Equações Diferenciais Lineares de 2ª ordem

$ay'' + by' + cy = g(x)$

Vamos supor a, b e c números reais. Para resolver uma EDOL de 2ª ordem, primeiro resolvemos a equação onde $g(x) = 0$, que é chamada de homogênea:

$ay'' + by' + cy = 0$

Observe que a solução desta equação diferencial depende das raízes do polinômio associado, que chamamos de polinômio característico:

$p(\lambda) = a\lambda^2 + b\lambda + c$

1º caso $\quad \lambda_1 \neq \lambda_2$ reais

$$\boxed{y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}}$$

2º caso $\quad \lambda_1 = \lambda_2$ reais

$$\boxed{y = C_1 e^{\lambda_1 x} + C_2 x e^{\lambda_1 x}} \qquad y = C_1 e^{\lambda_1 x} + C_2 \cdot x \cdot e^{\lambda_1 x}$$

3º caso $\quad \lambda = a \pm bi$

$$\boxed{y = e^{ax}(c_1 \cos(bx) + c_2 \operatorname{sen}(bx))}$$

Exemplos:

1) $y'' - 5y' + 6y = 0$

$$\lambda = \frac{5 \pm \sqrt{25 - 24}}{2} \qquad \lambda_1 = 3 \qquad \lambda_2 = 2$$

$$y = C_1 e^{3x} + C_2 e^{2x}$$

Equações Diferenciais Lineares de 2ª Ordem, não homogêneas

Vamos estudar a solução das equações do seguinte tipo:

$$\boxed{ay'' + by' + cy = g(x)} \quad (*)$$

onde $g(x)$ é uma função diferente de zero. Para resolver esta equação, primeiro resolvemos a homogênea associada

$$ay'' + by' + cy = 0$$

Depois, vamos procurar uma outra solução, sempre relacionada à função $g(x)$. Esta solução é chamada de solução particular: y_P

A solução de $(*)$ é:

$$y = y_H + y_P$$

Vamos estudar $g(x)$ em casos:

1º caso $g(x) = C$, C = constante

Vamos procurar $y_P = A$, A = constante, • se o polinômio da homogênea associada não tiver $\lambda = 0$ como raiz. • Se $\lambda = 0$ for raiz simples de $p(\lambda)$, $y_P = Ax$ e, • se $\lambda = 0$ for raiz dupla de $p(\lambda)$, então $y_P = Ax^2$

Exemplos

1) $y'' - 3y' + 2y = 7$ Homogênea associada
(*)
$$y'' - 3y' + 2y = 0$$

/ /

$p(\lambda) = \lambda^2 - 3\lambda + 2 \quad \begin{cases} \lambda = 1 \\ \lambda = 2 \end{cases}$

$y_H = C_1 e^{x} + C_2 e^{2x}$

$y_p = A$ — Substitui (*)

$y'_p = 0$ — $0 - 3.0 + 2A = 7$

$y''_p = 0$ — $A = 7/2$

$y_G = C_1 e^{x} + C_2 e^{2x} + 7/2$

2) $y'' - y' = 8/3$

Homogênea associada

$y'' - y' = 0$

$p(\lambda) = \lambda^2 - \lambda \qquad \lambda(\lambda - 1) = 0 \quad \begin{cases} \lambda = 0 \\ \lambda = 1 \end{cases}$

$y_H = C_1 + C_2 e^{x}$

Substituição

$y_p = Ax$ — $-A = 8/3$

$y'_p = A$ — $A = -8/3$

$y''_p = 0$

$y = C_1 + C_2 e^{x} - \frac{8x}{3}$

3) $y'' = 15/2$

Homogênea associada — $y_p = Ax^2$

$y'' = 0$ — $y'_p = 2Ax$

$p(\lambda) = y^2 \qquad y = 0 \text{ e } y = 0$ — $y''_p = 2A$

$y_H = C_1 e^{0x} + C_2 x e^{0x} = C_1 + x C_2$

Substituição

$2A = 15/2$

$A = 15/4$

$y_g = C_1 + xC_2 + \frac{15}{4}x^2$

2º caso: $g(x) = p_n(x)$ (polinômio de grau n)

Como já sabemos, vai depender das raízes de $p(\lambda)$ da homogênea associada. Então.

- Se $\lambda = 0$ não é raiz, $y_p = A_n x^n + A_{n-1}x^{n-1} + \dots + A_1 x + A_0$;
- Se $\lambda = 0$ for raiz simples, $y_p = (A_n x^n + A_{n-1}x^{n-1} + \dots + A_1 x + A_0).x$;
- Se $\lambda = 0$ for raiz dupla, $y_p = (A_n x^n + A_{n-1}x^{n-1} + \dots + A_1 x + A_0).x^2$.

Exemplos

1) $y'' - 2y' + 2y = 3x + 2$

homogênea associada:

$y'' - 2y' + 2y = 0$

$p(\lambda) = \lambda^2 - 2\lambda + 2$

$\lambda = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$

$y_H = e^x(C_1 \cos x + C_2 \sen x)$

$y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

Substituindo:

$0 - 2A + 2(Ax + B) = 3x + 2$

$-2A + 2Ax + 2B = 3x + 2$

$2A = 3 \Rightarrow A = 3/2$

$-2A + 2B = 2$

$-A + B = 1$

$-\frac{3}{2} + B = 1 \quad B = 5/2$

$y_? = e^x(C_1 \cos x + C_2 \sen x) + \frac{3x}{2} + \frac{5}{2}$

2) $y'' - 2y' = x^2+1$

Homogênea associada

$$y'' - 2y' = 0$$

$p(\lambda) = \lambda^2 - 2\lambda \qquad \lambda(\lambda-2)=0 \qquad y_h = C_1 + C_2 e^{2x}$

$\lambda = 0$ e $\lambda = 2$

$$y_p = (Ax^2 + Bx + C)x = Ax^3 + Bx^2 + Cx$$

$$y'_p = 3Ax^2 + 2Bx + C$$

$$y''_p = 6Ax + 2B$$

$$6Ax + 2B - 2(3Ax^2 + 2Bx + C) = x^2 + 1$$

$$6Ax + 2B - 6Ax^2 - 4Bx - 2C = x^2 + 1$$

$-6A = 1 \Rightarrow \underline{A = -1/6}$

$6A - 4B = 0 \qquad\qquad 2B - 2C = 1$

$-1 - 4B = 0 \Rightarrow \underline{B = -1/4} \qquad \frac{-1}{2} - 2C = 1 \Rightarrow C = \frac{-3}{4}$

$$y_p = \left(-\frac{1}{6}x^2 - \frac{1}{4}x - \frac{3}{4}\right)x$$

$$y_g = C_1 + C_2 e^{2x} + \left(-\frac{1}{6}x^2 - \frac{1}{4}x - \frac{3}{4}\right)x$$

Questão 151 errado

Equações Diferenciais de 2ª ordem, não homogêneas

3º caso: $g(x) = e^{ax}$

* $y_p = Ae^{ax}$, se $\lambda = a$ não for raiz de $p(x)$;
* $y_p = Axe^{ax}$, se $\lambda = a$ for raiz simples de $p(\lambda)$;
* $y_p = Ax^2e^{ax}$, se $\lambda = a$ for raiz dupla se $p(\lambda)$.

Exemplos:

1-) $y'' - 7y' + 12y = e^{2x}$

homogênea associada:

$y'' - 7y' + 12y = 0$

$p(\lambda) = \lambda'' - 7\lambda + 12$

$\lambda = \dfrac{7 \pm \sqrt{(7)^2 - 4.12}}{2}$

$\lambda_1 = 4 \quad \lambda_2 = 3$

$y_H = C_1e^{3x} + C_2e^{4x}$

$y_p = Ae^{2x}$

$y'p = 2Ae^{2x}$

$y''p = 4Ae^{2x}$

Substitui

$4Ae^{2x} - 7.2Ae^{2x} + 12Ae^{2x} = e^{2x}$

$2Ae^{2x} = e^{2x}$

$2A = 1 \quad ; \quad A = 1/2$

$y_H = C_1e^{3x} + C_2e^{4x} + \frac{1}{2}e^{2x}$

2-) $y'' - 4y' + 4y = 3e^{2x}$

homogênea associada

$y'' - 4y' + 4y$

$p(\lambda) = \lambda^2 - 4\lambda + 4$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = \frac{4 \pm \sqrt{16-16}}{2}$ $\lambda = 2$

$y_H = C_1 e^{2x} + C_2 e^{2x}$

$y_p = Ax^2 e^{2x}$

$y'_p = 2Axe^{2x} + Ax^2 . 2e^{2x} = 2Axe^{2x} + 2Ax^2 e^{2x}$

$y''_p = 2Ae^{2x} + 2Ax . 2e^{2x} + 4Axe^{2x} + 2Ax^2 . 2e^{2x}$

$= 2Ae^{2x} + 4Axe^{2x} + 4Axe^{2x} + 4Ax^2 e^{2x}$

$= 2Ae^{2x} + 8Axe^{2x} + 4Ax^2 e^{2x}$

Substitui

$2Ae^{2x} + \cancel{8Axe^{2x}} + 4Ax^2e^{2x} - \cancel{8Axe^{2x}} - 8Ax^2e^{2x} + 4Ax^2e^{2x} = 3e^{2x}$

$2A\cancel{e^{2x}} = 3e^{2x}$ $\therefore$ $A = \frac{3}{2}$

$y_g = C_1 e^{2x} + C_2 e^{2x} + \frac{3}{2} x^2 e^{2x}$

4º caso: $g(x) = \cos(bx)$ ou $\operatorname{sen}(bx)$

* $y_p = A\cos(bx) + B\operatorname{sen}(bx)$ se $\lambda = a \pm bi$ não for raiz de $p(\lambda)$.
* $y_p = Ax\cos(bx) + Bx\operatorname{sen}(bx)$ se $\lambda = a \pm bi$ for raiz de $p(\lambda)$

Exemplos:

1-) $y'' - 5y' + 6y = \cos(2x)$

homogênea associada

$y'' - 5y' + 6y$

$p(\lambda) = \lambda^2 - 5\lambda + 6$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda = \frac{5 \pm \sqrt{25-24}}{2}$ $\quad \lambda_1 = 3 \quad \lambda_2 = 2$

$y = C_1 e^{3x} + C_2 e^{2x}$

$y_p = A\cos(2x) + B\operatorname{sen}(2x)$

$y'_p = -2A\operatorname{sen}(2x) + 2B\cos(2x)$

$y''_p = -4A\cos(2x) - 4B\operatorname{sen}(2x)$

$-4A\cos(2x) - 4B\operatorname{sen}(2x) + 10A\operatorname{sen}(2x) - 10B\cos(2x) + 6A\cos(2x) + 6B\operatorname{sen}(2x) = \cos(2x)$

$2A\cos(2x) + 2B\operatorname{sen}(2x) + 10A\operatorname{sen}(2x) - 10B\cos(2x) = \cos(2x)$

$\begin{cases} 2A\cos(2x) - 10B\cos(2x) = \cos(2x) \\ 2B\operatorname{sen}(2x) + 10A\operatorname{sen}(2x) = 0 \end{cases}$

$\begin{cases} 2A - 10B = 1 \\ 10A + 2B = 0 \end{cases}$ $\quad \begin{cases} 2A - 10B = 1 \\ 50A + 10B = 0 \end{cases}$ $\quad \therefore 52A = 1$

$A = \frac{1}{52}$

$B = \frac{-10A}{2} = \frac{-10}{52} : 2 = \frac{-10}{104} = -\frac{5}{52}$

$y = C_1 e^{3x} + C_2 e^{2x} + \frac{1}{52}\cos(2x) - \frac{5}{52}\operatorname{sen}(2x)$

/ /

2) $y'' - 2y' + 2y = sen(3x)$

homogênea associada $\lambda^2 - 2\lambda + 2 = 0$

$y'' - 2y' + 2y$ $\lambda = \dfrac{2 \pm \sqrt{4-8}}{2}$ $\lambda = 1 \pm i$

$p(\lambda) = \lambda^2 - 2\lambda + 2$

$y_h = e^{x}(C_1 \cos(x) + C_2 sen(x))$

Não é este caso pois $b = 3$ da eq. é $\neq$ $1 \pm i$

, então utilizo:

$y_p = A\cos(3x) + B sen(3x)$

$y'_p = -3A sen(3x) + 3B\cos(3x)$

$y''_p = -9A\cos(3x) - 9B sen(3x)$

$-9A\cos(3x) - 9B sen(3x) + 6A sen(3x) - 6B\cos(3x) + 2A\cos(3x) + 2B sen(3x) = sen(3x)$

$-7A\cos(3x) - 7B sen(3x) + 6A sen(3x) - 6B\cos(3x) = sen(3x)$

$$\begin{cases} 6A sen(3x) - 7B sen(3x) = sen(3x) \\ -7A\cos(3x) - 6B\cos(3x) = 0 \end{cases}$$

$$\begin{cases} 6A - 7B = 1 \\ -7A - 6B = 0 \end{cases} \qquad A = \frac{1+7B}{6} \qquad B = \frac{-7A}{6}$$

$$B = \left(-7 \cdot \frac{(1+7B)}{6}\right) \div 6 = \frac{-7-49B}{36}$$

$36B = -7 - 49B$ $85B = -7$ $\therefore$ $B = -7/85$ $A = 6/85$

$$y_h = e^{x}(C_1\cos(x) + C_2 sen(x)) + \frac{6}{85}\cos(3x) - \frac{7}{85} sen(3x)$$

Exercícios

Resolva o problema de valor inicial

$$\begin{cases} y'' - 7y' = x - 1 + 4e^{7x} \\ y(0) = 1 \\ y'(0) = 1/7 \end{cases}$$

equação homogênea

$y'' - 7y'$

$p(\lambda) = \lambda^2 - 7\lambda = 0$ $\quad \lambda(\lambda - 7) = 0$

$\lambda = 0 \quad \lambda = 7$

$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$

$y = C_1 + C_2 e^{7x}$

$y_p = (Ax + B)x = Ax^2 + Bx$

$y'_p = 2Ax + B$

$y''_p = 2A$

Substituindo

$2A - 14Ax - 7B = x - 1$

$-14A = 1 \therefore A = -1/14$

$2 \cdot \left(\frac{-1}{14}\right) - 7B = -1 \therefore B = 6/49$

$\therefore y_p = \frac{-1}{14}x^2 + \frac{6}{49}x$

$y_p = Axe^{7x}$

$y'_p = A(e^{7x} + 7xe^{7x})$

$y''_p = A(7e^{7x} + 7(e^{7x} + 7xe^{7x}))$

$= A7e^{7x} + A7e^{7x} + 49Axe^{7x} = 14Ae^{7x} + 49xe^{7x}$

Substituindo.

$14Ae^{7x} + \cancel{49xe^{7x}} - 7Ae^{7x} - \cancel{49xe^{7x}} = 4e^{7x}$

$7A = 4 \quad \therefore A = \frac{4}{7}$

$\therefore y_{p2} = \frac{4}{7}xe^{7x}$

$$y = C_1 + C_2 e^{7x} - \frac{1}{14}x^2 + \frac{6}{49}x + \frac{4}{7}xe^{7x}$$

$$y(0) = C_1 + C_2 = 1$$

$$y' = 7C_2 e^{7x} - \frac{1}{7}x + \frac{6}{49} + \frac{4}{7}\left(e^{7x} + 7xe^{7x}\right)$$

$$y'(0) = 7C_2 + \frac{6+28}{49} = 7C_2 + \frac{34}{49} = \frac{1}{7}$$

$$C_2 = \left(\frac{1}{7} - \frac{34}{49}\right) \div 7 = \frac{-27}{343}$$

$$C_1 + C_2 = 1$$

$$C_1 = 1 - C_2$$

$$C_1 = 1 + \frac{27}{343} = \frac{370}{343}$$

$$y = \frac{370}{343} - \frac{27}{343}e^{7x} - \frac{1}{14}x^2 + \frac{6}{49}x + \frac{4}{7}xe^{7x}$$

P2

2.) Resolver

$$y'' - y = e^x(x-2) \Rightarrow y'' - y = xe^x - 2e^x$$

$$p(\lambda) = \lambda^2 - 1 = 0 \quad \therefore \lambda = \pm 1$$

$$y_h = C_1 e^{-x} + C_2 e^x$$

$$y_p = x(Ax+B)e^x = Ax^2e^x + Bxe^x$$

$$y'_p = A(2xe^x + x^2e^x) + B(e^x + xe^x)$$

$$y''_p = A(2(e^x + xe^x) + 2xe^x + x^2e^x) + B(e^x + e^x + xe^x))$$

$$= 2Ae^x + 2Axe^x + 2Axe^x + Ax^2e^x + 2Be^x + Bxe^x$$

$x e^x \Rightarrow (Ax+B)e^x$

Substituindo:

$2Ae^x + 4Axe^x + Ax^2e^x + 2Be^x + \cancel{Bxe^x} - Ax^2e^x - \cancel{Bxe^x} = xe^x - 2e^x$

$4A = 1 \quad \therefore \quad A = \frac{1}{4}$

$2A + 2B = -2$

$2 \cdot \frac{1}{4} + 2B = -2 \quad \therefore \quad B = \frac{-5}{4}$

$y_p = \frac{1}{4}x^2e^x - \frac{5}{4}xe^x$

$y_H = C_1e^{-x} + C_2e^x + \frac{1}{4}x^2e^x - \frac{5}{4}xe^x$

3) Resolver

$y'' + 4y' + 3y = 8xe^x - 6$

Homogenea associada

$p(\lambda) = \lambda^2 + 4\lambda + 3$

$\lambda^2 + 4\lambda + 3 = 0$

$\lambda = \frac{-4 \pm \sqrt{16-12}}{2} \qquad \lambda_1 = -1 \quad \lambda_2 = -3$

$y = C_1e^{\lambda_1 x} + C_2e^{\lambda_2 x} \Rightarrow y = C_1e^{-x} + C_2e^{-3x}$

$A = 1$

$B = \frac{-3}{4}$

$y_{p_1} = (Ax+B)e^x \Rightarrow Axe^x + Be^x$

$y'_{p_1} = A(e^x + xe^x) + Be^x$

$y''_{p_1} = A(e^x + e^x + xe^x) + Be^x$

$= 2Ae^x + Axe^x + Be^x$

$2Ae^x + Axe^x + Be^x + Ae^x + Axe^x + Be^x + Axe^x + Be^x = 8xe^x - 6$

$3Ae^x + 3Axe^x + 3Be^x = 8xe^x - 6$

$3A \quad + 3B \quad = 0$

$3A \qquad\qquad = 8 \qquad A = \frac{8}{3}$

(P2 · 2°/2008)

1ª Questão

Seja a função $f(x,y) = \ln(-x+y+8) + \dfrac{1}{\sqrt{x^2+y^2-64}}$ Determine o dominio o domínio de $f(x,y)$ e represente graficamente

2ª Questão (subs.t

Resolva: $y'' + 2y' + y = 3xe^{-x}$

3ª Questão

Determinar as equações dos planos tangentes à superfície $z = x^2+y^2$ que sejam paralelos a $8x - 4y + 2z - 1 = 0$

4ª Questão

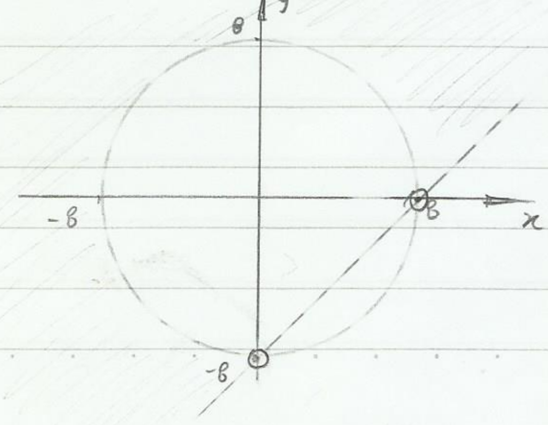
Encontre o valor mínimo de $z = x^3+y^3+xy$ sujeito a restrição $x+y-4=0$ $f(2,2)=20$

5ª Questão

Calcule o volume do sólido limitado superiormente pela superfície $z = 4 - x^2 - y^2$ e inferiormente por $R = \{(x,y) \in \mathbb{R}^2 / y = -2x+2,\ x \geq 0,\ y \geq 0\}$

$V = \frac{13}{6}$ u.v

1-) $D_f = \{(x,y) \in \mathbb{R}^2 / -x+y+8 > 0$ e $x^2+y^2-64 \geq 0\}$



$-x+y+8 = 0$ $\qquad$ $x^2+y^2-64=0$

se $x=0$ $\quad y=-8$ $\qquad$ se $x=0$ $\quad y=\pm 8$

se $y=0$ $\quad x=8$ $\qquad$ se $y=0$ $\quad x=\pm 8$

/ /

2.)

