112103 68-4

Rua Augusta 440

11-12 22 29

/ /

MA1110 - Cálculo 1

Turma 1

3ª feira: Marcos
4ª feira: J. Emilio
5ª feira: J. Emilio

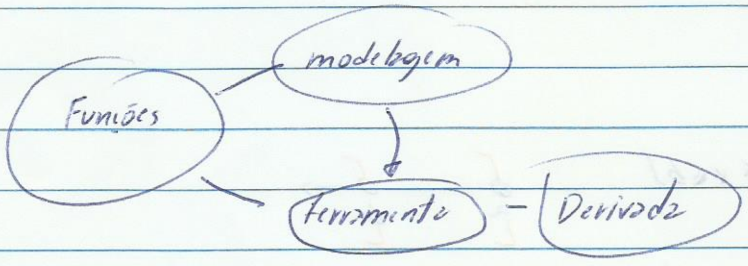
Livro texto

Cálculo básico: Teoria e Exercício
A. Novazzi / A. P. Loreto Jr LCTE editor

Média

$M = (0{,}4 \cdot P_1 + 0{,}6 P_2) \cdot \text{fator} \geq 5{,}0$ Aprovado

fator: atividades ⇔ Pelo menos 4 aceitos → f = 1.1.
Nos demais caso: f = 1.0.



1 - Funções

1.1 Conjunto dos nº Reais

$\mathbb{R}$

Subconjunto

a) $\mathbb{N}$: naturais: 1, 2, 3, 4, 5, 6...

b) $\mathbb{Z}$: inteiros: 0, ±1, ±2, ±3...

c) $\mathbb{Q}$: racionais: (fracionário)

$\to \dfrac{a/b}{c/d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{b \cdot c}$

$\to \dfrac{a/b}{c} = \dfrac{a}{b \cdot c}$

$\to \dfrac{a}{b/c} = \dfrac{a \cdot c}{b}$

$\to \dfrac{a \pm b}{c} = \dfrac{a}{c} \pm \dfrac{b}{c}$

$\to \dfrac{a \cdot b}{c} = \dfrac{a}{c} \cdot b = \dfrac{a \cdot b}{c}$

d) Intervalos:

* aberto: $a<b$

$]a,b[$

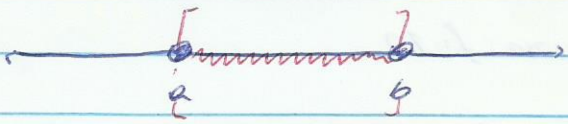
propriedade

$a<b = \{x \in \mathbb{R} \mid a<x<b\}$

pertence tal que tenha a

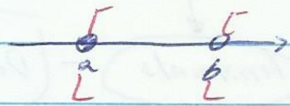
* fechado

$[a,b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

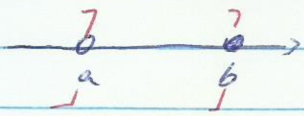


* semi-aberto / semi-fechado

$[a,b[= \{x \in \mathbb{R} \mid a \leq x < b\}$



$]a,b] = \{x \in \mathbb{R} \mid a < x \leq b\}$

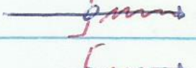


* Generalizados

$]-\infty, a[= \{x \in \mathbb{R} \mid x < a\}$ 

$]-\infty, a] = \{x \in \mathbb{R} \mid x \leq a\}$ 

$]a, +\infty[= \{x \in \mathbb{R} \mid x > a\}$ 

$[a, +\infty[= \{x \in \mathbb{R} \mid x \leq a\}$ 

Exercício: Escreva o conjunto

$A = \{x \in \mathbb{R} \mid x^2 - 1 < 0\}$

sob a forma de intervalo

Domínio

~~$x^2 - 1 < 0$~~

~~$x^2 < 1$~~

~~$x < \sqrt{1}$~~

~~$\sqrt{1}$~~

Resolução (Professor): $a^2 - b^2 = (a-b)(a+b)$ (fatoração)

$x^2 - 1 = (x-1)(x+1)$

Aplicar regra de sinais

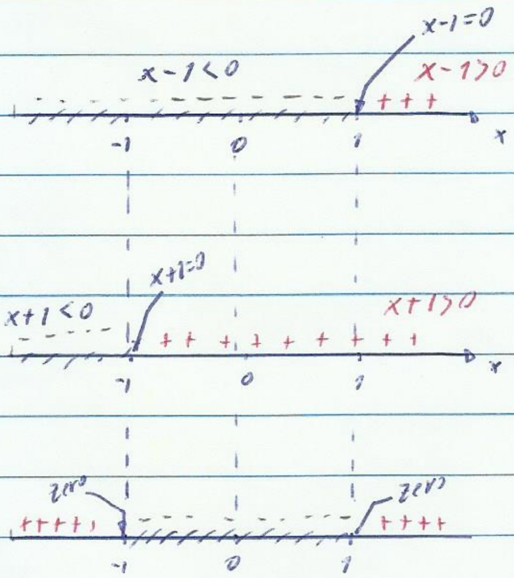
1) $x-1$

$\begin{cases} x-1 > 0 & x > 1 \\ x-1 = 0 & x = 1 \\ x-1 < 0 & x < 1 \end{cases}$

2) $x+1$

$\begin{cases} x+1 > 0 \Rightarrow x > -1 \\ \vdots \end{cases}$

3) produto: $(x-1)(x+1)$



Resp: $A =]-1, 1[$

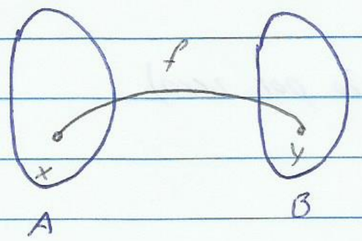
/ /

11210368 4

Funções

1. Conceito
2. Domínio
3. Imagem
4. Gráfico

1. Conceito



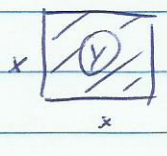
A,B: conjunto não vazios

f: relação entre A e B será função se,

"Todo elemento x de A tem em correspondência um único valor y de B, indicado por $y = f(x)$"

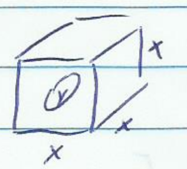
Exemplos

1) A área de um quadrado é função dos lados



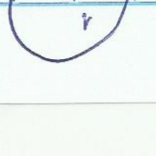
$y = x^2$

2) Volume de um cubo é função das arestas



$y = x^3$

3) O comprimento de uma circunferência é função do raio



$\ell = 2\pi . r$

(/ /)

2. Domínio : Conjunto A

Quando A não for indicado explicitamente será, por convenção, o "mais amplo" subconjunto dos nºs reais onde f tenha sentido ("existência")

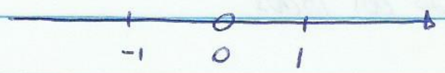
Notação: $A = D_f$

① $y = f(x) = \dfrac{1}{x}$

Condição de existência: $x \neq 0$ (não há divisão por zero)

$\boxed{\text{Erro} = \dfrac{1}{0} = 0}$

Domínio: $D_f = \{x \in \mathbb{R} \mid x \neq 0\} = \mathbb{R} - \{0\} =$
$=]-\infty, 0[\cup]0, +\infty[$

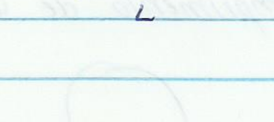


② $y = \sqrt{x-1}$

Condição de existência: $x - 1 \geq 0$ (Não há raiz quadrada de valores negativo)

$\therefore x - 1 \geq 0 \Rightarrow x \geq 1$

Domínio: $D_f = \{x \in \mathbb{R} \mid x \geq 1\} = [1, +\infty[$



tilibra

Exercício: Dê o domínio de

$$f(x) = \sqrt{\frac{x-1}{x+2}}$$

Sob a forma de intervalos.

Resolução:

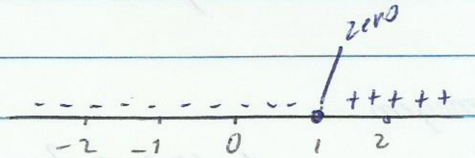
condições de existência:

① $\frac{x-1}{x+2} \geq 0$

② $x+2 \neq 0 \Rightarrow x \neq -2$

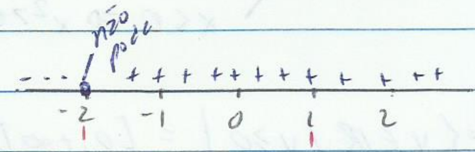
Grade de sinais: $\frac{x-1}{x+2}$

Num.: $x-1$

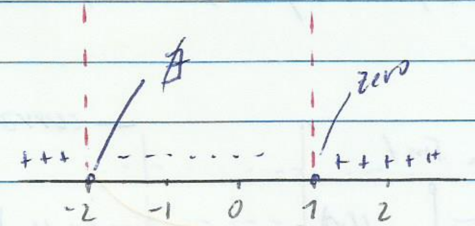


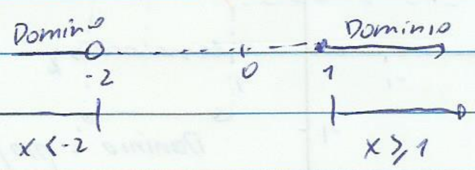
$\begin{cases} x-1 > 0 \Leftrightarrow x > 1 \\ x-1 = 0 \Leftrightarrow x = 1 \\ x-1 < 0 \Leftrightarrow x < 1 \end{cases}$

Den.: $x+2$



$\begin{cases} x+2 > 0 \Leftrightarrow x > -2 \\ x+2 \neq 0 \Leftrightarrow x \neq 0 \\ x+2 < 0 \Leftrightarrow x < -2 \end{cases}$



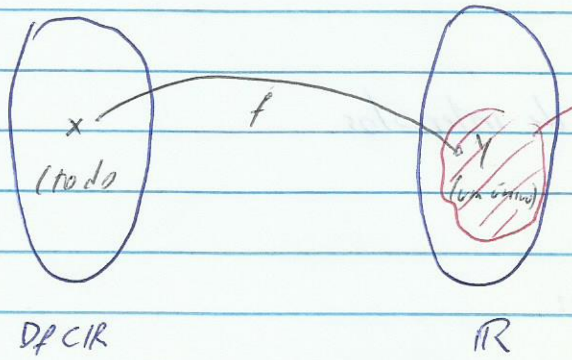


$D_f = \{x \in \mathbb{R} \mid x<-2 \text{ ou } x \geq 1\} =]-\infty, -2[\cup [1, +\infty[$

