - 2{,}7t^2)e_r + (2\pi - 3\pi t)e_\theta$

P/ $t = 1$

$v = -0{,}2\, e_r \left(\frac{m}{s}\right) + -\pi\, e_\theta \left(\frac{rad}{s}\right).$

$v = -0{,}2\, e_r - \pi \cdot (1{,}25t^2 - 0{,}9t^3)|^{t=1}$

$\boxed{v = -0{,}2\, e_r + 1{,}1\, e_\theta}$

$r = r e_r$

$v = \frac{dr}{dt} = \frac{dr}{dt} e_r + r\frac{de_r}{dt}$

$= \frac{dr}{dt} e_r + r \frac{de_r}{dt}\frac{d\theta}{d\theta}$

$\boxed{v = \dot{r} e_r + r \cdot \dot{\theta} e_\theta}$

$a = \frac{dv}{dt} = \ddot{r} e_r + \dot{r}\underbrace{\frac{de_r}{dt}}_{\dot{\theta} e_\theta} + \dot{r}\dot{\theta} e_\theta + r\ddot{\theta} e_\theta + r\dot{\theta}\underbrace{\frac{de_\theta}{dt}}_{-\dot{\theta} e_r}$

11.142) (Beer) - Determine velocidade e acelerações para $t = 0$ e $t = 0{,}25$

$r = 10t$

$\theta = 2\pi t$

$r = 10t$	$r(0) = 0$	$r(0{,}25) = 2{,}5$	[m]
$\dot{r} = 10$	$\dot{r}(0) = 10$	$\dot{r}(0{,}25) = 10$	[m/s]
$\ddot{r} = 0$	$\ddot{r}(0) = 0$	$\ddot{r}(0{,}25) = 0$	[m/s²]
$\theta = 2\pi t$	$\theta(0) = 0$	$\theta(0{,}25) = 0{,}5\pi$	[rad]
$\dot{\theta} = 2\pi$	$\dot{\theta}(0) = 2\pi$	$\dot{\theta}(0{,}25) = 2\pi$	[rad/s]
$\ddot{\theta} = 0$	$\ddot{\theta}(0) = 0$	$\ddot{\theta}(0{,}25) = 0$	[rad/s²]

$v = \dot{r} e_\theta + r\dot{\theta} e_\theta$

$a = (\cancel{\ddot{r}}^{\to 0} - r\dot{\theta}^2)e_r + (\cancel{r\ddot{\theta}}^{\to 0} + 2\dot{r}\dot{\theta})e_\theta$

$a = -r\dot{\theta}^2 e_r + 2\dot{r}\dot{\theta} e_\theta$

$a = -10t(2\pi)^2 + 2 \cdot 10 \cdot 2\pi$

$a = -40\pi^2 t + 40\pi\, e_\theta$

Para $t = 0$:

$v = 10 e_r$ (m/s)

$a = 40\pi\, e_\theta$ (m/s²)

Para $t = 0{,}25$

$v = 10 e_r + 5\pi\, e_\theta$ (m/s)

$a = -10\pi^2 e_r + 40\pi\, e_\theta$ (m/s²)

Capítulo 15 - Cinemática dos Corpos Rígidos

$$v = \frac{dr}{dt} = \omega \times r$$

$$a = \alpha * r + \omega \times (\omega \times r)$$

Poison: $\vec{v_P} = \vec{v_O} + \vec{\omega} \wedge (P-O)$

$$\vec{a_P} = \vec{a_O} + \dot{\vec{\omega}} \wedge (P-O) + \vec{\omega} \wedge [\vec{\omega} \wedge (P-O)]$$

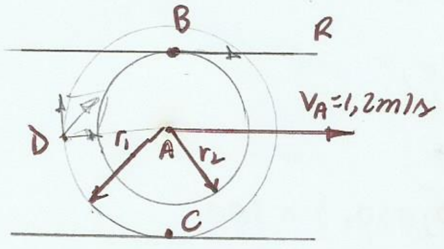
$$\vec{a_A} = \vec{a_B} + \dot{\vec{\omega}} \wedge (A-B) + \vec{\omega} \wedge (\omega \wedge (A-B)) + \ddot{x}$$

Problema Resolvido 15.2) (Beer)

Determinar:

a - velocidade angular da engrenagem

b - velocidades da cremalheira superior R e do ponto D da engrenagem



$r_1 = 150 mm$

$r_2 = 100 mm$

$v = \omega r$

$1,2 = \omega . 0,15 \quad \therefore \boxed{\omega = 8 \frac{rad}{s} \text{ ou } \vec{\omega} = -8k \frac{rad}{s}}$

$v_D = \omega . r_1$

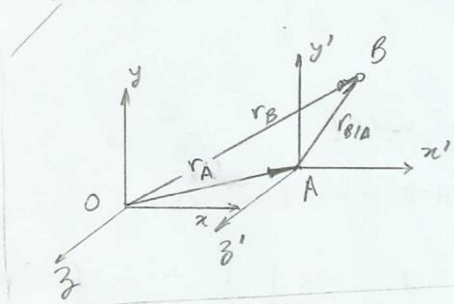
$v_R = v_B = v_A + v_{B/A}$

$= 1,2 + 8 \times 0,1 \quad \therefore \boxed{v_R = 2 m/s}$

$v_D = v_A + v_{D/A}$

$= 1,2 i + (-8k)(-0,15 i)$

$$\boxed{\begin{array}{l} v_D = 1,2\vec{i} + 1,2\vec{j} \ (m/s) \\ \|v_D\| = 1,7 \frac{m}{s^2} \end{array}}$$



$r_B = r_A + r_{B/A}$

Posição b em relação a A

$\overrightarrow{+}$ i j k i j $\overleftarrow{-}$

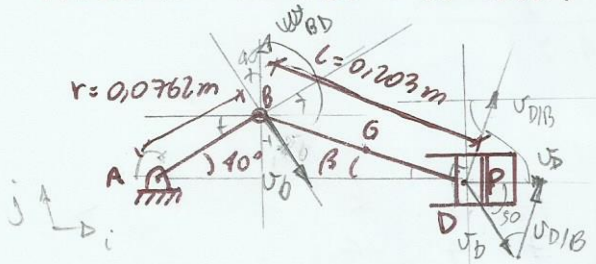
Lei dos cossinos

$a^2 = b^2 + c^2 - 2bc \cos \hat{A}$

Lei dos senos

$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} = \frac{c}{\operatorname{sen} C}$$

Problema Resolvido 15.3) (Beer)



$\omega_{AB} = 2\,000$ rpm

(a) Velocidade angular BD

(b) Velocidade do Pistão P

$rpm = \frac{rev}{min} = \frac{2\pi}{60} \left[\frac{rad}{s}\right] \quad \therefore \omega_{AB} = 2000 \text{ rpm} = 209,44 \text{ rad/s}$

$v = \omega r$

$v_b = \omega_{AB} . r$

$v_b = 15,96 \frac{m}{s}$

$\vec{v_b} = +10,26 i - 12,23 j$

$$\frac{0,203}{\operatorname{sen} 40} = \frac{0,0762}{\operatorname{sen} \beta}$$

$\beta = 13,96°$

$$\frac{v_{d/B}}{\operatorname{sen} 50} = \frac{v_b}{\operatorname{sen} 76,04} = \frac{v_d}{\operatorname{sen} 53,96}$$

$v_{D/B} = 12,6$ m/s

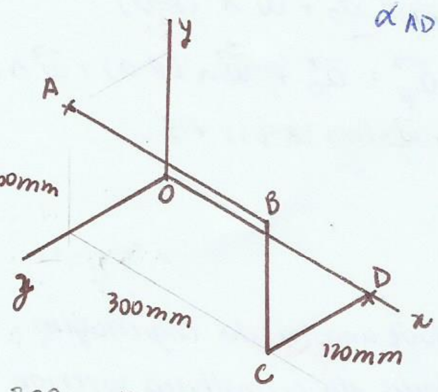
$$\boxed{v_D = 13,3 \frac{m}{s}}$$

Exercício 15.6) (Beer)

$w_{AD} = 95 \frac{rad}{s}$

$\alpha_{AD} = -380 \frac{rad}{s^2}$

Determinar aceleração e velocidade no vértice B.



$\overrightarrow{w}_{AD} = w \cdot \frac{(A-D)}{\|A-D\|}$

$\overrightarrow{w}_{AD} = 95 \times \frac{(-300i + 200j + 120k)}{380}$

$\overrightarrow{w}_{AD} = -75i + 50j + 30k$

$A = 200j + 120k$

$D = 300i$

$\vec{r} = (A-D) = -300i + 200j + 120k$

$\|r\| = 380$ mm

$r_{B/A} = 300i$

$v_B = \overrightarrow{w}_{AD} \wedge r_{B/A}$

$v_B = (-75i + 50j + 30k) \wedge 300i$

$$v_B = \begin{vmatrix} i & j & k \\ -75 & 50 & 30 \\ 300 & 0 & 0 \end{vmatrix} = +9000j - 15000k \quad \therefore \ \vec{v_B} = 9j - 15k \ (m/s)$$

$\alpha_{AB} = \alpha \cdot \frac{(A-D)}{\|(A-D)\|}$

$\alpha_{AB} = -380 \frac{(-300i + 200j + 120k)}{380}$

$\alpha_{AB} = 300i - 200j - 120k$

$a = \alpha \times r + w(w \times r)$

$a_b = \alpha_{AB} \times (A-D) + w_{AD} \times (\underbrace{w_{AD} \times r_{B/A}}_{v_B})$

$$a_b = \begin{vmatrix} i & j & k \\ 300 & -200 & -120 \\ -300 & 200 & 120 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -75 & 50 & 30 \\ 0 & 9 & -15 \end{vmatrix}$$

$= -1020i - 1125j - 675\vec{k} \quad \therefore \ a_b = -1{,}02i - 1{,}125j - 0{,}675k$

Exercício 15.8) (Beer)

$w_{ABC} = 9 rad/s$

$A = 100j$

$B = 175i + 100k$

$C = 350i - 100j + 200k$

$\vec{e} = \frac{(A-C)}{\|(A-C)\|} = \frac{-350i + 200j - 200k}{450}$

$\vec{e} = \frac{-7i + 4j - 4k}{9}$

$\vec{w} = -7i + 4j - 4k$

$\vec{v_F} = \vec{w}_{ABC} \wedge (A-F)$

$\vec{v_F} = (-7i + 4j - 4k) \wedge (0i + 100j - 200k)$

$$\vec{v_F} = \begin{vmatrix} i & j & k \\ -7 & 4 & -4 \\ 0 & 100 & -200 \end{vmatrix}$$

$\vec{v_F} = -400i - 1400j - 700k$

$\therefore \ \boxed{v_F = -0{,}4i - 1{,}4j - 0{,}7k}$

$a_F = \cancel{\alpha \wedge r}^{0} + w(\underbrace{w \times r}_{}) \quad w_F$

$$a_F = \begin{vmatrix} i & j & k \\ -7 & 4 & -4 \\ -0{,}4 & -1{,}4 & -0{,}7 \end{vmatrix} = -8{,}4i - 3{,}3j + 11{,}4k$$

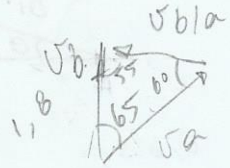
$\therefore \boxed{a_F = -8{,}4i - 3{,}3j + 11{,}4k}$

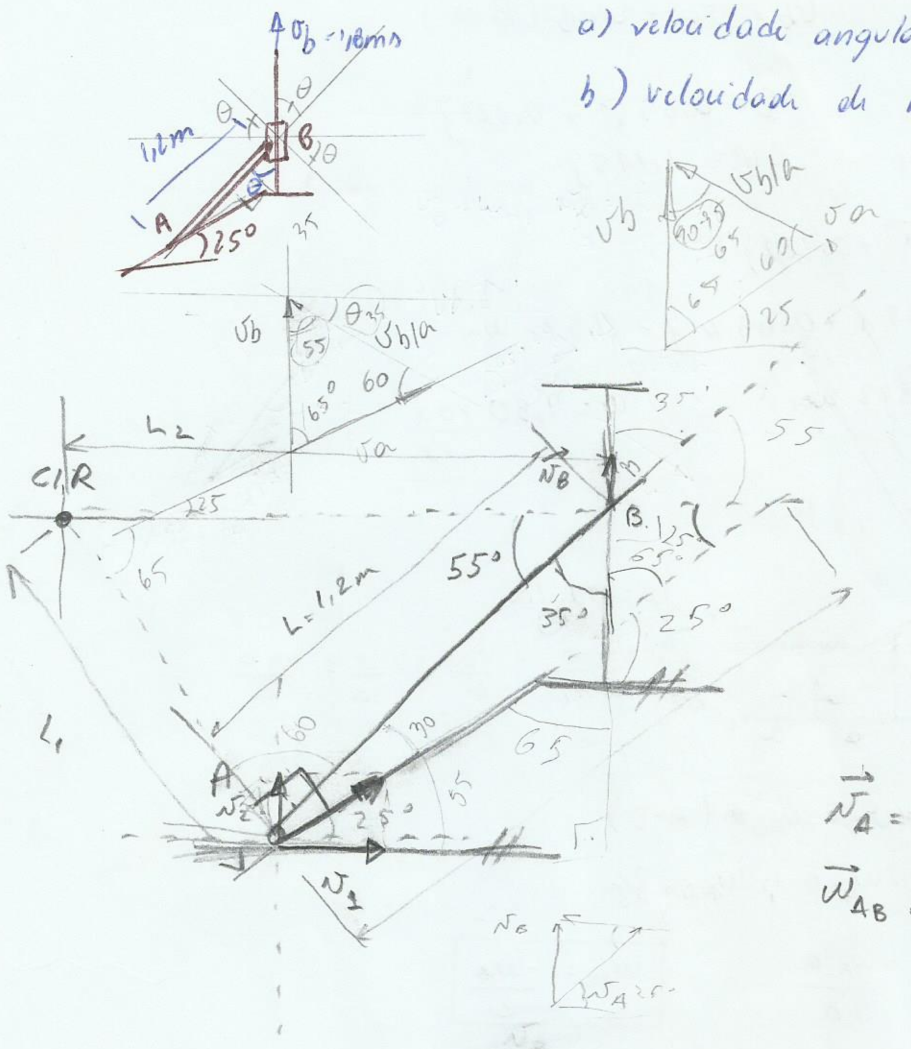
Exercício 15.32) (Beer)

Para $\theta = 35$

a) velocidade angular AB

b) velocidade de A





$\frac{1{,}8}{\text{sen}60} = \frac{v_a}{\text{sen}55} = \frac{v_{b/a}}{\text{sen}65}$

$v_{b/a} = 1{,}88 \ m/s$

$v_{b/a} = w_{ab} \times r$

$\vec{v}_B = 1{,}8\vec{j} \ m/s$

$\vec{v}_A = v_1\vec{i} + v_2\vec{j}$

$\vec{w}_{AB} = w\vec{k}$

$\frac{L_1}{\text{sen}55} = \frac{1{,}2}{\text{sen}65}$

$\therefore L_1 = 1{,}085 \ mm$

$L_1 = 0{,}458i - 0{,}98\vec{j} \ m$

$v_A = \cancel{v_{CIR}}^{0} + w_{AB} \wedge \overbrace{(A - CIR)}^{L_1}$

$v_A = \cancel{v_B}^{1{,}6j} + w_{AB} \wedge \underbrace{(A - B)}_{L}$

$w_{AB} \wedge (0{,}458i - 0{,}98j) = 1{,}8j + w_{AB} \wedge (-0{,}688i - 0{,}98j)$; $\vec{w}_{AB} = w_{AB}k$

$0{,}458j\, w_{AB} + 0{,}98i\, w_{AB} = 1{,}8j - 0{,}688j\, w_{AB} + 0{,}98w_{AB}i$

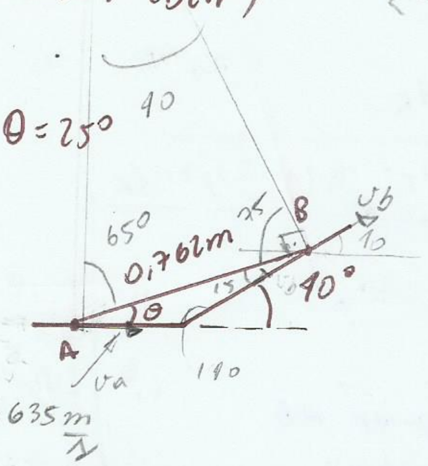
$0{,}458j\, w_{AB} = 1{,}8j - 0{,}688j\, w_{AB}$

$w_{AB} = 1{,}57$

Exercício 15.33) (Beer)

