Lista de exercício Integral Indefinida

001) $\int 2x\sqrt[3]{x}\,dx$

Admitindo $\sqrt[3]{x} = t$; $x = t^3$

derivando $x = t^3$;

$dx = 3t^2\,dt$

$$\int 2x\,x^{\frac{1}{3}}\,dx \Rightarrow \int 2x^{\frac{4}{3}}\,dx$$
$$2\int x^{4/3}\,dx = 2.\,x^{7/3} : \frac{7}{3} \Rightarrow$$
$$= \frac{6x^{7/3}}{7} = \frac{6x^2\sqrt[3]{x}}{7}$$

2ª Maneira

$$\int 2.t^3.t.3t^2\,dt \Rightarrow 6\int t^6\,dt = 6.\frac{t^7}{7}\,dt \Rightarrow$$

$$\Rightarrow \frac{6}{7}.(\sqrt[3]{x})^7 = \frac{6}{7}.x^{\frac{7}{3}} = \frac{6}{7}\sqrt[3]{x^3.x^3.x} = \frac{6x^2\sqrt[3]{x}}{7} + C$$

002) $\int \frac{2}{\sqrt{2+2x}.\sqrt{2-2x}}\,dx$

Elevando ao quadrado a integral obtimos:

$$\int \frac{4}{(2+2x)(2-2x)}\,dx^2 \Rightarrow \int \frac{4}{4-4x^2}\,dx^2 \Rightarrow \int \frac{4}{4(1-x^2)}\,dx^2 \Rightarrow$$

$$\Rightarrow \int \frac{1}{1-x^2}\,dx$$

Agora realiza a raiz quadrada da integral:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$$

003) $\int \frac{3}{5x\sqrt{9x^2-9}}\, dx$

$\int \frac{3}{5x\sqrt{3^2(x^2-1)}}\, dx \Rightarrow \int \frac{3}{15x\sqrt{x^2-1}}\, dx \Rightarrow \int \frac{1}{5x\sqrt{x^2-1}}\, dx \Rightarrow$

$\Rightarrow \frac{1}{5}\int \frac{1}{x\sqrt{x^2-1}}\, dx = \frac{1}{5} \text{ arc sec}(x) + C$

"Dúvida: Verificar as derivadas da tabela do número 16 ao 21"

004.) $\int \frac{sen(x)}{1-\cos^2(x)}\, dx$ "Verificar resposta!"

$\int \frac{sen(x)}{sen^2(x)}\, dx \Rightarrow \int \frac{1}{sen(x)}\, dx \Rightarrow \int cosec(x)\, dx = \ln|\csc(x)-\cot(x)|+C$

005) $\int (1-t)(2+t^2)\, dt$ (+-)

$\int (2-2t+t^2-t^3)\, dt = 2t - t^2 + \frac{t^3}{3} - \frac{t^4}{4} + C$

006) $\int \left(x + \frac{1}{x^3}\right) dx$

$\int x\, dx + \int x^{-3}\, dx = \frac{x^2}{2} + \frac{1}{x^2} : -2 = \frac{x^2}{2} - \frac{1}{2x^2} + C$

007) $\int \left(3x^2 + x + \frac{1}{x^3}\right) dx$ (+ -)

$\int 3x^2\, dx + \int x\, dx + \int x^{-3}\, dx = x^3 + \frac{x^2}{2} - \frac{1}{2x^2} + C$

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008) $\int (3\sqrt[5]{x^2}+3)\,dx$

$\int[3(\sqrt[5]{x^2}+1)]\,dx \Rightarrow 3\int(\sqrt[5]{x^2}+1)\,dx \Rightarrow 3\int(x^{\frac{2}{5}}+1)\,dx =$

$= 3.(x^{\frac{7}{5}} : \frac{7}{5} + x) + C = \frac{15}{7}.\sqrt[5]{x^7} + 3x + C$

009) $\int (x^2 + \sin(x))\,dx = \frac{x^3}{3} - \cos(x) + C$

Integral por substituição

024-) $\int \sqrt{9-x^2}\,(-2x)\,dx$

Admitindo. $\sqrt{9-x^2} = t$; $9-x^2 = t^2$

diferenciando $9-x^2=t^2$, temos: $-2x\,dx = 2t\,dt$

$\int t.2t\,dt \Rightarrow 2\int t^2\,dt = \frac{2}{3}t^3 + C$, Subsitituindo t

$\frac{2}{3}.(9-x^2)^{\frac{3}{2}} + C$

025-) $\int \frac{x}{(1-x^2)^3}\,dx$ Admitindo: $1-x^2 = t$

diferenciando obtemos: $-2x\,dx = dt \Rightarrow x\,dx = \frac{-1}{2}\,dt$

$\int(\frac{-1}{2} : t^3)\,dt \Rightarrow \int \frac{-1}{2t^3}\,dt \Rightarrow \frac{-1}{2}\int t^{-3}\,dt =$

$= -\frac{1}{2}.\frac{1}{t^2} : -2 = \frac{1}{4}.\frac{1}{t^2} = \frac{1}{4}.\frac{1}{(1-x^2)^2} + C$

026) $\int tg^4(x)\sec^2(x)\,dx$

Admitindo $tg(x)=t \rightarrow \sec^2(x)\,dx = dt$

$\int t^4\,dt = \frac{t^5}{5} + C = \frac{tg^5}{5} + C$

027-) $\int \frac{cosec^2(x)}{cotg^3(x)}\,dx$

Admitindo $cotg(x) = t \rightarrow -csc^2(x) = dt \Rightarrow csc^2(x) = -dt$

$\int \frac{-1}{t^3}\,dt \Rightarrow -1\int t^{-3}dt = -1.\frac{1}{2t^2} + C \Rightarrow \frac{1}{2\,ctg^2(x)} + C$

028-) $\int e^{\cos(x)}\,sen(x)\,dx$

Admitindo $e^{\cos(x)} = t \rightarrow e^{\cos(x)}.(-sen(x))\,dx = dt \Rightarrow$

$\Rightarrow e^{\cos(x)}\,sen(x)\,dx = (-1).dt$

$\int -1\,dt = -t + C = -e^{\cos(x)} + C$

029-) $\int \cos\left(\frac{x}{5}\right)dx$

Admitindo $\frac{x}{5} = t \Rightarrow \frac{1}{5}\,dx = dt \quad dx = 5dt$

$\int \cos t.5dt = 5\,sen\,t + C \Rightarrow$

$\Rightarrow 5\,sen\left(\frac{x}{5}\right) + C$

030) $\int \frac{x^2}{\sqrt{5x^3+4}}\, dx$

Admitindo $\sqrt{5x^3+4} = t \rightarrow 5x^3+4 = t^2 \rightarrow 15x^2\, dx = 2t\, dt \rightarrow$

$\rightarrow x^2\, dx = \frac{2}{15} t\, dt$

$\int \frac{2t}{15} : t\, dt = \frac{2}{15} t + C = \frac{2}{15} . \sqrt{5x^3+4} + C$.