Reunião

3. Contradomínio: Conjunto B

Quando não indicado será, sempre; $B = \mathbb{R}$



Conjunto-Imagem
$Im f = \{ y \in \mathbb{R} \mid y = f(x) \text{ p/ } x \in D_f \}$

Exemplo: $y = f(x) = x^2$

Domínio: $D_f = \mathbb{R}$
Contradomínio: $CD_f = \mathbb{R}$.

conjunto-imagem

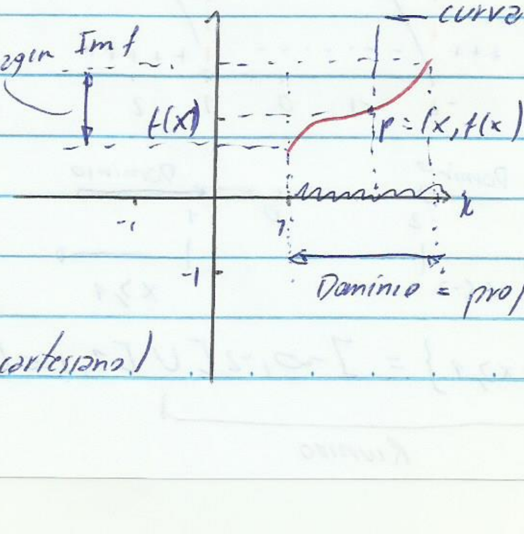
$$x \in D_f = \mathbb{R} \begin{cases} x > 0 \Rightarrow x^2 > 0 \Rightarrow y > 0 \\ x = 0 \Rightarrow 0^2 = 0 \Rightarrow y = 0 \\ x < 0 \Rightarrow x^2 > 0 \Rightarrow y > 0 \end{cases} \Bigg\} \; y \geq 0$$

$\therefore Im f = \{ y \in \mathbb{R} \mid y \geq 0 \} = [0, +\infty[$.

4. Gráfico

curva: Gráfico: $G_f = \{ (x, f(x)) \mid x \in D_f \}$

projeção do gráfico G_f sobre o eixo y (das ordenadas).



Domínio = projeção do gráfico sobre o eixo x. (eixo das abscissas)

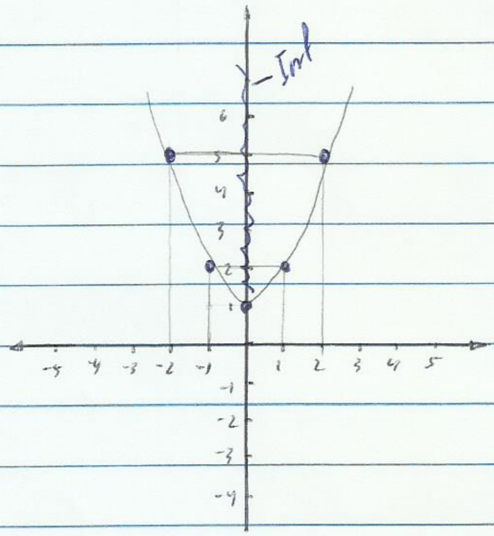
(plano cartesiano)

Exemplo = Gráfico de $y = 1 + x^2$

$x \in Df$	$y = 1 + x^2$
-2	5
-1	2
0	1
1	2
2	5

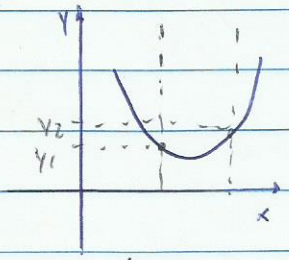
$Df = \mathbb{R}$

$CDf = \mathbb{R}$

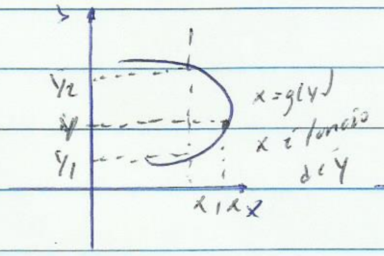


$Imf = [1, +\infty[$

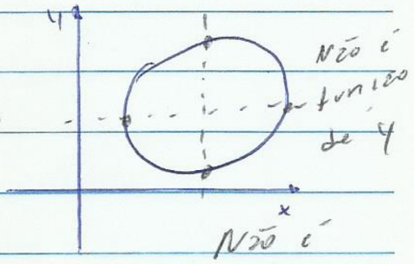
Obs.:



função em x



Não é uma função de x mas poderia ser de y



Não é função de x

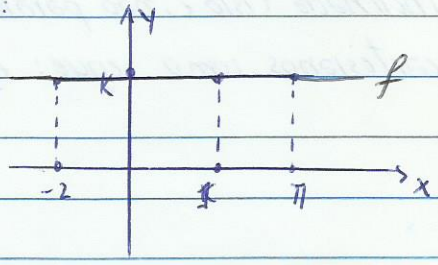
Uma curva será uma função de x se, e somente se, qualquer reta vertical "corta" o seu gráfico em no máximo em um ponto

Funções básicas

1. Função Constante: $f: A \subset \mathbb{R} \to \mathbb{R}$, $y = f(x) = K$, com K uma constante Real

(a) Domínio: $D_f = A = \mathbb{R}$

(b) Gráfico:



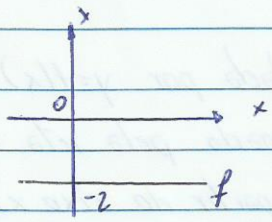
Trata-se de uma reta paralela ao eixo x que intercepta o eixo y no ponto K ($K>0$, $K=0$ ou $K<0$)

(c) Imagem: $Im_f = \{K\}$

ex1: $y = f(x) = -2$

$D_f = \mathbb{R}$

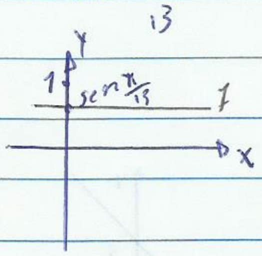
$Im_f = \{-2\}$



ex2: $y = f(x) = \text{sen}\,\frac{\pi}{13}$

$D_f = \mathbb{R}$

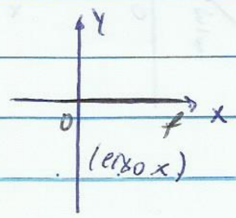
$Im_f = \{\text{sen}\,\frac{\pi}{13}\}$



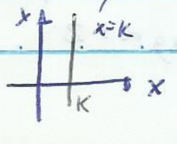
* ex3: $y = f(x) = 0$

$D_f = \mathbb{R}$

$Im_f = \{0\}$



Obs: Relações da forma $x = K$ não consistuem funções de x, mas representam retas paralelas ao eixo y.



em particular $\boxed{x=0}$ representa o eixo y

/ /

2. Função Polinômio de 1º grau

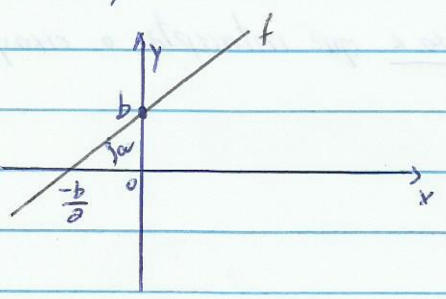
$y = f(x) = ax + b$, com a e b constantes reais e com $a \neq 0$

(a) Domínio = $Df = \mathbb{R}$

(b) Gráfico = Trata-se de uma reta inclinada (isto é, não paralela a nenhum dos eixos) que intercepta os eixos cartesianos como segue: eixo x ($y=0$):

$ax + b = 0 \Rightarrow x = \frac{-b}{a}$

eixo y ($x=0$): $y = a.0 + b = b$



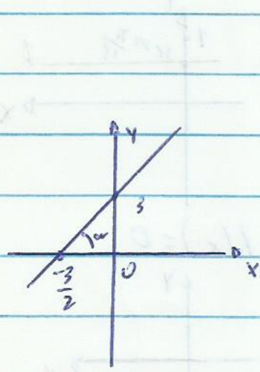
(c) Imagem - $Imf = \mathbb{R}$

Obs.: Chama-se coeficiente angular da reta dada por $y = f(x) = ax + b$, com $a \neq 0$, ao valor da tangente do ângulo formado pela reta e pelo eixo x. sempre contado positivo (sentido anti-horário, apartir do eixo x), isto é:

coef. ang = $tg\,\alpha = a$

$$\frac{b}{-(-\frac{b}{a})}$$

ex1: $y = f(x) = 2x + 3$



$Df = \mathbb{R}$

Gráfico: eixo x ($y=0$): $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

eixo y ($x=0$): $y = 2.0 + 3 = 3$

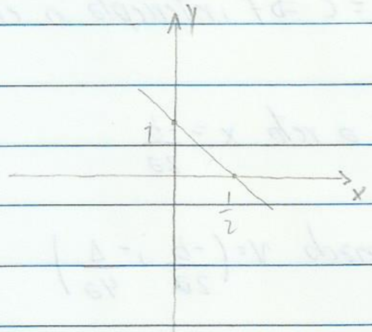
$Imf = \mathbb{R}$. $Tag\,\alpha = 2$ ($\alpha = arctg\,2$).

ex2: $y = f(x) = 1 - 2x$ $-2x + 1$

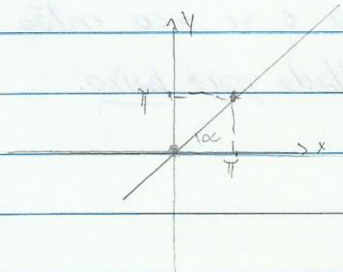
$Df = \mathbb{R}$ $Imf = \mathbb{R}$

Gráfico: $x = \frac{-1}{-2} = \frac{1}{2}$ $\alpha = -2$

$y = 1$



*ex3: $y = f(x) = x$



$Df = \mathbb{R}$ $Imf = \mathbb{R}$

$\alpha = 1$

$x = x$ $(\alpha = \frac{\pi}{4}$ ou $45°)$

$y = 0$

3 Função Polinômio de 2º grau

$$y = f(x) = ax^2 + bx + c, \text{ com } a, b, c \in \mathbb{R} \text{ e } a \neq 0$$

(a) Domínio = $Df = \mathbb{R}$

(b) Gráfico: Trata-se de uma parábola com as seguintes características:

(b_1) Intersecção com os eixos

eixo x ($y = 0$): $ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac$$

Se $\Delta > 0 \Rightarrow$ 2 raízes reais distintas $x_1 < x_2$

$\Rightarrow f$ intercepta o eixo x nos pontos x_1 e x_2

Se $\Delta = 0 \Rightarrow$ 2 raízes reais iguais $x_1 = x_2 = -\frac{b}{2a}$

$\Rightarrow f$ intercepta o eixo x apenas no ponto $\frac{-b}{2a}$

/ /

Se $\Delta<0 \Rightarrow \nexists$ raiz real $\Rightarrow f$ não intercepta o eixo x

eixo y $(x=0)$: $y=a\cdot 0^2+b\cdot 0+c=c \Rightarrow f$ intercepta o eixo y no ponto c

(b2) eixo de simetria: é a reta $x=\frac{-b}{2a}$

(b3) vértice: é o par ordenado $V=\left(\frac{-b}{2a},\frac{-\Delta}{4a}\right)$

Obs.: Se $a>0$ então o vértice é de mínimo $(V=V_{min})$ e a concavidade é voltada para cima e se $a<0$ então o vértice é de máximo $(V=V_{max})$ e a concavidade é voltada para baixo.