$i \; j \; k \; i \; j$

$\theta = 25^\circ$



$$\frac{CIR}{\operatorname{sen}75} = \frac{0,762}{\operatorname{sen}40} \quad \therefore CIR = 1,145m$$

$$\vec{v_b} = v_b\cos 40 + v_b \operatorname{sen} 40$$

$$v_b = v_{CIR} + \omega_{AB}\,(b - CIR)$$

$$v_b = v_a + \omega_{AB}\,(B - A)$$

$$B = 0,69\vec{i} + 0,322j$$

$$CIR = 1,145j$$

$$v_b = 0 + \omega k \wedge (0,69i - 0,82j)$$

$$v_b = 0,635i + \omega k \wedge (0,69\vec{i} + 0,322j)$$

$$0,69\omega j + 0,82\omega i = 0,635i + 0,69\omega j - 0,322\omega i$$

$$i: \quad 0,82\omega = 0,635 - 0,322\omega \quad \therefore \quad \boxed{\omega = 1,80 \, \frac{rad}{s}}$$

Exercício 15.38) (Beer)

$\frac{1}{2}\omega_A \quad \frac{1}{4}\omega_a$

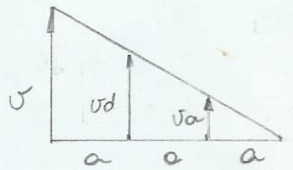
$v = \omega r$

$v_A = \omega_A . a$

$$\frac{v_d}{2a} = \frac{v_a}{a}$$

$\therefore v_d = 2v_a$

$v_d = 2\omega_A a$



$\omega = v_d . 2a$

$$\frac{v}{3a} = \frac{v_a}{a} \Rightarrow \frac{v}{3a} = \frac{\omega_A . a}{a}$$

$v = 3\omega_A a$

$$v_A = v_D + \omega_{AD} \wedge (A - D)$$

$$\omega_A . a = 2\omega_A . a + \omega_{AD} \times 2a$$

$$\omega_{AD} = \frac{-\omega_A . a}{2a} \quad \therefore \quad \boxed{\omega_{AD} = \frac{-\omega_A}{2}}$$

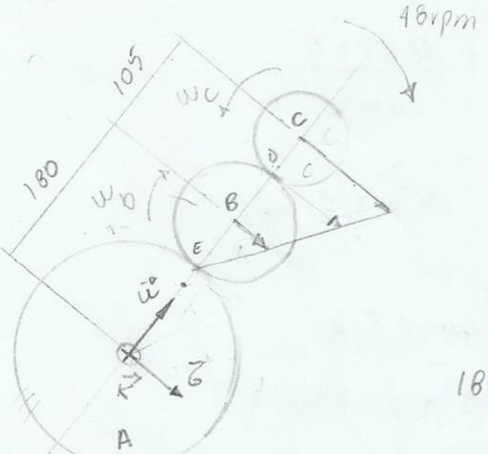
$$v_E = v_A + \omega_{EA} \wedge (E - A)$$

$$3\omega_A . a = \omega_A . a + \omega \times 2a$$

Exercício 15.42) (Beer)

$rpm = \frac{rev}{min} = \frac{2\pi}{60} \cdot \frac{\pi}{30} \frac{rad}{s}$

$w_{AC} = \frac{48\pi}{30} \therefore w_{AC} = 1{,}6\pi$



$w_{ABC} = 48 rpm$

$v_A = 0$

$v_B = 180\, w_{ABC}$

$v_C = 285\, w_{ABC}$

$v_B = v_E + w_B \wedge (B-E)$

$180 \times 48 = 0 + 60\, w_b \therefore \boxed{w_b = 144 rpm}$

$v_D = v_E + w_b \wedge (D-E)$

$v_D = 0 + 144\vec{k} \wedge 120\vec{u}$

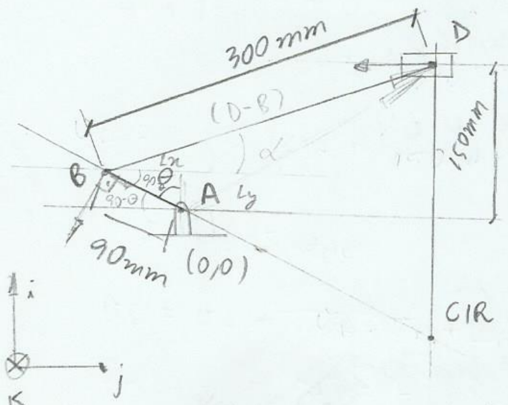
$v_D = 17280\,\vec{b}$

$v_C = v_D + w_C \wedge (C-D)$

$285 \times 48 = 17280 + 45\, w_C$

$\therefore \boxed{w_C = -80 rpm}$

Exercício 15.44) (Beer)



$w_{AB} = 200 rpm$

$\xrightarrow{+}$ $i \; j \; k \; i \; j$ $\xleftarrow{-}$

$(B-A) = \underbrace{90 \sin\theta}_{Lx}\, i + \underbrace{90\cos\theta}_{Ly}\, j$

$(D-B) = \underbrace{300 \cos\alpha}_{Rx}\, i + \underbrace{300 \sin\alpha}_{Ry}\, j$

$\alpha = \arcsin\left(\frac{150 - 90\cos\theta}{300}\right)$

$v_B = \cancel{v_A}^{\,0} + w_{BA} \wedge (B-A)$

$v_B = 200\,k \wedge (Lx_i + Ly_j)$

$v_B = 200\,Lx\,\vec{j} - 200\,Ly\,\vec{i}$

Para $\theta = 0$ $Lx = 0$ $Ly = 90$

$Rx = 293{,}94$ $Ry = 60$

Para $\theta = 90$ $Lx = 90$ $Ly = 0$

$Rx = 259{,}81$ $Ry = 150$

Para $\theta = 180$ $Lx = 0$ $Ly = -90$

$Rx = 180$ $Ry = 240$

$v_D = v_B + w_{DB} \wedge (D-B)$

$v_D = 200\,Lx\,\vec{j} - 200\,Ly\,\vec{i} + w\,k \wedge (Rx_i + Ry_j)$

$v_D = 200\,Lx\,\vec{j} + 200\,Ly\,\vec{i} + w\,Rx\,\vec{j} - w\,Ry\,i$

$v_D = (200\,Ly - w\,Ry)\vec{i} + (200\,Lx + w\,Rx)\vec{j}$

$1 rpm = \frac{\pi}{30} rad/s$

Para $\theta = 0$:

$200 \times 0 + w \times 293{,}94 = 0 \therefore$ $\boxed{w = 0}$

$v_D = 200 \times 90 = \frac{18000}{1000} \times \frac{\pi}{30} \therefore$ $\boxed{v_D = 1{,}88 m/s}$

Para $\theta = 90$:

$200 \times 90 + w \times 259{,}81 = 0 \therefore$ $\boxed{w = -7{,}26 \frac{rad}{s}}$

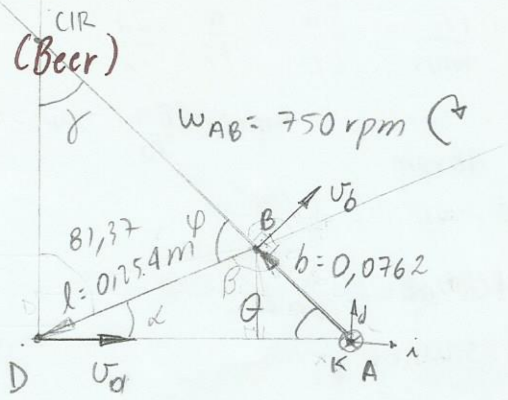
$v_D = 200 \times 0 + 7{,}26 \times \frac{150}{103}$ $\boxed{v_D = 1{,}09 m/s}$

Para $\theta = 180$:

$200 \times 0 + w \times 180 = 0$ $\boxed{w = 0}$

$v_D = 200 \times (-90) - 0 \times 240$ $\boxed{v_D = -1{,}88 \frac{m}{s}}$

15.46) (Beer)



$$\frac{0{,}254}{\operatorname{sen}\theta} = \frac{0{,}0762}{\operatorname{sen}\alpha}$$

p/ $\theta = 30 \to \alpha = 8{,}63$

$\beta = 180 - \alpha - \theta \quad \therefore \quad \beta = 141{,}37$

$\varphi = \dfrac{360 - 2\beta}{2} \therefore \varphi = 38{,}63°$ $\quad \gamma = 180 - (90 - \alpha) - \varphi = 60°$

$$\frac{0{,}254}{\operatorname{sen} 60} = \frac{(B - CIR)}{\operatorname{sen} 81{,}37} = \frac{(CIR - D)}{\operatorname{sen} 38{,}63}$$

$v_D = \cancel{v_{CIR}}^{\,0} + \omega_{DB} \wedge (D - CIR)$

$v_D = \omega k \wedge (-0{,}183 j) = 0{,}183\,\omega\, i \quad (1)$

$v_B = v_D + \omega_{DB} \wedge (B - D)$

$v_B = 0{,}183\,\omega\, i + \omega k \wedge (0{,}254 \cos\alpha\, i + 0{,}254 \operatorname{sen}\alpha\, j)$

$v_B = 0{,}183\,\omega\, i + 0{,}254 \cos\alpha\, \omega\, j - 0{,}254 \operatorname{sen}\alpha\, \omega\, i$

$v_B = \cancel{v_A}^{\,0} + \omega_{BA} \wedge (B - A)$

$v_B = -750\, k \wedge (0{,}0762 \cos 30\, i + 0{,}0762 \operatorname{sen} 30\, j)$

$v_B = -750 \times 0{,}0762 \cos 30\, \vec{j} + 750 \times 0{,}0762 \operatorname{sen} 30\, \vec{i}$

$-750 \times 0{,}0762 \cos 30 = 0{,}254 \cos\alpha\, \omega$

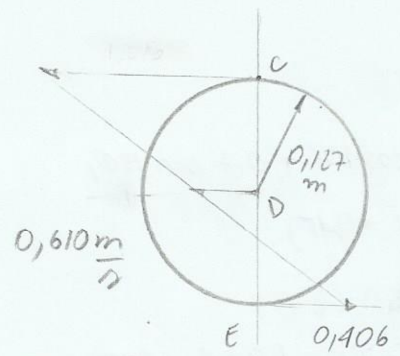
$$\boxed{\omega = -197{,}09\ rpm = -20{,}64\ \frac{rad}{s}}$$

$v_D = 0{,}183 \times 20{,}64$

$$\boxed{v_D = 3{,}78\ m/s}$$

Exercício 15.56) (Beer)

$\omega = 8 rad/s$



$v_E = v_D + \omega_{ED} \wedge (E-D)$

$v_E = -0,610 + 8 \times 0,127$

$\therefore \boxed{v_E = 0,406 \frac{m}{s}}$

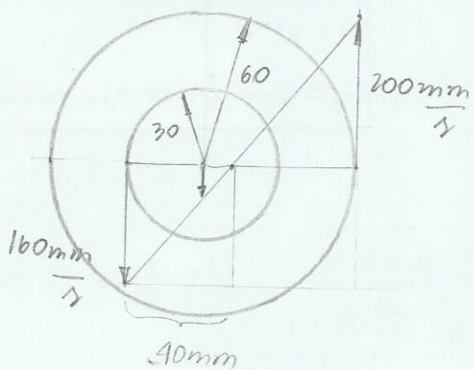
$v_C = v_D + \omega_{ED} \wedge (C-D)$

$v_C = 0,610 + 8 \times 0,127 = 1,626 \frac{m}{s}$

$$\frac{1,626}{0,127 + x} = \frac{0,610}{x} \Rightarrow 1,626x - 0,610x = 0,610 \times 0,127$$

$\therefore \boxed{x = 0,07625 \; m}$

Exercício 15.58) (Beer)

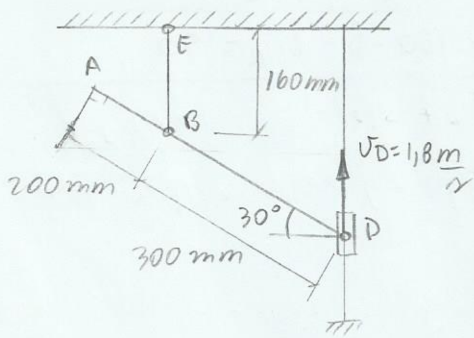


$$\frac{200+160}{60+30} = \frac{160}{CIR} \quad \therefore CIR = 40mm$$

$\therefore$ $\boxed{\text{10 mm para direita de A}}$

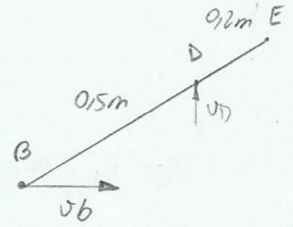
$$\frac{160}{40} = \frac{v_{bloco}}{10} \quad \therefore \boxed{v_{bloco} = 40 \frac{m}{s}}$$

Exercício 15.60) (Beer)



Exercício 15.64) (Beer)

$\xrightarrow{+}$ $i\ j\ k\ i\ j$ $\xleftarrow{-}$



$v_b = \omega_{BA} \times (B-A)$

$v_b = 5 \times 0{,}12 \therefore v_b = 0{,}6 \frac{m}{s}$

$(D-B) = 0{,}5 \cos 40 i + 0{,}5 \sin 40 j$

$(D-B) = 0{,}38 i + 0{,}32 j$

$v_D = v_B + \omega_{DB} \wedge (D-B)$

$v_D = 0{,}6 i + \omega k \wedge (0{,}38 i + 0{,}32 j)$

$v_D j = 0{,}6 i + 0{,}38 \omega j - 0{,}32 \omega i$

$0 = 0{,}6 - 0{,}32 \omega \therefore \boxed{\omega = 1{,}87 \frac{rad}{s}}$

$(E-B) = 0{,}7 \cos 40 i + 0{,}7 \sin (40) j$

$(E-B) = 0{,}54 i + 0{,}45 j$

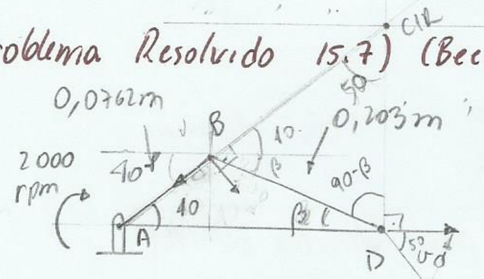
$v_E = v_B + \omega_{EB} \wedge (E-B)$

$v_E = 0{,}6 i + 1{,}87 k \wedge (0{,}54 i + 0{,}45 j)$

$v_E = 0{,}6 i + 1{,}87 \times 0{,}54 j - 1{,}87 \times 0{,}45 i$

$\therefore v_E = -0{,}24 i + j$

Problema Resolvido 15.7) (Beer)



$\alpha_{BD} = ?$ $a_D = ?$

$$\boxed{\begin{aligned} &a = \omega^2 r \\ &a_P = a_O + \dot{\omega} \wedge (P-O) + \omega \wedge [\omega \wedge (P-O)] \end{aligned}}$$

$rpm = \frac{\pi}{30} \frac{rad}{s}$

$a_b = \omega_{AB}^2 \cdot (B-A)$

$a_b = 2000^2 \times 0{,}0762 \times (\pi/32)^2$

$\therefore a_b = 3{,}34 \cdot 10^3 \ m/s^2$ e $\alpha_{AB} = 0$

$v_b = 2000 \frac{\pi}{30} \times 0{,}0762 = 15{,}96 \ m/s$

$v_B = 15{,}96 \cos 50 i - 15{,}96 \sin 50 j$

$\frac{0{,}203}{\sin 40} = \frac{0{,}0762}{\sin \beta} \therefore \beta = 13{,}96$

$\hat{B} = 180 - 40 - \beta \therefore \hat{\beta} = 126{,}04$

$\hat{D} = 90 - \beta = 76{,}03$

$\hat{B}_{CIR} = 40 + \beta = 53{,}96$

$CIR = 180 - \hat{D} - \hat{B}_{CIR} = 50$

$\frac{0{,}203}{\sin 50} = \frac{D-CIR}{\sin(40+\beta)} \therefore D-CIR = 0{,}214$

$v_D = v_{CIR} + \omega \wedge (D-CIR)$

$v_D i = 0 + \omega k \wedge (-0{,}214 j)$

$v_D i = 0{,}214 \omega i$

$v_D = v_B + \omega \wedge (D-B)$

$0{,}214 \omega i = 15{,}96 \cos 50 i + 15{,}96 \sin 50 j + \omega k \wedge (0{,}203 \cos \beta i - 0{,}203 \sin \beta j)$

$0{,}214 \omega = 15{,}96 \cos 50 + 0{,}203 \omega \sin \beta \therefore \omega = 62{,}06$