031) $\int x^3 \sqrt{2-2x^2}\, dx$

Admitindo $\sqrt{1-x^2} = t \rightarrow 1-x^2 = t^2 \rightarrow -2x\, dx = 2t\, dt \rightarrow$

$\rightarrow x\, dx = (-t)\, dt$ // $1-x^2 = t^2 \rightarrow x^2 = 1-t^2$

$\sqrt{2} \int (1-t^2) . t . (-t)\, dt \rightarrow \sqrt{2} \int (-t)^2 + t^4\, dt =$

$= \sqrt{2} . \left(\frac{(-t)^3}{3} + \frac{t^5}{5} \right) + C = \sqrt{2} . \left[\frac{(\sqrt{1-x^2})^5}{5} - \frac{(\sqrt{1-x^2})^3}{3} \right] + C$

032) $\int \frac{1+4x}{\sqrt{1+x+2x^2}}\, dx$ Admitindo $\sqrt{1+x+2x^2} = t \rightarrow$

$\rightarrow 1 + x + 2x^2 = t^2 \rightarrow (1+4x)\, dx = 2t\, dt$

$\int \frac{2t}{t}\, dt \rightarrow 2 \int 1\, dt = 2.t + C = 2\sqrt{1+x+2x^2} + C$

033) $\int \frac{3}{(2y+1)^5}\, dy$ Admitindo $2y+1 = t \rightarrow 2\, dy = dt \rightarrow dy = \frac{1}{2}\, dt$

$3 \int t^{-5} . \frac{1}{2}\, dt \rightarrow \frac{3}{2} . \int t^{-5}\, dt = \frac{3}{2} . \frac{1}{t^4} : -4 + C =$

$= \frac{3}{2} . \frac{1}{(-4) t^4} + C = -\frac{3}{8} . \frac{1}{t^4} + C = \frac{-3}{8(2y+1)^5} + C$

034) $\int \frac{(\ln(x))^2}{x} dx$

Admitindo $\ln(x) = t \Rightarrow \frac{1}{x} dx = dt \Rightarrow dx = x\,dt$

$\int \frac{t^2}{\cancel{x}} . \cancel{x} dt = \frac{t^3}{3} + C \Rightarrow \frac{[\ln(x)]^3}{3} + C$

035) $\int \frac{\cos(\sqrt{t})}{\sqrt{t}} dt$

Admitindo $\sqrt{t} = x \Rightarrow t = x^2 \Rightarrow dt = 2x\,dx$

$\int \frac{\cos x}{\cancel{x}} . 2\cancel{x} dx = 2 \operatorname{sen}(x) + C \Rightarrow 2 \operatorname{sen}(\sqrt{t}) + C$

Integrais por partes

036) $\int x^3 e^{x^2} dx$

$x^2 = t \Rightarrow 2x dx = dt \Rightarrow x dx = \frac{1}{2} dt$

$\int t\, e^t . \frac{1}{2} dt = \frac{1}{2} \int t e^t dt$

$u = t \Rightarrow du = dt$

$dv = e^t \Rightarrow v = \int e^t dt = e^t$

$\frac{1}{2} \int t e^t dt = t.e^t - \int e^t dt$

$\frac{1}{2} \int t e^t dt = \frac{1}{2} (t e^t - e^t) = \frac{1}{2} e^t (t-1) =$

$= \frac{1}{2} . e^{x^2} (x^2 - 1) + C$

037-) $\int x e^{2x} dx$

$t = 2x \Rightarrow dt = 2dx \Rightarrow \frac{1}{2} dt = dx$

$\hookrightarrow x = \frac{1}{2} t$

$$\int \frac{1}{2} t \cdot \frac{1}{2} e^t dt = \frac{1}{4} \int t e^t dt$$

$u = t \Rightarrow du = dt$

$dv = e^t dt \Rightarrow v = \int e^t dt = e^t$

$$\frac{1}{4} \int t e^t dt = \frac{1}{4} \cdot \left(t \cdot e^t - \int e^t dt \right) = \frac{1}{4} \left(2x e^{2x} - e^{2x} \right) + C$$

038-) $\int x^3 \cos(x^2) dx$

$t = x^2 \Rightarrow dt = 2x\, dx \Rightarrow x dx = \frac{1}{2} dt$

$$\int t \cos(t) \cdot \frac{1}{2} dt = \frac{1}{2} \int t \cos(t) dt$$

$u = t \Rightarrow du = dt$

$dv = \cos(t) \quad v = \int \cos(t) dt = sen(t)$

$$\frac{1}{2} \int t \cos(t) = \frac{1}{2} \cdot \left(t \cdot sen(t) - \int sen(t) dt \right) =$$

$$= \frac{1}{2} \left(t\, sen(t) - (-\cos(t)) \right) + C \Rightarrow$$

$$= \frac{1}{2} \left(x^2 sen(x^2) + \cos(x^2) \right) + C$$

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039-) $\int x^2 sen(x)\,dx$

$u = x^2 \Rightarrow du = 2x\,dx$

$dv = sen(x)\,dx \Rightarrow v = \int sen(x)\,dx = -\cos(x)$

$\int x^2 sen(x)\,dx = x^2.(-\cos(x)) - \int -\cos(x).2x\,dx \Rightarrow$

$\Rightarrow \int x^2 sen(x)\,dx = -x^2.\cos(x) + \int \cos(x).2x\,dx$ (I)

$\int \cos(x).2x\,dx = 2\int \cos(x)\,x\,dx$

$u = x \Rightarrow du = dx$

$dv = \cos(x)\,dx \Rightarrow v = \int \cos(x)\,dx = sen\,x$

$2\int \cos(x)\,x\,dx = 2.\left(x.sen\,x - \int sen\,x\,dx\right) =$

$= 2.\,(x\,sen\,x + \cos x)$ (II)

