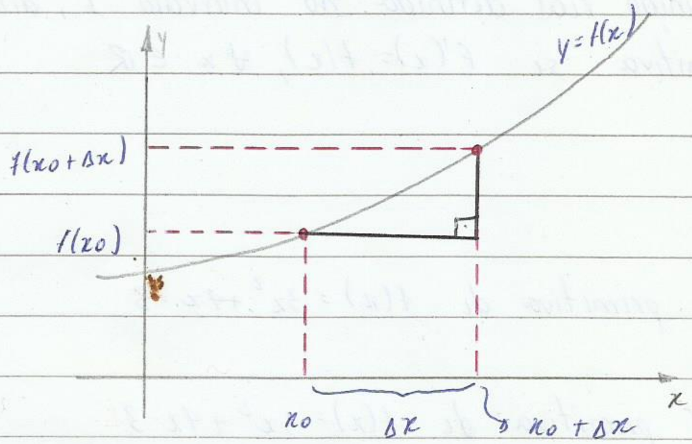
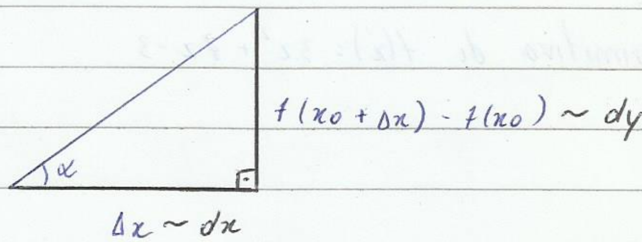
Integral

Cálculo II - Profª Suzana Abreu





$$tg\ \alpha = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

$$\frac{dy}{dx} = f'(x_0)$$

$\boxed{dy = f'(x_0)\,dx}$ diferencial de y

Primitiva de uma função

Dada $f: I \to \mathbb{R}$ uma função real definida no intervalo I, dizemos que $F: I \to \mathbb{R}$ é sua primitiva se $F'(x) = f(x), \forall x \in \mathbb{R}$

Exemplos:

1-) $F_1(x) = x^3 + 2x^2 - 3x + 1$ é primitiva de $f(x) = 3x^2 + 4x - 3$

2-) $F_2(x) = x^3 + 2x^2 - 3x - 4/3$ é primitiva de $f(x) = 3x^2 + 4x - 3$

3-) $F_3(x) = x^3 + 2x^2 - 3x + 20$ é primitiva de $f(x) = 3x^2 + 4x - 3$

Integral definida

Seja $f: I \to \mathbb{R}$ uma função contínua e $F: I \to \mathbb{R}$ uma de suas primitivas. A integral indefinida de $f(x)$ é o conjunto de todas as primitivas e é representada por:

$$\boxed{\int f(x)\,dx = F(x) + K, \quad K \in \mathbb{R}}$$

Propriedades:

1-) $\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

2-) $\int C \cdot f(x)\,dx = C \int f(x)\,dx, \quad C \in \mathbb{R}$

Importante

NÃO VALE !!!

$$\int f(x)\cdot g(x)\,dx \neq \int f(x)\,dx \cdot \int g(x)\,dx$$

$$\int \frac{f(x)}{g(x)}\cdot dx \neq \frac{\int f(x)\,dx}{\int g(x)\,dx}$$

Exemplos:

1-) $\int 3x^2 dx = x^3 + K$

2-) $\int 4x^3 dx = x^4 + K$

3-) $\int 5x^4 dx = x^5 + K$

4-) $\int x^2 dx = x^3/3 + K$

5-) $\int x^3 dx = x^4/4 + K$

6-) $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + K$

7-) $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + K$

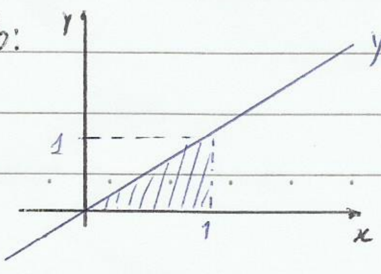
Integral definida

Dada: $f: [a,b] \to \mathbb{R}$ uma função contínua e $F: [a,b] \to \mathbb{R}$ uma de suas primitivas, a integral definida de $f(x)$ no intervalo $[a,b]$ é calculada por:

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

(Teorema Fundamental do Cálculo)

Exemplo:



$$A = \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 =$$

$$= \frac{1^2}{2} - \frac{0^2}{2} = 1/2$$

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Tabela de Integrais:

1) $I = \int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + K \ , \alpha \neq -1$

2) $I = \int \frac{1}{x} dx = \ln|x| + K$

3) $I = \int e^{x} dx = e^{x} + K$

4) $I = \int a^{x} dx = \frac{a^{x}}{\ln a} + K$

5) $I = \int \cos x \, dx = \operatorname{sen} x + K$

6) $I = \int \operatorname{sen} x \, dx = -\cos x + K$

7) $I = \int \sec^2 x \, dx = \operatorname{tg} x + K$

8) $I = \int \operatorname{cossec}^2 x \, dx = -\operatorname{cotg} x + K$

9) $I = \int \sec x \operatorname{tg} x \, dx = \sec x + K$

10) $I = \int \operatorname{cossec} x \cdot \operatorname{cotg} x \, dx = -\operatorname{cossec} x + K$

11) $I = \int \frac{1}{1+x^2} dx \quad \operatorname{arc} \operatorname{tg} x + K$

12) $I = \int \frac{1}{\sqrt{1-x^2}} dx \quad \operatorname{arc} \operatorname{sen} x + K$

13) $I = \int \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{arc} \sec x + K$

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Integral Definida

Definição:

$$\int f(x) dx = F(x) + C \iff (F(x) + C)' = f(x)$$

onde: $f(x)$: função integrada

$F(x)$: função primitiva

Exercícios

1) $\int 5 \sqrt[3]{x^2} \, dx \qquad 5 \int x^{\frac{2}{3}} dx = 5 \cdot \frac{x^{\frac{2}{3}+1}}{2/3+1} + C \Rightarrow$

$\Rightarrow 5x^{\frac{5}{3}} : \frac{5}{3} = 3x^{\frac{5}{3}} + C$

$\int (1-t)(2+t^2)\,dt$

$\int 2+t^2-2t-t^3\,dt = 2t + \frac{t^3}{3} - t^2 - \frac{t^4}{4} + C$

$\int \frac{3}{5x\sqrt{9x^2-9}}\,dx \Rightarrow \int \frac{3}{5x\sqrt{9(x^2-1)}}\,dx \Rightarrow$

$\int \frac{1}{5x\sqrt{x^2-1}}\,dx \Rightarrow \frac{1}{5}\int \frac{1}{x\sqrt{x^2-1}} = \frac{1}{5}.\,\text{arc sec}\,x + C$

Técnicas de Integração

1) Imediatas
2) Mudança de variável ou por substituição
3) por partes
... entre outras

2-) Mudança de variável ou por substituição: Fazer a mudança para obter uma $\int$ conhecida

$\int \frac{3x^2}{\sqrt[5]{4x^3-7}}\,dx$ Admitindo $\sqrt[5]{4x^3-7} = t$

$4x^3-7 = t^5$

Diferencia

$\Rightarrow 12x^2\,dx = 5t^4\,dt$

$x^2\,dx = \frac{5}{12}t^4\,dt$

$\int \frac{\frac{5}{4}t^4}{t}\,dt \Rightarrow$

$\frac{5}{4}\int t^3\,dt = \frac{5.t^4}{4.4} + C = \frac{5t^4}{16} \Rightarrow \frac{5}{16}.(\sqrt[5]{4x^3-7})^4 + C$

$$\int x^4 \sqrt{x^5-7}\, dx \qquad x^5 \left(x^4 \sqrt{x^5-7}\, dx\right)$$

Admitindo $\sqrt{x^5-7} = t$

$$x^5 - 7 = t^2, \Rightarrow x^5 = t^2 + 7$$

derivando $x^5 - 7 = t^2 = 5x^4\, dx = 2t\, dt$

$$x^4 dx = \frac{2t}{5} dt$$

$$\int x^5 \sqrt{x^5-7}.x^4\, dx \Rightarrow$$

$$\Rightarrow \int x^5 \sqrt{x^5-7}.\frac{2t}{5} dt \Rightarrow \frac{2}{5}\int (t^2+7).t^2\, dt \Rightarrow$$

$$\frac{2}{5}\int (t^4 + 7t^2)\, dt = \frac{2}{5}.\left(\frac{t^5}{5} + \frac{7}{3}t^3\right) = \frac{2}{25}t^5 + \frac{14}{15}t^3 + C$$

Técnica de Integração Por partes

$$\int u\, dv = ?$$

<u>Dedução da fórmula:</u>

Derivada de um produto

$$(uv)' = u'v + uv'$$

$$uv' = (uv)' - u'v$$

Integrando

$$\int uv' = \int (uv)' - \int u'v$$

Diferencial

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Exemplo

$$\int \underset{u}{x}\,\underbrace{\cos x\, dx}_{dv} = ?$$

Admitindo $x = u$

$dx = du$

$dv = \cos x\, dx \rightarrow v = \int \cos x\, dx = \operatorname{sen} x$

Fórmula:

$$\int u\,dv = uv - \int v\,du$$

$$\int x \cos x\, dx = x \operatorname{sen} x - \int \operatorname{sen} x\, dx = \underline{x \operatorname{sen} x + \cos x + C}$$

Exemplo

$$\int e^x \cos x\, dx:$$

$e^x = u \rightarrow e^x\, dx = du$

$\cos x\, dx = dv \rightarrow v = \int \cos x\, dx = \operatorname{sen} x$

$$\int u\, dv = uv - \int v\, du$$

$$\int e^x \cos x\, dx = e^x . \operatorname{sen} x - \int e^x \operatorname{sen} x\, dx$$

$\rightarrow 1$

Cálculo do $\int e^x \operatorname{sen} x\, dx$

$u = e^x \rightarrow du = e^x$

$dv = \operatorname{sen} x\, dx \rightarrow v = \int \operatorname{sen} x\, dx$

$$\int e^x \operatorname{sen} x\, dx = e^x (-\cos x) - \int (-\cos x) \operatorname{sen} x\, dx$$

$\rightarrow 2$

Substituindo 2 em 1

$$\int e^x \cos x \, dx = e^x \operatorname{sen} x - [e^x(-\cos x) + \int e^x \cos x \, dx]$$

$$\int e^x \cos x \, dx = e^x \operatorname{sen} x + e^x \cos x - \int e^x \cos x \, dx$$

$$\int e^x \cos x \, dx + \int e^x \cos x \, dx = e^x \operatorname{sen} x + e^x \cos x$$

$$2\int e^x \cos x \, dx = e^x \operatorname{sen} x + e^x \cos x$$

$$\int e^x \cos x \, dx = \frac{e^x(\operatorname{sen} x + \cos x)}{2} + C$$

Técnica de Integração

Integral de função racional

$$\int \frac{P(x)}{Q(x)}\,dx$$

1ª regra: Se o grau de $P(x)$ for maior ou igual ao grau de $Q(x)$:
"Divisão":

Exemplo

$$\int \frac{x^3+2}{x+1}\,dx$$

$$\begin{array}{l|l} x^3 \quad +2 & x+1 \\ -x^3-x^2 & x^2-x+1 \\ -x^2+2 & \\ x^2+x & \\ x+2 & \end{array}$$

$$\frac{x^3+2}{x+1} = x^2-x+1+\frac{1}{x+1}$$

$$\int x^2\,dx - \int x\,dx + \int dx + \int \frac{1}{x+1}\,dx \qquad \frac{-x-1}{1}$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x + \ln|x+1| + C$$

2º regra Se o grau de $P(x)$ for menor do que o grau de $Q(x)$:
Decomposição numa soma de frações simples:

$$x^2 - Sx + P$$

$$7 \quad + 12$$

$$3+4 \quad 3 \cdot 4$$

1º caso: fator simples

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{(x+a)(x+b)(x+c)} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{x+c}$$

Exemplo

$$\int \frac{x^2+x+1}{x^2-7x+12}\,dx$$

$$\begin{array}{l|l} x^2+x+1 & x^2-7x+12 \\ -x^2+7x-12 & 1 \\ 8x-11 & \end{array}$$

$$= 1 + \frac{8x-11}{x^2-7x+12} \Rightarrow$$

$$\Rightarrow \int 1 + \frac{8x-11}{(x-3)(x-4)}\,dx = \int dx + \int \frac{8x-11}{(x-3)(x-4)}\,dx$$

$$\frac{A}{x-3} + \frac{B}{x-4} = \frac{A(x-4)+B(x-3)}{(x-3)(x-4)} = \frac{Ax - A4 + Bx - 3B}{(x-3)(x-4)}$$

$$\frac{8x-11}{x^2-7x+12} = \frac{Ax - A4 + Bx - 3B}{(x-3)(x-4)}$$

$8x - 11 = Ax - A4 + Bx - 3B \Rightarrow 8x - 11 = A(x-4) + B(x-3)$

Admitindo $x = 3$

$13 = -A \Rightarrow \underline{A = -13}$

Admitindo $x = 4$

$21 = B \Rightarrow \underline{B = 21}$

$$\frac{8x-11}{x^2-7x+12} = \frac{21}{x-4} - \frac{13}{x-3}$$

$$\int \frac{8x-11}{x^2-7x+12}\,dx = 21\int \frac{1}{x-4}\,dx - 13\int \frac{1}{x-3}\,dx$$

$$= 21 \ln|x-4| - 13 \ln|x-3| + x + C$$

2º caso: Fator multiplo

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)^2} + \frac{C}{(x+b)}$$

3º caso Fator do 2º grau

Com $\Delta < 0$: $mx^2 + nx + p$

$$\frac{P(x)}{Q(x)} = \frac{Px}{(x+a)^3\,\underbrace{(mx^2+nx+p)}} \quad \Delta < 0$$

$$= \frac{A}{(x+a)^3} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)} + \frac{Dx+F}{mx^2+nx+p}$$

Equação Diferencial

Definição: É uma equação que liga a função, suas derivadas e variáveis

Exemplos:

1) $(y''')^2 - 3(y'')^5 + xy = 0$

$y = f(x)$; EDO3 Grau 2

2) $\frac{d^2x}{dt^2} + \frac{dx}{dt} = 0$

$x = f(t)$; EDO2 Grau 1

3) $xy' + (y''')^2 - 3(y')^5 + (y''')^4 = 0$

$y = f(x)$; EDO3 Grau 4

4) $\frac{\partial^2 w}{dx^2} + \frac{d^2 w}{dy^2} = 0$

$w = f(x,y)$; EDP

Tipos de equação diferencial:

1) EDO: Equação diferencial ordinária
Função envolvida depende de uma variável

2) EDP: Equação diferencial parcial
Função envolvida depende de duas ou mais variáveis

Classificação da Equação Diferencial Ordinária (EDO)

1) Ordem:

É a ordem da derivada, de maior ordem que nela aparece.

2) Grau de uma equação dif. ordinária (EDO)

É o maior expoente de um dos termos que da a ordem da EDO.

Estudo de uma EDO 1:

Tipos:

1) EDO1S: Eq. dif. ord. de 1ª ordem de variaveis separáveis

Depois de operações devemos obter uma equação

$$\frac{dy}{F(y)} + \frac{dx}{G(x)} = 0$$

Solução geral da EDO1S:

Integrando

$$\int \frac{dy}{F(y)} + \int \frac{dx}{G(x)} = C \ (cte)$$

Exercício:

Dar a solução particular da EDO1

$$y' - x^2 = x$$

Que satisfaça a condição $y(0) = 1$

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Exemplo:

134-) $x^2 y' = y - xy$

$$x^2 \frac{dy}{dx} = y(1-x)$$

$$\frac{dy}{dx} = \frac{y(1-x)}{x^2} \Rightarrow \frac{dy}{y} = \left(\frac{1-x}{x^2}\right) dx \Rightarrow$$

$$\Rightarrow \int \frac{1}{y}\, dy = \int \left(\frac{1-x}{x^2}\right) dx$$

$$\ln y + C = \int \frac{1}{x^2}\, dx - \int \frac{1}{x}\, dx$$

$$\ln|y| = -\frac{1}{x} - \ln|x| + c$$

$$\ln|y| = \ln e^{-\frac{1}{x}} - \ln|x| + \ln C$$

$$\ln|y| = \ln\left|\frac{e^{-\frac{1}{x}}}{x} . C\right|$$

$$y = \frac{1}{x} e^{-\frac{1}{x}} . C \rightarrow \text{Solução Geral}$$

Quando $y(-1) = -1$ $\begin{cases} x = -1 \\ y = -1 \end{cases}$ (Solução particular)

$$-1 = \frac{1}{-1} e . C \Rightarrow C = \frac{1}{e}$$

$$y = \frac{1}{x} e^{-\frac{1}{x}} . \frac{1}{e} = \frac{1}{xe^{\frac{1}{x}}} . \frac{1}{e} = \frac{1}{xe^{\frac{1}{x}+1}} = \frac{1}{x} e^{-\frac{1}{x}-1}$$

Propriedades do ln

1) $f(x) = \ln e^{f(x)}$

2) $C_1 = C_2 + \ln C_1 = C_2$

3) $\ln|A| - \ln|B| = \ln\left|\frac{A}{B}\right|$

4) $\ln|A| + \ln|B| = \ln|AB|$

Ex 139

$$x y' + 4x^2 y = e^{-2x^2}$$

$$y(-1) = \frac{1}{3}$$

Obs:

$$y' + P(x).y = Q(x)$$

Fator Integrante

$$I(x) = e^{\int P(x)dx}$$

divide por "x"

$$y' + 4xy = \frac{e^{-2x^2}}{x} \quad \text{I}$$

Fator Integrante

$$I(x) = e^{\int 4x\,dx} = e^{2x^2}$$

Multiplicando I por $I(x)$

$$e^{2x^2} y' + 4xy\,e^{2x^2} = \frac{1}{x}$$

$$(e^{2x^2} y)' = \frac{1}{x}$$

Integrando

$$\int (e^{2x^2} y)' = \int \frac{1}{x} dx$$

$$e^{2x^2} y = \ln|x| + C \quad \div\ e^{2x^2}$$

$$y = \frac{\ln|x|}{e^{2x^2}} + \frac{C}{e^{2x^2}} \quad \text{I} \quad \Rightarrow \text{Solução Geral}$$

Solução particular $y(-1) = \frac{1}{3}$ $\begin{cases} x = -1 \\ y = \frac{1}{3} \end{cases}$

$$\frac{1}{3} = \frac{\cancel{\ln|-1|}}{e^{2(-1)^2}} + \frac{C}{e^{2(-1)^2}} \Rightarrow \frac{1}{3} = \frac{C}{e^2} \Rightarrow C = \frac{e^2}{3}$$

R: $y = \dfrac{\ln|x|}{e^{2x^2}} + \dfrac{e^2}{3e^{2x^2}}$

Exemplo 140

$y' - y = 2xe^{2x}$, $y(0) = 1$

$I(x) = e^{\int -1\,dx} = e^{-x}$

Multiplicando por $I(x)$

$e^{-x}y' - e^{-x}y = 2xe^{2x}e^{-x}$

$(e^{-x}y)' = 2xe^{x}$

Integrando:

$\int (e^{-x}y)' = \int 2xe^{x}\,dx$ $\quad u = x \quad du = dx \quad dv = e^{x} \quad v = e^{x}dx$

$e^{-x}y = 2e^{x}(x-1) + C$ $\quad = xe^{x} - \int e^{x}\,dx$

Dividindo po e^{-x} $\quad = xe^{x} - e^{x} = e^{x}(x-1)$

$y = 2e^{2x}(x-1) + \dfrac{C}{e^{-x}}$ → Solução Geral

Solução Particular

$y(0) = 1$ $\quad \begin{cases} y = 1 \\ x = 0 \end{cases}$

$1 = 2.1(0-1) + \dfrac{C}{1} \Rightarrow C = 3$

$y = 2e^{2x}(x-1) + 3e^{x}$

Ex 136

$$\cos^2 x \frac{dy}{dx} + y = 1 \qquad y(0) = -3$$

$$y' + \frac{1}{\cos^2 x} y = \frac{1}{\cos^2 x} \qquad y' + y \sec^2 x = \sec^2 x$$

$$P(x) = \sec^2 x$$

$$I = e^{\int \sec^2 x \, dx} = e^{tg x}$$

$$y' e^{tg x} + y \sec^2 x \, e^{tg x} = \sec^2 x \, e^{tg x}$$

$$(y e^{tg x})' = \sec^2 x \, e^{tg x}$$

$$\int (e^{tg x} y)' = \int \sec^2 x \, e^{tg x} \, dx$$

$$e^{tg x} y = \int \sec^2 x \, e^{tg x} \, dx$$

$$tg x = t \qquad \sec^2 x \, dx = dt$$

$$\int \sec^2 x \, e^{tg x} \, dx = \int e^t \, dt = e^t$$

$$= e^{tg x} + C$$

$$e^{tg x} y = e^{tg x} + C$$

$$y = 1 + \frac{C}{e^{tg x}} \quad \text{Solução geral}$$

$$y(0) = -3 \quad \begin{cases} x = 0 \\ y = -3 \end{cases}$$

$$-3 = 1 + \frac{C}{e^0} \Rightarrow C = -4$$

$$y = 1 - \frac{4}{e^{tg x}}$$

Ex 143)

$$\frac{dy}{dx} = \sin x [\cos(2y) - \cos^2 y]$$

$$\frac{dy}{dx} = \sin x [\cancel{\cos^2 y} - \sin^2 y - \cancel{\cos^2 y}]$$

$$\frac{dy}{dx} = \sin x . (-\sin^2 y)$$

$$\frac{dy}{-\sin^2 y} = \sin x \, dx \Rightarrow -\csc^2 y \, dy = \sin x \, dx$$

$$-\int \csc^2 y \, dy = \int \sin x \, dx \Rightarrow \operatorname{ctg} y = -\cos x + C$$

Ex 141.) $\frac{x^2-1}{y^2+1} y' = \frac{x}{y}$

$$\frac{y' y}{y^2+1} = \frac{x}{x^2-1} \Rightarrow \frac{dy}{dx} \cdot \frac{y}{y^2+1} = \frac{x}{x^2-1}$$

$$\frac{y}{y^2+1} dy = \frac{x}{x^2-1} dx$$

$$\int \frac{y}{y^2+1} dy = \int \frac{x}{x^2-1} dx$$

$y^2+1 = t \qquad 2y \, dy = dt \qquad y \, dy = \frac{1}{2} dt \qquad \int \frac{y}{y^2+1} dy = \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \ln|y^2+1|$

$\int \frac{x}{x^2-1} dx \qquad x^2-1 = t \qquad 2x \, dx = dt \qquad x \, dx = \frac{1}{2} dt$

$$= \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \ln|x^2-1| + C$$

R: $\cancel{\frac{1}{2}} \ln|y^2+1| = \cancel{\frac{1}{2}} \ln|x^2-1| + C \Rightarrow y^2+1 = C(x^2-1)$

$\ln|y^2+1| = \ln|x^2-1| + C$

72-

Prof: Ex $\displaystyle\int \frac{1}{2+\operatorname{sen}(x)}\,dx$ Sugestão $tg\left(\frac{x}{2}\right)=t$

$$tg\left(\frac{x}{2}\right)=t$$

$$\frac{x}{2}=\operatorname{arctg} t$$

$$x = 2\operatorname{arctg} t$$

$$dx = \frac{2}{1+t^2}\,dt$$

$$\operatorname{sen}(x)=f(t)$$

$$tg^2\left(\frac{x}{2}\right)=t^2$$

$$\frac{\operatorname{sen}^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}=t^2$$

$$\frac{1+\cos(x)}{\cancel{2}} : \frac{1-\cos(x)}{\cancel{2}} = t^2$$

$$1+\cos x = t^2(1-\cos(x))$$

$$1+\cos x = t^2 - t^2\cos(x)$$

$$\cos x + t^2\cos(x) = t^2-1$$

$$\cos(x)(1+t^2) = t^2-1$$

$$\cos(x) = \frac{t^2-1}{t^2+1}$$

$$\operatorname{sen} x = ?$$

$$\operatorname{sen}^2 x = 1-\cos^2 x$$

$$\operatorname{sen}^2 x = 1-\left(\frac{t^2-1}{t^2+1}\right)^2$$

$$\operatorname{sen}^2 x = \frac{(t^2+1)^2-(t^2-1)^2}{(t^2+1)^2}$$

$$\operatorname{sen}^2 x = \frac{\cancel{t^4}+2t^2+\cancel{1}\cancel{-t^4}+2t^2\cancel{-1}}{(t^2+1)^2} = \frac{4t^2}{(t^2+1)^2}$$

$$\therefore \operatorname{sen} x = \frac{2t}{t^2+1}$$

$$\int \frac{\frac{2}{1+t^2}}{2+\frac{2}{1+t^2}}\,dt \sim \int \frac{2}{\cancel{1+t^2}}\cdot\frac{\cancel{1+t^2}}{2+2t^2+2}\,dt \sim \int \frac{1}{t^2+2}\,dt$$

$$\sim \int \frac{1}{t^2+(\sqrt{2})^2}\,dt = \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{t}{\sqrt{2}}\right)+C \sim \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{tg\frac{x}{2}}{\sqrt{2}}\right)+C$$

Ex 71. $\displaystyle\int \frac{1}{\sqrt{3}\cos(x) - \operatorname{sen}(x)}\,dx$

$\frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right) \qquad \frac{1}{2} = \operatorname{sen}\left(\frac{\pi}{6}\right)$

$$\int \frac{1/2}{\frac{\sqrt{3}}{2}\cos(x) - \frac{\operatorname{sen}(x)}{2}}\,dx$$

$$\frac{1}{2}\int \frac{1}{\cos\left(\frac{\pi}{6}\right)\cos(x) - \operatorname{sen}\left(\frac{\pi}{6}\right)\operatorname{sen}(x)}\,dx \sim$$

$$\frac{1}{2}\int \frac{1}{\cos\left(\frac{\pi}{6}+x\right)}\,dx \sim \frac{1}{2}\int \sec\left(\frac{\pi}{6}+x\right)dx =$$

$$= \frac{1}{2}\ln\left|\sec\left(\frac{\pi}{6}+x\right) + \operatorname{tg}\left(\frac{\pi}{6}+x\right)\right| + C$$

Ex 67 = $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt$ $\qquad t = 2 = \sec\theta = \frac{1}{\cos\theta} = 2 \quad \cos\theta = \frac{1}{2} \therefore \theta = \frac{\pi}{6}$

$t = \sec\theta \qquad dt = \sec\theta\,\operatorname{tg}\theta\,d\theta \qquad t = \sqrt{2} = \frac{1}{\cos\theta} = \sqrt{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{\pi}{4}$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{6}} \frac{\sec\theta\,\operatorname{tg}\theta}{\sec^3\theta\sqrt{\sec^2-1}}\,d\theta \sim \int_{\pi/4}^{\pi/6} \frac{\operatorname{tg}\theta}{\sec^2\theta\,\operatorname{tg}\theta}\,d\theta \sim \int_{\pi/4}^{\pi/6} \frac{1}{\sec^2\theta}\,d\theta$$

$$\int_{\pi/4}^{\pi/6} \cos^2\theta\,d\theta = \frac{1}{2}(\theta + \cos\theta\operatorname{sen}\theta)\Big|_{\pi/4}^{\pi/6}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{6} + \underbrace{\operatorname{sen}\frac{\pi}{6}}_{1/2}\,\underbrace{\cos\frac{\pi}{6}}_{\sqrt{3}/2}\right) - \left(\frac{\pi}{4} + \underbrace{\operatorname{sen}\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\,\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\right)\right] = \frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$$

$P_1)$

3-) $\displaystyle\int \frac{x+9}{x^2-3x+10}\,dx$

$$\int \frac{x+9}{(x-5)(x+2)}\,dx \Rightarrow$$

$$\frac{x+9}{(x-5)(x+2)} = \frac{A}{(x-5)} + \frac{B}{(x+2)} = \frac{A(x+2)+B(x-5)}{(x-5)(x+2)}$$

Admitindo $x=-2$

$B(-2-5) = -2+9$

$-7B = 7 \quad \therefore B = -1$

Admitindo $x = 5$

$A(5+2) = 5+9$

$7A = 14$

$A = 2$

$$\frac{x+9}{(x-5)(x+2)} = \frac{2}{x-5} - \frac{1}{x+2}$$

$$\therefore \int \frac{2}{x-5}\,dx - \int \frac{1}{x+2}\,dx = 2\ln|x-5| - \ln|x+2| + C$$

4-) $\displaystyle\int \frac{dx}{x\sqrt{1-x^2}} \approx \int \frac{x}{x^2\sqrt{1-x^2}}\,dx$

$\sqrt{1-x^2} = t$

$1-x^2 = t^2$

$-2x\,dx = 2t\,dt$

$x\,dx = -t\,dt$

$$\int \frac{-t}{(1-t^2)t}\,dt = \int \frac{-1}{1-t^2}\,dt = -\int \frac{1}{t^2-1}\,dt$$

$$= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + C$$

$=$

/ /

$$\int x e^{-3x} dx \qquad \int \frac{-1}{3} t . e^{t} \frac{-1}{3} dt = \frac{1}{9} \int t e^{t} dt$$

$-3x = t \qquad -3dx = dt$

$x = \frac{-1}{3} t \qquad dx = \frac{-1}{3} dt \qquad u = t \quad du = dt$

$dv = e^{t} \quad v = e^{t}$

$$\therefore \int x e^{-3x} dx = t . e^{t} - \int e^{t} dt$$

$$-(t e^{t} - e^{t}) . \frac{1}{9}$$

$$= \left(-3x e^{-3x} - e^{-3x}\right) \frac{1}{9} \Rightarrow e^{-3x} \left(\frac{-x}{3} - \frac{1}{9}\right) + C$$

83- $\displaystyle\int_{1}^{2} \frac{4y^2 - 7y - 12}{y(y+2)(y-3)} dy \qquad \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3}$

$$\frac{A(y+2)(y-3) + B(y-3)y + C(y+2)y}{y(y+2)(y-3)}$$

Admitindo $y = 0$

$A(0+2)(0-3) = 4.0^2 - 7.0 - 12$

$-6A = -12 \quad \therefore A = 2$

Adimtindo $y = -2$

$B(-2-3)-2 = 4(-2)^2 - 7(-2) - 12$

$+10B = 18$

$B = 1{,}8 = +9/5$

Admitindo $y = 3$

$C(3+2).3 = 4(3)^2 - 7.3 - 12$

$15C = 3$

$C = 1/5 = 0{,}2$

$$\int \frac{2}{y} dy = 2 \ln|y| + C_1 \qquad -\frac{9}{5} \int \frac{1}{y+2} dy = \frac{9}{5} \ln|y+2| + C_2$$

$$\frac{1}{5} \int \frac{1}{y-3} dy = \frac{1}{5} \ln|y-3| + C$$

$$I = 2\ln|y| - \frac{9}{5} \ln|y+2| + \frac{1}{5} \ln|y-3| \Big|_{1}^{2} = \frac{27}{5} \ln 2 - \frac{9}{5} \ln 3$$

$$\left(2\ln 2 + \frac{9}{5} \ln 4 + \frac{1}{5} \ln 1\right) - \left(\ln 1 + \frac{9}{5} \ln 3 + \frac{1}{5} \ln 2\right) = 2\ln 2 + \frac{18}{5} \ln 2 - \frac{9}{5} \ln 3 - \frac{9}{5} \ln 3 + \frac{1}{5} \ln 2$$

Ex 106

$$x = \frac{1}{3}(y^2+2)^{\frac{3}{2}} \qquad x' = \frac{1}{2}(y^2+2)^{\frac{1}{2}} 2y = y(y^2+2)$$

$$L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy' \qquad (x')^2 = y^2(y^2+2)+1$$
$$= y^4+2y^2+1$$

$$L = \int_0^4 \sqrt{y^4+2y^2+1}\, dy$$

$$= \int_0^4 \sqrt{(y^2+1)^2}\, dy = \int_0^4 y^2+1\, dy = \frac{y^3}{3}+y \Big|_0^4 = \frac{64}{3}+4 = \frac{76}{3}$$

Ex 142

$$\frac{dy}{dx} = \frac{1+2y^2}{y \sin x} \qquad dy \cdot y \sin x = (1+2y^2)\,dx$$
$$\frac{y\,dy}{1+2y^2} = \frac{1}{\sin x}\, dx$$

$$\int \frac{y}{1+2y^2}\, dy = \int \csc x\, dx$$

$$1+2y^2 = t \qquad \frac{1}{4}\int \frac{1}{t}\, dt = \ln|\csc x - \operatorname{ctg} x| + c$$
$$4y\,dy = dt$$

$$\frac{1}{4}\ln|1+2y^2| = \ln|\csc x - \operatorname{ctg} x| + c$$

$$\ln|1+2y^2| = 4\ln|\csc x - \operatorname{ctg} x| + 4c$$
$$\ln|1+2y^2| = \ln|\csc x - \operatorname{ctg} x|^4 + \ln k$$
$$\ln|1+2y^2| = \ln|(\csc x - \operatorname{ctg} x)c|^4$$
$$1+2y^2 = k(\csc x - \operatorname{ctg} x)^4$$

Equações separáveis

133) $y' - x^2 = x, \quad y(0) = 1$

$y' = x + x^2$

$\frac{dy}{dx} = x + x^2 \Rightarrow dy = x + x^2\, dx$

$\int dy = \int x + x^2\, dx$

$y = \frac{x^2}{2} + \frac{1}{3}x^3 + C$ (Solução Geral)

$y(0) = 1 \begin{cases} x = 0 \\ y = 1 \end{cases}$

$1 = \frac{0^2}{2} + \frac{1}{3}0^3 + C \quad \therefore \quad C = 1$

$y = \frac{x^2}{2} + + \frac{x^3}{3} + 1$ (Solução Particular)

134) $x^2 y' = y - xy, \quad y(-1) = -1$

$x^2 y' = y(1 - x)$

$\frac{y'}{y} = \frac{1 - x}{x^2}$

$\frac{dy}{dx} \cdot \frac{1}{y} = \frac{1 - x}{x^2}$

$\frac{1}{y} dy = \frac{1}{x^2} - \frac{1}{x}\, dx \Rightarrow \int \frac{1}{y} dy = \int \frac{1}{x^2} - \frac{1}{x}\, dx$

$\ln y = \frac{-1}{x} - \ln|x| + C$

$\ln y = \ln e^{\frac{-1}{x}} - \ln|x| + C$

$\ln y = \ln\left|\frac{e^{\frac{-1}{x}}}{x}\right| \cdot C \Rightarrow y = \frac{1}{x} \cdot e^{\frac{-1}{x}} \cdot C$