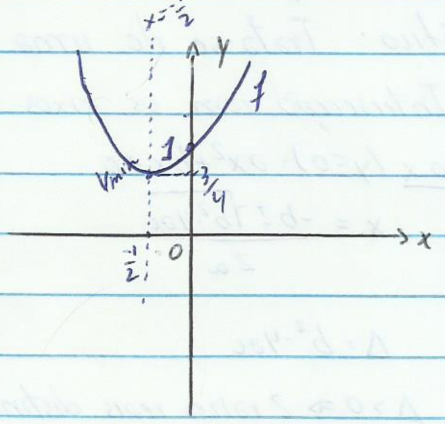
ex1: $y=f(x)=x^2+x+1$

$D_f=\mathbb{R}$

eixo x $(y=0)$: $x^2+x+1=0$

$$x=\frac{-1\pm\sqrt{1-4}}{2} \Rightarrow x=\frac{-1\pm\sqrt{-3}}{2}$$

$\Delta=-3<0$ (não corta o eixo x)



eixo y $(x=0)$: $y=1$

eixo simetria: $x=\frac{-b}{2a}=\frac{-1}{2}$

vértice: $V=\left(\frac{-1}{2},\frac{3}{4}\right)$

$a=1>0 \Rightarrow V=V_{min}$

e concavidade voltada para cima

$Im_f=\left[\frac{3}{4},+\infty\right[$

tilibra

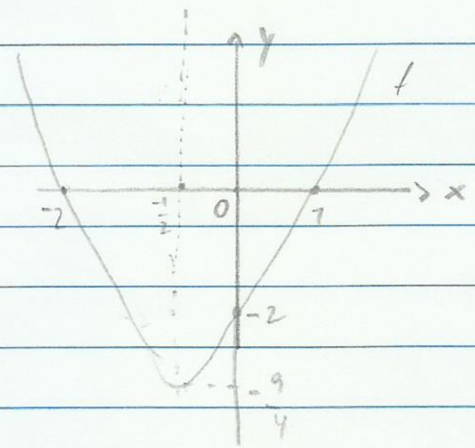
ex2: $y = f(x) = x^2 + x - 2$

eixo x: $x = \frac{-1 \pm \sqrt{1-4.1.-2}}{2.1}$ $\quad x = \frac{-1 \pm \sqrt{9}}{2}$

$x = \frac{-1 \pm \sqrt{1+8}}{2}$ $\quad x_1 = \frac{-1+3}{2} = \frac{2}{2} = 1$ $\quad x_2 = \frac{-1-3}{2} = \frac{-4}{2} = -2$

eixo y (x=0): $y = -2$ $\quad$ vértice = $V = \left(-\frac{1}{2}, -\frac{9}{4}\right)$

eixo simetria: $-\frac{1}{2}$



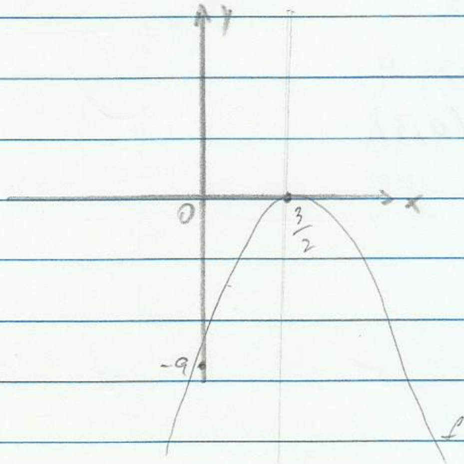
Im f = $\left[\frac{9}{4}, +\infty\right[$

ex3: $-4x^2 + 12x - 9$

eixo x = $x = \frac{-12 \pm \sqrt{12^2 - 4.(-4).(-9)}}{2.(-4)}$ $\quad \left\{ \frac{-12}{2.-4} = \frac{-12}{-8} = \frac{3}{2} \right\}$

$x = \frac{-12 \pm \sqrt{144-144}}{-8}$

$x = \frac{-12 \pm \sqrt{0}}{-8}$



eixo y = -9

eixo simetria = $\frac{3}{2}$

vértice = $\left(\frac{3}{2}, 0\right)$

Im f = $[0, -\infty[$

Funções Modulares

a) Módulo de um número Real

$|3| = +3$

$|-3| = +3$

$|\underbrace{1-\sqrt{3}}_{Neg}| = -(1-\sqrt{3}) = -1+\sqrt{3}$

$$DEF: \ |x| = \begin{cases} x, & se \ x \geq 0 \\ -x, & se \ x < 0 \end{cases}$$

$|\pi - 2| = \pi - 2$

$|\sqrt{3} - 10| = -(\sqrt{3}-10) = -\sqrt{3}+10$

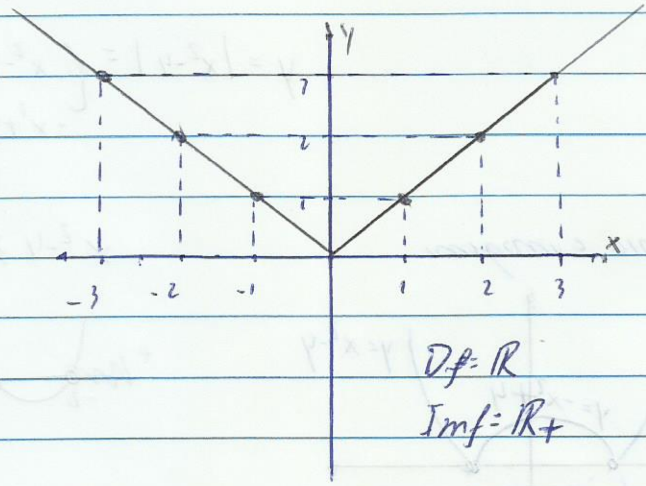
$$|x-3| = \begin{cases} x-3, & se \ x \geq 3 \\ -x+3, & se \ x < 3 \end{cases}$$

Função Modular

EX1

a) $y = |x| = \begin{cases} x, & se \ x \geq 0 \\ -x, & se \ x < 0 \end{cases}$

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3



$Df = \mathbb{R}$

$Imf = \mathbb{R}_+$

EX2 = $y = |2x-1|$

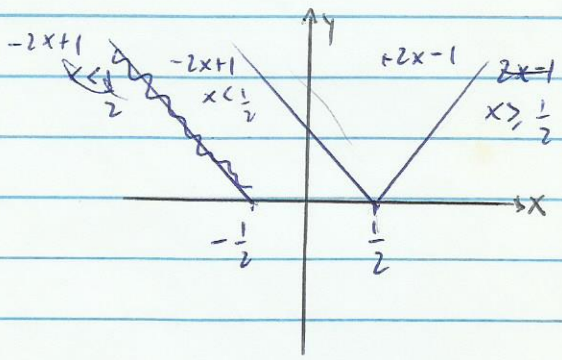
$2x-1=0$
$2x=1$
$x=\frac{1}{2}$

a) esboçar o gráfico
b) determinar domínio e imagem

$$y=|2x-1|=\begin{cases}2x-1, & se\ x \geq 0{,}5\\ -2x+1, & se\ x<0{,}5\end{cases}$$

ou

$$y=|2x-1|=\begin{cases}2x-1, & se\ 2x-1\geq 0\\ -2x+1, & se\ 2x-1<0\end{cases}$$



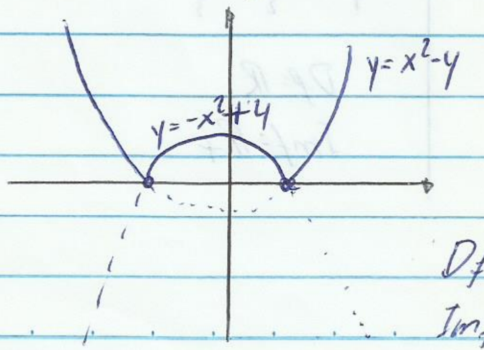
$D_f = \mathbb{R}$
$Im_f = \mathbb{R}_+$

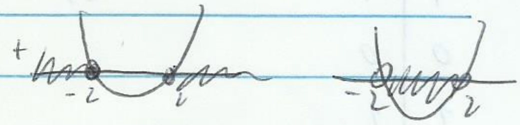
EX3 = $y=|x^2-4|$

$$y=|x^2-4|=\begin{cases}x^2-4, & se\ x\leq -2\ ou\ x\geq 2\\ -x^2+4, & se\ \cancel{x<2}\ -2<x<2\end{cases}$$

a) esboçar o gráfico
b) determinar domínio e imagem

$x^2-4\geq 0$ $\qquad$ $x^2-4<0$





$D_f = \mathbb{R}$
$Im_f = \mathbb{R}_+$

/ /

EX4: $y = x^2 - 3|x| + 2$

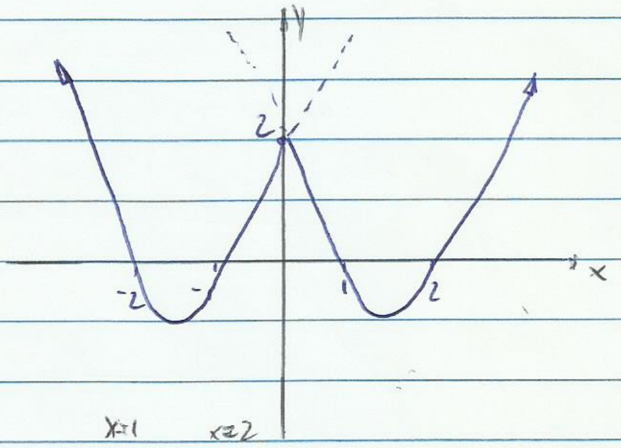
Raízes soma e produto

a) esboçar o gráfico
b) determinar o domínio e imagem

$$y = x^2 - 3|x| + 2 = \begin{cases} x^2 - 3x + 2, & \text{se } x \geq 0 \\ x^2 + 3x + 2, & \text{se } x < 0 \end{cases}$$