$y'' + 2y' + y = 3xe^{-x}$

homogênea associada

$p(\lambda) = \lambda^2 + 2\lambda + 1$ $\qquad y_h = C_1 e^{-x} + xC_2 e^{-x}$

$\lambda = \frac{-2 \pm \sqrt{4-4}}{2} = -1$

$y_p = (Ax+B)x^2 e^{-x} = Ax^3e^{-x} + Bx^2e^{-x}$

$y'_p = A(3x^2e^{-x} + x^3.(-1)e^{-x}) + B(2xe^{-x} + x^2(-1)e^{-x})$

$= A(3x^2e^{-x} - x^3e^{-x}) + B(2xe^{-x} - x^2e^{-x})$

$y''_p = A(3[2xe^{-x} + x^2(-1)e^{-x}] - (3x^2e^{-x} + x^3(-1)e^{-x}) + B(2[e^{-x} + x(-1)e^{-x}]$

$-(2xe^{-x} + x^2(-1)e^{-x})$

$= 6Axe^{-x} - 6Ax^2e^{-x} - 3Ax^2e^{-x} + Ax^3e^{-x} + 2Be^{-x} - 2Bxe^{-x} - 2Bxe^{-x} +$

Bx^2e^{-x}

$6A - 4B + 4Bx = 3 \quad \therefore \quad A = \frac{3}{6} = \frac{1}{2}$

ou

$y_p = (Ax+B)x^2e^{-x}$

$y'_p = A(x^2e^{-x}) + (Ax+B)(2xe^{-x} + x^2(-1)e^{-x})$

$y''_p = A(2xe^{-x} - x^2e^{-x}) + A(2xe^{-x} - x^2e^{-x}) + (Ax+B)(2(e^{-x} - xe^{-x}) -$

3-) $8x - 4y + 2z - 1 = 0$

$4x - 2y + z - \frac{1}{2} = 0 \quad \therefore \quad z = -4x + 2y + \frac{1}{2} \quad \vec{V}_o = (4, -2, 1)$

$3x = -4 \quad 2y = 2$

$fx = 2x = -4 \quad \therefore \quad x = -2$

$fy = 2y = 2 \quad \therefore \quad y = 1$

$z = 2^2 + 1^2 = 5$

$-4(x+2) + 2(y-1) - (z-5) = 0$

$-4x + 2y - z - 8 - 2 + 5 = 0$

$z = -4x + 2y + 5$

4-) $L(x, y, \lambda) = x^3 + y^3 + xy - 2 + \lambda(x + y - 4)$

$\frac{dL}{dx} = 3x^2 + y + \lambda \quad \frac{dL}{dy} = 3y^2 + x + \lambda \quad \frac{dL}{d\lambda} = x + y - 4$

$3x^2 + y + \lambda = 0 \qquad x = 4 - y$

$x + 3y^2 + \lambda = 0$

$x + y - 4 = 0$

$3(4-y)^2 + y = 4 - y + 3y^2$

$3(16 - 16y + y^2) + y = 4 - y + 3y^2$

$48 - 48y + \cancel{3y^2} + y - 4 + y \cancel{-3y^2} = 0$

$-46y + 44 = 0$

$y = \frac{44}{46} = \frac{22}{23}$

1.) $u = f(x,y,z) = y + \dfrac{y-x}{x-z}$

$Df = \{(x,y) \in \mathbb{R}^2 \mid x - z \neq 0\}$

b)

$\dfrac{\partial u}{dx} = \dfrac{-(x-z)-(y-x)}{(x-z)^2} = \dfrac{-x+x+z-y}{(x-z)^2} = \dfrac{z-y}{(x-z)^2}$

$\dfrac{\partial u}{dy} = 1 + \dfrac{1}{(x-z)} = \dfrac{x-z+1}{(x-z)}$

$\dfrac{du}{dz} = \dfrac{(y-x).(-1).(-1)}{(x-z)^2} = \dfrac{y-x}{(x-z)^2}$

$k = \dfrac{z-\cancel{y}}{(x-z)^2} + \dfrac{\cancel{y}-x}{(x-z)^2} + \dfrac{x-z+1}{(x-z)}$

$= \dfrac{\cancel{z}-\cancel{x}+x^2-xz+\cancel{x}-xz+z^2-\cancel{z}}{(x-z)^2}$

$= \dfrac{x^2-2xz+z^2}{x^2-2xz+z^2} = \underline{1}$

2) $y'' - 4y' = -32\cos(4x)$

Equação homogênea

$p(\lambda) = \lambda^2 - 4\lambda = 0$ $\qquad y_h = C_1 + C_2 e^{4x}$

$\lambda(\lambda - 4) = 0$

$\lambda = 0 \quad \lambda = 4$

$y_p = A\cos(4x) + B\sin(4x)$
$y'_p = -4A\sin(4x) + 4B\cos(4x)$
$y''_p = -16A\cos(4x) - 16B\sin(4x)$

$-16A\cos(4x) - 16B\sin(4x) + 16A\sin(4x) - 16B\cos(4x) = -32\cos(4x)$

$$\begin{cases} -16A - 16B = -32 \\ 16A - 16B = 0 \end{cases} \qquad A = \frac{16B}{16} = 1$$

$-32B = -32$

$B = 1$

$y = C_1 + C_2 e^{4x} + \sin(4x) + \cos(4x)$

4) $f(x,y) = y^3 + x^2 - xy - 4y + x$

$f_x(x,y) = 2x - y + 1$
$f_y(x,y) = 3y^2 - x - 4$

$$\begin{cases} 2x - y + 1 = 0 & \text{II} \\ 3y^2 - x - 4 = 0 & \text{I} \end{cases} \quad x = 3y^2 - 4$$

I e II $\quad 6y^2 - 8 - y + 1 = 0$
$6y^2 - y - 7 = 0$

$y = \frac{1 \pm \sqrt{1 + 178}}{12} \qquad y_1 = -1 \quad y_2 = \frac{7}{6}$

$x_1 = \frac{y-1}{2} \quad \frac{-1-1}{2} = -1 \qquad x_2 = \frac{\frac{7}{6} - 1}{2} = \frac{\frac{1}{6}}{2} \quad \frac{1}{12}$

$P_1($

Equação Diferencial Linear

133) $xy' - 2y = x^2$

$y' - \frac{2y}{x} = x$ $\qquad I = e^{\int \frac{-2}{x} dx} = e^{-2\int \frac{1}{x} dx} = e^{-2\ln(x)}$

$y' e^{-2\ln(x)} - \frac{2y e^{-2\ln(x)}}{x} = x e^{-2\ln(x)}$ $\qquad \int x e^{-2\ln(x)} dx$

$u = x \quad dv = e^{-2\ln(x)} dx$

$\int (y e^{-2\ln(x)})' = \int x e^{-2\ln(x)} dx$ $\qquad v = \int e^{-2\ln(x)} dx$

$y e^{-2\ln(x)} = -2 + 2\ln(x)$ $\qquad s = e^{-2\ln(x)} \quad dt = dx$

$ds = \frac{-2}{x} e^{-2\ln(x)} \quad t = x$

$\int e^{-2\ln(x)} dx = e^{-2\ln(x)} x - \int x$

134-) $xy' + y = \sqrt{x}$ $\qquad y' + \frac{1}{x} y = \frac{\sqrt{x}}{x}$ $\qquad \int \frac{\sqrt{x}}{x} e^{\ln(x)} dx$

$I = e^{\int \frac{1}{x} dx} = e^{\ln(x)}$ $\qquad \ln(x) = t \quad x = e^t$

$\frac{1}{x} dx = dt$

$y' e^{\ln(x)} + \frac{1}{x} y e^{\ln(x)} = \frac{\sqrt{x}}{x} e^{\ln(x)}$

$\int \sqrt{e^t} e^t dt = \int (e^t)^{\frac{3}{2}} dt$

$\int (y e^{\ln(x)})' = \int \frac{\sqrt{x}}{x} e^{\ln(x)} dx$

$\frac{3t}{2} = s \Rightarrow \frac{3}{2} dt = ds$

$y e^{\ln(x)} = \frac{2}{3} e^{\frac{3}{2}\ln(x)} + C$

$\int e^s \cdot \frac{2}{3} ds = \frac{2}{3} e^s = \frac{2}{3} e^{3t/2} = \frac{2}{3} e^{\frac{3\ln(x)}{2}}$

/ /

135-) $y' = x + y \; ; \; y(0) = 2$

$$I = e^{\int dx} = e^{x}$$

$$y' - y = x$$

$$y'e^{x} - ye^{x} = xe^{x}$$

$$\int (ye^{x})' = \int xe^{2x}\, dx$$

$$ye^{x} = e^{2x}(2x - 1)$$

$$y = \frac{e^{x}e^{x}(2x-1)}{e^{x}} + \frac{c}{e^{x}}$$

$$y = e^{x}(2x-1) + \frac{c}{e^{x}}$$

$$\int xe^{2x}\, dx$$

$2x = t \rightarrow x = \frac{t}{2}$

$2dx = dt$

$dx = \frac{1}{2} dt$

$$\int \frac{t}{2} e^{t} \frac{1}{2} dt$$

$$= \frac{1}{4} \int te^{t} dt$$

$u = t \quad dv = e^{t} dt$

$du = dt \quad v = e^{t}$

$$= te^{t} - e^{t} \Rightarrow 2xe^{2x} - e^{2x}$$

$$y(0) = 2$$

$$2 = e^{0}(2.0 - 1) + \frac{c}{e^{0}} \quad \therefore \quad c = 3$$

$$y = e^{x}(2x-1) + 3e^{-x}$$

136-)

(145) $\frac{d^2x}{dt^2} - 19\frac{dx}{dt} - 20x = -20t^2 - 70t - 96$

$x'' - 19x' - 20x = -20t^2 - 70t - 96$

homogênea associada

$p(\lambda) = \lambda^2 - 19\lambda - 20 = 0$

$\lambda = \frac{19 \pm \sqrt{361 + 80}}{2}$

$\lambda_1 = \frac{19+21}{2} = 20$

$\lambda_2 = \frac{19-21}{2} = -1$

$y_h = C_1 e^{20x} + C_2 e^{-x}$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

?

Substituindo

$2A - 38At - 19B - 20At^2 - 20Bt - 20C = -20t^2 - 70t - 96$

$-20At^2 = -20t^2$

$A = 1$

$-38At - 20Bt = -70t$

$-38A - 20B = -70$

$-20B = -70 + 38$

$\therefore B =$

(146) $y'' - 5y' + 6y = 5e^{2x}$

homogênea associada

R: $C_1 e^{3x} + C_2 e^{2x} - 5xe^{2x}$

$p(\lambda) = \lambda^2 - 5\lambda + 6 = 0$

$\lambda = \frac{5 \pm \sqrt{25 - 24}}{2}$ $\lambda_1 = 3$ $\lambda_2 = 2$

$y_p = xAe^{2x}$

$y'_p = A(e^{2x} + x.2e^{2x})$

$y''_p = A(2e^{2x} + 2(e^{2x} + 2xe^{2x}))$

$= 2Ae^{2x} + 2Ae^{2x} + 2Axe^{2x}$

$= 4Ae^{2x} + 2Axe^{2x}$

Substituindo

$4Ae^{2x} + 2Axe^{2x} - 5Ae^{2x} - 10xe^{2x} +$

$+ 6Axe^{2x} = 5e^{2x}$

$4A + 20Ax - 5A - 10x + 6Ax = 5$

$-A = 5$ $A = -5$

$y_p = -5xe^{2x}$

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147-) $y'' + 4y' + 3y = 8xe^{x} - 6$

homogenia associada

$p(\lambda) = \lambda^2 + 4\lambda + 3 = 0$

$\lambda = \dfrac{-4 \pm \sqrt{16-12}}{2}$ $\lambda_1 = -1$ $\lambda_2 = -3$

$y_h = C_1 e^{-x} + C_2 e^{-3x}$

$y_p = e^{x}(Ax + B) = Axe^{x} + Be^{x}$

$y'_p = A(e^{x} + xe^{x}) + Be^{x} = Ae^{x} + Axe^{x} + Be^{x}$

$y''_p = A(e^{x} + e^{x} + xe^{x}) + Be^{x} = 2Ae^{x} + Axe^{x} + Be^{x}$

Substituindo

$2Ae^{x} + Axe^{x} + Be^{x} + 4Ae^{x} + 4Axe^{x} + 4Be^{x} + 3Axe^{x} + 3Be^{x} = 8xe^{x} - 6$

$\underline{2Ae^{x}} + Be^{x} + \underline{4Ae^{x}} + 4Be^{x} + 3Be^{x} + Axe^{x} + 4Axe^{x} + 3Axe^{x} = 8xe^{x} - 6$

$6Ae^{x} + 8Be^{x} + 8Axe^{x} = 8xe^{x} - 6$

$\begin{cases} 6A + 8B = 0 \\ 8A = 8 \end{cases}$ $A = 1$ $B = \dfrac{-6A}{8} = \dfrac{-6}{8} = \dfrac{-3}{4}$

$y_p = xe^{x} - \dfrac{3}{4}e^{x}$

Substituindo

$0 + 4.0 + 3C = -6$

$\therefore C = -2$

$y_h = C_1 e^{-x} + C_2 e^{-3x} + \left(x - \dfrac{3}{4}\right)e^{x} - 2$

149-) $y'' + 9y = e^{3x} + \cos(3x)$

homogênea associada $\qquad y_h = C_1 e^{3} + C_2 e^{-3}$

$p(\lambda) = \lambda^2 + 9 = 0$

$\lambda = \pm 3$

$y_{p_1} = A x e^{3x}$

$y'_{p_1} = A(e^{3x} + 3x e^{3x}) = A e^{3x} + 3A x e^{3x}$

$y''_{p_2} = A(3e^{3x} + 3(e^{3x} + 3x e^{3x})) = 3A e^{3x} + 3A e^{3x} + 9A x e^{3x}$

$= 6A e^{3x} + 9A x e^{3x}$

Substituindo:

$6A e^{3x} + 9A x e^{3x} + 9A x e^{3x} = e^{3x}$

$6A + 9Ax + 9Ax = 1$

$6A = 1 \therefore A = \frac{1}{6} \qquad y_{p_1} = \frac{x e^{3x}}{6}$

$y_{p_2} = A\cos(3x) + B\sin(3x)$

$y'_{p_2} = -3A\sin(3x) + 3B\cos(3x)$

$y''_{p_2} = -9A\cos(3x) - 9B\sin(3x)$

Substituindo

$-9A\cos(3x) - 9B\sin(3x) + 9A\cos(3x) + 9B\sin(3x) = \cos(3x)$

Dúvida

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149-) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - y = 0$

$y'' - 2y' - y = 0$

homogênea associada

$p(\lambda) = \lambda^2 - 2\lambda - 1 = 0$

$\lambda = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$y_h = C_1 e^{(1+\sqrt{2})t} + C_2 e^{(1-\sqrt{2})t}$

8	2
4	2
2	2
1	$2^2 \cdot 2$

150-) $\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0$

$\sqrt{-3} = \sqrt{3}i$

$y'' + y' + y = 0$

homogenea associada

$p(\lambda) = \lambda^2 + \lambda + 1$

$\lambda = \frac{-1 \pm \sqrt{1-4}}{2}$

$y_h = e^{at}(C_1 \cos(bt) + C_2 \operatorname{sen}(bt))$

$y_h = e^{-\frac{t}{2}}\left(C_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 \operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)\right)$

151-) $4y'' - 4y' + y = 0$; $y(0) = 0$; $y'(0) = -1,5$

homogênea associada

$p(\lambda) = 4\lambda^2 - 4\lambda + 1 = 0$

$\lambda = \frac{4 \pm \sqrt{16-16}}{8}$ $\quad \lambda = \frac{1}{2}$

$y_h = C_1 e^{\frac{x}{2}} + x C_2 e^{\frac{x}{2}}$

$y(0) = C_1 = 0$

$y'h = \frac{1}{2} C_1 e^{\frac{x}{2}} + C_2 \left(e^{x/2} + \frac{x}{2} e^{\frac{x}{2}}\right)$

$y'h(0) = \frac{1}{2} C_1 + C_2(1 + 0)$; $C_1 = 0$

$C_2 = -1,5$

$y = -1,5 x e^{\frac{x}{2}}$

152-) $y'' + 2y' + 2y = 0$; $y(0) = 2$; $y'(0) = 1$

homogênea associada

$p(\lambda) = \lambda^2 + 2\lambda + 2 = 0$

$\lambda = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i$

$y = e^{-x}(c_1 \cos(x) + c_2 \sin(x))$

$y(0) = c_1 = 2$

$y' = c_1(-e^{-x}\cos(x) + e^{-x}\sin(x)) + c_2(-e^{-x}\sin(x) - e^{-x}\cos(x))$

$y'(0) = -c_1 - c_2 = 1 \therefore c_2 = -2-1 = -3$

$y = e^{-x}(2\cos(x) - 3\sin(x))$

153-) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = 5e^t$

$x'' - 2x' + x = 5e^t$

homogênea associada $\quad y_h = c_1 e^t + x c_2 e^t$

$p(\lambda) = \lambda^2 - 2\lambda + 1 = 0$

$\lambda = \frac{2 \pm \sqrt{4-4}}{2} = 1$

$y_p = At^2e^t$

$y_p' = A(2te^t + t^2e^t)$

$y''_p = A(2(e^t + te^t) + 2te^t + t^2e^t)$

$= 2Ae^t + 2Ate^t + 2Ate^t + At^2e^t$

$= 2Ae^t + 4Ate^t + At^2e^t$

Substituindo

$2Ae^t + 4Ate^t + At^2e^t -$
$- 4Ate^t - 2At^2e^t + At^2e^t =$
$5e^t$

$2A + \cancel{4At} + \cancel{At^2} - \cancel{4At} - 2At^2 + At^2 = 5$

$2A = 5 \therefore A = \frac{5}{2}$

$y_p = \frac{5}{2}t^2e^t \quad \therefore \quad y = c_1e^t + xc_2e^t + \frac{5}{2}t^2e^t$

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154-) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} = sen(3t)$

$x'' - 2x' = sen(3t)$ $\quad\quad y_h = C_1 + C_2 e^{2t}$

homogênea associada

$p(\lambda) = \lambda^2 - 2\lambda = 0$ $\quad \lambda = \frac{2 \pm \sqrt{4-4}}{2} = 1$ ou $\lambda(\lambda - 2) = 0$

$\lambda = 0 \quad \lambda = 2$

$y_p = A\cos(3t) + B sen(3t)$

$y'_p = -3A sen(3t) + 3B\cos(3t)$

$y''_p = -9A\cos(3t) - 9B sen(3t)$

Substituindo:

$-9A\cos(3t) - 9B sen(3t) + 6A sen(3t) - 6B\cos(3t) = sen(3t)$

$\begin{cases} 6A - 9B = 1 \\ -9A - 6B = 0 \end{cases} \sim \begin{cases} 6A - 9B = 1 \\ 0 - 117B = 9 \end{cases} \therefore B = \frac{1}{13} \quad A = \frac{11}{39}$

$y = C_1 + C_2 e^{2t} + \frac{11}{39}\cos(3t) + \frac{1}{13} sen(3t)$

155-) $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = e^{-3t}$; $x(0) = 0$; $\frac{dx}{dt}(0) = 1$

homogênea associada $\quad \lambda = \frac{-6 \pm \sqrt{36-36}}{2} \quad \lambda = -3$

$p(\lambda) = \lambda^2 + 6\lambda + 9$

$y_h = C_1 e^{-3t} + C_2 t e^{-3t}$

$y_p = At^2 e^{-3t}$

$y'_p = A(2te^{-3t} - 3t^2 e^{-3t})$

$y''_p = A(2(e^{-3t} - 3te^{-3t}) - 3(2te^{-3t} - 3t^2 e^{-3t}))$

$= 2Ae^{-3t} - 6Ate^{-3t} - 6Ate^{-3t} + 9At^2 e^{-3t} = 2Ae^{-3t} - 12Ate^{-3t} + 9At^2 e^{-3t}$

Substituindo:

$2Ae^{-3t} - 12Ate^{-3t} + 9At^2e^{-3t} + 12Ate^{-3t} - 18t^2e^{-3t} + 9At^2e^{-3t} = e^{-3t}$

$2A = 1 \quad \therefore \quad A = \frac{1}{2} \qquad Y_p = \frac{1}{2}t^2e^{-3t}$

$y = C_1e^{-3t} + C_2te^{-3t} + \frac{1}{2}t^2e^{-3t}$

$y(0) = C_1 = 0$

$y' = -3C_1e^{-3t} + C_2(e^{-3t} - 3te^{-3t}) + \frac{1}{2}(2te^{-3t} - 3t^2e^{-3t})$

$y'(0) = -3C_1 + C_2(1-0) + \frac{1}{2}(0-0) = 3C_1 + C_2 = 1 \;,\; C_1 = 0$

$\therefore C_2 = 1$

$y = te^{-3t} + \frac{1}{2}t^2e^{-3t} = te^{-3t}\left(1 + \frac{1}{2}t\right)$

156-) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} = 5e^{3t}; \; x(0) = 0; \; \frac{dx}{dt}(0) = 0$

homogênica associada $\qquad Y_h = C_1 + C_2e^{-4t}$

$p(\lambda) = \lambda^2 + 4\lambda$

$\lambda(\lambda+4) = 0 \quad \lambda_1 = 0 \quad \lambda_2 = -4 \qquad R: \frac{-5}{12} + \frac{5}{28}e^{-4t} + \frac{5}{21}e^{3t}$

$Y_p = Ae^{3t}$

$Y'_p = 3Ae^{3t}$

$Y''_p = 9Ae^{3t}$

Substituindo:

$9Ae^{3t} + 12Ae^{3t} = 5e^{3t}$

$21A = 5 \; \therefore \; A = \frac{5}{21} \qquad Y_p = \frac{5}{21}e^{3t}$

$y = C_1 + C_2e^{-4t} + \frac{5}{21}e^{3t} \qquad y' = -4C_2e^{-4t} + \frac{5}{7}e^{3t} \qquad y(0) = -4C_2 + \frac{5}{7} = 0$

$y(0) = C_1 + C_2 + \frac{5}{21} = 0 \qquad \therefore C_2 = \frac{5}{28} \qquad C_1 = \frac{-5}{21} - \frac{5}{28} = \frac{-5}{12}$

Equações Diferenciais Lineares de 2ª ordem

1º caso $\lambda_1 \neq \lambda_2$ reais

$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$

$ay'' + by' + cy = g(x)$, onde $g(x) = 0$

$ay'' + by' + cy = 0$

2º caso $\lambda_1 = \lambda_2$ reais

$y = C_1 e^{\lambda_1 x} + C_2 x e^{\lambda_1 x}$

homogênea associada

$p(\lambda) = a\lambda^2 + b\lambda + c$

3º caso $\lambda = a \pm bi$

$y = e^{ax}(C_1 \cos(bx) + C_2 \operatorname{sen}(bx))$

Equações Diferenciais lineares de 2ª ordem não homogênea

$ay'' + by' + cy = g(x)$, onde $g(x) \neq 0$

$y = y_h + y_p$ solução particular

Homogênea associada

1º Caso $g(x) = C$, C = constante

- $y_p = A$ (A const) se $\lambda \neq 0$
- $y_p = Ax$ se pelo menos uma das raízes for $\lambda = 0$
- $y_p = Ax^2$ se $\lambda = 0$ for raiz dupla

2º Caso $g(x) = p_n(x)$ (polinômio de grau n)

- $y_p = (A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x + A_0)$; se $\lambda \neq 0$
- $y_p = (A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x + A_0) x$; se pelo menos uma for $\lambda = 0$
- $y_p = (A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x + A_0) x^2$; se $\lambda = 0$ for raiz dupla

/ /

3º caso $g(x) = e^{ax}$

- $y_p = Ae^{ax}$, se $\lambda = a$ não for raiz de $p(\lambda)$
- $y_p = Axe^{ax}$, se $\lambda = a$ for raiz simples de $p(\lambda)$;
- $y_p = Ax^2e^{ax}$, se $\lambda = a$ for raiz dupla de $p(\lambda)$.

4º caso $g(x) = \cos(bx)$ ou $\sin(bx)$

- $y_p = A\cos(bx) + B\sin(bx)$ se $\lambda = a \pm bi$ não for raiz de $p(\lambda)$
- $y_p = Ax\cos(bx) + Bx\sin(bx)$ se $\lambda = a \pm bi$ for raiz de $p(\lambda)$

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Equação diferencial de 2ª ordem

145) $x'' - 19x' - 20x = -20t^2 - 78t - 96$

homogênea associada

$p(\lambda) = \lambda^2 - 19\lambda - 20 = 0$

$\lambda = \dfrac{19 \pm \sqrt{361 + 80}}{2}$

$\lambda = \dfrac{19 + 21}{2} \Rightarrow \lambda_1 = 20 \quad \lambda_2 = -1$

$y_h = C_1 e^{20x} + C_2 e^{-x}$

$$\begin{array}{r} 19 \\ \underline{19} \\ 171 \\ \underline{19} \\ 361 \end{array} \qquad \begin{array}{r} 361 \\ \underline{80} \\ 441 \end{array} \qquad \begin{array}{r} 21 \\ \underline{21} \\ 21 \\ \underline{42} \\ 441 \end{array}$$

Solução particular

$y_p = (Ax^2 + Bx + C)$

$y'_p = 2Ax + B$

$y''_p = 2A$

Substituindo:

$2A - 38Ax - 19B - 20Ax^2 - 20Bx - 20C = -20x^2 - 78x - 96$

$-20A = -20 \quad \therefore A = 1$

$-38A - 20B = -78 \quad \therefore B = \dfrac{-78 + 38}{-20} = 2$

$2A - 19B - 20C = -96$

$2 - 38 - 20C = -96 \quad \therefore C = \dfrac{-96 + 38 - 2}{-20} = 3$

R: $y_h = C_1 e^{20x} + C_2 e^{-x} + x^2 + 2x + 3$

146-) $y'' - 5y' + 6y = 5e^{2x}$

homogênea associada

$p(\lambda) = \lambda^2 - 5\lambda + 6 = 0$

$\lambda = \dfrac{5 \pm \sqrt{25 - 24}}{2} \quad \lambda_1 = 3 \quad \lambda_2 = 2$

$y_h = C_1 e^{3x} + C_2 e^{2x}$

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Solução particular

$y_p = Axe^{2x}$

$y'_p = A(e^{2x} + 2xe^{2x}) = Ae^{2x} + 2Axe^{2x}$

$y''_p = 2Ae^{2x} + 2A(e^{2x} + 2xe^{2x}) = 2Ae^{2x} + 2Ae^{2x} + 4Axe^{2x}$

$\therefore \; y''_p = 4Ae^{2x} + 4Axe^{2x}$

Substituindo

$4Ae^{2x} + 4Axe^{2x} - 5Ae^{2x} - 10Axe^{2x} + 6Axe^{2x} = 5e^{2x}$

$4A + 4Ax - 5A - 10Ax + 6Ax = 5$

$4A - 5A = 5 \quad \therefore \quad A = -5$

$4Ax - 10Ax + 6Ax = 0$

$y = C_1e^{3x} + C_2e^{2x} - 5xe^{2x}$

197.) $y'' + 4y' + 3y = 8xe^{x} - 6$

homogênea associada

$p(\lambda) = \lambda^2 + 4\lambda + 3 = 0$

$\lambda = \dfrac{-4 \pm \sqrt{16-12}}{2} \qquad \lambda_1 = -1 \quad \lambda_2 = -3 \qquad y_h = C_1e^{-x} + C_2e^{-3x}$

Solução particular

$y_p = (Ax + B)e^{x}$

$y'_p = Ae^{x} + (Ax+B)e^{x}$

$y''_p = Ae^{x} + Ae^{x} + (Ax+B)e^{x} = 2Ae^{x} + Axe^{x} + Be^{x}$

Substituindo temos:

$2Ae^{x} + Axe^{x} + Be^{x} + 4Ae^{x} + 4Axe^{x} + 4Be^{x} + 3Axe^{x} + 3Be^{x} = 8xe^{x} - 6$

$2Ae^{x} + Be^{x} + 4Ae^{x} + 4Be^{x} + 3Be^{x} = 0 \quad \therefore \quad 6Ae^{x} + 8Be^{x} = 0$

$Axe^{x} + 4Axe^{x} + 3Axe^{x} = 8xe^{x}$

$8A = 8 \quad \therefore A = 1$

$6A + 8B = 0 \quad \therefore B = \frac{-6}{8} = \frac{-3}{4}$

Solução particular II

$y_p = A$

$y'_p = 0$ Substituindo

$y''_p = 0$ $0 + 4.0 + 3A = -6$

$\therefore A = -2$

$$y = C_1 e^{-x} + C_2 e^{-3x} + \left(x - \frac{3}{4}\right)e^{x} - 2$$

148-) $y'' + 9y = e^{3x} + \cos(3x)$

homogênea associada

$p(\lambda) = \lambda^2 + 9 = 0 \quad \therefore y_h = C_1 e^{3x} + C_2 e^{-3x}$

$\lambda = \pm 3$

Solução particular I

$y_p = Axe^{3x}$

$y'_p = A(e^{3x} + 3xe^{3x}) = Ae^{3x} + 3Axe^{3x}$

$y''_p = 3Ae^{3x} + 3A(e^{3x} + 3xe^{3x}) = 6Ae^{3x} + 9Axe^{3x}$

Substituindo:

$6Ae^{3x} + 9Axe^{3x} + 9Axe^{3x} = e^{3x}$

$6A + 18Ax = 1$

$\therefore A = \frac{1}{6}$

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Solução Particular II

$y_p = A\cos(3x) + B\sin(3x)$

$y'_p = -3A\sin(3x) + 3B\cos(3x)$

$y''_p = -9A\cos(3x) - 9B\sin(3x)$

Substituindo:

$-9A\cos(3x) - 9B\sin(3x) + 9A\cos(3x) + 9B\sin(3x) = \cos(3x)$

$-9A + 9A = 1$

$\nexists$

149) $y'' - 2y' - y = 0$

$$\begin{array}{c|c} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \qquad 2^2 \cdot 2 \qquad \sqrt{8} = 2\sqrt{2}$$

homogênea associada

$p(\lambda) = \lambda^2 - 2\lambda - 1 = 0$

$\lambda = \frac{2 \pm \sqrt{4+4}}{2} \quad \lambda = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$y = C_1 e^{(1+\sqrt{2})t} + C_2 e^{(1-\sqrt{2})t}$

150) $y'' + y' + y = 0$

homogênea associada

$p(\lambda) = \lambda^2 + \lambda + 1 = 0$

$\lambda = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2}$

$y_h = e^{-1/2} \left(C_1 \cos\left(\frac{\sqrt{3}}{2}\right) + C_2 \sin\left(\frac{\sqrt{3}}{2}\right) \right)$

151-) $4y'' - 4y' + y = 0$ $\quad y(0) = 0$

$y'(0) = -1,5$

$p(\lambda) = 4\lambda^2 - 4\lambda + 1 = 0$

$\lambda = \frac{4 \pm \sqrt{16-16}}{8} = \frac{1}{2}$ $\quad y_h = C_1 e^{0,5x} + x C_2 e^{0,5x}$

$y(0) = C_1 e^{0,5.0} + 0.C_2 e^{0,5.0} = 0$

$C_1 = 0$

$y'(x) \; 0,5 C_1 e^{0,5x} + C_2 (e^{0,5x} + 0,5 x e^{0,5x})$

$y'(0) = 0,5 C_1 e^{0,5.0} + C_2 (e^{0,5.0} + 0,5.0 e^{0,5.0}) = -1,5$

$0,5 C_1 + C_2 = -1,5$

$C_2 = -1,5 + 0,5.0 \quad \therefore \quad C_2 = -1,5$

$y = -1,5 x e^{\frac{x}{2}}$

152-) $y'' + 2y' + 2y = 0$ $\quad y(0) = 2$

$y'(0) = 1$

homogênea associada

$p(\lambda) = \lambda^2 + 2\lambda + 2 = 0$

$\lambda = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

$y_h = e^{-x} (C_1 \cos(x) + C_2 \sin(x))$

$y(0) = e^{-0} (C_1 \cos(0) + C_2 \sin(0) = 2 \quad \therefore \quad C_1 = 2$

$y'_h = C_1 (-e^{-x} \cos(x) - e^{-x} \sin(x)) + C_2 (-e^{-x} \sin(x) + e^{-x} \cos(x))$

$y'(0) = 2 (-e^{0} \cos(0) - e^{-0} \sin(0)) + C_2 (-e^{-0} \sin(0) + e^{-0} \cos(0)) = 1$

$-2 + C_2 = 1 \quad \therefore \quad C_2 = -3$

$y = e^{-x} (2\cos(x) + 3\sin(x))$

/ /

153) $x'' - 2x' + x = 5e^t$

homogênea associada

$p(\lambda) = \lambda^2 - 2\lambda + 1 = 0$

$\lambda = \dfrac{2 \pm \sqrt{4-4}}{2} = 1 \qquad y_h = C_1 e^x + x C_2 e^x$

$y_p = At^2 e^t$

$y'_p = A(2te^t + t^2 e^t) = 2Ate^t + At^2 e^t$

$y''_p = 2A(e^t + te^t) + 2Ate^t + At^2e^t = 2Ae^t + 4Ate^t + At^2e^t$

Substituindo

$2Ae^t + 4Ate^t + At^2e^t - 4Ate^t - 2At^2e^t + At^2e^t = 5e^t$

$2A + 4At + At^2 - 4At - 2At^2 + At^2 = 5$

$A = \dfrac{5}{2}$

$y = C_1 e^x + x C_2 e^x + \dfrac{5}{2} t^2 e^t$

154-) $x'' - 2x' = \operatorname{sen}(3t)$

homogênea associada

$p(\lambda) = \lambda^2 - 2\lambda = 0 \qquad y_h = C_1 + C_2 e^{2x}$

$\lambda(\lambda - 2) = 0 \quad \lambda_1 = 0 \quad \lambda_2 = 2$

$y_p = A\cos(3t) + B\operatorname{sen}(3t)$

$y'_p = -3A\operatorname{sen}(3t) + 3B\cos(3t)$

$y''_p = -9A\cos(3t) - 9B\operatorname{sen}(3t)$

Substituindo

$-9A\cos(3t) - 9B\operatorname{sen}(3t) + 6A\operatorname{sen}(3t) - 6B\cos(3t) = \operatorname{sen}(3t)$

$\begin{cases} -9B + 6A = 1 \\ -6B - 9A = 0 \end{cases} \qquad \begin{cases} -54B + 36A = 6 \\ 54B + 81A = 0 \end{cases} \qquad \therefore A = \dfrac{2}{39}$

$y_h = C_1 + C_2 e^{2x} + \dfrac{2}{39}\cos(3t) + \dfrac{1}{13}\operatorname{sen}(3t) \qquad B = \dfrac{1}{13}$

155) $x'' + 6x' + 9x = e^{-3t}$ $\qquad x(0) = 0$

$x'(0) = 1$

homogênea associada

$p(\lambda) = \lambda^2 + 6\lambda + 9 = 0$

$\lambda = \dfrac{-6 \pm \sqrt{36-36}}{2} = -3$ $\qquad y_h = C_1 e^{-3x} + x C_2 e^{-3x}$

$y_p = A t^2 e^{-3t}$

$y'p = A(2te^{-3t} + t^2 . -3\, e^{-3t}) = 2Ate^{-3t} - 3At^2 e^{-3t}$

$y''p = 2A(e^{-3t} - 3te^{-3t}) - 3A(2te^{-3t} + t^2.(-3)e^{-3t})$

$= 2Ae^{-3t} - 6Ate^{-3t} - 6Ate^{-3t} + 9At^2e^{-3t}$

$= 2Ae^{-3t} - 12Ate^{-3t} + 9At^2e^{-3t}$

Substituindo:

$2Ae^{-3t} - 12Ate^{-3t} + 9At^2e^{-3t} + 12Ate^{-3t} - 18At^2e^{-3t} + 9At^2e^{-3t} = e^{-3t}$

$2A - 12At + 9At^2 + 12At - 18At^2 + 9At^2 = 1$

$2A = 1 \quad \therefore \quad A = \dfrac{1}{2}$

$y = C_1 e^{-3t} + t C_2 e^{-3t} + \dfrac{1}{2} t^2 e^{-3t}$

$y(0) = C_1 e^0 + 0 C_2 e^0 + \dfrac{1}{2} 0^2 e^0 = 0 \qquad C_1 = 0$

$y'(t) = -3C_1 e^{-3t} + C_2(e^{-3t} - 3te^{-3t}) + \dfrac{1}{2}(2te^{-3t} - 3te^{-3t})$

$y'(0) = C_2 = 1$

$x = te^{-3t} + \dfrac{1}{2} t^2 e^{-3t} = te^{-3t}\left(1 + \dfrac{1}{2} t\right)$

156-) $x'' + 4x' = 5e^{3t}$; $x(0) = 0$

$x'(0) = 0$

homogênea homogênea

$p(\lambda) = \lambda^2 + 4\lambda = 0$

$\lambda(\lambda + 4) = 0 \quad \lambda_1 = 0 \quad \lambda_2 = -4$

$y_h = C_1 + C_2 e^{-4t}$

$y_p = Ae^{3t}$

$y'_p = 3Ae^{3t}$

$y''_p = 9Ae^{3t}$

Substituindo

$9Ae^{3t} + 12Ae^{3t} = 5e^{3t}$

$9A + 12A = 5$

$21A = 5 \quad \therefore A = \frac{5}{21}$

$y = C_1 + C_2 e^{-4t} + \frac{5}{21} e^{3t}$

$y(0) = C_1 + C_2 + \frac{5}{21} = 0 \quad C_1 = \frac{-5}{21} - \frac{5}{28} = \frac{-140 - 105}{588} = \frac{245}{588} = \frac{-5}{12}$

$y' = -4C_2 e^{-t} + \frac{5}{7} e^{3t} = 0$

$y'(0) = -4C_2 + \frac{5}{7} = 0 \quad \therefore C_2 = \frac{5}{28}$

$\therefore R: \; y = \frac{-5}{12} + \frac{5}{28} e^{-4t} + \frac{5}{21} e^{3t}$

157.) $y'' + 2y' + 2y = xe^{-x}$ $\quad y(0)=0$

$y'(0)=0$

homogênea associada

$p(\lambda) = \lambda^2 + 2\lambda + 2 = 0$ $\quad y_h = e^{-x}\left[C_1\cos(x) + C_2\sin(x)\right]$

$\lambda = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

$a \pm bi$

e^{ax}

Solução particular

$y_p = (Ax+B)e^{-x}$

$y'_p = Ae^{-x} - (Ax+B)e^{-x}$

$y''_p = -Ae^{-x} - \left[Ae^{-x} + (Ax+B).(-1)e^{-x}\right]$

$= -Ae^{-x} - Ae^{-x} + Axe^{-x} + Be^{-x}$

$= -2Ae^{-x} + Axe^{-x} + Be^{-x}$

Substituindo, temos

$-2Ae^{-x} + Axe^{-x} + Be^{-x} + 2Ae^{-x} - 2Axe^{-x} - 2Be^{-x} + 2Axe^{-x} + 2Be^{-x} = xe^{-x}$

$A - 2A + 2A = 1 \quad \therefore A = 1$

$B - 2A + 2A - 2B + 2B = 0 \quad \therefore B = 0$

$y(x) = e^{-x}\left[C_1\cos(x) + C_2\sin(x)\right] + xe^{-x}$

$y(0) = C_1 = 0$

$y'(x) = C_1\left[-e^{-x}\cos(x) - e^{-x}\sin(x)\right] + C_2\left[-e^{-x}\sin(x) + e^{-x}\cos(x)\right] + e^{-x} - xe^{-x}$

$y'(0) = -C_1 + C_2 + 1 = 0 \quad \therefore C_2 = -1$

$y = -e^{-x}\sin(x)$

/ /

158-) $y'' + 4y = x^2 + 3e^x$, $y(0) = 0$ $y'(0) = 2$

homogênea associada

$p(\lambda) = \lambda^2 + 4 = 0$

$\lambda = \pm 2i$ $y_h = C_1 \cos(2x) + C_2 \sin(2x)$

Solução particular

$y_p = (Ax^2 + Bx + C) + De^x$

$y'_p = 2Ax + B + De^x$

$y''_p = 2A + De^x$ Substituindo, temos

$2A + De^x + 4Ax^2 + 4Bx + 4C + 4De^x = x^2 + 3e^x$

$5De^x = 3e^x$

$\therefore D = \frac{3}{5}$

$2A + 4C = 0$

$4A = 1$ $\therefore$ $A = 1/4$ $\frac{1}{2} + 4C = 0$ $\therefore C = -\frac{1}{8}$

$4Bx = 0$

$\therefore B = 0$

$y_G = C_1 \cos(2x) + C_2 \sin(2x) + \frac{1}{4}x^2 - \frac{1}{8} + \frac{3}{5}e^x$

$y'_G = -2C_1 \sin(2x) + 2C_2 \cos(2x) + \frac{1}{2}x + \frac{3}{5}e^x$

$y_G(0) = C_1 + \frac{3}{5} = \frac{1}{8} = 0$ $\therefore C_1 = \frac{1}{8} - \frac{3}{5} = \frac{5-24}{40} = \frac{-19}{40}$

$y'_G(0) = 2C_2 + \frac{3}{5} = 2$ $\therefore C_2 = (2 - \frac{3}{5}) : 2 = \left(\frac{10-3}{5}\right) : 2 = \frac{7}{5} : 2 = \frac{7}{10}$

$y = \frac{-19}{40}\cos(2x) + \frac{7}{10}\sin(2x) + \frac{1}{4}x^2 - \frac{1}{8} + \frac{3}{5}e^x$

(Exercício de prova)

$$3y'' + 12y = 5\,sen(2x)$$

$$3y'' + 12y = 5\,sen(2x)$$

homogênea associada

$p(\lambda) = 3\lambda^2 + 12 = 0 \qquad y_h = C_1 \cos(2x) + C_2\, sen(2x)$

$\lambda = \pm 2i$

Solução Particular

$y_p = Ax\cos(2x) + Bx\,sen(2x)$

$y'_p = A(\cos(2x) - 2x\,sen(2x)) + B(sen(2x) + 2x\cos(2x))$

$= A\cos(2x) - 2Ax\,sen(2x) + B\,sen(2x) + 2Bx\cos(2x)$

$y''_p = -2A\,sen(2x) - 2A(sen(2x) + 2x\cos(2x)) + 2B\cos(2x) + 2B(\cos(2x) - 2x\,sen(2x))$

$= -2A\,sen(2x) - 2A\,sen(2x) - 4Ax\cos(2x) + 2B\cos(2x) + 2B\cos(2x) - 4Bx\,sen(2x)$

$= -4A\,sen(2x) - 4Ax\cos(2x) + 4B\cos(2x) - 4Bx\,sen(2x)$

Substituindo:

$-12A\,sen(2x) - 12x\cos(2x) + 12B\cos(2x) - 12Bx\,sen(2x) + 12x\cos(2x) + Bx\,sen(2x) = 5\,sen(2x)$

$-12A = 5 \quad \therefore \quad A = -\frac{5}{12}$

$12B = 0 \quad \therefore \quad B = 0$

$y_h = C_1\cos(2x) + C_2\,sen(2x) - \frac{5}{12}x\cos(2x)$

/ /

(Exercício de prova)

$y'' + 2y' + y = 3xe^{-x}$

homogênea associada

$p(\lambda) = \lambda^2 + 2\lambda + 1$

$\lambda = \frac{-2 \pm \sqrt{4-4}}{2} = -1$

$y_h = C_1 e^{-x} + x C_2 e^{-x}$

Solução particular

$y_p = (Ax + B)x^2 e^{-x}$

$y'_p = Ax^2 e^{-x} + (Ax+B)(2xe^{-x} - x^2 e^{-x})$

$y''_p = A(2xe^{-x} - x^2 e^{-x}) + (2xe^{-x} - x^2 e^{-x}) + (Ax+B)[2(e^{-x} - xe^{-x}) - (2xe^{-x} - x^2 e^{-x})]$

$= 2Axe^{-x} - Ax^2 e^{-x} + 2xe^{-x} - x^2 e^{-x} + (Ax+B)[2e^{-x} - 2xe^{-x} - 2xe^{-x} + x^2 e^{-x}]$

$= 2Axe^{-x} - Ax^2 e^{-x} + 2xe^{-x} - x^2 e^{-x} + 2Axe^{-x} - 2Ax^2 e^{-x} - 2x^2 e^{-x} + 2x^3 e^{-x} +$

$2Be^{-x} - 2Bxe^{-x} - 2Bxe^{-x} + Bx^2 e^{-x}]$

$= 4Axe^{-x} - 3Ax^2 e^{-x} + Bx^2 e^{-x} - 4Bxe^{-x} + 2Be^{-x} + 2xe^{-x} - 3x^2 e^{-x} + 2x^3 e^{-x}$

$4Axe^{-x} - 4Bxe^{-x} + 2xe^{-x} + 4Bxe^{-x} = 3xe^{-x}$

$4A - \cancel{4B} + 2 + \cancel{4B} = 3$

$A = \frac{1}{4}$

$2Be^{-x} = 0 \quad \therefore \quad B = 0$

$y = C_1 e^{-x} + x C_2 e^{-x} + \frac{1}{4} x^3 e^{-x}$

Revisão Gráfico de função de Duas variáveis

$z = f(x,y) = \arcsin(x+y)$

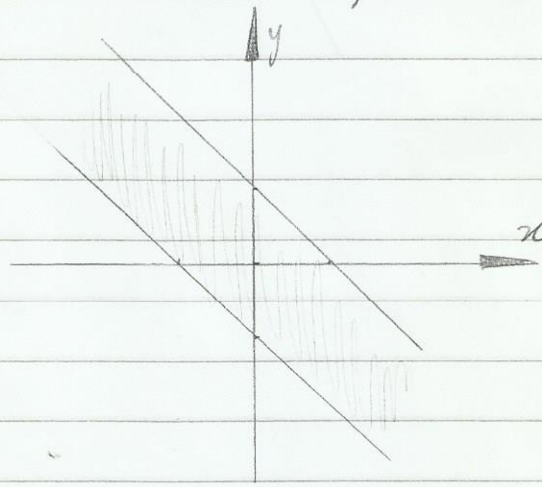
$D_f: \{(x,y) \in \mathbb{R}^2 \mid -1 \leq x+y \leq 1\}$

$-1 \leq x+y$ e $x+y \leq 1$

$x+y=-1$ $\qquad$ $x+y=1$

se $x=0 \to y=-1$ $\qquad$ se $x=0$ $y=1$

se $y=0 \to x=-1$ $\qquad$ se $y=0$ $x=1$



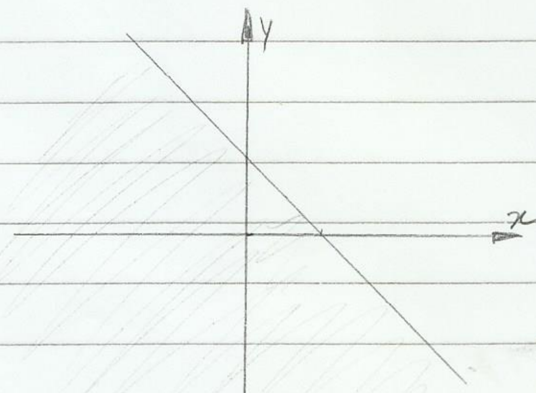
170.) $x+y-1+z^2=0$; $z \geq 0$

$z = \sqrt{1-x-y}$ $\qquad$ $D_f = \{(x,y) \in \mathbb{R}^2 \mid 1-x-y \geq 0\}$

$1-x-y=0$

se $x=0$ $y=1$

se $y=0$ $x=1$

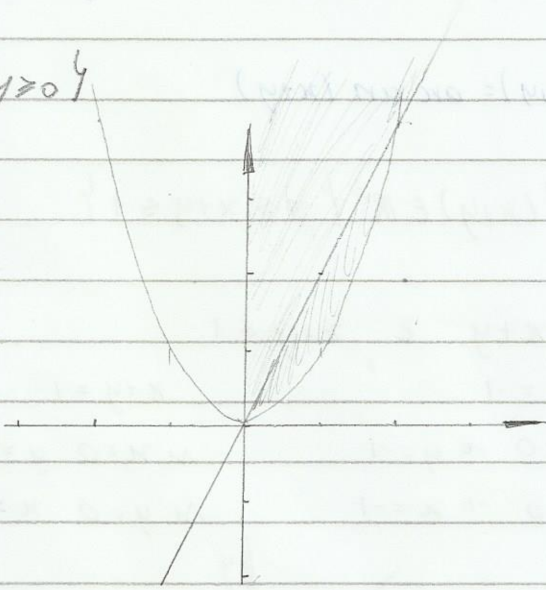


$z = f(x,y) = \sqrt{y-x^2} + \sqrt{2x-y}$

$D_f = \{(x,y) \in \mathbb{R}^2 \mid y-x^2 \geq 0 \text{ e } 2x-y \geq 0\}$

$y - x^2 = 0 \qquad 2x - y = 0$

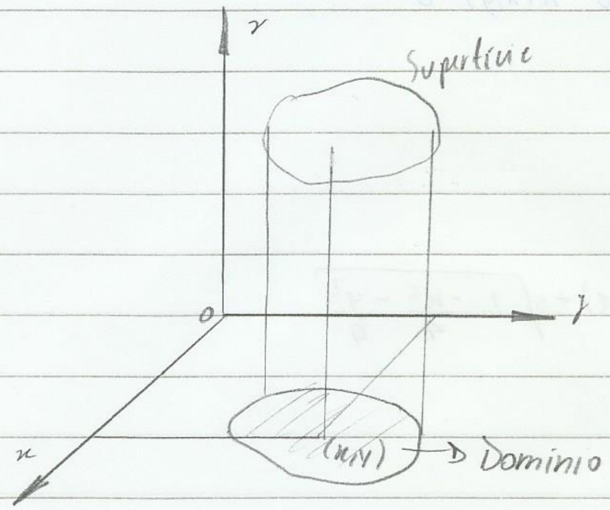
$y = x^2 \qquad y = 2x$



1º (y-2x) em evidência   Moodle / Exercícios 169 ao 204

2º ED 2Ordem

Função de duas variáveis $z = f(x,y)$



Definição

Se para par $(x,y) \in \mathbb{R}^2$ corresponder um e somente um $z \in \mathbb{R}$ Então dizemos que z é uma função das duas ~~duas~~ variáveis independentes x e y e escrevemos

$$z = f(x,y)$$

Definição

Domínio

$$D_f(x,y) = \{ (x,y) \in \mathbb{R}^2 \mid \exists z \in \mathbb{R} \text{ e } z = f(x,y) \}$$

Imagem

$$Imf(x,y) = \{ z \in \mathbb{R} \mid \exists (x,y) \in \mathbb{R}^2 \text{ e } z = f(x,y) \}$$

Problemas de domínio

1) $z = f(x,y) = \sqrt{g(x,y)}$ condição: $g(x,y) \geq 0$

2) $z = f(x,y) = \dfrac{g(x,y)}{h(x,y)}$ condição $h(x,y) = 0$

3) funções conhecidas

Ex 177: corrigir:

$$z = f(x,y) = \ln(x^2+y^2-4) + \sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}}$$

a) Analítica:

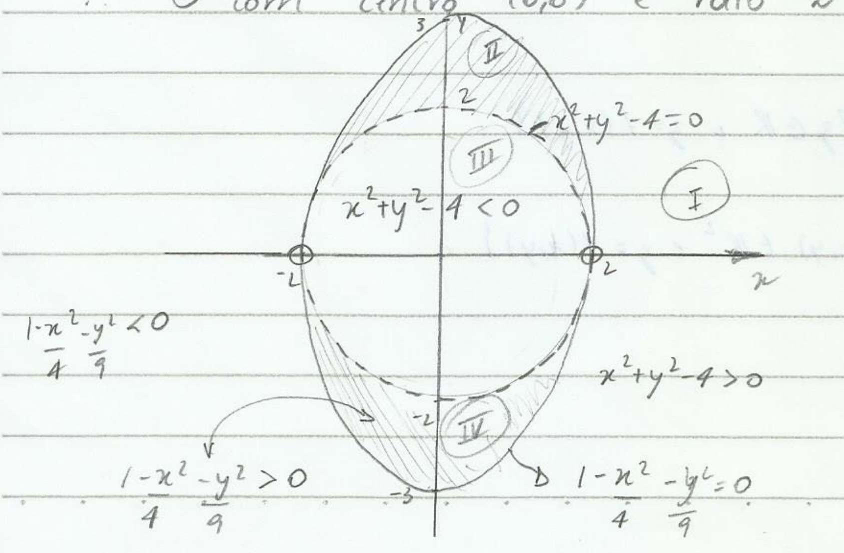
$$D_f(x,y) = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2-4 > 0 \text{ e } 1 - \frac{x^2}{4} - \frac{y^2}{9} \geq 0\}$$

b) Gráfico

I: $x^2+y^2-4=0$

$x=0 \begin{cases} y=-2 \\ y=2 \end{cases}$ $\quad y=0 \begin{cases} x=-2 \\ x=2 \end{cases}$

$\therefore$ ⊙ com centro $(0,0)$ e raio 2



$II: \quad 1 - \frac{x^2}{4} - \frac{y^2}{9} = 0$

$x=0 \begin{cases} y = 3 \\ y = -3 \end{cases} \qquad y=0 \begin{cases} x = 2 \\ x = -2 \end{cases}$

Estudo das regiões

$I \begin{cases} 1 - \frac{x^2}{4} - \frac{y^2}{9} < 0 \\ x^2 + y^2 - 4 > 0 \end{cases}$ ∴ não serve

$II \begin{cases} 1 - \frac{x^2}{4} - \frac{y^2}{9} > 0 \\ x^2 + y^2 - 4 > 0 \end{cases}$ ∴ serve

$III \begin{cases} 1 - \frac{x^2}{4} - \frac{y^2}{9} > 0 \\ x^2 + y^2 - 4 < 0 \end{cases}$ ∴ não serve

$IV \begin{cases} 1 - \frac{x^2}{4} - \frac{y^2}{9} > 0 \\ x^2 + y^2 - 4 > 0 \end{cases}$ ∴ serve

Derivadas de $z = f(x,y)$ em relação a x e em relação a y:

a) $f_x = \frac{df}{dx}$; varia só x

b) $f_y = \frac{df}{dy}$; varia só y

Dada a função

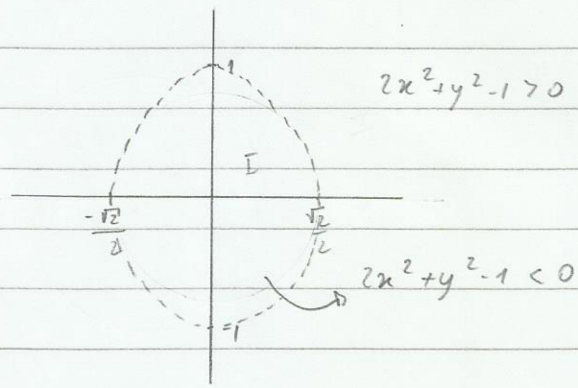
$z = f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}$, Verificar se a expressão $E = x \frac{dz}{dx} + y \frac{dz}{dy}$ é válida

(169-) $z = f(x,y) = \ln(2x^2+y^2-1)$

$D_f = f(x,y) = \{(x,y) \in \mathbb{R}^2 \mid \exists z \in \mathbb{R} \text{ e } 2x^2+y^2-1>0\}$

$2x^2+y^2-1=0$

$x=0 \begin{cases} y=1 \\ y=-1 \end{cases} \qquad y=0 \begin{cases} x=\sqrt{2}/2 \\ x=-\sqrt{2}/2 \end{cases}$



Estudo da região I

$(x,y) = (0,0) \Rightarrow 2\cdot 0^2+0^2-1 = -1$

$I: 2x^2+y^2-1<0$

(170-) $x+y-1+z^2=0$; $z \geq 0$

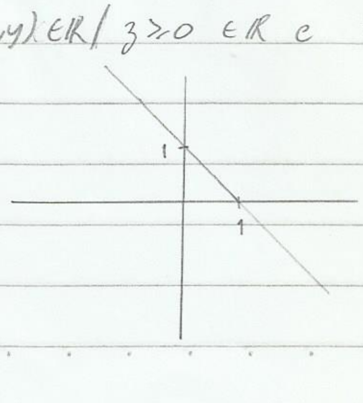
$z = \sqrt{1-x-y}$

$D_f = f(x,y) = \{(x,y) \in \mathbb{R}^2 \mid z \geq 0 \in \mathbb{R} \text{ e } 1-x-y \geq 0\}$

$1-x-y=0$

$x=0 \Rightarrow y=1$

$y=0 \Rightarrow x=1$



(171) $z = f(x,y) = \sqrt{y-x^2} + \sqrt{2x-y}$

$Df = f(x,y) = \{(x,y) \in \mathbb{R}^2 \mid \exists z \in \mathbb{R} \text{ e } y-x^2 \geq 0 \text{ e } 2x-y \geq 0\}$

$y - x^2 = 0$ $\quad$ $2x - y = 0$

$x = 0 \quad y = 0$ $\quad$ $x = 0 = y = 0$

$y = 0 \quad x = 0$

181) $f(x,y) = \dfrac{x-y}{x+y}$

$\dfrac{dz}{dx} = f(x) = \dfrac{(x+y)-(x-y)}{(x+y)^2} = \dfrac{2y}{(x+y)^2}$

$\dfrac{dz}{dy} = f(y) = \dfrac{-1(x+y)-(x-y)}{(x+y)^2} = \dfrac{-2x}{(x+y)^2}$

182-) $f(x,y,z) = xy^2z^3 + 3yz$

$\dfrac{df}{dx} = f(x) = y^2z^3$

$\dfrac{df}{dy} = f(y) = 2yxz^3 + 3z$

$\dfrac{df}{dz} = f(z) = 3xy^2z^2 + 3y$

183-) $f(x,y,z) = \cos(4x+3y+2z)$, f_{xyz} ? f_{yzz}

$f(x) = -\sin(4x+3y+2z).4 = -4\sin(4x+3y+2z)$

$f(y) = -\sin(4x+3y+2z).3 = -3\sin(4x+3y+2z)$

$f(z) = -\sin(4x+3y+2z).2 = -2\sin(4x+3y+2)$

$f_{xyz} = -24\sin(4x+3y+2z)$

$f_{yzz} = -12\sin(4x+3y+2z)$

184.) $u = e^{r\theta} \operatorname{sen}(\theta)$; calcular $\dfrac{\partial^3 u}{\partial r^2 \partial \theta}$

185.) $z = \dfrac{xy^2}{x^2+y^2}$ $\qquad x\dfrac{dz}{dx} + y\dfrac{dz}{dy} = z$

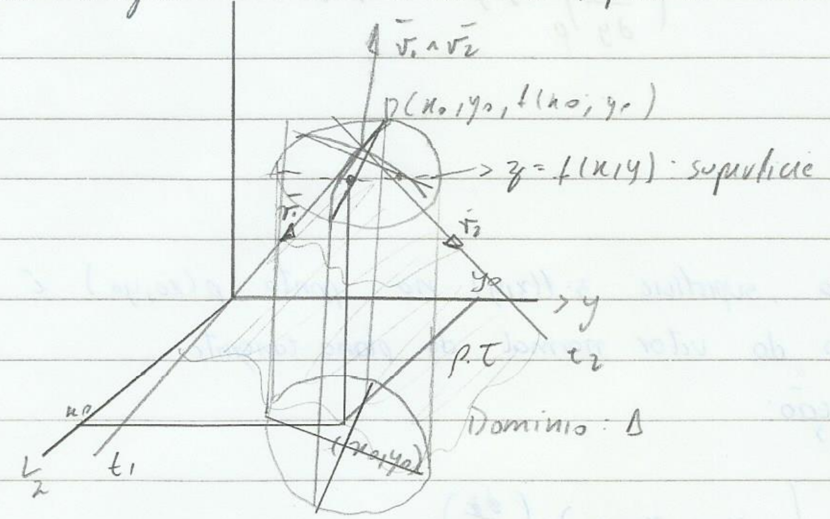
$$\frac{dz}{dx} = \frac{y^2(x^2+y^2) - xy^2 \cdot 2x}{(x^2+y^2)^2} = \frac{x^2y^2 + y^4 - 2x^2y^2}{(x^2+y^2)^2} = \frac{-x^2y^2+y^4}{(x^2+y^2)^2}$$

$$\frac{dz}{dy} = \frac{2yx(x^2+y^2) - (xy^2)2y}{(x^2+y^2)^2} = \frac{2yx^3 + 2y^3x - 2xy^3}{(x^2+y^2)^2} = \frac{2yx^3}{(x^2+y^2)^2}$$

$$x\frac{(-x^2y^2+y^4)}{(x^2+y^2)^2} + y\frac{(2yx^3)}{(x^2+y^2)^2} = \frac{-x^3y^2+xy^4}{(x^2+y^2)^2} + \frac{2y^2x^3}{(x^2+y^2)^2} =$$

$$= \frac{y^2x^3 + xy^4}{(x^2+y^2)^2}$$

Plano tangente e reta normal à superfície num ponto $p(x_0, y_0, z_0)$



Definição

$f_x(x_0, y_0) = \lim_{h \to 0} \dfrac{f(x_0 + h, y_0) - f(x_0, y_0)}{h}$, se existir.

$y = y_0$: pl // pl. x, z que intercepta a superfície numa curva $z = f(x, x_0)$

$m_{t_1} = f_x(x_0, y_0)$

Definição

$f_y(x_0, y_0) = \lim_{k \to 0} \dfrac{f(x_0, y_0 + k) - f(x_0, y_0)}{k}$, se existir

$x = x_0$: pl // pl (y, z) que intercepta a superfície $z = f(x, y)$ numa curva: $f(x_0, y)$:

Definição

Plano tangente à superfície $z = f(x,y)$ num ponto $p(x_0, y_0, z_0)$
É o plano determinado pelas retas t_1 e t_2 e tem para equação geral:

/ /

$$\left(\frac{\partial z}{\partial x}\right)_p (x-x_0) + \left(\frac{\partial z}{\partial y}\right)_p (y-y_0) - (z-z_0) = 0$$

Definição:

Reta normal a superfície $z=f(x,y)$ no ponto $p(x_0,y_0)$ é a reta que tem a direção do vetor normal do plano tangente.

que tem equação:

1) paramétricas
$$\begin{cases} x = x_0 + \lambda \left(\frac{\partial z}{\partial x}\right)_p \\ y = y_0 + \lambda \left(\frac{\partial z}{\partial y}\right)_p \\ z = z_0 - \lambda \end{cases}$$

2) simétrica:
$$\frac{x-x_0}{\left(\frac{\partial z}{\partial x}\right)_p} = \frac{y-y_0}{\left(\frac{\partial z}{\partial y}\right)_p} = \frac{z-z_0}{-1}$$

3) vetorial:
$$(x,y,z) = (x_0,y_0,z_0) + \lambda \left(\left(\frac{\partial z}{\partial x}\right)_p, \left(\frac{\partial z}{\partial y}\right)_p, -1\right) \quad \lambda \in \mathbb{R}$$

/ /

Máximo relativo, Mínimo relativo e ponto de sela de $z=f(x,y)$ hessiano

1) Pontos críticos:

Resolver o sistema:

$$(S)\begin{cases}\dfrac{dz}{dx}=0\\[2mm]\dfrac{dz}{dy}=0\end{cases}$$

2) ~~Devida.~~ Derivadas segundas em cada ponto crítico

3) Calcular o hessiano em cada ponto P_i(crítico)

$$H(x_i;z_i)=\begin{vmatrix}\left(\dfrac{\partial^2 z}{dx^2}\right)_{pi}=A & \left(\dfrac{d^2 z}{dy\,dx}\right)_{pi}=C\\[3mm] \left(\dfrac{d^2 z}{dx\,dy}\right)_{pi}=C & \left(\dfrac{d^2 z}{dy^2}\right)_{pi}=B\end{vmatrix}=\underline{AB-C^2}$$

$$AB-C^2\begin{cases}>0\begin{cases}\text{se } A>0 \to P_i(x_i,y_i;z_i) \text{ Min relativo}\\ \text{se } A<0 \to P_i(x_i;y_i;z_i) \text{ Máx relativo}\end{cases}\\ <0 \quad P_i(x_i;y_i;z_i) \text{ Ponto de sela}\\ =0 \quad \text{nada se conclue}\end{cases}$$

/ /

Exemplo Ex 210

$z = f(x,y) = 2x^3 + 2y^3 - 6x - 6y$

1) Resolver o sistema

$$(S)\begin{cases} \dfrac{\partial z}{\partial x} = 6x^2 - 6 = 0 & \begin{cases} x = 1 \\ x = -1 \end{cases} \\ \dfrac{\partial z}{\partial y} = 6y^2 - 6 = 0 & \begin{cases} y = 1 \\ y = -1 \end{cases} \end{cases}$$

Pontos críticos

$P_1(1,1)$; $P_2(1,-1)$; $P_3(-1,1)$; $P_4(-1,-1)$

2) Derivadas segundas.

$\dfrac{d^2z}{dx^2} = 12x$

$\left(\dfrac{d^2z}{dx}\right)_{P_1} = 12$

$\left(\dfrac{d^2z}{dy}\right)_{P_2} = 12$

$\left(\dfrac{d^2z}{dx}\right)_{P_3} = -12$

$\left(\dfrac{d^2z}{dx}\right)_{P_4} = -12$

$\rightarrow A$

$\dfrac{d^2z}{dy^2} = 12y$

$\left(\dfrac{d^2z}{dy^2}\right)_{P_1} = 12$

$\left(\dfrac{d^2z}{dy^2}\right)_{P_2} = -12$

$\left(\dfrac{d^2z}{dy^2}\right)_{P_3} = 12$

$\left(\dfrac{d^2z}{dy^2}\right)_{P_4} = -12$

$\rightarrow B$

$\dfrac{d^2z}{dy\,dx} = \dfrac{d^2z}{dx\,dy} = 0$ (Calcularmos pontos P_1; P_2, P_3, P_4)

$\rightarrow D = 0 - C$

1º coloco $(y-2x)$ em evidência

1-) $x \; \frac{e^{y^2}}{2} \cdot y = K$

2)

3-) Hessiano

$H(1,1) = \begin{vmatrix} 12 & 0 \\ 0 & 12 \end{vmatrix} = 144 > 0 \; ; \; A > 0$

$P_1 (1,1, f(1,1))$ mínimo

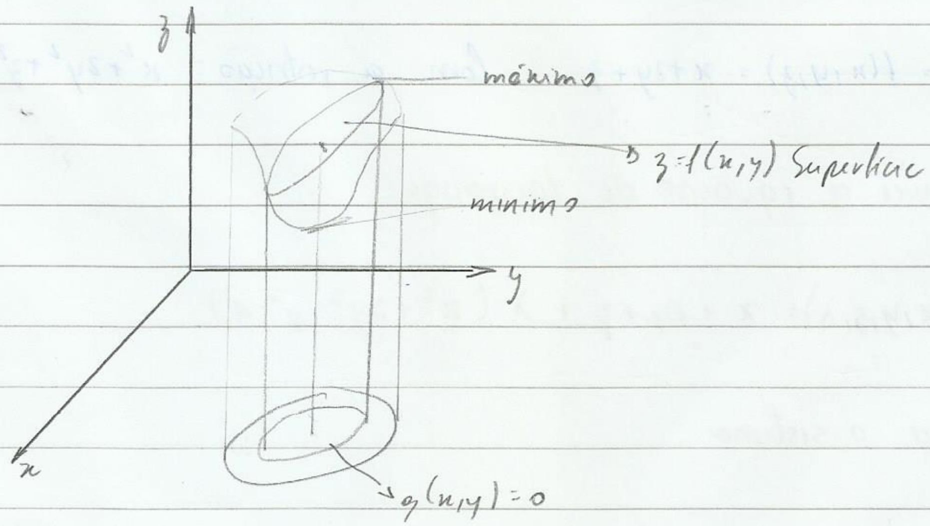
$H(1,-1) = \begin{vmatrix} 12 & 0 \\ 0 & -12 \end{vmatrix} = -144 < 0 \; ; \; P_2(1,-1, f(1,-1))$ Ponto de sela

$H(-1,1) = \begin{vmatrix} -12 & 0 \\ 0 & 12 \end{vmatrix} = -144 < 0 \; ; \; P_3(-1,1, f(-1,1))$ Ponto de sela

$H(-1,-1) = \begin{vmatrix} -12 & 0 \\ 0 & -12 \end{vmatrix} = 144 > 0 \; ; \; A < 0$

$P_4(-1,-1, f(-1,-1))$ máximo

Máximo e mínimo condicionado com uma restrição $g(x,y) = 0$



Solução

1) Equação de Lagrange

$L(x,y,\lambda) = f(x,y) + \lambda g(x,y) \; , \; \lambda \neq 0$

/ /

2) Resolver o sistema

$$(S)\begin{cases}\dfrac{\partial z}{dx}=0\\ \dfrac{dz}{dy}=0\end{cases}\qquad \text{Solução } p(x_i,y_i)$$

$$\frac{dz}{dx}=0$$

3) Calcular a função nas soluções

$$f(x_i,y_i) = \begin{cases} V_{max}: \text{maior } f(x_i,y_i) \\ V_{min}: \text{menor } f(x_i,y_i) \end{cases}$$

Exemplo: Ex 220 (Ver solução no MUPAD)

Ache o valor máximo e mínimo da função

$w = f(x,y,z) = x+2y+z$ com a restrição: $x^2+2y^2+z^2=4$

1) Escrever a equação de Lagrange:

$$L(x,y,z,\lambda) = x+2y+z+\lambda(x^2+2y^2+z^2-4)$$

2) Resolver o sistema

$$\frac{dL}{dx} = 1+2x\lambda=0 \quad \text{(I)} \qquad \frac{dL}{d\lambda} = x^2+2y^2+z^2-4=0 \quad \text{(IV)}$$

$$\frac{dL}{dy} = 2+4y\lambda=0 \quad \text{(II)}$$

$$\frac{dL}{dz} = 1+2z\lambda=0 \quad \text{(III)}$$

$\lambda = \frac{-1}{2x} = \frac{-2}{4y} = \frac{-1}{2z}$

$\therefore x = y = z$

$x^2 + 2x^2 + x^2 - 4 = 0$

$4x^2 - 4 = 0$

$x^2 = \frac{4}{4} \quad = \quad x = \pm 1$

I − III ou V − II

$2x(x-z) = 0 \qquad 4\lambda(x-y) = 0$

$x = z \qquad x = y$

$P_1(1,1,1, f(1,1,1))$

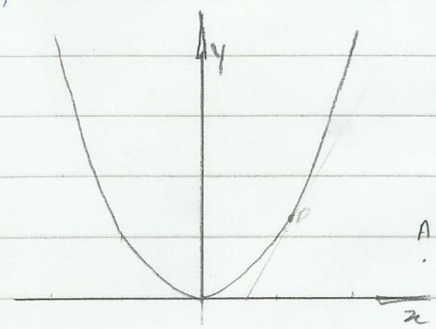
$P_2(-1,-1,-1, f(-1,-1,-1))$

Calcular $f(1,1,1) = 4$ Vmáx

$f(-1,-1,-1) = -4$ Vmin

218-) Determine o ponto da parábola mais próximo de A(14,1)

$y = x^2 \qquad P = (x_0, x_0^2)$



$(y - y_0) = -\frac{1}{f'(x_0)}(x - x_0)$

$y - x_0^2 = \frac{-1}{2x_0}(x - x_0)$

$1 - x_0^2 = \frac{-1}{2x_0}(14 - x_0)$

$1 - x_0^2 = \frac{-7}{x_0} + \frac{1}{2}$

$-x_0^2 + \frac{7}{x_0} + \frac{1}{2} = 0$

$= \frac{-2x_0^3 + 14 + x_0 = 0}{2x_0}$

$-2x_0^3 + x_0 + 14 = 0$

$2x_0^3 - x_0 - 14 = 0$

	2	0	−1	−14
1	2	2	1	−13
2	2	4	7	0

(Respostas "trabalho")

1-) $x \frac{e^{y^2}}{2} \cdot y = k$

2-) $y = \frac{5}{2e^2-1} e^{4x} + \frac{5}{1-2e^2} e^{2x} + (x^2+x) 2e^{2x}$

3-) $f_x = 0$; $f_y = e$

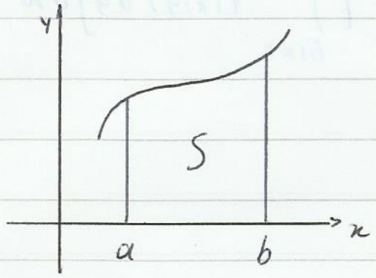
4-) $V_{máx} = 2$ $\quad V_{min} = -2$

5-) $V = 6 u.v$

Integral Dupla

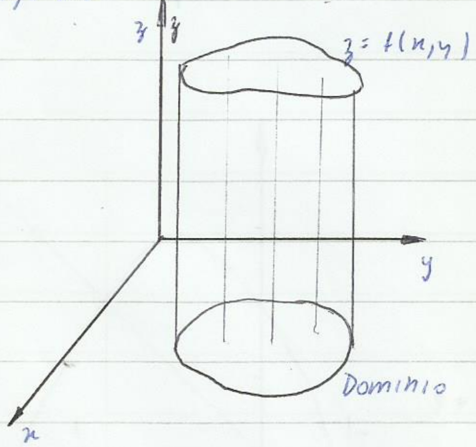
1) $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

Geometricamente: $f(x) > 0$



Área limitada da região por $x = a$, $x = b$, $y = 0$, e $y = f(x)$.