$0 = 15{,}96 \sin 50 + 0{,}203 \omega \cos \beta \therefore \omega = 62{,}06$

$$a_D = a_B + \dot{\omega} \wedge (D-B) + \omega \wedge [\omega \wedge (D-B)]$$

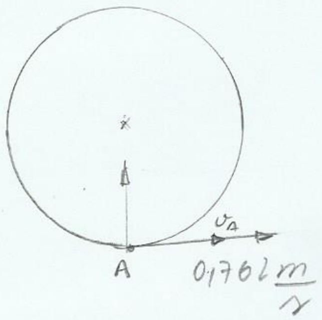
$$a_D = 3{,}34.10^3 (-\cos 40 i - \operatorname{sen} 40 j) + 62{,}06_k \wedge [62{,}06_k \wedge (0{,}203 \cos\beta i - 0{,}203 \operatorname{sen}\beta j)]$$

$$a_D = -3{,}34.10^3 \cos 40 + 62{,}06^2 . 0{,}203 \cos\beta + 2 \times 62{,}06 \times$$

Exercício 15.82) (Beer)

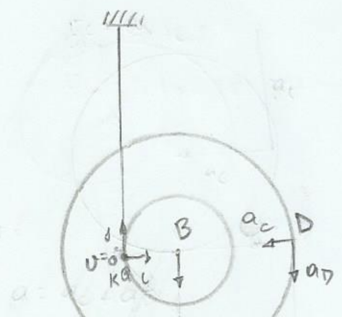
$\alpha_A = 7 \frac{rad}{s^2}$

$$a_c = \frac{v^2}{R} = \frac{(\omega R)^2}{R} = \omega^2 R$$



$$a_A = a_t + a_c$$

Exercício 15.84) (Beer)



$$\boxed{\begin{aligned} a_B &= a_A + a_{B/A} \\ &= a_A + (a_{B/A})_n + (a_{B/A})_T \\ a_B &= a_A + \alpha k \times r_{B/A} - \omega^2 r_{B/A} \end{aligned}}$$

$v_b = \omega . r_b$

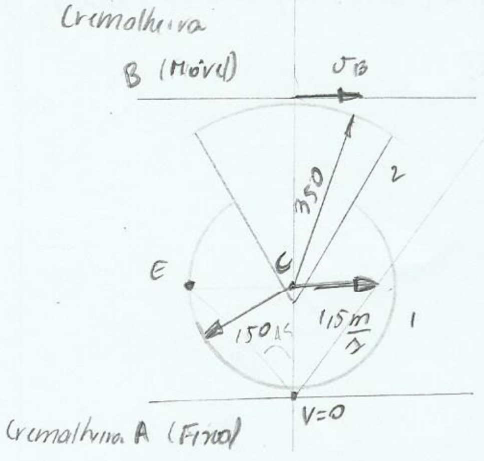
$0{,}6 = \omega . 0{,}08$

$\therefore \omega = 7{,}5 \frac{rad}{s}$

$v_b = 0{,}6 m/s$

$a_b = 2{,}4 m/s^2$

Questão 01) (27-03-2013)



$V_C = V_{CIR} + \omega_1 \wedge (C - CIR)$

$1{,}5\,i = 0 + \omega_1 k \wedge (0{,}15\,j)$

$1{,}5\,i = -\omega_1 \times 0{,}15\,i$ $\quad \therefore \boxed{\omega_1 = -10 \,\frac{rad}{s}}$

$v_B = v_{CIR} + \omega_1 \wedge (B - CIR)$

$v_B = -10\,k \wedge (0{,}5\,j)$

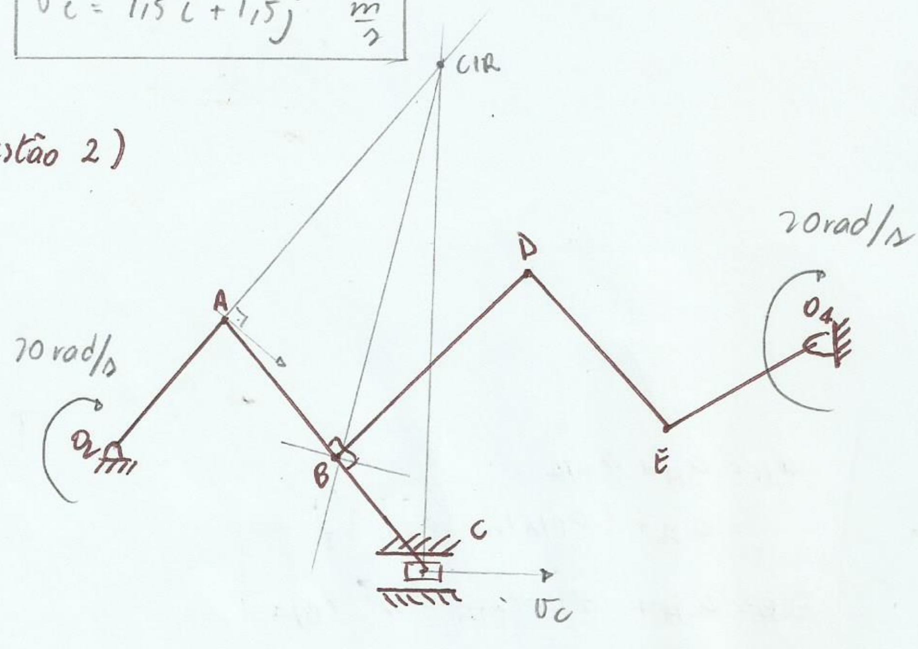
$\boxed{v_B = 5\,i \ m/s}$

$v_e = v_{CIR} + \omega_1 \wedge (E - CIR)$

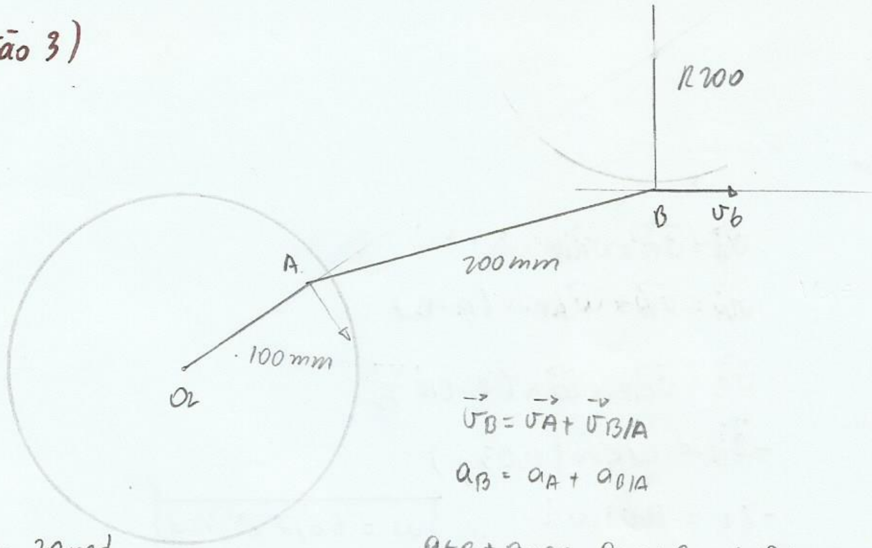
$v_e = -10\,k \wedge (-0{,}15\,i + 0{,}15\,j)$

$\boxed{v_e = 1{,}5\vec{i} + 1{,}5\vec{j} \quad \frac{m}{s}}$

Questão 2)



Questão 3)



$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

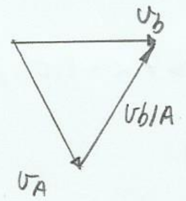
$a_B = a_A + a_{B/A}$

$a_{tB} + a_{nB} = a_{tA} + a_{nA} + a_{tB/A} + a_{nB/A}$

$\omega_2 = 20 \frac{rad}{s}$ $\alpha_2 = 80 \frac{rad}{s^2}$

$v_A = \omega_2 \cdot r_2$

$v_a = 20 \times 0{,}1 \therefore v_a = 2\,m/s$



Questão 1) P1-ME7810-2°2011

$i\,j\,k\,i\,j$ (+ / −)

$v_A = v_C + \omega \wedge (A-C)$

$0{,}25i = -0{,}15i + \omega k \wedge (0{,}15j)$

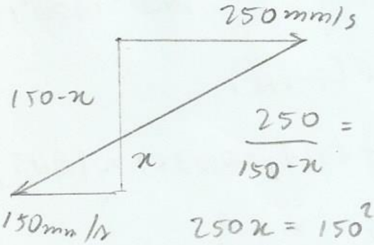
$0{,}25i = -0{,}15i + 0{,}15(-\omega)i$

$\therefore \omega = -\frac{8}{3}k = -2{,}67k \left(\frac{rad}{s}\right)$

$a_B = a_A + \vec{\alpha} \wedge (B-A) + \omega \wedge [\omega \wedge (B-A)]$

Aceleração angular

$\vec{a}_t = \vec{\alpha} \wedge \vec{r}_{A/CIR}$



$\frac{250}{150-x} = \frac{150}{x}$

$250x = 150^2 - 150x$

$\therefore x = 56{,}25$

$-0{,}9i = \vec{\alpha} \wedge (150-56{,}25) \cdot 10^{-3}$

$\therefore \vec{\alpha} = 9{,}6K$

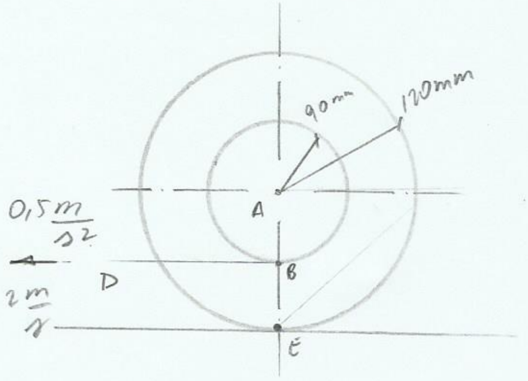
$\vec{a}_B = \cancel{\vec{a}_{CIR}}^{=0} + \vec{\alpha} \wedge (B-CIR) + \vec{\omega} \wedge [\vec{\omega} \wedge (B-CIR)]$

$= 9{,}6K \wedge (-0{,}15i + 0{,}094j) + (-2{,}67)k \wedge [(-2{,}67k) \wedge (-0{,}15i + 93{,}75j)]$

$= -1{,}44j - 0{,}9i + 1{,}07i - 0{,}67j$

$\therefore \boxed{\vec{a}_B = 0{,}17i - 2{,}11j} \ \frac{m}{s^2}$

Questão 1) (19/09/2012) (Prova)



$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$
$$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \wedge (A-B)$$
$$\vec{v}_B = \vec{v}_{CIR} + \vec{\omega} \wedge (B - CIR)$$
$$-2i = \omega k \wedge (0{,}03j)$$
$$-2i = -0{,}03\omega i \quad \therefore \boxed{\omega = 66{,}7\vec{k}\ \frac{rad}{s}}$$

$$\vec{v}_A = -2i + 66{,}7k \wedge (0{,}09j)$$
$$\vec{v}_A = -2i - 6i \quad \therefore \boxed{\vec{v}_A = -8i\ \frac{m}{s}}$$

$$\vec{a}_B = \vec{a}_{CIR} + \vec{\alpha} \wedge (B - CIR) + \vec{\omega} \wedge [\vec{\omega} \wedge (B - CIR)]$$

$$\vec{a}_t = \vec{\alpha} \wedge (B - CIR)$$
$$-0{,}5i = \alpha k \wedge 0{,}03j$$
$$\therefore \alpha = 16{,}67k\ \frac{rad}{s^2}$$

$$\vec{a}_B = 16{,}67k \wedge 0{,}03j + 66{,}67k \wedge [66{,}67k \times 0{,}03j]$$
$$\vec{a}_B = -16{,}67 \times 0{,}03i - 66{,}67^2 \times 0{,}03j$$
$$\therefore \boxed{\vec{a}_b = -0{,}5i - 133{,}33j\ \frac{m}{s^2}}$$

Questão 01) (2º 2013)

$$\|\omega\| = 10\ rad/s$$

$$A - C = 500i - 300j - 400k$$

$$\vec{\omega} = \frac{(A-C)}{\|(A-C)\|}\omega = 10 \times \frac{500i - 300j - 400k}{\sqrt{500^2 + 300^2 + 400^2}} \quad \therefore \vec{\omega} = 7{,}07i - 4{,}24j - 5{,}67k$$

$$\vec{v}_F = \cancel{\vec{v}_{CIR}}^{\,0} + \vec{\omega} \wedge (F - CIR)$$

$$\vec{v}_F = (7{,}07i - 4{,}24j - 5{,}67k) \wedge (0{,}15j - 0{,}2k)$$

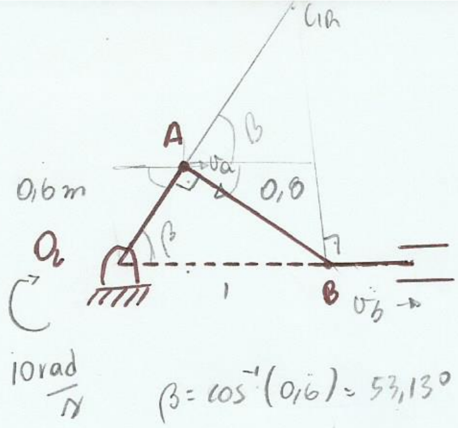
$$v_F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7{,}07 & -4{,}24 & -5{,}67 \\ 0 & 0{,}15 & -0{,}2 \end{vmatrix} = 1{,}98\vec{i} + 1{,}41\vec{j} + 1{,}06\vec{k}$$

$$\vec{a}_F = \cancel{\vec{a}_{CIR}}^{\,0} + \cancel{\vec{\alpha} \wedge (F - CIR)}^{\,0} + \vec{\omega} \wedge \overbrace{[\vec{\omega} \wedge (F - CIR)]}$$

$$\vec{a}_F = (7{,}07i - 4{,}24j - 5{,}67) \wedge (1{,}98i + 1{,}41j + 1{,}06k)$$

$$\therefore \boxed{a_F = 3{,}1i - 17{,}8j + 17{,}6k}\ \frac{m}{s^2}$$

$v = \omega r$



$\|v_a\| = \omega . r_a$

$\|v_a\| = 10 \times 0{,}6 = 6\ m/s$

$\beta = \cos^{-1}(0{,}6) = 53{,}13°$

$\cos\beta = \frac{1}{CIR-O} \therefore CIR-O = 1{,}67 = \frac{5}{3}$

$tag\beta = \frac{CIR-B}{1} \therefore CIR-B = 1{,}33 = \frac{4}{3}$

$CIR-A = \frac{5}{3} - 0{,}6 = 1{,}067 = \frac{16}{15}$

$v_a = \omega\, r_{a,cir}$

$6 = \omega \times \frac{16}{15} \therefore \omega = 5{,}63\, k$

$\vec{v}_b = \vec{v}_{cir} + \omega \wedge (B - CIR)$

$\vec{v}_b = 5{,}63\, k \wedge \frac{4}{3}(-j)$

$\therefore \boxed{\vec{v}_b = 7{,}5\, i\ \frac{m}{s}}$

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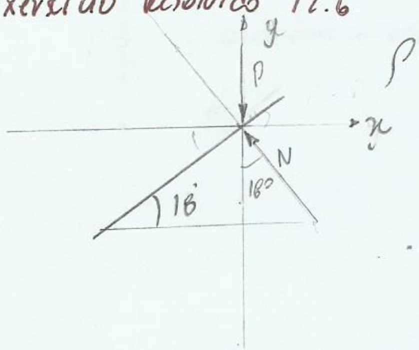
Exercício Resolvido 12.6



$\rho = 120\, m$

$y: \quad N\cos 18 - mg = 0 \quad \therefore N = \frac{mg}{\cos 18}$

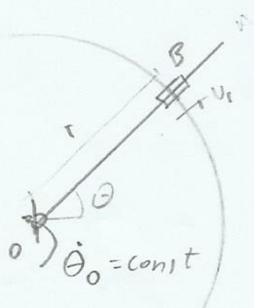
$x: \quad \sum \vec{F}_x = m\,\vec{a}$

$N \sin 18 = m . \frac{v^2}{\rho} \qquad \therefore v^2 = tg 18\, \rho . g$

$\frac{mg}{\cos 18} \sin 18 = \cancel{m} \frac{v^2}{\rho}$

Exercício Resolvido 12.7

$a = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{Radial} + \underbrace{(2\dot{r}\dot{\theta} + r\ddot{\theta})}$



$\sum F_r = m . a_r$

$0 = m(\ddot{r} - r\dot{\theta}^2)$

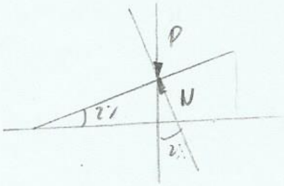
$\sum F_\theta = m . a_\theta$

$F = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$

Exercício 12.8)

(1)