$\int x^2 sen(x)\,dx = -x^2\cos(x) + 2\,(x\,sen(x) + \cos(x)) + C$

040-) $\int x\cos(x)\,dx$

$u = x \Rightarrow du = dx$

$dv = \cos(x)\,dx \Rightarrow v = \int \cos(x)\,dx = sen\,x$

$\int x\cos(x)\,dx = x.sen\,x - \int sen\,x\,dx =$

$= x\,sen(x) + \cos(x) + C$

041-) $\int arctg(x)\,dx$

Integral Imediata $= x\,arctg\,x - \frac{1}{2}\ln(1+x^2) + C$

042-) $\int e^{2x} sen(x)\,dx$

$u = e^{2x} \Rightarrow du = 2e^{2x}\,dx$

$dv = sen(x)\,dx \Rightarrow v = \int sen(x)\,dx = -\cos(x)$

$\int e^{2x} sen(x)\,dx = e^{2x}.-\cos(x) - \int -\cos(x).2e^{2x}\,dx$

$\int e^{2x} sen(x)\,dx = -e^{2x}\cos(x) + 2\int \cos(x)\,e^{2x}\,dx$ (I)

$\int \cos(x)\, e^{2x}\, dx$

$u = e^{2x} \Rightarrow du = 2e^{2x}\, dx$

$dv = \cos(x)dx \Rightarrow v = \int \cos(x)\, dx = \sin(x)$

$\int \cos(x)\, e^{2x}\, dx = e^{2x}.\sin(x) - \int \sin(x).2e^{2x}\, dx$ (II)

$\int e^{2x}\sin(x)dx = -e^{2x}\cos(x) + 2.\left(e^{2x}\sin(x) - \int \sin(x).2e^{2x}\, dx\right)$

$\int e^{2x}\sin(x)\,dx = -e^{2x}\cos(x) + 2e^{2x}\sin(x) - 2\int \sin(x).2e^{2x}\, dx$

$5\int e^{2x}\sin(x)dx = -e^{2x}\cos(x) + 2e^{2x}\sin(x)$

$\int e^{2x}\sin(x)\,dx = \frac{1}{5}.e^{2x}\left(2\sin(x) - \cos(x)\right) + C$

043-) $\int x^3 \sin(x)\, dx$

$u = x^3 \Rightarrow du = 3x^2\, dx$

$dv = \sin(x)\, dx \Rightarrow v = \int \sin(x)\, dx = -\cos(x)$

$\int x^3\sin(x)\,dx = x^3.-\cos(x) - \int -\cos(x).3x^2\, dx$

$\int x^3 \sin(x)\, dx = -x^3.\cos(x) + 3\int \cos(x)\, x^2\, dx$ (I)

$\int \cos(x)\, x^2\, dx$

$u = x^2 \Rightarrow du = 2x\, dx$

$dv = \cos(x)\, dx \Rightarrow v = \int \cos(x)\, dx = \sin x$

$\int \cos(x)\, x^2 dx = x^2 \sin x - \int \sin x.\, 2x.\, dx$

$\int \cos(x)\, x^2 dx = x^2 \sin x - 2\int \sin(x).\, x\, dx$ (II)

$\int \sin(x).\, x\, dx$

$u = x \Rightarrow du = dx$

$dv = \sin(x)\, dx \Rightarrow v = \int \sin(x)\, dx = \cos(x)$

$\int \sin(x).x\, dx = x.\cos(x) - \int \cos(x)\, dx = x\cos(x) - \sin(x)$ (III)

$\int \cos(x)\, x^2 dx = x^2 \sin x - 2.\left(x\cos(x) - \sin(x)\right)$

$\int \cos(x)\, x^2 dx = x^2 \sin x - 2x\cos(x) + 2\sin(x)$

$\int x^3 sen(x) = -x^3\cos(x) + 3\left(x^2 sen x - 2x\cos(x) + 2 sen(x)\right)$

$\int x^3 sen(x) = -x^3\cos(x) + 3x^2 sen x - 6x\cos(x) + 6 sen(x)$

$\int x^3 sen(x) = \cos(x)(-x^3 - 6x) + sen(x)(3x^2 + 6) + C$

44) $\int x \sec^2(x)\,dx$

$u = x \Rightarrow du = dx$

$dv = \sec^2(x) \Rightarrow v = \int \sec^2(x)\,dx = \tan(x)$

$\int x \sec^2(x)\,dx = x.\,tg(x) - \int \tan(x)\,dx$

$\int x \sec^2(x)\,dx = x\,tg(x) - \ln(\sec(x)) + C$

045-) $\int \frac{\ln(x)}{x^4}\,dx \Rightarrow \int x^{-4}\ln(x)\,dx$

$u = \ln(x) \Rightarrow du = \frac{1}{x}dx$

$dv = x^{-4}dx \Rightarrow$

$v = \int x^{-4}dx = \frac{x^{-3}}{-3}$

$\int x^{-4}\ln(x)\,dx = \frac{x^{-3}}{-3}.\ln(x) - \int \frac{x^{-3}}{-3}.\frac{1}{x}\,dx =$

$= \frac{x^{-3}}{-3}\ln(x) - \frac{1}{3}\int x^{-4}dx = \frac{x^{-3}}{-3}\ln(x) + \frac{1}{3}.\frac{x^{-3}}{-3} =$

$= -\frac{1}{3x^3}.\ln(x) + \frac{1}{3x^3} + C$

046-) $\int x\, arctg(x)\,dx$

$u = arctg(x)$, $\quad dv = x\,dx$

$du = \frac{1}{1+x^2}\,dx \Rightarrow v = \int x\,dx = \frac{x^2}{2}$

$\int x\, arctg(x)\,dx = (arctg(x)).\frac{x^2}{2} - \int \frac{x^2}{2}.\frac{1}{1+x^2}\,dx$

$\int x\, arctg(x)\,dx = arctg(x).\frac{x^2}{2} - (x - arctg\,x)\cdot\frac{1}{2} + C$