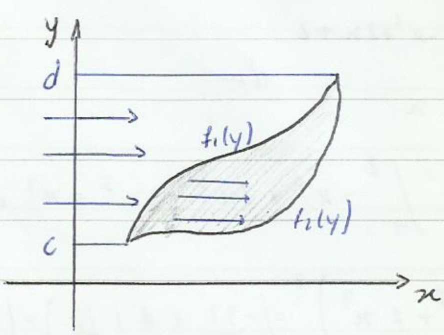
2) $\iint_D f(x,y)\,dA$



Geometricamente: volume de um ~~inter-valo~~ sólido limitado pelo domínio D e a superfície $z = f(x,y)$, se $f(x,y) > 0$

Calculo da $\iint_D f(x,y)\,dA$

1.º caso:



$c \leq y \leq d$

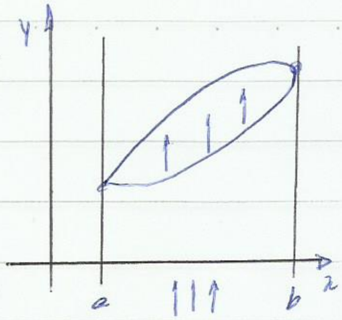
$f_1(y) \leq x \leq f_2(y)$

Calculo da $\iint_D f(x,y)\,dA$

$$\iint_D f(x,y)\,dA = \int_c^d \left[\int_{f_1(y)}^{f_2(y)} f(x,y)\,dx \right] dy$$

/ /

2º caso



$a \le x \le b$ $\qquad \iint_D f(x,y)\,dA = \int_a^b \left[\int_{f_1(x)}^{f_2(x)} f(x,y)\,dy \right] dx$

Ex: 235

Sendo $z = f(x,y) = x^2$ e D o conjunto de todos os pares ordenados (x,y) tais que

$x \le y \le -x^2 + 2x + 2$

$-x^2 + 2x + 2 = x$

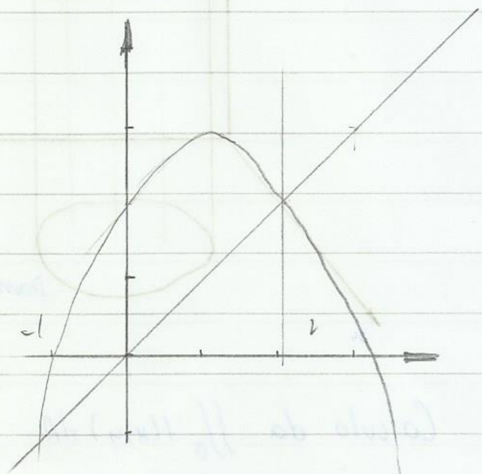
$-x^2 + x + 2 = 0$

$x = \dfrac{-1 \pm \sqrt{1+8}}{-2}$

$\therefore -1 \le x \le 2$

$x_1 = \dfrac{-1+3}{-2} = \dfrac{2}{-2} = -1$ $\qquad x \le y \le -x^2 + 2x + 2$

$x_2 = \dfrac{-1-3}{-2} = \dfrac{-4}{-2} = 2$



$\int_{-1}^{2} \left[\int_{x}^{-x^2+2x+2} x^2\,dy \right] dx = \int_{-1}^{2} \left[x^2 y \right]_{x}^{-x^2+2x+2} dx$

$= \int_{-1}^{2} \left[x^2(-x^2+2x+2) - x^3 \right] dx = \int_{-1}^{2} -x^4 + 2x^3 + 2x^2 - x^3\,dx$

$= \int_{-1}^{2} -x^4 + x^3 + 2x^2\,dx = \left. -\frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{2}{3}x^3 \right|_{-1}^{2} = \left(-\frac{32}{5} + 4 + \frac{16}{3} \right) - \left(\frac{1}{5} + \frac{1}{4} - \frac{2}{3} \right) = \frac{63}{20}$

(23-)

$w = f(x,y,z) = xy + yz$ com vínculos $\pi_1 \quad x + 2y = 6$

$\pi_2 \quad x - 3z = 0$

$$L(x,y,z,a,b) = xy + yz + a(x+2y-6) + b(x-3z)$$

$\frac{dL}{dx} = y + a + b = 0 \qquad \frac{dL}{dy} = x + z + 2a = 0 \qquad \frac{dL}{dz} = y - 3b = 0$

$\frac{dL}{da} = x + 2y - 6 = 0 \qquad \frac{dL}{db} = x - 3z = 0$

$$\begin{cases} y + a + b = 0 \\ x + z + 2a = 0 \\ y - 3b = 0 & y = 3b \\ x + 2y - 6 = 0 & y = \frac{6-x}{2} \\ x - 3z = 0 & x = 3z \quad z = \frac{x}{3} \end{cases}$$

$3b + a + b = 0 \qquad a = -4b$

$x + z - 8b = 0 \qquad b = \frac{y}{3}$

$x + z - \frac{8}{3}y = 0 \Rightarrow 3x + 3z - 8y = 0$

$3x + x - 24 + 4x = 0$

$8x = 24 \qquad x = \frac{24}{8} = 3$

$y = \frac{6-3}{2} = \frac{3}{2} \qquad z = 1$

$P\left(3, \frac{3}{2}, 1\right)$

Exercícios treino

- $z = f(x,y) = x^3y^2$

$f_x(x,y) = 3x^2y^2$
$f_y(x,y) = 2x^3y$

- $z = x^3y^2 + \sqrt{x} + \ln y$

$f_x(x,y) = 3x^2y^2 + \frac{1}{2\sqrt{x}}$

$f_y(x,y) = 2x^3y + \frac{1}{y}$

- $z = \sqrt{x^2-y^2} = (x^2-y^2)^{\frac{1}{2}}$

$f_x(x,y) = \frac{1}{\cancel{2}}(x^2-y^2)^{-\frac{1}{2}} \cdot \cancel{2}x = \frac{x}{\sqrt{x^2-y^2}}$

$f_y(x,y) = \frac{1}{2}(x^2-y^2)^{-\frac{1}{2}} \cdot (-2y) = \frac{-y}{\sqrt{x^2-y^2}}$

- $z = \frac{1}{\sqrt{x^2-y^2}} = (x^2-y^2)^{-\frac{1}{2}}$

$f_x(x,y) = -\frac{1}{\cancel{2}}(x^2-y^2)^{-\frac{3}{2}} \cdot \cancel{2}x = \frac{-x}{\sqrt{(x^2-y^2)^3}}$

$f_y(x,y) = -\frac{1}{2}(x^2-y^2)^{-\frac{3}{2}} \cdot (-2y) = \frac{y}{\sqrt{(x^2-y^2)^3}}$

/ /

- $z = \ln \dfrac{(x^2-y^2)^3}{\sqrt{x^3+y^3}} = 3\ln(x^2-y^2) - \frac{1}{2}\ln(x^3+y^3)$

$f_x(x,y) = 3 \cdot \dfrac{2x}{x^2-y^2} - \dfrac{1}{2} \cdot \dfrac{3x^2}{x^3+y^3} = \dfrac{6x}{x^2-y^2} - \dfrac{3x^2}{2(x^3+y^3)}$

$f_y(x,y) = 3 \cdot \dfrac{(-2y)}{x^2-y^2} - \dfrac{1}{2} \cdot \dfrac{3y^2}{x^3+y^3} = \dfrac{-6y}{x^2-y^2} - \dfrac{3y^2}{2(x^3+y^3)}$

Lista

181-) $f(x,y) = \dfrac{x-y}{x+y}$

$f_x(x,y) = \dfrac{(x+y)-(x-y)}{(x+y)^2} = \dfrac{2y}{(x+y)^2}$

$f_y(x,y) = \dfrac{-(x+y)-(x-y)}{(x+y)^2} = \dfrac{-2x}{(x+y)^2}$

182-) $f(x,y,z) = xy^2z^3 + 3yz$

$f_x(x,y,z) = y^2z^3$

$f_y(x,y,z) = 2xyz^3 + 3z$

$f_z(x,y,z) = 3xy^2z^2 + 3y$

183-) $f(x,y,z) = \cos(4x+3y+2z)$; f_{xyz}; f_{yzz}

$f_x(x,y,z) = -4\sin(4x+3y+2z)$ $f_{xyz} = -24\sin(4x+3y+2z)$

$f_y(x,y,z) = -3\sin(4x+3y+2z)$ $f_{yzz} = -12\sin(4x+3y+2z)$

$f_z(x,y,z) = -2\sin(4x+3y+2z)$

184) $u = e^{r\theta} \operatorname{sen}(\theta)$; calcular $\dfrac{\delta^3 u}{\delta r^2 \delta \theta}$

185) $z = \dfrac{xy^2}{x^2+y^2}$, Verifique que: $x\dfrac{dz}{dx} + y\dfrac{dz}{dy} = z$

$$f_x(x,y) = \frac{2y^2(x^2+y^2) - xy^2 . 2x}{(x^2+y^2)^2} = \frac{\cancel{2y^2x^2} + 2y^4 - \cancel{2x^2y^2}}{(x^2+y^2)^2} = \frac{y^4 - x^2y^2}{(x^2+y^2)^2}$$

$$f_y(x,y) = \frac{2xy(x^2+y^2) - xy^2 . 2y}{(x^2+y^2)^2} = \frac{2x^3y + \cancel{2xy^3} - \cancel{2xy^3}}{(x^2+y^2)^2} = \frac{2x^3y}{(x^2+y^2)^2}$$

$$x\frac{dz}{dx} + y\frac{dz}{dy} = z$$

$$x\frac{y^4 - x^2y^2}{(x^2+y^2)^2} + y\frac{2x^3y}{(x^2+y^2)^2} = \frac{xy^4 - x^3y^2 + 2x^3y^2}{(x^2+y^2)^2} = \frac{xy^4 + x^3y^2}{(x^2+y^2)^2}$$

/ /

186-) $z = x \sin\left(\frac{x}{y}\right)$

$$f_x(x,y) = \sin\left(\frac{x}{y}\right) + \frac{x}{y}\cos\left(\frac{x}{y}\right)$$

$$f_y(x,y) = x \cdot \frac{(-x)}{y^2}\cos\left(\frac{x}{y}\right) = \frac{-x^2}{y^2}\cos\left(\frac{x}{y}\right)$$

$$x\sin\left(\frac{x}{y}\right) + \frac{x^2}{y}\cos\left(\frac{x}{y}\right) - \frac{x^2}{y}\cos\left(\frac{x}{y}\right) = x\sin\left(\frac{x}{y}\right)$$

187-) $z = f(x,y) = \frac{x\sin(y)}{\cos(x^2+y^2)}$

$$f_x(x,y) = \frac{\sin(y)\cos(x^2+y^2) + x\sin(y)\,2x\sin(x^2+y^2)}{\cos^2(x^2+y^2)}$$

$$= \frac{\sin(y)\left[\cos(x^2+y^2) + 2x^2\sin(x^2+y^2)\right]}{\cos^2(x^2+y^2)}$$

$$f_y(x,y) = \frac{x\cos(y)\cos(x^2+y^2) + x\sin(y)\,2y\sin(x^2+y^2)}{\cos^2(x^2+y^2)}$$

$$= \frac{x\left[\cos(y)\cos(x^2+y^2) + 2y\sin(y)\sin(x^2+y^2)\right]}{\cos^2(x^2+y^2)}$$

188) $z =$

188) $z = f(x,y) = x^2 \ln(1+x^2+y^2)$

$$\frac{dz}{dx} = 2x\ln(1+x^2+y^2) + x^2 \cdot \frac{2x}{1+x^2+y^2} = 2x\ln(1+x^2+y^2) + \frac{2x^3}{1+x^2+y^2}$$

$$\frac{dz}{dy} = x^2 \cdot \frac{2y}{1+x^2+y^2} = \frac{2x^2y}{1+x^2+y^2}$$

189-) $f(x,y) = e^{-(x^2+y^2)}$

$$f_x(x,y) = -2x e^{-(x^2+y^2)}$$

$$f_y(x,y) = -2y e^{-(x^2+y^2)}$$

190-) $z = f(x,y) = \ln\left(\frac{x+y}{x-y}\right)$

$$f_x(x,y) = [(x-y)+(x+y)] : \frac{x+y}{x-y}$$

$$= 2x : \frac{x+y}{x-y} = \frac{2x(x-y)}{x+y} = \frac{2x^2-2xy}{x+y}$$

$$f_y(x,y) = [(x-y)-(x+y)] : \frac{x+y}{x-y}$$

$$= \frac{-2y(x-y)}{x+y} = \frac{-2yx+2y^2}{x+y}$$

191-) $z = f(x,y) = \ln(\sqrt{x^2+y^2})$

$$f_x(x,y) = \left[\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x\right] : \sqrt{x^2+y^2} = \frac{x}{x^2+y^2}$$

$f_y(x,y) = \left[\frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}} \cdot \cancel{2}y\right] : \sqrt{x^2+y^2} = \frac{y}{x^2+y^2}$

192-) $z = f(x,y) = \sqrt{x^2+y^2}$

$f_x(x,y) = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f_y(x,y) = \frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}} \cancel{2}y = \frac{y}{\sqrt{x^2+y^2}}$

193-) $f(x,y) = xe^{x+y}$, $P1(1,1,f(1,1))$; $P2(1,0,f(1,0))$

$f_x(x,y) = e^{x+y} + xe^{x+y} = e^{x+y}(1+x)$

$f_x(1,1) = 2e^2$

$f_y(x,y) = xe^{x+y}$ $\qquad f(1,1) = e^2$

$f_y(1,1) = e^2$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$2e^2(x-1) + e^2(y-1) - (z-e^2) = 0$

$2e^2x + e^2y - z - 2e^2 - e^2 + e^2 = 0$

$\therefore$ no ponto $P1$: $2e^2x + e^2y - z - 2e^2 = 0$

$f_x(1,0) = 2e$ $\qquad f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$f_y(1,0) = e$ $\qquad 2e(x-1) + e(y-0) - (z-e) = 0$

$f(1,0) = e$ $\qquad 2ex + ey - z - 2e + e = 0$

$\therefore$ no ponto $P2$: $2ex + ey - z - e = 0$

194-) $f(x,y) = e^{(-x^2-y^2)}$, $P(0,0,1)$

$f_x(x,y) = -2x\,e^{(-x^2-y^2)}$ $\qquad f_x(0,0) = 0$
$f_y(x,y) = -2y\,e^{(-x^2-y^2)}$ $\qquad f_y(0,0) = 0$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$
$0(x-0) + 0(y-0) - (z-1) = 0$

$\therefore -z+1=0$ ou $z-1=0$

195-) $f(x,y) = e^x \ln(y)$; $P(3,1,f(3,1))$

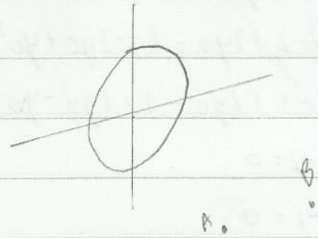
$f_x(x,y) = e^x \ln(y)$ $\qquad f_x(3,1) = 0$
$f_y(x,y) = e^x \dfrac{1}{y}$ $\qquad f_y(3,1) = e^3$

$f(3,1) = 0$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$0(x-3) + e^3(y-1) - (z-0) = 0$
$e^3 y - z - e^3 = 0$

196) $f(x,y) = x^2+y^2$ $\quad A(1,1,0)$; $B(2,1,4)$



$f_x(x,y) = 2x_0$
$f_y(x,y) = 2y_0$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$
$2x_0(x-x_0) + 2y_0(y-y_0) - (z-z_0) = 0$

/ /

197-) $z = f(x,y) = 4x^2 - y^2 + 2y$; $P(-1, 2, f(-1,2))$

$f_x(x,y) = 8x$ $\quad$ $f_x(-1,2) = -8$

$f_y(x,y) = -2y + 2$ $\quad$ $f_y(-1,2) = -2$

$f(x,y) = 4x^2 - y^2 + 2y$ $\quad$ $f(-1,2) = 4 - 4 + 4$

$f_x(x - x_0) + f_y(y - y_0) - (z - z_0) = 0$

$-8(x - (-1)) - 2(y - 2) - (z - 4) = 0$

$-8x - \cancel{8} - 2y + \cancel{4} - z + \cancel{4} = 0$

$-8x - 2y - z = 0 \quad \therefore z = -8x - 2y$

198-) $z = f(x,y) = 9x^2 + y^2 + 6x - 3y + 5$, $P(1, 2, f(1,2))$

$f_x(x,y) = 18x + 6$ $\quad$ $f_x(1,2) = 24$ $\quad$ $f(x,y) = 18$

$f_y(x,y) = 2y - 3$ $\quad$ $f_y(1,2) = 1$

$f_x(x - x_0) + f_y(y - y_0) - (z - z_0) = 0$

$24(x - 1) + (y - 2) - (z - 18) = 0$

$24x - 24 + y - 2 - z + 18 = 0$

$24x + y - z - 8 = 0$

199-) $z = f(x,y) = \sqrt{4 - x^2 - 2y^2}$; $P(1, -1, f(1,-1))$

$f_x(x,y) = \frac{1}{2}(4 - x^2 - 2y^2)^{-\frac{1}{2}} \cdot (-2x) = \dfrac{-x}{\sqrt{4 - x^2 - 2y^2}}$

$f_y(x,y) = \frac{1}{2}(4 - x^2 - 2y^2)^{-\frac{1}{2}}(-4y) = \dfrac{-2y}{\sqrt{4 - x^2 - 2y^2}}$

$f_x(1,-1) = -1$

$f_y(1,-1) = 2$

$f(1,-1) = 1$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$-(x-1) + 2(y+1) - (z-1) = 0$

$-x + 2y - z + 4 = 0$

200) $z = f(x,y) = y\cos(x-y)$; $P(2,2,f(2,2))$

$f_x(x,y) = -y\,sen(x-y)$ $\quad$ $f_x(2,2) = 0$

$f_y(x,y) = \cos(x-y) + y\,sen(x-y)$ $\quad$ $f_y(2,2) = 1$

$f(2,2) = 2$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$0(x-2) + (y-2) - (z-2) = 0$

$y - z = 0$

201-) $f(x,y) = x^2 + y^2$ $\quad$ $z = 2x + y$

$f_x(x,y) = 2x$ $\quad$ $z_x = 2$

$f_y(x,y) = 2y$ $\quad$ $z_y = 1$

$2x = 2 \therefore x = 1$ $\quad$ $f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$2y = 1 \therefore y = \frac{1}{2}$ $\quad$ $2(x-1) + 1(y - \frac{1}{2}) - (z - \frac{9}{4}) = 0$

$2x + y - z + z_0 = \frac{5}{4} + \frac{9}{4} =$

$x^2 + y^2 = 2x + y + z_0 - \frac{5}{4}$

$\therefore x = 1 \quad y = \frac{1}{2}$

$1 + \frac{1}{4} = 2 + \frac{1}{4} + z_0 - \frac{5}{4} \Rightarrow \frac{5}{4} - \frac{9}{4} + \frac{5}{4} = z_0$

202) $z = 2x+y \qquad A(1,1,3)$

$$f_x(1,1)(x-1) + f_y(1,1)(y-1) - (z-3) = 0$$

$$x f_x(1,1) - f_x(1,1) + y f_y(1,1) - f_y(1,1) - z + 3 = 0$$

$$z = x f(\cancel{x},1) - f(x)(1,1) + y f_y(1,1) - f_y(1,1) - z + 3 = 2x+y$$

$$\frac{dz}{dx} f(1,1) = 2 \qquad \frac{dz}{dy} = 1$$

203-) $f(x,y) = x^2+y^2 \qquad x+y+z-3=0 \cap z=0$

$$x+y+z-3 = z$$

$$x+y-3=0$$

204.) $f(x,y) = \text{arctg}(x-2y)$ Ponto $A\left(2, \frac{1}{2}, f(2,\frac{1}{2})\right)$

$f(2,\frac{1}{2}) = \text{arctg}(2-1) = \frac{\pi}{4}$

$f_x = \dfrac{1}{1+(x-2y)^2} \Rightarrow f_x(2,\frac{1}{2}) = \frac{1}{2}$

$f_y = \dfrac{-2}{1+(x-2y)^2} \Rightarrow f_y(2,\frac{1}{2}) = -1$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$\frac{1}{2}(x-2) - (y-\frac{1}{2}) - (z-\frac{\pi}{4}) = 0$

$\frac{x}{2} - y - z - 1 + \frac{1}{2} + \frac{\pi}{4} = 0$ $\vec{V} = (\frac{1}{2}, -1, -1)$

$2x - 4y - 4z - 2 + \pi = 0$ $\therefore$ $4z = \underline{2x - 4y + \pi - 2}$

$r: X = (2, \frac{1}{2}, \frac{\pi}{4}) + \lambda(\frac{1}{2}, -1, -1), \lambda \in \mathbb{R}$

205-) $f(x,y) = 2x^2 + 2xy + y^2 + 2x - 3$

$f_x(x,y) = 4x + 2y + 2$ $\qquad$ $f_y(x,y) = 2x + 2y$

$4x + 2y + 2 = 0$ $\qquad$ $2x + 2y = 0$

$\begin{cases} 4x + 2y = -2 \\ 2x + 2y = 0 \end{cases} \sim \begin{cases} 4x + 2y = -2 \\ 0 - 2y = -2 \end{cases}$ $\quad \therefore y = 1$ $\qquad 4x + 2 = -2$ $\quad x = -1$

Ponto crítico $(-1, 1)$

$\dfrac{d^2z}{dx^2} = 4$ $\qquad$ $\dfrac{d^2z}{dy^2} = 2$ $\qquad$ R: Mínimo Relativo $(-1, 1, -4)$

$f(-1,1) = 2(-1)^2 + 2(-1)1 + 1^2 + 2(-1) - 3$

206-) $f(x,y) = -5x^2 + 4xy - y^2 + 16x + 10$

$f_x(x,y) = -10x + 4y + 16$ $\qquad$ $f_y(x,y) = 4x - 2y$

$-10x + 4y + 16 = 0$ $\qquad$ $4x - 2y = 0$

$\begin{cases} -10x + 4y = -16 \\ 4x - 2y = 0 \end{cases} \sim \begin{cases} -10x + 4y = -16 \\ -2x + 0 = -16 \end{cases}$ $\quad \therefore x = 8$ $\qquad -80 + 4y = -16$ $\quad \therefore y = \dfrac{64}{4} = 16$

Ponto crítico $P_1 = (8, 16)$

$\dfrac{d^2z}{dx^2} = -10$ $\qquad$ $\dfrac{d^2z}{dy^2} = -2$

$f(8,16) = 74$ $\qquad$ R: Ponto Máximo Relativo $(8, 16, 74)$

/ /

207-) $f(x,y) = 2x^2 + 3y^2 - 4x - 12y + 13$

$f_x(x,y) = 4x - 4$ $\qquad$ $f_y(x,y) = 6y - 12$

$4x - 4 = 0$ $\qquad$ $6y - 12 = 0$

$x = 1$ $\qquad$ $y = 2$

$\dfrac{d^2 z}{dx^2} = 4 \qquad \dfrac{d^2 z}{dy^2} = 6 \qquad f(1,2) = 2 + 12 - 4 - 24 + 13 = -1$

R: Ponto Mínimo Relativo $(1,2,-1)$

208-) $f(x,y) = 2\sqrt{x^2+y^2+3}$

$f_x(x,y) = \dfrac{2 \cdot 2x}{\sqrt{x^2+y^2+3}} \Rightarrow \dfrac{4x}{\sqrt{x^2+y^2+3}} = 0 \quad \therefore \quad x = 0$

$f_y(x,y) = \dfrac{2 \cdot 2y}{\sqrt{x^2+y^2+3}} \Rightarrow \dfrac{4y}{\sqrt{x^2+y^2+3}} = 0 \quad \therefore \quad y = 0$

$\dfrac{d^2 z}{dx^2} = 4 \cdot \left[\sqrt{x^2+y^2+3} - \dfrac{x \cdot 2x}{2\sqrt{x^2+y^2+3}} \right] : (x^2+y^2+3)$

$= 4 \cdot \left[\dfrac{\cancel{2}(x^2+y^2+3) - \cancel{2}x^2}{\cancel{2}\sqrt{x^2+y^2+3}} \right] : (x^2+y^2+3)$

$= \dfrac{4 \quad (y^2+3)}{\sqrt{(x^2+y^2+3)^3}}$

$\left(\dfrac{d^2 z}{dx^2} \right)_{(0,0)} = \dfrac{12}{\sqrt{27}} = \dfrac{12}{3\sqrt{3}} = \dfrac{4}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$

$$\frac{d^2 z}{dy^2} = 4 \cdot \left[\sqrt{x^2+y^2+3} - y \cdot \frac{2y}{2\sqrt{x^2+y^2+3}} \right] : (x^2+y^2+3)$$

$$= 4 \cdot \left[\frac{(x^2+y^2+3) - y^2}{\sqrt{x^2+y^2+3}} \right] : (x^2+y^2+3)$$

$$\left(\frac{d^2 z}{dy^2} \right)_{(0,0)} = 4 \cdot \frac{3}{\sqrt{3}} : 3 = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

$$H(0,0) = \begin{vmatrix} \frac{4\sqrt{3}}{3} & 0 \\ 0 & \frac{4\sqrt{3}}{3} \end{vmatrix} = \left(\frac{4\sqrt{3}}{3} \right)^2 = \frac{16.3}{9} = \frac{16}{3} > 0$$

$\therefore$ Ponto Minimo Relativo $(0,0,2\sqrt{3})$

209) $f(x,y) = 4x^2 + 3xy + y^2 + 12x + 2y + 1$

$f_x(x,y) = 8x + 3y + 12$ $\quad$ $f_y(x,y) = 3x + 2y + 2$

$8x + 3y + 12 = 0$ $\quad$ $3x + 2y + 2 = 0$

$$\begin{cases} 8x + 3y = -12 \\ 3x + 2y = -2 \end{cases} \sim \begin{cases} 8x + 3y = -12 \\ 0 + 7y = 20 \end{cases} \quad \therefore y = \frac{20}{7}$$

$x = \left(-12 - \frac{3.20}{7}\right) : 8$

$x = \frac{-18}{7}$

$$\frac{d^2 z}{dx^2} = 8 \quad \frac{d^2 z}{dy^2} = 2$$

$$H = \begin{vmatrix} 8 & 0 \\ 0 & 2 \end{vmatrix} = 16 > 0$$

$\therefore$ Ponto minimo relativo

$\left(\frac{-18}{7}, \frac{20}{7}, f\left(\frac{-18}{7}, \frac{20}{7}\right) \right)$