$\Sigma F_n = m \cdot a \quad ; \; a = 0$



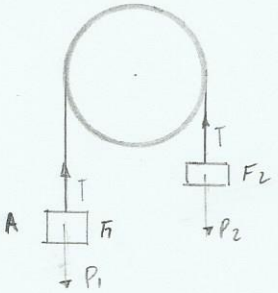
$F_c - N$

Exercício 12.16)

$$\begin{cases} F_1 - T = m_1 \cdot a \\ T - F_2 = m_2 \cdot a \end{cases}$$

$$F_1 - F_2 = a(m_1 + m_2)$$

$$F_1 - F_2 = a\left(\frac{F_1 + F_2}{g}\right)$$



$$\boxed{a = \frac{(F_1 - F_2)}{(F_1 + F_2)} \cdot g}$$

$$v^2 = v_0^2 + 2a\Delta s$$

$$\therefore \boxed{v^2 = 2a\Delta s}$$

$$v = v_0 + at$$

$$\boxed{v(t) = at}$$

Exercício 12.24)

$F_r = 2{,}50\,v^2$

$F_p = 50\,kN$

$$50 \cdot 10^3 - 2{,}50\,v^2 = 30 \cdot 10^3 a \quad ; \quad a = \frac{dv}{dt}$$

$$50 \cdot 10^3 - 2{,}50\,v^2 = 30 \cdot 10^3 \frac{dv}{dt}$$

$$\int dt = 30 \cdot 10^3 \int \frac{1}{50 \cdot 10^3 - 2{,}5\,v^2}\,dv$$

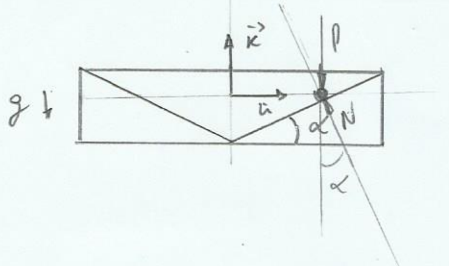
$$t - t_0 = 30 \cdot 10^3 \int \frac{1}{125\,000 - v^2}\,dv$$

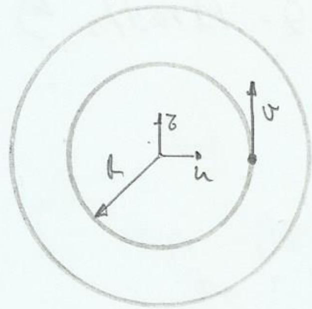
$$t = \frac{12000}{\sqrt{125000}} \cdot \ln\left(\frac{\sqrt{125\,000} + v}{\sqrt{125\,000} - v}\right)$$

$$\therefore \; t\left(270 \frac{Km}{h}\right) \; \therefore \; t\left(75 \frac{m}{s}\right) = 14{,}62\,s$$

Prova P1-2012

Questão 1)





Dados:
massa: m
R const
v: const

Eixo y: $N\cos\alpha - P = 0$; $P = mg$

$\therefore \boxed{N = \dfrac{m.g}{\cos\alpha}}$ (a)

Eixo x:

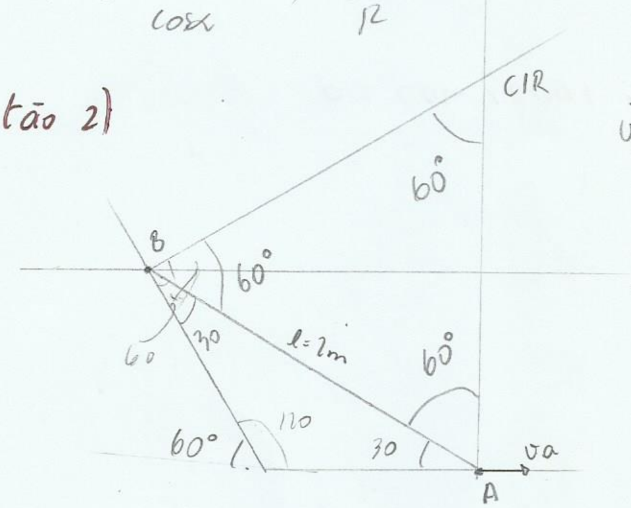
$N\sin\alpha = m.a$

$a = \ddot{r} - \dfrac{v^2}{R}$

$-N\sin\alpha = m\left(-\dfrac{v^2}{R}\right)$

$\therefore \cancel{m}g\dfrac{\sin\alpha}{\cos\alpha} = \cancel{m}\dfrac{v^2}{R}$ $\therefore v^2 = Rg\tan\alpha$

Questão 2)



$\dfrac{2}{\sin 60} = \dfrac{CIR-A}{\sin 60}$ $\therefore CIR-A = 2$

$\vec{v}_A = 3i$ m/s

$\vec{v}_A = \vec{v}_{CIR} + \vec{\omega} \wedge (A - CIR)$

$3i = \omega k \wedge (-2j)$

$3i = 2\omega i$ $\therefore \boxed{\vec{\omega} = 1{,}5k \text{ rad/s}}$ (a)

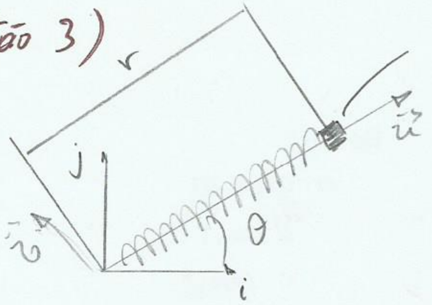
$\vec{v}_B = \vec{v}_{CIR} + \vec{\omega} \wedge (B - CIR)$

$\vec{v}_B = 1{,}5k \wedge (-2\cos 30 i - 2\sin 30 j)$

$\vec{v}_B = -3\cos 30 j + 3\sin 30 i$

$\therefore \boxed{\vec{v}_B = 1{,}5i - 2{,}6j \text{ (m/s)}}$ (b)

Questão 3)



$m = 0{,}2\,Kg$

$K = 100\,N/m$

$r_0 = 0{,}2$

$\dot{\theta} = 10\ rad/s$

$\vec{r} = r u$

$\dot{\vec{r}} = \dot{r} u + r\dot{\theta} u$

$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\vec{u} + (2\dot{r}\dot{\theta} + \cancel{r\ddot{\theta}}^{\,0})\vec{\tau}$

ΣF_u:

$m.\ddot{r} = -K.(r - r_0)$

$m[(\ddot{r} - r\dot{\theta}^2)u + 2\dot{r}\dot{\theta}\tau] = -K(r - r_0)\vec{u}$

$m\ddot{r} - mr\dot{\theta}^2 = -K.(r - r_0)$

$0{,}2\ddot{r} - 0{,}2 \cdot r.10^2 = -100(r - 0{,}2)$

$0{,}2\ddot{r} - 20r = -100r + 20$

$0{,}2\ddot{r} + 80r - 20 = 0 \quad \therefore \quad \ddot{r} + 400r - 100 = 0$

— // —

$\vec{r} = 0{,}25 + 0{,}1\cos(20t)$

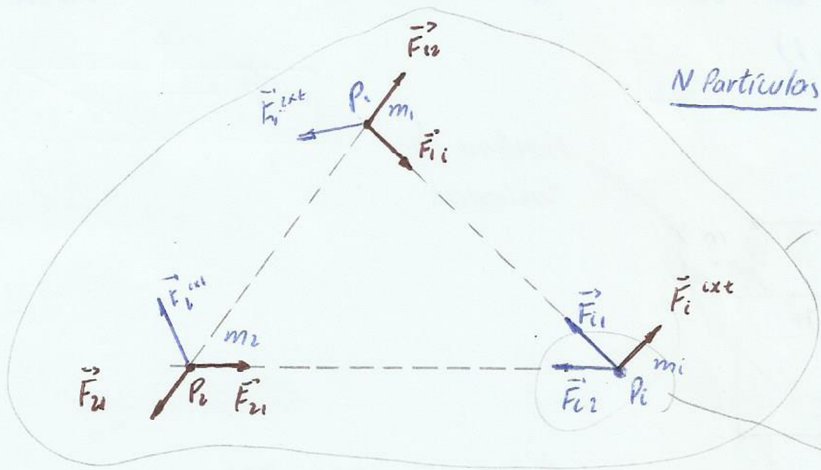
$\dot{\vec{r}} = -2\sin(20t)$

$\ddot{\vec{r}} = -40\cos(20t)$

Dinâmica dos Sistemas Materiais

Franco & Matsumura, 2011 Pag 173-184
e Pg 205-209

Dinâmica de um sistema de partículas (Teoria do Movimento do Baricentro)



Aplicando a 2ª lei de Newton para a partícula i:

$$m_i \vec{a_i} = \sum_{\substack{j=1 \\ (i\neq j)}}^{n} \vec{F_{ij}} + \vec{F_i}^{Ext}$$

Somando para todas as partículas:

$$\sum_{i=1}^{N} m_i \vec{a_i} = \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{n} \vec{F_{ij}}}_{\vec{0}} + \sum_{i=1}^{N} \vec{F_i}^{Ext}$$

A partir da definição do centro de massa G:

$$\sum_{i=1}^{N} m_i \vec{a_i} = \left(\sum_{i=1}^{N} m_i\right)\vec{a_G}$$

$$\therefore \underbrace{\left(\sum_{i=1}^{N} m_i\right)}_{m} \vec{a_G} = \sum_{i=1}^{N} \vec{F_i}^{ext}$$

$$(G-O) = \frac{1}{\sum_i^N m_i} \sum_{i=1}^{N} m_i (P_i - O)$$

$O \equiv G$

$$\vec{O} = \frac{1}{\sum_i^N m_i} \sum_{i=1}^{N} m_i (P_i - G)$$

$$\sum_{i=1}^{N} m_i (P_i - G) = 0$$

$$\sum_{i=1}^{N} m_i (\vec{a_i} - \vec{a_G}) = 0$$

$$\sum_{i=1}^{N} m_i \vec{a_i} = \left(\sum_{i=1}^{N} m_i\right)\vec{a_G}$$

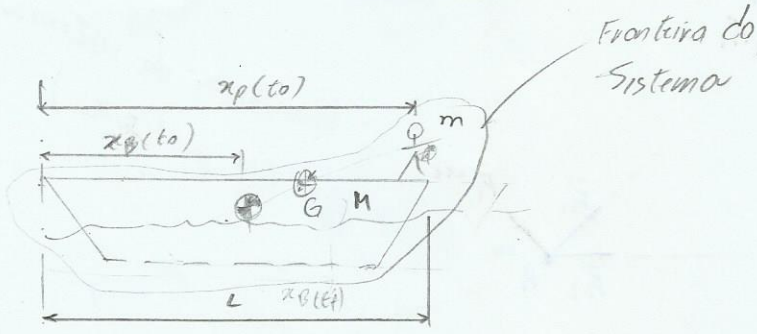
$$\boxed{m\vec{a_G} = \sum_{i=1}^{N} \vec{F_i}^{Ext}}$$ TMB (Teorema do Movimento do Baricentro)

Portanto, o baricentro G de qualquer sistema material move-se como se fosse um ponto material (em G), de massa igual a m, do sistema, sujeito à resultante das forças externas ao sistema.

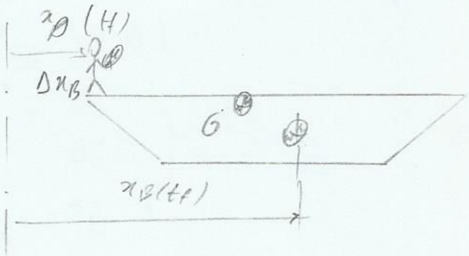
Nota: Este sistema de partículas pode constituir um corpo rígido, caso em que o lado direito do TMB corresponde à resultante das forças aplicadas ao corpo rígido.

$$\boxed{m\vec{a_G} = \vec{R}}$$

Exercício: Considere um barco de massa M e uma pessoa de massa m, a qual se encontra inicialmente na popa do barco. O sistema está inicialmente em repouso. Se a pessoa deslocar-se da popa para a proa, calcule o deslocamento do barco. (Obs.: Considere o atrito entre o barco e a água desprezível)



$$\Delta x_B = x_B(t_f) - x_B(t_0)$$



Analizando o problema na horizontal:

$$(M+m)\, a_{G,x} = F_x^{ext} \to 0 \quad \text{(atrito desprezível)}$$

$$a_{G,x} = 0$$

$$\int_{t_0}^{t_f} a_{G,x}\, dt = 0 \quad v_{G,x}(t_f) - \cancel{v_{G,x}(t_0)}^{\,0} = 0 \quad \rightarrow \quad x_G(t_f) = x_G(t_0)$$

$$x_G = \frac{1}{(M+m)} (M x_B + m x_p)$$

$$x_G(t_0) = \frac{1}{(M+m)} (M x_B(t_0) + m x_p(t_0))$$

$$x_G(t_f) = \frac{1}{(M+m)} (M x_B(t_f) + m x_p(t_f))$$

$$M x_B(t_f) + m x_p(t_f) = M x_B(t_0) + m x_p(t_0)$$

$$M \underbrace{(x_B(t_f) - x_B(t_0))}_{\Delta x_B} = m \underbrace{x_p(t_0)}_{L} - m \underbrace{x_p(t_f)}_{\Delta x_B}$$

$$(M+m)\, \Delta x_B = mL$$

$$\boxed{\Delta x_B = \frac{m}{M+m} L}$$

Teorema do momento Angular (TMA)

ou Teorema da variação da quantidade de movimento angular)

$\vec{p} = m\vec{v}$

$\frac{d\vec{p}}{dt} = \sum \vec{F_i} = R$

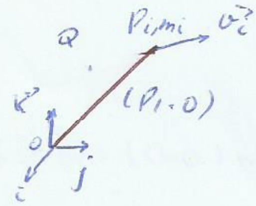
$m \cdot \vec{a}$

Quantidade de movimento angular:

N partículas

ou "l"

$$\vec{K_0} \triangleq \sum_{i=1}^{n} (P_i - O) \wedge m_i \vec{v_i}$$



Derivando no tempo

$$\frac{d\vec{K_0}}{dt} = \dot{\vec{K_0}} = \sum_{i=1}^{n} (\vec{v_i} - \vec{v_0}) \wedge m_i \vec{v_i} + \sum_{i=1}^{n} (P_i - O) \wedge m_i \vec{a_i}$$

$$\dot{\vec{K_0}} = \underbrace{\sum_{i=1}^{n} -\vec{v_0} \wedge m_i \vec{v_i}}_{\left(\sum_{i=1}^{n} m_i \vec{v_i}\right) \wedge \vec{v_0}} + \sum (P_i - O) \wedge m_i \vec{a_i}$$

$(a \wedge b = -b \wedge a)$

Mas:

$$\sum_{i=1}^{n} m_i \vec{v_i} = \left(\sum_{i=1}^{n} m_i\right) \vec{v_G}$$

$$m_i \vec{a_i} = \vec{F_i}^{ext} + \sum_{j=1}^{n} \vec{F_{ij}}$$

$$\dot{\vec{K_0}} = \left(\sum_i m_i\right) \vec{v_G} \wedge \vec{v_0} + \sum_{i=1}^{n} (P_i - O) \wedge \vec{F_i}^{ext} + \underbrace{\sum_{i=1}^{n} \sum_{j=1}^{n} (P_i - O) \wedge F_{ij}}_{\to 0}$$

$$\boxed{\dot{\vec{K_0}} = (\sum m_i) \vec{v_G} \wedge \vec{v_0} + \underbrace{\sum_{i=1}^{n} (P_i - O) \wedge \vec{F_i}^{ext}}_{\vec{M_0}^{ext}}}$$

TMA

$$\dot{\vec{K}}_0 = m \cdot \vec{v_G} \wedge \vec{v_0} + M_0^{ext}$$

Partículas (Importante!

c/ $\vec{K} = \sum_{i=1}^{n} (P_i - O) \wedge m_i \vec{v}$

Quantidade de movimento angular para um corpo rígido

$\vec{\omega}$

$$\vec{K_0} \triangleq \sum_{i=1}^{n} (P_i - O) \wedge m_i \vec{v_i}$$

$$\vec{v_i} = \vec{v_0} + \vec{\omega} \wedge (P_i - O) \quad \text{c/ } O \in \text{ao corpo rígido}$$

$$\Rightarrow \vec{K_0} = \sum_{i=1}^{n} (P_i - O) \wedge m_i (\vec{v_0} + \vec{\omega} \wedge (P_i - O))$$

$$\vec{K_0} = \underbrace{\sum_{i=1}^{n} m_i (P_i - O) \wedge \vec{v_0}}_{\left(\sum_{i=1}^{n} m_i\right)(G-O)} + \underbrace{\sum_{i=1}^{n} m_i (P_i - O) \wedge \vec{\omega} \wedge (P_i - O)}_{\Sigma_2}$$

$$\Sigma_2 = \sum_{i=1}^{n} m_i (P_i - O) \wedge \vec{\omega} \wedge (P_i - O)$$

$$(P_i - O) = x_i \vec{i} + y_i \vec{j} + z_i \vec{k}$$

$$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

$$\Sigma_2 = [\vec{i}\ \vec{j}\ \vec{K}] \underbrace{\begin{bmatrix} \sum_i (y_i^2+z_i^2)m_i & -\sum_i x_i y_i m_i & -\sum_i x_i z_i m_i \\ -\sum_i x_i y_i m_i & \sum_i (x_i^2+z_i^2)m_i & -\sum y_i z_i m_i \\ -\sum_i x_i z_i m_i & -\sum_i y_i z_i m_i & \sum_i (x_i^2+y_i^2)m_i \end{bmatrix}}_{\substack{\text{Matriz de Inércia} \\ J_O}} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

$$\vec{K_O} = m(G-O)\wedge\vec{v_O} + [\vec{i}\ \vec{j}\ \vec{K}]\ J_O \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

c/ $J_O = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{xy} & J_y & -J_{yz} \\ -J_{xz} & -J_{yz} & J_z \end{bmatrix}$