$\int x\, arctg(x)\,dx = \frac{1}{2}\cdot(arctg(x).x^2 - x + arctg(x)) + C$

$x^2 \;|\; x^2+1$

$-x^2-1$ $\quad$ 1 quociente

-1 resto

$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$

$\frac{7}{3}$ $\quad$ $7 \;|\; 3$, $1 \quad 2$

$\frac{7}{3} = 2\frac{1}{3}$

017-) $\int e^x \cos(x)\, dx$

$u = \cos(x) \Rightarrow du = -sen(x)\, dx$

$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$

$\int e^x \cos(x)\, dx = e^x \cos(x) - \int e^x . -sen(x)\, dx$

$\int e^x \cos(x)\, dx = e^x \cos(x) + \int e^x sen(x)\, dx$ I

$\int e^x sen(x)\, dx \Rightarrow u = sen(x) \quad du = \cos(x)\, dx$

$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$

$\int e^x sen(x)\, dx - e^x sen(x) - \int e^x . \cos(x)\, dx$ II

I e II

$\int e^x \cos(x) dx = e^x \cos(x) + e^x sen(x) - \int e^x \cos(x)\, dx$

$2\int e^x \cos(x)\, dx = e^x(\cos(x) + sen(x))$

$\int e^x \cos(x)\, dx = \frac{1}{2} e^x (\cos(x) - sen(x)) + C$

18-) $\int sec^3(x)\, dx$

$\int sec^3(x)\, dx \sim \int sec^2(x) sec(x)\, dx$

$u = sec(x) \Rightarrow du = sec(x) . tg(x)\, dx$

$dv = sec^2(x)\, dx \Rightarrow v = \int sec^2(x)\, dx = tg(x)$

$\int sec^3(x)\, dx = sec(x) . tg(x) - \int tg^2(x) sec(x)\, dx$ I

$\int tg^2(x) sec(x) \Rightarrow u = sec(x) \quad du = sec(x)\, tg(x)$

$dv = tg^2(x) \quad v = \int tg^2(x)\, dx = tg x - x$

$\int tg^2(x) sec(x) = sec(x)(tg(x) - x) - \int (tg(x) - x) sec(x)\, tg(x)\, dx$

$\int tg^2(x) sec(x) = sec(x) tg(x) - x\, sen(x) - \int sec(x) tg^2(x) - x\, sec(x)\, tg(x)\, dx$

$2\int tg^2(x) sec(x) = sec(x) tg(x) - x\, sen(x) + \int x\, sec(x)\, tg(x)\, dx$

$\int tg^2(x) sec(x) = \frac{1}{2} . \left[sec(x) tg(x) - x\, sen(x) + \int x\, sec(x)\, tg(x)\, dx \right]$ II

$\int x \sec(x)\, tg(x)\, dx$

$u = x \Rightarrow du = dx$

$dv = \sec(x)\, tg(x) \Rightarrow v = \int \sec(x)\, tg(x)\, dx = \sec(x)$

$\int x \sec(x)\, tg(x) = x \sec(x) - \int \sec(x)\, dx$

$\int x \sec(x)\, tg(x) = x \sec(x) - \ln|\sec(x) + tg(x)|$ III

II e III

$\int tg^2(x) \sec(x) = \frac{1}{2} [\sec(x)\, tg(x) - \cancel{x \sec(x)} + \cancel{x \sec(x)} - \ln|\sec(x) + tg(x)|]$

$\int tg^2(x) \sec(x) = \frac{1}{2} [\sec(x)\, tg(x) - \ln|\sec(x) + tg(x)|]$ IV

IV e I

$\int \sec^3(x) = \sec(x)\, tg(x) - \frac{1}{2} [\sec(x)\, tg(x) - \ln|\sec(x) + tg(x)|]$

$\int \sec^3(x) = \frac{1}{2} \cdot [\sec(x)\, tg(x) + \ln|\sec(x) + tg(x)|]$

049) $\int sen^3(x)\sqrt{\cos(x)}\ dx$

$\cos(x) = t^2 \rightarrow -sen(x)dx = 2t\,dt \sim sen(x)\,dx = -2t\,dt$

$\int sen(x)sen^2(x)\sqrt{\cos(x)}\,dx \sim \int sen\,(1-\cos^2(x))\sqrt{\cos x}\ dx$

$\sim \int (1-t^4)t^2(-2)\,dt \rightarrow -2\int t^2 - t^6\,dt = -2\left(\frac{t^3}{3} - \frac{t^7}{7}\right) + C$

$= -\frac{2}{3}\cos(x)\sqrt{\cos x} + \frac{2}{7}\cos^3(x)\sqrt{\cos(x)}$

$= \left(-\frac{2}{3}\cos(x) + \frac{2}{7}\cos^3(x)\right)\sqrt{\cos(x)} + C$

050) $\int sec^2(x) \cdot tg(x)\,dx \sim \int sec(x)\,sec(x)\,tg(x)\,dx$

$sec(x) = t \qquad sec(x)\,tg(x)\,dx = dt$

$\int t\,dt = \frac{t^2}{2} = \frac{sec^2(x)^2}{2}$

ou

$tg(x) = t \qquad sec^2(x)\,dx = dt$

$\int sec^2(x)\,tg(x)\,dx = \int t\,dt$

$= \frac{t^2}{2} + C$

$= \frac{1}{2}tg^2(x) + C$

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051-) $\int_0^{\frac{\pi}{3}} tg^5(x) sec^4(x)\,dx \sim \int_0^{\frac{\pi}{3}} tg^5(x) sec^2(x) sec^2(x)\,dx$

$tg(x) = t \quad sec^2(x)\,dx = dt$

$x = \frac{\pi}{3} \;;\; t = tg(\frac{\pi}{3}) = \sqrt{3}$

$x = 0 \;;\; t = tg(0) = 0$

$\int_0^{\sqrt{3}} t^5(1+t^2)\,dt \sim \int_0^{\sqrt{3}} t^5 + t^7\,dt =$

$= \frac{t^6}{6} + \frac{t^8}{8} \Big|_0^{\sqrt{3}} = \frac{3^{\frac{1}{2}\cdot 6}}{6} + \frac{3^{\frac{1}{2}\cdot 8}}{8}$

$= \frac{3^3}{6} + \frac{3^4}{8} = \frac{117}{8}$

052-) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} cotg^2(x)\,dx = -cot(x) - x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= -cotg(\frac{\pi}{2}) - \frac{\pi}{2} + cotg(\frac{\pi}{2}) + \frac{\pi}{6} = \frac{\pi - 3\pi}{6} = \frac{-2\pi}{6} = -\frac{\pi}{3}$