$D_f = \mathbb{R}$

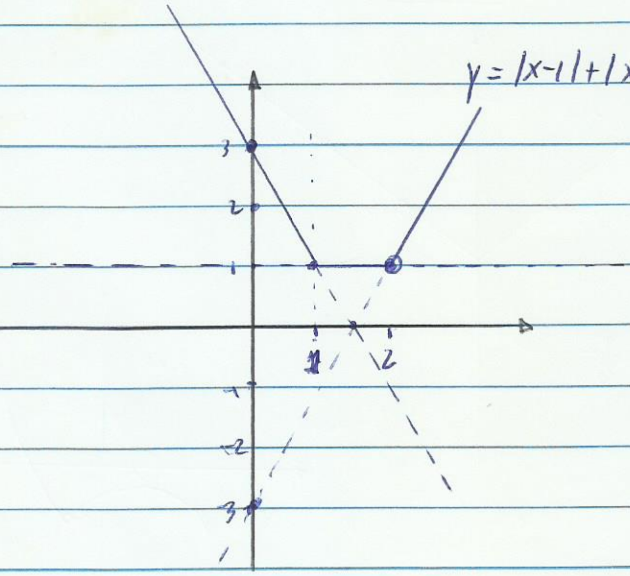
$Im_f = \{ y \in \mathbb{R} \mid y \geq -\frac{1}{4} \}$



EX5: $y = |x-1| + |x-2|$

a) esboçar o gráfico
b) determinar o domínio e imagem

$$y = |x-1| + |x-2| = \begin{cases} -x+1-x+2, & \text{se } x \leq 1 \\ x-1-x+2, & \text{se } 1 < x < 2 \\ x-1+x-2, & \text{se } x \geq 2 \end{cases}$$



$$y = |x-1| + |x-2| = \begin{cases} -2x+3, & \text{se } x \leq 1 \\ 1, & \text{se } 1 < x < 2 \\ 2x-3, & \text{se } x \geq 2 \end{cases}$$

$D_f = \mathbb{R}$

$Im_f = \{ y \in \mathbb{R} \mid y \geq 1 \}$

Exercícios

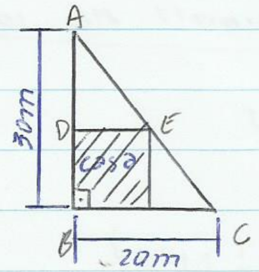
1) A conta mensal de um telefone é dada por $y = m.x + n$ onde x é o número de ~~tham~~ impulsos (nº inteiro) e y é o preço a ser pago em reais. Sabe-se que 30 impulsos custam R$ 23,00 e 40 impulsos custam R$ 24,00.

a) Determinar m e n

b) Esboçar o gráfico da função

c) Calcular o preço de 200 impulsos

2) Num terreno com a forma de um triângulo retângulo deseja-se construir uma casa retangular conforme a figura. ~~Calcular as dimensões x e y da casa de modo que a área ocupada pela casa seja máximo~~



a) escreva y em função de x

b) calcular x e y de modo que a área ocupada pela casa seja máxima

3) Determinar o domínio das funções

a) $y = \sqrt{\dfrac{x^2-4x+3}{x^2-4x}} \geq 0$

b) $y = \dfrac{\sqrt{x^2-4x+3}}{\sqrt{x^2-4x}}$ ≥ 0 ≥ 0

1) $y = m x + n$

$\begin{cases} 30m + n = 23 \ (-1) \\ 40m + n = 24 \end{cases}$

$\begin{cases} -30m - n = -23 \\ 40m + n = 24 \end{cases}$

$10m = 1$

$m = \frac{1}{10} = 0,1$

$30m + n = 23$

$30 \cdot 0,1 + n = 23$

$3 + n = 23$

$n = 23 - 3$

$n = 20$

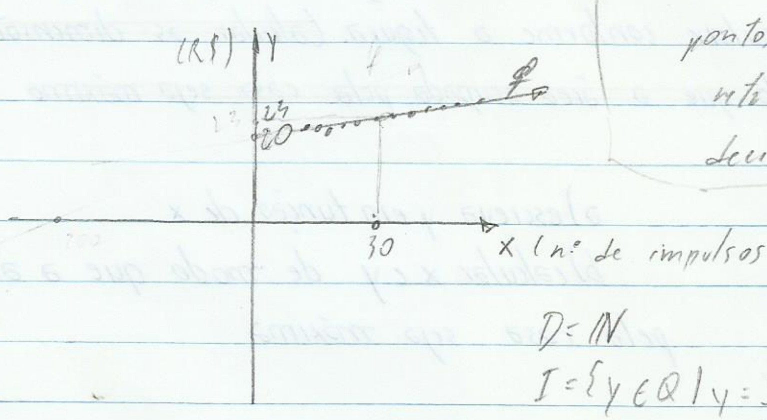
a) $m = 0,1$

$n = 20$

$y = 0,1x + 20$

* Por ser números reais deve ser realizados por pontos e não por retas, pois os valores decimais não somam

b)



$D = \mathbb{N}$

$I = \{ y \in \mathbb{Q} \mid y = \frac{1}{10} x + 20 \}$

c) $y = m \cdot x + n$

$y = 0,1 \cdot 200 + 20$

$y = 20 + 20$

$y = 40$

2) Da figura, temos $\Delta ABC \sim ADE$

$\frac{y}{20} = \frac{(30 - x)}{30}$ $\quad 30y = 600 - 20x$

$y = \frac{600 - 20x}{30} \Rightarrow y = \frac{60 - 2x}{30}$

a) $y = \frac{60 - 2x}{30}$

b) Equação da área

$A = x \cdot y$

$A = x \cdot \left(\frac{60-2x}{3}\right) \Rightarrow x \cdot \left(\frac{60}{3} - \frac{2x}{3}\right)$

$A = \frac{60x}{3} - \frac{2x^2}{3} = \frac{60x - 2x^2}{3}$

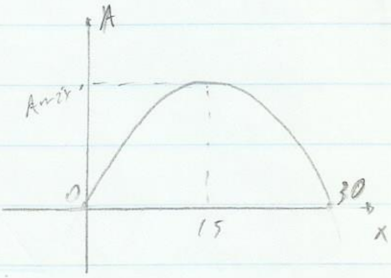
$A = \frac{-2}{3}x^2 + 20x$ ou $\frac{60x-2x^2}{3}$

Raízes

$0 = \frac{60x - 2x^2}{3} = \frac{\overset{0}{\overset{\shortparallel}{2x}}\overset{0}{\overset{\shortparallel}{(30-x)}}}{3} = 0$

$x_1 = 2x = 0 \Rightarrow x_1 = 0$

$x_2 = 30 - x = 0 \Rightarrow x_2 = 30$



$\frac{-b}{2a}; \frac{-\Delta}{4a}$

$x = 15m$

$y = 10m$

$y = \frac{60-2x}{3} =$

$= \frac{60 - 2.15}{3} =$

$\frac{60-30}{3} = \frac{30}{3} = 10$

$Df = \mathbb{R}$

$Im f = \{y \in \mathbb{R} \mid 0 < y \leq 10\}$

3) a) $y=\sqrt{\dfrac{x^2-4x+3}{x^2-4x}}$

$x^2-4x+3=0$

$x=\dfrac{4\pm\sqrt{(-4)^2-4.1.3}}{2.1}$

$x=\dfrac{4\pm\sqrt{16-12}}{2}$

$x_1=\dfrac{4+2}{2}=\dfrac{6}{2}=3$

$x_2=\dfrac{4-2}{2}=\dfrac{2}{2}=1$

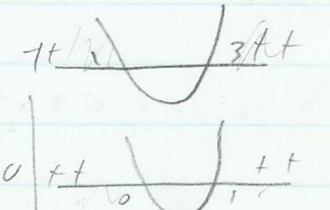
$x^2-4x=0$

condição

$\dfrac{x^2-4x+3}{x^2-4x}\geq 0$

		0	1	3	4
N	+	+	-	+	+
D	+	-	-	-	+
R	+	-	+	-	+

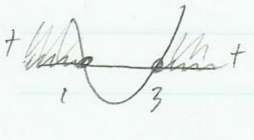
$D_f=\{x\in\mathbb{R} \mid x<0$ ou $1\leq x\leq 3$ ou $x>4\}$



b) $y=\dfrac{\sqrt{x^2-4x+3}}{\sqrt{x^2-4x}}$

1ª condição

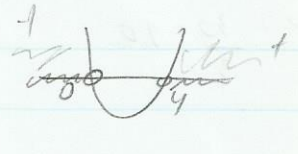
$x^2-4x+3\geq 0$



$x\leq 1$ ou ≥ 3

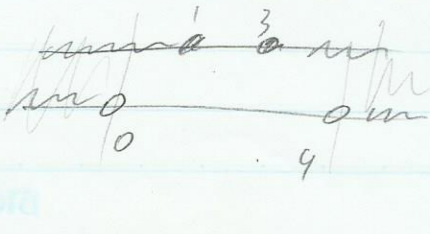
2ª condição

$x^2-4x>0$



$x<0$ ou $x>4$

intersecção



$D=\{x\in\mathbb{R} \mid x<0$ ou $x>4\}$

Ex:

4) Uma máquina custa R$ 20.000,00 e sofre uma depreciação linear até valer R$ 1000,00 após 10 anos.

a) escreva a expressão do valor V da máquina em função do tempo t

b) calcule o valor da máquina após 4 anos

Resp. a) $V = -1900t + 20000$
b) R$ 12400,00

5) Um fabricante estima que o lucro L obtido com a venda de um produto em função do preço de venda p é dado por $L(p) = (p-20)(120-p)$ reais.

a) Esboce o gráfico da função

b) Estime o preço p para o qual o lucro é máximo.