Momentos de Inércia

$J_x = \sum_i (y_i^2+z_i^2)m_i$

$J_y = \sum_i (x_i^2+z_i^2)m_i$

$J_z = \sum_i (x_i^2+y_i^2)m_i$

Produtos de Inércia

$J_{xy} = \sum_i x_i y_i m_i$

$J_{xz} = \sum_i x_i z_i m_i$

$J_{yz} = \sum_i y_i z_i m_i$

Teorema do momento angular (TMA)

(Para corpo rígido)

$$\vec{\dot{K}_O} = m\vec{v_G}\wedge\vec{v_O} + \sum_{i=1}^{n} (P_i - O)\wedge \vec{F_i}^{Ext}$$

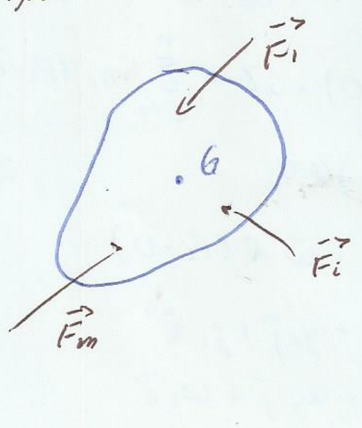
$$\vec{\dot{K}_O} = m(\vec{v_G}-\vec{v_O})\wedge\vec{v_O} + \frac{d}{dt}\left([\vec{i}\ \vec{j}\ \vec{K}]\ J_O \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}\right) + m(G-O)\wedge\vec{a_O}$$

$$\boxed{m(G-O)\wedge\vec{a_O} + \frac{d}{dt}\left([\vec{i}\ \vec{j}\ \vec{K}]\ J_O \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}\right) = \vec{M_O}^{Ext}}$$

TMA Geral p/ corpo rígido

$$\boxed{m\vec{a_G} = \vec{R}}$$

TMB



Teorema do Momento Angular (P/ corpo rígido)

$$m\,(G-O)\wedge\vec{a}_O+\frac{d}{dt}\left([\vec{i}\ \vec{j}\ \vec{k}]\ J_O\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}\right)=\vec{M}_O^{Ext}$$

Onde: $J_O=\begin{bmatrix}J_x & -J_{xy} & -J_{xz}\\ -J_{xy} & J_y & -J_{yz}\\ -J_{xz} & -J_{yz} & J_z\end{bmatrix}$ $(O\,\vec{i}\,\vec{j}\,\vec{k})$

$\dot{\vec{i}}=\vec{\omega}\wedge\vec{i}$

$\dot{\vec{j}}=\vec{\omega}\wedge\vec{j}$

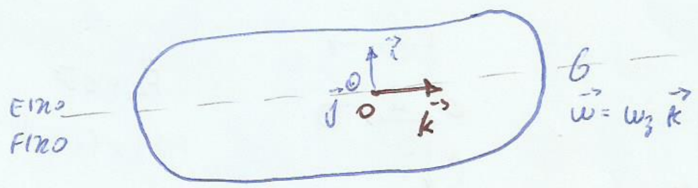
$\dot{\vec{k}}=\vec{\omega}\wedge\vec{k}$

Para $(O\vec{i}\vec{j}\vec{k})$ Fixa ao corpo rígido

$$m\,(G-O)\wedge\vec{a}_O+\vec{\omega}\wedge[\vec{i}\ \vec{j}\ \vec{k}]\ J_O\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}+[\vec{i}\ \vec{j}\ \vec{k}]\ J_O\begin{bmatrix}\dot\omega_x\\ \dot\omega_y\\ \dot\omega_z\end{bmatrix}=M^{ext}$$

Rotação em torno de um eixo fixo $(O\vec{k})$

c/ $(O\vec{i}\vec{j}\vec{k})$ Base solidária ao ~~Fixa~~ corpo rígido



Note

$J_z=\sum_i m_i(x_i^2+y_i^2)$

$J_z=\sum_i m_i\,d_i^2$

$J_z\int_V m(x_i^2+y_i^2)\,dm$

$\vec{a}_O=\vec{0}$

$\omega_x=\omega_y=0$; $\dot\omega_x=\dot\omega_y=0$

$$\omega_z\vec{k}\wedge[\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix}J_x & -J_{xy} & -J_{xz}\\ -J_{xy} & J_y & -J_{yz}\\ -J_{xz} & -J_{yz} & J_z\end{bmatrix}\begin{bmatrix}0\\0\\ \omega_z\end{bmatrix}+[\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix}J_x & -J_{xy} & -J_{xz}\\ -J_{xy} & J_y & -J_{yz}\\ -J_{xz} & -J_{yz} & J_z\end{bmatrix}\begin{bmatrix}0\\0\\ \dot\omega_z\end{bmatrix}=\vec{M}_O^{ext}$$

$$[\omega_z\vec{j}\ \ -\omega_z\vec{i}\ \ 0]\begin{bmatrix}-J_{xz}\,\omega_z\\ -J_{yz}\,\omega_z\\ J_z\,\omega_z\end{bmatrix}+[\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix}-J_{xz}\,\dot\omega_z\\ -J_{yz}\,\dot\omega_z\\ J_z\,\dot\omega_z\end{bmatrix}=\vec{M}_O^{Ext}$$

Em $\vec{i}$:

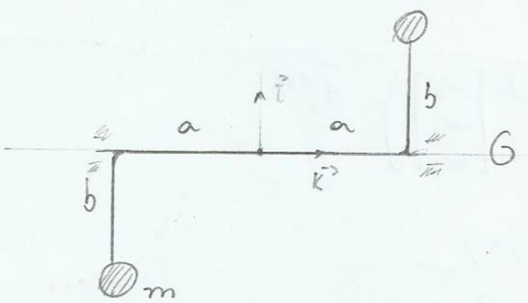
$$J_{yz}\,\omega_z^2-J_{xz}\,\dot\omega_z=M_{Ox}^{Ext}$$

Em $\vec{j}$:

$$-J_{xz}\,\omega_z^2-J_{yz}\,\dot\omega_z=M_{Oy}^{Ext}$$

Em $\vec{k}$:

$$J_z\,\dot\omega_z=M_{Oz}^{Ext}$$



$\vec{\omega} = \omega_z \vec{k}$

mais difícil de rodar

Momento de Inércia

$J_x = \sum_i m_i (y_i^2 + z_i^2) = 2ma^2$

$J_y = \sum_i m_i (x_i^2 + z_i^2) = 2m(a^2+b^2)$

$J_z = \sum_i m_i (x_i^2 + y_i^2) = 2mb^2$

Produtos de Inércia

$J_{xy} = \sum_i m_i x_i y_i = 0$

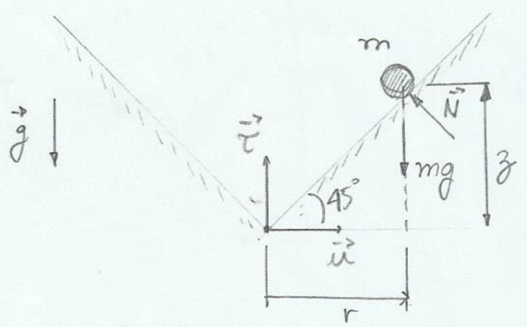
$J_{yz} = \sum_i m_i y_i z_i = 0$

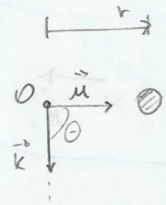
$J_{xz} = \sum_i m_i x_i z_i = 2mab$

$-2mab\,\dot{\omega}_z = M_{Ox}$; $-2mab\,\omega_z^2 = M_{Oy}$; $2mb^2\,\dot{\omega}_z = M_{Oz}$

Exercício Programa 3

- Usar coord. cilíndrica





$\vec{F}_{at} = c\vec{v}$

$[N] = [c]\left[\frac{m}{s}\right]$

Como rampa = 45° → r = z

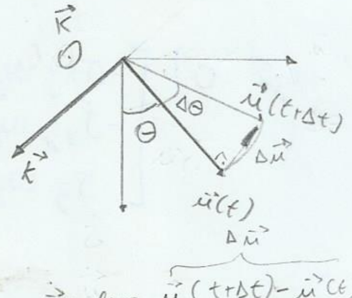
Posição da bola: $f(r, \theta)$

Aplicando 2ª Lei de Newton em m:

$m.\vec{a} = \sum F$

$m.\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{at}$

$m.\vec{a} = -mg\vec{z} - \frac{\sqrt{2}}{2}N\vec{u} + \frac{\sqrt{2}}{2}N\vec{z} - c\vec{v}$



Posição da bolinha:

$\vec{r} = r\vec{u} + z\vec{z}$; $r = z$

$\vec{r} = r\vec{u} + r\vec{z}$

Velocidade: $\dot{\vec{r}} = \dot{r}\vec{u} + r\dot{\vec{u}} + \dot{r}\vec{z} + r\dot{\vec{z}}$

$\dot{\vec{r}} = \dot{r}\vec{u} - r\dot{\theta}\vec{k} + \dot{r}\vec{z}$

aceleração: $\vec{a} = \ddot{r}\vec{u} + \dot{r}\dot{\vec{u}} - \dot{r}$

$\dot{\vec{u}} = \lim_{\Delta t \to 0} \frac{\overbrace{\vec{u}(t+\Delta t) - \vec{u}(t)}^{\Delta\vec{u}}}{\Delta t}$

$\dot{\vec{u}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = -\dot{\theta}\vec{k}$

$\dot{\vec{z}} = 0$ $\dot{\vec{k}} = \dot{\theta}\vec{u}$

$\omega_0 = \sqrt{\frac{g}{R}}$

$0 = \frac{g}{R} + r$

$$\omega_0 = \sqrt{\frac{g}{r}}$$

$$\theta = \frac{g}{r} + r$$

Em u:

$$N = \left(-c\dot{r} - m(\ddot{r} - r\dot{\theta}^2)\right) \times \frac{2}{\sqrt{2}}$$

em $\vec{z}$: $N = \left(m\ddot{r} + c\dot{r} + mg\right) \times \frac{2}{\sqrt{2}}$

$$(m \cdot \ddot{x} - mg + c \cdot \dot{x}) \cdot \frac{2}{\sqrt{2}}$$

$$m\ddot{r} - mr\dot{\theta}^2 = \frac{-N\sqrt{2}}{2} - c\dot{r}$$

$$N = (-c\dot{r} - m\ddot{r} + mr\dot{\theta}^2) \times \frac{2}{\sqrt{2}}$$

$$m(\ddot{r} - r\dot{\theta}^2) - m\ddot{r} = mg - N\sqrt{2}$$

$$N = \frac{m(g + \ddot{r} - \ddot{r} + r\dot{\theta}^2)}{\sqrt{2}} \qquad N = \frac{m(g + r\dot{\theta}^2)}{\sqrt{2}}$$

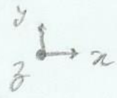
Dinamica dos Sistemas

~~Teorema do Centro de Massa (TCM)~~

Quantidade de movimento $L = \sum_{i=1}^{n} m_i v_i$

Momento Angular $H_O = \sum_{i=1}^{n} (r_i \times m_i v_i)$

Problema Resolvido 14.1



$$\left[\begin{array}{l} t=0 \text{ e } (x,y,z)=(0,0,0) \\ m = 200\,Kg \text{ e } v_0 = 150\,\frac{m}{s}\ (\vec{i}) \end{array}\right]$$ Instante Inicial

Depois da explosão $t = 2{,}5\,s$

$P_A\ (555, -180, 240)$	$m_A = 100\,Kg$
$P_B\ (255, 0, -120)$	$m_B = 60\,Kg$
$P_C\ (\ ?\ )$	$m_C = 40\,Kg$

Atraves da conservação de Quantidade de Movimento

$$L = \sum_{i=1}^{n} m_i v_i$$

$$200 \times 150\,\vec{i} = 100 \times \left(\frac{555\,\vec{i} - 180\,\vec{j} + 240\,\vec{k}}{2{,}5}\right) + 60\left(\frac{255\,\vec{i} - 120\,\vec{k}}{2{,}5}\right) + 40\left(\frac{P_{cx}\,\vec{i} + P_{cy}\,\vec{j} + P_{cz}\,\vec{k}}{2{,}5}\right)$$

$$75000\,\vec{i} = (70\,800 + 40\,P_{cx})\,\vec{i} + (-18\,000 + 40\,P_{cy})\,\vec{j} + (16800 + 40\,P_{cz})\,\vec{k}$$

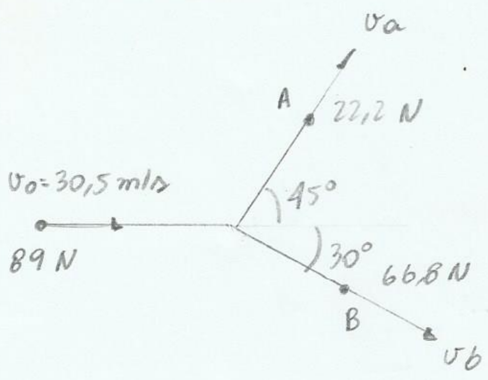
$$P_{cx} = \frac{75\,000 - 70\,800}{40} = 105\ m$$

$$P_{cy} = \frac{18\,000}{40} = 450\ m$$

$$P_{cz} = \frac{-16\,800}{40} = -420\ m$$

$$P_c\,(105, 450, -420)\ [m]$$

Problema Resolvido 14.2



Determinar va e vb

$F = m \cdot a \Rightarrow m = \frac{F}{a}, a = g$

$m v_0 = m_A v_a + m_B v_B$

$\frac{89}{g} \cdot 30{,}5 \vec{i} = \frac{22{,}2}{g} v_a (\cos 45 \vec{i} + \sin 45 \vec{j}) + \frac{66{,}8}{g} v_b (\cos 30 i - \sin 30 j)$

Eixo i: $2714{,}5 = 22{,}2 \cos 45\, v_a + 66{,}8 \cos 30\, v_b$

Eixo j: $0 = 22{,}2 \sin 45\, v_a - 66{,}8 \sin 30\, v_b$

$$v_a = \left\{ \frac{\begin{vmatrix} 22{,}2 \cos 45 & 66{,}8 \cos 30 \\ 22{,}2 \sin 45 & -66{,}8 \sin 30 \end{vmatrix}}{\begin{vmatrix} 2714{,}5 & 66{,}8 \cos 30 \\ 0 & -66{,}8 \sin 30 \end{vmatrix}} \right\}^{-1}$$

$v_a = 63{,}29$ m/s

$v_b = -29{,}75$ m/s

Questão 01) (Prova)

$\curvearrowleft i \; j \; k$

(a) $2m \cdot v = m_1 v_1 + m_2 v_2$

$v = \frac{m_1 v_1 + m_2 v_2}{2m} \therefore v = \frac{-1+3}{2} = 1 \frac{m}{s} (j)$

(b) $\vec{v_1} = 1\vec{i} - 0{,}73\vec{j}$ (Como não existe força externa v_a = const)

$2\cancel{m} v_0 = \cancel{m} v_1 + \cancel{m} v_2$

$2 \cdot 1j = 1\vec{i} - 0{,}73 j + v_2 \therefore v_2 = -1\vec{i} + 2{,}73\vec{j}$

(c) S/ força externa

$\vec{v_C}$ = const

$\vec{w}$ = const

$\vec{v_1} = \vec{v_2} + w \wedge (P_1 - P_2)$

$-j = 3j + w_k \wedge (-2i)$

$w = 2\vec{K}$

$\vec{v_1} = \vec{v_G} + w \wedge (P_1 - G)$

$\vec{v_1} = 1j + 2k \wedge (-\cos 60 \hat{i} - \sin 60 \hat{j})$

$\vec{v_1} = 1j - 1j + 1{,}732 i$

$v_1 = 1{,}732 i$ (m/s) //

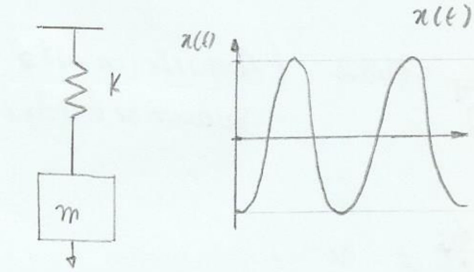
Vibrações em Sistemas Mecânicos

Frequência de oscilação $f = \frac{1}{T}$ $[f] = Hz$

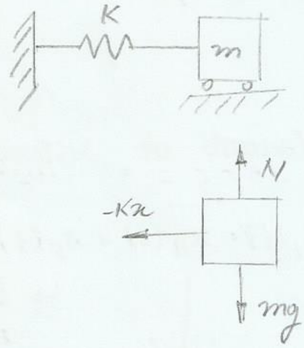
Frequência angular, ou de oscilação $\omega = 2\pi f = \frac{2\pi}{T}$ $[\omega] = rad/s$