053-) $\int sen(7x)\cos(2x)\,dx$

$sen(A+B) = senA\cos B + senB\cos A$

$sen(A-B) = senA\cos B - senB\cos A$

$sen(A+B) + sen(A-B) = 2\,senA\cos B$

$senA\cos B = \frac{sen(A+B) + sen(A-B)}{2}$

$sen(7x)\cos(2x) = \frac{sen(9x) + sen(5x)}{2}$

$\int sen(7x)\cos(2x)\,dx = \frac{1}{2}\int sen(9x) + sen(5x)\,dx$

$= \frac{1}{2}\cdot\left(-\frac{\cos(9x)}{9} - \frac{\cos(5x)}{5}\right) = -\frac{\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$

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54-) $\int \cos(x)\, sen(2x)\, dx$

$$\int \cos(x)\, sen(2x)\, dx = \int sen(2x)\cos(x)\, dx =$$
$$= \int \frac{sen(3x) + sen(x)}{2}\, dx$$
$$= 1/2 \int sen(3x) + sen(x)\, dx$$
$$= \frac{1}{2}\left(\frac{-\cos(3x)}{3} - \cos(x)\right)$$
$$= \frac{-1}{6}\cos(3x) - \frac{1}{2}\cos(x) + C$$

55-) $\int \cos^2(2x)\, sen^2(2x)\, dx$

$$\int \frac{1+\cos(4x)}{2} \cdot \frac{1-\cos(4x)}{2}\, dx$$
$$\int \frac{1-\cos^2(4x)}{4}\, dx = \frac{1}{4}\int 1 - \cos^2(4x)\, dx$$
$$= \frac{1}{4}x - \frac{1}{4}\int \frac{1+\cos(8x)}{2}\, dx = \frac{1}{4}x - \frac{1}{4}\cdot\frac{1}{2}\left(x + \frac{sen(8x)}{8}\right)$$
$$\int \cos^2(2x)\, sen^2(2x)\, dx = \frac{x}{8} - \frac{sen(8x)}{64} + C$$

56-) $\int \frac{\cos^3(x)}{sen^7(x)}\, dx \sim \int \frac{\cos^3(x)}{sen^3(x)} \cdot sen^{-4}(x)\, dx =$

$$= \int \cot^3(x) \cdot \csc^4(x)\, dx \sim \int \cot^3(x)(1+\cot^2(x))\csc^2(x)\, dx$$

$\cot(x) = t \quad -\csc^2(x)\, dx = dt \quad \csc^2(x)\, dx = -dt$

$$\int t^3(1+t^2)(-1)\, dt = -\int t^3 + t^5\, dt$$
$$= -\left(\frac{t^4}{4} + \frac{t^6}{6}\right) + C$$
$$= -\frac{\cot^4(x)}{4} + \frac{\cot^6(x)}{6} + C$$
$$= -\frac{1}{4\, sen^4(x)} - \frac{1}{6\, sen^6(x)} + C$$

57-) $\int sen^5(x)\cos^2(x)\,dx$

$\cos(x) = t \quad -sen(x)\,dx = dt \quad sen(x)\,dx = -dt$

$\int (1-\cos^2(x))^2 sen(x)\cos^2(x)\,dx$

$-\int (1-t^2)^2 t^2\,dt = -\int (1-2t^2+t^4)\,t^2\,dt = -\int t^2-2t^4+t^6\,dt$

$= -\left(\frac{t^3}{3} - \frac{2}{5}t^5 + \frac{1}{7}t^7\right) + C$

$= -\frac{1}{3}\cos^3(x) + \frac{2}{5}\cos^5(x) - \frac{1}{7}\cos^7(x) + C$

58-) $\int \cos^3(\theta)\sqrt{sen(\theta)}\,d\theta \sim \int \cos^3(x)\sqrt{sen(x)}\,dx \sim \int (1-sen^2(x))\cos(x)\sqrt{sen(x)}\,dx$

$sen(x) = t^2 \quad \cos(x)\,dx = 2t\,dt \qquad " \quad t = sen^{\frac{1}{2}}(x)$

$\int (1-t^4)\,t \cdot 2t\,dt \sim 2\int t^2 - t^6\,dt =$

$2\cdot\left(\frac{t^3}{3} - \frac{t^7}{7}\right) + C$

$\frac{2}{3} sen^{\frac{3}{2}}(x) - \frac{2}{7} sen^{\frac{7}{2}}(x) + C$

59-) $\int sen^2(\alpha)\cos^2(\alpha)\,d\alpha$

$\int \frac{1-\cos(2x)}{2} \cdot \frac{1+\cos(2x)}{2}\,dx =$

$\int \frac{1-\cos^2(2x)}{4}\,dx \sim \frac{1}{4}\int 1-\cos^2(2x)\,dx =$

$= \frac{1}{4}\int 1 - \frac{1-\cos(4x)}{2}\,dx = \frac{1}{4}\int \frac{2-1-\cos(4x)}{2}\,dx$

$= \frac{1}{8}\left(x - \frac{sen(4x)}{4}\right) + C$

60-) $\int x\,sen^2(x)\,dx$

$$\int x\,\frac{(1-\cos(2x))}{2}\,dx = \frac{1}{2}\int x - x\cos(2x)\,dx$$

$\int x\,dx = \frac{x^2}{2}$ $\quad \int x\cos(2x)\,dx$

$u = x \quad du = dx$

$dv = \cos(2x) \quad v = \int \cos(2x)\,dx = \frac{sen(2x)}{2}$

$$\int x\cos(2x)\,dx = \frac{x\,sen(2x)}{2} - \frac{1}{2}\int sen(2x)\,dx$$

$$= \frac{x\,sen(2x)}{2} - \frac{1}{2}\cdot -\frac{\cos(2x)}{2} = \frac{x\,sen(2x)}{2} + \frac{\cos(2x)}{4}$$

$$\int x\,sen^2(x)\,dx = \frac{1}{2}\left(\frac{x^2}{2} - \frac{x\,sen(2x)}{2} - \frac{\cos(2x)}{4}\right)$$

$$= \frac{1}{4}x^2 - \frac{1}{4}x\,sen(2x) - \frac{1}{8}\cos(2x) + C$$

Substituições Trigonométricas

61-) $\int \frac{1}{x\sqrt{x^2+1}}\,dx$

$x = tg(\theta) \quad dx = sec^2(\theta)\,d\theta$

$$\int \frac{sec^2\theta}{tg(\theta)\sqrt{tg^2\theta+1}}\,d\theta \sim \int \frac{sec^2\theta}{tg(\theta)\,sec(\theta)}\,d\theta \sim \int \frac{sec\,\theta}{tg(\theta)}\,d\theta$$

$$\int \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{sen\,\theta}\,d\theta = \int \frac{1}{sen\,\theta}\,d\theta = \ln|\csc(\theta) - \cot(\theta)| + C$$

$$\csc^2\theta = 1 + \cot^2\theta = 1 + \frac{1}{tg^2\theta} = 1 + \frac{1}{x^2} = \frac{x^2+1}{x^2}$$