$y = \frac{1}{x} e^{\frac{-1}{x}} \cdot C$

$y(-1) = -1 \quad \begin{cases} x = -1 \\ y = -1 \end{cases}$

$-1 = \frac{1}{-1} e^{\frac{-1}{-1}} \cdot C \quad \therefore \quad C = \frac{1}{e}$

$y = \frac{1}{x} e^{\frac{-1}{x}} \cdot \frac{1}{e} \Rightarrow y = \frac{1}{x} e^{-1} e^{\frac{-1}{x}} \sim y = \frac{1}{x} e^{-1 -\frac{1}{x}}$ (Solução Particular)

135-) $y' = 2y + x(e^{3x} - e^{2x})$; $y(0) = 2$

$I(x) = e^{\int -2\,dx} \sim I(x) = e^{-2x}$

$y' e^{-2x} - 2y e^{-2x} = x(e^{3x} - e^{2x}) e^{-2x}$

$(y e^{-2x})' = x(e^{3x} - e^{2x}) e^{-2x}$

$(y e^{-2x})' = x(e^{x} - 1)$

$\int (y e^{-2x})' = \int x(e^{x} - 1)\,dx$

$y e^{-2x} = \int x e^{x}\,dx - \int x\,dx$

$y e^{-2x} = e^{x}(x-1) - \frac{x^2}{2}$

dividindo por e^{-2x}

$y = e^{3x}(x-1) - \frac{1}{2} x^2 e^{2x} + \frac{C}{e^{-2x}}$

$y = e^{2x}(x e^{x} - e^{x} - \frac{1}{2} x^2 + C)$ (Solução Geral)

$\int x e^{x}\,dx =$

$u = x \quad du = dx$

$dv = e^{x} \quad v = e^{x}$

$\int x e^{x}\,dx = x e^{x} - \int e^{x}\,dx$

$= x e^{x} - e^{x} =$

$= e^{x}(x-1)$

$y(0) = 2 \quad \begin{cases} x = 0 \\ y = 2 \end{cases}$

$2 = 1(-1) + \frac{C}{1} \quad \therefore \quad C = 3$

$y = e^{2x}(x e^{x} - e^{x} - \frac{1}{2} x^2 + 3)$

136-) $\cos^2(x)\dfrac{dy}{dx} + y = 1$; $y(0) = -3$

$\int \sec^2(x)\, e^{tg(x)}\, dx$

$tg(x) = t \quad \sec^2(x)dx = dt$

$\int e^t dt = e^t = e^{tg(x)} + C$

$\cos^2(x)\, y' + y = 1 \qquad \div \cos^2(x)$

$y' + \dfrac{1}{\cos^2(x)} \cdot y = \dfrac{1}{\cos^2(x)}$

$y' + \sec^2(x)\, y = \sec^2(x)$

$I(x) = e^{\int \sec^2(x)dx} = e^{tg x}$

$y' e^{tg x} + \sec^2(x)\, y\, e^{tg x} = \sec^2(x)\, e^{tg x}$

$(y\, e^{tg x})' = \sec^2(x)\, e^{tg(x)}$

$\int (y\, e^{tg x})' = \int \sec^2(x)\, e^{tg(x)}\, dx$

$y\, e^{tg(x)} = e^{tg(x)} + C$

Dividindo por $e^{tg(x)}$

$y = 1 + C\, e^{-tg(x)}$ (Solução Geral)

$y(0) = -3 \begin{cases} x = 0 \\ y = -3 \end{cases}$

$-3 = 1 + C\, e^0 \quad \therefore \quad C = -4$

$y = 1 - \dfrac{4}{e^{tg(x)}}$ (Solução Particular

137-) $\dfrac{dy}{dt} + ty = t$, $y(1) = 3$

$y' = t - ty$

$y' = t(1-y)$

$\dfrac{y'}{1-y} = t$

$\dfrac{1}{1-y}\, dy = t\, dt$

$\int \dfrac{1}{1-y}\, dy = \int t\, dt$

$1 - y = t \quad -dy = dt$

$-\int \dfrac{1}{t}\, dt = \dfrac{t^2}{2} + C \Rightarrow$

$\Rightarrow -\ln|1-y| = \dfrac{t^2}{2} + C$

$-\ln|1-y| = \ln e^{\frac{t^2}{2}} \cdot C$

$y - 1 = e^{\frac{t^2}{2}} \cdot C$

$y = 1 + e^{\frac{t^2}{2}} \cdot C$

spiral

$y = 1 + e^{\frac{t^2}{2}} . c$ (Solução Geral)

$y(1) = 3 \begin{cases} t = 1 \\ y = 3 \end{cases}$

$3 = 1 + e^{\frac{1}{2}} . c \quad \therefore \quad c = \dfrac{2}{e^{\frac{1}{2}}} = 2e^{-\frac{1}{2}} = \dfrac{2}{e^{\frac{1}{2}}} = \dfrac{2}{\sqrt{e}}.$

$y = 1 + e^{\frac{t^2}{2}} . \dfrac{2}{\sqrt{e}}$

ou

$\dfrac{dy}{dt} + ty = t, \; y(1) = 3$

$y' + ty = t$

$I = e^{\int t\,dt} = e^{\frac{t^2}{2}}$

$y' e^{\frac{t^2}{2}} + ty e^{\frac{t^2}{2}} = t e^{\frac{t^2}{2}}$

$(y e^{\frac{t^2}{2}})' = t e^{\frac{t^2}{2}}$

$\int (y e^{\frac{t^2}{2}})' = \int t e^{\frac{t^2}{2}} dt$

$y e^{\frac{t^2}{2}} = e^{\frac{t^2}{2}} + c$

$\int t e^{\frac{t^2}{2}} dt$

$\dfrac{t^2}{2} = s \qquad t\,dt = ds$

$\int t e^{\frac{t^2}{2}} dt = \int e^{s} ds = e^{s} = e^{\frac{t^2}{2}}$

$y = 1 + \dfrac{c}{e^{\frac{t^2}{2}}}$ (Solução Geral)

$y(1) = 3 \begin{cases} x = 1 \\ y = 3 \end{cases}$

$3 = 1 + \dfrac{c}{e^{\frac{1}{2}}} \Rightarrow c = 2\sqrt{e}$

$y = 1 + \dfrac{2\sqrt{e}}{e^{\frac{t^2}{2}}}$ (Solução Particular)

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138) $xy' + (x+1)y = x, \quad y(\ln 2) = 1$

dividindo por x

$$y' + \frac{x+1}{x} y = 1$$

$$\int \frac{x+1}{x} dx = \int dx + \int \frac{1}{x} dx$$

$$= x + \ln|x| + c$$

Fator Integrante

$$I = e^{\int \frac{x+1}{x} dx} = e^{x + \ln|x|}$$

$$y' \cdot e^{x+\ln(x)} + \frac{x+1}{x} y e^{x+\ln(x)} = e^{x+\ln|x|}$$

$$(y e^{x+\ln(x)})' = e^{x+\ln(x)}$$

$$\int (y e^{x+\ln(x)})' = \int e^{x+\ln(x)} dx$$

$$y e^{x+\ln(x)} = e^{x}(x-1) + C$$

$$y = \frac{e^{x}(x-1)}{e^{x} \cdot e^{\ln x}} + \frac{C}{e^{x} \cdot x}$$

$$y = \frac{x-1}{x} + \frac{C}{x e^{x}} \quad \text{(Solução Geral)}$$

$$\int e^{x + \ln(x)} dx$$

$$\int e^{x} \cdot e^{\ln(x)} dx$$

$$\int e^{x} \cdot x \, dx =$$

$$= x e^{x} - e^{x} = e^{x}(x-1) + C$$

$$y(\ln 2) = 1 \begin{cases} x = \ln 2 \\ y = 1 \end{cases}$$

$$1 = \frac{\ln 2 - 1}{\ln 2} + \frac{C}{\ln 2 \cdot e^{\ln 2}} \qquad C = \frac{1}{\ln 2} \cdot 2 \ln 2 = 2$$

$$y = \frac{x-1}{x} + \frac{2}{x e^{x}} \quad \text{(Solução Particular)}$$

139-) $xy' + 4x^2y = e^{-2x^2}$, $y(-1) = \frac{1}{3}$

divide por x

$$y' + 4xy = \frac{e^{-2x^2}}{x}$$

$$I(x) = e^{\int 4x\,dx} = e^{2x^2}$$

$$y'e^{2x^2} + 4xye^{2x^2} = e^{2x^2}\cdot\frac{1}{e^{2x^2}x}$$

$$(ye^{2x^2})' = 1/x$$

$$\int (ye^{2x^2})' = \int \frac{1}{x}\,dx$$

$$ye^{2x^2} = \ln|x| + C$$

$$y = \frac{\ln|x|}{e^{2x^2}} + \frac{c}{e^{2x^2}} \quad \text{(Solução Geral)}$$

$$y(-1) = 1/3 \begin{cases} x = -1 \\ y = 1/3 \end{cases}$$

$$\frac{1}{3} = \frac{0}{e^2} + \frac{c}{e^2} \quad \therefore \quad c = \frac{e^2}{3}$$

$$y = \frac{\ln|x|}{e^{2x^2}} + \frac{e^2}{3} : e^{2x^2}$$

$$= e^{-2x^2}\left(\ln|x| + \frac{e^2}{3}\right)$$

140-) $y' - y = 2xe^{2x}$, $y(0) = 1$

$$I(x) = e^{\int -1\,dx} = e^{-x}$$

$$y'e^{-x} - ye^{-x} = 2xe^{2x}e^{-x}$$

$$(ye^{-x})' = 2xe^{x}$$

$$\int (ye^{-x})' = \int 2xe^{x}\,dx$$

$$ye^{-x} = e^{x}(2x-2) + c$$

$$y = \frac{e^x}{e^{-x}}(2x-2) + \frac{c}{e^{-x}} \sim y = e^{2x}(2x-2) + c.e^{x}$$

(Solução Geral)

$2\int xe^x\,dx$

$u = x \quad du = dx$

$dv = e^x \quad v = e^x$

$2\int xe^x\,dx = 2(xe^x - e^x) + c$

$y(0)=1 \begin{cases} x=0 \\ y=1 \end{cases}$

$1 = e^0(2.0-2)+C. \quad \therefore \quad C=3$

$y = e^{2x}(2x-2)+3e^x$ ou $y = e^x[(2x-2)e^x+3]$

141-) $\frac{x^2-1}{y^2+1}\, y' = \frac{x}{y}$

$\frac{y' \cdot y}{y^2+1} = \frac{x}{x^2-1} \sim \frac{dy}{dx} \cdot \frac{y}{y^2+1} = \frac{x}{x^2-1}$

$\frac{y}{y^2+1}\, dy = \frac{x}{x^2-1}\, dx$

$\int \frac{y}{y^2+1}\, dy = \int \frac{x}{x^2-1}\, dx$

$\frac{1}{2} \ln|y^2+1| = \frac{1}{2} \ln|x^2-1| + c$

$\ln|y^2+1| = \ln|x^2-1| + c$

$y^2+1 = (x^2-1).c$ (Solução Geral)

$\rightarrow \int \frac{y}{y^2+1}\, dy$

$y^2+1 = t \quad 2y\,dy = dt$

$= \frac{1}{2} \int \frac{1}{t}\, dy = \frac{1}{2} \ln|y^2+1|$

$\rightarrow \int \frac{x}{x^2-1}\, dx$

$x^2-1 = s \quad 2x\,dx = ds$

$= \frac{1}{2} \int \frac{1}{s}\, ds = \frac{1}{2} \ln|x^2-1|$

142-) $\frac{dy}{dx} = \frac{1+2y^2}{y \operatorname{sen} x}$

$\frac{y}{1+2y^2}\, dy = \frac{1}{\operatorname{sen} x}\, dx$

$\int \frac{y}{1+2y^2}\, dy = \int \csc(x)\, dx$

$\frac{1}{4} \ln|1+2y^2| = \ln|\csc(x) - \cot(x)| + c$

$\frac{1}{4}(1+2y^2) = (\csc(x) - \cot(x))\, c$

$\int \frac{y}{1+2y^2}\, dy =$

$1+2y^2 = t \quad 4y\,dy = dt$

$= \frac{1}{4} \int \frac{1}{t}\, dt = \frac{1}{4} \ln|1+2y^2|$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(2y) = \cos^2 y - \sin^2 y$

143) $\frac{dy}{dx} = \sin x \left(\cos(2y) - \cos^2 y\right)$

$\frac{dy}{dx} = \sin x\left(\cancel{\cos^2 y} - \sin^2 y - \cancel{\cos^2 y}\right)$

$\frac{dy}{dx} = \sin x\left(-\sin^2 y\right)$

$-\frac{1}{\sin^2 y}\, dy = \sin x \, dx \quad \sim \quad -\csc^2 y \, dy = \sin x \, dx$

$-\int \csc^2 y \, dy = \int \sin x \, dx \quad \sim \quad \cot(x) = \cos(x) + C$

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074-) $\displaystyle\int \frac{x^2+12x+12}{x^3-4x}\,dx$

$$\frac{x^2+12x+12}{x^3-4x} = \frac{A}{x} + \frac{Bx+C}{x^2-4} = \frac{A(x^2-4)+(Bx+c)x}{x(x^2-4)}$$

Admitindo $x=0$ $\qquad A(x^2-4)+Bx^2+Cx$

$A(-4)=12$

$A=-3$

Admitindo $x=2$ $\qquad$ Admitindo $x=-2$

$4B+2C=40$ $\qquad 4B-2C=-8$

$$\begin{cases} 4B+2C=40 \\ 4B-2C=-8 \end{cases} \qquad \begin{matrix} 8B=32 \\ B=4 \end{matrix} \qquad C=\frac{40-4B}{2}=\frac{40-4.4}{2}=12$$

$$\int\left(\frac{-3}{x}+\frac{4x+12}{x^2-4}\right)dx = -3\int\frac{1}{x}\,dx + \underbrace{\int\frac{4x+12}{x^2-4}\,dx}_{I_1=?}$$

I_1: $\displaystyle\int\frac{4x}{x^2-4}\,dx + 12\int\frac{1}{x^2-4}\,dx$

$x^2-4=t$

$2x\,dx=dt$ $\qquad \displaystyle\int\frac{4x}{x^2-4}\,dx = \frac{4}{2}\int\frac{1}{t}\,dt = 2\ln|x^2-4|$

$x\,dx=\frac{1}{2}dt$

$$12\int\frac{1}{x^2-4}\,dx = 12.\frac{1}{2.2}\ln\left|\frac{x-2}{x+2}\right|$$

$$\therefore \int\frac{x^2+12x+12}{x^3-4x}\,dx = -3\ln|x| + 2\ln|x^2-4| + 3\ln\left|\frac{x-2}{x+2}\right| + C$$

$$= \ln|x^{-3}| + \ln|(x^2-4)^2| + \ln\left|\frac{(x-2)^3}{(x+2)^3}\right| + \ln C$$

$= \ln|$

/ /

Exemplo: $\int \frac{4x^2+2x-1}{x^3+x^2}\,dx$

Decomposição $\frac{Ax+B}{x^2} + \frac{C}{x+1} = \frac{(Ax+B)(x+1)+Cx^2}{x^2(x+1)}$

Admitindo $x=0$

$B=-1$

Admitindo $x=-1$

$C=1$

$\frac{4x^2+2x-1}{x^3+x^2} = \frac{3x-1}{x^2} + \frac{1}{x+1}$

Admitindo $x=1$

$(A+B)(1+1)+C = 4+2-1$

$2A+2B+C=5$

$2A-2+1=5$

$A=\frac{6}{2}=3$

$\int \frac{4x^2+2x-1}{x^3+x^2}\,dx = \int \frac{3x-1}{x^2}\,dx + \int \frac{1}{x+1}\,dx$ α

$\alpha = \ln|x+1|$ $\int \frac{3x-1}{x^2}\,dx \sim \int \frac{3x}{x^2}\,dx - \int \frac{1}{x^2}\,dx = \ln|x| + \frac{1}{x}$

$I = \ln|x+1| + 3\ln|x| + \frac{1}{x} + C =$

$= \ln|x+1| + \ln|x^3| + \ln e^{1/x} + \ln k =$

$= \ln|(x+1)x^3 . e^{1/x} . k|$

$\rightarrow \int \cos(11x)\, sen(5x)\, dx$

transformação de um produto numa soma:

$sen(A+B) = sen A \cos B + sen B \cos A$ (I)

$sen(A-B) = sen A \cos B - sen B \cos A$ (II)

Somando $I + II$

$sen(A+B) + sen(A-B) = 2\, sen A \cos B$

$$sen A \cos B = \frac{sen(A+B) + sen(A-B)}{2}$$

$$\int \cos(11x)\, sen(5x)\, dx = \frac{1}{2} \int sen(11x+5x) + sen(5x-11x)$$

$$= \frac{1}{2} \int sen(16x) + sen(-6x)\, dx$$

$$= \frac{1}{2} \int (sen(16x) - sen(6x))\, dx$$

$$= \frac{1}{2}\left(-\frac{\cos(16x)}{16} + \frac{\cos(6x)}{6} \right) + C$$

$$= \frac{\cos(6x)}{12} - \frac{\cos(16x)}{32}$$

$\rightarrow \int \cos^2(7x)\, sen^2(7x)\, dx$

$$\int \frac{1+\cos(14x)}{2} \cdot \frac{1-\cos(14x)}{2}\, dx \sim \frac{1}{4} \int 1 - \cos^2(14x)\, dx$$

$$\frac{1}{4} \int sen^2(14x)\, dx \sim \frac{1}{4} \int \frac{1-\cos(28x)}{2}\, dx \sim$$

$$\frac{1}{8} \left(\int dx - \int \cos(28x)\, dx \right) = \frac{1}{8}\left(x - \frac{sen(28x)}{28} \right) + C$$

Arco Duplo

$\cos(A+B) = \cos A \cos B - \operatorname{sen} A \operatorname{sen} B$

$A = B = \alpha$

$\cos(2\alpha) = \cos^2\alpha - \operatorname{sen}^2\alpha$

$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$ (I)

$\cos^2\alpha - \operatorname{sen}^2\alpha = \cos(2\alpha)$ (II)

Somando I e II

$2\cos^2\alpha = 1 + \cos(2\alpha)$

$\cos^2\alpha = \dfrac{1+\cos(2\alpha)}{2}$

Subtraindo I e II

$2\operatorname{sen}^2\alpha = 1 - \cos(2\alpha)$

$\operatorname{sen}^2\alpha = \dfrac{1-\cos(2\alpha)}{2}$

→ $\int x^3 \operatorname{sen}(x^2)\,dx$

$x^2 = t \Rightarrow 2x\,dx = dt \qquad x\,dx = \frac{1}{2}dt$

$\int t \operatorname{sen}(t)\,\frac{1}{2}\,dt \sim \frac{1}{2}\int t \operatorname{sen} t\,dt$

$u = t \Rightarrow du = dt$

$dv = \operatorname{sen} t \Rightarrow v = \int \operatorname{sen} t = -\cos(t)$

$\int t \operatorname{sen} t\,dt = -t\cos(t) + \int \cos(t)\,dt$

$= -t\cos(t) + \operatorname{sen}(t)$

$\int x^3 \operatorname{sen}(x^2)\,dx = \frac{1}{2}\left(\operatorname{sen}(x^2) - x^2\cos(x^2)\right) + C$

$\frac{4}{4-4x^2}$ $\frac{1}{1-x^2}$

www.abacoaulas.com/matematica/trigononometrica

$$\int_0^{\frac{\pi}{3}} tg^5 x \sec^4 x \, dx$$

$$\int_0^{\frac{\pi}{3}} tg^5 x \sec^2 x \sec^2 x \, dx \Rightarrow \int_0^{\frac{\pi}{3}} tg^5 x \, (tg^2 x + 1) \sec^2 x \, dx$$

$tg\,x = t \qquad \sec^2 x \, dx = dt \qquad tg\,0 \Rightarrow t = 0$

$tg\,\frac{\pi}{3} \Rightarrow t = \sqrt{3}$

$$\int_0^{\sqrt{3}} t^5 (t^2+1)\, dt \Rightarrow$$

$$\Rightarrow \int_0^{\sqrt{3}} t^7 + t^5 \, dt = \frac{t^8}{8} + \frac{t^6}{6}\Bigg|_0^{\sqrt{3}} = \frac{(\sqrt{3})^8}{8} + \frac{(\sqrt{3})^6}{6} =$$

$$= \frac{3^4}{8} + \frac{3^3}{6} = \frac{81}{8} + \frac{27}{6} = \frac{108 + 243}{24} = \frac{351}{24}$$

Revisão

1) $\sin^2 x + \cos^2 x = 1$
2) $\sec^2 x = 1 + tg^2 x$
3) $tg^2 x = \sec^2 x - 1$
4) $\csc^2 x = ctg^2 x + 1$
5) $tg^2 x = \sec^2 x - 1$

Arco duplo

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

Integral Definida

010) $\int_{-1}^{3} x^5\,dx$

$$\int_{-1}^{3} x^5\,dx = \frac{x^6}{6}\Big|_{-1}^{3} = \frac{3^6}{6} - \frac{(-1)^6}{6} = \frac{728}{6} = \frac{364}{3}$$

011) $\int_0^1 x^{\frac{4}{5}}\,dx$

$$\int_0^1 x^{\frac{4}{5}}\,dx = \frac{5x^{9/5}}{9}\Big|_0^1 = \frac{5}{9}$$

012) $\int_0^{\frac{\pi}{4}} sec^2(t)\,dt$

$$\int_0^{\frac{\pi}{4}} sec^2(t)\,dt = Tag(x)\Big|_0^{\frac{\pi}{4}} = tag(\tfrac{\pi}{4}) = 1$$

013) $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{6}{\sqrt{1-t^2}}\,dt$

$$6\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-t^2}}\,dt = 6\, arc\,sen(x)\Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} = 360 - 180 = 180\text{ rad} = \pi$$

014) $\int_1^4 (5x + \sqrt{x})\,dx$

$$\int_1^4 (5x+\sqrt{x})\,dx = \frac{5}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}}\Big|_1^4 = \left(\frac{5.16}{2} + \frac{2}{3}.1^{\frac{3}{2}}\right) - \left(\frac{5.1}{2} + \frac{2}{3}.1^{\frac{2}{3}}\right)$$

$$\int_1^4 (5x+\sqrt{x})\,dx = \frac{253}{6}$$

spiral

015) $\int_0^1 (x+1)\,dx$

$$\int_0^1 (x+1)\,dx = \frac{x^2}{2} + x \Big|_0^1 = \frac{1}{2} + 1 = \frac{3}{2}$$

016) $\int_1^2 \frac{1+3x^2}{x}\,dx$

$$\int_1^2 x^{-1}(1+3x^2)\,dx \Rightarrow \int_1^2 x^{-1} + 3x \;\; dx =$$

$$= \frac{3x^2}{2}\Big|_1^2 = \frac{3 \cdot 4}{2} - \frac{3 \cdot 1}{2} = \frac{9}{2} =$$

Fundamentos da matemática
- Cálculo e Análise
Cálculo Diferencial e Integral a Uma variável
Barboni, Ayrton

Identidades trigonométricas

$sen^2(x) + cos^2(x) = 1$

$tg^2(x) + 1 = sec^2(x)$

$1 + ctg^2(x) = csc^2(x)$

$$sen^2(x) = \frac{1-\cos(2x)}{2}$$

$$cos^2(x) = \frac{1+\cos(2x)}{2}$$

Integração por substituição trigonométrica

$\sqrt{a^2-x^2}$ $\quad x = a\,sen\,\theta \quad \to a\cos\theta$

$\sqrt{a^2+x^2}$ $\quad x = a\,tg\,\theta \quad \to a\sec\theta$

$\sqrt{x^2-a^2}$ $\quad x = a\sec\theta \quad \to a\,tg\,\theta$

Integração de função trignométricas

1º Modelo

- $\int sen(kx)\,dx$ $\qquad \int sen(t)\frac{1}{k}\,dt \sim \frac{1}{k}\int sen(t)\,dt \Rightarrow$

$kx = t \qquad k\,dx = dt$

$\Rightarrow \frac{1}{k} \cdot -cos(t) + C \quad -\frac{1}{k} \cdot cos(kx) + C$

- $\int cos\left(\frac{x}{2}\right)dx$ $\qquad \int cos(t)\,2\,dt \sim 2\int cos(t)\,dt \Rightarrow$

$\frac{x}{2} = t \Rightarrow \frac{1}{2}dx = dt$ $\qquad 2\,sen(t) + C$

$2\,sen\left(\frac{x}{2}\right) + C$

$dx = 2\,dt$

- $\int sen(\pi x)\,dx$ $\qquad \int sen(t)\frac{1}{\pi}\,dt \sim \frac{1}{\pi} \cdot \int sen(t)\,dt \Rightarrow$

$\pi x = t$, $dx = \frac{1}{\pi}dt$

$\pi\,dx = dt$ $\qquad \Rightarrow \frac{1}{\pi} \cdot -cos(\pi x) + C = \frac{-cos(\pi x)}{\pi} + C$

2º Modelo

- $\int cos^3 x\,dx \sim \int cos^2(x)\,cos(x)\,dx \sim \int (1 - sen^2(x))\,cos(x)\,dx$

$sen(x) = t \qquad cos(x)\,dx = dt$

$\int (1 - sen^2(x))\,cos(x)\,dx = \int (1 - t^2)\,dt$

$= \int dt - \int t^2\,dt$

$= t - \frac{t^3}{3}$

$= sen(x) - \frac{sen^3(x)}{3} + C$

- $\int \cos^5(x)\,dx$

$\int \cos^5(x)\,dx = \int \cos^4(x)\cos(x)\,dx$ $\quad$ $sen(x) = t$

$= \int (1-sen^2(x))^2 \cos(x)\,dx$ $\quad$ $\cos(x)\,dx = dt$

$= \int (1-t^2)^2\,dt$

$= \int 1 - 2t^2 + t^4\,dt$

$= t - \frac{2}{3}t^3 + \frac{t^5}{5} + C = sen(x) - \frac{2}{3}sen^3(x) + \frac{1}{5}sen^5(x) + C$

- $\int sen^3(2x)\,dx \sim \int sen^2(2x)\,sen(2x)\,dx$ $\quad$ $\cos(2x) = t$

$\int sen^3(2x)\,dx = \int (1-\cos^2(2x))\,sen(2x)\,dx$ $\quad$ $-2.\,sen(2x)\,dx = dt$

$= \int (1-t^2)\,-\frac{1}{2}\,dt$ $\quad$ $sen(2x)\,dx = -\frac{1}{2}\,dt$

$= \int -\frac{1}{2} + \frac{t^2}{2}\,dt$

$= -\frac{1}{2}t + \frac{t^3}{6} + C = -\frac{1}{2}\cos(2x) + \frac{\cos^3(2x)}{6} + C$

- $\int sen^5x\,dx \sim \int sen^4(x)\,sen(x)\,dx$ $\quad$ $\cos(x) = t$

$\int sen^5x\,dx = \int (1-\cos^2(x))^2 sen(x)\,dx$ $\quad$ $-sen(x)\,dx = dt$

$= \int (1-t^2)^2.(-1).dt$ $\quad$ $sen(x)\,dx = -dt$

$= -\int 1 - 2t^2 + t^4\,dt$

$= -t + \frac{2}{3}t^3 - \frac{t^5}{5}\,dt = -\cos(x) + \frac{2}{3}\cos^3(x) - \frac{1}{5}\cos^5(x) + C$

- $\int \cos^2x\,dx \sim \int \frac{1+\cos(2x)}{2}\,dx$ $\quad$ $2x = t$

$\int \cos^2x\,dx = \frac{1}{2}\int 1 + \cos(2x)\,dx$ $\quad$ $2dx = dt$

$= \frac{1}{2}\int (1+\cos(t))\,\frac{1}{2}\,dt$ $\quad$ $dx = \frac{1}{2}\,dt$

$= 1/4 \left(\int dt + \int \cos(t)\,dt\right.$

$= 1/4\,(.\,t - sen(t)) + C$

$= 1/4\,(.\,2x - sen(2x)) + C$

$= \frac{1}{2}x - \frac{1}{4}sen(2x) + C$

- $\int sen^2(x)\,dx$

$2x = t$

$2dx = dt$

$dx = 1/2 \cdot dt$

$$\int sen^2(x)\,dx = \int \frac{1-\cos(2x)}{2}\,dx$$

$$= 1/2 \int 1-\cos(2x)\,dx$$

$$= 1/2 \int (1-\cos(t))\ 1/2\,dt$$

$$= 1/4 \int 1-\cos(t)\,dt$$

$$= 1/4\,(t + sen(t)) + c$$

$$= 1/4\,(2x + sen(2x)) + c = \frac{1}{2}x + \frac{1}{4}sen(2x) + c$$

- $\int sen^4 x\,dx$?

$2x = t$

$2dx = dt$

$dx = 1/2\,dt$

$$\int sen^4 x\,dx = \int \left(\frac{1-\cos(2x)}{2}\right)^2 dx$$

1

$$= 1/4 \int 1 - 2\cos(2x) + \cos^2(2x)\,dx$$

$$= 1/4 \int dx - 2\int \cos(2x)\,dx + \int \cos^2(2x)\,dx$$

$$= \frac{1}{4}x + sen(2x) + \frac{1}{2}(x + sen(x)\cos(x))$$

3º Modelo

- $\int sen^2(x)\cos^2(x)\,dx$

$4x = t$

$4dx = dt$

$dx = 1/4\,dt$

$$\int \frac{1-\cos(2x)}{2} \cdot \frac{1+\cos(2x)}{2}\,dx$$

$$= 1/4 \int 1-\cos^2(2x)\,dx$$

$$= 1/4 \int sen^2(2x)\,dx$$

$$= 1/4 \int \frac{1-\cos(4x)}{2}\,dx$$

$$= 1/8 \int 1-\cos(4x)\,dx$$

$$= 1/8 \int 1 - \cos(t) \cdot 1/4\,dt$$

$$= 1/8 \int \frac{4-\cos(t)}{4}\,dt$$

$$= 1/32 \int 4 - \cos(t)\,dt$$

$$= 1/8\,t - 1/32\,sen(t) + C$$

$$= \frac{1}{2}x - \frac{1}{32}sen(4x) + C$$

spiral

/ /

- $\int sen^4 x . \cos^2 x \, dx$

$\int \left(\frac{1-\cos(2x)}{2}\right)^2 \frac{1+\cos(2x)}{2} \, dx$

$= 1/8 \int (1 - 2\cos(2x) + \cos^2(2x)) (1 + \cos(2x)) \, dx$

$= 1/8 \int 1 - 2\cos(2x) + \cos^2(2x) + \cos(2x) - 2\cos^2(2x) + \cos^3(2x) \, dx$

$\Rightarrow \int \cos(2x) \, dx \qquad 2x = t$

$\int \cos(t) \frac{1}{2} dt \qquad 2dx = dt$

$= 1/2 . sen(t) \qquad dx = 1/2 \, dt$

$= \frac{1}{2} sen(2x)$

$\Rightarrow \int \cos^2(2x) dx \qquad 4x = t$

$\int \frac{1 + \cos(4x)}{2} dx \qquad 4dx = dt$

$dx = 1/4 \, dt$

$1/8 \int 1 + \cos(t) \, dt$

$= 1/8 (t + sen(t)) = 1/8 (4x + sen(4x)) = \frac{1}{2} x + \frac{1}{8} sen(4x)$

$\Rightarrow \int \cos^3(2x) dx \sim \int \cos^2(2x) \cos(2x) \, dx \sim$

$\sim \int (1 - sen^2(2x)) \cos(2x) \, dx \qquad t = sen(2x)$

$\int (1 - t^2) \frac{1}{2} \, dt \Rightarrow \qquad dt = 2\cos(2x) \, dx$

$\Rightarrow 1/2 \int 1 - t^2 \, dt = 1/2 \left(t - \frac{t^3}{3}\right) = \frac{sen(2x)}{2} - \frac{sen^3(2x)}{6}$

$\int sen^4 x \cos^2 x \, dx = 1/8 \int 1 - \cos(2x) - \cos^2(2x) + \cos^3(2x) \, dx$

$= 1/8 \left(x - \frac{1}{2} sen(2x) - \frac{1}{2} x - \frac{1}{8} sen(4x) + \frac{sen(2x)}{2} - \frac{sen^3(2x)}{6} \right)$

$= \frac{1}{8} \cdot \left(\frac{x}{2} - \frac{1}{8} sen(4x) - \frac{1}{6} sen^3(2x) \right)$

spiral

- $\int sec^4(x)\,dx$

$\int sec^4(x)\,dx = \int sec^2(x)\, sec^2(x)\,dx$ $\quad t = tg(x)$

$= \int (tg^2(x)+1)\, sec^2(x)\,dx$ $\quad dt = sec^2(x)\,dx$

$= \int (t^2+1)\,dt$

$= \frac{t^3}{3} + t + C = \frac{tg^3(x)}{3} + tg(x) + C$

- $\int tg^4(x)\,dx$

$\int (tg^2(x))^2\,dx \Rightarrow \int (sec^2(x)-1)^2\,dx$

$\int sec^4(x) - 2sec^2(x) + 1\,dx$

$\int sec^4(x) - 2\int sec^2(x) + \int 1\,dx$

$\int sec^4(x)\,dx \Rightarrow \int sec^2(x)\, sec^2(x)\,dx \sim \int (1+tg^2(x))\, sec^2(x)\,dx$

$tg(x) = t \qquad sec^2(x)\,dx = dt$

$\int (1+t^2)\,dt = t + \frac{t^3}{3} + C_1$

$\int tg^4(x)\,dx = tg(x) + \frac{tg^3(x)}{3} - 2\,tg(x) + x$

$= \frac{1}{3}\,tg^3(x) - tg(x) + x + C$

- $\int tg^3(x)\,dx \quad \int tg^2(x)\, tg(x)\,dx \sim \int (sec^2(x)-1)\, tg(x)\,dx$

$\int sec^2(x)\, tg(x) - tg(x)\,dx$

$\int sec^2(x)\, tg(x) - \int tg(x)\,dx$ ①

$\int sec^2(x)\, tg(x)\,dx \sim \int sec(x)\, sec(x)\, tg(x)\,dx$

$sec(x) = t \quad sec(x)\, tg(x)\,dx = dt$ | $tg(x) = t \quad sec^2(x)\,dx = dt$

$\int t\,dt = \frac{t^2}{2} = \frac{sec^2(x)}{2}$ | $\int t\,dt = \frac{t^2}{2} + C \Rightarrow \frac{tg^2(x)}{2} + C$