Resp. b) R$ 70,00

6) Determinar o domínio

a) $f(x) = \sqrt{|5-2x|-7}$ Resp $D = \{x \in \mathbb{R} \mid x \leq -1 \text{ ou } x \geq 6\}$

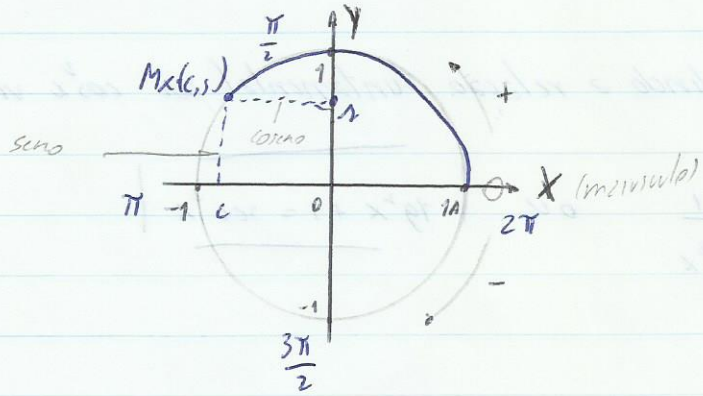
b) $f(x) = \dfrac{x-3}{\sqrt{|x-2|}}$ Resp $D = \{x \in \mathbb{R} \mid x \neq 2\}$

c) $f(x) = \dfrac{|x-3|}{\sqrt{1-|x|}}$ Resp $D = \{x \in \mathbb{R} \mid -1 < x < 1\}$

Funções trigonométricas

seno = ord
cos = abs

Num sistema cartesiano ortogonal, considere a circunferência C de centro na origem O=(0,0) e raio R=1 unidade e sobre esta circunferência considere o ponto A=(1,0)



Arcos desta circunferência são medidos em radianos (rad) e são contados positivos a partir do ponto A=(1,0) no sentido antihorário e negativos também a partir de A porém no sentido horário. Assim um arco completo de A até A no sentido antihorário terá o comprimento da circunferência C, isto é, $2\pi.1 = 2\pi$ rad ou 360°.

Dado $x \in \mathbb{R}$, seja $M_x=(c,s)$ o ponto da circunferência C de modo que o arco $\widehat{AM_x}$, tenha comprimento ou medida x, respeitadas as convenções de sinais.

Assim, por definição:

(a) seno: $\text{sen}\, x = s$ (ordenada de M_x)

(b) cosseno: $\cos x = c$ (abscissa de M_x)

(c) tangente: $tg\, x = \tan x = \frac{\text{sen}\, x}{\cos x}$, $\cos x \neq 0$

(d) cotangente: $\text{cotg}\, x = \cot x = \frac{\cos x}{\text{sen}\, x}$, $\text{sen}\, x \neq 0$

(e) secante: $\sec = \frac{1}{\cos x}$, $\cos x \neq 0$

(f) cossecante: $\text{cosec} = \csc x = \frac{1}{\text{sen}\, x}$, $\text{sen}\, x \neq 0$

Propriedade:

1ª Relação Fundamental

$\forall x \in \mathbb{R},\ sen^2 x + \cos^2 x = 1$

consequências:

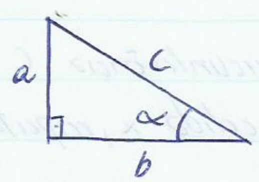
(a) Se $\cos x \neq 0$, então dividindo a relação fundamental por $\cos^2 x$ vem:

$$\frac{sen^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \quad \text{ou} \quad \boxed{tg^2 x + 1 = sec^2 x}$$

(b) Idem para $sen\, x \neq 0$:

$$\frac{sen^2 x}{sen^2 x} + \frac{\cos^2 x}{sen^2 x} = \frac{1}{sen^2 x} \quad \text{ou} \quad \boxed{1 + cotg^2 x = cosec^2 x}$$

2ª Relações trignométricas num triângulo retângulo

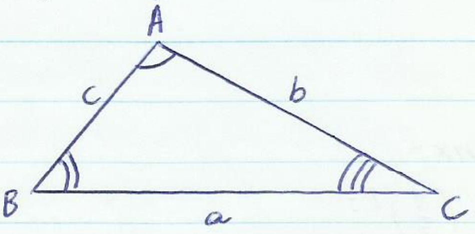


$$sen\, \alpha = \frac{a}{c} = \frac{cat.\ oposto}{hipotenusa}$$

$$\cos \alpha = \frac{b}{c} = \frac{cat.\ adjacente}{hipotenusa}$$

$$tag\, \alpha = \frac{a}{b} = \frac{cat.\ oposto}{cat.\ adjacente}$$

* 3º) Relações trigonométricas num triangulo qualquer



(a) <u>Lei dos senos</u>: $\frac{sen\hat{A}}{a} = \frac{sen\,\hat{B}}{b} = \frac{sen\,\hat{C}}{c}$

(b) <u>lei dos cosenos</u>:

$$a^2 = b^2 + c^2 + 2bc.\cos\hat{A}$$
$$b^2 = a^2 + c^2 + 2ac\cos\hat{B}$$
$$c^2 = a^2 + b^2 + 2ac\cos\hat{C}$$

$-$

4º) Seno e Cosseno da adição

$sen\,(A \pm B) = sen\,A \cdot \cos B \pm sen\,B \cos A$

$\cos\,(A \pm B) = \cos A \cdot \cos B \mp sen\,A\, sen\,B$ (Inverte o sinal)

<u>Exercícios</u>

1) Sendo x um arco do 2º quadrante com $sen\,x = \frac{3}{5}$, determinar

(a) $\cos x$, $tg\,x$, $cotg\,x$, $sec\,x$ e $cosec\,x$

(b) $sen\,2x$, $\cos 2x$ e o quadrante do arco $2x$

(a) $sen^2 x + \cos^2 x = 1$ $\qquad \cos x = \frac{-4}{5}$, $tag = \frac{3}{5} / \frac{-4}{5} = \frac{-3}{4}$; $cotg\,x = \frac{-4}{5}$

$\frac{9}{25} + \cos^2 x = 1$

$\cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$ $\qquad sec\,x = \frac{1}{\cos x} = \frac{-5}{4}$, $cosec\,x = \frac{5}{3}$

$\cos x = \begin{cases} +4/5 \\ -4/5 \end{cases}$

(b) $\operatorname{sen} 2x = \operatorname{sen}(x+x) = \operatorname{sen} x \cdot \cos x + \operatorname{sen} x \cdot \cos x = 2 \operatorname{sen} x \cos x =$

$$2 \cdot \frac{3}{5}\left(\frac{-4}{5}\right) = \frac{-24}{25}$$

$\cos 2x = \cos(x+x) = \cos x \cdot \cos x - \operatorname{sen} x \cdot \operatorname{sen} x =$

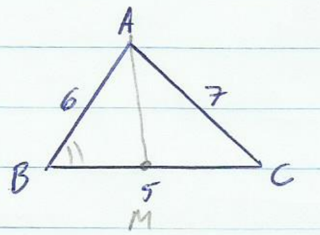
$$= \cos^2 x - \operatorname{sen}^2 x = \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 =$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

O arco $(2x)$ está no 4º quadrante

(seno < 0 e cosseno > 0)

2) Dado o ΔABC



a) Se M é o ponto médio de BC determinar $\overline{AM}$

b) Se X está entre B e C com $\overline{BX} = 3$, determinar $\overline{AX}$

c) Determinar a área ΔAXM

$7^2 = 6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos \hat{B}$

$49 = 36 + 25 - 60 \cdot \cos \hat{B}$

$\cancel{49 = 61 - 60 \cos \hat{B}}$

$\cancel{49 = 1 \cdot \cos \hat{B}}$

$\cancel{\cos \hat{B} = 49}$

$6^2 = 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cdot \cos C$

$36 = 25 + 49 - 70 \cos C$

$36 = 74 - 70 \cos C$

$38 = 70 \cos C$

$\cos C = \frac{38}{70} = \frac{19}{35}$

$m^2 = 49 + \frac{25}{4} - 2 \cdot 7 \cdot \frac{5}{2} \cdot \frac{19}{35}$

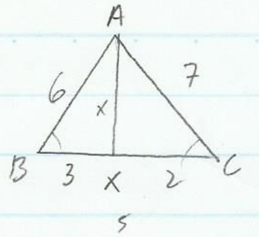
$m^2 = 49 + \frac{25}{4} - 35 \cdot \frac{19}{35}$

$m^2 = 49 + \frac{25}{4} - 19 = \frac{30 + 25}{4}$

$\frac{120 + 25}{4} = \frac{145}{4}$

$m = \sqrt{\frac{145}{4}}$ ou $\frac{\sqrt{145}}{2}$

(b)



$\cos C = \frac{19}{35}$

$x^2 = 7^2 + 2^2 - 2.7.2 . \frac{19}{35}$

$x^2 = 49 + 4 - 28 . \frac{19}{35}$

$x^2 = 53 - \frac{372}{35} = \sqrt{\frac{1855}{35}} = \sqrt{53}$

$x = \sqrt{53}$

$\begin{array}{r} 28 \\ \times 19 \\ \hline 192 \\ 26 \\ \hline 372 \end{array}$

$\begin{array}{r} 53 \\ \times 35 \\ \hline 265 \\ 159 \\ \hline 1855 \\ \hline 35 \end{array}$

$\begin{array}{l|l} 1855 & 35 \\ 175 & 53 \\ 105 & \\ 105 & \end{array}$

Composição e inversão de funções

* Chama-se composição $(\cancel{f \circ g}\ g \circ f)(x) = g(f(x))$ A ~~composição~~ função cujo domínio é formado pelos elementos x do domínio de f, tais que f(x) pertence ao domínio de g.

Função Sobrejetora — Não pode se alterar

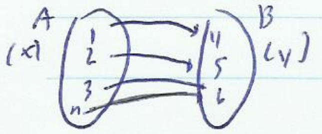
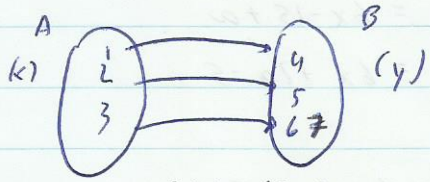


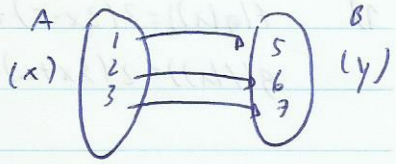
Imagem = contradomínio

Função Injetora — não pode



$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Função bijetora — Pode alterar



É sobrejetora e injetora

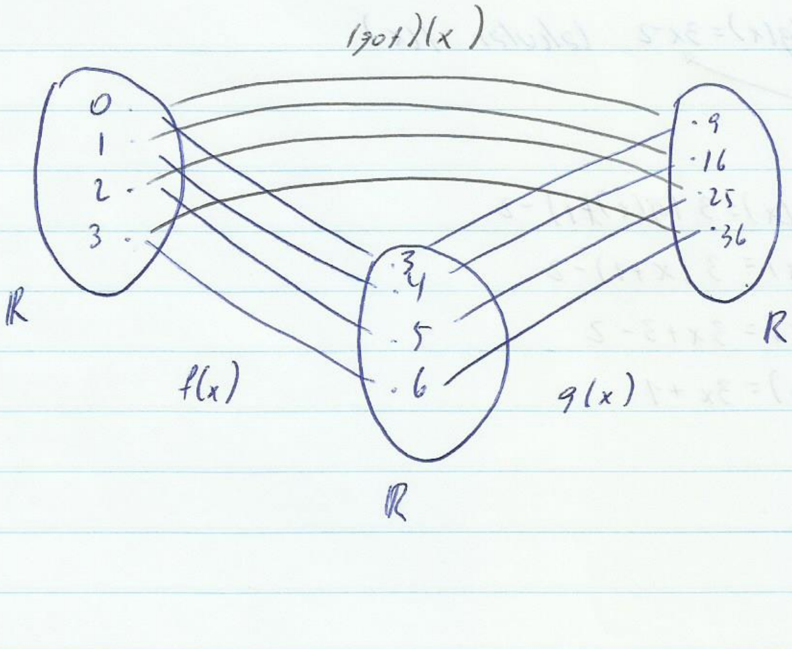
TEM INVERSA

Continua ⇒

Exemplo: seja $f(x) = x+3$ de $\mathbb{R}$ em $\mathbb{R}$ e $g(x) = x^2$ de $\mathbb{R}$ em $\mathbb{R}$. Determinar a composição $(g \circ f)(x) = g(f(x))$