$\csc\theta = \frac{\sqrt{x^2+1}}{x} \qquad \cot\theta = \frac{1}{tg\theta} = \frac{1}{x} \qquad R: \ln\left|\frac{\sqrt{x^2+1}-1}{x}\right| + C$

063-) $\displaystyle\int \frac{x^2}{\sqrt{x^2+9}}\, dx$

$x = 3\,tg\,\theta \qquad dx = 3\sec^2\theta\, d\theta$

$$\int \frac{9\,tg^2\theta}{\sqrt{9\,tg^2\theta+9}} \cdot 3\sec^2\theta\, d\theta \sim \int \frac{9\,tg^2\theta\; 3\sec^2\theta}{\cancel{3}\sqrt{tg^2\theta+1}}\, d\theta \sim$$

$$9\int \frac{tg^2\theta \sec^2\theta}{\sec\theta}\, d\theta \sim 9\int tg^2\theta \sec\theta\, d\theta$$

$$9\int tg^2\theta\sec\theta\, d\theta = 9\int (\sec^2\theta - 1)\sec\theta\, d\theta =$$
$$= 9\int (\sec^3\theta - \sec\theta)\, d\theta =$$
$$= 9\int \sec^3\theta\, d\theta. \quad - 9\int \sec\theta\, d\theta.$$

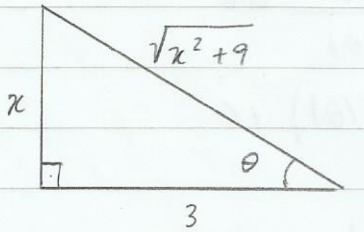
$$= 9\left(\frac{\sec\theta\, tg\theta + \ln|\sec\theta + tg\theta|}{2} - \ln|\sec\theta + tg\theta|\right.$$

$$= 9\left[\frac{1}{2}\left(\sec\theta\, tg\theta - \ln|\sec\theta + tg\theta|\right)\right] + C$$

$$= \cancel{9}\left[\frac{1}{2}\left(\frac{x\sqrt{x^2+9}}{\cancel{9}} - \ln\left|\frac{\sqrt{x^2+9}+x}{3}\right|\right)\right. + C$$

$$= \frac{1}{2}\, x\sqrt{x^2+9} - \ln|\sqrt{x^2+9} + x| - \cancel{\ln|3|} + C$$

$$= \frac{1}{2}\, x\sqrt{x^2+9} - \ln|\sqrt{x^2+9} + x| + C$$



$tg\theta = \frac{x}{3} \qquad \sec\theta = \frac{1}{\cos\theta} = 1 : \frac{3}{\sqrt{x^2+9}} = \frac{\sqrt{x^2+9}}{3}$

$$= \frac{1}{10}\left(\frac{\sqrt{x^2+9}}{3}\cdot\frac{x}{3} - \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right|\right) + C$$

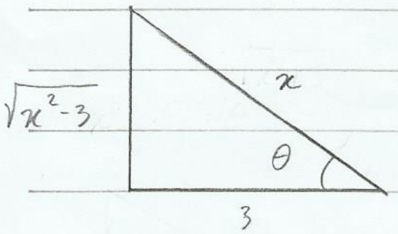
063) $\int \frac{1}{x^3\sqrt{x^2+9}}\,dx$ Caso seja $\int \frac{1}{x^3\sqrt{x^2-9}}\,dx = I$

Substituindo $x = 3\sec\theta$ $\quad dx = 3\sec\theta\, tg\theta\, d\theta$

$$\int \frac{3\sec\theta\, tg\theta}{27\sec^3\theta\sqrt{9(\sec^2+1)}}\,d\theta \sim \frac{1}{27}\int \frac{\sec\theta\, tg\theta}{\sec^3\theta\; tg\theta}\,d\theta \sim \frac{1}{27}\int \frac{1}{\sec^2\theta}\,d\theta$$

$$\frac{1}{\sec^2\theta} = 1 : \frac{1}{\cos^2\theta} = \cos^2\theta \quad \therefore \quad \frac{1}{27}\int \frac{1}{\sec^2\theta}\,d\theta \sim \frac{1}{27}\int \cos^2\theta\,d\theta$$

$$\int \frac{1}{x^3\sqrt{x^2-9}}\,dx = \frac{1}{27}\cdot\frac{1}{2}\left(\theta + \sin\theta\cos\theta\right)$$



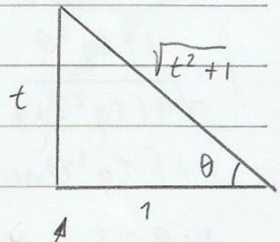
$\sec\theta = \frac{x}{3} = \frac{1}{\cos\theta}$ $\quad x\cos\theta = 3$ $\quad \cos\theta = \frac{3}{x}$

$\theta = \text{arcsec}\left(\frac{x}{3}\right)$ $\quad \sin\theta\cos\theta = \frac{\sqrt{x^2-3}}{x}\cdot\frac{3}{x}$

$$\therefore\ I = \frac{1}{54}\left(\text{arcsec}\left(\frac{x}{3}\right) + \frac{3\sqrt{x^2-3}}{x^2}\right) + C$$

064) $\int e^x\sqrt{1+e^{2x}}\,dx = I$

$e^x = t \quad e^x\,dx = dt$



$\int \sqrt{1+t^2}\,dt$ $\quad$ Substituindo $t = tg\theta$ $\quad dt = \sec^2\theta\,d\theta$

$\int \sqrt{1+tg^2\theta}\,\sec^2\theta\,d\theta \sim \int \sec^3\theta\,d\theta \sim \int \sec\theta\,\sec^2\theta\,d\theta$

$u = \sec\theta \quad du = \sec\theta\,tg\theta\,d\theta \quad dv = \sec^2\theta\,d\theta \quad v = tg\theta$

$$\boxed{\begin{array}{l}\sec\theta = \sqrt{t^2+1} = \sqrt{e^{2x}+1} \\ tg\theta = t = e^x\end{array}}$$

$\int \sec^3\theta\,d\theta = \sec\theta\,tg\theta - \int tg^2\theta\,\sec\theta\,d\theta$