Vibração Livre de Sistemas Não-Amortecidos

$x(t) = x_0 \cos(\omega t - \phi)$

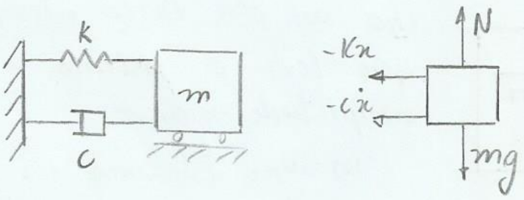


K: rigidez da mola



frequência natural da mola $\boxed{\omega_n = \sqrt{\frac{K}{m}}}$

$$x(t) = X_0 \cos(\omega t - \phi) \Rightarrow \boxed{\begin{array}{l} X_0 = \sqrt{x_0^2 + \left(\frac{x_0}{\omega_n}\right)^2} \\ \theta = \operatorname{arctg}\left(\frac{v_0}{x_0 \omega_n}\right) \end{array}}$$



$F_{amort}(t) = -c\dot{x}(t)$

c: coeficiente de amortecimento

$$m\ddot{x} = -Kx - c\dot{x}$$
$$m\ddot{x} + c\dot{x} + Kx = 0$$
$$\left.\begin{array}{l} x(t) = e^{\lambda t} \\ \dot{x}(t) = \lambda e^{\lambda t} \\ \ddot{x}(t) = \lambda^2 e^{\lambda t} \end{array}\right\} \rightarrow m\lambda^2 + c\lambda + K = 0$$

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mK}}{2m}$$

Coeficiente de amortecimento crítico

$$c^2 - 4mK = 0$$
$$\boxed{c = \sqrt{4mK}} \rightarrow c = \sqrt{2^2 \frac{m^2}{m} K}$$
$$c = 2m\sqrt{\frac{K}{m}}$$
$$c = 2m\omega_n$$

Fator de Amortecimento $\boxed{\zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega_n}}$

Portanto

$$\lambda_{1,2} = -\zeta w_n \pm w_n\sqrt{\zeta^2-1}$$

$\zeta < 1$ - Subamortecido
$\zeta = 1$ - Criticamente amortecido
$\zeta > 1$ - Superamortecido

1.5 Exercícios Propostos

Exercício 1)

Dados: $m = 1Kg$
$c = 2\ Kg/s$
$K = 10\ N/m$

$$\zeta = \frac{c}{C_{cr}} \ ; \ C_{cr} = \sqrt{4mK}$$

$$\zeta = \frac{2}{\sqrt{4\times1\times10}} = 0{,}32$$ (Amortecimento subamortecido)

Vibração Forçada de Sistemas amortecidos

$$x_t(t) = x_H(t) + x_p(t)$$

$x_p(t)$ → Estado estacionário ou vibração em regime

$x_H(t)$ → transiente ou vibração livre

$$x_p(t) = X_0 \cos(wt - \phi)$$

$$\boxed{X_0 = \frac{F_0}{\sqrt{(K-m\omega^2)+(cw)^2}}} \quad e \quad \boxed{\phi = arctg\left(\frac{cw}{K-m\omega^2}\right)}$$

Sabendo que $r = \frac{w}{w_n}$

$$\boxed{G = \frac{X_0}{X_{est}} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}}$$

A frequência de ressonância w_{res} é o valor de frequência de trabalho w da força externa $F(t)$ que leva o sistema a vibrar com amplitude máxima

$w = w_{res}$ (Sistema em ressonância)

Ressonância em Sistemas com $\zeta = 0$

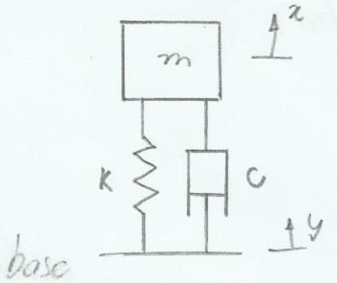
Para sistemas sem amortecimento a frequência de ressonância w_{res} é dada pela frequência natural w_n. Logo, a condição de ressonância $w = w_n$ é válida apenas para $\zeta = 0$. Amplitude tende ao infinito

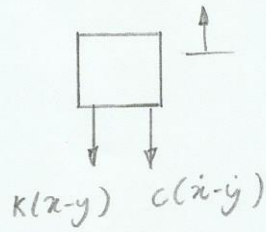
Ressonância em Sistemas com $\zeta \neq 0$

Para máximo valor do ganho G → $w = w_{res}$ → $r_{res} = \sqrt{1-2\zeta^2}$
(Sistema em ressonância) $w_{res} = w_n\sqrt{1-2\zeta^2}$

Caso $\zeta > \frac{\sqrt{2}}{2}$ então w_{res} não se aplica

Oscilação de Base





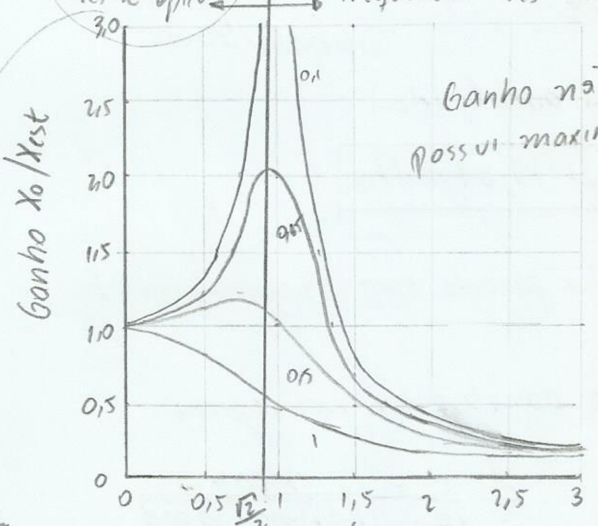
$$m\ddot{x} + c\dot{x} + Kx = Ky + c\dot{y}$$

Transmissibilidade de deslocamento

$$\boxed{Td = \frac{X_0}{Y_0} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}}$$

frequencia w_{res} se aplica

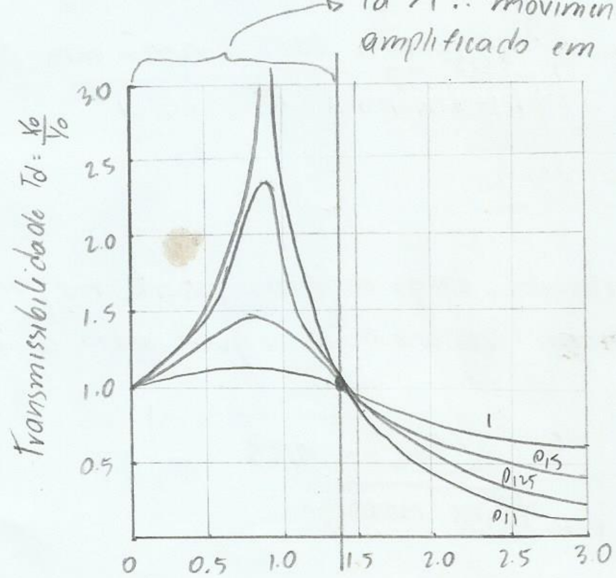
frequência w_{res} não se aplica



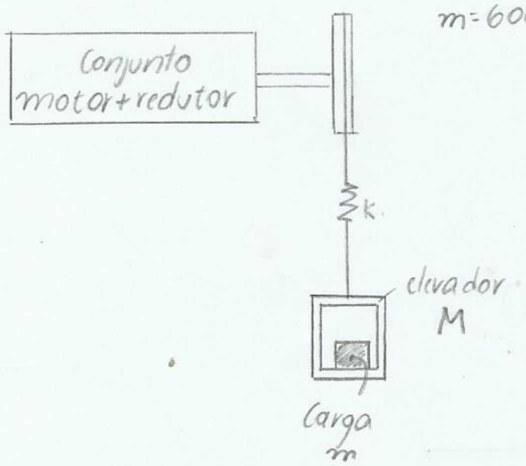
$w_{res} = w_n\sqrt{1-2\zeta^2}$

∴ Amplitude de vibração possui um valor máximo (limito)

Td >1 ∴ movimento da massa m está amplificado em relação ao movimento y(t)



Questão de Prova



$M = 600\,kg$ $\quad K_{max} = 240\,000\ N/m$ (Último andar)

$m = 600\,kg$ $\quad K_{min} = 30\,000\ N/m$ (térreo)

$c = 3\,000\ Ns/m$

(a) frequência de vibração do elevador

$$\xi = \frac{c}{C_{cr}} = \frac{c}{\sqrt{4mK}} = \frac{3\,000}{\sqrt{4 \times 600 \times 240\,000}} \therefore \xi = 0{,}125 < 1$$

$\therefore$ Subamortecida

$$w_a = w_n\sqrt{1-\xi^2} \quad \text{(frequencia natural amortecida)}$$

$$w_a = \sqrt{\frac{240\,000}{600}} \cdot \sqrt{1-0{,}125^2} \therefore \boxed{w_a = 19{,}84\ rad/s}$$

(b) Pessoa pula em 1Hz (Quando elevador parado com carga maxima)

$F(t) = 800\cos(2\pi f t)$

Determine a amplitude de vibração do elevador em regime.

$$w_n = \sqrt{\frac{30\,000}{1200}} = 5\ rad/s \qquad \xi = \frac{c}{C_{cr}} = \frac{3\,000}{\sqrt{4 \times 1200 \times 30\,000}}$$

$$r = \frac{w}{w_n} = \frac{2\pi}{5} \therefore r = 1{,}26\left(\frac{w}{w_n}\right) \qquad \therefore \xi = 0{,}25$$

$$x(t) = X_0\cos(wt - \phi)$$

$$X_0 = \frac{x_{est}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \; ; \; x_{est} = \frac{F_0}{K} = \frac{800}{30\,000} = 0{,}02\bar{6}\ mm$$

$$\boxed{X_0 = 0{,}031\,mm}$$

(c) Qual a frequencia para (elevador carga máxima parado no andar térreo) para que o elevador entre em ressonância e qual seria a amplitude de vibração correspondente?

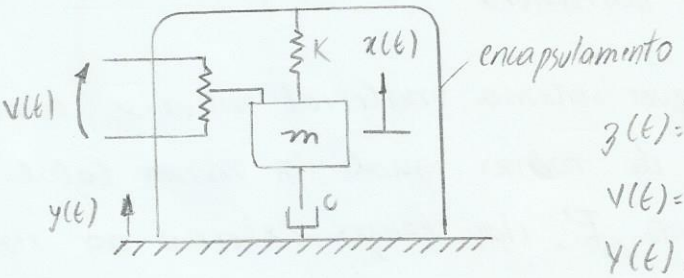
$$w_n = \sqrt{\frac{30\,000}{1200}} = 5\,rad/s \qquad \xi = \frac{3000}{\sqrt{4 \cdot 1200 \cdot 30000}} = 0{,}25$$

$$w_{res} = w_n\sqrt{1-2\xi^2}$$

$$w_{res} = 5\sqrt{1-2 \cdot 0{,}25^2}$$

$$\therefore w_{res} = 4{,}68\ rad/s$$

Questão de Prova

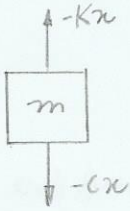


$z(t) = x(t) - y(t)$

$V(t) = b \cdot z(t)$

$Y(t) = Y_0 \cos(wt)$

(a)



$m\ddot{x} = -K(x-y) - c(\dot{x}-\dot{y})$

(b) $w < 0{,}2\, w_n$, Determine K/m, Para $w = 200 Hz$

$2\pi f < 0{,}2 \sqrt{\frac{K}{m}}$ $\therefore$ $\frac{K}{m} > 39\,478{,}42$

$\hookrightarrow 200\ Hz$

(c) $\zeta = 0{,}5$

$K = 4 \cdot 10^6\ N/m$

$m = 0{,}1\ Kg$

$Y_0 = 0{,}01 m$

Para $r_{ress} \approx 0{,}8$

$w = 0{,}8\, w_n$

$w = 0{,}8 \sqrt{\frac{4 \cdot 10^6}{0{,}1}}$ $\therefore w = 5\,059{,}65$

(d) $w = 0{,}1\, w_n$

$Y_0 = 0{,}01$

$V(t) = b(\,x(t) - y(t))$

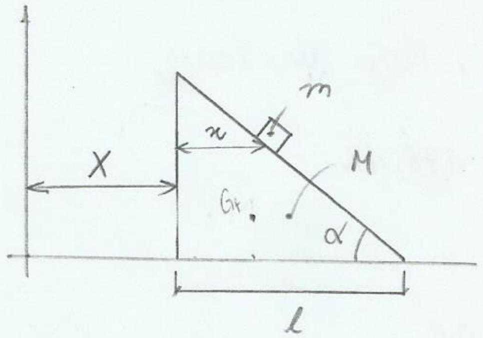
Cap 10 - Dinâmica dos Sistemas

Teorema do Movimento do Baricentro

"O baricentro G de qualquer sistema material move-se como se fosse um ponto material, de massa igual à massa total m, do sistema, sujeito à resultante, $\vec{F}$, das forças externas ao sistema

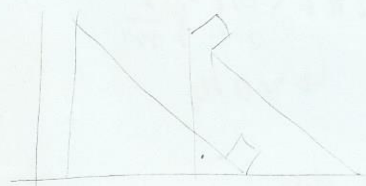
$$\boxed{m\,\vec{a}_G = \sum_i \vec{F}_e}$$

Exemplo 10.1



(a) Velocidade relativa v do ponto material

(b) Deslocamento d, quando atingir o plano



$$\sum F = (M+m)\,a_G = F_{ext} \quad , \quad F_{ext} = 0$$

$$a = 0$$

velocidade inicial nula

$$\int_{t_0}^{t} a_G\,dt = 0 \Rightarrow v_G(t) - \cancel{v_G(t_0)}^{\,\to 0} = 0 \;\therefore\; \text{O Baricentro não se move } (x_G(t) = x_G(t_0))$$

Através da conservação da quantidade de movimento

$$(M+m)\,x_G = M(b+X) + m(X+x)$$

$$x_G(t) = \frac{M(b+X) + m(X+x)}{M+m}$$

Como $x_G(t_f) = x_G(t_0)$

$$\cancel{Mb} + MX(t_f) + mX(t_f) + mx(t_f) = \cancel{Mb} + MX(t_0) + mX(t_0) + mx(t_0)$$

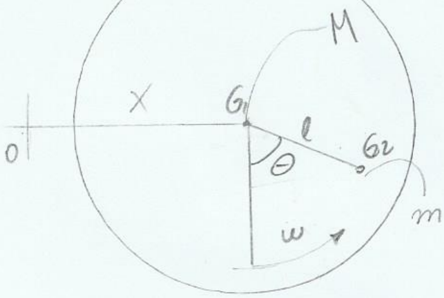
$$M(\underbrace{X(t_f) - X(t_0)}_{\Delta X}) = m(\underbrace{X(t_0) - X(t_f)}_{-\Delta X} + \underbrace{x(t_0) - x(t_f)}_{-\Delta x})$$

$$M\Delta X + m\Delta X = -m\Delta x$$

$$\boxed{\Delta X = \frac{-m\Delta x}{M+m}}$$

Para $\Delta x = l$ então $\boxed{d = \frac{-ml}{M+m}}$

Exemplo 10.2



$G_1 G_2 = l$

θ ângulo de rotação do rotor

ω velocidade angular

(1) Velocidade de deslocamento

(2) Deslocamento do estator

(3) Reação vertical na base

(4) Componente horizontal

$$MX + m(X + l\sin\theta) = ml$$

— // —

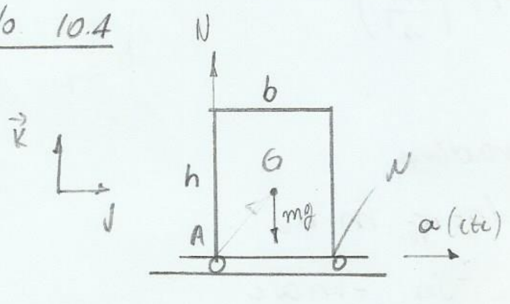
Teorema do Momento Angular

$$\vec{H}_O = \sum_i (P_i - O) \wedge m_i \vec{v}_i \quad \text{(Momento Angular)}$$

$$\vec{H}_O = \vec{M}_O^{ext} + m\vec{v}_G \wedge \vec{v}_O$$

Quando $O = G$ e $\vec{v}_i = \vec{0}$ $\therefore$ $\vec{H}_O = \vec{M}_O^{ext}$

Exemplo 10.4



Calcular valor máximo de a para
(1) sem escorregamento
(2) sem tombamento

$\overset{+}{\vec{\imath} \times \vec{\jmath}}$

$N = mg$

Caso o bloco esteja na iminência de escorregar, temos $F_{atr} = \mu N$

$F_{at} = m.a$

$\mu N = m.a$

$\mu mg = ma$

$\therefore \boxed{a = \mu g}$

Para a iminência do tombamento, calcula o momento angular em A.