$(g \circ f)(x) = (x+3)^2$

$(g \circ f)(x) = x^2 + 6x + 9$



/ /

Exercício

1) Dados $f(x) = 3x + a$ e $g(x) = 2x - 5$ calcular $\underline{a}$ tal que $f(g(x)) = g(f(x))$

2) Sendo $f(x) = 3x + 1$ e $f(g(x)) = 3x - 2$ calcular $g(x)$

1)
$f(g(x)) = 3(2x-5) + a = 6x - 15 + a$
$g(f(x)) = 2(3x+a) - 5 = 6x + 2a - 5$

Logo $\cancel{6x} - 15 + a = \cancel{6x} + 2a - 5$
$\underline{\underline{-10 = a}}$

2) $f(x) = 3x + 1$ (I) igualar I e II
$f(g(x)) = 3g(x) + 1$ (II)

$f(g(x)) = 3x - 2$

$3g(x) + 1 = 3x - 2$
$3g(x) = 3x - 3$
$g(x) = x - 1$

3) Sendo $g(x) = x - 1$ e $f(g(x)) = 3x - 2$ calcular $f(x)$

$x = g(x) + 1$

$f(g(x)) = 3(g(x) + 1) - 2$
$f(x) = 3(x+1) - 2$
$f(x) = 3x + 3 - 2$
$f(x) = 3x + 1$

tilibra

* Se $y=f(x)$ é uma função bijetora, então $\overset{-1}{f}(x)$ é a inversa. Portanto

$(x,y) \in f \iff (y,x) \in f^{-1}$

Neste caso, temos:

* $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
* $D_f = Im_{f^{-1}}$ e $D_{f^{-1}} = I_f$
* Os gráficos de f e f^{-1} são simétricos em relação à bissetriz $y=x$

Exemplo: $y=f(x)=2x+1$

$x = \frac{-b}{a} = -\frac{1}{2}$

$y=1$

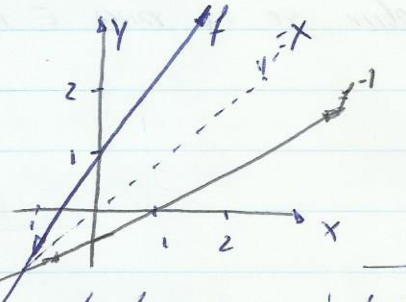


Imagem = contradomínio | (É bijetora) |

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

cálculo da inversa f $\boxed{y=2x+1}$ (trocar $x \rightleftarrows y$)

$x = 2y+1$ (isolar y)

f^{-1} $\boxed{y=\frac{x-1}{2}}$

Exemplo: $y = x^2 - 4x + 3$

$y = x^2 - 4x + 3$

Raízes $\begin{cases} 1 \\ 3 \end{cases}$

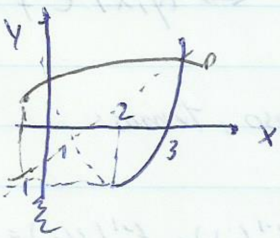


Imagem = contradomínio $= [-1, +\infty[$

Domínio = $[2; +\infty[$

É bijetora

Inversa

$y = x^2 - 4x + 3$

trocar $x \rightleftarrows y$

$x = y^2 - 4y + 3$

isolar y

$y^2 - 4y + (3 - x) = 0$

$y = \dfrac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot (3-x)}}{2 \cdot 1}$

$y = \dfrac{4 \pm \sqrt{16 - 12 + 4x}}{2}$

$y = \dfrac{4 \pm \sqrt{4 + 4x}}{2}$

~~$y = 4 \pm \sqrt{4}$~~

$y = 2 \pm \sqrt{1 + x}$

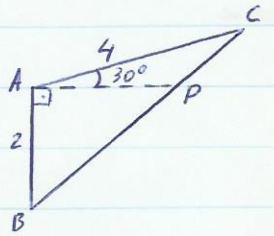
$y = 2 + \sqrt{1 + x}$

Realizar os testes com os valores para descobrir se as raízes é soma ou ~~adição~~ subtração

Exercícios

3) O triângulo ABC é retângulo em A e os catetos AB e AC medem, respectivamente, 6 m e 8 m. Se M é ponto médio da hipotenusa, determinar o comprimento de AM e a área do AMB

4) Considere

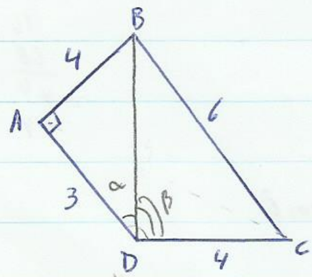


(a) Comprimento de BC

(b) sen $\hat{C}$ e cos $\hat{C}$

(c) Comprimento de AP

5) Considere



(a) sen $\hat{D}$ = sen ADC e cos $\hat{D}$ = cos ADC

(b) comprimento de AC

6) Usando a definição, calcule sen $\frac{\pi}{3}$ e cos $\frac{\pi}{3}$

7) Prove que

(a) $\sec\theta = \operatorname{sen}\theta \, tg\theta + \cos\theta$

(b) $(\operatorname{cosec}\theta - \operatorname{cotg}\theta)(\sec\theta + 1) = tg\theta$

(c) $\dfrac{\operatorname{sen}\theta}{\sec\theta+1} + \dfrac{\operatorname{sen}\theta}{\sec\theta-1} = 2\operatorname{cotg}\theta$

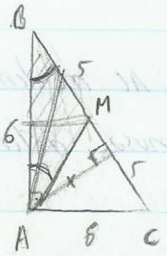
(d) $\dfrac{\cos^4\theta - \operatorname{sen}^4\theta}{1 - tg^4\theta} = \cos^4\theta$

(e) $\cos A - \cos B = -2\operatorname{sen}\dfrac{A+B}{2} \cdot \operatorname{sen}\dfrac{A-B}{2}$

Respostas

$$\begin{array}{r} 136 \\ 64 \\ \hline 72 \end{array}$$

3-)



$AC^2 = 6^2 + 8^2$ $\quad \bar{AC} = 10$

$AC^2 = 36 + 64$

$AC = \sqrt{100} = 10$

área do triângulo

$A = \frac{b \cdot h}{2}$

$A = \frac{5 \cdot h}{2}$

$sen\ \hat{B} = \frac{h}{6} = \frac{8}{10} = \frac{4}{5}$

$h = \frac{6 \cdot 4}{5} = \frac{24}{5}$ $\quad A = \frac{\cancel{5}}{2} \cdot \frac{24}{\cancel{5}} = 12\ m^2$

$\Delta AMC = \ m^2 = 5^2 + 8^2 - 2 \cdot 5 \cdot 8 \cdot \cos \alpha$

$m^2 = 89 - 80 \cos \alpha$

$\Delta ABC = \ \cos \alpha = \frac{8}{10} = \frac{4}{5}$

$m^2 = 89 - 80 \cdot \frac{4}{5} = 89 - 64$

$m^2 = 25$

$m = 5mm$

$$\begin{array}{r} 16 \\ 28 \\ \hline 44 \end{array}$$

4-) $x^2 = 2^2 + 4^2 - 2 \cdot 2 \cdot 4 \cdot \cos 120°$

$x^2 = 20 - 16 \cdot (-\frac{1}{2})$

$x^2 = 20 + 8$

$x = \sqrt{28} = 2\sqrt{7}$ //

$2^2 = 4^2 + (\sqrt{28})^2 - 2 \cdot 4 \cdot \sqrt{28} \cdot \cos \hat{C}$

$4 = 16 + 28 - 8\sqrt{28} \cdot \cos \hat{C}$

$4 - 16 - 28 = -8\sqrt{28} \cdot \cos \hat{C}$

$40 = -8\sqrt{28} \cdot \cos \hat{C}$

$\cos \hat{C} = \frac{40}{8\sqrt{28}} = \frac{5}{\sqrt{28}}$ //

$sen \hat{C} = \sqrt{1 - \cos^2 C}$ $\quad sen\ \hat{C} = \sqrt{\frac{3}{28}}$

$sen \hat{C} = \sqrt{1 - \frac{25}{28}}$

$16 = 4 + 28 - 2 \cdot 2 \cdot \sqrt{28} \cdot \cos B$

$16 = 32 - 4\sqrt{28} \cdot \cos B$

$\cos B = \frac{16}{4\sqrt{28}} = \frac{4}{\sqrt{28}} = \frac{4}{2\sqrt{7}} = \frac{2}{\sqrt{7}}$

$\cos \hat{B} = \frac{2}{BP}$

$\overline{BP} = \frac{2}{\cos B} = \frac{2}{\frac{2}{\sqrt{7}}} = \sqrt{7}$

Pitagoras

$AP^2 = (\sqrt{7})^2 - 2^2 = 3$

$AP = \sqrt{3}$

$\begin{array}{r} 25 \\ 16 \\ \hline 41 \end{array}$

5-) $BD^2 = 3^2 + 4^2$ $\quad$ $sen\,\alpha = \frac{4}{5}$ $\quad$ $\cos\alpha = \frac{3}{5}$