$= \sec\theta\,tg\theta - \int (\sec^2-1)\sec\theta\,d\theta$

$= \sec\theta\,tg\theta - \int \sec^3\theta - \sec\theta\,d\theta$

$2\int \sec^3\theta\,d\theta = \sec\theta\,tg\theta + \ln|\sec\theta + tg\theta|$

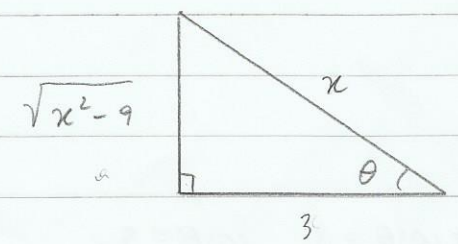
$$\int e^x\sqrt{1+e^{2x}}\,dx = \frac{1}{2}\left(\sec\theta\,tg\theta + \ln|\sec\theta + tg\theta|\right) = \frac{1}{2}\left(e^x\sqrt{e^{2x}+1} + \ln\left|\sqrt{e^{2x}+1}+e^x\right|\right) + C$$

065) $\int \frac{1}{x^2\sqrt{x^2-9}}\, dx$

$x = 3\sec\theta \quad dx = 3\sec\theta \, tg\theta \, d\theta$

$\int \frac{3\sec\theta \, tg\theta}{9\sec^2\theta\sqrt{9(\sec^2\theta - 1)}}\, d\theta \sim \frac{1}{9}\int \frac{\sec\theta \, tg\theta}{\sec^2\theta \, tg\theta}\, d\theta$

$\int \frac{1}{x^2\sqrt{x^2-9}}\, dx = \frac{1}{9}\int \cos\theta \, d\theta = \frac{1}{9}\operatorname{sen}\theta + C$



$\sec\theta = \frac{x}{3} \quad \cos\theta = \frac{1}{\sec\theta} = \frac{3}{x} = \cos\theta$

$\operatorname{sen}\theta = \frac{\sqrt{x^2-9}}{x}$

$\int \frac{1}{x^2\sqrt{x^2-9}}\, dx = \frac{\sqrt{x^2-9}}{9x} + C$

066) $\int \frac{x^3}{\sqrt{x^2+9}}\, dx$

$x = 3\, tg\,\theta \quad dx = 3\sec^2\theta \, d\theta$

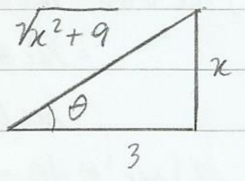
$\int \frac{27\, tg^3\theta}{\sqrt{9(tg^2+1)}} \cdot 3\sec^2\theta \, d\theta = 27\int \frac{tg^3\theta \sec^2\theta}{\sec\theta}\, d\theta$

$27\int tg^3\theta \sec\theta \, d\theta \sim 27\int (\sec^2\theta - 1)\sec\theta \, tg\theta \, d\theta$

$\sec\theta = t \quad \sec\theta \, tg\theta \, d\theta = dt$

$\therefore 27\int (\sec^2\theta - 1)\sec\theta \, tg\theta \, d\theta \sim 27\int (t^2-1)\, dt = \frac{27t^3}{3} - 27t + C$

$\int \frac{x^3}{\sqrt{x^2+9}}\, dx = 9\sec^3\theta - 27\sec\theta + C$



$= \frac{9.(x^2+9)\sqrt{x^2+9}}{27} - \frac{27}{3}\sqrt{x^2+9} + C$

$\sec\theta = \frac{1}{\cos} = \frac{\sqrt{x^2+9}}{3}$

$= \frac{1}{3}(x^2+9)\sqrt{x^2+9} - 9\sqrt{x^2+9} \Rightarrow \frac{x^2}{3}\sqrt{x^2+9} + C$

$$067\text{-})\int_{\sqrt{2}}^{2}\frac{1}{t^3\sqrt{t^2-1}}\,dt$$

$t = \sec\theta \quad dt = \sec\theta\, tg\theta\, d\theta$

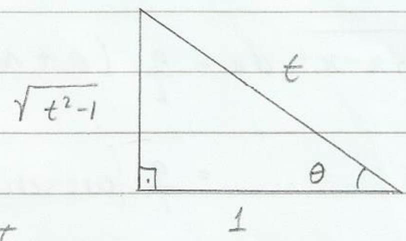
$$\int_{\sqrt{2}}^{2}\frac{\sec\theta\, tg\theta}{\sec^3\theta\sqrt{\sec^2\theta-1}}\,d\theta \sim \int_{\sqrt{2}}^{2}\frac{tg\theta}{\sec^2\theta\,.\,tg\theta}\,d\theta \sim \int_{\sqrt{2}}^{2}\frac{1}{\sec^2\theta}\,d\theta$$

$$\Rightarrow \int_{\sqrt{2}}^{2}\cos^2\theta\,d\theta = \frac{1}{2}\int 1+\cos(2\theta)\,d\theta = \frac{1}{2}\theta + \frac{\operatorname{sen}(2\theta)}{4}\Bigg|_{\sqrt{2}}^{2}$$

$\operatorname{sen}(A+B) = \operatorname{sen}A\cos B + \operatorname{sen}B\cos A$

$\operatorname{sen}(\theta+\theta) = \operatorname{sen}\theta\cos\theta + \operatorname{sen}\theta\cos\theta$

$\therefore \operatorname{sen}(2\theta) = 2\operatorname{sen}\theta\cos\theta$



$$= \frac{1}{2}\theta + \frac{1}{\cancel{4}_2}\cancel{2}\operatorname{sen}\theta\cos\theta\Bigg|_{\sqrt{2}}^{2}$$