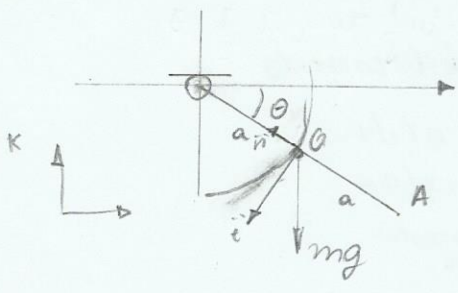
$$\vec{H}_A = (G - A) \wedge m.\vec{v}$$
$$= \left(\frac{b}{2}\vec{\jmath} + \frac{h}{2}\vec{k}\right) \wedge v\vec{\jmath} m = -\frac{mhv}{2}\vec{k}$$

$$\dot{\vec{H}}_A = -\frac{mha}{2}$$

$$\vec{M}_A = -\frac{mgb}{2}\vec{k} \quad \therefore \quad -\frac{mgb}{2} = -\frac{mha}{2} \quad \therefore \quad \boxed{a = \frac{gb}{h}}$$

Dinâmica dos Sólidos

Ex 12.1



$\vec{R} = T\vec{t} - N\vec{n}$

$m\vec{a} = -mg\vec{k} + \vec{R}$

$m\vec{a} = -mg\vec{k} + T\vec{t} + N\vec{n}$

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Questão 2 (Prova)

$M = 100\,kg$

$r = 0{,}3\,m \quad m = 5\,kg$

$J = 0{,}5\,kg\,m^2$

$F = -120\,i\,N$

(a) $(4m + M)\,\vec{a}_G = \vec{F}$

$(4 \times 5 + 100)\,\vec{a}_G = -120$

$\therefore \vec{a}_G = -1\,i \left(\frac{m}{s^2}\right)$

(b)

TMB no chassi

$-120 + 4\,fr = M.a_c$

TMA (Roda)

$-r\vec{j} \wedge fatr\,\vec{i} = J.\dot{\vec{\omega}}_r\,\hat{k}$

$r\,fatr = J.\dot{\omega}_r$

$\dot{\omega}_K \wedge r\vec{j} = -a_c\,\hat{i}$

$\dot{\omega} = \frac{-a_c}{r}$

$fat = \frac{J\dot{\omega}_r}{r}$

$fat = \frac{-Ja_c}{r^2}$

TMB na roda

$-fr + fatr = m.a_c$

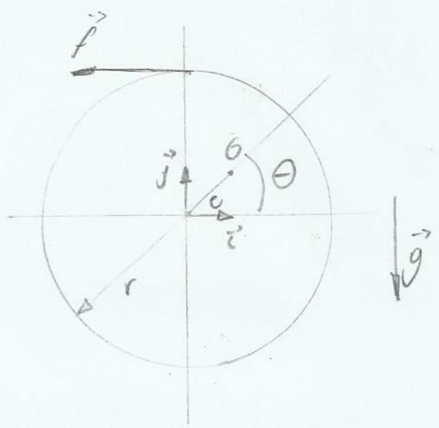
$fr = \frac{-Ja_c}{r^2} - ma_c$

$-120 = Ma_c + \frac{4Ja_c}{r^2} + ma_c$

$-120 = \left(M + \frac{4J}{r^2} + 4m\right)a_c$

$a_c = -0{,}81\,\frac{m}{s^2}$

Questão 1 (Prova)



$f = -50\vec{i}$ N

$r = 0{,}5$

$m = 20$ Kg

$J_0 = 5$ Kgm^2

$e = 0{,}1$m

$\overset{+}{\longrightarrow}$ $i\, j\, k\, i\, j$ $\underset{-}{\longleftarrow}$

(a)

TMA:

$$J_0\,\ddot{\theta} = r\vec{j}\wedge f(-\vec{i}) + (e\cos\theta\,\vec{i} + e\sin\theta\,\vec{j})\wedge m\,g(-j)$$

$$J_0\,\ddot{\theta}\,\vec{k} = rf\,\vec{k} - emg\cos\theta\,\vec{k}$$

$$\therefore\ \alpha = \ddot{\theta} = \frac{rf - emg\cos\theta}{J_0} = \frac{50\times0{,}5 - 0{,}1\times20\times9{,}81\cos\theta}{5}$$

$$\therefore\ \boxed{\alpha(\theta) = 5 - 3{,}924\cos\theta}$$

(b)

TMB:

$$\vec{f} + m\vec{g} + fr = m\,a$$

$$-f\,\vec{i} - mg\,\vec{j} + \vec{fr} = m\vec{a}_G$$

$$\vec{a}_G = \cancel{\vec{a}_0}^{\,0} + \ddot{\vec{\theta}}\wedge(G-O) + \dot{\theta}\wedge[\dot{\theta}\wedge(G-O)]$$

$$\vec{a}_G = (5 - 3{,}9\cos\theta)\,\vec{k}\wedge(e\cos\theta\,\vec{i} + e\sin\theta\,\vec{j})\wedge$$

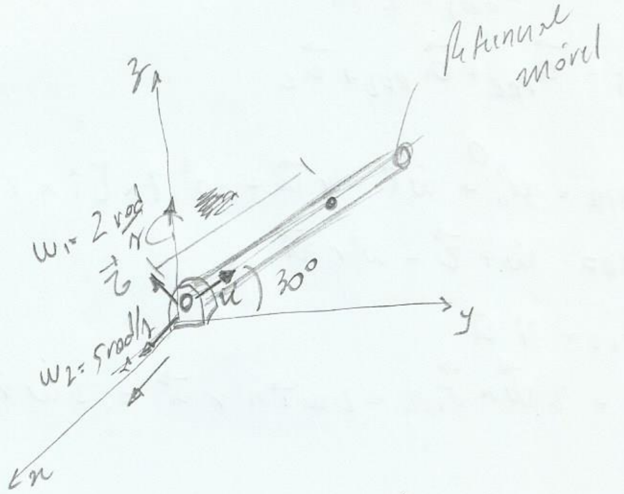
(c)

$$L = \Sigma mv$$

$$(m + m_b)\,x_G' = me + m_b \times d$$

$$\therefore\ d = -\frac{m}{m_b}\,e$$

Exemplo 1



Adaptado do Hilber 12. ed Problema 2051

2º lei : $m \cdot \vec{a}_{abs} = \sum_i \vec{F_i}$

$\vec{a}_{abs} = \vec{a}_{ARR} + \vec{a}_{REL} + \vec{a}_c$

$(O\ \vec{i}\,\vec{j}\,\vec{k})$ - Solidária à base da peça

$(O\ \vec{u}\,\vec{z}\,\vec{i})$ - Solidária ao tubo

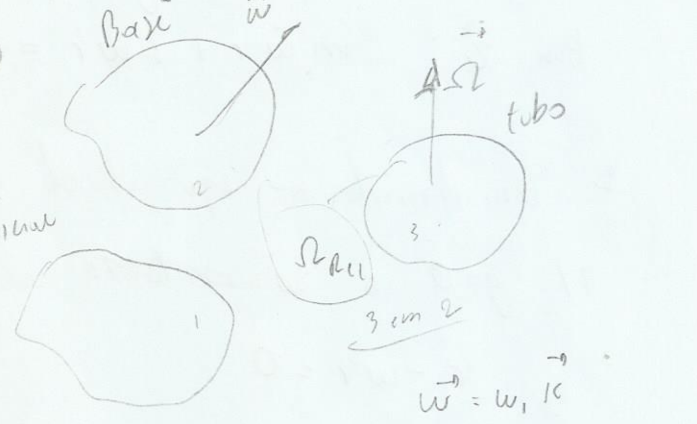
$\vec{a}_{ARR} = \cancel{\vec{a}_O}^{\,0} + \dot{\vec{\Omega}} \wedge (B-O) + \vec{\Omega} \wedge [\vec{\Omega} \wedge (B-O)]$

Arrastamento: É a velocidade que a bolinha teria se fixada ao referencial móvel (tubo).

$\vec{a}_{ARR} = w_1 w_2 \vec{j} \wedge r\vec{u} + (w_2 \vec{i} + w_1 \vec{k}) \wedge [(w_2 \vec{i} + w_1 \vec{k}) \wedge r\vec{u}]$

$\vec{a}_{rel} = \ddot{r}\vec{u}$ (Bolinha vista pelo tubo)

$\vec{a}_c = 2\vec{\Omega} \wedge \vec{v}_{rel} = 2(w_2\vec{i} + w_1\vec{k}) \wedge \dot{r}\vec{u}$



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$\sum_i \vec{F_i} = -mg\vec{k} + F_z\vec{z} + F_x\vec{i}$

$\vec{w} = w_1 \vec{k}$

$\vec{\Omega}_{rel} = w_2 \vec{i}$

$\vec{\Omega} = \vec{w} + \vec{\Omega}_{rel}$

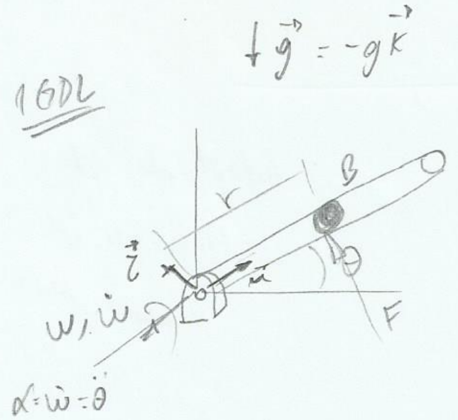
$= w_1\vec{k} + w_2\vec{i}$

$\dot{\vec{\Omega}} = \cancel{\dot{\vec{\Omega}}_{rel}}^{\,0} + \cancel{\dot{\vec{w}}}^{\,0} + \vec{w} \wedge \vec{\Omega}_{rel}$ Pq $\vec{i}$ em relação a base

$\dot{\vec{\Omega}} = \vec{w} \wedge \vec{\Omega}_{rel}$

$= w_1\vec{k} \wedge w_2\vec{i} = w_1 w_2 \vec{j}$

- Encontrar r
- Depois substituir r nas outras equações



$$m \cdot \vec{a}_{obs} = \sum_i \vec{F}_i$$

$$\vec{a}_{obs} = \vec{a}_{ARR} + \vec{a}_{REL} + \vec{a}_c$$

$$\vec{a}_{ARR} = \overset{0}{\vec{a}_o} + \dot{w}\vec{i} \wedge r\vec{u} + w^2 \vec{i} \wedge [\vec{i} \wedge r\vec{u}]$$

$\rightarrow \vec{a}_{ARR} = \dot{w} r \vec{z} - w^2 r \vec{u}$

$\rightarrow \vec{a}_{rel} = \ddot{r}\vec{u}$

$\rightarrow \vec{a}_c = 2\vec{w} \wedge \vec{v}_{rel} = 2w\vec{i} \wedge \dot{r}\vec{u} = 2w\dot{r}\vec{z}$

$$\sum_i F_i = -mg\vec{k} + F\vec{z}$$

$$m \cdot (\dot{w} r \vec{z} - w^2 r \vec{u} + \ddot{r}\vec{u} + 2w \cdot \dot{r} \cdot \vec{z}) = -mg\vec{k} + F\vec{z}$$

$$(m\dot{w}r + 2w\dot{r})\vec{z} + (m\ddot{r} - mw^2 r)\vec{u} = -mg\vec{k} + F\vec{z} \quad (\vec{k} = \cos\theta\,\vec{z} + \sin\theta\,\vec{u})$$

Em $\vec{u}$: $m\ddot{r} - mw^2 r = -mg\sin\theta$

$\ddot{r} - w^2 r = g\sin\theta$ Equação do movimento

Em $\vec{z}$: $m\dot{w}r + 2w\dot{r} = -mg\cos\theta + F$ (Equação vincular)

▷ Linearizando a equação do movimento

P/ $g=0$ e w = constante

P/ $g=0$ e $\dot{w}=0$

$F = 2w\dot{r}$

$$\ddot{r} - w^2 r = 0$$

$r(t) = A r_1(t) + B r_2(t)$

$r(t) = e^{\lambda t}$; $\ddot{r}(t) = \lambda^2 e^{\lambda t}$

▷ $\lambda^2 e^{\lambda t} - w^2 \cdot e^{\lambda t} = 0 \Rightarrow \lambda = \pm w$

$r_1(t) = e^{wt}$; $r_2(t) = -e^{wt}$ $\quad\therefore\ r(t) = A e^{wt} + B e^{-wt}$

$r(t=0) = r_0 = A + B$

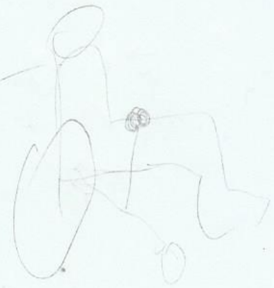
$\dot{r}(t) = Aw e^{wt} - Bw e^{-wt}$ $\quad \dot{r}(t=0) = 0 \rightarrow A = B$

logo: $A = B = \frac{r_0}{2}$ $\quad\therefore\ r(t) = \frac{r_0}{2}\left(e^{wt} + e^{-wt}\right)$

Se $\theta(t=0) = 0$: $\theta = wt$; $r(t) = \frac{r_0}{2}\left(e^{\theta} + e^{-\theta}\right)$

Sistemas Multicorpo

1º Descrever a topologia (vetores posições, rotações) p/ cada corpo



Sist. 3D: 6p $f = 6p - q$ (

Sist 2D: 3p $f = 3p - q$

Exemplo: Junta esférica

$$\vec{r}_i + \vec{\rho}_i = \vec{r}_K + \vec{\rho}_K$$

$${}^I\underline{r}_i + {}^I\underline{\rho}_i = {}^I\underline{r}_K + {}^I\underline{\rho}_K$$

$${}^I\underline{\rho}_i = R^{Ii}\,{}^i\underline{\rho}_i$$

$${}^I\underline{r}_i + \underline{\underline{R}}^{Ii}\,{}^i\underline{\rho}_i - \left({}^I\underline{r}_K + \underline{\underline{R}}^{IK}\,{}^K\underline{\rho}_K\right) = 0$$

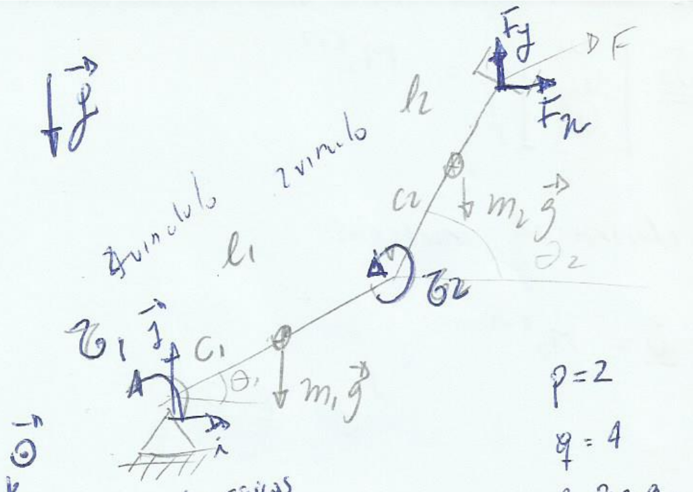
$$\underline{\underline{R}}^{IK}(\alpha_K, \beta_K, \gamma_K)$$

$$\underline{\underline{R}}^{Ii}(\alpha_i, \beta_i, \gamma_i)$$

$f = 6p - q$

q: número de vínculos

f: nº de graus de liberdade



$$\overset{2\ \text{corpos}}{f = 3p - q} = 2\ \text{GdL}$$

4 vínculos

x

$p = 2$ corpos

$q = 4$ vínculos

$f = 3p - q = 2$ GdL

Coordenadas físicas

$$\underline{x} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \theta_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} c_1 \cos\theta_1 \\ c_1 \sin\theta_1 \\ l_1 \cos\theta_1 + c_2 \cos\theta_2 \\ l_2 \sin\theta_1 + c_2 \cos\theta_2 \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\underline{x} = \underline{x}(q) \qquad q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\underline{r}_i = r_i(q, t)$$

$$\dot{r}_i = \frac{d}{dt}\underline{r}_i = \underbrace{\frac{\partial r_i}{\partial q^T}}_{J_{T,i}} \dot{q} + \frac{\partial \underline{r}_i}{\partial t}$$

$$\underline{r}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} c_1 \cos\theta_1 \\ c_1 \sin\theta_1 \end{bmatrix}$$

$$\underline{\dot{r}_1}\ \frac{dr_1}{dt} = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial r_{1,1}}{\partial q_1} & \frac{\partial r_{1,1}}{\partial q_2} \\ \frac{\partial r_{1,2}}{\partial q_1} & \frac{\partial r_{1,2}}{\partial q_2} \end{bmatrix}}_{\underline{\underline{J_{T,1}}}} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$J_{T,1} = \begin{bmatrix} -c_1 \sin\theta_1 & 0 \\ c_1 \cos\theta_1 & 0 \end{bmatrix}$$

$$\partial r_i = J_{T,i}\, q$$ deslocamento virtual

$$\underline{\ddot{r}_i} = \underline{\underline{J}}_{T,i}\, \ddot{q}_i + \underline{\underline{\dot{J}}}_{T,i}\, \dot{q} + \frac{d^2 r}{dt}$$