$\int tg^3(x)\,dx = \frac{sec^2(x)}{2} - \ln|sec(x)| + C$

/ /

- $\int sec(x)\,tg^3(x)\,dx \sim \int sec(x)\,tg^2(x)\,tg(x)\,dx \sim$

$\int sec(x)\,(sec^2(x)-1)\,tg(x)\,dx$

$t = sec(x) \quad dt = sec(x)\,tg(x)\,dx$

$\Rightarrow \int (t^2-1)\,dt = \frac{t^3}{3} - t + C = \frac{sec^3(x)}{3} - sec(x) + C$

- $\int sec^3(x)\,tg(x)\,dx \sim \int sec^2(x)\,sec(x)\,tg(x)\,dx$

$t = sec(x) \quad dt = sec(x)\,tg(x)\,dx$

$\Rightarrow \int t^2\,dt = \frac{t^3}{3} = \frac{sec^3(x)}{3} + C$

- $\int sec(x)\,tg^5(x)\,dx \sim \int sec(x)\,tg^2(x)\,tg^2(x)\,tg(x)\,dx \sim$

$\sim \int sec(x)\,(sec^2(x)-1)^2\,tg(x)\,dx$

$sec(x) = t \quad sec(x)\,tg(x)\,dx = dt$

$\int (t^2-1)^2\,dt \sim \int t^4 - 2t^2 + 1\,dt \sim \int t^4\,dt - 2\int t^2\,dt + \int dt =$

$= \frac{t^5}{5} - \frac{2}{3}t^3 + t + C = \frac{sec^5(x)}{5} - \frac{2}{3}sec^3(x) + sec(x) + C$

- $\int tg^2(x)\,sec(x)\,dx \sim \int (sec^2(x)-1)\,sec(x)\,dx \sim \int sec^3(x) - sec(x)\,dx$

$\int tg^2(x)\,sec(x)\,dx = \int sec^3(x)\,dx - \int sec(x)\,dx$

$= \int sec^2(x)\,sec(x)\,dx - \int sec(x)\,dx$

"

$u = sec(x) \quad du = sec(x)\,tg(x)\,dx$

$dv = sec^2(x) \quad v = tg(x)$ "

$= sec(x)\,tg(x) - \int tg^2(x)\,sec(x)\,dx - \int sec(x)\,dx$

$2\int tg^2(x)\,sec(x)\,dx = sec(x)\,tg(x) - \ln|sec(x) + tg(x)|$

$\int tg^2(x)\,sec(x)\,dx = \frac{1}{2}\cdot\left(sec(x)\,tg(x) - \ln|sec(x)+tg(x)|\right) + C$

spiral

$$-\int tg^2(x)\,sec^3(x)\,dx \sim \int (sec^2(x)-1)\,sec^3(x)\,dx$$

$$\Rightarrow \int sec^5(x)\,dx - \int sec^3(x)\,dx \quad (1)$$

$$\int sec^5(x)\,dx \sim \int sec^3(x)\,sec^2(x)\,dx$$

$$u = sec^3(x) \qquad du = 3\,sec^3(x)\,tg(x)\,dx$$

$$dv = \int sec^2(x)\,dx \;.\; v = tg(x)$$

$$\int sec^5(x)\,dx = sec^3(x)\,tg(x) - \int tg(x)\,.\,3\,sec^3(x)\,tg(x)\,dx$$

$$= sec^3(x)\,tg(x) - 3\int sec^3(x)\,(sec^2(x)-1)\,dx$$

$$= sec^3(x)\,tg(x) - 3\int sec^5(x)\,dx + 3\int sec^3(x)\,dx$$

$$4\int sec^5(x)\,dx = sec^3(x)\,tg(x) + 3\int sec^3(x)\,dx \quad (II)$$

$$\int sec^3(x)\,dx \sim \int sec^2(x)\,sec(x)\,dx$$

$$u = sec(x) \qquad du = sec(x)\,tg(x)\,dx$$

$$dv = sec^2(x) \qquad v = \int sec^2(x)\,dx = tg(x)$$

$$\int sec^3(x)\,dx = sec(x)\,tg(x) - \int tg^2(x)\,sec(x)\,dx$$

$$= sec(x)\,tg(x) - \int (sec^2(x)-1)\,sec(x)\,dx$$

$$= sec(x)\,tg(x) - \int sec^3(x)\,dx + \int sec(x)\,dx$$

$$2\int sec^3(x)\,dx = sec(x)\,tg(x) + \ln|sec(x)+tg(x)|$$

$$\int sec^3(x)\,dx = \frac{1}{2}\,.\left(sec(x)\,tg(x) + \ln|sec(x)+tg(x)|\right) \quad (III)$$

II e III

$$4\int sec^5(x)\,dx = sec^3(x)\,tg(x) + 3\,.\,\frac{1}{2}\left(sec(x)\,tg(x) + \ln|sec(x)+tg(x)|\right)$$

$$\int sec^5(x)\,dx = \frac{1}{4}\,.\left(sec^3(x)\,tg(x) + \frac{3}{2}\left(sec(x)\,tg(x) + \ln|sec(x)+tg(x)|\right)\right)$$

(II e III) I

$$\int tg^2(x)\,sec^3(x)\,dx = \frac{1}{4}\,sec^3(x)\,tg(x) + \frac{3}{8}\,sec(x)\,tg(x) + \frac{3}{8}\ln|sec(x)+tg(x)| -$$

$$-\frac{1}{2}\,sec(x)\,tg(x) - \frac{1}{2}\,.\,\ln|sec(x)+tg(x)| -$$

$$\int tg^2(x)\,sec^3(x)\,dx = \frac{1}{4}\,sec^3(x)\,tg(x) - \frac{1}{8}\,sec(x)\,tg(x) - \frac{1}{8}\ln|sec(x)+tg(x)|$$

Exercício de aplicação

1) $\int \cos(2x)\,dx$

$2x = t \qquad 2dx = dt \quad \rightarrow dx = \frac{1}{2}dt$

$\int \cos(t)\cdot\frac{1}{2}dt \sim \frac{1}{2}\int \cos(t)\,dt = \frac{1}{2}\operatorname{sen}(t) + C$

$\int \cos(2x)\,dx = \frac{1}{2}\operatorname{sen}(2x) + C$

2) $\int \cos(x/3)\,dx$

$\frac{x}{3} = t \qquad \frac{1}{3}dx = dt \quad dx = 3dt$

$\int \cos(t)\,3dt \sim 3\int \cos(t)\,dt = 3.\operatorname{sen}(t) + C$

$\int \cos(x/3)\,dx = 3\operatorname{sen}\left(\frac{x}{3}\right) + C$

3) $\int \operatorname{sen}(5x)\,dx$

$5x = t \qquad 5dx = dt \qquad dx = 1/5\,dt$

$\int \operatorname{sen}(t)\,\frac{1}{5}dt \sim \frac{1}{5}\int \operatorname{sen}(t)\,dt = -\frac{1}{5}\cos(t) + C$

$\int \operatorname{sen}(5x)\,dx = -\frac{1}{5}\cos(5x) + C$

4) $\int \cos^5 x\,dx$

$\int \cos^5(x)\,dx \sim \int (1 + \operatorname{sen}^2(x))^2 \cos(x)\,dx$

$\operatorname{sen}(x) = t \qquad \cos(x)\,dx = dt$

$\int \cos^5(x)\,dx = \int (1 - t^2)^2\,dt$

$= \int (1 - 2t^2 + t^4)\,dt$

$= t - \frac{2}{3}t^3 + \frac{t^5}{5} + C$

$\int \cos^5(x)\,dx = \operatorname{sen}(x) - \frac{2}{3}\operatorname{sen}^3(x) + \frac{1}{5}\operatorname{sen}^5(x) + C$

5-) $\int sen^3(x)\,dx$

$$\int sen^3(x)\,dx = \int sen^2(x)\,sen(x)\,dx$$
$$\int (1-\cos^2(x))\,sen(x)\,dx$$

$\cos(x) = t \quad -sen(x)\,dx = dt \quad sen(x)\,dx = -dt$

$$\int sen^3(x)\,dx = \int (1-t^2)\cdot -1\,dt$$
$$= \int (t^2-1)\,dt$$
$$= \frac{t^3}{3} - t + C$$

$$\int sen^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

6-) $\int \cos^4(x)\,dx$

$$\int \cos^4(x)\,dx = \int \left(\frac{1+\cos(2x)}{2}\right)^2 dx$$

$$= \tfrac{1}{4}\int (1 + 2\cos(2x) + \cos^2(2x))\,dx$$
$$= \tfrac{1}{4}\left(\int dx + 2\int \cos(2x)\,dx + \int \frac{1+\cos(4x)}{2}\,dx\right.$$

$$\int \cos^4(x)\,dx = \frac{1}{4}x + \frac{1}{4}sen(2x) + \frac{1}{8}x + \frac{1}{32}\cos(4x) + C$$

$$\int \cos^4(x)\,dx = \frac{3}{8}x + \frac{1}{4}sen(2x) + \frac{1}{32}\cos(4x) + C$$

7-) $\int sen^7(x)\cos(x)\,dx$

$sen(x) = t \quad \cos(x)\,dx = dt$

$$\int sen^7(x)\cos(x)\,dx = \int t^7\,dt$$
$$= \frac{t^8}{8} + C$$

$$\int sen^7(x)\cos(x)\,dx = \frac{sen^8(x)}{8} + C$$

8.) $\int \cos^8(x)\sin^3(x)\,dx$

$$\int \cos^8(x)\sin^3(x)\,dx = \int \cos^8(x)(1-\cos^2(x))\sin(x)\,dx$$

$\cos(x)=t \quad -\sin(x)\,dx = dt \quad \sin(x)\,dx = (-1)\,dt$

$$\int \cos^8(x)\sin^3(x)\,dx = \int t^8(1-t^2)(-1)\,dt$$
$$= \int(-t^8+t^{10})\,dt$$
$$= \frac{t^{11}}{11} - \frac{t^9}{9} + C$$
$$= \frac{\cos^{11}(t)}{11} - \frac{\cos^9(t)}{9} + C$$

9.) $\int \cos^2(x)\sin^5(x)\,dx$

$$\int \cos^2(x)\sin^5(x)\,dx = \int(1-\sin^2(x))\sin^5(x)\,dx$$
$$= \int(\sin^5(x) - \sin^7(x))\,dx$$
$$= \int \sin^5(x)\,dx - \int \sin^7(x)\,dx$$

* $\int \sin^5(x)\,dx = \int(1-\cos^2(x))^2\sin(x)\,dx$

$\cos(x)=t \quad -\sin(x)\,dx = dt \quad \sin(x)\,dx = (-1)\,dt$

$$\int \sin^5(x)\,dx = \int(1-t^2)^2(-1)\,dt$$
$$= \int(1-2t^2+t^4)(-1)\,dt$$
$$= \int(2t^2-t^4-1)\,dt$$
$$= \frac{2}{3}t^3 - \frac{t^5}{5} - t + C \Rightarrow \frac{2}{3}\cos^3(x) - \frac{1}{5}\cos^5(x) - \cos(x) + C$$

* $\int \sin^7(x)\,dx = \int(1-\cos^2(x))^3\sin(x)\,dx$

$\cos(x)=t \quad -\sin(x)\,dx = dt \quad \sin(x)\,dx = (-1)\,dt$

$$\int \sin^7(x)\,dx = \int(1-t^2)^3(-1)\,dt$$
$$= -\int(1-3t^2+3t^4-t^6) + C$$
$$= -t + t^3 - \frac{3}{5}t^5 - \frac{t^7}{7} + C = -\cos(x) + \cos^3(x) - \frac{3}{5}\cos^5(x) - \frac{1}{7}\cos^7(x)$$

$$\int \cos^2(x)\sin^5(x)\,dx = \frac{1}{7}\cos^7(x) - \frac{1}{3}\cos^3(x) + \frac{2}{5}\cos^5(x) + C$$

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10-) $\int \cos^4 x \cos^2 x \, dx$

$$\int \cos^4 x \cos^2 x \, dx = \int \left(\frac{1 + \cos(2x)}{2} \right)^2 \frac{1 + \cos(2x)}{2} \, dx$$

$$= 1/8 \int ((1 + 2\cos(2x) + \cos^2(2x))(1 + \cos(2x)))\, dx$$

$$= 1/8 \int (1 + 2\cos(2x) + \cos^2(2x) + \cos(2x) + 2\cos^2(2x) + \cos^3(2x))\, dx$$

$$= \frac{1}{8} x + \frac{1}{8} \operatorname{sen}(2x) + \frac{1}{16} x + \frac{1}{64} \operatorname{sen}(4x) + \frac{1}{16} \operatorname{sen}(2x) + \frac{x}{8} + \frac{\operatorname{sen}(4x)}{32} + \frac{1}{16} \operatorname{sen}(2x)$$

$$- \frac{1}{48} \operatorname{sen}^3(2x)$$

$$\int \cos^2(2x)\, dx = \int \frac{1 + \cos(4x)}{2} \, dx$$

$$= 1/2 \int 1 + \cos(4x)\, dx$$

$$= \frac{1}{2} x + \frac{1}{8} \operatorname{sen}(4x)$$

$$\int \cos^3(2x)\, dx \sim \int \cos^2(2x) \cos(2x)\, dx \sim \int (1 - \operatorname{sen}^2(2x)) \cos(2x)\, dx$$

$t = \operatorname{sen}(2x) \quad dt = 2\cos(2x)\, dx$

$$\int (1 - t^2) \frac{1}{2} dt = \frac{1}{2}\left(t - \frac{t^3}{3}\right) = \frac{t}{2} - \frac{t^3}{6}$$

$$= \frac{\operatorname{sen}(2x)}{2} - \frac{\operatorname{sen}^3(2x)}{6}$$

$$\int \cos^4 x \cos^2(x)\, dx = \frac{3}{16} x + \frac{1}{4} \operatorname{sen}(2x) + \frac{3}{64} \operatorname{sen}(4x) - \frac{1}{48} \operatorname{sen}^3(2x) + C$$

Integração por substituição trigonométrica

1-) $\int \sqrt{a^2-x^2}\,dx$

$x = a\,sen\,\theta \qquad dx = a\cos\theta\,d\theta$

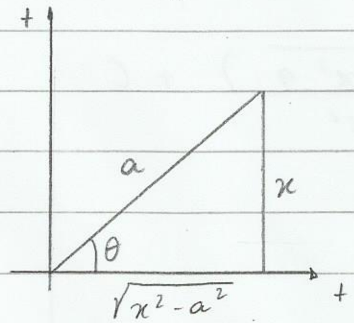
$\int \sqrt{a^2-a^2sen^2\theta}\;a\cos\theta\,d\theta =$

$\int a\sqrt{1-sen^2\theta}\;a\cos\theta\,d\theta = \int a^2\cos^2\theta\,d\theta$

$\int \sqrt{a^2-x^2}\,dx = a^2\int \cos^2\theta\,d\theta$

$= a^2 \int \frac{1+\cos(2\theta)}{2}\,d\theta$

$= \frac{a^2}{2}\left(\theta + \frac{sen\,\theta}{2}\right) + C$

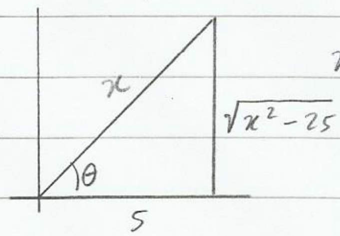


4-) $\int \frac{1}{\sqrt{x^2-25}}\,dx$

$x = 5\sec\theta \qquad dx = 5\sec\theta\,tg\,\theta\,d\theta$

$\int \frac{5\sec\theta\,tg\,\theta}{\sqrt{25\sec^2\theta-25}}\,d\theta = \int \frac{\sec\theta\,\cancel{tg\,\theta}}{\cancel{tg\,\theta}}\,d\theta = \ln|\sec(\theta)+tg(\theta)| + C$

$= \ln\left|\frac{x}{5} + \frac{\sqrt{x^2-25}}{5}\right| + C$



$x = 5\frac{1}{\cos\theta} \qquad \cos\theta . x = 5$

$\cos\theta = \frac{5}{x}$

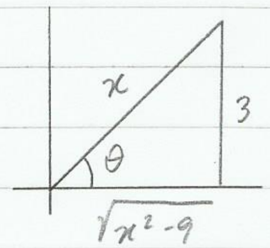
$\frac{1}{\cos\theta} = \frac{x}{5} \qquad tg\,\theta = \frac{sen\,\theta}{\cos\theta} = \frac{\sqrt{x^2-25}}{x} : \frac{5}{x}$

5) $\displaystyle\int \frac{1}{x^3\sqrt{x^2-9}}\,dx$

$x = 3\sec\theta \qquad dx = 3\sec\theta\, tg\theta\, d\theta$

$$\int \frac{\cancel{3}\sec\theta\, tg\theta}{27\sec^3\theta \,.\, \cancel{3}\, \cancel{tg\theta}}\,d\theta = \frac{1}{27}\int \frac{1}{\sec^2\theta}\,d\theta = \frac{1}{27}\int 1 : \frac{1}{\cos^2\theta}\,d\theta =$$

$$= \frac{1}{27}\int \cos^2\theta\,d\theta = \frac{1}{54}\left(\theta + \operatorname{sen}\theta\cos\theta\right).$$



$x = 3\sec\theta \qquad \cos\theta = \frac{3}{x}$

$x = 3 \,.\, \frac{1}{\cos\theta}$

$\theta = \operatorname{arcsec}\left(\frac{x}{3}\right)$

$$\int \frac{1}{x^3\sqrt{x^2-9}}\,dx = \frac{1}{54}\left(\operatorname{arcsec}\left(\frac{x}{3}\right) + \frac{3\sqrt{x^2-9}}{x^2}\right) + C$$

Ex: $\int \frac{3x+2}{x^2+2x+2}\,dx \sim \int \frac{3x+2}{(x+1)^2+1}\,dx$

$x+1=t \qquad dx = dt$

$x = t-1$

$\int \frac{3(t-1)+2}{t^2+1}\,dt \sim 3\int \frac{t}{t^2+1}\,dt - 1\int \frac{1}{t^2+1}\,dt$

$t^2+1 = s$

$2t\,dt = ds \qquad t\,dt = \frac{1}{2}ds$

$= \frac{3}{2}\int \frac{ds}{s}\,ds - \operatorname{arctg}(t)$

$= \frac{3}{2}\ln|s| - \operatorname{arctg}(t)$

$\int \frac{3x+2}{x^2+2x+2}\,dx = \frac{3}{2}\ln|x^2+2x+2| - \operatorname{arctg}(x+1) + C$

→ $\int \frac{x\,dx}{(1-x^2)^3}$

$t = 1-x^2 \qquad dt = -2x\,dx \qquad x\,dx = \frac{-1}{2}dt$

$\int \frac{1}{t^3}\cdot\frac{-1}{2}dt \sim \frac{-1}{2}\int t^{-3}dt = \frac{-1}{2}\frac{t^{-2}}{-2} = \frac{t^{-2}}{4} + C$

$\int \frac{x\,dx}{(1-x^2)^3} = \frac{1}{4t^2} = \frac{1}{2(1-x^2)^2} + C$

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* $\int tg^5 x \cdot sec^4 x \, dx$

$tg\,x = t \qquad sec^2 x \, dx = dt$

$\int t^5 (1+t^2)\, dt \sim \int t^5 + t^7 \, dt = \frac{t^6}{6} + \frac{t^8}{8} + c$

$\int tg^5 x \, x^4 x \, dx = \frac{tg^6 x}{6} + \frac{tg^8 x}{8} + c$

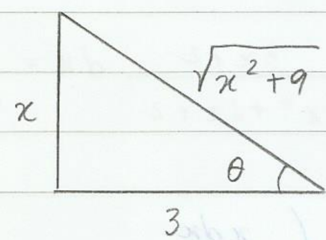
* $\int \frac{x^2}{\sqrt{x^2+9}} dx \qquad x = 3\,tg\,\theta \qquad dx = 3\,sec^2\theta\, d\theta$

$\int \frac{9\,tg^2\theta\, 3sec^2\theta}{\sqrt{9\,tg\,\theta + 9}} d\theta \sim 9 \int \frac{tg^2\theta\, sec^2\theta}{sec\,\theta} d\theta \sim 9 \int tg^2\theta\, sec\,\theta\, d\theta$

$9\int (sec^2\theta - 1)\, sec\,\theta\, d\theta \sim 9 \int sec^3\theta - sec\,\theta\, d\theta$

$= \frac{1}{9} \left(\frac{sec\,\theta\, tg\,\theta + \ln|sec\,\theta + tg\,\theta|}{2} - \ln|sec\,\theta + tg\,\theta| \right)$

$= \frac{1}{9} \left(\frac{sec\,\theta\, tg\,\theta}{2} - \frac{\ln|sec\,\theta + tg\,\theta|}{2} \right)$



$tg\,\theta = \frac{x}{3} \qquad sec\,\theta = \frac{1}{cos} = 1 : \frac{3}{\sqrt{x^2+9}} = \frac{\sqrt{x^2+9}}{3}$

$= 9 \left(\frac{x\sqrt{x^2+9}}{18} - \ln\left| \frac{\sqrt{x^2+9}}{3} + x \right| : 2 \right) \quad sec\,\theta\, tg\,\theta = \frac{\sqrt{x^2+9}}{3} \cdot \frac{x}{3} = \frac{x\sqrt{x^2+9}}{9}$

$= \frac{1}{2} x \sqrt{x^2+9} - 3^2$

Integrais de Funções Racionais

073-) $\int \frac{3}{x^2+x-2}\,dx \sim \int \frac{3}{(x-1)(x+2)}\,dx$

$$\frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{(x+2).(x-1)}$$

Admitindo $x=-2$ Admitindo $x=1$

$B(-2-1)=3$ $A(1+2)=3$

$B=-1$ $A=1$

$$\int \frac{3}{x^2+x-2}\,dx = \int \frac{1}{x-1}\,dx - \int \frac{1}{x+2}\,dx =$$

$$= \ln|x-1| - \ln|x+2| + c$$

$$= \ln\left(\frac{x-1}{x+2}\right) + C$$

074-) $\int \frac{x^2+12x+12}{x^3-4x}\,dx \sim \int \frac{x^2+12x+12}{x(x^2-4)}\,dx$

$$\frac{A}{x} + \frac{Bx+C}{x^2-4} = \frac{x^2+12x+12}{x^3-4x}$$

$$A(x^2-4)+(Bx+C)x = x^2+12x+12$$

Admitindo $x=0$

$A(-4)=12$ $-3(x^2-4)+(Bx+C)x$

$A=-3$ $-3x^2+12+(Bx+C)x = x^2+12x+12$

$Bx^2+Cx = 4x^2+12x$

Admitindo $x=1 \rightarrow B+C=16$ I

Admitindo $x=-1 \rightarrow B-C=-8$ II

I + II $\rightarrow 2B=8$ $B=16-C$

$B=4$ $B=12$

$\frac{A}{x} + \frac{Bx+C}{x^2-4} \sim \frac{-3}{x} + \frac{4x+12}{x^2-4}$

$\int \frac{x^2+12x+12}{x^3-4x}\,dx = -3\int \frac{1}{x}\,dx + 4\int \frac{x}{x^2-4}\,dx + 12\int \frac{1}{x^2-4}\,dx$

$\int \frac{x}{x^2-4}\,dx = ?$ $\quad x^2-4=t$ $\quad \int \frac{x}{x^2-4}\,dx = \frac{1}{2}\int \frac{1}{t}\,dt =$

$2x\,dx = dt$

$x\,dx = \frac{1}{2}dt$ $\quad = \frac{1}{2}\ln|x^2-4|$

$$\int \frac{x^2+12x+12}{x^3-4x}\,dx = -3\ln|x| + 2\ln|x^2-4| + 12.\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$$

$$= \ln|x^{-3}| + \ln|(x^2-4)^2| + \ln\left|\frac{(x-2)^3}{(x+2)^3}\right| + C$$

$$= \ln\left|\frac{(x^2-4)^2}{x^3}\,\frac{(x-2)^3}{(x+2)^3}\right| + C$$

075-) $\int \frac{4x^2+2x-1}{x^3+x^2}\,dx$ $\qquad (x^3+x^2) = x^2(x+1)$

$$\frac{4x^2+2x-1}{x^3+x^2} = \frac{A}{x+1} + \frac{Bx+C}{x^2} = \frac{Ax^2+(Bx+C)(x+1)}{(x+1)x^2}$$

Admitindo $x=0$ $\qquad Ax^2+(Bx-1)(x+1)$

$C=-1$ $\qquad Ax^2+Bx^2+Bx-x-\cancel{1} = 4x^2+2x-\cancel{1}$

$Ax^2 + x(Bx+B-1) = 4x^2+2x$

Admitindo $x=1$ $\;->\;$ $A+2B=7$

Admitindo $x=-1$ $\;->\;$ $A=1$ e $B=3$

$$\int \frac{4x^2+2x-1}{x^3+x^2}\,dx = \int \frac{1}{x+1}\,dx + \int \frac{3x-1}{x^2}\,dx$$

$\hookrightarrow \ln|x+1|$

$$\int \frac{3x-1}{x^2}\,dx = 3\int \frac{1}{x}\,dx - \int \frac{1}{x^2}\,dx =$$

$$= 3\ln|x| + \frac{1}{x}$$

$$\int \frac{4x^2+2x-1}{x^3+x^2}\,dx = \ln|x+1| + 3\ln|x| + \frac{1}{x} + C$$

$$= \ln|(x+1)x^3| + \frac{1}{x} + C$$

076) $\int \frac{x^2}{x^4-2x^2-8}\,dx$ $\qquad x^4-2x^2-8 \quad ; \quad t = x^2$

$t^2 - 2t - 8 = 0$

$t = \frac{2 \pm \sqrt{4+32}}{2} \Rightarrow t_1 = 4 \rightarrow x = \pm 2$

$t_2 = -2 \rightarrow x = \nexists \in \mathbb{R}$

$(x-2)(x+2)$

077.) $\int \frac{dx}{(x^2-5x+6)(x-1)}$ $\qquad x^2-5x+6=0$

$x = \frac{5 \pm \sqrt{25-24}}{2} \qquad x = \frac{5 \pm 1}{2} \quad x_1 = 3, \; x_2 = 2$

$x^2-5x+6 = (x+3)(x+2)$

$$\int \frac{dx}{(x+3)(x+2)(x-1)} \Rightarrow \frac{A}{x+3} + \frac{B}{x+2} + \frac{C}{x-1}$$

$$\frac{A(x+2)(x-1) + B(x+3)(x-1) + C(x+3)(x+2)}{(x^2-5x+6)(x-1)} = \frac{1}{(x^2-5x+6)(x-1)}$$

Admitindo $x=-2$ $\qquad$ Admitindo $x=-3$ $\qquad$ Admitindo $x=1$

$B(-2+3)(-2-1) = 1 \qquad A(-3+2)(-3-1) = 1 \qquad C(1+3)=1$

$B = -1/3 \qquad A = 1/4 \qquad C = 1/4$

$$\int \frac{dx}{(x^2-5x+6)(x-1)} = \frac{1}{4}\int \frac{1}{x+3}\,dx + \frac{1}{3}\int \frac{1}{x+2}\,dx + \frac{1}{4}\int \frac{1}{x-1}\,dx$$

$$= \frac{1}{4}\ln|x+3| - \frac{1}{3}\ln|x+2| + \frac{1}{4}|x-1| + C$$

$$= \ln|(x+3)^{1/4}| + \ln|(x+2)^{-1/3}| + \ln|(x-1)^{1/4}| + C$$

$$= \ln\left|\frac{(x+3)^{1/4}(x-1)^{1/4}}{(x+2)^{1/3}}\right| + C$$

078-) $\int \frac{dx}{(x^2+1)(x^2-1)}$ $\qquad \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2-1} =$

$$\frac{(Ax+B)(x^2-1) + (Cx+D)(x^2+1)}{(x^2+1)(x^2-1)} = \frac{1}{(x^2+1)(x^2-1)}$$

$Ax^3 + Bx^2 - Ax - B + Cx^2 + Dx + Cx + D = 1$

$A(x^3-x) + B(x^2-1) + C(x^2+1) + D(x+1) = 1$

Admitindo $x=-1$

$C(1+1) = 1 \quad \rightarrow \quad C = 1$

$A(x^3-x) + B(x^2-1) + D(x+1) = x^2$

Admitindo $x=1$

$D(1+1) = 1 \quad \rightarrow \quad D = 1/2$

$A(x^3-x)+B(x^2-1)=x^2-\frac{1}{2}x-\frac{1}{2}$

Admitindo $x=2$ Admitindo $x=-2$

$4A+B=5/2$ $-6A+3B=9/2$

$\begin{cases} 8A+2B=5 \\ -12A+6B=9 \end{cases}$ $\begin{cases} 8A+2B=5 \\ 88B=132 \end{cases}$ $B=3/2$ $A=1/4$

b) $x = \frac{y^2}{2}$ com $0 \leq y \leq 1$

$f(y) = \frac{y^2}{2}$ $\quad f(0) = 0$

$y^2 = 2x$ $\quad f(1) = 1/2$

$y = \sqrt{2x}$ $\quad$ Intervalo $0 \leq x \leq 1/2$

$y' = \frac{\cancel{2}}{2\sqrt{2x}} = \frac{1}{\sqrt{2x}}$ $\quad (y')^2 = \frac{1}{2x}$ $\quad (y')^2 + 1 = \frac{1+2x}{2x}$

$\sqrt{\frac{1+2x}{2x}} = \frac{\sqrt{1+2x}}{\sqrt{2x}}$ $\quad f(x) . \sqrt{1+(f'(x))^2} = \sqrt{2x} . \frac{\sqrt{1+2x}}{\sqrt{2x}}$

$S_x = 2\pi \int_0^{1/2} \sqrt{1+2x}\, dx \sim 2\pi \int_0^{1/2} \sqrt{t}\, dt = 2\pi . \frac{2}{3} t^{\frac{3}{2}} =$

$1 + 2x = t$ $\quad \frac{4\pi}{3} (1+2x)^{\frac{3}{2}} \Big|_0^{1/2} =$

$2dx = dt$

$dx = \frac{1}{2} dt$ $\quad \frac{4\pi}{3} (2\sqrt{2} - 1)$ u.a

c) $y = x^3$ $\quad 0 \leq x \leq 2$

$y' = 3x^2$ $\quad (y')^2 = 9x^4$ $\quad (y')^2 + 1 = 9x^4 + 1$ $\quad \sqrt{(y')^2+1} = \sqrt{9x^4+1}$

$S_x = 2\pi \int_0^2 x^3 . \sqrt{9x^4+1}\, dx \sim 2\pi \int_0^2 \sqrt{t}\, \frac{1}{36} dt =$

$9x^4 + 1 = t$ $\quad = \frac{\pi}{18} . \frac{2}{3} (9x^4+1)^{\frac{3}{2}} \Big|_0^2 =$

$36x^3 dx = dt$

$x^3 dx = \frac{1}{36} dt$ $\quad = \frac{\pi}{27} (145 . \sqrt{145} - 1)$ u.a

/ /

d) $y^2 = 12x$ $\quad 0 \leq x \leq 3$ $\quad S_x = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2}\, dx$

$y = \sqrt{12x}$

$y' = \frac{12}{2\sqrt{12x}} = \frac{6}{\sqrt{12x}}$ $\quad (y')^2 = \frac{36}{12x} = \frac{3}{x}$ $\quad (y')^2 + 1 = \frac{3}{x} + 1 = \frac{3+x}{x}$

$S_x = 2\pi \int_0^3 \sqrt{12}\, \sqrt{x} \cdot \frac{\sqrt{3+x}}{\sqrt{x}}\, dx \sim S_x = 2\sqrt{12}\,\pi \int_0^3 \sqrt{3+x}\, dx$

$3 + x = t$ $\quad \therefore S_x = 2\sqrt{12}\,\pi \int_0^3 \sqrt{t}\, dt = 2\sqrt{12}\,\pi \left| \frac{2}{3}(3+x)^{\frac{3}{2}} \right|_0^3 =$

$dx = dt$

$2\sqrt{12}\,\pi\Big(-$

e) $f(x) = \frac{x^3}{3}$ $\quad 1 \leq x \leq 2$ em torno do eixo x

$f'(x) = \frac{1}{3} \cdot 3x^2 = x^2$ $\quad (f'(x))^2 = x^4$ $\quad (f'(x))^2 + 1 = x^4 + 1$

$S_x = 2\pi \int_1^2 \frac{x^3}{3} \cdot \sqrt{x^4+1}\, dx \sim S_x = \frac{2\pi}{3} \int \sqrt{t} \cdot \frac{1}{4}\, dt =$

$x^4 + 1 = t$ $\quad = \frac{\pi}{6} \cdot \left| \frac{2}{3} (x^4+1)^{\frac{3}{2}} \right|_1^2 = \frac{\pi}{9}\left(17\sqrt{17} - 2\sqrt{2}\right)$ u.a

$4x^3\, dx = dt$

1.) $y = \cos(x) \quad 0 \leq x \leq \frac{\pi}{2}$ em torno do eixo x

$y' = -\operatorname{sen}(x) \quad (y')^2 = \operatorname{sen}^2(x) \quad (y')^2 + 1 = \operatorname{sen}^2(x) + 1 =$

$$S_x = 2\pi \int_0^{\frac{\pi}{2}} \cos(x) \sqrt{\operatorname{sen}^2(x) + 1}\, dx$$

$\operatorname{sen}(x) = t$ $\qquad S_x = 2\pi \int_0^{\frac{\pi}{2}} \sqrt{t^2+1}\, dt =$