$BD^2 = 9 + 16$

$BD^2 = 25$ $\quad$ $6^2 = 4^2 + 5^2 - 2.4.5.\cos\beta$

$BD = 5$ $\quad$ $36 = 41 - 40.\cos\beta$

$\cos\beta = \frac{5}{40} = \frac{1}{8}$

$sen\,\hat{D} = sen(\alpha + \beta) =$

$sen\alpha . \cos\beta + sen\beta \cos\alpha$

$\cos\hat{D} = \cos(\alpha + \beta) =$

$\cos\alpha + \cos\beta - sen\,\alpha + sen\,\beta$

$sen\,\hat{D} = sen(\alpha + \beta)$

$sen\alpha . \cos\beta + sen\beta \cos\alpha$

$\frac{4}{5} \cdot \frac{1}{8} + \sqrt{\frac{63}{64}} \cdot \frac{1}{8}$

$sen\,\hat{D} = \frac{1}{10} + \frac{1}{8}\sqrt{\frac{63}{64}}$ //

Relação Fundamental

$sen^2 x + \cos^2 x = 1$ $\quad$ $sen^2 x = 1 - \frac{1}{64}$

$sen^2 x + \left(\frac{1}{8}\right)^2 = 1$

$sen^2 x + \frac{1}{64} = 1$ $\quad$ $sen\,x = \sqrt{\frac{63}{64}}$

$\cos\hat{D} = \cos(\alpha + \beta)$

$\cos\alpha + \cos\beta - sen\alpha + sen\,\beta$

$\frac{3}{5} + \frac{1}{8} - \frac{4}{5} + \sqrt{\frac{63}{64}} = \sqrt{\frac{63}{64}} - \frac{3}{40}$ //

$\frac{3}{5} + \frac{1}{8} = \frac{24+5}{40} = \frac{29}{40} - \frac{32}{40} = -\frac{3}{40}$

Respostas

(a) $sen\,\hat{D} = \frac{4 + 9\sqrt{7}}{40}$ $\quad$ (b) $\overline{AC} = \sqrt{\frac{776 + 36\sqrt{7}}{5}}$

$\cos\hat{D} = \frac{3 - 12\sqrt{7}}{40}$

$x^2 = 3^2 + 4^2 - 2.3.4.\left(\sqrt{\frac{63}{64}} - \frac{3}{40}\right)$

$x^2 = 9 + 16 - 24\left(\sqrt{\frac{63}{64}} - \frac{3}{40}\right)$

$x^2 = 25 - 24\sqrt{\frac{63}{64}} + \frac{72}{40}$ $\quad$ $x = \sqrt{\frac{134}{5} - \left(\frac{24\sqrt{63}}{64}\right)}$

$x^2 = 25 - 24\sqrt{\frac{63}{64}} + \frac{9}{5}$

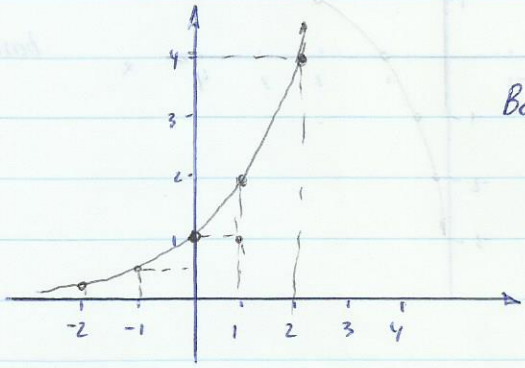
$25 + \frac{9}{5} = \frac{125 + 9}{5} = \frac{134}{5}$

Função Exponencial

É a função dada por $y = a^x$ onde $1 \neq a > 0$ $(1 \neq a > 0)$

Ex: $y = 2^x$

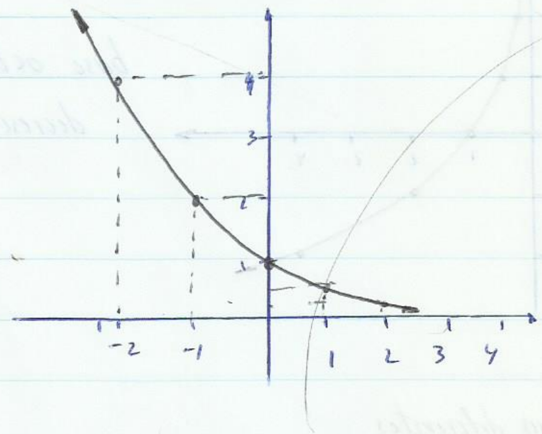
x	y
-2	1/4
-1	1/2
0	1
1	2
2	4



Base > 1 crescente

Ex: $\left(\frac{1}{2}\right)^x$

x	y
-2	4
-1	2
0	1
1	1/2
2	1/4



0 < Base < 1 Decrescente

Consideremos $y = a^x$, $1 \neq a > 0$ Domínio = $\mathbb{R}$ } É bijetora

Contradomínio = $\mathbb{R}_+^*$

Cálculo da Inversa

$f: \boxed{y = a^x}$ $1 \neq a > 0$

$x = a^y$

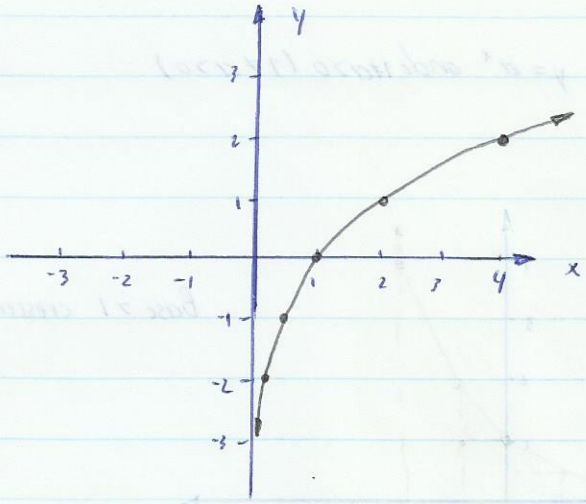
$f^{-1}: \boxed{y = \log_a x}$ $1 \neq a > 0$

Domínio = $\mathbb{R}_+^*$

Contradomínio = $\mathbb{R}$

Ex: $y = \log_2 x$

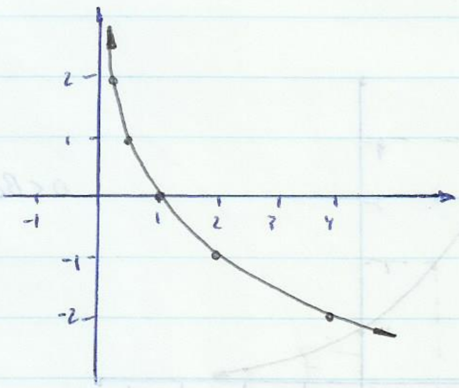
x	y
1/4	-2
1/2	-1
0	0
2	1
4	2



base > 1 crescente

Ex: $y = \log_{1/2} x$

x	y
4	-2
2	-1
1	0
1/2	1
1/4	2



base 0 < base < 1
decrescente

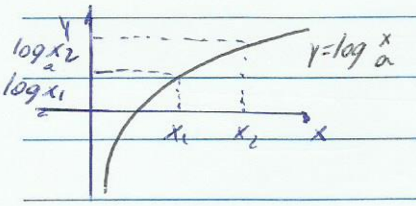
Cálculo de logaritmos com bases diferentes
mudança de base

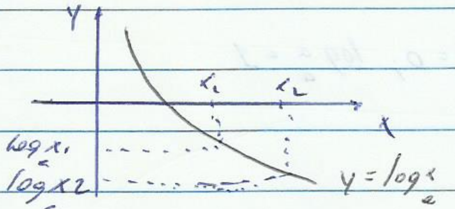
$$\log_a b = \frac{\log_? b}{\log_? a}$$

$$\log_5 7 = \frac{\log_{10} 7}{\log_{10} 5} = \frac{0,8450}{0,6989} = 1,209$$

$$\log_5 7 = \frac{\cancel{\log} 7}{\ln 5} = \frac{1,9459}{1,1609} = 1,209$$

Inequações logaritmicas





$\log_a x_2 > \log_a x_1$

$x_2 > x_1$

base > 1

Mantem a desigualdade

$\log_a x1 > \log_a x_2$

$x1 < x_2$

0 < base < 1

Inverte a desigualdade

Ex: $2^x > 2^5$ | $\log_7 x > \log_7 8$ | $\log_{\frac{1}{5}} x > \log_{\frac{1}{5}} 9$

$x > 5$ | $x > 8$ | $x < 9 \Rightarrow 0 < x < 9$

condição de existência do logaritimo

$x > 0$

Exercícios

1) Resolver a inequação

$\log_3 (x^2-1) \leq 1$

2) Determinar o domínio das funções

a) $y = \log\left(\frac{2+x}{2-x}\right)$

c) $y = \log(|x|-1)$

b) $y = \frac{1}{\sqrt{1-2^x}}$

Propriedades

1) $a^{\log_a b} = b$, $\log_a 1 = 0$; $\log_a a = 1$

2) $\log_a x^n = n . \log_a x$

3) $\log_a (x.y) = \log_a x + \log_a y$

4) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

5) $\log_a b = \dfrac{\log_? b}{\log_? a}$

Respostas

1) $\log_3 (x^2-1) \leq 1$

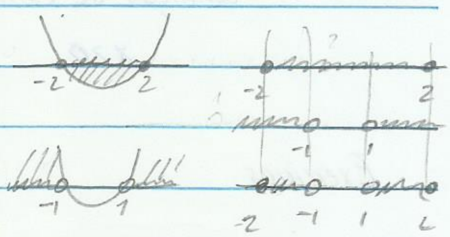
$\log_3 (x^2-1) \leq \log_3 3$

$x^2-1 \leq 3$

$x^2-1 > 0$

$x^2-1 \leq 3$

$x^2-1 > 0$



$S = \{x \in \mathbb{R} \mid -2 \leq x < -1 \text{ ou } 1 < x \leq 2\}$

2) a-) $y = \log\left(\frac{2+x}{2-x}\right)$ $\frac{2+x}{2-x} > 0$

$D = \{x \in \mathbb{R} \mid -2 < x < 2\}$

		-2		2	
$2+x$	−		+		+
$2-x$	+		+		−
R	−	0	+	0	−

b) $y = \dfrac{1}{\sqrt{1-2^x}}$

$1 - 2^x > 0$

$-2^x > 1$

$2^x < 1$

$2^x < 2^0$

$x < 0$

$D = \{x \in \mathbb{R} \mid x < 0\}$

c) $y = \log(|x| - 1)$

$|x| - 1 > 0$

$|x| > 1$

$-1 \quad 1$

$D = \{x \in \mathbb{R} \mid x < -1 \text{ ou } x > 1\}$

Exercícios

1) Esboçar os gráficos das funções

a) $y = (x+1).|x+2|$

b) $y = |\operatorname{sen} x| + \operatorname{sen} x$

2) Determinar o domínio das funções

a) $y = \sqrt{|5-2x| - 7}$

b) $y = \sqrt{\log_{1/2}(x-1)}$

3) Se $f(x) = |x-3|$ e $g(x) = 3x-4$ Calcular x tal que $(f \circ g)(x) \geq 2$

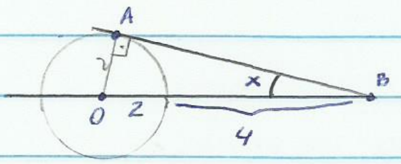
4) Se $f(x) = \ln x$, $x > 0$ e $g(f(x)) = 5x$, Calcular $g(x)$