$\sec\theta = t$

$$= \frac{1}{2}\left(\theta + \operatorname{sen}\theta\cos\theta\right)\Bigg|_{\sqrt{2}}^{2} \qquad \cos\theta = \frac{1}{t}$$

$$= \frac{1}{2}\left(\operatorname{arc\,tg}\left(\sqrt{t^2-1}\right) + \frac{\sqrt{t^2-1}}{t^2}\right)\Bigg|_{\sqrt{2}}^{2}$$

$$= \frac{1}{2}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4} - \frac{\pi}{4} - \frac{1}{2}\right) \Rightarrow \frac{1}{2}\left(\frac{4\pi-3\pi}{12} + \frac{\sqrt{3}}{4} - \frac{1}{2}\right)$$

$$\int_{\sqrt{2}}^{2}\frac{1}{t^3\sqrt{t^2-1}}\,dt = \frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$$

$$(x-2)^2 = x^2-4x+4$$
$$(x^2-4x-5) = (x-2)^2-9$$

068) $\int \sqrt{5+4x-x^2}\,dx$

$$\int \sqrt{-(x^2-4x-5)}\,dx \sim \int \sqrt{-(x-2)^2+9}\,dx$$

$x-2 = t \qquad dx = dt$

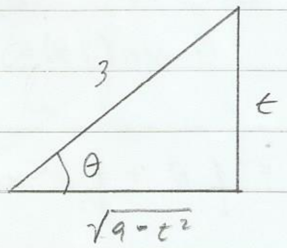
$$\int \sqrt{-t^2+9}\,dt \sim \int \sqrt{9-t^2}\,dt$$

$t = 3\,\text{sen}\,\theta \qquad dt = 3\cos\theta$

$$\int \sqrt{9-t^2}\,dt \sim \int \sqrt{9-9\,\text{sen}^2\theta}\,.\,3\cos\theta\,d\theta \sim \int \sqrt{9(1-\text{sen}^2\theta)}\,3\cos\theta\,d\theta$$

$$9\int \cos^2\theta\,d\theta = \frac{9}{2}\left(\theta+\text{sen}\,\theta\cos\theta\right)$$

$$\int \sqrt{5+4x-x^2}\,dx = \frac{9}{2}\left(\theta+\text{sen}\,\theta\cos\theta\right)$$



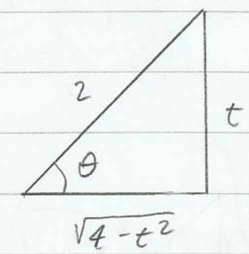
$$= \frac{9}{2}.\left(\text{arcsen}\left(\frac{t}{3}\right)+\frac{t\sqrt{9-t^2}}{9}\right)$$

$$= \frac{9}{2}.\left(\text{arcsen}\,\frac{(x-2)}{3}+\frac{(x-2)\sqrt{5+4x-x^2}}{9}\right)$$

$$\int \sqrt{5+4x-x^2}\,dx = \frac{9}{2}\,\text{arcsen}\left(\frac{x-2}{3}\right)+\frac{1}{2}(x-2)\sqrt{5+4x-x^2}+C$$

069) $\int \frac{\cos(x)}{4-\text{sen}^2(x)}\,dx$

$t = \text{sen}(x) \qquad dt = \cos(x)\,dx$



$\int \frac{dt}{4-t^2}$ Admitindo $t = 2\,\text{sen}\,\theta \qquad dt = 2\cos\theta\,d\theta$

$$\int \frac{2\cos\theta}{4(1-\text{sen}^2\theta)}\,d\theta \sim \frac{1}{2}\int \frac{\cos\theta}{\cos^2\theta}\,d\theta = \frac{1}{2}\int \sec\,d\theta =$$

$$= \frac{1}{2}\ln|\sec\theta+tg\theta|+C = \frac{1}{2}\ln\left|\frac{2}{\sqrt{4-t^2}}+\frac{t}{\sqrt{4-t^2}}\right| =$$

$$\frac{1}{2}\ln\left|\frac{2+t}{\sqrt{4-t^2}}.\frac{\sqrt{4-t^2}}{\sqrt{4-t^2}}\right| = \frac{1}{2}\ln\left|\frac{\cancel{(2+t)}\sqrt{4-t^2}}{\cancel{(2+t)}(2-t)}\right| = \frac{1}{2}\ln\left|\frac{\sqrt{4-\text{sen}^2(x)}}{2-\text{sen}(x)}\right|+C$$

spiral

070-) $\int \frac{sen(2x)}{1+cos(x)}\,dx$ $\qquad sen(A+B) = senA\cos B + senB\cos A$

$sen(2A) = senA\cos A + senA\cos A$

$\sim \int \frac{2\,sen(x)\cos(x)}{1+\cos(x)}\,dx \qquad sen(2x) = 2\,sen(x)\cos(x)$

$t = 1+\cos(x) \qquad dt = sen(x)\,dx$

$\int \frac{2(t-1)}{t}\,dt \sim 2\int \frac{t-1}{t}\,dt = 2\int dt - 2\int \frac{1}{t}\,dt$

$= 2t - \ln|t| + c = 2(t - \ln|t|) + c$

$= 2(1+\cos(x) - \ln|1+\cos(x)|) + c$

$= 2(\cos(x) - \ln|1+\cos(x)|) + c$

071-) $\int \frac{1}{\sqrt{3}\cos(x) - sen(x)}\,dx$

Integrais de funções Racionais

073-) $\int \frac{3}{x^2+x-2}\,dx \sim \int \frac{3}{(x-1)(x+2)}\,dx \sim$

$$\frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{(x-1)(x+2)} =$$

$$\frac{3}{(x-1)(x+2)} = \frac{Ax+2A+Bx-B}{(x-1)(x+2)}$$

$$A(x+2)+B(x-1)=3$$

Admitindo $x=-2$

$B(-2-1)=3 \Rightarrow B=-1$

Admitindo $x=1$

$A(1+2)=3 \Rightarrow A=1$

$$\int \frac{3}{x^2+x-2}\,dx = \frac{+1}{x-1} - \frac{1}{x+2} = -\int \frac{1}{x+2}\,dx + \int \frac{1}{x-1}\,dx$$

$$= \ln|x-1| - \ln|x+2| + C$$

$$= \frac{\ln|x-1|}{\ln|x+2|} + C$$

074-) $\int \frac{x^2+12x+12}{x^3-4x}\,dx \Rightarrow \int \frac{x^2+12x+12}{x(x^2-4)}\,dx \Rightarrow \int \frac{x^2+12x+12}{x(x+2)(x-2)}\,dx$

$$\frac{A}{x} + \frac{B}{(x+2)} + \frac{C}{(x-2)} = \frac{A(x+2)(x-2)+B(x-2)x+C(x+2)x}{x(x+2)(x-2)}$$

$$A(x+2)(x-2)+B(x-2)x+C(x+2)x = x^2+12x+12$$

Admitindo $x=-2$

$B(-2-2)-2 = 4-24+12 \quad \therefore B=-1$

Admitindo $x=2$

$C(2+2).2 = 4+24+12 \quad \therefore C=5$

$A(x^2-4x)$