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210-) $f(x,y) = 2x^3 + 2y^3 - 6x - 6y$

$f_x(x,y) = 6x^2 - 6$ $\quad\quad$ $f_y(x,y) = 6y^2 - 6$

$6x^2 - 6 = 0$ $\quad\quad$ $6y^2 - 6 = 0$

$x = \pm 1$ $\quad\quad$ $y = \pm 1$

Pontos criticos

$P_1 = (1,1)$ $P_2(1,-1)$ $P_3(-1,1)$ $P_4(-1,-1)$

$\frac{d^2z}{dx^2} = 12x$ $\quad$ $\frac{d^2z}{dy^2} = 12y$

$H(1,1) = \begin{vmatrix} 12 & 0 \\ 0 & 12 \end{vmatrix} = 144 > 0$ $\therefore$ Ponto Mínimo relativo no Ponto: $(1,1,-8)$

$H(1,-1) = \begin{vmatrix} 12 & 0 \\ 0 & -12 \end{vmatrix} = -144 < 0$ $\therefore$ Ponto de sela no Ponto $(1,-1,0)$

$H(-1,1) = \begin{vmatrix} -12 & 0 \\ 0 & 12 \end{vmatrix} = -144 < 0$ $\therefore$ Ponto de sela no Ponto $(-1,1,0)$

$H(-1,-1) = \begin{vmatrix} -12 & 0 \\ 0 & -12 \end{vmatrix} = 144 > 0$ $\therefore$ Ponto Máximo no Ponto $(-1,-1,8)$

211-) $f(x,y) = \frac{x^4}{9} - x^2y + 3y^3 - x^2 + 4$

$f_x(x,y) = \frac{4}{9}x^3 - 2xy - 2x$ $\quad\quad$ $f_y(x,y) = -x^2 + 9y^2$

$= \frac{4x^3 - 18xy - 18x}{9} = 0$ $\quad\quad$ $= -x^2 + 9y^2 = 0$

$$\begin{cases} \dfrac{4x^3 - 18xy - 18x = 0}{9} & x(4x^2 - 18y - 18) = 0 \quad \therefore \; x = 0 \\ -x^2 + 9y^2 = 0 & x^2 = 9y^2 \end{cases}$$

$4x^2 - 18y - 18 = 0$

$36y^2 - 18y - 18 = 0$

$2y^2 - y - 1 = 0$

$y = \dfrac{1 \pm \sqrt{1+8}}{4}$

$y_1 = \dfrac{1+3}{4} = 1 \quad y_2 = \dfrac{1-3}{4} = -\dfrac{1}{2}$

$P_1(0,1) \; P_2(0,-\frac{1}{2})$

$P_3(\frac{3}{2},1) \; P_4(\frac{3}{2},-\frac{1}{2})$

$\therefore \; x = \sqrt{9.1} = 3$

$x = \sqrt{9.\frac{1}{4}} = \frac{3}{2}$

$\dfrac{d^2 z}{dx^2} = \dfrac{4}{3}x^2 - 2y - 2$

$\dfrac{d^2 z}{dy^2} = 18y$

$\dfrac{d^2 z}{dx dy} = -2 \qquad \dfrac{d^2 z}{dy dx} = 0$

/ /

212-) $f(x,y) = 2x^3 + 2y^3 - 6x - 6y$

$f_x(x,y) = 6x^2 - 6$ $\qquad$ $f_y(x,y) = 6y^2 - 6$

$6x^2 - 6 = 0$ $\qquad$ $6y^2 - 6 = 0$

$x = \pm 1$ $\qquad$ $y = \pm 1$

$P_1(1,1)$ $P_2(1,-1)$ $P_3(-1,1)$ $P_4(-1,-1)$

$\frac{d^2 z}{dx^2} = 12x$ $\qquad$ $\frac{d^2 z}{dy^2} = 12y$ $\qquad$ $\frac{d^2 z}{dx\,dy} = 0$ $\qquad$ $\frac{d^2 z}{dy\,dx} = 0$

$H(1,1) = \begin{vmatrix} 12 & 0 \\ 0 & 12 \end{vmatrix} = 144 > 0$ Ponto mínimo relativo $(1,1,-8)$

$H(1,-1) = \begin{vmatrix} 12 & 0 \\ 0 & -12 \end{vmatrix} = -144 < 0$ Ponto de sela

$H(-1,1) = \begin{vmatrix} -12 & 0 \\ 0 & 12 \end{vmatrix} = -144 < 0$ Ponto de sela

$H(-1,-1) = \begin{vmatrix} -12 & 0 \\ 0 & -12 \end{vmatrix} = 144 > 0$ Ponto máximo relativo $(-1,-1,8)$

213) $f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

$f_x(x,y) = -2 - 2x$ $\qquad$ $f_y(x,y) = 4 - 8y$

$-2x - 2 = 0$ $\qquad$ $4 - 8y = 0$

$x = -1$ $\qquad$ $y = \frac{1}{2}$

$P\left(-1, \frac{1}{2}\right)$

$\frac{d^2 z}{dx^2} = -2$ $\frac{d^2 z}{dy^2} = -8$ $\frac{d^2 z}{dx dy} = 0$ $\frac{d^2 z}{dy dx} = 0$

$H\left(-1, \frac{1}{2}\right) = \begin{vmatrix} -2 & 0 \\ 0 & -8 \end{vmatrix} = 16 > 0$ Ponto mínimo relativo

$P \cdot \left(-1, \frac{1}{2}, 11\right)$

$f\left(-1, \frac{1}{2}\right) = 9 + 2 + 2 - 1 - 4 \cdot \frac{1}{4} = 11$

214) $f(x,y) = x \operatorname{sen}(y)$

$f_x(x,y) = \operatorname{sen}(y)$ $f_y(x,y) = x\cos(y)$

$\operatorname{sen}(y) = 0$ $x\cos(y) = 0$

$y = n\pi$ $x\cos(n\pi) = 0$ $\therefore x = 0$

$\frac{d^2 z}{dx^2} = \cos(y)$ $\frac{d^2 z}{dy^2} = -x\operatorname{sen}(y)$

$\frac{d^2 z}{dx dy} = -\operatorname{sen}(y)$ $\frac{d^2 z}{dy dx} = -\operatorname{sen}(y)$

$H(0, n\pi) = \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix}$

?

215-) $f(x,y) = x^2 + y^2 + x^2y + 4$ em $D\ \{(x,y),\ |x| \leq 1 \text{ e } |y| \leq 1\}$

$f_x(x,y) = 2x + 2xy$ $\qquad$ $f_y(x,y) = 2y + x^2$

$2x + 2xy = 0$ $\qquad$ $2y + x^2 = 0$

$$\begin{cases} 2x + 2xy = 0 & \rightarrow 2x + x.(-x^2) = 0 \quad 2x - x^3 = 0 \\ 2y + x^2 = 0 & 2y = -x^2 \end{cases}$$

$x(2 - x^2) = 0$

$x = 0 \quad x = \cancel{\pm\sqrt{2}}$

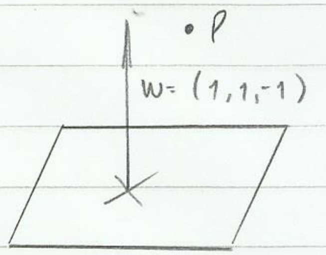
$2y + 0 = 0 \quad \therefore y = 0$

$2y + 2 = 0 \quad \therefore y = -1$

$P_1 = (0,0) \quad P_2 = (1;1) \quad P_2 = (-1,-1)$

$\dfrac{d^2z}{dx^2} = 2 + 2y \qquad \dfrac{d^2z}{dy^2} = 2 \qquad \dfrac{dz}{dx\,dy} = 2$

216-)



217-) $f(x,y) = x^2 + 2xy + y^2$ $\to$ restrição $x + 2y - 1 = 0$

$$L(x,y,\lambda) = x^2 + 2xy + y^2 + \lambda(x + 2y - 1) \; ; \; \lambda \in \mathbb{R}$$

$$\frac{dL}{dx} = 2x + 2y + \lambda \qquad \frac{dL}{dy} = 2x + 2y + 2\lambda \qquad \frac{dL}{d\lambda} = x + 2y - 1$$

$$\begin{cases} 2x + 2y + \lambda = 0 \\ 2x + 2y + 2\lambda = 0 \\ x + 2y - 1 = 0 \end{cases} \qquad \begin{cases} 2x + 2y = \lambda \\ -2x - 2y = 2\lambda \end{cases}$$

$$0 = 3\lambda \therefore \lambda = 0$$

$$2x + 2y = 0 \qquad x + 2y - 1 = 0$$

$$x + y = 0 \quad x = -y \qquad -y + 2y - 1 = 0$$

$$y = 1 \quad \therefore x = -1$$

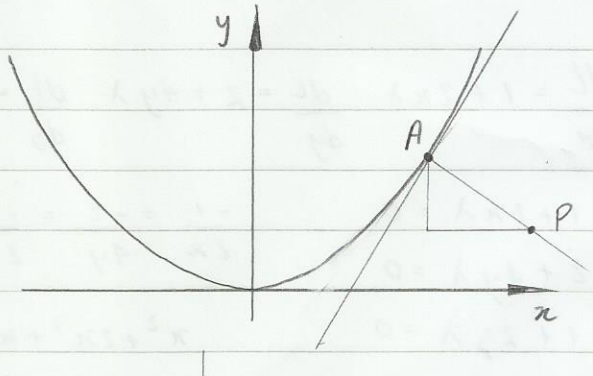
$P(-1, 1, f(x,y))$

$$f(-1,1) = (-1)^2 + 2(-1).1 + 1^2 = 0$$

$P(-1, 1, 0)$

$\therefore$ Ponto Mínimo

218-) $y = x^2$ $\quad P = (14, 1)$



$$(y - f(x_0)) = -\frac{1}{f'(x_0)}(x - x_0)$$

$$(1 + x_0^2) = -\frac{1}{2x_0}(14 - x_0)$$

$$1 - x_0^2 = -\frac{7}{x_0} + \frac{1}{2} \qquad \frac{-2x_0^3 + 14 + x_0}{2x_0} = 0$$

$$1 - x_0^2 + \frac{7}{x_0} - \frac{1}{2} = 0$$

	-2	0	1	14
1	-2	-2	-1	13
2	-2	-4	-7	0

$x_0 = 2$ $\quad y_0 = 4$

$P(2, 4)$

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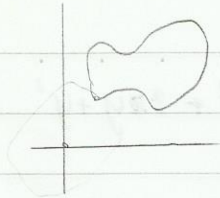


219.) $x + 2y - 3z = 4$

$$z = \frac{4 - x - 2y}{-3}$$

$f_x(x,y) = 1/3$ $\quad 1/3(0-x_0) + 2/3(0-y_0) - (0-z_0) = 0$

$f_y(x,y) = 2/3$

$$\begin{cases} -\frac{1}{3}x_0 - \frac{2}{3}y_0 + z_0 = 0 \\ x_0 + 2y_0 - 3z_0 = 4 \end{cases} \qquad \begin{cases} -x_0 - 2y_0 + 3z_0 = 0 \\ x_0 + 2y_0 - z_0 = 0 \end{cases}$$

220.) $f(x,y,z) = x + 2y + z \qquad x^2 + 2y^2 + z^2 - 4 = 0$

$$L(x,y,z,\lambda) = x + 2y + z + \lambda(x^2 + 2y^2 + z^2 - 4)$$

$$\frac{dL}{dx} = 1 + 2x\lambda \quad \frac{dL}{dy} = 2 + 4y\lambda \quad \frac{dL}{dz} = 1 + 2z\lambda \quad \frac{dL}{d\lambda} = x^2 + 2y^2 + z^2 - 4$$

$$\begin{cases} 1 + 2x\lambda = 0 \\ 2 + 4y\lambda = 0 \\ 1 + 2z\lambda = 0 \\ x^2 + 2y^2 + z^2 - 4 = 0 \end{cases}$$

$$\frac{-1}{2x} = \frac{-2}{4y} = \frac{-1}{2z} \Rightarrow \frac{-1}{2x} = \frac{-1}{2y} = \frac{-1}{2z} \therefore x = y = z$$

$$x^2 + 2x^2 + x^2 - 4 = 0$$

$$4x^2 = 4 \Rightarrow x = \pm 1$$

$P_1(1,1,1) \Rightarrow f(1,1,1) = 4 \therefore$ Valor máximo é 4 sendo atingido neste ponto

$P_1(-1,-1,-1) \Rightarrow f(-1,-1,-1) = -4 \therefore$ Valor mínimo é -4 sendo atingido neste ponto

iii) $f(x,y,z) = xyz \qquad x+y+z=32 \qquad x-y+z=0$

$L(x,y,z,a,b) = xyz + a(x+y+z-32) + b(x-y+z) \; ; \; a,b \in \mathbb{R}$

$\frac{dL}{dx} = yz + a + b \qquad \frac{dL}{dy} = xz + a - b \qquad \frac{dL}{dz} = xy + a + b$

$\frac{dL}{da} = x+y+z-32 \qquad \frac{dL}{db} = x-y+z$

$$\begin{cases} yz + a + b = 0 & \text{I} \\ xz + a - b = 0 & \text{III} \\ xy + a + b = 0 & \text{II} \\ x+y+z-32 = 0 \\ x - y + z = 0 \end{cases} \qquad \begin{cases} yz + a + b = 0 \\ xz + a + b = 0 \end{cases} \sim \begin{cases} yz + a + b = 0 \\ -xz - a - b = 0 \end{cases}$$

$yz - xz = 0 \quad z(y-x) = 0$

$\therefore z = 0$

$x = y$

$$\begin{cases} x+y+z = 32 \\ x-y+z = 0 \quad (-1) \end{cases} \qquad \begin{cases} x+y+z = 32 \\ -x+y-z = 0 \end{cases}$$

$2y = 32$

$y = \frac{32}{2} = 16$

/ /

222.) $f(x,y,z) = x^2+y^2+z^2 \qquad x+2z=6 \quad x-3z=0$

$L(x,y,z,\lambda) = x^2+y^2+z^2 + a(x+2z-6) + b(x-3z)$

$\frac{dL}{dx} = 2x+a+b \qquad \frac{dL}{dy} = 2y \qquad \frac{dL}{dz} = 2z+2a-3b$

$\frac{dL}{da} = x+2z-6 \qquad \frac{dL}{db} = x-3z$

$$\begin{cases} 2x+a+b=0 \\ 2y=0 \\ 2z+2a-3b=0 \\ x+2z-6=0 & \text{I} \\ x-3z=0 & \text{II} \end{cases} \qquad \begin{cases} x+2z=6 \\ x-3z=0 \end{cases} \sim \begin{cases} x+2z=6 \\ -x+3z=0 \end{cases}$$

$5z = 6$

$z = \frac{6}{5}$

$x = 3 \cdot \frac{6}{5} = \frac{18}{5}$

$f(x,y,z) = f\left(\frac{18}{5}, 0, \frac{6}{5}\right) = \frac{360}{25}$

223.) $f(x,y,z) = xy+yz \qquad x+2y-6=0 \qquad x-3z=0$

$L(x,y,z,a,b) = xy+yz + a(x+2y-6) + b(x-3z)$

$\frac{dL}{dx} = y+a+b \qquad \frac{dL}{dy} = x+z+2a \qquad \frac{dL}{dz} = y-3b \qquad \frac{dL}{da} = x+2y-6 \qquad \frac{dL}{db} = x-3z$

$$\begin{cases} y+a+b=0 \\ x+z+2a=0 \\ y-3b=0 \\ x-3z=0 \end{cases}$$
$x+2y=6$

$a = -b-y$

$b = \frac{y}{3}$

$\Rightarrow a = -\frac{y}{3} - 1 = -\frac{4}{3}y$

$x+z+2a=0$

$x+z-\frac{8}{3}y=0$

$x+z-\frac{8}{3}y=0 \qquad x=3z$

$4z-\frac{8}{3}y=0 \qquad z=\frac{2}{3}y \qquad \therefore x=3.\frac{2}{3}y \quad \therefore x=2y$

$x+2y=6 \qquad \therefore x=2.\frac{3}{2}=3$

$2y+2y=6$

$y=\frac{6}{4}=\frac{3}{2} \qquad \therefore z=\frac{2}{3}.\frac{3}{2}=1$

R: $f(3,\frac{3}{2},1)=6$ Valor máximo

224-) $f(x,y,z)=xyz \qquad x^2+z^2=5$ e $x-2y=0$

$L(x,y,z,a,b)=xyz+a(x^2+z^2-5)+b(x-2y)$

$\frac{dL}{dx}=yz+2xa+b \qquad \frac{dL}{dy}=xz-2b \qquad \frac{dL}{dz}=xy+2za$

$\frac{dL}{da}=x^2+z^2-5 \qquad \frac{dL}{db}=x-2y$

$$\begin{cases} yz+2xa+b=0 & \text{I} \\ xz-2b=0 \\ xy+2za=0 \\ x^2+z^2-5=0 \\ x-2y=0 \end{cases}$$

$b=\frac{xz}{2}$

$a=\frac{-xy}{2z}$

I : $yz+2x.\frac{(-xy)}{2z}+\frac{xz}{2}=0$

$\Rightarrow yz-\frac{x^2y}{z}+\frac{xz}{2}=0$

$\frac{2yz^2-x^2y+xz^2}{2z}=0$

$\Rightarrow 2yz^2-x^2y+xz^2=0$

225) $f(x,y,z) = x^2 y \; ; \; x^2 + 2y^2 = 6$

$L(x,y,\lambda) = x^2 y + \lambda(x^2 + 2y^2 - 6) \; ; \; \lambda \in \mathbb{R}$

$\frac{dL}{dx} = 2xy + 2x\lambda \qquad \frac{dL}{dy} = x^2 + 4y\lambda \qquad \frac{dL}{d\lambda} = x^2 + 2y^2 - 6$

$\begin{cases} 2xy + 2x\lambda = 0 \\ x^2 + 4y\lambda = 0 & \text{I} \\ x^2 + 2y^2 - 6 = 0 & \text{II} \end{cases}$ $\qquad 2xy + 2x\lambda = 0 \; (\div 2x)$

$y + \lambda = 0$

$y = -\lambda \iff \lambda = -y$

$\text{I} \Rightarrow x^2 + 4y\lambda = 0$

$x^2 - 4y^2 = 0$

$\begin{cases} x^2 - 4y^2 = 0 \\ x^2 + 2y^2 = 6 \end{cases} \sim \begin{cases} x^2 - 4y^2 = 0 \\ -6y^2 = -6 \end{cases} \quad \therefore y = \pm 1$

$x^2 - 4 = 0$

$x = \pm 2 \qquad P_1 = (2,1) \; P_2 = (-2,1) \; P_3 = (2,-1) \; P_3 = (-2,-1)$

$f(2,1) = 4 \quad f(-2,1) = 4 \quad f(2,-1) = -4 \quad f(-2,-1) = -4$

Valor Máximo $f(\pm 2, 1) = 4$

Valor Mínimo $f(\pm 2, -1) = -4$

226-) $f(x,y,z) = x^2 + y^2 + z^2 \; ; \; x^4 + y^4 + z^4 = 1$

$L(x,y,z,\lambda) = x^2 + y^2 + z^2 + \lambda(x^4 + y^4 + z^4 - 1) \; ; \; \lambda \in \mathbb{R}$

$\frac{dL}{dx} = 2x + 4x^3\lambda \qquad \frac{dL}{dy} = 2y + 4y^3\lambda \qquad \frac{dL}{dz} = 2z + 4z^3\lambda \qquad \frac{dL}{d\lambda} = x^4 + y^4 + z^4 - 1$

$\begin{cases} 2x + 4x^3\lambda = 0 \\ 2y + 4y^3\lambda = 0 \\ 2z + 4z^3\lambda = 0 \end{cases}$ $\qquad \frac{-2x}{4x^3} = \frac{-2y}{4y^3} = \frac{-2z}{4z^3} \Rightarrow \frac{-1}{2x^2} = \frac{-1}{2y^2} = \frac{-1}{2z^2}$

$\therefore x^2 = y^2 = z^2$

$$(x^2)^2 + (y^2)^2 + (z^2)^2 = 1$$

$$x^4 + x^4 + x^4 = 1$$

$$3x^4 = 1 \qquad x^4 = \frac{1}{3}$$

227-) α: $x + y + 2z = 2$

r: $z = x^2 + y^2$

$$L(x,y,z,\lambda) = x + y + 2z - 2 + \lambda(x^2 + y^2 - z)$$

$$\frac{dL}{dx} = 1 + 2x\lambda \qquad \frac{dL}{dy} = 1 + 2y\lambda \qquad \frac{dL}{dz} = 2 - \lambda \qquad \frac{dL}{d\lambda} = x^2 + y^2 - z$$

$$\begin{cases} 1 + 2x\lambda = 0 \\ 1 + 2y\lambda = 0 \\ 2 - \lambda = 0 \\ x^2 + y^2 - z = 0 \end{cases}$$

$\lambda = 2$

$1 + 2x.2 = 0$

$\therefore x = \frac{-1}{4}$

$1 + 2.y.2 = 0$

$y = \frac{-1}{4}$

$x^2 + y^2 - z = 0$

$\therefore z = x^2 + y^2$

$z = \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$

228-) $x^2+y^2 \leq 1$

$T(x,y) = x^2 - x + 2y^2$

$L(x,y,\lambda) = x^2+y^2-1 + a(x^2-x+2y^2)$

$\frac{dL}{dx} = 2x + 2xa - a \qquad \frac{dL}{dy} = 2y + 4ya \qquad \frac{dL}{da} = x^2 - x + 2y^2$

$2x + 2xa - a = 0 \qquad \text{I}$

$2y + 4ya = 0 \Rightarrow 2y(1+2a) = 0 \therefore a = -1/2$

$x^2 - x + 2y^2 = 0 \qquad \text{II}$

$\text{I} \quad 2x - x + \frac{1}{2} = 0$

$x = -\frac{1}{2}$

$\text{II} \quad \frac{1}{4} + \frac{1}{2} + 2y^2 = 0$

$\frac{1+2}{4} = \frac{3}{4} + 2y^2 = 0$

/ /

Integral Dupla

229-) $\iint_R 3\,dA,\ R = (x,y);\ -2 \leq x \leq 2;\ 1 \leq y \leq 6\}$

$$\int_{-2}^{2}\left[\int_{1}^{6} 3\,dA\right]dA = \int_{-2}^{2}\left[3A\Big|_{1}^{6}\right]da = \int_{-2}^{2}(18-3)\,dA = \int_{-2}^{2} 15\,dA = 15A\Big|_{-2}^{2}$$

$15.2 - (15.-2) = \underline{60}$

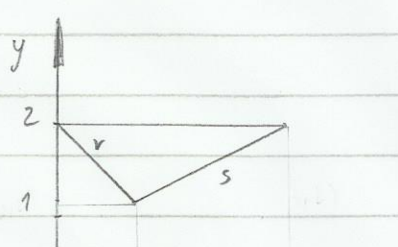
230-) $\iint_R (5-x)\,dA,\ R = \{(x,y),\ 0 \leq x \leq 5,\ 0 \leq y \leq 3\}$

$$\int_0^5\left[\int_0^3 5-x\,dA\right]dx$$

$$\int_0^3 5-x\,dA = 5A - xA\Big|_0^3 = 15-3x$$

$$\int_0^5 (15-3x)\,dx = 15x - \frac{3x^2}{2}\Big|_0^5 = 75 - \frac{75}{2} = \frac{75}{2}$$

231-) $\iint_D y^3\,dA$; D é a região $(0,2); (1,1); (3,2)$



r: $2 = b$ $\therefore a = 1-b = 1-2 = -1$

$1 = a+b$ $\therefore y = -x+2$

s: $\begin{cases} 2 = 3a+b \\ 1 = a+b \end{cases} \sim \begin{cases} 2 = 3a+b \\ 1 = 2a \end{cases} \therefore a = \frac{1}{2}$

$b = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ s: $y = \frac{x+1}{2}$

r: $y = -x+2$ $\therefore$ $x = 2-y$

s: $y = \frac{x+1}{2}$ $\therefore$ $x = 2y-1$

$$\int_1^2 \left[\int_{2-y}^{2y-1} y^3 \, dx \right] dy$$

$$\int_{2-y}^{2y-1} y^3 \, dx = xy^3 \Big|_{2-y}^{2y-1} = y^3(2y-1+y-2) = y^3(3y-3)$$