$\cos(x)\, dx = dt$

$$= \frac{t}{2}\sqrt{t^2+1} + \frac{1}{2} \ln \left| t + \sqrt{t^2+1} \right| =$$

$$= 2\pi \left| \frac{\operatorname{sen}(x)}{2} \sqrt{\operatorname{sen}^2(x)+1} + \frac{1}{2} \ln \left| \operatorname{sen}(x) + \sqrt{\operatorname{sen}^2(x)+1} \right| \right|_0^{\frac{\pi}{2}} =$$

$$= 2\pi \left(\frac{1}{2}\sqrt{2} + \frac{1}{2} \ln|1+\sqrt{2}| \right) = \pi \left(\sqrt{2} + \ln|1+\sqrt{2}| \right) \text{ u.a}$$

g) $y = 4x + 3 \quad 1 \leq x \leq 2$ em torno do eixo x

$y' = 4 \quad (y')^2 = 16 \quad (y')^2 + 1 = 17$

$$S_x = 2\pi \int_1^2 (4x+3) \cdot \sqrt{17}\, dx = 2\sqrt{17}\,\pi \int_1^2 4x + 3\, dx =$$

$$2\sqrt{17}\,\pi \cdot \left| 2x^2 + 3x \right|_1^2 = 2\sqrt{17}\,\pi \left| 14 - 5 \right| = 18\sqrt{17}\,\pi \text{ u.a}$$

h) $y = \sqrt{9-x^2}$ $\quad -2 \leq x \leq 2$ em torno do eixo x

$$(y') = \frac{-2x}{2\sqrt{9-x^2}} = \frac{-x}{\sqrt{9-x^2}} \qquad (y')^2 = \frac{x^2}{9-x^2}$$

$$(y')^2 + 1 = \frac{x^2}{9-x^2} + 1 = \frac{9}{9-x^2} = \left(\frac{3}{\sqrt{9-x^2}}\right)^2$$

$$S_x = 2\pi \int_{-2}^{0} \cancel{\sqrt{9-x^2}} \cdot \frac{3}{\cancel{\sqrt{9-x^2}}}\, dx = 2\pi . 3x = 6\pi x \Big|_{-2}^{0} + 6\pi x \Big|_{0}^{2}$$

$$= -(-12\pi) + 12\pi = 24\pi \text{ u.a}$$

i) $f(x) = 2\sqrt{x}$ $\quad x \in [3,8]$ em torno do eixo x

$$f'(x) = \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}} \qquad (f'(x))^2 = \frac{1}{x} \qquad (f'(x))^2 + 1 = \frac{1}{x} + 1 = \frac{1+x}{x}$$

$$S_x = 2\pi \int_3^8 2\cancel{\sqrt{x}} \cdot \frac{\sqrt{1+x}}{\cancel{\sqrt{x}}}\, dx = 4\pi \int_3^8 \sqrt{1+x}\, dx =$$

$$4\pi \left| \frac{2}{3}(1+x)^{\frac{3}{2}} \right|_3^8 = 4\pi \left(18 - \frac{16}{3}\right) = \frac{152}{3}\pi \text{ u.a}$$

j) $f(x) = \sqrt{2x+4}$ $\quad x \in [2,10]$ em torno do eixo x

$$f'(x) = \frac{2}{2\sqrt{2x+4}} = \frac{1}{\sqrt{2x+4}} \qquad (f'(x))^2 = \frac{1}{2x+4} \qquad (f'(x))^2 + 1 = \frac{2x+5}{2x+4}$$

$$S_x = 2\pi \int_2^{10} \cancel{\sqrt{2x+4}} \cdot \frac{\sqrt{2x+5}}{\cancel{\sqrt{2x+4}}}\, dx \sim S_x = 2\pi . \frac{1}{2} \int \sqrt{t}\, dt =$$

$2x+5 = t$

$$= \pi \left| \frac{2}{3}(2x+5)^{\frac{3}{2}} \right|_2^{10} = \pi \left(\frac{250}{3} - 18 \right) = \frac{196\pi}{3} \text{ u.a}$$

$2dx = dt$

$dx = \frac{1}{2} dt$

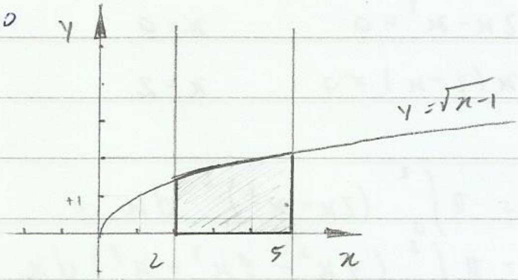
Calcule o volume do sólido gerado pela rotação da curva

a) (em torno de x) $y = \sqrt{x-1}$ $x=2$ $x=5$ $y=0$

$x = y^2 + 1$

$y^2 + 1 = 0$

$y = \pm 1$



$$V_x = \pi \int_2^5 x-1 \, dx = \pi \left| \frac{x^2}{2} - x \right|_2^5 =$$

$$\pi \left(\frac{25}{2} - 5 - \frac{4}{2} + 2 \right) = \frac{15}{2} \pi \ u.v$$

b) em torno do eixo x, a região entre $y = \frac{x^2}{4}$ $y = \frac{x}{2}$

$$\frac{x^2}{4} = \frac{x}{2} \Rightarrow 2x^2 = 4x$$
$$x^2 = 2x$$

/ $x^2 - 2x = 0$

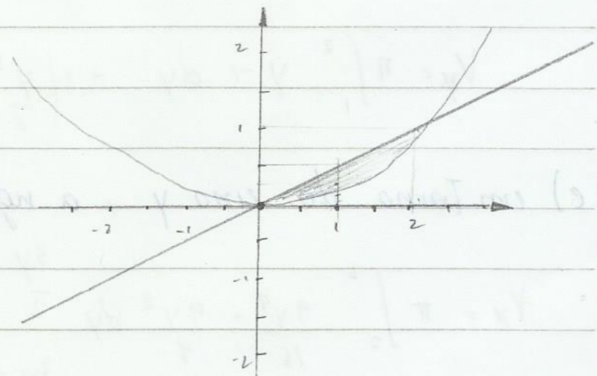
$x(x-2) = 0$

$x = 0$ $x = 2$

$$V_x = \pi \int_0^2 \left(\frac{x}{2}\right)^2 - \left(\frac{x^2}{4}\right)^2 dx$$

$$= \pi \int_0^2 \frac{x^2}{4} - \frac{x^4}{16} \, dx =$$

$$= \pi \left| \frac{x^3}{12} - \frac{x^5}{80} \right|_0^2 =$$



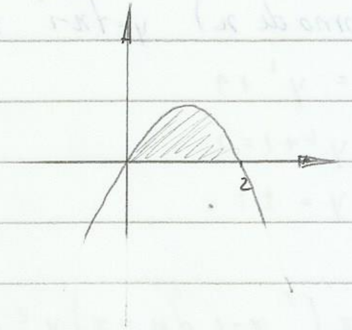
$$= \pi \left| \frac{8}{12} - \frac{32}{80} \right| = \pi \left| \frac{2}{3} - \frac{2}{5} \right| =$$

$$\pi \left| \frac{10-6}{15} \right| = \frac{4}{15} \pi \ u.v$$

c) em torno do eixo x, a região limitada por $y = 2x - x^2$ $y = 0$

$2x - x^2 = 0$ $\qquad x = 0$

$x(2-x) = 0$ $\qquad x = 2$



$$V_x = \pi \int_0^2 (2x - x^2)^2 \, dx =$$

$$= \pi \int_0^2 (4x^2 - 4x^3 + x^4) \, dx =$$

$$= \pi \left(\frac{4}{3}x^3 - x^4 + \frac{1}{5}x^5 \right|_0^2 = \pi \left(\frac{32}{3} - 16 + \frac{32}{5} \right) = \pi \left(\frac{160 - 240 + 96}{15} \right) =$$

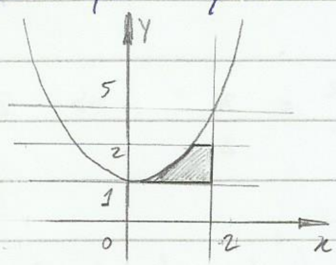
$$= \frac{16}{15} \pi \ u.v$$

d) em torno do eixo y, a região limitada por $y = x^2 + 1$ $\quad 0 \leq x \leq 2$

$1 \leq y \leq 2$ $\qquad x = \sqrt{y-1}$

$x^2 + 1 = 0$

$V = \left(\frac{-b}{2a}, \frac{-\Delta}{4a} \right)$



$$V_y = \pi \int_1^2 y - 1 \, dy = \pi \left(\frac{y^2}{2} - y \right|_1^2 = \pi \left| \frac{2^2}{2} - 2 - \frac{1}{2} + 1 \right| = \frac{\pi}{2}$$

e) em Torno do eixo y, a região limitada por $x = \frac{3y}{2}$ $\quad x = \frac{3y^2}{4}$

$$V_x = \pi \int_0^2 \frac{9y^4}{16} - \frac{9y^2}{4} \, dy$$

$\frac{3y}{2} = \frac{3y^2}{4}$

$3y = \frac{3y^2}{2} \Rightarrow 6y = 3y^2$

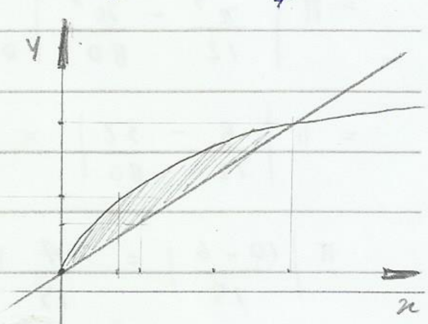
$2y = y^2$

$y(2-y) = 0$

$y = 0 \quad y = 2$

$$= \pi \left(\frac{9}{80} y^5 - \frac{9}{12} y^3 \right|_0^2$$

$$= \frac{12}{5} \pi \ u.v$$



f) em torno do eixo x, da região limitada pela curva $y=\sqrt{x(e^x+1)}$ e o eixo x, com $0 \le x \le 1$

$$V_x = \pi \int_0^1 x(e^x+1)\,dx$$

$$= \pi \int_0^1 xe^x + x\,dx$$

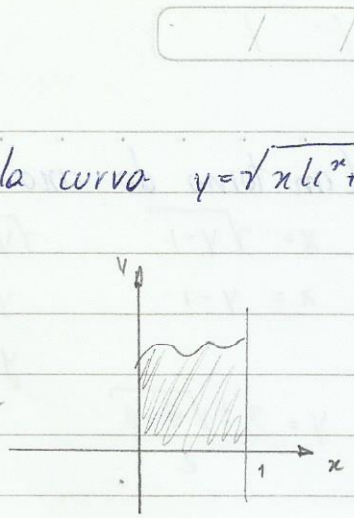
$\int xe^x\,dx$

$u = x \quad du = dx$

$dv = e^x \quad v = e^x$

$\int xe^x\,dx = xe^x - e^x =$

$e^x(x-1)$

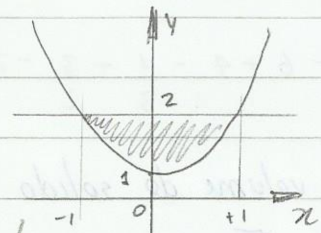


$$= \pi \left| e^x(x-1) + \frac{x^2}{2} \right|_0^1 \Rightarrow \pi \left| \frac{1}{2} - (-1) \right| = \frac{3}{2}\pi \text{ u.v}$$

g) em torno do eixo x, da região limitada pela curva $y = x^2+1$ $y=2$

$x^2+1=0$ $\qquad 2 = x^2+1$

$\Delta = (0, 1)$ $\qquad x = \pm 1$



$$V_{xI} = \pi \int_{-1}^{0} (x^2+1)^2\,dx = \pi \int_{-1}^{0} x^4+2x^2+1\,dx =$$

$$= \pi \left| \frac{1}{5}x^5 + \frac{2}{3}x^3 + x \right|_{-1}^{0} = \pi \left| -\left(-\frac{1}{5} - \frac{2}{3} - 1\right) \right| = \left(\frac{3+10+15}{15} \right)\pi = \frac{28}{15}\pi$$

$$V_{xII} = \pi \int_0^1 x^4+2x^2+1\,dx = \pi \left| \frac{1}{5}x^5 + \frac{2}{3}x^3 + x \right|_0^1 = \frac{28}{15}\pi$$

$$V = \frac{28}{15}\pi + \frac{28\pi}{15} = \frac{56}{15}\pi$$

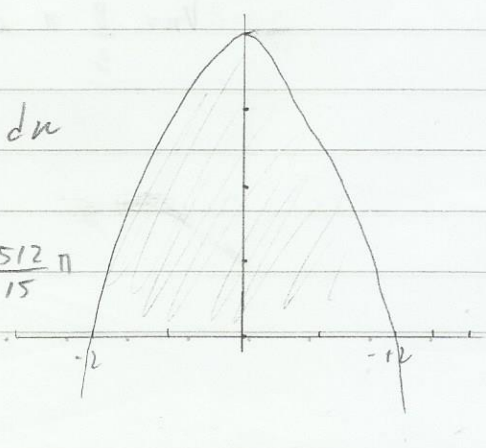
h) em torno do eixo x $\quad f(x) = 4 - x^2 \quad x \in [-2, 2]$

$4 - x^2 = 0$ $\qquad V_x = \pi \int_{-2}^{0} (4-x^2)\,dx =$

$x = \pm 2$

$$V_x = \pi \int_{-2}^{0} (16 - 8x^2 + x^4)\,dx$$

$$= \pi \left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right|_{-2}^{0} = \frac{256}{15} \cdot 2 = \frac{512}{15}\pi$$



/ /

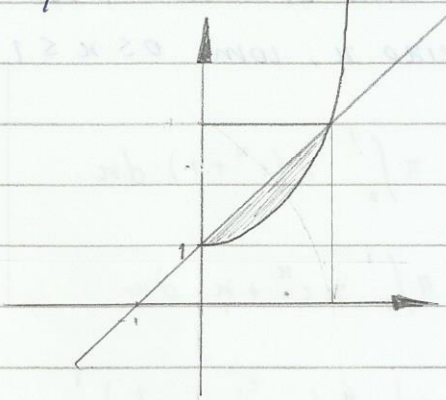
i) (em torno do eixo y) $y = x^2+1$ $y = x+1$

$x = \sqrt{y-1}$ $\sqrt{y-1} = y-1$

$x = y-1$ $y-1 = y^2-2y+1$

$y^2-3y+2=0$

$y = \dfrac{3 \pm \sqrt{9-8}}{2}$



$y_1 = 2$ $y_2 = 1$ $[1,2]$

$V_y = \pi\int_1^2 (y-1)^2 - (y-1)\, dy \sim \pi\int_1^2 y^2-2y+1-y+1\, dy =$

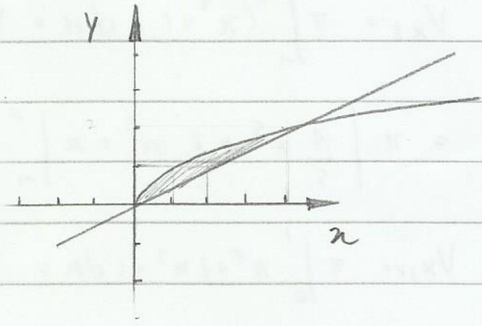
$V_y = \pi\int_1^2 y^2-3y+2\, dy = \pi\left|\frac{1}{3}y^3 - \frac{3}{2}y^2 + 2y\right|_1^2 =$

$= \pi\left|\frac{8}{3} - 6 + 4 - \frac{1}{3} + \frac{3}{2} - 2\right| = \frac{\pi}{6}$ u.v

j) Encontrar volume do sólido obtido pela rotação em torno do eixo x

$y = \sqrt{x}$ $y = \frac{x}{2}$

$\sqrt{x} = \frac{x}{2} \Rightarrow x = \frac{x^2}{2}$



$2x - x^2 = 0$

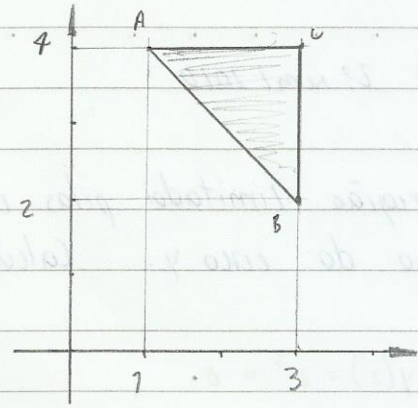
$x(2-x) = 0$ $V_x = \pi\int_0^2 (\sqrt{x})^2 - \left(\frac{x}{2}\right)^2 dx =$

$x=0$ $x=2$

$V_x = \pi\int_0^2 x - \frac{x^2}{2}\, dx = \pi\left|\frac{x^2}{2} - \frac{x^3}{6}\right|_0^2 = \pi\left|\frac{2^2}{2} - \frac{2^3}{6}\right| =$

$V_x = \frac{8}{3}\pi$ u.v

i) $A(1,4)\quad B(3,2)\quad C(3,4)$



$\therefore [1,3] \qquad f(x) = 4$

$\qquad\qquad\quad f(x) = -x+5$

$V_x = \pi \int_1^3 4^2 - (-x+5)^2 \, dx =$

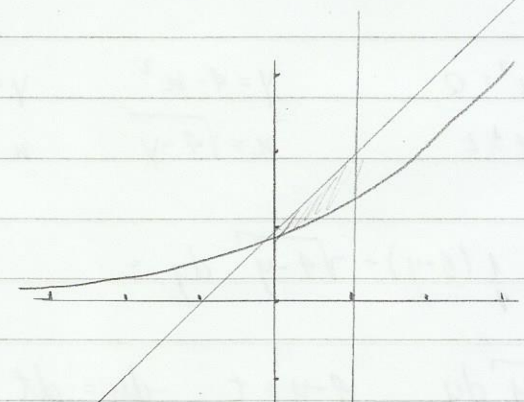
$= \pi \int_1^3 16 - (x^2 - 10x + 25) \, dx \qquad (y - y_0) = a(x - x_0)$

$= \pi \int_1^3 -x^2 + 10x - 9 \, dx \qquad y - 4 = -1(x-1)$

$y = -x + 5$

$= \pi \left(-\frac{1}{3}x^3 + 5x^2 - 9x \right|_1^3 = \pi \left(-9 + 45 - 27 + \frac{1}{3} - 5 + 9 \right) = \frac{40}{3} \pi \text{ u.v}$

m) $y = e^x \qquad y = x+1 \qquad x = 1$



$V_x = \pi \int_0^1 (x+1)^2 - e^{2x} \, dx$

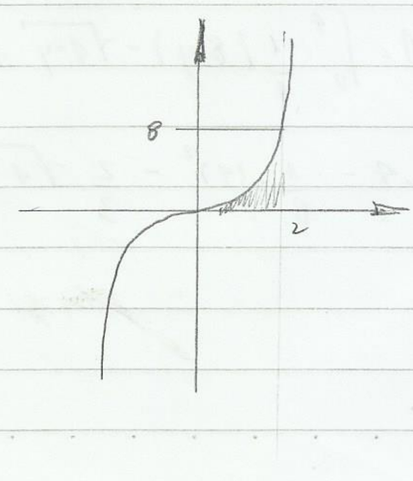
$= \pi \int_0^1 x^2 + 2x + 1 - e^{2x} \, dx$

$= \pi \left| \frac{1}{3}x^3 + x^2 + x - \frac{1}{2}e^{2x} \right|_0^1$

$= \pi \left| \frac{1}{3} + 1 + 1 - \frac{e^2}{2} + \frac{1}{2} \right| = \left(\frac{17}{6} - \frac{e^2}{2} \right) \pi = \frac{17 - 3e^2}{6} \pi \text{ u.v}$

n) $y = x^3 \qquad y = 0 \qquad x = 2$

$x = \sqrt[3]{y} \qquad y(2) = 2^3 = 8$



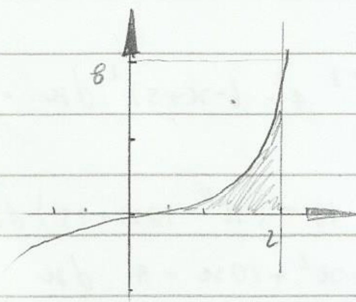
$V_y = \pi \int_0^8 4 - y^{\frac{2}{3}} \, dy = \pi \left| 4y - \frac{3}{5} y^{\frac{5}{3}} \right|_0^8 =$

$= \frac{64}{5} \pi \text{ u.v}$

/ /

→ P: 2º sem/ 2010

A região limitada pelas curvas $y = x^3$ $y = 0$ $x = 2$ gira em torno do eixo y. Calcular o volume do sólido

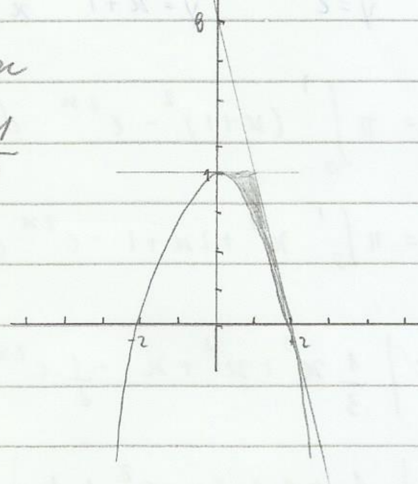


$y(2) = 2^3 = 8$

$V_y = \pi \int_0^8 4 - y^{\frac{2}{3}} \, dy = \pi \left(4y - \frac{3}{5} y^{\frac{5}{3}} \right) \Big|_0^8$

$= \pi \left(4 \cdot 8 - \frac{3}{5} 8^{\frac{5}{3}} \right) = \pi \left(32 - \frac{3}{5} \cdot 32 \right) = \frac{64}{5} \pi$ u.v

→ Calcular a área da região limitada pelas curvas $y = 4 - x^2$, $y = 8 - 4x$, $y = 4$



$4 - x^2 = 0$ $\qquad y = 4 - x^2$ $\qquad y = 8 - 4x$

$x = \pm 2$ $\qquad x = \sqrt{4 - y}$ $\qquad x = \frac{8 - y}{4}$

$A = \int_0^4 \frac{1}{4}(8 - y) - \sqrt{4 - y} \, dy =$

$\int \sqrt{4 - y} \, dy \qquad 4 - y = t \qquad -dy = dt$

$dy = -dt$

$-\int \sqrt{t} \, dt = -\frac{2}{3} (4 - y)^{\frac{3}{2}}$

→ $A = \int_0^4 \frac{1}{4}(8 - y) - \sqrt{4 - y} \, dy = \frac{1}{4}\left(8y - \frac{1}{2} y^2 \right) + \frac{2}{3} (4 - y)^{\frac{3}{2}} \Big|_0^4 =$

$= 2 \cdot 4 - \frac{1}{8} \cdot (4)^2 - \frac{2}{3} \sqrt{4^3} = \frac{2}{3}$ u.a

→ Calcular o comprimento de arco da curva

$$y = \frac{2}{9}\sqrt{27x^3} \quad \text{com} \quad x \in [0,1]$$

$$L = \int_a^b \sqrt{1 + (f'(x))^2}$$

$$f(x) = \frac{2}{9}\sqrt{27x^3} = \frac{2}{9}\sqrt{(3x)^3}$$

$$f'(x) = \frac{\cancel{2}}{\cancel{9}} \cdot \frac{3}{\cancel{2}}(3x)^{1/2} \cdot \cancel{3} = \sqrt{3x} \quad \text{ou} \ (3x)^{1/2}$$

$$(f'(x))^2 = 3x \qquad (f'(x))^2 + 1 = 3x + 1$$

$$L = \int_0^1 \sqrt{3x+1}\,dx \qquad 3x + 1 = t \qquad \int \sqrt{t} \cdot \frac{1}{3}\,dt = \frac{1}{3} \cdot \frac{2}{3} \cdot t^{3/2} + C$$

$$3dx = dt$$

$$= \frac{2}{9}(3x+1)^{3/2}\Big|_0^1 \qquad dx = \frac{1}{3}dt$$

$$= \frac{2}{9}\sqrt{4^3} - \frac{2}{9} = \frac{16}{9} - \frac{2}{9} = \frac{14}{9} \ \text{u.c}$$

ou

$$\int_0^1 \sqrt{3x+1}\,dx \qquad 3x+1 = t^2$$

$$3dx = 2t\,dt$$

$$\int \sqrt{3x+1}\,dx \sim \int t \cdot \frac{2}{3} t^2\,dt \sim \qquad dx = \frac{2}{3}t\,dt$$

$$\frac{2}{3}\int t^2\,dt = \frac{2}{3} \cdot \frac{t^3}{3} = \frac{2}{9}t^2 \cdot t = \frac{2}{9}(3x+1)\sqrt{3x+1} =$$

$$\frac{2}{9}(3x+1)^{\frac{3}{2}}\Big|_0^1 = \frac{14}{9} \ \text{u.c}$$

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2ª Questão: Calcule a área da superfície gerada pela rotação em torno do eixo Ox, do gráfico $f(x) = \frac{e^{x} + e^{-x}}{2}$ com $-1 \leq x \leq 1$

$f'(x) = \frac{1}{2}(e^{x} - e^{-x}) \qquad (f'(x))^2 = \frac{1}{4}(e^{2x} + e^{-2x} - 2)$

$(f'(x))^2 + 1 = \frac{e^{2x} + e^{-2x}}{4} + \frac{1}{2} = \frac{1}{4}(e^{2x} + e^{-2x} + 2) = \left(\frac{e^{x} + e^{-x}}{2}\right)^2$

$Sx = \cancel{2}\pi \int_{-1}^{0} \frac{e^{x} + e^{-x}}{\cancel{2}} \cdot \left(\frac{e^{x} + e^{-x}}{2}\right) dx =$

$Sx = \pi \int_{-1}^{0} e^{2x} + 1 + 1 + e^{-2x}\, dx = \pi \int e^{2x} + e^{-2x} + 2\, dx$

$2x = t \qquad \int e^{2x} dx = \int e^{t} \frac{1}{2} dt = \frac{1}{2} e^{t} = \frac{1}{2} e^{2x}$

$dx = \frac{1}{2} dt$

$-2x = s \qquad \int e^{-2x} dx = \int e^{s} \frac{-1}{2} ds = \frac{-1}{2} e^{-2x}$

$dx = \frac{-1}{2} ds$

$Sx = \pi \left(\frac{1}{2} e^{2x} - \frac{1}{2} e^{-2x} + 2x \right) \Big|_{-1}^{0} = \pi \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} - \frac{1}{2e^2} + \frac{e^2}{2} + 2 \right)$

$= \pi \left(\frac{e^2}{2} + 2 - \frac{e^{-2}}{2} \right) \cdot 2 = \pi \left(e^2 + 4 - e^{-2} \right)$ u.a

- Calcule o comprimento da curva dada em forma paramétrica

$x = 3t$ e $y = 2t^{3/2}$ $\quad 0 \le t \le 1$

$\frac{dx}{dt} = 3 \quad \frac{dy}{dt} = \cancel{2}\cdot\frac{3}{\cancel{2}}t^{1/2} = 3\sqrt{t}$

$$L = \int_0^1 \sqrt{9 + 9t}\,dt \sim \int_0^1 \sqrt{3^2(1+t)}\,dt =$$

$$= 3\int_0^1 \sqrt{1+t}\,dt \qquad 1+t = s$$

$$dt = ds$$

$$= 3\int_0^1 \sqrt{s}\,ds = 3\cdot\left(\frac{2}{3}s^{3/2}\right) = 3\left(\frac{2}{3}\cdot(1+t)^{3/2}\right)\Bigg|_0^1 =$$

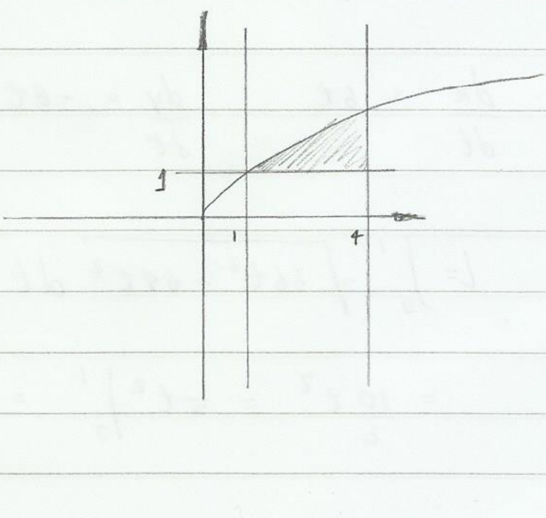
$$3\left(\frac{2}{3}\cdot 2\sqrt{2} - \frac{2}{3}\right) = \cancel{3}\left(\frac{4\sqrt{2}-2}{\cancel{3}}\right) = 4\sqrt{2} - 2 \text{ u.c}$$

- Calcule o volume do sólido gerado pela rotação, em torno do eixo Ox, do conjunto de todos os pares (x,y) tais que $1 \le x \le 4$ e $1 \le y \le \sqrt{x}$

$$V_x = \pi\int_1^4 (\sqrt{x})^2 - 1\,dx =$$

$$= \pi\int_1^4 x - 1\,dx = \pi\left(\frac{x^2}{2} - x\right)\Bigg|_1^4$$

$$= \pi\left(\frac{16}{2} - 4 - \frac{1}{2} + 1\right) = \frac{9}{2}\pi \text{ u.v}$$



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- Obter a área da superfície gerada pela rotação em torno do eixo x, do gráfico de $f(x) = 2\sqrt{x}$; $x \in [3, 8]$

$f'(x) = \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$ $\quad f'(x)^2 = \frac{1}{x}$ $\quad (f'(x)^2) + 1 = \frac{1}{x} + 1 = \frac{1+x}{x}$

$S_x = 2\pi \int_3^8 2\sqrt{x} \cdot \frac{\sqrt{1+x}}{\sqrt{x}} \, dx = 4\pi \int \sqrt{1+x} \, dx$

$1+x = t$
$dx = dt$

$S_x = 4\pi \int \sqrt{t} \, dt = 4\pi \cdot \left. \frac{2}{3} \cdot (1+x)^{3/2} \right|_3^8$

$= 4\pi \left(\frac{2}{3} \cdot 27 - \frac{2}{3} \cdot 8 \right) = \frac{152}{3} \pi$ u.a

- Obter o comprimento de arco da curva dada por

$\begin{cases} x(t) = 4 + 3t^2 \\ y(t) = 6 - 4t^2 \end{cases}$, $t \in [0,1]$

$\frac{dx}{dt} = 6t$ $\quad \frac{dy}{dt} = -8t$

$L = \int_0^1 \sqrt{36t^2 + 64t^2} \, dt = \int_0^1 \sqrt{100t^2} \, dt = \int_0^1 10t \, dt =$

$= \frac{10}{2} t^2 = \left. 5t^2 \right|_0^1 = 5$ u.c

→ Obter o volume do sólido de rotação em torno do eixo x dado por: $f(x) = 4 - x^2$; $x \in [-2, 2]$

$$V_x = \pi \int_{-2}^{2} (4-x^2)^2 \, dx = \pi \int_{-2}^{2} 16 - 8x^2 + x^4 \, dx =$$

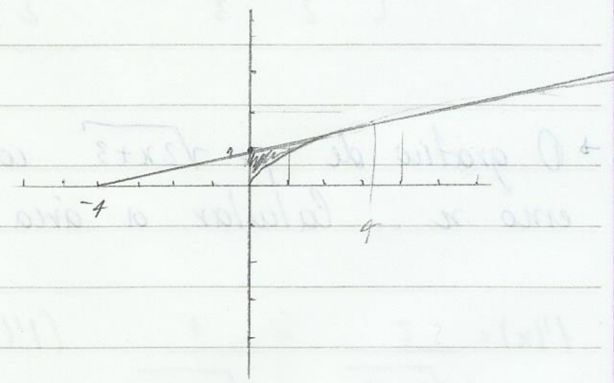
$$= \pi \left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right) \Big|_{-2}^{2}$$

$$= \pi \left[\left(32 - \frac{64}{3} + \frac{32}{5} \right) - \left(-32 + \frac{64}{3} - \frac{32}{5} \right) \right]$$

$$= \pi \left(32 - \frac{64}{3} + \frac{32}{5} + 32 - \frac{64}{3} + \frac{32}{5} \right) = \frac{512}{15} \pi \text{ u.v}$$

→ Calcular a área da região do plano Oxy limitada pelas curvas:

$y = \sqrt{x}$ $\quad y = \frac{x}{4} + 1 \quad x = 0$



$$\sqrt{x} = \frac{x}{4} + 1 \qquad x = \frac{x^2}{16} + \frac{x}{2} + 1$$

$$\frac{x^2}{16} - \frac{x}{2} + 1 = 0$$

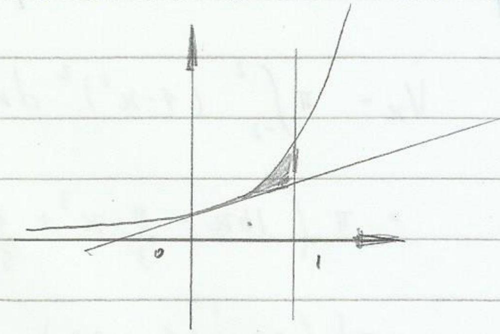
$$x^2 - 8x + 16 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 64}}{2} \rightarrow x = 4$$

$$\int_0^4 \frac{x}{4} + 1 - \sqrt{x} \, dx = \frac{1}{8}x^2 + x - \frac{2}{3}x^{3/2} \Big|_0^4$$

$$= 2 + 4 - \frac{16}{3} = \frac{2}{3} \text{ u.a}$$

→ A região limitada pelas curvas $y = e^x$, $y = x+1$ e $x = 1$ gira em torno do eixo x. Calcular o volume do sólido resultante



$$V_x = \pi \int_0^1 e^{2x} - (x+1)^2 \, dx$$

$$= \pi \int_0^1 e^{2x} - x^2 - 2x - 1 \, dx$$

$$2x = t \qquad \int e^{2x} dx = \int e^t \cdot \frac{1}{2} dt =$$

$$dx = \frac{1}{2} dt$$

$$= \frac{1}{2} e^{2x} + C$$

$$V_x = \pi \cdot \left(\frac{1}{2} e^{2x} - \frac{1}{3} x^3 - x^2 - x \right) \Big|_0^1$$

$$= \pi \left[\frac{e^2}{2} - \frac{1}{3} - 1 - 1 - \frac{1}{2} \right] = \left[\frac{e^2}{2} - \frac{17}{6} \right] \pi = \frac{3e^2 - 17}{6} \pi \text{ u.v}$$

→ O gráfico de $y = \sqrt{2x+3}$ com $x \in [0,6]$ gira em torno do eixo x. Calcular a área da superfície de revolução resultante

$$f'(x) = \frac{2}{2\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}} \qquad (f'(x))^2 = \frac{1}{2x+3} \qquad (f'(x))^2 + 1 = \frac{2x+4}{2x+3}$$

$$S_x = 2\pi \int_0^6 \cancel{\sqrt{2x+3}} \cdot \frac{\sqrt{2x+4}}{\cancel{\sqrt{2x+3}}} \, dx$$

$$2x + 4 = t \qquad \int \sqrt{2x+4} \, dx = \frac{1}{2} \int \sqrt{t} \, dt = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} t^{3/2} + C$$

$$dx = \frac{1}{2} dt$$

$$S_x = 2\pi \left(\frac{1}{3} (2x+4)^{3/2} \right) \Big|_0^6 = 2\pi \left(\frac{64}{3} - \frac{8}{3} \right) = \frac{112}{3} \pi \text{ u.a}$$