Equações de "Newton-Euler"

$$m(\cancel{G-O}) \wedge \vec{a}_O + \frac{d}{dt}\left([\vec{i}\ \vec{j}\ \vec{k}]\ \underline{J_O} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \right) = \vec{M}_O^{EXT}$$

Se $(O\ \vec{i}\ \vec{j}\ \vec{k})$ fixo ao eixo referencial inercial

$$[\vec{i}\ \vec{j}\ \vec{k}]\ \dot{\underline{J_O}}\ \underline{w} + [\vec{i}\ \vec{j}\ \vec{k}]\ \underline{J_O}\ \dot{\underline{w}} = \vec{M}_O^{EXT}$$

$${}^{I}\underline{J}_O = \underline{R}\ {}^{I}\underline{J}_O\ \underline{R}^T$$

$${}^{I}\dot{\underline{J}}_O\ \underline{w} + {}^{I}\underline{J}_O\ {}^{I}\dot{\underline{w}} = {}^{I}M_O^{EXT}$$

$$\hookrightarrow \overline{(\underline{R}\ {}^{I}\underline{J}_O\ \underline{R}^T)}\ {}^{I}\underline{w} + \underline{R}\ {}^{I}\underline{J}_O\ \underline{R}^T\ {}^{I}\dot{\underline{w}} = {}^{I}M_O^{EXT}$$

$$\dot{R}\ {}^{I}J_O\ R^T = R\ {}^{I}J_O\ R^T$$

—//—

Vínculo 1 : $x_1 = c_1 \cos\theta_1$

$y_1 = c_1 \sin\theta_1$

Vínculo 2 : $x_1 + (l_1 - c_1)\cos\theta_1 = x_2 - c_2\cos\theta_2$

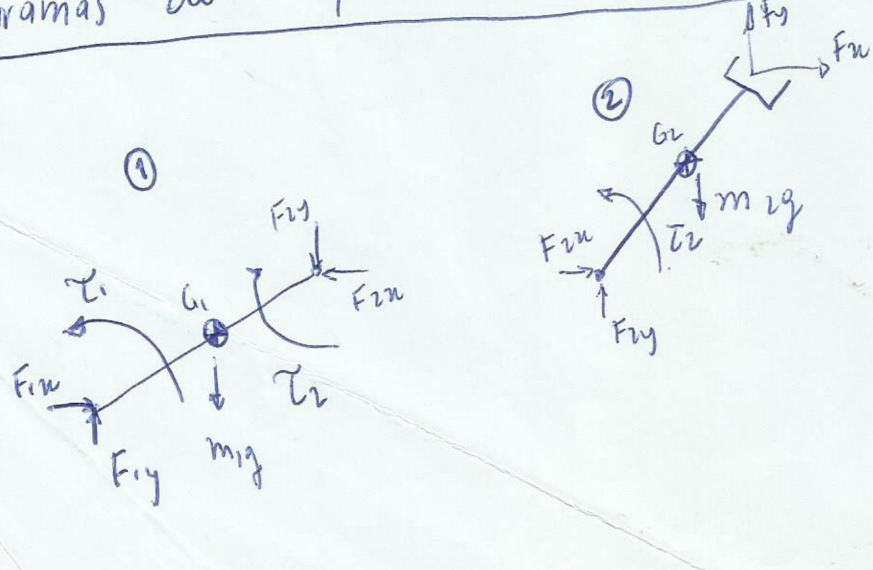
$y_1 + (l_1 - c_1)\sin\theta_1 = y_2 - c_1 \sin\theta_2$

$\Big\}\ q = 4$

Equações Vínculos :

$$\Phi = \begin{bmatrix} x_1 - c_1\cos\theta_1 \\ y_1 - c_1\sin\theta_1 \\ x_1 + (l_1 - c_1)\cos\theta_1 - x_2 + c_2\cos\theta_2 \\ y_1 + (l_1 - c_1)\sin\theta_1 - y_2 + c_2\sin\theta_2 \end{bmatrix} = 0$$

Diagramas de corpo livre



TMA: $J_0\ddot{\theta} = \sum_i M_{0,i}$

TMB:

$$m_1 \cdot \ddot{x}_1 = F_{1x} - F_{2x}$$
$$m_1 \, \ddot{y}_1 = F_{1y} - F_{2y} - m_1 g$$
$$m_2 \cdot \ddot{x}_2 = F_{2x} + F_x$$
$$m_2 \cdot \ddot{y}_2 = F_{2y} + F_y - m_2 g$$

vetor coluna

$$\dot{\underline{v}} = \frac{\partial v}{\partial x^+} \, \dot{\underline{x}}$$

TMA:

$$J_1 \ddot{\theta}_1 = \tau_1 - \tau_2 + \boxed{F_{1x} \cdot c_1 \sin\theta_1 - F_{1y} \cdot c_1 \cos\theta_1 + F_{2x}(l_1 - c_1)\sin\theta_1 + F_{2y}(l_1 - c_1)\cos\theta_1} \; *$$

$$J_2 \ddot{\theta}_2 = \tau_2 + F_{2x} c_2 \sin\theta_2 - F_{2y} c_2 \cos\theta_2 - F_x (l_2 - c_2)\sin\theta_2 + F_y (l_2 - c_2)\cos\theta_2$$

Incognitas

$$\underbrace{\begin{bmatrix} m_1 & & & & & \\ & m_1 & & & \underline{\underline{0}} & \\ & & m_2 & & & \\ & \underline{\underline{0}} & & m_2 & & \\ & & & & J_1 & \\ & & & & & J_2 \end{bmatrix}}_{\underline{\underline{M}}} \underbrace{\begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}}_{\ddot{\underline{x}}} = \underbrace{\begin{bmatrix} 0 \\ -m_1 g \\ F_x \\ -m_2 g + F_y \\ \tau_1 - \tau_2 \\ \tau_2 - F_x(l_2 - c_2)\sin\theta_2 + F_y(l_2 - c_2)\cos\theta_2 \end{bmatrix}}_{\underline{F}^e} + \underbrace{\begin{bmatrix} F_{1x} - F_{2x} \\ F_{1y} - F_{2y} \\ F_{2x} \\ F_{2y} \\ * \\ F_{2x} c_2 \sin\theta_2 - F_{2y} c_2 \cos\theta_2 \end{bmatrix}}_{\underline{F}^v}$$

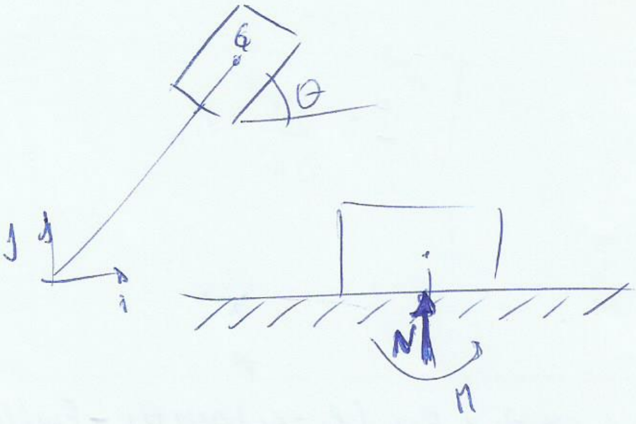
Forças ñ-vinculadas / avinculos

Forças vinculares

Incognitas $\ddot{x}$ + forças vinculares

$$\underline{F}^v = \underbrace{\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ c_1 \sin\theta_1 & -c_1\cos\theta_1 & (l_1 - c_1)\sin\theta_1 & -(l_1 c_1)\cos\theta_1 \\ 0 & 0 & c_2 \sin\theta_2 & -c_2\cos\theta_2 \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \end{bmatrix}}_{g}$$

Matriz de distribuição



$$x = \begin{bmatrix} x_B \\ y_B \\ \theta_B \end{bmatrix}$$

$y_B = 0$

$\theta = 0$

1 Grau de liberdade

$$F_Y = \begin{bmatrix} 0 \\ N \\ M \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\underline{\underline{Q}}} \begin{bmatrix} N \\ M \end{bmatrix}$$

$g_{g \cdot 1}$

$g = 2$

$3p = 3$

$f = 3p - g = 1\,GdL$

$q = [x_B]$

$$\underline{\underline{J}}^T \cdot \underline{\underline{Q}} = [1\,0\,0] \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = [0\,0]$$

$$\delta x = \begin{bmatrix} \delta x_B \\ \delta y_B \\ \delta \theta \end{bmatrix} = \begin{bmatrix} \delta x_B \\ 0 \\ 0 \end{bmatrix}$$

$$\delta x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \underbrace{\delta x_B}_{\delta q}$$

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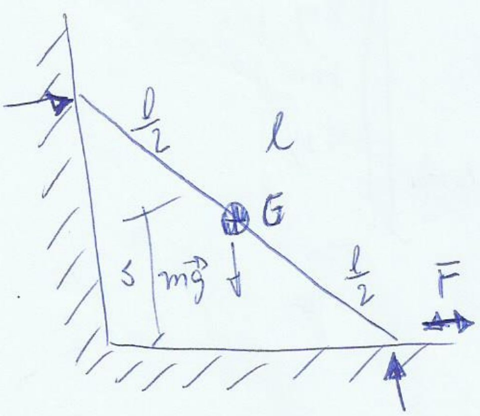

$-mg\,\delta v + F\,\delta h = 0 \quad (I)$

$$\begin{cases} v = \frac{l}{2}\cos\theta \\ h = l \sin\theta \end{cases}$$

$$\delta v = \frac{\partial}{\partial \theta}\left(\frac{l}{2}\cos\theta\right)\delta\theta = -\frac{l}{2}\sin\theta\,\delta\theta$$

$$\delta h = \frac{\partial}{\partial \theta}(l \sin\theta)\,\delta\theta = l\cos\theta\,\delta\theta$$

logo (I): $-mg\left(-\frac{\cancel{l}}{2}\sin\theta\,\cancel{\delta\theta}\right) + F\left(\cancel{l}\cos\theta\,\cancel{\delta\theta}\right) = 0$

$$\boxed{F = -mg\,tg\,\theta}$$

Braço Robótico

$p=2$

$$\underbrace{f_1^{e^T} \cdot \delta r_1}_{\text{produto escalar}} + l_1^{e^T} \delta s_1 + f_2^{e^T} \cdot \delta r_2 + l_2^{e^T} \delta s_2 = 0$$

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$ coordenadas generalizadas

$$\delta r_1 = \underbrace{J_{T1}}_{\frac{\delta r_1}{\delta q^T}} \delta q$$

$$r_1 = \begin{bmatrix} c_1 \cos\theta_1 \\ c_1 \sin\theta_1 \end{bmatrix} \quad ; \; \alpha_1 = \theta_1$$

$$r_2 = \begin{bmatrix} l_1 \cos\theta_1 + c_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + c_2 \sin(\theta_1+\theta_2) \end{bmatrix} ; \alpha_2 = \theta_1+\theta_2$$

$$J_{T1} = \begin{bmatrix} \frac{\delta r_{11}}{\delta q_1} & \frac{\delta r_{11}}{\delta q_2} \\ \frac{\delta r_{12}}{\delta q_1} & \frac{\delta r_{12}}{\delta q_2} \end{bmatrix} = \begin{bmatrix} -c_1 \sin\theta_1 & 0 \\ c_1 \cos\theta_1 & 0 \end{bmatrix}$$

$$\delta r_2 = \underbrace{J_{T2}}_{\frac{\delta r_2}{\delta q^T}} \delta q$$

$$J_{T2} = \begin{bmatrix} \frac{\delta r_{21}}{q_1} & \frac{\delta r_{21}}{q_2} \\ \frac{d r_{22}}{\delta q_1} & \frac{\delta r_{22}}{\delta q_2} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\theta_1 - c_2 \sin(\theta_1+\theta_2) & -c_2 \sin(\theta_1+\theta_2) \\ l_1 \cos\theta_1 + c_2 \cos(\theta_1+\theta_2) & c_2 \cos(\theta_1+\theta_2) \end{bmatrix}$$

$$\delta s_1 = J_{R1} \, \delta q \qquad w_1 = J_{R1} \dot{q} + \cancel{\frac{\delta s_1}{\delta t}}$$

$$\delta\alpha_1 = J_{R1} \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \end{bmatrix}$$

$$\delta\alpha_1 = \delta\theta_1 = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{J_{R1}} \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \end{bmatrix}$$

$$\delta\alpha_2 = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{J_{R2}} \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \end{bmatrix}$$

$$\left(\underline{f}_1^{eT}\,\underline{\underline{J}}_{T1} + \underline{l}_1^{eT}\,\underline{\underline{J}}_{R1} + \underline{f}_2^{eT}\,\underline{\underline{J}}_{T2} + \underline{l}_2^{eT}\,\underline{\underline{J}}_{R2}\right)_{1\times 2}\ \underline{\delta}_{q\,2\times 1} = 0$$

$$\underline{f}_1^{e} = \begin{bmatrix} 0 \\ -m_1 g \end{bmatrix}; \qquad \underline{f}_2^{e} = \begin{bmatrix} F_x \\ F_y - m_2 g \end{bmatrix}$$

$$l_1^{e} = 0$$

$$l_2^{e} = F_x (l_2 - c_2)\sin(\theta_1+\theta_2) + F_y (l_2 - c_2)\cos(\theta_1+\theta_2)$$

$\dot\theta \quad \varphi \quad \dot\varphi \quad \varphi$

$$\theta \begin{bmatrix} \theta \\ \dot\theta \\ \varphi \\ \dot\varphi \end{bmatrix}$$

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$$\underline{\underline{J}}^T \wedge \left(\underline{\underline{\bar M}}\,\underline{\underline{J}}\,\ddot q + \underline{\underline{\bar M}}\,\underline{\underline{\dot J}}\,\dot q + \underline{\underline{\bar M}} \begin{bmatrix} \bar a \\ \bar\alpha \end{bmatrix} + \underline{\bar k} = \underline{F}^e + \underline{\underline{Q}}\, g \right.$$

$$\underbrace{\underline{\underline{J}}^T \underline{\underline{\bar M}}\,\underline{\underline{J}}}_{\underline{\underline{M}}}\,\ddot q + \underbrace{\underline{\underline{J}}^T \underline{\underline{\bar M}}\,\underline{\underline{\dot J}}\,\dot q + \underline{\underline{J}}^T \underline{\underline{\bar M}} \begin{bmatrix} \bar a \\ \bar\alpha \end{bmatrix} + \underline{\underline{J}}^T \underline{\bar k}}_{\underline{k}(q,\dot q)} = \underbrace{\underline{\underline{J}}^T \underline{F}^e + \underline{\underline{J}}^T \underline{\underline{Q}}\, g}_{\text{Forças novas}} = 0$$

Matriz de Massa

~~Forças que não ...~~

Forças inerciais generalizadas de coriolis e centrípetas

~~Forças generalizadas não l...~~

Forças generalizadas

$$\underline{y} = \begin{bmatrix} q \\ q_d \end{bmatrix}$$

$$\underline{\underline{M}}(q)\,\ddot q + \underline{k}(q,\dot q,t) = \underline{k}^e(q,\dot q,t)$$

$$q_d = \dot q$$

$$\underline{\dot y} = \begin{bmatrix} \dot q \\ \dot q_d \end{bmatrix} = \begin{bmatrix} q_d \\ \underline{\underline{M}}^{-1}\left[\underline{k}^e(q,q_d,t) - k(q,q_d,t)\right] \end{bmatrix}$$

$$\begin{cases} \dot q = q_d \\ \underline{\underline{M}}(q)\,\dot q_d = k(q,q_d,t) = k^e(q,q_d,t) \end{cases}$$