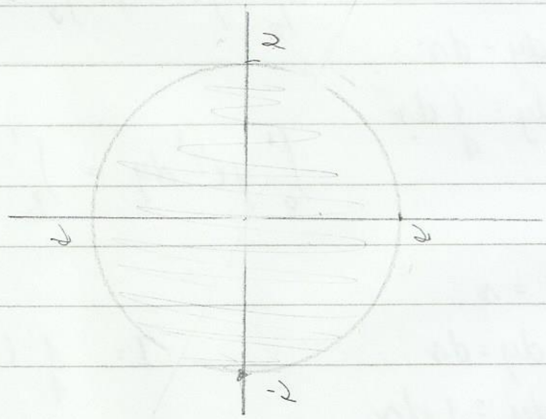
$$= 3y^4 - 3y^3$$

$$\int_1^2 3(y^4 - y^3)\, dy = 3\left(\frac{1}{5}y^5 - \frac{1}{4}y^4\right)\Big|_1^2 = \left(\frac{32}{5} - \frac{16}{4} - \frac{1}{5} + \frac{1}{4}\right)3$$

$$= \frac{49}{20}$$

232) $\iint_D (2x-y)\, dA$, $D = x^2+y^2-4=0$

$$y = \sqrt{4-x^2}$$



$$\int_{-2}^{2} \left(\int_{\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x-y)\, dy \right) dx$$

$$237)\ f(x,y) = y^3 e^{xy^2} \qquad 0 \le x \le 1 \quad e \quad 1 \le y \le 2$$

$$\int_0^1 \left[\int_1^2 y^3 e^{xy^2} dx \right] dy$$

$$\int_1^2 y^3 e^{xy^2} dx \qquad \int_1^2 \frac{1}{y^2} y^3 e^{s} ds = y \int_1^2 e^{s} = y e^{xy^2} \Big|_1^2 =$$

$$xy^2 = s \qquad = y e^{2y^2} - y e^{y^2}$$

$$y^2 dx = ds$$

$$\int_0^1 y e^{2y^2} - y e^{y^2} dy = \int_0^1 y e^{2y^2} dy - \int_0^1 y e^{y^2} dy$$

$$2y^2 = s \qquad \int_0^1 y e^{2y^2} dy = \int_0^1 e^{s} \frac{1}{4} ds = \frac{1}{4} e^{2y^2} \Big|_0^1 = \frac{e^2}{4} - \frac{1}{4}$$

$$4y\, dy = ds$$

$$y\, dy = \frac{1}{4} ds$$

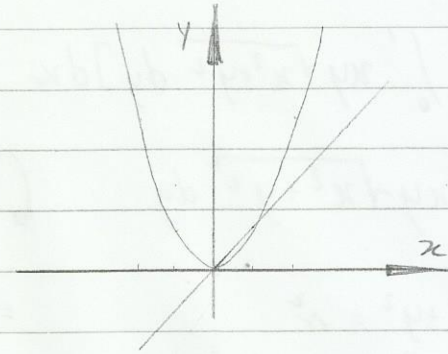
$$\int_0^1 y e^{y^2} dy = \int_0^1 e^{s} \frac{1}{2} ds = \frac{1}{2} e^{y^2} \Big|_0^1 = \frac{e-1}{2}$$

$$y^2 = s$$

$$2y\, dy = ds \qquad \therefore I = \frac{1}{4}(e^2 - 1) - \frac{1}{2}(e-1) = \frac{1}{4}(e^2 - 1 - 2e + 2)$$

$$y\, dy = \frac{1}{2} ds$$

234-) $f(x,y) = x+y$; $y=x$ $y=x^2$ com $0 \leq x \leq 1$



$x = x^2$ $\quad$ $y=0$ $\quad y=1$

$x - x^2 = 0$

$x(1-x) = 0$

$x=0 \quad x=1$

$$\int_0^1 \left[\int_{x^2}^{x} x+y \, dy \right] dx$$

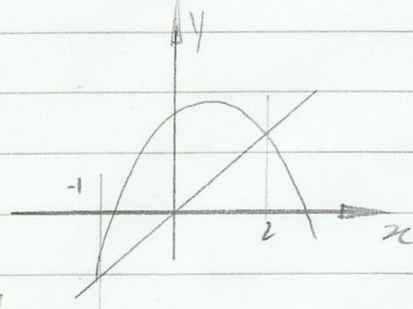
$$\int_{x^2}^{x} x+y \, dy = xy + \frac{1}{2} y^2 \Big|_{x^2}^{x} = x^2 + \frac{x^2}{2} - x^3 - \frac{x^4}{2}$$

$$= \frac{3x^2}{2} - x^3 - \frac{x^4}{2} = \frac{3x^2 - 2x^3 - x^4}{2}$$

$$\int_0^1 \frac{1}{2} (3x^2 - 2x^3 - x^4) \, dx = \frac{1}{2} \left(x^3 - \frac{1}{2} x^4 - \frac{1}{5} x^5 \right) \Big|_0^1$$

$$= \frac{1}{2} - \frac{1}{4} - \frac{1}{10} = \frac{3}{20}$$

235) $f(x,y) = x^2$ $\quad x < y < -x^2 + 2x + 2$



$-x^2 + 2x + 2 = x$

$-x^2 + x + 2 = 0 \quad x = -1 \quad x = 2$

$$\int_{-1}^{2} \left[\int_{x}^{-x^2+2x+2} x^2 \, dy \right] dx = \int_{-1}^{2} (x^2 y) \Big|_{x}^{-x^2+2x+2} dx$$

$$\int_{-1}^{2} x^2(-x^2 + 2x + 2 - x) \, dx = \int_{-1}^{2} -x^4 + x^3 + 2x^2 \, dx = \frac{-1}{5} x^5 + \frac{1}{4} x^4 + \frac{2}{3} x^3 \Big|_{-1}^{2}$$

$$= \frac{-32}{5} + \frac{16}{4} + \frac{16}{3} - \left(\frac{1}{5} + \frac{1}{4} - \frac{2}{3} \right) = \frac{63}{20}$$

236-) $f(x,y) = xy\sqrt{x^2+y^2}$ $0 \le x \le 1$ e $0 \le y \le 1$

$\int_0^1 \left[\int_0^1 xy\sqrt{x^2+y^2}\, dy \right] dx$

$\int_0^1 xy\sqrt{x^2+y^2}\, dy$ $\int_0^1 x . s^2\, ds = x . \frac{1}{3} s^3 \Big|_0^1$

$x^2+y^2 = s^2$ $= \frac{1}{3} x \, (x^2+y^2)(x^2+y^2)^{\frac{1}{2}} \Big|_0^1$

$2y\,dy = 2s\,ds$

$y\,dy = s\,ds$ $= \frac{1}{3} x (x^2+1)(x^2+1)^{\frac{1}{2}} - \frac{1}{3} x . (x^2) . x$

$= \frac{1}{3} x (x^2+1)(x^2+1)^{1/2} - \frac{1}{3} x^4$

$= \frac{1}{3}\left(x^2\sqrt{(x^2+1)^3} - x^4\right)$

$\int_0^1 \frac{1}{3}\left(x^2\sqrt{(x^2+1)^3} - x^4\right) dx = \frac{1}{3}\left(\int_0^1 x^2\sqrt{(x^2+1)^3}\, dx - \int x^4 dx\right)$

$x^2+1 = s^2$ $= \frac{1}{3}\left(\sqrt{s^2-1} . (s)\right)$

$2x\,dx = 2s\,ds$

$x\,dx = s\,ds$

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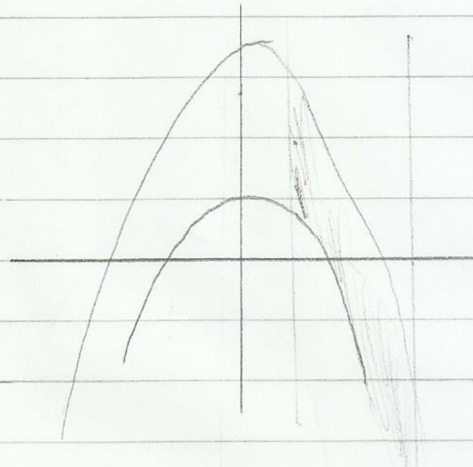
237-) $\iint_R x^3\, dA \qquad y = 6 - x^2;\ y = 2 - x^2;\ x = 1 \quad x = 3$

$6 - x^2 = 0 \qquad 2 - x^2 = 0$

$x = \pm\sqrt{6} \qquad x = \pm\sqrt{2}$



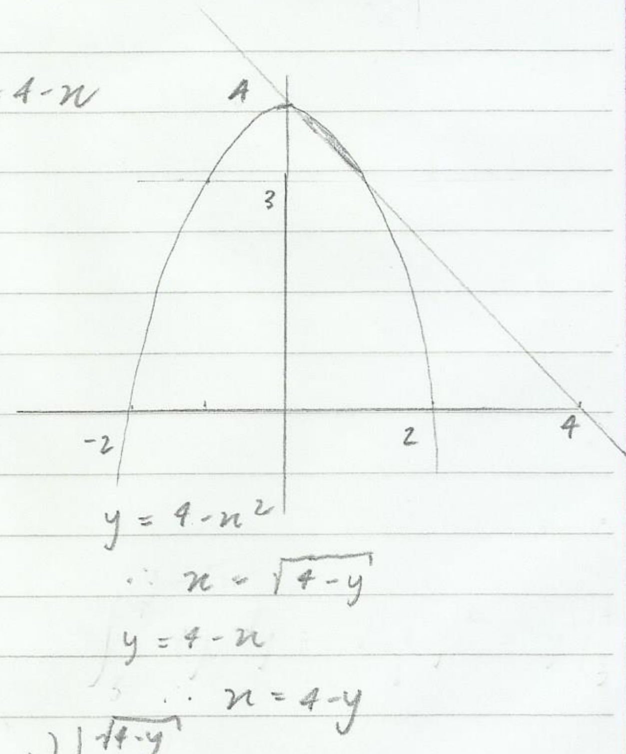
$\int_1^3 \left[\int_{2-x^2}^{6-x^2} x^3\, dy\right] dx$

$\int_{2-x^2}^{6-x^2} x^3\, dy = x^3 y \Big|_{2-x^2}^{6-x^2}$

$x^3(6 - x^2 - 2 + x^2) = x^3 \cdot 4$

$\int_1^3 4x^3\, dx = 4 \cdot \frac{1}{4} x^4 \Big|_1^3 = 3^4 - 1 = \underline{80}$

238-) $\iint_R -2y\ln(x)\, dA \qquad y = 4 - x^2 \quad y = 4 - x$



$4 - x^2 = 0 \qquad 4 - x = 0$

$x = \pm 2 \qquad x = 4$

$4 - x^2 = 4 - x$

$-x^2 + x = 0$

$x(-x+1) = 0 \qquad y(0) = 4$

$x = 0 \quad x = 1 \qquad y(1) = 3$

$y = 4 - x^2$

$\therefore x = \sqrt{4-y}$

$y = 4 - x$

$\therefore x = 4 - y$

$\int_3^4 \left[\int_{4-y}^{\sqrt{4-y}} -2y\ln(x)\, dx\right] dy$

$\int_{4-y}^{\sqrt{4-y}} -2y\ln(x)\, dx = -2yx(\ln x - 1)\Big|_{4-y}^{\sqrt{4-y}}$

$-2y\sqrt{4-y}\,(\ln\sqrt{4-y} - 1)$

Plano tangente e Reta Normal

Determinar o plano que contenha os pontos $A(1,1,0)$ e $B(2,1,4)$ que seja tangente ao gráfico de $f(x,y) = x^2+y^2$

$f_x(x,y) = 2x_0$

$f_y(x,y) = 2y_0$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$2x_0(1-x_0) + 2y_0(1-y_0) - (0-z_0) = 0$

$2x_0 - 2x_0^2 + 2y_0 - 2y_0^2 + z_0 = 0$ (I)

$2x_0(2-x_0) + 2y_0(1-y_0) - (4-z_0) = 0$

$4x_0 - 2x_0^2 + 2y_0 - 2y_0^2 - 4 + z_0 = 0$ (II)

$$\begin{cases} 4x_0 - 2x_0^2 + 2y_0 - 2y_0^2 + z_0 = 4 \\ 2x_0 - 2x_0^2 + 2y_0 - 2y_0^2 + z_0 = 0 \quad (x-1) \end{cases}$$

$2x_0 = 4 \therefore x_0 = 2$

$2.2 - 2.4 + 2y_0 - 2y_0^2 + z_0 = 0$

$-2y_0^2 + 2y_0 - 4 + z_0 = 0 \quad \therefore z_0 = 2y_0^2 - 2y_0 + 4$

$\therefore 2y_0^2 - 2y_0 + 4 - x_0^2 - y_0^2 = 0$

$y_0^2 - 2y_0 = 0 \Rightarrow y_0(y-2) = 0 \quad \therefore y_0 = 0 \quad y_0 = 2$

$4(x-2) - (z-4) = 0 \rightarrow r_1: 4x - z - 4 = 0$

$4(x-2) + 4(y-2) - (z-8) = 0$

$r_2: 4x + 4y - z - 8 = 0$

$z = f(x,y) = \sqrt{4-x^2-2y^2}$; $P(1,-1,f(1,-1))$

$f_x(x,y) = \dfrac{-2x}{2\sqrt{4-x^2-2y^2}} = \dfrac{-x}{\sqrt{4-x^2-2y^2}}$ $\quad f_x(1,-1) = -1$

$f_y(x,y) = \dfrac{-4y}{2\sqrt{4-x^2-2y^2}} = \dfrac{-2y}{\sqrt{4-x^2-2y^2}}$ $\quad f_y(1,-1) = 2$

$f(1,-1) = 1$

$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$

$-(x-1) + 2(y+1) - (z-1) = 0$

$-x+2y-z+1+2+1 = 0 \quad \therefore \quad -x+2y-z+4=0$

Determine o plano que seja paralelo ao plano $z = 2x+y$ e tangente ao gráfico de $f(x,y) = x^2+y^2$

Resolução Anexo

229-) $\iint_R 3dA$, $R = \{(x,y) \,;\, -2 \leq x \leq 2 \,;\, 1 \leq y \leq 6\}$

$$\int_{-2}^{2}\left[\int_{1}^{6} 3\,dy\right]dx = \int_{-2}^{2}\left(3y\Big|_{1}^{6}\right)dx = \int_{-2}^{2} 15\,dx = 15x\Big|_{-2}^{2}$$

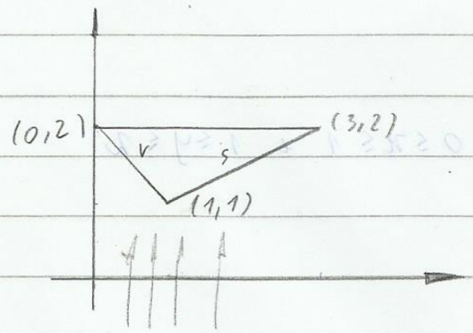
$$= 30 - (-30) = 60$$

230-) $\iint_R (5-x)\,dA$, $R = \{(x,y) \,;\, 0 \leq x \leq 5 \,;\, 0 \leq y \leq 3\}$

$$\int_{0}^{3}\left[\int_{0}^{5}(5-x)\,dx\right]dy = \int_{0}^{3}\left(5x - \frac{x^2}{2}\Big|_{0}^{5}\right)dy = \int_{0}^{3} 25 - \frac{25}{2}\,dy$$

$$= \int_{0}^{3}\frac{25}{2}\,dy = \frac{25x}{2}\Big|_{0}^{3} = \frac{75}{2}$$

231) $\iint_D y^3\,dA$, D é a região (limitada) triangular com vértices $(0,2)$; $(1,1)$; $(3,2)$



$y = ax + b$

r: $2 = a.0 + b \quad \therefore b = 2$

$1 = a + b \quad a = -1$

$\therefore r$: $y = -x + 2 \quad x = 2 - y$

s: $\begin{cases} 1 = a + b \\ 2 = 3a + b \end{cases} \sim \begin{cases} 1 = a + b \\ 1 = 2a \end{cases} \quad \therefore a = \frac{1}{2}$

$\therefore s$: $y = \frac{1}{2}x + \frac{1}{2} \quad b = \frac{1}{2}$

$y = \frac{x+1}{2} \quad \therefore x = 2y - 1$

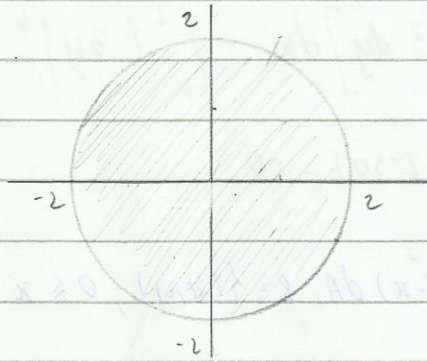
$$\int_{1}^{2}\left(\int_{2-y}^{2y-1} y^3\,dx\right)dy$$

$$\int_{1}^{2} xy^3\Big|_{2-y}^{2y-1}\,dy$$

$$\int_{1}^{2}\left((2y-1)y^3 - (2-y)y^3\right)dy \sim \int_{1}^{2} y^3(2y-1-2+y)\,dy \sim \int_{1}^{2} y^3(3y-3)\,dy$$

$$= \int_{1}^{2} 3y^4 - 3y^3\,dy = \frac{3}{5}y^5 - \frac{3}{4}y^4\Big|_{1}^{2} = \frac{96}{5} - \frac{48}{4} - \frac{3}{5} + \frac{3}{4} = \frac{147}{20}$$

232) $\iint_D (2x-y)\,dA$ limitada pelo círculo de centro (0,0) e raio 2



$(x-a)^2+(y-b)^2=R^2$

$x^2+y^2=4$

$\therefore x=\sqrt{4-y^2}$

$$\int_{-2}^{2}\left(\int_{\sqrt{4-y^2}}^{\sqrt{4-y^2}} 2x-y\,dx\right)dy$$

$$\int_{-2}^{2} x^2-y\Big|_{\sqrt{4-y^2}}^{\sqrt{4-y^2}} dy = \int_{-2}^{2} (4-y^2)-y-(4-y^2)+y\,dy$$

$$\int_{-2}^{2} \cancel{4}-\cancel{y^2}-\cancel{y}-\cancel{4}+\cancel{y^2}+y\,dy = y\Big|_{-2}^{2} = 2-(-2) = 0$$

232) $f(x,y)=y^3e^{xy^2}$ D é o retângulo $0\le x\le 1$ e $1\le y\le 2$

$$\int_1^2\left(\int_0^1 y^3e^{xy^2}\,dx\right)dy$$

$$\int_0^1 y^3e^{xy^2}dx = \int_0^1 y^{\cancel{3}}e^r\frac{1}{\cancel{y^2}}dr = ye^{xy^2}\Big|_0^1 = ye^{y^2}-y$$

$xy^2=r$

$y^2dx=dr$

$dx=\frac{1}{y^2}dr$

$$\int_1^2 ye^{y^2}dy-\int_1^2 y\,dy = \int_1^2 ye^{y^2}dy-\frac12 y^2\Big|_1^2$$

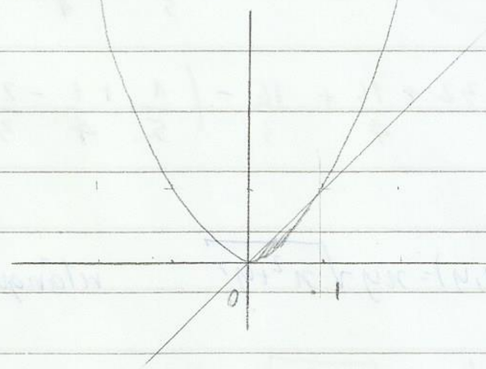
$$\int_1^2 ye^{y^2}dy \sim \int_1^2 e^r\cdot\frac12 dr = \frac12 e^{y^2}\Big|_1^2 = \frac{e^4}{2}-\frac{e}{2}-2-\frac12$$

$y^2=r$

$$=\frac{e^4-e}{2}-\frac32 = \frac12(e^4-e-3)$$

$2y\,dy=dr \quad y\,dy=\frac12 dr$

234) $f(x,y) = x+y \qquad y = x \quad y = x^2 \quad 0 \le x \le 1$



$$\int_0^1 \left(\int_{x^2}^{x} x+y \, dy \right) dx$$

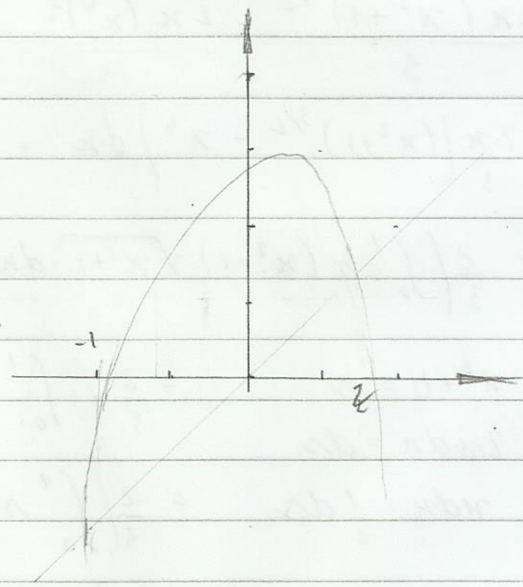
$$\int_0^1 xy + \frac{y^2}{2} \Big|_{x^2}^{x} dx$$

$$\int_0^1 \left(x^2 + \frac{x^2}{2}\right) - \left(x^3 + \frac{x^4}{2}\right) dx$$

$$\int_0^1 \left(\frac{3x^2}{2} - x^3 - \frac{x^4}{2} \right) dx$$

$$= \frac{3x^3}{2 \cdot 3} - \frac{1}{4}x^4 - \frac{x^5}{10} \Big|_0^1 = \frac{10x^3 - 5x^4 + 2x^5}{20} \Big|_0^1 = \frac{10-5-2}{20} = \frac{3}{20}$$

235) $f(x,y) = x^2 \qquad x \le y \le -x^2 + 2x + 2$



$$V = \left(\frac{-b}{2a}, -\frac{\Delta}{4a} \right) \qquad \Delta = 4+8 = 12$$

$$= \left(\frac{-2}{-2}, \frac{12}{4} \right)$$

$$= (1, 3)$$

$$x = -x^2 + 2x + 2$$

$$-x^2 + x + 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1+8}}{-2} \qquad x_1 = -1 \quad x_2 = 2$$

$$\int_{-1}^{2} \left(\int_{x}^{-x^2+2x+2} x^2 \, dy \right) dx = \int_{-1}^{2} x^2 y \Big|_{x}^{-x^2+2x+2} dx =$$

$$\int_{-1}^{2} x^2(-x^2+2x+2) - x^3 \, dx = \int_{-1}^{2} -x^4 + 2x^3 + 2x^2 - x^3 \, dx$$

$$\int_{-1}^{2} -x^4 + x^3 + 2x^2\, dx = -\frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{2}{3}x^3 \Big|_{-1}^{2} =$$

$$= -\frac{1}{5}\cdot 32 + \frac{16}{4} + \frac{16}{3} - \left(\frac{1}{5} + \frac{1}{4} - \frac{2}{3}\right) = \frac{63}{20}$$

236-) $f(x,y) = xy\sqrt{x^2+y^2}$ retângulo $0 \le x \le 1$ e $0 \le y \le 1$.

$$\int_0^1 \left(\int_0^1 xy\sqrt{x^2+y^2}\, dy \right) dx$$

$x^2+y^2 = s$ $\qquad \int_0^1 \left(\int_0^1 x\sqrt{s}\, ds \right) dx$

$2y\, dy = ds$

$y\, dy = \frac{1}{2} ds$ $\qquad \int_0^1 x \frac{(x^2+y^2)^{\frac{3}{2}}}{(3/2)} \Big|_0^1 dx$

$$\int_0^1 \frac{2x(x^2+1)^{3/2}}{3} - \frac{2x(x^2)^{\frac{3}{2}}}{3}\, dx$$

$$\int_0^1 \frac{2x}{3}\left((x^2+1)^{3/2} - x^3\right) dx = \int_0^1 \frac{2x\sqrt[2]{(x^2+1)^3} - x^3}{3}\, dx$$

$$= \frac{2}{3}\left(\int_0^1 x(x^2+1)\sqrt{x^2+1}\, dx - \int x^3\, dx \right)$$