1) $\int (x+3)^5 \, dx$ $\qquad \int t^5 \, dt = \frac{1}{6} t^6 + C$

$x+3 = t$

$dx = dt$ $\qquad \int (x+3)^5 \, dx = \frac{1}{6} (x+3)^6 + C$

2) $\int \frac{1}{x \ln(x)} \, dx$ $\qquad \int \frac{1}{t} \, dt = \ln|\ln|x|| + C$

$\ln(x) = t$

$\frac{1}{x} \, dx = dt$

3) $\int x \sqrt{5x^2+3} \, dx$ $\qquad \int \sqrt{t} \, \frac{1}{10} \, dt = \frac{1}{\underset{5}{10}} \cdot \frac{2}{3} t^{3/2} = \frac{1}{15} (5x^2+3)^{3/2} + C$

$5x^2 + 3 = t$

$10x \, dx = dt$

$x \, dx = \frac{1}{10} \, dt$

4) $\int \ln(|\cos(x)|) \tan(x) \, dx$ $\sim \int \ln|\cos(x)| \frac{\operatorname{sen}(x)}{\cos(x)} \, dx$

$\cos(x) = t$ $\qquad -\int \ln|t| \frac{1}{t} \, dt$

$-\operatorname{sen}(x) \, dx = dt$ $\qquad -\int s \, ds = \frac{-s^2}{2}$

$\operatorname{sen}(x) \, dx = -dt$ $\qquad \ln|t| = s$

$\frac{1}{t} \, dt = ds$ $\qquad = -\frac{(\ln|\cos(x)|)^2}{2} + C$

5) $\int x^{7/5} \sqrt{x} + 4x \, dx$ $\sim \int x^{19/10} + 4x \, dx$

$= \frac{10}{29} x^{29/10} + 2x^2 + C$

/ /

6) $\int x e^{x^2} dx$

$x^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{1}{2} dt$

$$\int e^t \frac{1}{2} dt = \frac{1}{2} e^t + C$$

$$= \frac{1}{2} e^{x^2} + C$$

7) $\int \sqrt{x} \ln(x)\, dx$ $\qquad x^{3/2} = x . x^{1/2}$

$u = \ln(x) \qquad dv = \sqrt{x}$

$du = \frac{1}{x} dx \qquad v = \frac{2}{3} x^{3/2}$

$$\int u\,dv = \ln x . \frac{2}{3} x^{3/2} - \int \frac{2}{3} x^{3/2} . \frac{1}{x} dx$$

$$= \frac{2}{3} \ln|x| . x^{3/2} - \frac{2}{3} \int x^{1/2} dx$$

$$= \frac{2}{3} \ln|x|\, x^{3/2} - \frac{2}{3} . \frac{2}{3} x^{3/2} + C$$

$$= \frac{2}{3} \ln|x|\, x^{3/2} - \frac{4}{9} x^{3/2} + C$$

8) $\int x \sin(5x)\, dx$

$5x = t$

$5dx = dt$

$dx = \frac{1}{5} dt$

$$\int \sin(t) \frac{1}{5} t . \frac{1}{5} dt = \frac{1}{25} \int \sin(t)\, t\, dt$$

$u = t \quad du = dt \quad dv = \sin(t) \quad v = \cos(t)$

$$\int u\,dv = t . \cos(t) - \int \cos(t)\, dt$$

$$= t\cos(t) - \sin(t)\, dt$$

$$\int x \sin(5x)\, dx = \frac{1}{25} . (5x \cos(5x) - \sin(5x)) + C$$

$$= \frac{x}{5} \cos(5x) - \frac{1}{25} \sin(5x) + C$$

9) $\int x e^{5x} dx$

$u = x \qquad dv = e^{5x} \qquad t = 5x \qquad dx = \frac{1}{5} dt$

$du = dx \qquad v = \int e^{5x} dx \qquad dt = 5dx$

$\int e^{5x} dx = \int e^{t} \cdot \frac{1}{5} dt = \frac{1}{5} e^{t}$

$\therefore v = \frac{1}{5} e^{5x}$

$\int u dv = x \cdot \frac{1}{5} e^{5x} - \frac{1}{5} \int e^{5x} dx$

$= \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} + C$

10) $\int e^{e^x} e^x dx \qquad \int e^t dt = e^t + C$

$e^x = t \qquad = e^{e^x} + C$

$e^x dx = dt$

11) $\int \frac{\ln(\ln(x))}{x \ln(x)} dx \sim \int \frac{\ln(t)}{t} dt \sim \int s ds = \frac{s^2}{2}$

$\ln(x) = t \qquad \ln(t) = s$

$\frac{1}{x} dx = dt \qquad \frac{1}{t} dt = ds$

$\frac{s^2}{2} = \frac{(\ln(t))^2}{2} = \frac{\ln|\ln(x))|^2}{2} + C$

12) $\int \frac{x dx}{x^2 + 7x + 12} \qquad x^2 + 7x + 12$

$x = \frac{-7 \pm \sqrt{49 - 48}}{2}$

$x_1 = \frac{-7+1}{2} = \frac{-6}{2} = -3$

$x_2 = \frac{-7-1}{2} = \frac{-8}{2} = -4$

$x^2 + 7x + 12 = (x+3)(x+4)$

$\int \frac{x}{x^2+7x+12} dx = \frac{A}{(x+3)} + \frac{B}{(x+4)} = \frac{A(x+4) + B(x+3)}{(x+3)(x+4)}$

Admitindo $x = -4$

$B(-4+3) = -4 \qquad B(-1) = -4 \qquad B = 4$

/ /

Admitindo $x=-3$ $\quad \int \frac{x}{x^2+7x+12}\,dx = \frac{-3}{x+3} + \frac{-4}{x+4}$

$A(-3+4)=-3$

$A=-3$

$-3\int \frac{1}{x+3}\,dx - 4\int \frac{1}{x+4}\,dx = -3\ln|x+3| - 4\ln|x+4| + C$

13) $\int x\sqrt{x+1}\,dx$ $\quad \int (t-1)\sqrt{t}\,dt \sim \int t\sqrt{t} - \sqrt{t}\,dt$

$x+1=t \to x=t-1$

$dx=dt$ $\quad \int t^{\frac{3}{2}} - t^{\frac{1}{2}}\,dt = \frac{2}{5}t^{\frac{5}{2}} - \frac{2}{3}t^{\frac{3}{2}} + C$

R: $\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$

ou

$x+1=t^2$ $\quad \int (t^2-1)t.2t\,dt = 2\int t^4 - t^2\,dt =$

$dx = 2t\,dt$

$\frac{2}{5}t^5 - \frac{2}{3}t^3 + C$

$\frac{2}{5}(x+1)^2\sqrt{x+1} - \frac{2}{3}(x+1)\sqrt{x+1} + C$

$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$

14-) $\int \frac{dx}{x^2+10x+34}$ $\quad x = t - \frac{b}{2a} \quad t = x + \frac{b}{2a} \quad = (x+5)^2$

$x^2+10x+34 = (x+5)^2 + 9 = (x+5)^2 + 3^2$

$\int \frac{dx}{(x+5)^2+3^2} = \frac{1}{3}\operatorname{arctg}\left(\frac{x+5}{3}\right) + C$

15) $\int \frac{2x+3}{x^3+x^2-6x}\,dx$ $\quad x^3+x^2-6x = x(x^2+x-6)$

$x^2+x-6=0$

$x = \frac{-1 \pm \sqrt{1+24}}{2}$ $\quad x_1 = \frac{-1+5}{2} = 2$ $\quad x_2 = \frac{-1-5}{2} = -3$

$x^2+x-6 = (x+3)(x-2)$

$$\frac{2x+3}{x^3+x^2-6x} = \frac{A}{(x+3)} + \frac{B}{(x-2)} + \frac{C}{x}$$

$$\frac{Ax(x-2)+Bx(x+3)+C(x+3)(x-2)}{x^3+x^2-6x} = \frac{2x+3}{x^3+x^2-6x}$$

Admitindo $x=2$ $\quad$ Admitindo $x=-3$ $\quad$ Admitindo $x=0$

$10B = 7$ $\quad$ $15A = -3$ $\quad$ $-6C = 3$

$B = 7/10$ $\quad$ $A = \frac{-3}{15} = -\frac{1}{5}$ $\quad$ $C = -1/2$

$$I = -\frac{1}{5}\int \frac{1}{x+3}\,dx + \frac{7}{10}\int \frac{1}{x-2}\,dx - \frac{1}{2}\int \frac{1}{x}\,dx$$

$$= -\frac{1}{5}\ln|x+3| + \frac{7}{10}\ln|x-2| - \frac{1}{2}\ln|x| + C$$

16) $\int \frac{\ln(x)}{\sqrt{x}}\,dx$

$u = \ln(x)$ $\quad dv = \frac{1}{\sqrt{x}}$ $\quad v = 2\sqrt{x}$

$du = \frac{1}{x}\,dx$

$$\int u\,dv = \ln x \cdot 2\sqrt{x} - 2\int \sqrt{x}\cdot\frac{1}{x}\,dx$$

$$= 2\sqrt{x}\ln x - 2\int x^{-\frac{1}{2}}\,dx$$

$$= 2\sqrt{x}\ln|x| - 2\cdot 2\, x^{\frac{1}{2}} + C$$

$$= 2\sqrt{x}\ln|x| - 4x^{1/2} + C$$

17-) $\int \frac{x}{\sqrt{x^2+x+1}}\,dx$

$x = u \qquad dv = \frac{1}{\sqrt{x^2+x+1}} \qquad v = \int \frac{1}{\sqrt{x^2+x+1}}\,dx$

$dx = du$

$x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4} = \left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$\int \frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}} = \ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|$

$\int u\,dv = x\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| - \int \ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|dx$

?

18-) $\int \frac{2x-1}{(x-1)(x+1)}\,dx \qquad \frac{A}{x-1} + \frac{B}{x+1} = \frac{2x-1}{(x-1)(x+1)}$

$A(x+1) + B(x-1) = 2x-1$

Admitindo $x = 1$ — Admitindo $x = -1$

$2A = 1 \qquad -2B = -3$

$A = \frac{1}{2} \qquad B = \frac{3}{2}$

$\frac{1}{2}\int \frac{1}{x-1}\,dx + \frac{3}{2}\int \frac{1}{x+1}\,dx = \frac{1}{2}\ln|x-1| + \frac{3}{2}\ln|x+1| + C$

/ /

19) $\int x\sqrt{x} + x^2\, dx = \int x^{\frac{3}{2}} + x^2\, dx = \frac{2}{5} x^{5/2} + \frac{1}{3} x^3 + C$

20) $\int \frac{\ln(x)}{x^3}\, dx$

$u = \ln(x) \qquad dv = \frac{1}{x^3} \qquad v = \frac{x^{-2}}{-2}$

$du = \frac{1}{x}\, dx$

$\int u\, dv = \frac{x^{-2}}{2} \ln(x) - \frac{1}{2} \int x^{-2} \frac{1}{x}\, dx$

$= \frac{x^{-2}}{2} \ln(x) - \frac{1}{2} \int x^{-3}\, dx$

$= \frac{x^{-2}}{2} \ln(x) + \frac{1}{2} \cdot \frac{x^{-2}}{-2} = \frac{x^{-2}}{2} \ln(x) - \frac{1}{4} x^{-2} + C$

21-) $\int \frac{x^2}{x^3+2}\, dx \qquad \frac{1}{3} \int \frac{1}{t}\, dt =$

$x^3 + 2 = t \qquad \frac{1}{3} \ln|x^3+2| + C$

$3x^2\, dx = dt$

$x^2\, dx = \frac{1}{3}\, dt$

22-) $\int \operatorname{sen}^3(x)\, dx \qquad \int \operatorname{sen}^2(x) \operatorname{sen}(x)\, dx \sim \int (1-\cos^2(x)) \operatorname{sen}(x)\, dx$

$\cos(x) = t$

$\operatorname{sen}(x)\, dx = -dt \qquad \int (1-t^2) -dt = -t + \frac{1}{3} t^3 + C$

$\therefore\ -\cos(x) + \frac{1}{3} \cos^3(x) + C$

23) $\int \frac{3x^2 - x - 7}{x^2 - x - 2}\, dx$

$$\begin{array}{r|l} 3x^2 - x - 7 & x^2 - x - 2 \\ \hline -3x^2 + 3x - 1 & 3 \\ \hline 2x - 8 & \end{array}$$

$$\frac{3x^2 - x - 7}{x^2 - x - 2} = 3 + \frac{2x - 8}{x^2 - x - 2}$$

$$\int \frac{3x^2 - x - 7}{x^2 - x - 2}\, dx = \int 3\, dx + \int \frac{2x - 8}{x^2 - x - 2}\, dx$$

$x^2 - x - 2 = 0$

$x = \frac{1 \pm \sqrt{1+8}}{2}$ $\begin{cases} x_1 = 2 \\ x_2 = -1 \end{cases}$ $x^2 - x - 2 = (x-2)(x+1)$

$$\int \frac{2x - 8}{x^2 - x - 2}\, dx = \frac{A(x+1) + B(x-2)}{x^2 - x - 2} = \frac{2x - 8}{x^2 - x - 2}$$

Admitindo $x = 2$

$3A = 4$

$A = -\frac{4}{3}$

Admitindo $x = -1$

$-3B = -10$

$B = \frac{10}{3}$

$$-\frac{4}{3} \int \frac{1}{x-2}\, dx + \frac{10}{3} \int \frac{1}{x+1}\, dx + 3 \int dx =$$

$$= -\frac{4}{3} \ln|x-2| + \frac{10}{3} \ln|x+1| + 3x + C$$

24) $\displaystyle\int \frac{2x+1}{x^2+x}\,dx$ $\qquad \displaystyle\int \frac{1}{t}\,dt \quad \ln|x^2+x|+C$

$x^2+x=t$

$(2x+1)\,dx=dt$

25) $\displaystyle\int \left(9t^2+\frac{1}{\sqrt{t^3}}+te^t\right)dt$ $\qquad \displaystyle\int te^t\,dt.$

$u=t \qquad dv=e^t$

$du=dt \qquad v=e^t$

$= 3t^3-2t^{-1/2}+\displaystyle\int te^t\,dt$

$= 3t^3-2t^{-1/2}+e^t(t-1)+C$

$\displaystyle\int u\,dv = te^t-\int e^t\,dt$

$= te^t-e^t = e^t(t-1)+C$

26) $\displaystyle\int \left(x\sqrt[3]{x}+\frac{2}{\cos^2(x)}+x^2\ln(x)\right)dx$

$\displaystyle\int x^{\frac{3}{4}}\,dx + 2\int \sec^2(x)\,dx + \int x^2\ln(x)\,dx$

$= \frac{4}{7}x^{7/4}+2\,tg(x)+\displaystyle\int x^2\ln(x)\,dx$

$u=\ln(x) \qquad dv=x^2$

$du=\frac{1}{x}\,dx \qquad v=\frac{1}{3}x^3$

$\displaystyle\int u\,dv=\frac{1}{3}x^3\ln(x)-\frac{1}{3}\int x^3\frac{1}{x}\,dx$

$=\frac{1}{3}x^3\ln(x)-\frac{1}{3}\cdot\frac{x^3}{3}+C$

$\therefore\ I=\frac{4}{7}x^{7/4}+2\,tg(x)+\frac{1}{3}x^3\ln(x)-\frac{1}{9}x^3+C$

27) $\int \left(\frac{x^3 + 2x^2}{x^4} + 2xe^x \right) dx$

$\int \frac{x^3}{x^4} dx + 2\int \frac{x^2}{x^4} dx + 2\int xe^x dx$

$\ln|x| - \frac{2}{x} + 2\, e^x(x-1) + C$

28) $\int \frac{1}{\sqrt{1-4x^2}} dx \qquad \int \frac{1}{\sqrt{1-4.\left(\frac{1}{2}\sin\theta\right)^2}} dx$

$x = \frac{1}{2}\sin\theta$

$\int \frac{1}{\sqrt{1-\sin^2\theta}} dx \sim \int \frac{1}{\cos(x)} dx$

$\sim \int \sec(x)\, dx = \ln|\sec(x) + tg(x)| + C$

29) $\int \left(\frac{1+\cos^2(x)}{\cos^2(x)} + \frac{4\ln(x)}{x^2} \right) dx$

Integração de Funções Racionais

078-) $\displaystyle\int \frac{dx}{(x^2+1)(x^2-1)}$

$$\frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2-1)} \Rightarrow \frac{(Ax+B)(x^2-1)+(Cx+D)(x^2+1)}{(x^2+1)(x^2-1)}$$

$Ax^3 - Ax + Bx^2 - B + Cx^3 + Cx + Dx^2 + D$

$-B + D = 1 \qquad -B - B = 1 \qquad -2B = 1$

$-A + C = 0 \qquad\qquad B = -1/2 \qquad D = 1/2$

$B + D = 0 \qquad B = -D \qquad D = -B$

$A + C = 0$

$$\int \frac{dx}{(x^2+1)(x^2-1)} = -\frac{1}{2}\int \frac{1}{x^2+1}\,dx + \frac{1}{2}\int \frac{1}{x^2-1}\,dx$$

$$= -\frac{1}{2}\cdot \operatorname{arctg}(x) + \frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$

$$= -\frac{1}{2}\operatorname{arctg}(x) + \ln\left|\sqrt[4]{\frac{x-1}{x+1}}\right| + C$$

079-) $\displaystyle\int \frac{3x+2}{x^2+2x+2}\,dx \qquad x^2+2x+2 = (x+1)^2+1$

/ /

080) $\displaystyle\int \frac{x+1}{x^2-6x+8}\,dx$

$x^2-6x+8=0$

$x=\dfrac{6\pm\sqrt{36-32}}{2}$

$x_1=\dfrac{6+2}{2}=\dfrac{8}{2}=4$

$x_2=\dfrac{6-2}{2}=\dfrac{4}{2}=2$

$x^2-6x+8=(x-4)(x-2)$

$\dfrac{x+1}{x^2-6x+8}=\dfrac{A}{x-4}+\dfrac{B}{x-2}=\dfrac{A(x-2)+B(x-4)}{(x-4)(x-2)}$

Admitindo $x=2$

$-2B=3$

$B=\dfrac{-3}{2}$

Admitindo $x=4$

$2A=5$

$A=\dfrac{5}{2}$

$\dfrac{5}{2}\displaystyle\int \frac{1}{x-4}\,dx-\frac{3}{2}\int\frac{1}{x-2}\,dx=\frac{5}{2}\ln|x-4|-\frac{3}{2}\ln|x-2|+C$

083) $\displaystyle\int_1^2 \frac{4x^2-7x-12}{x(x+2)(x-3)}\,dx=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{x-3}$

$A(x+2)(x-3)+B(x-3)x+Cx(x+2)$

Admitindo $x=3$

$15C=3$

$C=1/5$

Admitindo $x=-2$

$10B=18$

$B=\dfrac{9}{5}$

Admitindo $x=0$

$-6A=-12$

$A=2$

$2\displaystyle\int\frac{1}{x}\,dx+\frac{9}{5}\int\frac{1}{x+2}\,dx+\frac{1}{5}\int\frac{1}{x-3}\,dx=2\ln|x|+\frac{9}{5}\ln|x+2|+\frac{1}{5}\ln|x-3|\Big|_1^2$

$=2\ln(2)+\dfrac{9}{5}\ln(4)-\dfrac{9}{5}\ln(3)$

$1 + \cot g\theta - \operatorname{cossec}\theta$

$\rightarrow \int \frac{\sqrt{4-x^2}}{x}\,dx \sim \int \frac{\sqrt{4(1-\operatorname{sen}^2\theta)}}{2\operatorname{sen}\theta} \cdot \cos\theta\, d\theta$

$x = 2\operatorname{sen}\theta$

$dx = \cos\theta\, d\theta$

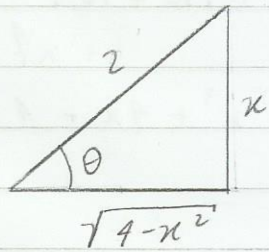
$\int \frac{\cos^2\theta}{\operatorname{sen}\theta}\, d\theta \sim \int \frac{1-\operatorname{sen}^2\theta}{\operatorname{sen}\theta}\, d\theta$

$\int \operatorname{cossec}\theta\, d\theta - \int \operatorname{sen}\theta\, d\theta$

$= \ln|\csc\theta - \cot\theta| + \cos\theta$

$= \ln\left|\frac{2}{x} - \frac{\sqrt{4-x^2}}{x}\right| + \frac{\sqrt{4-x^2}}{2}$

$= \ln\left|\frac{2-\sqrt{4-x^2}}{x}\right| + \frac{\sqrt{4-x^2}}{2} + C$



$\rightarrow \int \frac{5}{x^3-5x}\,dx \qquad \frac{5}{x(x^2-5)} = \frac{A}{x} + \frac{Bx+C}{x^2-5} = \frac{A(x^2-5)+(Bx+C)x}{x(x^2-5)}$

Admitindo $x = 0$ Admitindo $x = -\sqrt{5}$ Admitindo $x = \sqrt{5}$

$-5A = 5 \qquad 5B + \sqrt{5}C = 5 \qquad 5B - \sqrt{5}C = 5$

$A = -1$

$\begin{cases} 5B + \sqrt{5}C = 5 \\ 5B - \sqrt{5}C = 5 \end{cases} \qquad 10B = 10 \qquad B = 1$

$C = \frac{5-5B}{\sqrt{5}} = \frac{5-5\cdot 1}{\sqrt{5}} = 0$

$-\int \frac{1}{x}\,dx + \int \frac{x}{x^2-5}\,dx = -\ln(x) + \int \frac{x}{x^2-5}\,dx$

$x^2 - 5 = t \qquad \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln(x^2-5)$

$2x\,dx = dt$

$x\,dx = \frac{1}{2}\,dt \qquad \therefore\ I = -\ln(x) + \frac{1}{2}\ln(x^2-5) + C$

ou

$\ln|x^{-1} + \sqrt{x^2-5}| + C$

$x = x$

$a = 2$

$\rightarrow \int \frac{x^4}{x^2-4}\,dx$

$$\begin{array}{r|l} x^4 & x^2-4 \\ \hline \underline{-x^4+4x^2} & x^2+4 \\ 4x^2 & \\ \underline{-4x^2+16} & \\ 16 & \end{array}$$

$\int x^2+4+\frac{16}{x^2-4}\,dx$

$= \frac{1}{3}x^3+4x+16\int\frac{1}{x^2-4}\,dx$

$= \frac{1}{3}x^3+4x+16\cdot\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

$= \frac{1}{3}x^3+4x+4\ln\left|\frac{x-2}{x+2}\right|+C$

$$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} \int x e^{2x} dx$$

$u = x \qquad dv = e^{2x}$

$du = dx \qquad v = \frac{1}{2} e^{2x}$

$$\int u\,dv = x \cdot \frac{1}{2} e^{2x} - \frac{1}{2} \int e^{2x} dx$$

$$= \frac{1}{2} x e^{2x} - \frac{1}{2} \cdot \frac{1}{2} e^{2x} = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$$

$$\therefore = \frac{x^2}{2} e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} = \frac{e^{2x}}{2} \left(x^2 - x + \frac{1}{2} \right) + C$$

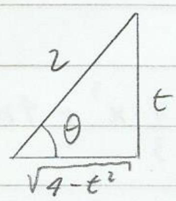
$$\rightarrow \int \frac{x\,dx}{\sqrt{-x^2 - 2x + 3}} \qquad x = t - \frac{b}{2a} = t - 1$$

$x = t - 1 \qquad t = x + 1$

$dx = dt$

$$\int \frac{(t-1)}{\sqrt{-(t-1)^2 - 2(t-1) + 3}} dt = \int \frac{t-1}{\sqrt{-t^2 + \cancel{2t} - 1 - \cancel{2t} + 2 + 3}} dt =$$

$$\int \frac{t-1}{\sqrt{-t^2 + 4}} dt \sim \int \frac{t}{\sqrt{2^2 - t^2}} dt - \int \frac{1}{\sqrt{2^2 - t^2}} dt$$



$$I_1 = \int \frac{1}{\sqrt{2^2 - t^2}} dt = \arcsin\left(\frac{x+1}{2} \right)$$

$$I_2 = \int \frac{t}{\sqrt{2^2 - t^2}} dt \qquad t = 2 \sin\theta \qquad \int \frac{2 \sin\theta}{2 \sin\theta} 2\cos\theta\, d\theta$$

$dt = 2\cos\theta\, d\theta$

$$= 2 \sin\theta = 2 \cdot \frac{t}{2} = x + 1$$

$\to \displaystyle\int \frac{dx}{x^2-3x+5} \qquad x = t - \frac{b}{2a} \qquad t = x + \frac{b}{2a} = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$

$$\int \frac{dx}{\left(x-\frac{3}{2}\right)^2 + \left(\frac{\sqrt{11}}{2}\right)^2} = \frac{2}{\sqrt{11}} \operatorname{arctg}\left(\frac{2x-3}{\sqrt{11}}\right) + C$$

$\to \displaystyle\int x e^{5x} dx$

$u = x \qquad dv = e^{5x} dx \qquad 5x = t$

$du = dx \qquad v = \int e^{5x} dx \qquad dx = \frac{1}{5} dt$

$v = \frac{1}{5} \int e^t dt = \frac{1}{5} e^{5x}$

$$\int u\,dv = x \cdot \frac{1}{5} e^{5x} - \int \frac{1}{5} e^{5x} dx$$

$$= \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} + C$$

$\to \displaystyle\int \frac{x^4}{x^2-4} dx$

$$\begin{array}{l|l} x^4 & x^2-4 \\ \hline \underline{-x^4+4x^2} & x^2+4 \\ 4x^2 & \\ \underline{-4x^2+4} & \\ 4 & \end{array}$$

$\to \displaystyle\int x^2+4+\frac{4}{x^2-4}\, dx$

$$= \frac{1}{3}x^3 + 4x + 4\int \frac{1}{x^2-2^2} dx$$

$$= \frac{1}{3}x^3 + 4x + 4 \cdot \frac{1}{4} \ln\left|\frac{x-2}{x+2}\right| + C$$

$$= \frac{1}{3}x^3 + 4x + \ln\left|\frac{x-2}{x+2}\right| + C$$

$$\rightarrow \int \frac{2}{x^3+2x}\,dx \qquad \frac{2}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$$

$$\frac{A(x^2+2) + Bx^2 + Cx}{x(x^2+2)} = \frac{2}{x(x^2+2)}$$

Admitindo $x = 0$ $\qquad Bx^2 + Cx = -x^2$

$$2A = 2 \qquad x = -1 \rightarrow \begin{cases} B - C = -1 \\ B + C = -1 \end{cases} \qquad C = -1 - B$$

$$A = 1 \qquad x = 1 \rightarrow \qquad C = 0$$

$$2B = -2 \rightarrow B = -1$$

$$\int \frac{2}{x^3+2x}\,dx = \int \frac{1}{x}\,dx - \int \frac{x}{x^2+2}\,dx$$

$$= \ln|x| - \int \frac{x}{x^2+2}\,dx$$

$$x^2+2 = t \qquad \int \frac{x}{x^2+2}\,dx \sim \int \frac{1}{2}\cdot\frac{1}{t}\,dt = \frac{1}{2}\ln|x^2+2)$$

$$2x\,dx = dt$$

$$x\,dx = \frac{1}{2}\,dt$$

$$= \ln|x| - \frac{1}{2}\ln|x^2+2| + C$$

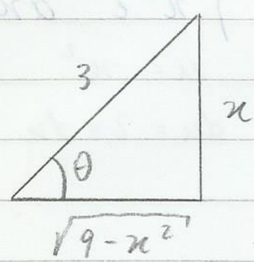
$$\rightarrow \int \frac{\sqrt{9-x^2}}{x}\,dx \qquad \int \frac{\sqrt{9-9\sin^2\theta}}{3\sin\theta}\cdot 3\cos\theta\,d\theta \sim$$

$$x = 3\sin\theta \qquad 3\int \frac{\cancel{3}\sqrt{1-\sin^2\theta}}{\cancel{3}\sin\theta}\cos\theta\,d\theta \sim$$

$$dx = 3\cos\theta\,d\theta$$

$$3\int \frac{\cos^2\theta}{\sin\theta}\,d\theta \sim 3\int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta$$

$$= 3\left[\int \operatorname{cossec}\theta\,d\theta - \int \sin\theta\,d\theta\right]$$



$$= 3\left[\ln|\operatorname{csc}\theta - \operatorname{cotg}\theta| + \cos\theta\right]$$

$$= 3\left[\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + \frac{\sqrt{9-x^2}}{3}\right] + C$$

$\rightarrow$ Calcular $\int x \operatorname{arccotg}\left(\frac{1}{x}\right) dx$

$u = \operatorname{arccotg}\left(\frac{1}{x}\right)$

$du = -\left(-\frac{1}{x^2}\right) : 1 + \frac{1}{x^2}$

$du = \frac{1}{x^2} : \frac{x^2+1}{x^2} = \frac{1}{x^2+1} dx$

$dv = x\,dx \quad v = \frac{x^2}{2} \qquad \int u\,dv = \operatorname{arccotg}\left(\frac{1}{x}\right)\frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x^2+1} dx$

$\int u\,dv = \frac{x^2}{2} \operatorname{arccotg}\left(\frac{1}{x}\right) - \frac{1}{2} \int \frac{x^2}{x^2+1} dx$

$\int \frac{x^2}{x^2+1} dx \sim \int 1 - \frac{1}{x^2+1} dx \sim \int dx - \int \frac{1}{x^2+1} dx$

$= \frac{x^2}{2} \operatorname{arccotg}\left(\frac{1}{x}\right) - \frac{1}{2} x + \frac{1}{2} \operatorname{arctg}\left(\frac{x}{1}\right) + C$

$\rightarrow \int \frac{x}{\sqrt{1-x^2}} dx \qquad 1 - x^2 = t$

$-2x\,dx = dt$

$x\,dx = -\frac{1}{2} dt$

$\int \frac{x}{\sqrt{1-x^2}} dx \sim \int -\frac{1}{2} \cdot \frac{1}{\sqrt{t}} dt = -\frac{1}{2} \int t^{-1/2} dt = -\frac{1}{2} 2 t^{1/2}$

$= -(1-x^2)^{1/2} = -\sqrt{1-x^2} + C$

$\rightarrow \int x^2 e^{2x} dx$

$u = x^2 \qquad dv = e^{2x} dx \qquad 2x = t \qquad \frac{1}{2}\int e^t dt$

$du = 2x\,dx \qquad v = \int e^{2x} dx \qquad 2dx = dt \qquad \frac{1}{2} e^{2x}$

$v = \frac{1}{2} e^{2x} \qquad dx = \frac{1}{2} dt$

$\int u\,dv = x^2 \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x}\, 2x\,dx$

$$\int \frac{x^3\,dx}{\sqrt{9-x^2}}$$

$9-x^2=t^2 \quad | \quad x^2 = 9-t^2$

$-2x\,dx = 2t\,dt$

$x\,dx = -t\,dt$

$$\int \frac{(9-t^2)-t}{t}\,dt$$

$$\int \frac{-9t+t^3}{t}\,dt$$

$$= \int -9+t^2\,dt = -9t + \frac{1}{3}t^3 + C$$

$$= -9\sqrt{9-x^2} + \frac{1}{3}(9-x^2)\sqrt{9-x^2} + C$$

ou

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$$\int \frac{27\sin^3\theta}{3\cancel{\cos\theta}}\,3\cancel{\cos\theta}\,d\theta$$

$$= 27\int \sin^3\theta\,d\theta = 27\int (1-\cos^2\theta)\sin\theta\,d\theta$$

$\cos\theta = t$

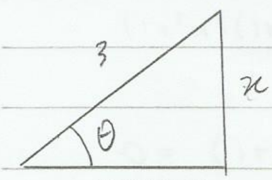
$-\sin\theta\,d\theta = dt$

$\sin\theta\,d\theta = -dt$

$$= 27\int (1-t^2)\,-dt$$

$$= -27\left(t - \frac{1}{3}t^3\right) + C$$

$$= -27\left(\cos\theta - \frac{1}{27}\cos^3\theta\right)$$



$$= -27\left(\frac{\sqrt{9-x^2}}{3} - \frac{1}{27}\cdot\frac{1}{3}(9-x^2)\sqrt{9-x^2}\right) + C$$

$$= -9\sqrt{9-x^2} + \frac{1}{3}(9-x^2)\sqrt{9-x^2} + C$$

/ /

3- $\int \sqrt[5]{x^2} \ln x \, dx$

$\int x^{2/5} \ln x \, dx$

$u = \ln x \qquad dv = x^{2/5} dx$

$du = \frac{1}{x} dx \qquad v = \frac{5}{7} x^{7/5}$

$\int u dv = \frac{5}{7} \ln|x| x^{7/5} - \frac{5}{7} \int x^{7/5} \cdot \frac{1}{x} \, dx$

$= \frac{5}{7} \ln(x) x^{7/5} - \frac{5}{7} \cdot \frac{5}{7} x^{7/5}$

$= \frac{5}{7} x^{7/5} \ln(x) - \frac{25}{49} x^{7/5} + C$

$= \frac{5}{7} x^{7/5} \left(\ln(x) - \frac{5}{7} \right) + C$

- $\int \frac{\sqrt[5]{\ln(x)}}{x} dx \sim \int t^{1/5} dt = \frac{5}{6} t^{6/5} + C$

$\ln(x) = t \qquad = \frac{5}{6} (\ln(x))^{6/5} + C$

$\frac{1}{x} dx = dt$

- $\int \frac{2x^2 \, dx}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1) + (Bx+C)(x+1)}{(x+1)(x^2+1)}$

Admitindo $x = -1$ $\qquad A(x^2+1) + Bx^2 + Bx + Cx + C$

$2A = 2 \qquad A(x^2+1) + B(x^2+x) + C(x+1) = 0$

$A = 1$

Admitindo $x = 0$ $\qquad$ Admitindo $x = 1$

$A + C = 0 \qquad 2A + 2B + 2C = 2$

$C = -A = -1 \qquad A + B + C = 1$

$1 + B - 1 = 1 \quad \therefore B = 1$

$$I = \int \frac{1}{x+1}\,dx + \int \frac{x-1}{x^2+1}\,dx$$

$$= \ln|x+1| + \int \frac{x}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx$$

$$= \ln|x+1| - \operatorname{arctg}(x) + \int \frac{x}{x^2+1}\,dx$$

$x^2+1 = t$ $\qquad \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|x^2+1| + C$

$2x\,dx = dt$

$x\,dx = \frac{1}{2}\,dt$

$$= \ln|x+1| + \frac{1}{2}\ln|x^2+1| - \operatorname{arctg}(x) + C$$

$-\int x^5 e^{x^3}\,dx$ $\qquad \int x^3 x^2 e^{x^3}\,dx$

$\frac{1}{3}\int t e^t\,dt =$

$x^3 = t$ $\qquad u = t \qquad dv = e^t$

$3x^2\,dx = dt$ $\qquad du = dt \qquad v = e^t$

$x^2\,dx = \frac{1}{3}\,dt$ $\qquad \int u\,dv = te^t - \int e^t\,dt$

$= (te^t - e^t) \cdot \frac{1}{3}$

$$= (x^3 e^{x^3} - x^3)\,\frac{1}{3} + C$$

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Equações separaveis

133) $y' - x^2 = x, \; y(0) = 1$

$y' = x + x^2$

$\frac{dy}{dx} = x + x^2$

$dy = (x + x^2)\,dx$

$\int dy = \int (x + x^2)\,dx$

$y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$ (solução geral)

Quando $x = 0 \rightarrow y = 1$

$1 = \frac{1}{2}0^2 + \frac{1}{3}0^3 + C \quad \therefore C = 1$

$y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + 1$ (solução particular)

$x^2 y' = y - xy \;,\; y(-1) = -1$

$x^2 y' = y(1 - x)$

$\frac{y'}{y} = \frac{1 - x}{x^2}$

$\frac{1}{y}\,dy = \frac{1 - x}{x^2}\,dx$

$\int \frac{1}{y}\,dy = \int \frac{1 - x}{x^2}\,dx$

$\ln|y| = \int \frac{1}{x^2} - \int \frac{1}{x}\,dx$

$\ln|y| = -\frac{1}{x} - \ln|x| + C$

$\ln|y| = \ln e^{-1/x} - \ln|x| + \ln C$

$\ln|y| = \ln\left|\frac{e^{-1/x}}{x}\right| \cdot C \Rightarrow y = \frac{1}{x} e^{-1/x} \cdot C$ (solução Geral

Quando $x = -1 \quad y = -1$

$-1 = \frac{1}{-1} e^{-1/-1} \cdot C$

$\therefore C = \frac{1}{e}$

$y = \frac{1}{x} e^{-1/x} e^{-1}$

$y = \frac{1}{x} e^{-1/x - 1}$

135) $y' = 2y + x(e^{3x} - e^{2x})$; $y(0) = 2$

I) $y' + P(x)\,y = Q(x)$

Fator Integrante

$I(x) = e^{\int P(x)\,dx}$

Depois multiplico o fator Integrande por I e Integra

$$y' + P(x)\,y = Q(x)$$
$$y' - 2y = x(e^{3x} - e^{2x})$$

$$I(x) = e^{\int -2\,dx} = e^{-2x}$$

$$y'e^{-2x} - 2ye^{-2x} = x(e^{3x} - e^{2x})e^{-2x}$$
$$(ye^{-2x})' = x(e^{3x} - e^{2x})e^{-2x}$$
$$\int (ye^{-2x})' = \int x(e^{3x} - e^{2x})e^{-2x}\,dx$$
$$ye^{-2x} = \int xe^{x} - x\,dx$$
$$ye^{-2x} = e^{x}(x-1) - \frac{x^2}{2} + C$$
$$y = \frac{e^{x}}{e^{-2x}}(x-1) - \frac{x^2}{2e^{-2x}} + \frac{C}{e^{-2x}}$$

$\int xe^{x}\,dx$

$u = x \quad dv = e^{x}$

$du = dx \quad v = e^{x}$

$\int u\,dv = xe^{x} - \int e^{x}\,dx$

$= xe^{x} - e^{x}$ ou

$e^{x}(x-1)$

$$y = e^{3x}(x-1) - \frac{x^2}{2}.e^{2x} + C.e^{+2x}$$

$$y = e^{2x}\left(-\frac{x^2}{2} + xe^{x} - e^{x} + C\right)$$ (Solução Geral)

Quando $x = 0 \Rightarrow y = 2$

$$2 = e^{0}\left(-\frac{0^2}{2} + 0 - 1 - 0 + C\right)$$
$$3 = C$$

$$y = e^{2x}\left(-\frac{x^2}{2} + e(x-1) + 3\right)$$ (Solução Particular)

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136) $\cos^2(x)\,\frac{dy}{dx} + y = 1$; $y(0) = -3$

$\cos^2(x)\,y' = 1 - y$

$y' = \frac{1-y}{\cos^2(x)}$

$\frac{1}{1-y}\,dy = \frac{1}{\cos^2(x)}\,dx$

$\int \frac{1}{1-y}\,dy = \int \sec^2(x)\,dx$

$\ln|1-y| = \tan(x) + c$

$\ln|1-y| = \ln e^{tg(x)} + \ln c$

$\ln|1-y| = \ln|e^{tg(x)}.|c$

$1-y = e^{tg(x)}.\,c$

$y = 1 - e^{tg\,x}.\,c$ (Solução Geral)

Quando $x = 0 \to y = -3$

$-3 = 1 - e^{tg\,0}\,c$

$\therefore c = 4$

$y = 1 - e^{tg\,t}.\,4$

137) $\frac{dy}{dt} + ty = t \quad y(1)=3$ $\quad y' + P(x)y = Qx$

$I = e^{\int P(x)\,dx}$

$y' + ty = t$

$I = e^{\int t\,dt} = e^{t^2/2}$

$y'e^{t^2/2} + tye^{t^2/2} = te^{t^2/2}$ $\quad s = \frac{t^2}{2} \quad ds = t\,dt$

$(ye^{t^2/2})' = te^{t^2/2}$

$\int (ye^{t^2/2})' = \int te^{t^2/2}\,dt$ $\quad \int e^s\,ds = e^s = e^{t^2/2}$

$ye^{t^2/2} = \int te^{t^2/2}\,dt$

$ye^{t^2/2} = e^{t^2/2} + C$

$y = 1 + \frac{C}{e^{t^2/2}}$ (Solução Geral)

Quando $x=1 \to y=3$

$3 = 1 + \frac{C}{e^{1/2}}$

$2 \cdot e^{1/2} = C$

$y = 1 + \frac{2e^{1/2}}{e^{t^2/2}}$

138) $xy' + (x+1)y = x \quad y, \ln(2) = 1$

$y' + \frac{(x+1)}{x}y = 1 \quad I(x) = e^{\int \frac{x+1}{x}dx} = e^{x+\ln|x|}$

$\int \frac{x+1}{x}\,dx \sim \int dx + \int \frac{1}{x}\,dx = x + \ln|x|$

$y'e^{x+\ln|x|} + \frac{(x+1)}{x}y \cdot e^{x+\ln|x|} = e^{x+\ln|x|}$

$$(y e^{x+\ln(x)})' = e^{x+\ln(x)}$$

$$\int (y e^{x+\ln x})' = \int e^{x+\ln(x)} dx$$

$$y e^{x+\ln(x)} = \int e^{x+\ln(x)} dx$$

$$y e^{x+\ln(x)} = e^{x}(x-1) + C$$

$$y = \frac{e^{x}(x-1)}{e^{x+\ln x}} + \frac{C}{e^{x+\ln(x)}}$$

$$= \frac{e^{x}(x-1)}{e^{x}.e^{\ln x}} + \frac{C}{e^{x}.e^{\ln(x)}}$$

$$= \frac{x-1}{x} + \frac{C}{xe^{x}} \quad \text{(Solução Geral)}$$

$$\int e^{x+\ln(x)} dx = \int e^{x}.e^{\ln(x)} dx$$

$$\int e^{x} x\, dx$$

$$u = x \qquad dv = e^{x} dx$$

$$du = dx \qquad v = e^{x}$$

$$\int u\,dv = xe^{x} - e^{x} \quad \text{ou} \quad e^{x}(x-1)$$

Quando $x = \ln 2 \Rightarrow y = 1$

$$1 = \frac{\ln(2) - 1}{\ln(2)} + \frac{C}{\ln 2\, e^{\ln 2}}$$

$$C = \left(1 - \frac{\ln(2)-1}{\ln(2)}\right) \ln 2 . e^{\ln 2}$$

$$C = \frac{\ln(2) - \ln(2) - 1}{\ln(2)} . \ln(2)\, e^{\ln 2}$$

$$C = -e^{\ln 2} = -2$$

$$y = \frac{x-1}{x} - \frac{2}{xe^{x}}$$

139-) $xy' + 4x^2 y = e^{-2x^2}$, $y(-1) = \frac{1}{3}$

$y' + 4xy = \frac{e^{-2x^2}}{x}$

$I(x) = e^{\int 4x\,dx} = e^{2x^2}$

$y' e^{2x^2} + 4xye^{2x^2} = \frac{e^{-2x^2} \cdot e^{2x^2}}{x}$

$(ye^{2x^2})' = \frac{1}{x} \rightarrow \int (ye^{2x^2})' = \int \frac{1}{x}\,dx$

$ye^{2x^2} = \ln(x) + C$

$y = \frac{\ln(x)}{e^{2x^2}} + \frac{C}{e^{2x^2}}$ (Solução Geral)

Quando $x = -1 \rightarrow y = \frac{1}{3}$

$-\frac{1}{3} = \frac{\ln|-1|}{e^{2(-1)^2}} + \frac{C}{e^{2(-1)^2}}$

$\therefore C = -\frac{e^2}{3}$

$\therefore y = \frac{\ln(x)}{e^{2x^2}} - \frac{-e^2}{3} : e^{2x^2}$

$= e^{-2x^2}\left(\ln(x) + \frac{e^2}{3}\right)$

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140-) $y' - y = 2xe^{2x}$, $y(0) = 1$

$y' - y = 2xe^{2x}$ $\qquad I(x) = e^{\int -1\,dx} = e^{-x}$

$y'e^{-x} - ye^{-x} = 2xe^{2x}\,e^{-x}$

$(ye^{-x})' = 2xe^{x}$

$\int (ye^{-x})' = \int 2xe^{x}\,dx$

$ye^{-x} = 2e^{x}(x-1) + C$

$y = \dfrac{2e^{x}(x-1)+C}{e^{-x}} = 2e^{2x}(x-1) + e^{x}C$ (Solução Geral

Quando $x = 0$ $\quad y = 1$

$1 = 2.e^{0}(0-1) + e^{0}C$

$C = 1 + 2 = 3$ $\qquad \therefore y = 2e^{2x}(x-1) + 3e^{x}$

141) $\dfrac{x^2-1}{y^2+1}\,y' = \dfrac{x}{y}$ $\qquad y^2+1 = t$ $\qquad x^2-1 = s$

$2y\,dy = dt$ $\qquad 2x\,dx = ds$

$\dfrac{y'}{y^2+1}\,y = \dfrac{x}{x^2-1}$ $\qquad y\,dy = \frac{1}{2}\,dt$ $\qquad x\,dx = \frac{1}{2}\,ds$

$\int \dfrac{y}{y^2+1}\,dy = \int \dfrac{x}{x^2-1}\,dx$

$\int \frac{1}{2}\,\frac{1}{t}\,dy = \frac{1}{2}\int \frac{1}{s}\,ds$

$\frac{1}{2}\ln|y^2+1| = \frac{1}{2}\ln|x^2-1| + C$

$y^2+1 = (x^2-1).C$ (Solução Geral)

142-) $\dfrac{dy}{dx} = \dfrac{1+2y^2}{y\,sen\,x}$

$\dfrac{1}{1+2y^2} \cdot y\,dy = \dfrac{1}{sen\,x}\,dx$

$\displaystyle\int \frac{y}{1+2y^2}\,dy = \int cossec\,x\,dx$

$1+2y^2 = t$ $\quad \dfrac{1}{4}\displaystyle\int \frac{1}{t}\,dt = \frac{1}{4}\ln|1+2y^2| = \ln|csc(x) - cotg(x)| + C$

$4y\,dy = dt$

$y\,dy = \dfrac{1}{4}\,dt$ $\quad 1+2y^2 = 4\,(csc(x) - cotg(x)).\,C$

143-) $\dfrac{dy}{dx} = sen\,x\,[\cos(2y) - \cos^2 y]$

$\cos(A+B) = \cos A.\cos B - sen A\, sen B$

$\cos(Y+Y) = \cos^2 y - sen^2 y$

$\displaystyle\int \frac{1}{\cos(2y) - \cos^2 y}\,dy = \int sen\,x\,dx$

$\displaystyle\int \frac{1}{(\cos^2 y - sen^2 y - \cos^2 y)}\,dy = -\cos(x) + C$

$\displaystyle\int -cossec^2 y\,dy = -\cos(x) + C$

$cotg(x) = -\cos(x) + C$

Prova P1

- $\int x e^{-3x}\,dx$

$u = x \qquad dv = e^{-3x}\,dx \qquad t = -3x$

$du = dx \qquad v = \int e^{-3x}\,dx \qquad dt = -3dx$

$dx = -\frac{1}{3}\,dt.$

$= -\frac{1}{3}\int e^{t}\,dt = -\frac{1}{3} e^{-3x}$

$\int u\,dv = x \cdot -\frac{1}{3} e^{-3x} + \frac{1}{3}\int e^{-3x}\,dx$

$= -\frac{1}{3} x e^{-3x} + \frac{1}{3} \cdot -\frac{1}{3} e^{-3x} + C$

$= \frac{-e^{-3x}}{9}\left(3x + 1\right) + C$

- $\int \frac{x^3}{\sqrt{5+x^2}}\,dx$

$5 + x^2 = t^2 \rightarrow x^2 = t^2 - 5$

$2x\,dx = 2t\,dt$

$x\,dx = t\,dt \qquad = \int \frac{(t^2-5) \cdot t}{t}\,dt =$

$= \frac{1}{3} t^3 - 5t + C$

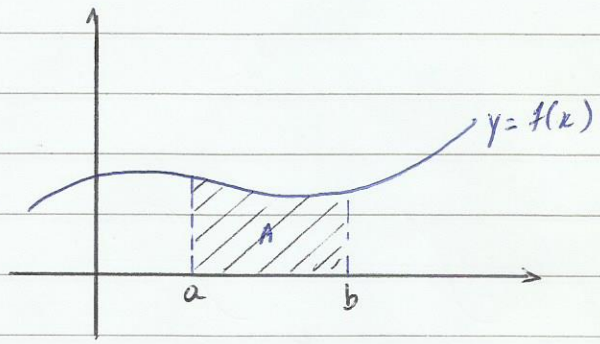
$= \frac{1}{3}(5+x^2) \cdot \sqrt{5+x^2} - 5\sqrt{5+x^2} + C$

$= \frac{5}{3}\sqrt{5+x^2} + \frac{x^2}{3}\sqrt{5+x^2} - 5\sqrt{5+x^2} + C$

$= \frac{-10}{3}\sqrt{5+x^2} + \frac{x^2}{3}\sqrt{5+x^2} + C = \frac{\sqrt{5+x^2}}{3}\left(-10 + x^2\right) + C$

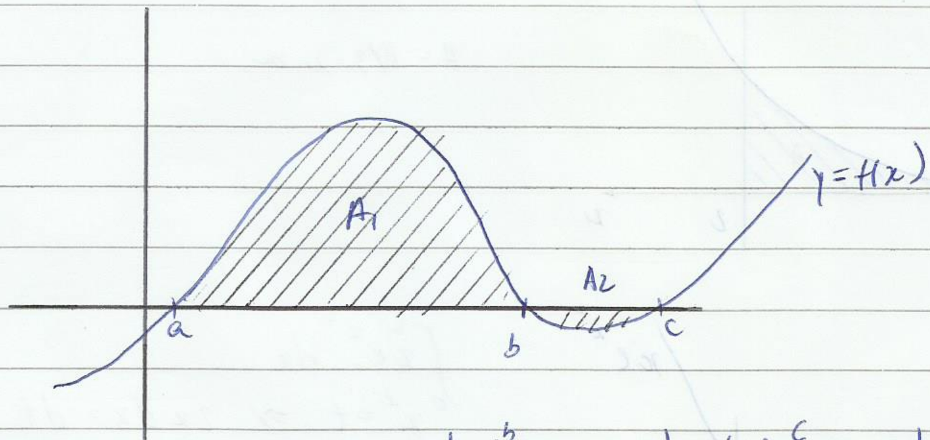
Cálculo de Áreas

1-)



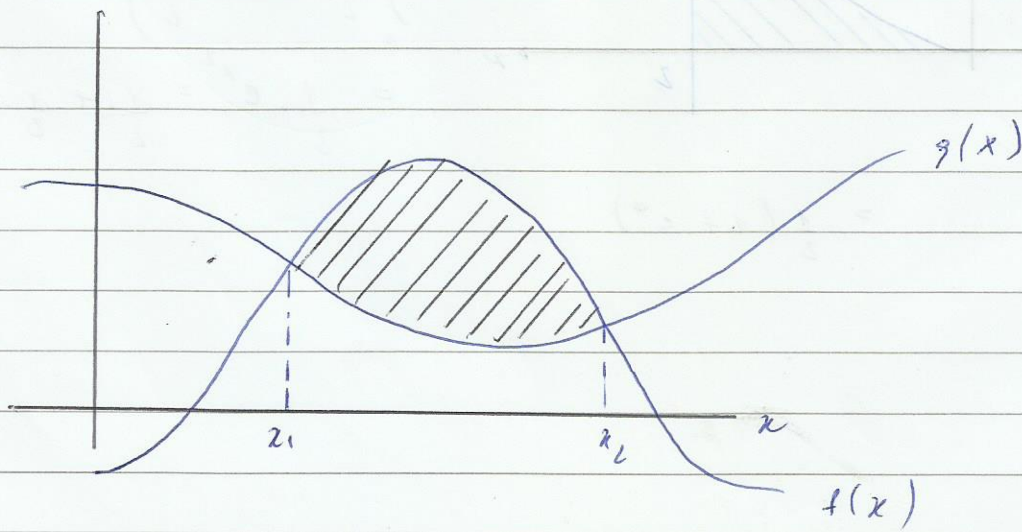
$$A = \left| \int_a^b f(x)\,dx \right|$$

2)

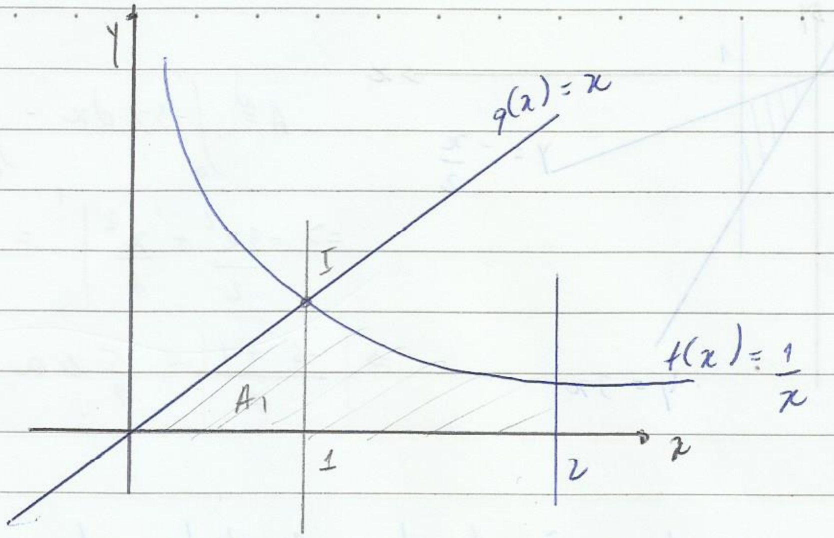


$$A = A_1 + A_2 = \left| \int_a^b f(x)\,dx \right| + \left| \int_a^c f(x)\,dx \right|$$

3-)



3)



$I = g(x) = f(x) \Rightarrow$

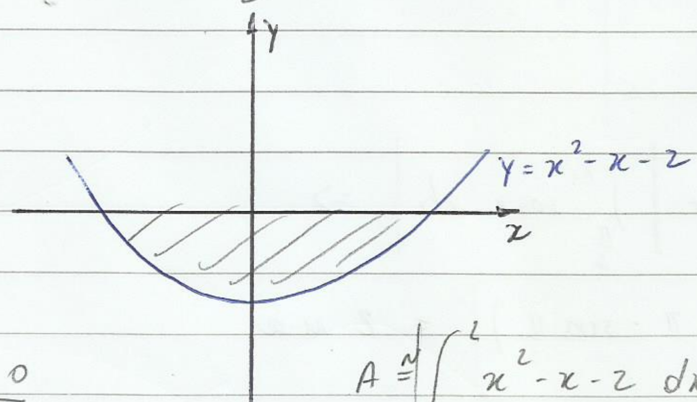
$\Rightarrow x = \frac{1}{x} = x^2 = 1 \Rightarrow x = \pm 1$

$A_1 \cong \int_0^1 x \, dx = \frac{x^2}{2}\Big|_0^1 = \frac{0^2}{2} + \frac{1^2}{2} = \frac{1}{2}$ u.a

$A_2 \cong \int_1^2 \frac{1}{x} \, dx = \ln x \Big|_1^2 = \cancel{\ln 1} - \ln 2 = \ln 2$ u.a

$A = \left(\ln 2 + \frac{1}{2}\right)$ u.a

4)



$x^2 - x - 2 = 0$

$x = \frac{1 \pm \sqrt{1+8}}{2}$

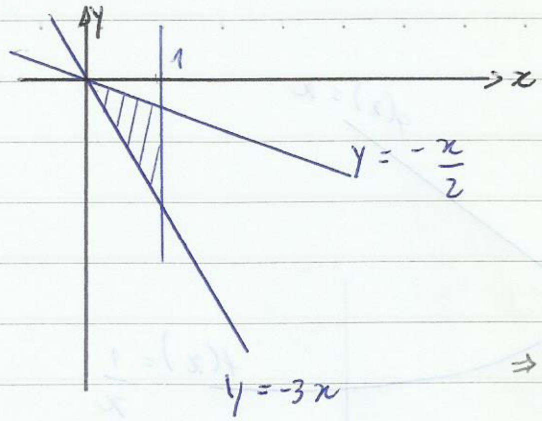
$x_1 = 2 \quad x_2 = -1$

$A \cong \left| \int_{-1}^{2} x^2 - x - 2 \, dx \right| \Rightarrow$

$\Rightarrow \frac{x^3}{3} - \frac{x^2}{2} - 2x \Big|_{-1}^{2} \Rightarrow$

$\Rightarrow \left| \left(\frac{2^3}{3} - \frac{2^2}{2} - 2.2 \right) - \left(\frac{(-1)^3}{3} - \frac{(-1)^2}{2} - 2.(-1) \right) \right| \cong \frac{27}{6}$ u.a

5

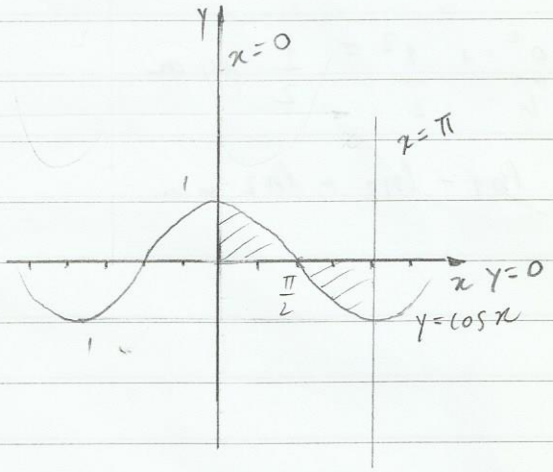


$$A = \left| \int_0^1 -3x \, dx - \int_0^1 -\frac{x}{2} \, dx \right| \Rightarrow$$

$$\Rightarrow -\frac{3x^2}{2} + \frac{x^2}{4} \Big|_0^1 = -\frac{3}{2} + \frac{1}{4} \Rightarrow$$

$$\Rightarrow \left| \frac{-6+1}{4} \right| = \frac{5}{4} \text{ u.a}$$

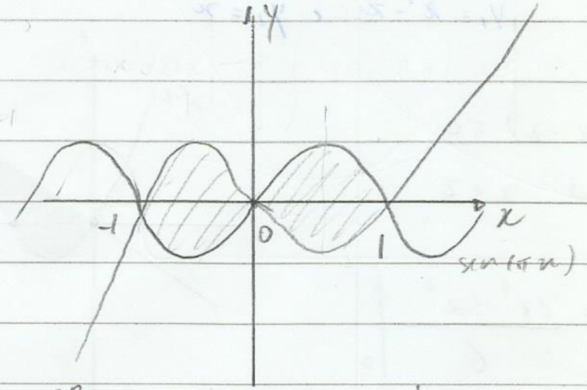
87-) Calcule a área da região do plano limitado pelas retas $x = 0$, $x = \pi$, $y = 0$ e pelo gráfico de $y = \cos x$:



$$A = \left| \int_0^{\pi/2} \cos x \, dx \right| + \left| \int_{\frac{\pi}{2}}^{\pi} \cos x \, dx \right| \Rightarrow$$

$$\Rightarrow \left| \operatorname{sen} \frac{\pi}{2} \right| + \left| \left(\operatorname{sen} \pi - \operatorname{sen} \frac{\pi}{2} \right) \right| = 2 \text{ u.a}$$

88) Calcule a área da região do plano limitado pelos gráficos de $y = x^3 - x$, $y = sen(\pi x)$ com $-1 \leq x \leq 1$



$\frac{\pi + 8}{2\pi}$

$$A_1 = \left| \int_{-1}^{0} (x^3 - x - sen(\pi x))\, dx \right| \Rightarrow \quad \pi x = t \Rightarrow \pi\, dx = dt$$

$x = 0 \to t = 0$

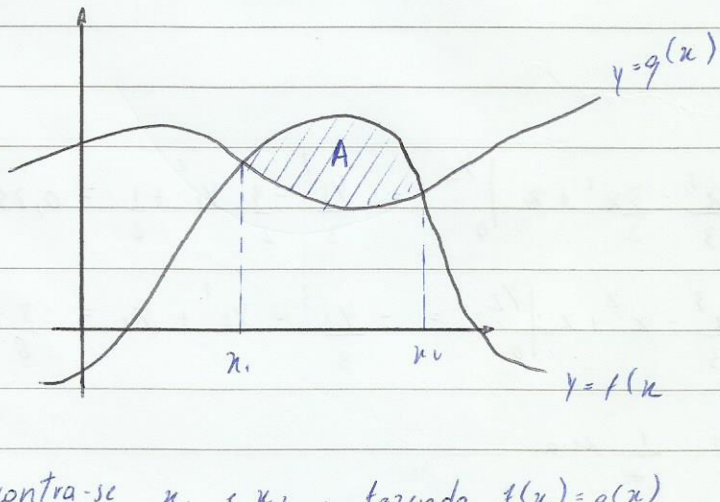
$x = -1 \to t = -\pi$

$$\frac{x^4}{4} - \frac{x^2}{2} \Big|_{-1}^{0} - \int_{-\pi}^{0} sen\, t\ dt \Rightarrow \quad \frac{\pi^4}{4} - \frac{x^2}{2} \Big|_{-1}^{0} + \cos t \Big|_{-\pi}^{0}$$

$$\Rightarrow \cancel{-\frac{1}{4} + |1 - 1| = \frac{1}{4}}$$

Área entre Curvas

Como já vimos



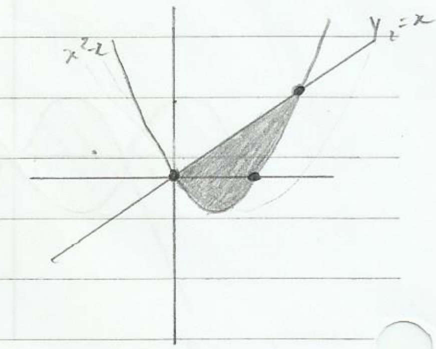
Encontra-se x_1 e x_2, fazendo $f(x) = g(x)$

$$A = \int_{x_1}^{x_2} (f(x) - g(x))\, dx$$

(/ /)

Exemplos:

1-) Calcule a área entre as curvas $y_1 = x^2 - x$ e $y_2 = x$



$$x^2 - x = x \qquad x(x-2) = 0$$
$$x^2 = 2x \qquad x = 0 \quad x = 2$$

$$\int_0^2 (x^2 - x)\,dx = \frac{x^3}{3} - \frac{x^2}{2}\Bigg|_0^2 = \frac{2x^3 - 3x^2}{6}\Bigg|_0^2$$

$$\frac{2 \cdot 2^3 - 3 \cdot 2^2}{6} = \frac{16 - 12}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\int_0^2 x\,dx = \frac{x^2}{2}\Bigg|_0^2 = \frac{4}{2} = 2$$

$$A = 2 - \frac{2}{3} = \frac{6-2}{3} = \frac{4}{3}\ \mu.a$$

2-) Calcule a área entre as curvas $y_1 = x^2 - 3x + 1$ e $y_2 = -x^2 - 2x + 1$

$$x^2 - 3x + 1 = -x^2 - 2x + 1$$
$$2x^2 - x = 0$$
$$x(2x - 1) = 0$$
$$x = 0 \quad x = 1/2$$

$$\int_0^{1/2} (x^2 - 3x + 1)\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + x\Bigg|_0^{1/2} = \frac{1/2^3}{3} - \frac{3}{2}\cdot 1/2^2 + \frac{1}{2} = 0,25$$

$$\int_0^{1/2} (-x^2 - 2x + 1)\,dx = \frac{-x^3}{3} - x^2 + x\Bigg|_0^{1/2} = \frac{-1/2^3}{3} - 1/2^2 + 1/2 = \frac{-5}{8}$$

$$\frac{1}{4} - \frac{5}{24} = \frac{6-5}{24} = \frac{1}{24}\ \mu.a$$

/ /

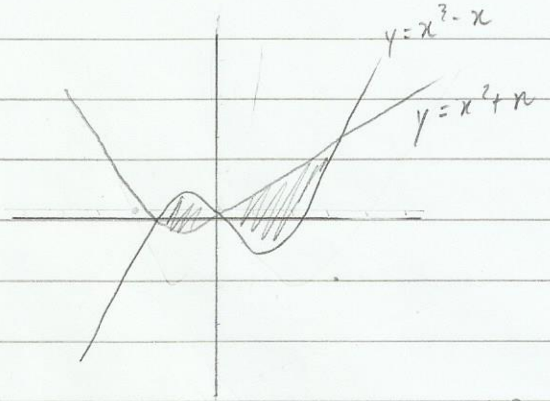
3-) Calcule a área entre as curvas $y_1 = x^3 - x$ e $y_2 = x^2 + x$

$x^3 - x = x^2 + x$ $\quad x = 0$

$x^3 - x^2 - 2x = 0$ $\quad x_1 = 2$

$x(x^2 - x - 2) = 0$ $\quad x_2 = -1$

$x = \frac{1 \pm \sqrt{1+8}}{2}$



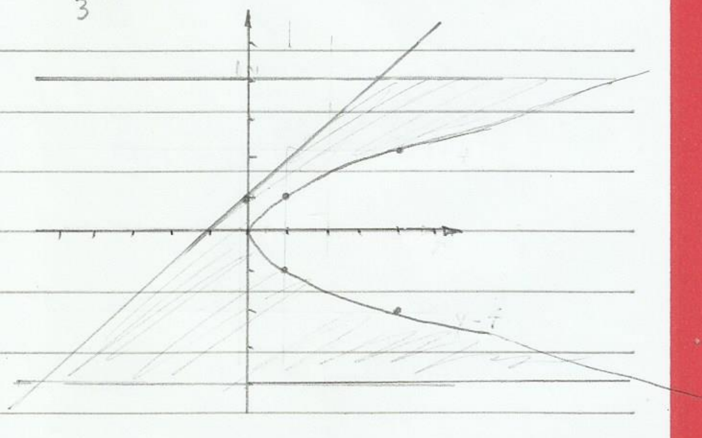
$$\int_{-1}^{0} (x^3 - x - x^2 - x)\, dx \sim \int_{-1}^{0} x^3 - x^2 - 2x\, dx = \frac{x^4}{4} - \frac{x^3}{3} - x^2 \Big|_{-1}^{0} =$$

$$= -\left(\frac{1}{4} + \frac{1}{3} - 1\right) = \frac{-1}{4} - \frac{1}{3} + 1 = \frac{-3-4+12}{12} = \frac{5}{12}$$

$$\int_{0}^{2} \frac{2^4}{4} - \frac{2^3}{3} - 2^2 = 4 - \frac{8}{3} - 4 = \frac{-8}{3}$$

$$= \frac{5+32}{12} = \frac{37}{12} \text{ u.a}$$

93-) $y = -4$ $\quad y = 4$ $\quad x = y^2$ $\quad y = x + 1$



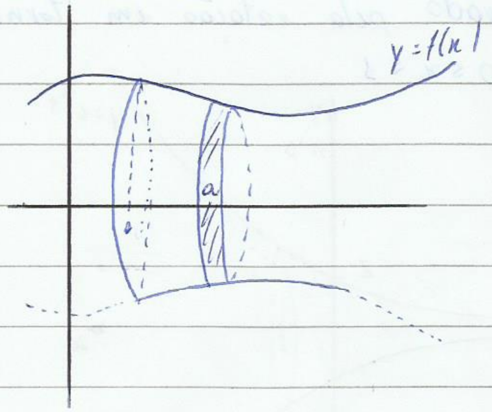
$x = y^2$ $\quad x = y - 1$

$$\int_{-4}^{4} (y^2 - y + 1)\, dy =$$

$$= \frac{y^3}{3} - \frac{y^2}{2} + y \Big|_{-4}^{4} = \left(\frac{4^3}{3} - \frac{4^2}{2} + 4\right) - \left(\frac{-4^3}{3} - \frac{(-4)^2}{2} - 4\right) = \frac{152}{3} \text{ u.a}$$

spiral

Volume de um sólido de Rotação



Seja $y = f(x)$, definida no intervalo $[a,b]$ e giramos a função em torno do eixo dos x.

Dividindo o intervalo $[a,b]$ em n partes iguais chamadas Δx, temos "pequenos" cilindros cujo volume é: $V = \pi r^2 h$.