$= x^2+1 = s$ $\qquad = \frac{2}{3}\left(\int_0^1 s\sqrt{s}\cdot\frac{1}{2}\, ds - \frac{1}{4}x^4 \Big|_0^1 \right.$

$2x\, dx = ds$

$x\, dx = \frac{1}{2} ds$ $\qquad = \frac{2}{3}\left(\int_0^1 s^{\frac{3}{2}}\cdot\frac{1}{2}\, ds - \frac{1}{4} \right)$

$$= \frac{2}{3}\left(\frac{2}{5}(x^2+1)^{5/2}\cdot\frac{1}{2}\Big|_0^1 - \frac{1}{4} \right)$$

$$= \frac{2}{3}\left(\frac{\cancel{2}}{5}\sqrt{2^2 2^2 2}\cdot\frac{1}{\cancel{2}} - \frac{\cancel{2}}{5}\cdot\frac{1}{\cancel{2}}\,1 - \frac{1}{4} \right)$$

$$= \frac{2}{3}\left(\frac{4\sqrt{2}}{5} - \frac{1}{5} - \frac{1}{4} \right) = \frac{8\sqrt{2}}{15} - \frac{2}{15} - \frac{1}{2} =$$

$$= 2\left(\frac{y^2+3}{(x^2+y^2+3)^{\frac{3}{2}}}\right) = \frac{2y^2+6}{(x^2+y^2+3)^{\frac{3}{2}}}$$

$$\frac{d^2z}{dy^2} = 2\left(\sqrt{x^2+y^2+3} - \frac{y \cdot y}{\sqrt{x^2+y^2+3}}\right) \div (x^2+y^2+3)$$

$$= 2\left(\frac{(x^2+y^2+3)-y^2}{(x^2+y^2+3)\sqrt{x^2+y^2+3}}\right)$$

$$= \frac{2x^2+6}{(x^2+y^2+3)^{\frac{3}{2}}}$$

$$\left(\frac{d^2z}{dx^2}\right)_{(0,0)} = \frac{6}{\sqrt{3^3}} = \frac{6}{3\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\left(\frac{d^2z}{dy^2}\right)_{(0,0)} = \frac{6}{\sqrt{3^3}} = \frac{6}{3\sqrt{3}} = \frac{2}{\sqrt{3}}\,\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{2}}{3}$$

Ponto: $(0,0,2\sqrt{3})$

Determinar a equação dos planos tangentes à superfície dada por $z = y^2x - x$ que sejam paralelos ao plano $3x - 4y - z + 8 = 0$

$3x - 4y - z + 8 = 0 \qquad \vec{W} = (3,-4,-1)$

$$\frac{\delta z}{\delta x} = y^2 - 1 \qquad \frac{\delta z}{dy} = 2yx$$

$y^2 - 1 = 3 \quad \therefore y = \pm 2 \qquad 2xy = -4 \qquad$ Quando $y = 2 \ \therefore x = -1 \quad z = -3$

$y = -2 \ \therefore x = 1 \quad z = 3$

$P_1 = (-1, 2, -3)$

$P_2 = (1, -2, 3)$

$\left(\frac{dz}{dx}\right) = 3 \quad \left(\frac{dz}{dy}\right) = -4$

$3(x+1) - 4(y-2) - (z+3) = 0$

$3x + 2y - z + 8 = 0$

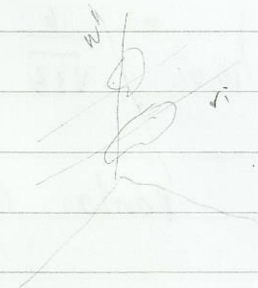
$3(x-1) - 4(y+2) - (z-3) = 0$

$3x - 4y - z - 8 = 0$

- Determine o plano que seja paralelo ao plano $z = 2x + y$ e tangente ao gráfico de $f(x,y) = x^2 + y^2$

$z = 2x + y$

$2x + y - z = 0 \quad \vec{w} = (2, 1, -1)$



$f_x(x,y) = 2x = 2 \quad \therefore \quad x = 1$

$f_y(x,y) = 2y = 1 \quad \therefore \quad y = 1/2$

$P_1 = \left(1, \frac{1}{2}, \frac{5}{4}\right)$

$f_x(x - x_0) + f_y(y - y_0) - (z - z_0) = 0$

$2(x-1) + \left(y - \frac{1}{2}\right) - \left(z - \frac{5}{4}\right) = 0 \qquad \frac{-8 - 2 + 5}{4} = -\frac{5}{4}$

$2x + y - z - 2 - \frac{1}{2} + \frac{5}{4} = 0 \qquad 2x + y - z - \frac{5}{4} = 0$

- Determine o plano que contem os pontos $A(1,1,0)$ e $B(2,1,4)$ e que seja tangente ao gráfico de $f(x,y) = x^2 + y^2$

$f(1,1) = 1^2 + 1^2 = 2 \quad \therefore$ o ponto A não pertence ao $f(x,y)$

$f(2,1) = 2^2 + 1^2 = 5 \quad \therefore$ " " B " " " "

$f_x(x,y) = 2x_0$

$f_y(x,y) = 2y_0$

$2x_0(1-x_0)+2y_0(1-y_0)-(0-z_0)=0$

$2x_0+2y_0+z_0-2x_0^2-2y_0^2=0$

$2x_0(2-x_0)+2y_0(1-y_0)-(4-z_0)=0$

$4x_0+2y_0+z_0-2x_0^2-2y_0^2-4=0$

$$\begin{cases} 4x_0+2y_0+z_0-2x_0^2-2y_0^2=4 \\ -2x_0-2y_0-z_0+2x_0^2+2y_0^2=0 \end{cases}$$

$2x_0=4$

$\therefore x_0=2$

$2x_0+2y_0+x_0^2+y_0^2-2x_0^2-2y_0^2=0$ $\qquad P_1=(2,0,4)$

$4+2y_0+4+y_0^2-8-2y_0^2=0$ $\qquad P_2=(2,2,8)$

$-y_0^2+2y_0=0$

$y_0(-y_0+2)=0 \qquad y_{0_1}=0 \quad y_{0_2}=2$

$2.2(x-2)+2.0(y-0)-(z-4)=0$

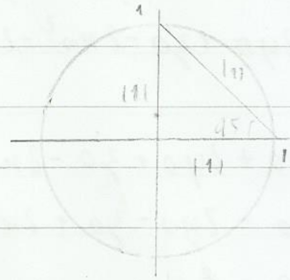
$\underline{4x-z-4=0}$

$2.2(x-2)+2.2(y-2)-(z-8)=0$

$\underline{4x+4y-8=0}$

Determine as equações do plano tangente e da reta normal ao gráfico da função $f(x,y) = \operatorname{arctg}(x-2y)$ no ponto $A\left(2, \frac{1}{2}, f\left(2, \frac{1}{2}\right)\right)$

$f\left(2, \frac{1}{2}\right) = \operatorname{arctg}\left(2 - \frac{2}{2}\right) = \operatorname{arctag} 1 = \frac{\pi}{4}$



$f_x(x,y) = \dfrac{1}{1+(x-2y)^2}$ $\qquad f_y(x,y) = \dfrac{2}{1+(x-2y)^2}$

$f_x\left(2, \frac{1}{2}\right) = 1/2$ $\qquad f_y\left(2, \frac{1}{2}\right) = 1$

$\frac{1}{2}(x-2) + \left(y - \frac{1}{2}\right) - \left(z - \frac{\pi}{4}\right) = 0$

$\frac{1}{2}x + y - z - 1 - \frac{1}{2} + \frac{\pi}{4} = 0 \quad \therefore \quad \frac{x}{2} + y - z - \frac{3}{2} + \frac{\pi}{4}$

$X = \left(2, \frac{1}{2}, \frac{\pi}{4}\right) + \lambda\left(\frac{1}{2}, 1, -1\right), \lambda \in R$

Determinar a equação geral do plano tangente à superfície $z = f(x,y) = x^2 + xy - y^2$ que seja paralelo ao plano: $3x + 4y + 2z - 5 = 0$

$3x + 4y + 2z - 5 = 0$ $\qquad 1 - 1 \cdot \frac{1}{2} - \frac{1}{4} =$ $\frac{4-2-1}{4} = \frac{1}{4}$

$(\div 2) \rightarrow \frac{3}{2}x + 2y + z - 5 = 0 \quad (x-1)$

$-\frac{3}{2}x - 2y - z + 5 = 0 \quad \therefore \quad \vec{w} = \left(-\frac{3}{2}, -2, -1\right)$

$f_x(x,y) = 2x + y = -\frac{3}{2}$ $\qquad \begin{cases} 4x + 2y = -3 \\ x - 2y = -2 \end{cases}$ $\qquad y = -\frac{3}{2} - 2x$

$f_y(x,y) = x - 2y = -2$ $\qquad 5x = -5$ $\qquad y = -\frac{3}{2} + 2 = \frac{1}{2}$

$x = -1$

$-\frac{3}{2}(x+1) - 2\left(y - \frac{1}{2}\right) - \left(z - \frac{5}{4}\right) = 0$ $\qquad f\left(-1, \frac{1}{2}\right) = \frac{1}{4}$

$-\frac{3}{2}x - 2y - z - \frac{3}{2} + \frac{2}{2} + \frac{1}{4} = 0 \quad \therefore \quad -\frac{3}{2}x - 2y - z - \frac{1}{4} = 0$ (R: $-6x - 8y - 4z - 1 = 0$

P2 (04/06/2010)

1ª Questão Resolver $y'' - 2y' - 3y = x^2 - 2x - 3$

homogênea associada

$p(\lambda) = \lambda^2 - 2\lambda - 3 = 0$ $\qquad y_h = C_1 e^{3x} + C_2 e^{-x}$

$\lambda = \frac{2 \pm \sqrt{4+12}}{2}$ $\qquad \lambda_1 = 3 \quad \lambda_2 = -1$

$y_p = (Ax^2 + Bx + C)$ $\qquad 2A - 4Ax - 2B - 3Ax^2 - 3Bx - 3C = x^2 - 2x - 3$

$y'p = 2Ax + B$ $\qquad -3Ax^2 = x^2$

$y''p = 2A$ $\qquad A = \frac{-1}{3}$

$-4Ax - 3Bx = -2x$ ∴

$-4 \cdot \left(\frac{-1}{3}\right) - 3B = -2$

∴ $B = \left(-2 - \frac{4}{3}\right) \div -3 = \frac{-6-4}{3} \div -3 = \frac{10}{9}$

$2A - 2B - 3C = -3$ $\qquad$ ∴ $C = \left(-3 + \frac{2}{3} + \frac{20}{9}\right) \div -3$

$-\frac{2}{3} - \frac{20}{9} - 3C = -3$

$C = \left(\frac{-27 + 6 + 20}{9}\right) \div -3 = \frac{-1}{9.(-3)} = \frac{1}{27}$

$y_G = C_1 e^{3x} + C_2 e^{-x} - \frac{x^2}{3} + \frac{10x}{9} + \frac{1}{27}$

/ /

3ª Questão

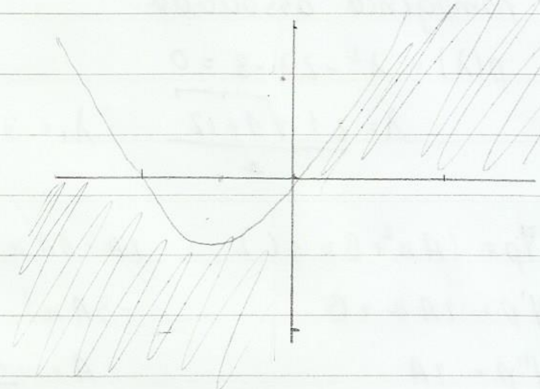
$z = f(x,y) = \ln(x^2+x-y) + \sqrt{xy}$

$D_f(x,y) = \{(x,y) \in \mathbb{R}^2 \mid x^2+x-y > 0 \text{ e } xy \geq 0$

$x^2+x-y=0$ $\qquad$ $x^2+x=0$

$x^2+x=y$ $\qquad$ $x(x+1)=0$

$x_1=0 \quad x_2=-1$



$xy \geq 0$

se $x \geq 0$ e $y \geq 0$ ou

$x \leq 0$ e $y \leq 0$

4º Questão

$z = f(x,y) = e^{y/x}$ $\qquad$ $E = x \dfrac{d^2z}{dx^2} + y \dfrac{d^2z}{dx\,dy}$

$\dfrac{dz}{dx} = -yx^{-2}e^{yx^{-1}}$ $\qquad$ $\dfrac{d^2z}{dx^2} = -y(-2x^{-3}e^{yx^{-1}} + x^{-2}.-yx^{-2}e^{yx^{-1}})$

$= -y(-2x^{-3}e^{yx^{-1}} - yx^{-4}e^{yx^{-1}})$

$= 2yx^{-3}e^{yx^{-1}} + y^2x^{-4}e^{yx^{-1}} = e^{yx^{-1}}(2yx^{-3}+y^2x^{-4})$

$\dfrac{d^2z}{dx\,dy} = x^{-2}(-e^{yx^{-1}} - y.x^{-1}e^{yx^{-1}}) = -x^{-2}e^{yx^{-1}} - x^{-3}ye^{yx^{-1}}$

$E = e^{yx^{-1}}(2yx^{-2}+y^2x^{-3}) - x^{-2}ye^{yx^{-1}} - x^{-3}y^2e^{yx^{-1}}$

$= e^{yx^{-1}}(2yx^{-2}+y^2x^{-3} - x^{-2}y - x^{-3}y^2)$

$= e^{yx^{-1}}(yx^{-2})$

$= \dfrac{y}{x^2}e^{yx^{-1}}$

5ª Questão

Determinar a equação dos planos tangentes à superfície dada por $z = y^2x - x$ que sejam paralelo ao plano $3x - 4y - z + 6 = 0$

$(3, -4, -1)$

$f_x(x,y) = y^2 - 1 = 3 \qquad y = \pm 2$

$f_y(x,y) = 2xy = -4$

se $y = 2 \quad x = -1 \quad f(-1,2) = -3 \qquad P_1 = (-1, 2, -3)$

se $y = -2 \quad x = 1 \quad f_y(1,-2) = 3 \qquad P_2 = (1, -2, 3)$

$3(x+1) - 4(y-2) - (z+3) = 0$

$3x - 4y - z + 6 = 0$

$3(x-1) - 4(y+2) - (z-3) = 0$

$3x - 4y - z - 6 = 0$

P2 2º sem 2010

1ª Questão Determinar a equação geral do plano tangente à superfície $z = f(x,y) = x^2 + xy - y^2$ que seja paralelo: $3x + 4y + 2z - 5 = 0$

$3x + 4y + 2z - 5 = 0 \quad (x - \frac{1}{2}) \Rightarrow -\frac{3}{2}x - 2y - z + \frac{5}{2} \quad (-\frac{3}{2}, -2, -1)$

$f_x(x,y) = 2x + y = -\frac{3}{2} \Rightarrow 4x + 2y = -3 \qquad z = \frac{1}{4}$

$f_y(x,y) = x - 2y = -2$

$y = -\frac{3}{2} - 2x = -\frac{3}{2} + 2 = \frac{1}{2}$

$\begin{cases} 4x + 2y = -3 \\ x - 2y = -2 \end{cases}$

$5x = -5$

$x = -1$

$-\frac{3}{2}(x+1) - 2(y - \frac{1}{2}) - (z + \frac{1}{4}) = 0$

$-\frac{3}{2}x - 2y - z + \frac{1}{4} = 0 \Rightarrow -6x - 8y - 4z + 1 = 0$

/ /

2º Questão

Determinar e representar graficamente o domínio mais amplo de

$$z = f(x,y) = \frac{\sqrt{xy}}{\sqrt{x^2+y^2-16}-3}$$

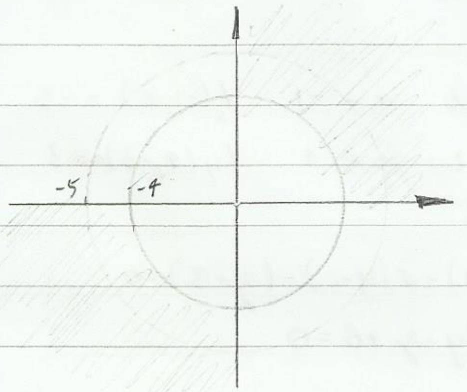
$D_{f(x,y)} = \{ (x,y) \in \mathbb{R}^2 \mid xy \geq 0 \text{ e } x^2+y^2-16 \geq 0 \text{ e } x^2+y^2-16 \neq 9$

$x^2+y^2-16=0$

$x^2+y^2=16$

$x^2+y^2-16-9=0$

$x^2+y^2=25$



3º Questão

$y''-y = tgx$

$y''-y' = e^x(x+1)$

homogenea associada

$p(\lambda) = \lambda^2 - \lambda = 0$

$\lambda(\lambda-1)=0 \quad \lambda_1 = 0 \quad \lambda_1 = 1$

$y_h = C_1 + C_2 e^x$

Solução particular

$y_p = (Ax+B)xe^x$

$y'_p = Axe^x + (Ax+B)(e^x+xe^x)$

$= Axe^x + Axe^x + Be^x + Ax^2e^x + Bxe^x$

$= 2Axe^x + Ax^2e^x + Bxe^x + Be^x$

$y''_p = 2A(e^x+xe^x) + A(2xe^x + x^2e^x) + B(e^x+xe^x) + Be^x$

$= 2Ae^x + 2Axe^x + 2Axe^x + Ax^2e^x + Be^x + Bxe^x + Be^x$

$= 2Ae^x + 4Axe^x + Ax^2e^x + 2Be^x + Bxe^x$

$2Ae^x + 4Axe^x + Ax^2e^x + 2Be^x + Bxe^x + Ax^2e^x + Bxe^x = xe^x + e^x$

$2A + 4Ax + Ax^2 + 2B + Bx + Ax^2 + Bx = x+1$

$$\begin{cases} 4A + 2B = 1 \\ 2A + 2B = 1 \end{cases} \qquad \begin{cases} 4A + 2B = 1 \\ -2A - 2B = -1 \end{cases}$$

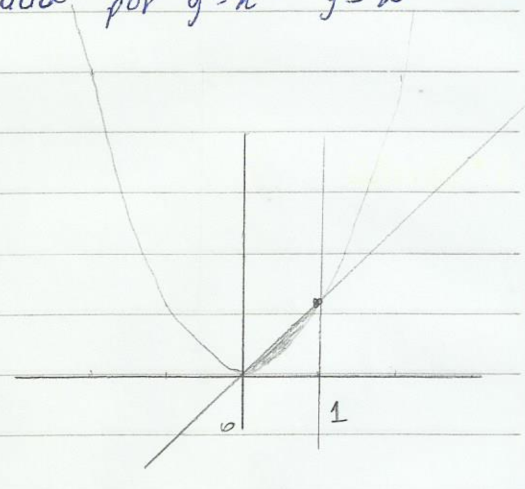
$\therefore\ 2A = 0 \quad \therefore\ A = 0 \quad e \quad B = \frac{1}{2}$

$$y_p = \frac{xe^x}{2}$$

$$y = C_1 + C_2 e^x + \frac{xe^x}{2}$$

4ª Questão

$\iint_D \sqrt[3]{x}\, dx dy$, onde D é a região limitada por $y = x^2$ $y = x$



$$\int_0^1 \left(\int_{x^2}^{x} x^{\frac{1}{3}}\, dy \right) dx$$

$$\int_0^1 y\, x^{1/3} \Big|_{x^2}^{x} dx$$

$$\int_0^1 x^{\frac{4}{3}} - x^{\frac{7}{3}}\, dx = \frac{3}{7} x^{7/3} - \frac{3}{10} x^{10/3} \Big|_0^1$$

$$= \frac{3}{7} - \frac{3}{10} = \frac{30 - 21}{70} = \frac{9}{70}$$

5ª Questão

$$z = f(x,y) = e^y (yx + x^2)$$

$$\frac{dz}{dx} = e^y (y + 2x) = ye^y + 2xe^y = 0$$

$$\frac{dz}{dy} = e^y (yx + x^2) + xe^y = 0$$

$$\begin{cases} ye^y + 2xe^y = 0 \\ yxe^y + e^y x^2 + e^y x = 0 \end{cases}$$

$$\begin{cases} y + 2x = 0 \\ yx + x^2 + x = 0 \end{cases}$$

$$\begin{cases} y + x + 1 = 0 \\ y + 2x = 0 \end{cases}$$

$$\begin{cases} y + x = -1 \quad (-1) \\ y + 2x = 0 \end{cases} \qquad \begin{cases} -y - x = 1 \\ y + 2x = 0 \end{cases} \qquad \underline{x = 1}$$

$$\underline{y = -2}$$

$$P(1, -2)$$

$$\frac{d^2 z}{dx^2} =$$

Determine os valores máximos e mínimos locais e pontos de Sela

$f(x,y) = 2x^2 + 2xy + y^2 + 2x - 3$ $\quad 2 - 2 + 1 - 2 - 3$

$f_x(x,y) = 4x + 2y + 2 = 0 \qquad 4x + 2y = -2$
$f_y(x,y) = 2x + 2y = 0$

$f(-1,1) = -4)$

$\begin{cases} -4x - 2y = 2 \\ 2x + 2y = 0 \end{cases} \qquad y = -x \qquad y = 1 \qquad$ Ponto crítico: $(-1,1,$

$-2x = 2$

$x = -1$

$\dfrac{d^2z^2}{dx^2} = 4 \qquad \dfrac{d^2z}{dy^2} = 2 \qquad \dfrac{dz}{dxdy} = 2 \qquad \dfrac{dz}{dydx} = 2$

$H = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix} = 4 > 0$, logo Ponto mínimo, pois $4 > 0$

Mínimo relativo $(-1,1,-4)$

$f(x,y) = -5x^2 + 4xy - y^2 + 16x + 10$

$f_x(x,y) = -10x + 4y + 16$
$f_y(x,y) = 4x - 2y$

$\begin{cases} -10x + 4y = -16 \\ 4x - 2y = 0 \end{cases} \sim \begin{cases} -10x + 4y = -16 \\ 8x - 4y = 0 \end{cases}$

$-2x = -16 \qquad y = \dfrac{-16 + 10x}{4} = \dfrac{-16 + 80}{4} = \dfrac{64}{4} = 16$

$x = 8$

$f(8,16) = 74 \qquad$ Ponto crítico: $(8,16,74)$

$\frac{d^2z}{dx^2} = -10$ $\frac{d^2z}{dy^2} = -2$ $\frac{dz}{dxy} = 4$ $\frac{dz}{dydx} = 4$

$H = \begin{vmatrix} -10 & 4 \\ 4 & -2 \end{vmatrix} = 4 > 0$ Ponto Máximo relativo $P(8,16,74)$

$f(x,y) = 2x^2 - 4x - 12y + 13 + 3y^2$

$fx(x,y) = 4x - 4$ $4x - 4 = 0$ $-12 + 6y = 0$
$fy(x,y) = -12 + 6y$ $x = 1$ $y = 2$

$\frac{d^2z}{dx} = 4$ $\frac{d^2z}{dy} = 6$ $\frac{dz}{dxdy} = 0$ $\frac{dz}{dydx} = 0$

$H = \begin{vmatrix} 4 & 0 \\ 0 & 6 \end{vmatrix} = 24 > 0$, logo Ponto Mínimo, $4 > 0$

$f(1,2) = -1$

$P = (1,2,-1)$

$f(x,y) = 2\sqrt{x^2+y^2+3}$

$fx = 2 \cdot \frac{2x}{2\sqrt{x^2+y^2+3}}$ $fy = 2 \cdot \frac{2y}{2\sqrt{x^2+y^2+3}}$

$\therefore x = 0$ $\therefore y = 0$

$\frac{d^2z}{dx^2} = 2\left(\frac{\sqrt{x^2+y^2+3} - x \cdot \frac{2x}{2\sqrt{x^2+y^2+3}}}{(x^2+y^2+3)}\right) = 2 \cdot \left(\frac{\frac{x^2+y^2+3-x^2}{\sqrt{x^2+y^2+3}}}{(x^2+y^2+3)}\right)$