Neste caso, $r = f(x)$ e $h = \Delta x$, então

$$\Delta V = \pi (f(x))^2 \Delta x \quad \div \Delta x$$

$$\frac{\Delta V}{\Delta x} = \pi (f(x))^2$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \lim_{\Delta x \to 0} \pi (f(x))^2$$

$$\frac{dV}{dx} = \pi (f(x))^2$$

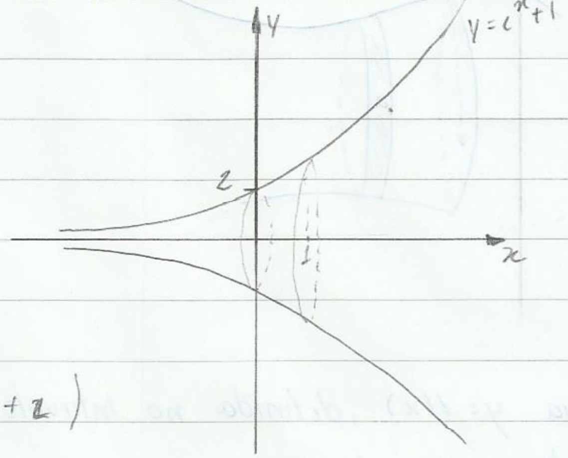
Integrando em $[a,b]$

$$\boxed{V = \pi \int_a^b (f(x))^2 \, dx}$$

/ /

Exemplos:

1-) Calcular o volume do sólido gerado pela rotação em torno do eixo x, da função $y = e^x + 1$, em $0 \le x \le 1$



$$V = \pi \int_0^1 (e^x + 1)^2 \, dx$$

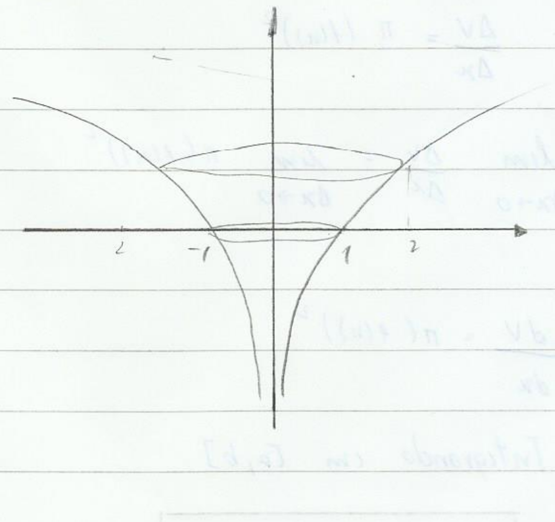
$$= \pi \int_0^1 (e^{2x} + 2e^x + 1)\, dx =$$

$$= \pi \left(\frac{1}{2} e^{2x} + 2e^x + x \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{2} e^2 + 2e + 1 \right) - \pi \left(\frac{1}{2} + 2 \right)$$

$$= \pi \left(\frac{1}{2} e^2 + 2e + 1 - \frac{1}{2} - 2 \right) = \pi \left(\frac{1}{2} e^2 + 2e - \frac{3}{2} \right) \text{ u.v.}$$

2-) Calcular o volume do sólido gerado pela rotação em torno do eixo dos y da função $y = \ln x$, $1 \le x \le 2$



$$y = \ln x$$

$$e^y = e^{\ln x}$$

$$e^y = x$$

$$V = \pi \int_0^{\ln 2} (e^y)^2 \, dy = \pi \frac{1 e^{2y}}{2} \Big|_0^{\ln 2}$$

$$= \pi \frac{((e^y)^2)}{2} \Big|_0^{\ln 2}$$

$$= \pi \left(2 - \frac{1}{2} \right) = \frac{3}{2} \pi \text{ u.v}$$

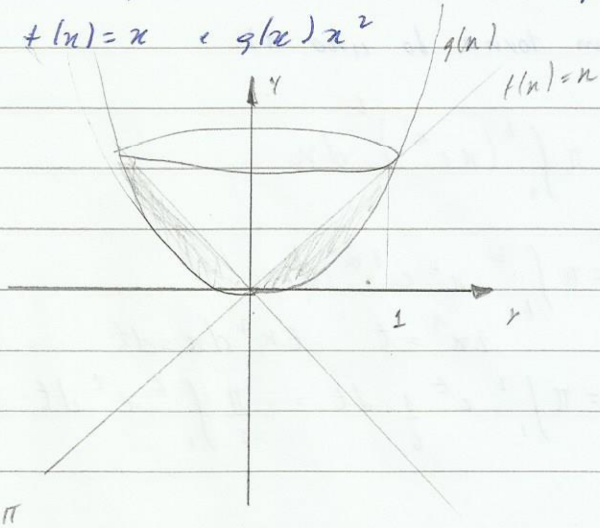
3-) Calcule o volume gerado pela rotação em torno do eixo y da região limitada pelas curvas $f(x) = x$ e $g(x)\ x^2$

$g(x) = x^2$ $\qquad x = x^2$

$y = x^2$ $\qquad x^2 - x = 0$

$x = \sqrt{y}$ $\qquad x(x-1) = 0$

$x = 0$ e $x = 1$



$f(x) \Rightarrow V = \pi \int_0^1 y^2\,dy = \frac{y^3}{3}\Big|_0^1 = \frac{1}{3}\pi$

$g(x) \Rightarrow V = \pi \int_0^1 (\sqrt{y})^2\,dy = \frac{y^2}{2}\Big|_0^1 = \frac{1}{2}\pi$

$V = \pi \left| \frac{1}{3} - \frac{1}{2} \right| = \left| \frac{2-3}{6} \right| = \frac{\pi}{6}$ u.v

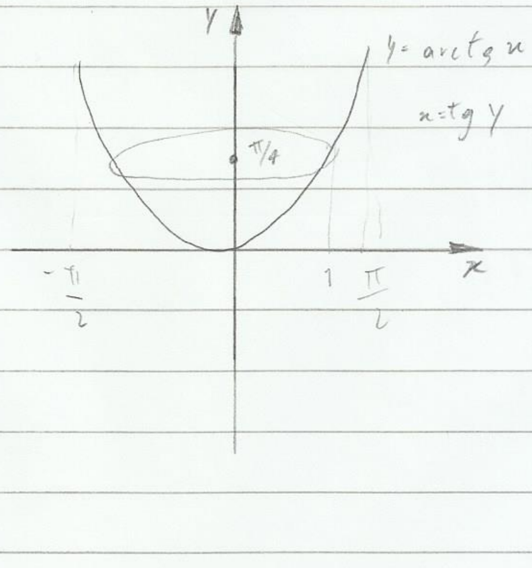
4-) Encontre o volume do sólido pela rotação em torno do eixo y de $0 \leq y \leq \operatorname{arctg} x$, onde $0 \leq x \leq 1$

$V = \pi \int_0^1 tg^2 y\,dy$

$= \pi \int_0^{\frac{\pi}{4}} (\sec^2 y - 1)\,dy$

$= \pi \left(\tan y - y \right)\Big|_0^{\frac{\pi}{4}}$

$= \pi \left(1 - \frac{\pi}{4} \right) = \left(\pi - \frac{\pi^2}{4} \right)$ u.v



5) Encontre o volume do sólido obtido pela rotação de $y = xe^{x^3}$ entre 1 e 2? (em torno do eixo x):

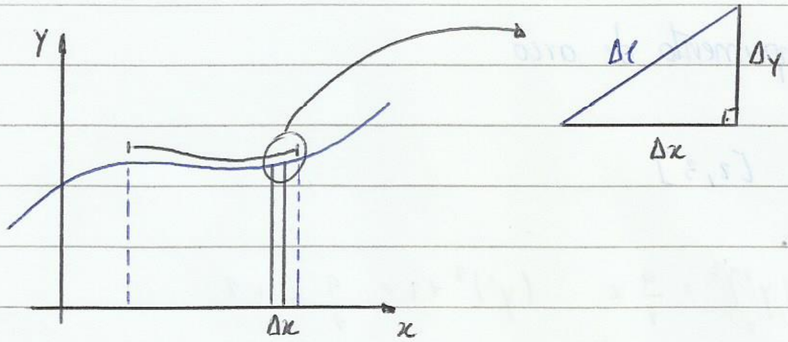
$$V = \pi \int_1^2 \left(xe^{x^3}\right)^2 dx$$

$$= \pi \int_1^2 x^2 e^{2x^3} dx$$

$2x^3 = t \qquad 6x^2 dx = dt$

$$= \pi \int_1^2 e^t \frac{1}{6} dt = \frac{\pi}{6} \int_1^2 e^t dt = \frac{\pi}{6} \left(e^{2x^3} \right|_1^2 = \frac{\pi e^{16}}{6} - \frac{\pi e^2}{6} \text{ u v}$$

Comprimento de arco



Para se calcular o comprimento de arco (l), dividimos o intervalo $[a,b]$ em n partes de "tamanho" Δx. Estes "pedaços" são tão pequenos, que no destaque, temos um triângulo retângulo:

Aplicando o teorema de Pitágoras:

$$(\Delta l)^2 = (\Delta x)^2 + (\Delta y)^2 \qquad \div (\Delta x)^2$$

$$\left(\frac{\Delta l}{\Delta x}\right)^2 = 1 + \left(\frac{\Delta y}{\Delta x}\right)^2$$

$$\lim_{\Delta x \to 0} \left(\frac{\Delta l}{\Delta x}\right)^2 = \lim_{\Delta x \to 0} \left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right]$$

$$\left(\frac{dl}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\frac{dl}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$dl = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Integrando ambos os lados:

$$\boxed{l = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx}$$

Exemplos:

Calcule o comprimento de arco

1- $y = x^{3/2}$ em $[2,3]$

$y' = \frac{3}{2} x^{\frac{1}{2}}$ $(y')^2 = \frac{9}{4} x$ $(y')^2 + 1 = \frac{9}{4} x + 1$

$l = \int_2^3 \sqrt{\frac{9}{4} x + 1} \, dx$

$\frac{9}{4} x + 1 = t$ $\frac{9}{4} dx = dt$ $dx = \frac{4}{9} dt$

$x = 2 \to t = \frac{9}{4} . 2 + 1 = \frac{11}{2}$

$x = 3 \to t = \frac{9}{4} . 3 + 1 = \frac{31}{4}$

$\int_{\frac{11}{2}}^{\frac{31}{4}} t^{1/2} \frac{4}{9} dt = \frac{4}{9} \frac{t^{3/2}}{3/2} \Big|_{11/2}^{31/4} = \frac{8}{27} \left(\left(\frac{31}{4}\right)^{3/2} - \left(\frac{11}{2}\right)^{3/2} \right) uc$

$= \frac{8}{27} \left(\frac{31\sqrt{31}}{8} - \frac{11}{2} \frac{\sqrt{11}}{\sqrt{2}} \right) uc$

2-) $y = \frac{x^4}{8} + \frac{1}{4x^2}$ em $[1,2]$

$y_1' = \frac{1}{2} x^3 - \frac{1}{2x^3}$ $(y')^2 = \left(\frac{1}{2} x^3 - \frac{1}{2x^3} \right)^2 = \frac{1}{4} x^6 - \frac{1}{2} + \frac{1}{4x^6}$

$(y')^2 + 1 = \frac{1}{4} x^6 + \frac{1}{2} + \frac{1}{4} x^{-6}$

$$l = \int_1^2 \sqrt{\frac{1}{4}x^6 + \frac{1}{2} + \frac{1}{4}x^{-6}}\, dx \sim \int_1^2 \sqrt{\left(\frac{x^3}{2} + \frac{x^{-3}}{2}\right)^2}\, dx \sim$$

$$\sim \int_1^2 \frac{x^3}{2} + \frac{x^{-3}}{2}\, dx \sim \frac{1}{2}\int_1^2 x^3 + x^{-3}\, dx = \frac{1}{2}\frac{x^4}{4} + \frac{1}{2}\frac{x^{-2}}{-2}\Big|_1^2$$

$$= \frac{1}{8}x^4 - \frac{1}{+2}x^{-2}\Big|_1^2 = \frac{1}{8}.2^4 - \frac{1}{2}\frac{2^{-2}}{2} - \frac{1}{8}1^4 + \frac{1}{2}1^{-2} = 2 - \frac{1}{16} - \frac{1}{8} + \frac{1}{4} = \frac{33}{16}\ u.c$$

3-) $y = \frac{1}{3}(x-3)\sqrt{x}$ em $[0,3]$

$$f(x) = \frac{1}{3}(x-3)x^{\frac{1}{2}} = \frac{x^{\frac{3}{2}}}{3} - x^{\frac{1}{2}} \qquad y' = \frac{x^{\frac{1}{2}}}{2} - \frac{1}{2}x^{-1/2}$$

$$(y')^2 = \left(\frac{x^{\frac{1}{2}}}{2} - \frac{x^{-1/2}}{2}\right)^2 = \frac{x}{4} + \frac{x^{-1}}{4} - \frac{1}{2} \qquad (y')^2 + 1 = \frac{x}{4} + \frac{x^{-1}}{4} + \frac{1}{2}$$

$$\int_0^3 \sqrt{\frac{x}{4} + \frac{x^{-1}}{4} + \frac{1}{2}}\, dx \sim \int_0^3 \sqrt{\left(\frac{x^{1/2}}{2} + \frac{x^{-1/2}}{2}\right)^2}\, dx \sim \int_0^3 \frac{x^{\frac{1}{2}}}{2} + \frac{x^{-\frac{1}{2}}}{2}\, dx =$$

$$\cancel{2}.\frac{1}{\cancel{2}}\frac{x^{\frac{3}{2}}}{3} + \cancel{2}\frac{1}{\cancel{2}}\frac{x^{\frac{1}{2}}}{1} = \frac{x^{3/2}}{3} + x^{1/2}\Big|_0^3 = \frac{\sqrt{x^3}}{3} + \sqrt{x}\Big|_0^3 = \sqrt{3} + \sqrt{3} = 2\sqrt{3}\ u.c$$

4) $y = \frac{1}{2}(e^x + e^{-x})$ em $[0,1]$

$$y' = \frac{1}{2}(e^x - e^{-x}) \quad (y')^2 = \frac{1}{4}(e^{2x} + e^{-2x} - 2) \quad (y')^2 + 1 = \frac{e^{2x}}{4} + \frac{e^{-2x}}{4} + \frac{1}{2}$$

$$\int_0^1 \sqrt{\frac{e^{2x}}{4} + \frac{e^{-2x}}{4} + \frac{1}{2}}\, dx = \int_0^1 \sqrt{\left(\frac{1}{2}(e^x + e^{-x})\right)^2}\, dx \sim \int_0^1 \frac{(e^x + e^{-x})}{2}\, dx$$

$$= \frac{1}{2}(e^x - e^{-x})\Big|_0^1 = \left(\frac{e}{2} - \frac{e^{-1}}{2}\right) - \left(\frac{1}{2} - 1\right) \Rightarrow \frac{1}{2}(e - e^{-1} + 1)$$

spiral®

5) $y = \ln(\cos x)$ on $[0, \pi/4]$

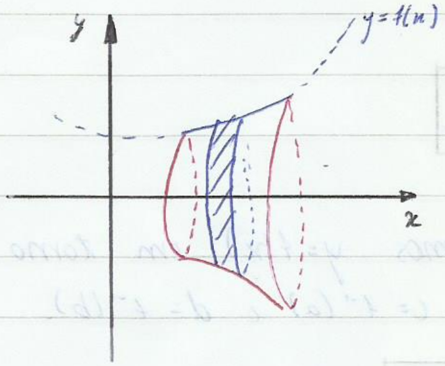
$y' = \frac{-\operatorname{sen} x}{\cos x} = -tg(x)$ $\quad (y')^2 = tg^2(x)$ $\quad (y')^2 + 1 = tg^2(x) + 1 = \sec^2(x)$

$l = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2(x)}\, dx = \int_0^{\frac{\pi}{4}} \sec(x)\, dx = \ln|\sec(x) + tg(x)|\Big|_0^{\frac{\pi}{4}}$

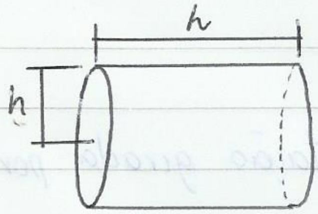
$\ln|\sqrt{2} + 1| - \cancel{\ln}|1+0| = \ln|\sqrt{2}+1|$ uc

Área da Superfície de Rotação

Seja $y = f(x)$, uma função contínua definida no intervalo $[a,b]$. Se girarmos a função em torno do eixo x, temos uma superfície de rotação, e queremos calcular a área lateral.



Para isto, dividimos o intervalo $[a,b]$ em n partes de "tamanho" Δx. Assim, obtemos n cilindro cuja área lateral é dada por:



$A_L = 2\pi r h$

Calculando este "pedaço" de área:

$\Delta S = 2\pi f(x) . \Delta l$, onde Δl é o "pedaço" de arco

$$\frac{\Delta S}{\Delta x} = 2\pi f(x) \frac{\Delta l}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta S}{\Delta x} = 2\pi \lim_{\Delta x \to 0} f(x) \frac{\Delta l}{\Delta x}$$

$$\frac{ds}{dx} = 2\pi f(x) \frac{dl}{dx}$$

/ /

Como já vimos, $\frac{dl}{dx} = \sqrt{1 + [f'(x)]^2}$

$$\frac{dS}{dx} = 2\pi f(x) \sqrt{1 + [f'(x)]^2}$$

Integrando em $[a,b]$

$$\boxed{S_x = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\, dx}$$

De modo análogo, se girarmos $y = f(x)$ em torno do eixo y, devemos tomar $x = f^{-1}(y) = g(x)$, $c = f^{-1}(a)$ e $d = f^{-1}(b)$.

$$\boxed{S_y = 2\pi \int_c^d g(y) \sqrt{1 + [g'(x)]^2}\, dy}$$

Exemplos:

1-) Calcule a área da superfície de rotação gerada por $y = x^3$ em $[0,1]$, em torno do eixo x:

$y' = 3x^2$ $\qquad\qquad 1 + 9x^4 = t$

$1 + (y')^2 = 1 + 9x^4$ $\qquad\qquad 36x^3\,dx = dt$

$\qquad\qquad x^3\,dx = \frac{1}{36}\,dt$

$$S_x = 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4}\, dx$$

$$= 2\pi \int_0^1 \frac{1}{36} \sqrt{t}\, dt = \frac{2\pi}{36} \cdot \frac{2}{3} t^{\frac{3}{2}} \Big|_0^1 = \frac{1\pi}{27} \sqrt[2]{(1+9x^4)^3} \Big|_0^1$$

$$\pi \frac{1}{27} \sqrt[2]{1000} - \frac{1\pi}{27} = \frac{1\pi}{27}\left(1 - \sqrt[2]{1000}\right) = \frac{1\pi}{27}(1 - 10\sqrt{10})\ u.a$$

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2-) Calcule a área da superfície obtida pela revolução do gráfico de $f(x) = x^2$, $0 \le x \le \sqrt{2}$, ao redor do eixo y

$y = x^2 \Rightarrow x = \sqrt{y}$

$y = 0^2 = 0$

$y = (\sqrt{2})^2 = 2$

$$S_y = 2\pi \int_b^d g(y)\sqrt{1 + [g'(y)]^2}\, dy$$

$x = \sqrt{y} \quad x' = \frac{1}{2\sqrt{y}}$

$1 + (x')^2 = 1 + \frac{1}{4y}$

$$S_y = 2\pi \int_0^2 \sqrt{y}\sqrt{1 + \frac{1}{4y}}\, dy \sim 2\pi \int_0^2 \cancel{\sqrt{y}} \cdot \frac{1}{2} \frac{\sqrt{4y+1}}{\cancel{\sqrt{y}}} \frac{dy}{4}$$

$$S_y = \pi \int_0^2 \sqrt{4y+1}\, dy \sim \pi \int_1^9 \sqrt{t}\, \frac{dt}{4} = \frac{2}{3} t^{\frac{3}{2}} \Big|_1^9 =$$

$4y + 1 = t \quad 4\,dy = dt$

Quando $y = 2 \Rightarrow t = 4.2 + 1 = 9$

$y = 0 \rightarrow t = 4.0 + 1 = 1$

$$= \pi \frac{2}{3} \sqrt[2]{t^3} \Big|_1^9 = \frac{2\pi}{6}(27 - 1) =$$

$$\frac{2.26\pi}{6} = \frac{13}{3}\pi \text{ u.a}$$

3-) Calcule a área lateral da superfície formada por $y = \sqrt{x+1}$, $1 \le x \le 5$, em torno do eixo x

$y = \sqrt{x+1} \Rightarrow y' = \frac{1}{2}(x+1)^{-1/2} = \frac{1}{2\sqrt{x+1}}$

$1 + (y')^2 = 1 + \frac{1}{4(x+1)}$ ou $1 + \frac{1}{4x+4}$ ou $\frac{4x+4+1}{4(x+1)}$

$$S_x = 2\pi \int_1^5 \sqrt{x+1}\, \frac{\sqrt{4x+4+1}}{\sqrt{(x+1)}} \cdot \frac{1}{2}\, dx \sim \pi \int_1^5 \sqrt{4x+4+1}\, dx =$$

$$\frac{2}{3} \sqrt[2]{(4x+5)^3} \Big|_1^5 = \frac{2\pi}{3}(125 - 27) = \frac{196}{3}\pi = \text{u.A}$$

4) Calcule a área da superfície de revolução de $y = \frac{x^3}{6} + \frac{1}{2x}$, $1 \le x \le 2$, em torno do eixo x

$$y = \frac{x^3}{6} + \frac{1}{2x} = \frac{x^3}{6} + \frac{x^{-1}}{2} \quad \therefore \quad y' = \frac{1}{2}x^2 - \frac{x^{-2}}{2}$$

$$1 + (y')^2 = 1 + \left(\frac{1}{2}x^2 - \frac{x^{-2}}{2}\right)^2 = 1 + \left[\frac{1}{2}\left(x^2 - x^{-2}\right)\right]^2 = 1 + \frac{1}{4}\left(x^4 - 2 + x^{-4}\right)$$

$$= 1 + \frac{x^4}{4} - \frac{1}{2} + \frac{x^{-4}}{4} = \frac{x^4}{4} + \frac{1}{2} + \frac{x^{-4}}{4} = \frac{1}{2}\left(\frac{x^4}{2} + 2 + \frac{x^{-4}}{2}\right) = \left(\frac{x^2}{2} + \frac{x^{-2}}{2}\right)^2$$

$$= 2\pi \int_1^2 \left(\frac{x^3}{6} + \frac{1}{2x}\right) \sqrt{\left(\frac{x^2}{2} + \frac{x^{-2}}{2}\right)^2}\, dx$$

$$= 2\pi \int_1^2 \left(\frac{x^3}{6} + \frac{x^{-1}}{2}\right)\left(\frac{x^2}{2} + \frac{x^{-2}}{2}\right) dx$$

=

P_1-

1ª Questão

Calcule as integrais indefinidas

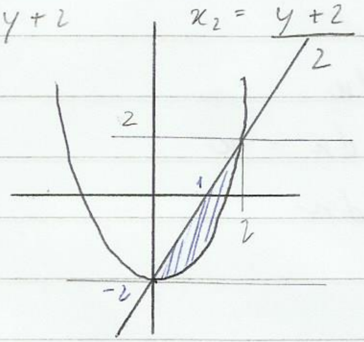
a) $I = \int \frac{(2x+3)}{x^2+4x+5}\,dx$ b) $I = \int \ln(x^2+4)\,dx$

2ª Questão

Determine a área da região plana delimitada pelas curvas

$y = x^2 - 2$ e $y = 2x - 2$

$x_1 = \sqrt{y+2}$ $x_2 = \frac{y+2}{2}$



3ª Questão

Calcule a área da superfície formada pela rotação em torno do eixo x, da curva $y = \sqrt{9-x^2}$ com $-2 \leq x \leq 2$

$x = \sqrt{9-y^2}$

4ª Questão

Obtenha o volume do sólido de revolução em torno do eixo x, da região limitada pela curva $y = \sqrt{x(e^x+1)}$ e o eixo x, com $0 \leq x \leq 1$

1.1 a) $I = \int \frac{(2x+3)}{x^2+4x+5}\,dx$ $\qquad x^2+4x+5 = (x+2)^2+1$

$$\int \frac{2x+3}{(x+2)^2+1}\,dx \;\therefore\; \int \frac{2(t-2)+3}{t^2+1}\,dt \sim \int \frac{2t-1}{t^2+1}\,dt$$

$x+2 = t$

$dx = dt$

$$\int \frac{2t}{t^2+1}\,dt - \int \frac{1}{t^2+1}\,dt$$

$$\ln|(x+2)^2+1| - \operatorname{arctg}(x+2) + C$$

b) $\int \ln(x^2+4)\,dx$

$u = \ln(x^2+4) \qquad du = \frac{2x}{x^2+4} \qquad dv = dx$

$\qquad\qquad\qquad\qquad\qquad\qquad v = \int dx = x$

$$\int \ln(x^2+4)\,dx = \ln(x^2+4)\,x - \int x \cdot \frac{2x}{x^2+4}\,dx$$

$$2\int \frac{x^2}{x^2+4}\,dx \sim \frac{A(x+2)+B(x-2)}{(x-2)(x+2)} = \frac{x^2}{x^2+4}$$

Admitindo $x = 2$ $\qquad$ Admitindo $x = -2$

$A(2+2) = 4 \qquad B(-2-2) = 4$

$A = 1 \qquad B = -1$

$$2\int \frac{x^2}{x^2+4}\,dx \sim 2\int \frac{x+2-x+2}{(x^2+4)}\,dx \sim 2\int \frac{4}{x^2+4}\,dx = 8\int \frac{1}{x^2+4}$$

$$\therefore \int \ln(x^2+4)\,dx = x\ln(x^2+4 - 2\operatorname{arctg}\left(\frac{x}{4}\right)$$

$$\mathrm{I} \qquad \mathrm{II}$$

2-) $x^2-2 = 2x-2$

$x^2 - 2x = 0 \qquad x = 0$

$x(x-2) = 0 \qquad x = 2$

$$\int_0^2 2x-2 - x^2+2 \, dx \sim \int_0^2 2x - x^2 \, dx =$$

$$x^2 - \frac{x^3}{3} \Big|_0^2 = 4 - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3} \ u.A$$

$$\therefore \ A = \frac{4}{3} \ u.A$$

3-) $S_x = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} \, dx$

$$y' = \frac{1}{2}(9-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{9-x^2}} \qquad \left(\frac{-x}{\sqrt{9-x^2}}\right)^2 = \frac{x^2}{9-x^2}$$

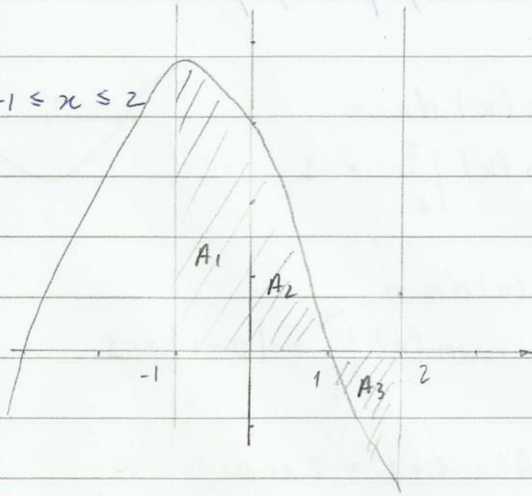
$$\frac{x^2}{9-x^2} + 1 = \frac{x^2 + 9 - x^2}{9-x^2} = \frac{9}{9-x^2} = \left(\frac{3}{\sqrt{9-x^2}}\right)^2$$

$$\therefore \ S = 2\pi \int \sqrt{9-x^2} \cdot \frac{3}{\sqrt{9-x^2}} \, dx = 2\pi \int 3 \, dx = 2\pi \cdot 3x \Big|_{-2}^{2}$$

$$\therefore \ S = 12\pi - (12\pi) = 24\pi$$

Áreas entre Curvas

085-) $y = 0 \quad y = 3 - 2x - x^2$, com $-1 \leq x \leq 2$



$-x^2 - 2x + 3$

$\Delta = 16$

$x_1 = 1 \quad x_2 = -3$

$$A_1 = \int_{-1}^{0} (3 - 2x - x^2)\, dx$$

$$= 3x - x^2 - \frac{1}{3}x^3 \Big|_{-1}^{0} =$$

$$= \left| -3 - 1 + \frac{1}{3} \right| = \frac{11}{3}$$

$$A_2 = \int_0^1 (3 - 2x - x^2)\, dx = 3x - x^2 - \frac{1}{3}x^3 \Big|_0^1 = 3 - 1 - \frac{1}{3} = \frac{5}{3}$$

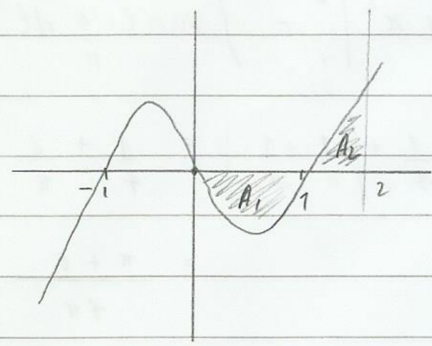
$$A_3 = \int_1^2 (3 - 2x - x^2) dx = 3x - x^2 - \frac{1}{3}x^3 \Big|_1^2 = \left(6 - 4 - \frac{8}{3}\right) - \left(3 - 1 - \frac{1}{3}\right) = \left| -\frac{7}{3} \right| = \frac{7}{3}$$

$$A_T = \frac{11 + 5 + 7}{3} = \frac{23}{3} \text{ u.a}$$

86-) $y = 0$, $y = x^3 - x$, com $0 \leq x \leq 2$

$x(x^2 - 1) = 0$

$x = 0 \quad x = \pm 1$



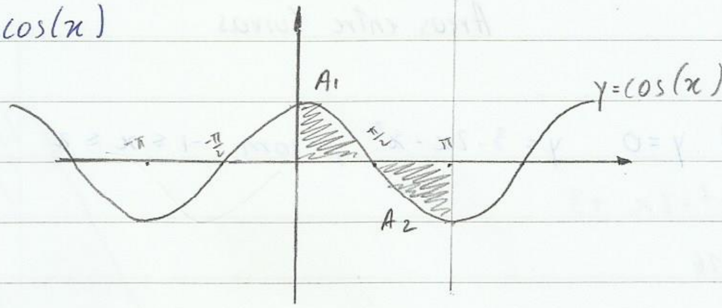
$$A_1 = \int_0^1 x^3 - x \; dx = \frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_0^1 = \left| -\frac{1}{4} \right| = \frac{1}{4}$$

$$A_2 = \int_1^2 x^3 - x \; dx = \left| \frac{1}{4}x^4 - \frac{1}{2}x^2 \right|_1^2 =$$

$$\left| (4 - 2) - \left(\frac{1}{4} - \frac{1}{2} \right) \right| = \frac{9}{4}$$

$$A_t = \frac{9}{4} + \frac{1}{4} = \frac{10}{4} = \frac{5}{2} \text{ u.a}$$

87-) $x=0$; $x=\pi$; $y=0$; $y=\cos(x)$



$$A_1 = \int_0^{\frac{\pi}{2}} \cos(x)\,dx = \operatorname{sen}(x)\Big|_0^{\frac{\pi}{2}} = 1$$

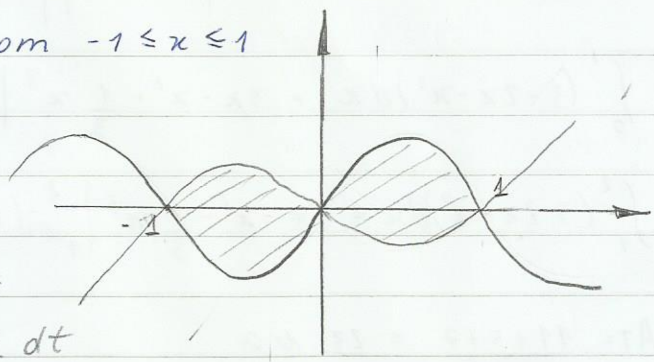
$$A_2 = \int_{\frac{\pi}{2}}^{\pi} \cos(x)\,dx = \operatorname{sen}(x)\Big|_{\frac{\pi}{2}}^{\pi} = |0-1| = 1$$

$$A_t = 1+1 = 2\ u.a$$

88-) $y=x^3-x$; $y=\operatorname{sen}(\pi x)$ com $-1 \leq x \leq 1$

$x(x^2-1)=0$ $\quad x=1,\ y=?$

$x=0 \quad x=\pm 1$



$$\int_{-1}^{1} (x^3-x) - (\operatorname{sen}(\pi x))\,dx =$$

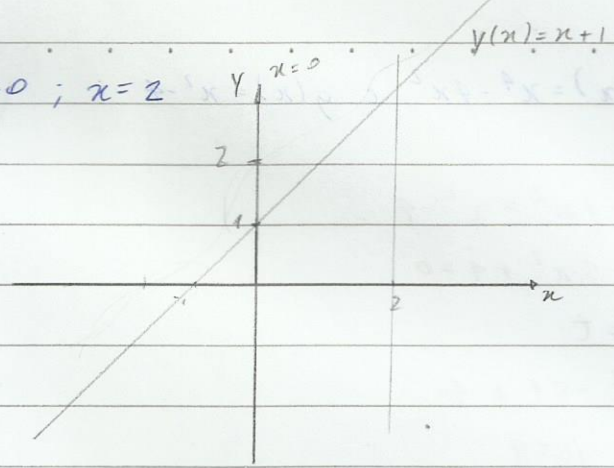
$\pi x = t \quad \pi\,dx = dt \quad dx = \frac{1}{\pi}\,dt$

$$\frac{1}{4}x^4 - \frac{1}{2}x^2\Big|_{-1}^{0} - \int \operatorname{sen} t\,\frac{1}{\pi}\,dt = \frac{1}{4}x^4 - \frac{1}{2}\Big|_{-1}^{0} + \frac{1}{\pi}\cos(\pi x)\Big|_{-1}^{1}$$

$$\frac{1}{4} + \left|\frac{1}{\pi} + \frac{1}{\pi}\right| = \frac{1}{4} + \frac{2}{\pi}$$

$$= \frac{\pi + 8}{4\pi}\ u.a$$

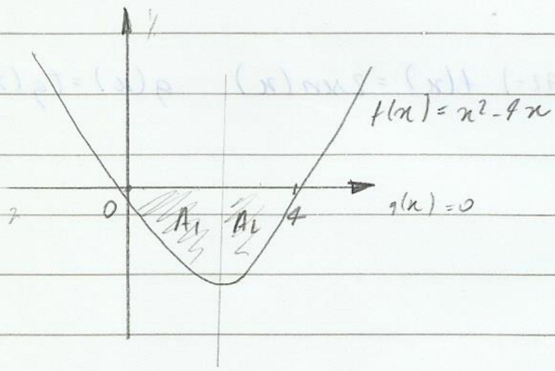
089) $y = \frac{1}{2}x^3 + 2$; $y = x+1$; $x=0$; $x=2$



090-) $f(x) = x^2 - 4x$ e $g(x) = 0$

$x(x-4) = 0$

$x=0 \quad x=4$



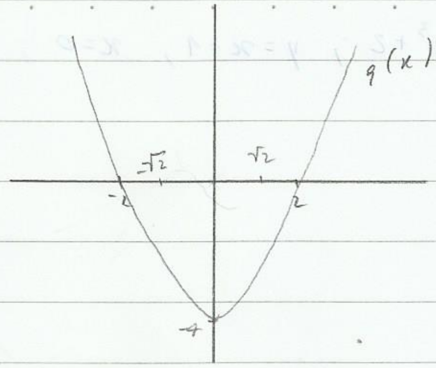
$A_1 = \int_0^2 (x^2 - 4x)\,dx = \frac{1}{3}x^3 - 2x^2 \Big|_0^2$

$\left| \frac{8}{3} - 8 \right| = \frac{16}{3}$

$A_2 = \int_2^4 (x^2 - 4x)\,dx = \frac{1}{3}x^3 - 2x^2 \Big|_2^4 = \left| \frac{8}{3} - 8 \right| = \frac{16}{3}$

$A_T = \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$ u.a

91-) $f(x) = x^4 - 4x^2$ e $g(x) = x^2 - 4$



$x^4 - 4x^2 = x^2 - 4$

$x^4 - 5x^2 + 4 = 0$

$x^2 = t$

$t^2 - 5t + 4$

$\Delta = 25 - 16 = 9$

$t_1 = \frac{5+3}{2} = \frac{8}{2} = 4 \qquad t_2 = \frac{5-3}{2} = \frac{2}{2} = 1 \qquad x_1 = \sqrt{4} = \pm 2$

$x_2 = \sqrt{1} = \pm 1$

$$\int_1^2 x^4 - 4x^2 - x^2 + 4 \, dx \sim \int_1^2 x^4 - 5x^2 + 4 \, dx =$$

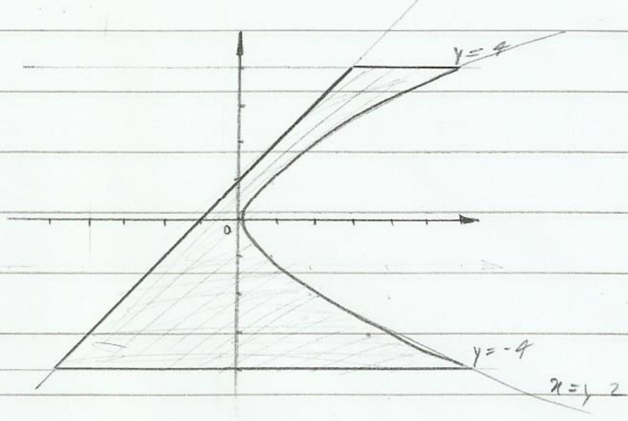
$$= \frac{1}{5} x^5 - \frac{5}{3} x^3 + 4x \Big|_1^2 = \left| \left(\frac{32}{5} - \frac{40}{3} + 4 \right) - \left(\frac{1}{5} - \frac{5}{3} + 4 \right) \right| = \frac{22}{15}$$

092-) $f(x) = 2sen(x)$ $\quad g(x) = tg(x)$ com $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

093) $y=-4$; $y=4$; $x=y^2$ $y=x+1$

$y=x+1$

$x=y-1$ e $x=y^2$



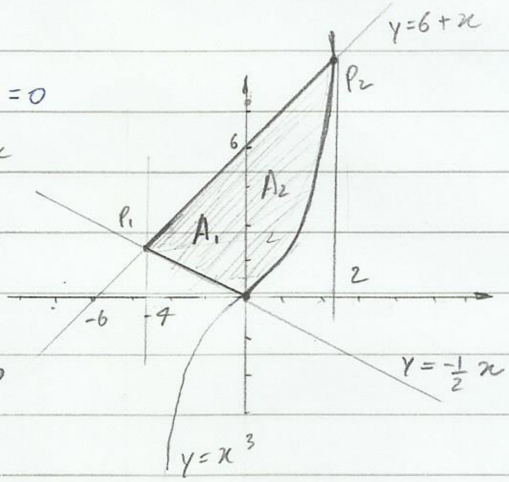
$$\int_{-4}^{4} (y^2-y+1)\,dy =$$

$$= \frac{1}{3}y^3 - \frac{1}{2}y + y \Big|_{-4}^{4} =$$

$$= \frac{4^3}{3} - \frac{4}{2} + 4 + \frac{4^3}{3} - \frac{4}{2} + 4 = \frac{152}{3} \text{ u.a}$$

094-) $y-x=6$; $y-x^3=0$; $2y+x=0$

$y=6+x$ $\quad y=-\frac{1}{2}x$



$$6+x = -\frac{1}{2}x \Rightarrow 6 = -\frac{3}{2}x \Rightarrow x=-4$$

$$A_1 = \int_{-4}^{0} 6+x+\frac{1}{2}x\,dx =$$

$$\int_{-4}^{0} 6+\frac{3}{2}x\,dx = 6x+\frac{3}{4}x^2 \Big|_{-4}^{0}$$

$$\left| -\left(6.(-4)+\frac{3}{4}(-4)^2\right) \right| = 36$$

$$6+x = x^3 \Rightarrow x^3-x-6=0$$

$$P_2 = x - \frac{x^3-x-6}{3x^2-1} \Rightarrow P_2 = 2$$

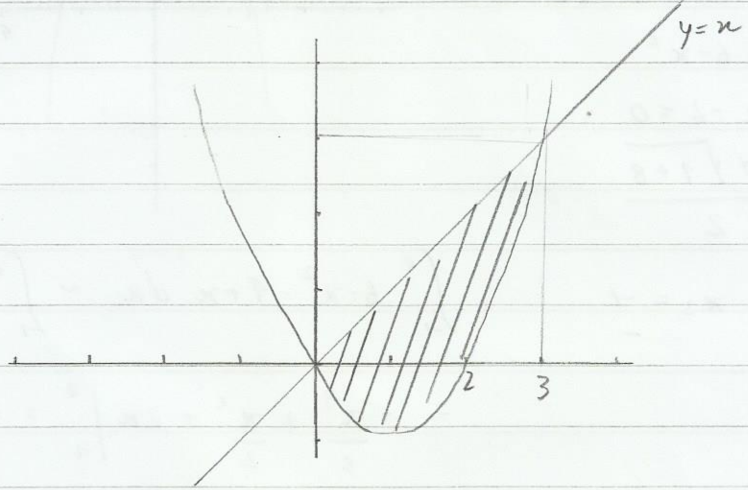
$$A_2 = \int_0^2 6+x-x^3\,dx = 6x+\frac{1}{2}x^2-\frac{1}{4}x^4 \Big|_0^2 = 10$$

$$A_t = 36+10 = 46 \text{ u a}$$

Cálculo 1

1) Calculo de área

a) $y = x \quad y = x^2 - 2x$



$x(x-2) = 0$

$x = 0 \quad x = 2$

$x = x^2 - 2x$

$x^2 - 3x = 0$

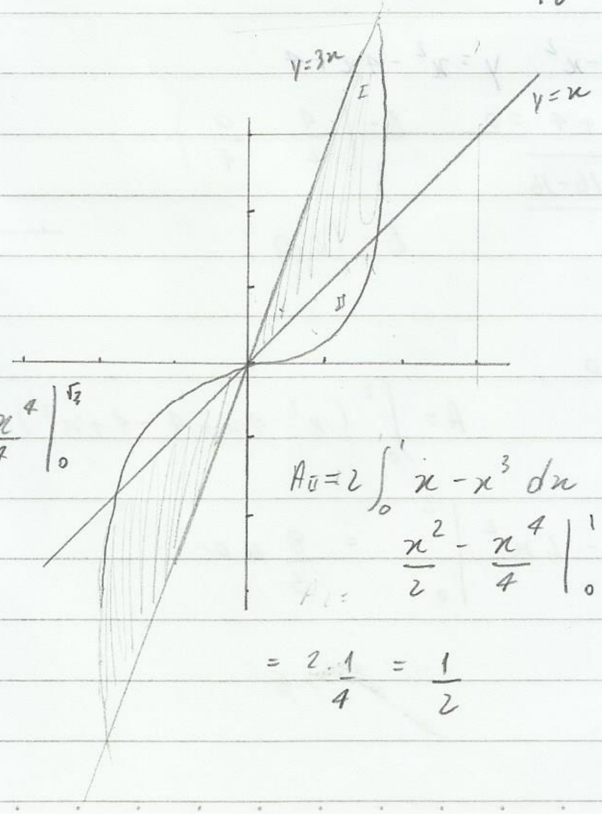
$x(x-3) = 0$

$x = 0 \quad x = 3$

$$A = \int_0^3 x - x^2 + 2x \, dx \sim \int_0^3 -x^2 + 3x \, dx = -\frac{x^3}{3} + \frac{3}{2}x^2 \Big|_0^3$$

$$-\frac{(3)^3}{3} + \frac{3}{2} \cdot 3^2 = \frac{9}{2} \; u.a$$

b) $y = x^3 \quad y = 3x \quad y = x$



$x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x = 0 \quad x = \sqrt{3}$

$$A = 2\int_0^{\sqrt{3}} 3x - x^3 \, dx = \frac{3}{2}x^2 - \frac{x^4}{4} \Big|_0^{\sqrt{3}}$$

$A_I = \frac{9}{2}$

$x^3 = x$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0 \quad x = \pm 1$

$$A_{II} = 2\int_0^1 x - x^3 \, dx = \frac{x^2}{2} - \frac{x^4}{4} \Big|_0^1$$

$$= 2 \cdot \frac{1}{4} = \frac{1}{2}$$

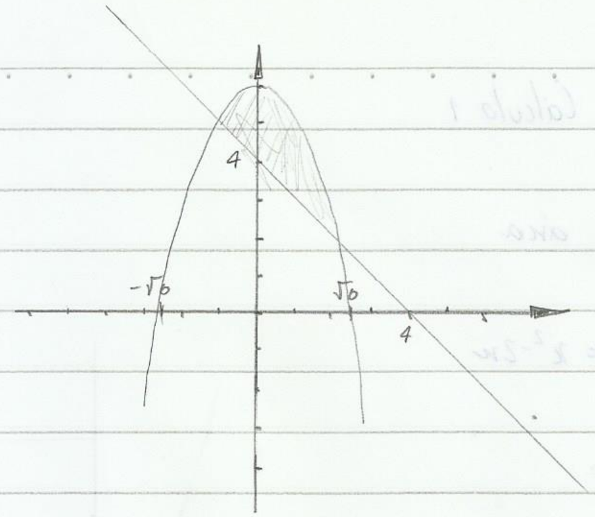
$$At = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4 \; u.a$$

c) $x + y = 4$ $y = 6 - x^2$

$y = 4 - x$ $6 - x^2 = 0$

$6 = x^2$

$x = \sqrt{6}$



$4 - x = 6 - x^2$

$x^2 - x - 2 = 0$

$x = \dfrac{1 \pm \sqrt{1+8}}{2}$

$x_1 = 2$ $x_2 = -1$

$$\int_{-1}^{0} 6 - x^2 - 4 + x \, dx \sim \int_{-1}^{0} -x^2 + x + 2 \, dx =$$

$$= -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{0} = -\left(\frac{1}{3} + \frac{1}{2} - 2\right) = \frac{7}{6}$$

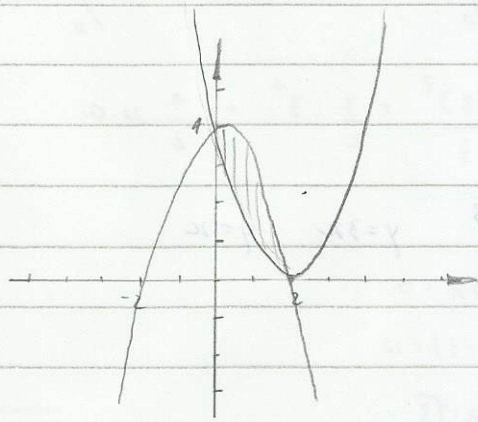
$$\int_{0}^{2} -x^2 + x + 2 \, dx = -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{0}^{2} = -\frac{8}{3} + \frac{4}{2} + 4 = \frac{20}{6}$$

$$A_t = \frac{20}{6} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2} \text{ u.a}$$

d) $y = 4 - x^2$ $y = x^2 - 4x + 4$

$x^2 - 4x + 4 = 0$ $\Delta - \left(\frac{4}{2}, \frac{0}{4}\right)$

$x = \dfrac{4 \pm \sqrt{16-16}}{2}$ $\Delta = (2,0)$



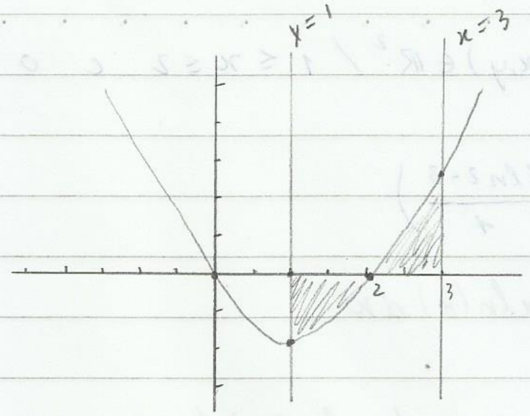
$x = 2$

$4 - x^2 = 0$

$x = \pm 2$ $A = \int_{0}^{2} (x^2 - 4x + 4 - 4 + x^2) \, dx \sim \int_{0}^{2} 2x^2 - 4x \, dx =$

$$= \frac{2x^3}{3} - 2x^2 \Big|_{0}^{2} = \frac{8}{3} \text{ u.a}$$

e) $y = x^2 - 2x \quad x = 1 \quad x = 3$



$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad x = 2$

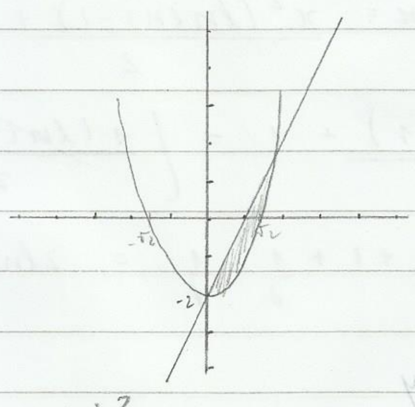
$A_1 = \int_1^2 x^2 - 2x \, dx = \frac{x^3}{3} - x^2 \Big|_1^2$

$\frac{2^3}{3} - 2^2 - \left(\frac{1}{3} - 1 \right) = \frac{2}{3}$

$A_2 = \int_2^3 x^2 - 2x \, dx = \frac{x^3}{3} - x^2 \Big|_2^3 \Rightarrow \frac{3^3}{3} - 3^2 - \left(\frac{2^3}{3} - 2^2 \right) = \frac{4}{3}$

$A_t = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2 \text{ u.a}$

f) $y = x^2 - 2 \quad y = 2x - 2$



$x^2 - 2 = 2x - 2$

$x = \pm\sqrt{2} \quad x^2 - 2x = 0$

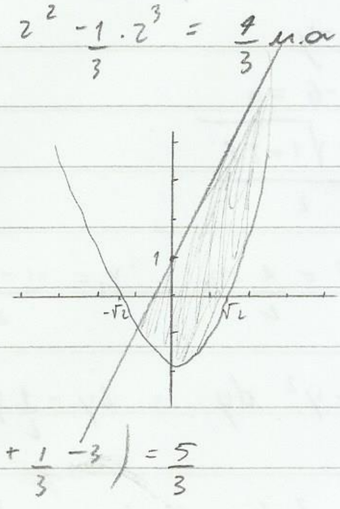
$x(x-2) = 0$

$x = 0 \text{ e } x = 2$

$\int_0^2 (2x - 2 - x^2 + 2) \, dx$

$\int_0^2 2x - x^2 \, dx = x^2 - \frac{1}{3} x^3 \Big|_0^2 = 2^2 - \frac{1}{3} \cdot 2^3 = \frac{4}{3} \text{ u.a}$

g) $y = x^2 - 2 \quad y = 2x + 1$



$x = \pm\sqrt{2} \quad x^2 - 2 = 2x + 1$

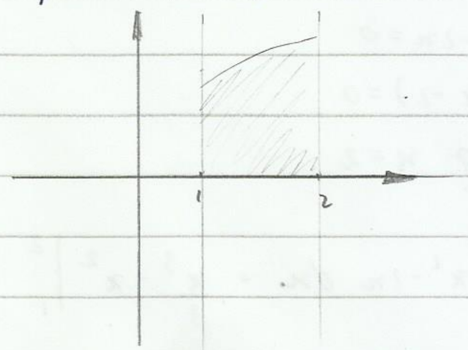
$x^2 - 2x - 3 = 0 \quad \begin{cases} x_1 = 3 \\ x_2 = -1 \end{cases}$

$x = \frac{2 \pm \sqrt{4 + 12}}{2}$

$\int_{-1}^0 2x - x^2 + 3 \, dx = x^2 - \frac{1}{3} x^3 + 3x \Big|_{-1}^0 = -\left(1 + \frac{1}{3} - 3 \right) = \frac{5}{3}$

$x^2 - \frac{1}{3} x^3 + 3x \Big|_0^3 = 3^2 - \frac{1}{3} 3^3 + 3.3 = 9 \qquad A = \frac{5}{3} + 9 = \frac{5 + 27}{3} = \frac{32}{3} \text{ u.a}$

$h)\quad R=\{(x,y)\in\mathbb{R}^2 \,/\, 1\le x\le 2 \ \text{e} \ 0\le y\le x\ln(x)\}$



$A = \left(\frac{8\ln 2-3}{4}\right)$

$A=\int_1^2 x\ln(x)\,dx$

$u=x \qquad dv=\ln(x)\,dx$

$du=dx \qquad v=\int \ln(x)\,dx = x(\ln x-1)$

$\int x\ln(x) = x^2(\ln x-1)-\int x(\ln x-1)\,dx$

$= x^2(\ln x-1)-\int x\ln x\,dx+\int x\,dx$

$2\int x\ln x = x^2(\ln x-1)+\frac{x^2}{2}$

$\int x\ln(x)\,dx = \frac{x^2(\ln(x)-1)}{2}+\frac{x^2}{4}\Big|_1^2$

$= \frac{4(\ln(2)-1)}{2}+1-\left(\frac{1(\ln(1)-1)}{2}+\frac{1}{4}\right)$

$= 2\ln(2)-2+1+\frac{1}{2}-\frac{1}{4} = 2\ln(2)-\frac{3}{4} = \frac{8\ln(2)-3}{4}$ u.a

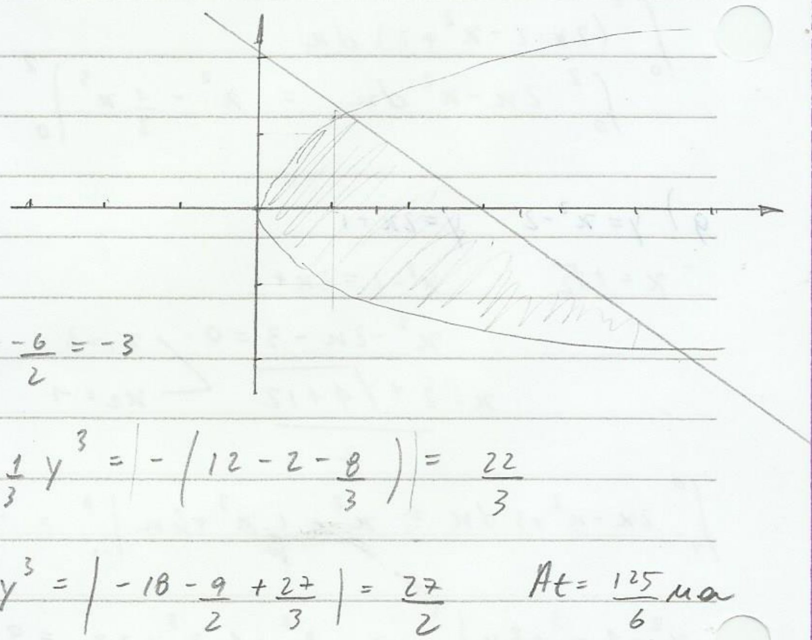
$i)\quad y^2=x \qquad x=6-y$

$y^2=6-y$

$y^2+y-6=0$

$y=\frac{-1\pm\sqrt{1+24}}{2}$

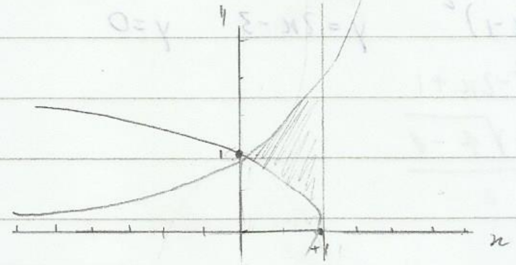
$y_1=\frac{-1+5}{2}=\frac{4}{2}=2 \qquad y_2=\frac{-1-5}{2}=\frac{-6}{2}=-3$



$\int_2^0 6-y-y^2\,dy = 6y-\frac{1}{2}y^2-\frac{1}{3}y^3 = \left|-\left(12-2-\frac{8}{3}\right)\right| = \frac{22}{3}$

$\int_0^{-3} 6-y-y^2\,dy = 6y-\frac{1}{2}y^2-\frac{1}{3}y^3 = \left|-18-\frac{9}{2}+\frac{27}{3}\right| = \frac{27}{2} \qquad At=\frac{125}{6}$ u.a

j) $y = e^x \qquad y = \sqrt{1-x}$



$$A = \int_0^1 e^x - \sqrt{1-x} \, dx =$$

$$\int \sqrt{1-x}\, dx \Rightarrow -\int \sqrt{t}\, dt =$$

$1 - x = t \qquad -\frac{2}{3} t^{\frac{3}{2}} = -\frac{2}{3}(1-x)^{\frac{3}{2}}$

$-dx = dt$

$$\Rightarrow A = e^x + \frac{2}{3}(1-x)^{\frac{3}{2}} \Big|_0^1 = e - 1 - \frac{2}{3} = e - \frac{5}{3} = \frac{3e-5}{3} \text{ u.a}$$

k) $x^2 + y^2 = 2 \qquad y = \sqrt{x} \qquad y = 0$

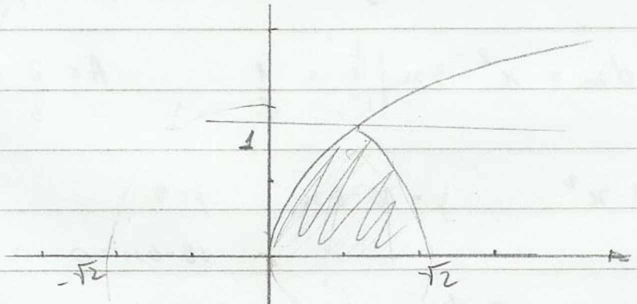
$x = \sqrt{2-y^2} \qquad x = y^2$

$2 - y^2 = y^4$

$y^4 + y^2 - 2 = 0 \qquad y^2 = t$

$t^2 + t - 2 = 0$

$t = \frac{-1 \pm \sqrt{1+8}}{2} \qquad t_1 = 1 \qquad t_2 = -2$



$$\int_0^1 \left(\sqrt{2-y^2} - y^2\right) dy \sim \int_0^1 \sqrt{2-y^2}\, dy - \int_0^1 y^2\, dy \rightarrow \frac{1}{3} y^3$$

$$\int_0^1 \sqrt{2-y^2}\, dy \sim \int_0^1 \sqrt{(\sqrt{2})^2 - y^2}\, dy = \frac{y}{2}\sqrt{2-y^2} + \arcsin\left(\frac{y}{\sqrt{2}}\right)$$

$$A = \frac{y}{2}\sqrt{2-y^2} + \arcsin\left(\frac{y}{\sqrt{2}}\right) - \frac{1}{3} y^3 \Big|_0^1$$

$$A = \frac{1}{2} + \frac{\pi}{4} - \frac{1}{3} = \frac{1}{6} + \frac{\pi}{4} = \frac{2+3\pi}{12} \text{ u.a}$$

l) $y=(x-1)^2 \qquad y=2x-3 \qquad y=0$

$y=x^2-2x+1$

$x=\frac{2\pm\sqrt{4-4}}{2}$

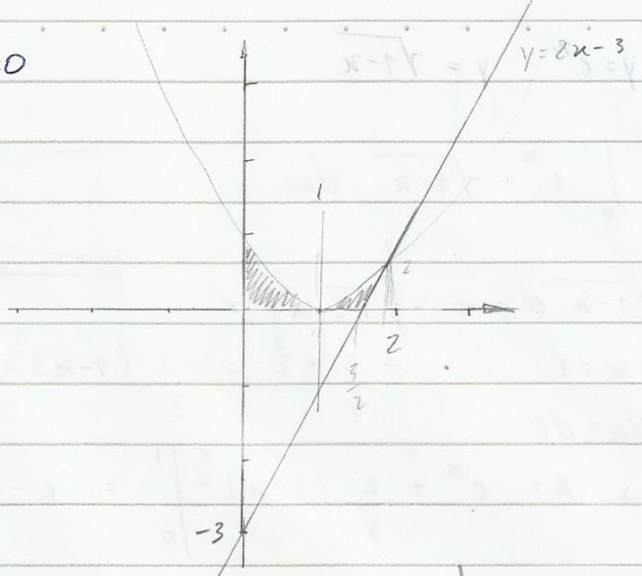
$x=1$

$x^2-2x+1=2x-3$

$x^2-4x+4=0$

$x=\frac{4\pm\sqrt{16-16}}{2}$

$x=\frac{4}{2}=2$



$\int_0^1 x^2-2x+1\,dx = \frac{1}{3}x^3-x^2+x\Big|_0^1 = \frac{2}{3}$

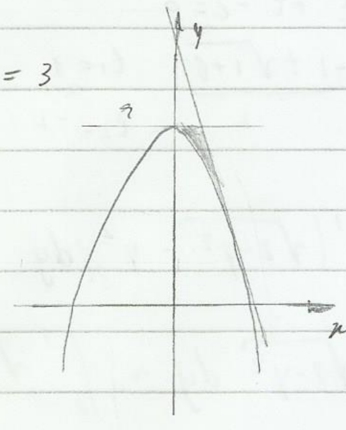
$\int_1^2 x^2-2x+1-2x-3\,dx \sim \int_1^2 x^2-4x-2\,dx =$

$= \frac{1}{3}x^3-2x^2-2x\Big|_1^2 = \frac{17}{3}$

$\int_1^{\frac{3}{2}} 2x-3\,dx = x^2-3x\Big|_1^{\frac{3}{2}} = \frac{1}{4} \qquad A=\frac{2}{3}+\frac{17}{3}-\frac{1}{4}=$

m) $y=9-x^2 \qquad y=18-6x \qquad y=9$

$18-6x=0 \quad \therefore x=3$



$A_1=\int_0^3 18-6x-9+x^2\,dx \sim \int_0^3 x^2-6x+9\,dx =$

$\frac{1}{3}x^3-3x^2+9\Big|_0^3 = 18$

$18-6x=9 \quad \therefore x=3/2$

$A_2=\int_0^{\frac{3}{2}} 18-6x\,dx = 18x-3x^2\Big|_0^{\frac{3}{2}} = 27-\frac{27}{4}=\frac{81}{4}$

$A = 18-\frac{81}{4}$

Comprimento de arco

a) $x = t^2 \qquad y = \frac{\sqrt{3}\,(3+4t)^{3/2}}{6}$

$y' = \frac{\sqrt{3}}{6} \cdot \frac{3}{2}(3+4t)^{1/2} = \left(\frac{\sqrt{3}}{4}(3+4t)^{1/2}\right)^2 = \frac{3}{16}(3+4t)$

$= \frac{9+12t}{16} + 1 = \frac{12t+25}{16}$

$\sqrt{\frac{12t+25}{16}} = \frac{\sqrt{12t+25}}{4}$

$\frac{1}{4}\int_0^4 \sqrt{12t+25}\,dt \sim \frac{1}{4}\int_0^4 \sqrt{u} \cdot \frac{1}{12}\,du = \frac{1}{48}\int_0^4 \sqrt{u}\,du =$

$12t + 25 = u$

$12\,dt = du$

$dt = \frac{1}{12}\,du$

$\frac{1}{48} \cdot \frac{2}{3}(12t+25)^{\frac{2}{3}}\Big|_0^1 =$

?

c) = $y = \frac{x^2}{2} - \frac{\ln(x)}{4}$

$y' = x - \frac{1}{4} \cdot \frac{1}{x} = x - \frac{1}{4x}$

$\left(x - \frac{1}{4x}\right)^2 = x^2 - \frac{1}{2} + \frac{1}{16x^2}$

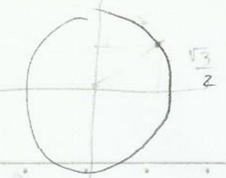
$x^2 + \frac{1}{16x^2} - \frac{1}{2} + 1 = x^2 + \frac{1}{16x^2} + \frac{1}{2} = \left(x + \frac{1}{4x}\right)^2$

$L = \int_2^4 \sqrt{\left(x + \frac{1}{4x}\right)^2}\,dx \sim \int_2^4 x + \frac{1}{4x}\,dx = \frac{x^2}{2} + \frac{1}{4}\ln|x|\Big|_2^4$

$8 + \frac{1}{4}\ln(4) - 2 - \frac{1}{4}\ln(2) = \boxed{6 + \frac{1}{4}\ln(2)}$ u.c

$\sin^2 x + \cos^2 x \equiv 1$

$tg^2 x + 1 = \sec^2 x$



b) $y = \ln(\cos(x))$

$y' = \dfrac{-\operatorname{sen} x}{\cos x} = -tg\,x \qquad (y')^2 = tg^2 x \qquad (y')^2 + 1 = tg^2 x + 1 = \sec^2 x$

$L = \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 x}\, dx \sim \int_0^{\frac{\pi}{3}} \sec(x)\, dx = \ln|\sec(x) + tg(x)| \Big|_0^{\frac{\pi}{3}}$

$\sec(x) = \dfrac{1}{\cos(x)} = \dfrac{1}{0{,}5} = 2$

$tg(x) = \sqrt{3}$

	30	45	60
sen	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tg	$\sqrt{3}/3$	1	$\sqrt{3}/1$

$L = \ln(2 + \sqrt{3})$ u.c

→ $y = f(x) = \dfrac{1}{6}\left(\sqrt{(4x+3)^3}\right)$

$f'(x) = \dfrac{1}{6} \cdot \dfrac{3}{2} \cdot (4x+3)^{1/2} \cdot 4 = (4x+3)^{1/2}$

$(f'(x))^2 = 4x + 3 \qquad (f'(x))^2 + 1 = 4x + 4 = 4(x+1)$

$\int_0^3 \sqrt{4(x+1)}\, dx \sim \int_0^3 2\sqrt{x+1}\, dx$

$x + 1 = t$ $\qquad 2\int \sqrt{t}\, d = 2 \cdot \dfrac{2}{3} (x+1)^{\frac{3}{2}} \Big|_0^3 =$

$dx = dt$

$4^{\frac{3}{2}} = \sqrt{4^3}$

$\dfrac{4}{3}(4)^{\frac{3}{2}} - \dfrac{4}{3} = \dfrac{28}{3}$ u.c

→ Calcule o comprimento de arco da curva:

- $x = t^2 \qquad y = \frac{\sqrt{3}(3+4t)^{3/2}}{6} \qquad 0 \leq t \leq 4$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = \frac{\sqrt{3}}{\cancel{6}} \cdot \frac{3}{2}(3+4t)^{\frac{1}{2}} \cdot 4 = \sqrt{3}(3+4t)^{1/2}$$

$$L = \int_0^4 \sqrt{(2t)^2 + (\sqrt{3}(3+4t)^{1/2})^2}\, dt$$

$$= \int_0^4 \sqrt{4t^2 + 3(3+4t)}\, dt$$

$$= \int_0^4 \sqrt{4t^2 + 9 + 12t}\, dt \qquad 4t^2 + 12t + 9 = (2t+3)^2$$

$$= \int_0^4 2t+3\, dt = t^2 + 3t \Big|_0^4 = 4^2 + 3.4 = 28 \text{ u.c}$$

- $x(t) = \frac{t^2}{2} \qquad y(t) = \frac{2t^{5/2}}{5} \qquad 0 \leq t \leq 1$

$$\frac{dx}{dt} = \frac{1}{2} \cdot 2t = t \qquad \frac{dy}{dt} = \frac{2}{5} \cdot \frac{5}{2} t^{\frac{3}{2}} = t^{\frac{3}{2}}$$

$$L = \int_0^1 \sqrt{t^2 + t^3}\, dt \sim L = \int_0^1 \sqrt{t^2(1+t)}\, dt$$

$$L = \int_0^1 t\sqrt{1+t}\, dt \sim L = \int_0^1 (s-1)\sqrt{s}\, ds \sim L = \int_0^1 s\sqrt{s} - \sqrt{s}\, ds$$

$1+t = s$

$dt = ds$

$t = s-1$

$$= \frac{2(1+t)^{\frac{5}{2}}}{5} - \frac{2}{3}(1+t)^{\frac{3}{2}} \Big|_0^1 = \frac{4}{15}(\sqrt{2}+1) \text{ u.c}$$

$\to$ $y = f(x) = \frac{1}{96}(\sqrt{16x})^3$ $\qquad (\sqrt{16x})^3 = 16x^{\frac{1}{2}\cdot 3} = (16x)^{\frac{3}{2}}$

$f'(x) = \frac{1}{96} \cdot \frac{3}{2} (16x)^{\frac{1}{2}} \cdot 16 = \frac{1}{4}(16x)^{\frac{1}{2}}$

$(f'(x))^2 = \frac{1}{16} 16x = x$

$(f'(x))^2 + 1 = x+1$

$L = \int_0^3 \sqrt{x+1}\, dx$

$x+1 = t$ $\qquad \int \sqrt{t}\, dt = \frac{2}{3}(x+1)^{\frac{3}{2}} \Big|_0^3 =$

$dx = dt$

$\frac{2}{3}\sqrt{4^3} - \frac{2}{3}\sqrt{1} = \frac{2}{3} \cdot 7 = \frac{14}{3}$ u.c

$\to$ $y = f(x) = \frac{1}{3}(\sqrt{2x-1})^3$ $\qquad x \in [0,2]$

$f'(x) = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} (2x-1)^{\frac{1}{2}} \cdot 2 = (2x-1)^{\frac{1}{2}}$

$(f'(x))^2 = 2x-1$

$(f'(x))^2) + 1 = 2x$

$L = \int_0^2 \sqrt{2x}\, dx \sim \sqrt{2} \int_0^2 \sqrt{x}\, dx =$

$\sqrt{2} \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_0^2 = \sqrt{2} \cdot \frac{2}{3}\sqrt{8} = \frac{2}{3} \cdot \sqrt{2} \cdot 2\sqrt{2} = \frac{8}{3}$ u.c