5) Resolver as inequações

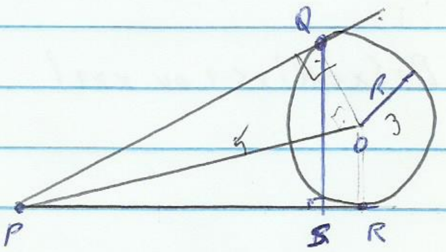
a) $|x^2+10| > |7x|$

b) $|3x-2| > |x|$ → tirar 3 equações

6) Calcular $\sec x$



7) Calcular $x = \overline{QS}$ na figura abaixo sendo $PO=5$ e $R=3$



Respostas:

$(x+1)(x+2)$

$a>0$

$a<0$

1-) a) $y=(x+1)|x+2|$ $\begin{cases} (x+1)x+2 = x^2+x+2, & se\ x \geq -2 \\ (x+1).(-x)-2 = -x^2-x-2, & se\ x < -2 \end{cases}$

$x^2+x+2=0$

$x = \dfrac{-1 \pm \sqrt{1^2-4.2}}{2}$

$x = \dfrac{-1 \pm \sqrt{-7}}{2}$

$\Delta = -7$

$V = \dfrac{-b}{2a} = \dfrac{-1}{2}$

$\dfrac{-\Delta}{4a} = -\left(\dfrac{-7}{4}\right) = \dfrac{7}{4}$

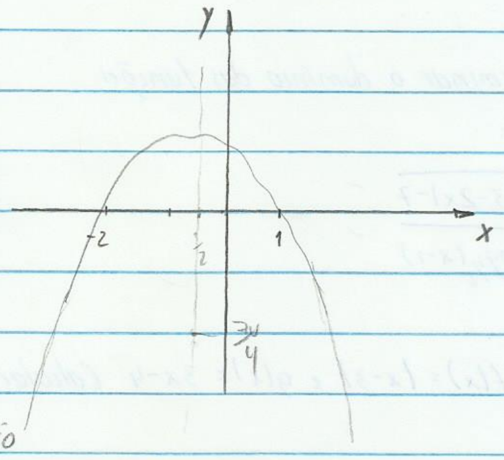
$-x^2-x-2$

$x = \left(1 \pm \sqrt{1^2-4.(-1).(-2)}\right)/-2$

$x = \left(1 \pm \sqrt{1+8}\right)/-2$

$x_1 = \dfrac{1+3}{-2} = \dfrac{4}{-2} = -2$

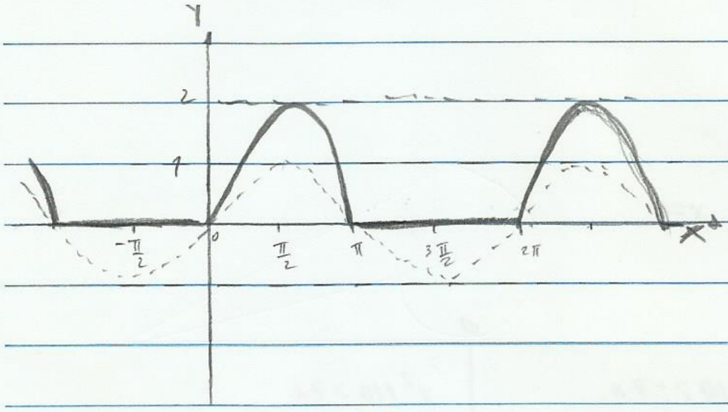
$x_2 = \dfrac{1-3}{-2} = \dfrac{-2}{-2} = 1$



Realizar a multiplicação com todos os dados do módulo

1-) b) $y = |\text{sen}\, x| + \text{sen}\, x$

$$y = \begin{cases} \text{sen}\, x + \text{sen}\, x, & \text{se } \text{sen}\, x \geq 0 \\ -\text{sen}\, x + \text{sen}\, x, & \text{se } \text{sen} < 0 \end{cases} \qquad y = \begin{cases} 2\,\text{sen}\, x, & \text{se } \text{sen}\, x \geq 0 \\ 0, & \text{se } \text{sen}\, x < 0 \end{cases}$$



2) b) $y = \sqrt{\log_{1/2}(x-1)}$

1ª condição $]-\infty, 2]$

$\log_{1/2}(x-1) \geq 0$

$\log_{\frac{1}{2}}(x-1) \geq \log_{\frac{1}{2}} 1$

$x - 1 \leq 1$

$x \leq 2$

2ª condição $]1, +\infty[$

$x - 1 > 0$

$x > 1$

Varal

1 2

$D = \{x \in \mathbb{R} \mid 1 < x \leq 2\}$ $]1, 2]$

a) $y = \sqrt{|5-2x| - 7}$

1ª condição

$|5-2x| - 7 \geq 0$

$D = \{x \in \mathbb{R} \mid x \leq -1 \text{ ou } x > 6\}$

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \begin{cases} 5 - 2x - 7, & \text{se } 5 - 2x \geq 0 \\ -5 + 2x - 7, & \text{se } 5 - 2x < 0 \end{cases}$$

$\text{I} = -2x - 2 = x = -1$ $\quad -2x - 2 \geq 0$ $\quad 2x - 12 < 0$

$\text{II} = 2x - 12 = x = 6$ $\quad -2x \geq 2$ $\quad 2x < 12$

$x \leq -1$ $\quad x < 6$

tilibra

| / / |

| Estudar logaritmo |

4) $f(x) = \ln x \; ; \; x > 0$ $\qquad f(x) = \ln x \Rightarrow \log_e x = f(x) \Rightarrow x = e^{f(x)}$

$g(f(x)) = 5x$

$g(x) = ?$

$$g(f(x)) = 5x$$

$$g(f(x)) = 5e^{f(x)}$$

$$g(x) = 5e^{x}$$

5) a) $|x^2+10| > |7x|$

Anula quando $x=0$

$\cancel{x^2+10=0}$ $\quad$ Não é possível anular

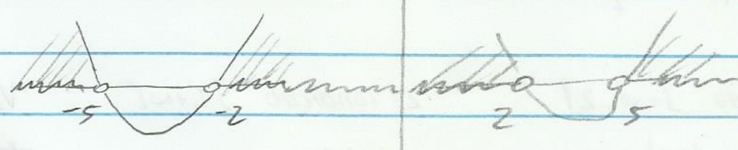
$\cancel{x^2=-10}$

$\cancel{x=\sqrt{-10}}$

(Realizar baskará quando ocorrer isso)

$\Delta = -4.1.10 = -40$

0	
$x^2+10 > -7x$	$x^2+10 > 7x$
$\underline{x^2+7x+10>0}$	$\underline{x^2-7x+10>0}$
parábola pos	parábola pos



$$S = \{x \in \mathbb{R} \mid x < -5 \text{ ou } -2 < x < 2 \text{ ou } x > 5\}$$

6. $\operatorname{sen} x = \frac{2}{6} = \frac{1}{3}$ $\qquad \operatorname{sen}^2 x + \cos^2 x = 1$ $\qquad \cos^2 x = 1 - \frac{1}{9} = \frac{8}{9}$

$\cos^2 x = 1 - \operatorname{sen}^2 x$

$\cos^2 x = 1 - \frac{1}{3}^2$ $\qquad \cos x = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

$$\begin{array}{c|c} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \quad 2^2 \cdot 2$$

$\frac{2\sqrt{2}}{3}$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{1}{1} : \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\sec x = \frac{3\sqrt{2}}{4}$$

3) $f(x) = |x-3|$ $\qquad (f \circ g) = f(g(x)) = |(3x-4)-3|$

$g(x) = 3x-4$

$x = ?$ $\qquad f(x) = x-3 \Rightarrow 3x-4-3 = 3x-7 = (f \circ g)$

$(f \circ g)(x) \geq 2$ $\qquad$ ou $= -x+3 \Rightarrow -(3x-4)+3 \Rightarrow -3x+4+3 = -3x+7 = (f \circ g)$

$3x-7 \geq 2$ ou $-3x+7 \geq 2$

$3x \geq 9$ $\qquad -3x \geq -5$ $\qquad$ R: $\{x \in \mathbb{R} \mid x \geq 3$ ou $x \leq \frac{5}{3}\}$

$x \geq 3$ $\qquad x \leq \frac{-5}{-3}$

1-) a) $y = (x+1).|x+2|$

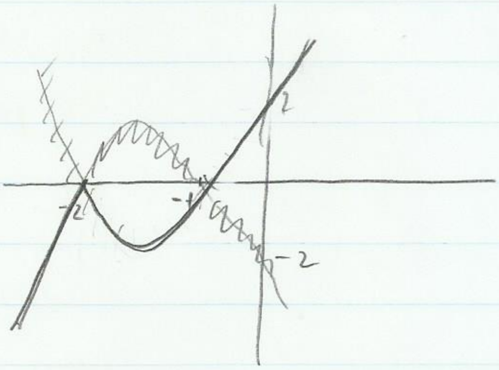
$$\begin{cases} (x+1)(x+2) = x^2+2x+x+2 \Rightarrow x^2+3x+2, & \text{se } x \geq -2 \\ (x+1)(-x-2) = -x^2-2x-x-2 \Rightarrow -x^2-3x-2, & \text{se } x < -2 \end{cases}$$

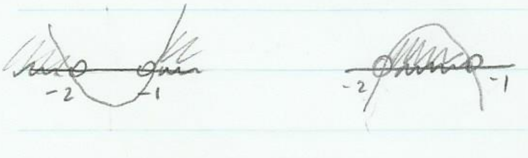
x^2+3x+2 $\qquad -x^2-3x-2$

$x = \frac{-3 \pm \sqrt{9-4.2}}{2}$ $\qquad x = \frac{3 \pm \sqrt{9-4.(-1).(-2)}}{-2}$

$x_1 = \frac{-3+1}{2} = \frac{-2}{2} = -1$ $\qquad x_1 = \frac{3+1}{-2} = \frac{4}{-2} = -2$

$x_2 = \frac{-3-1}{2} = \frac{-4}{2} = -2$ $\qquad x_2 = \frac{3-1}{-2} = \frac{2}{-2} = -1$





Exercícios

e - intersecção

ou - união

Obs.: Determinação de domínios

Restrições

1) $\sqrt{M}$: $M \geq 0$

2) $\ln M$ ou $\log_a M$: $M > 0$, $1 \neq a > 0$

3) $\frac{1}{M}$, $M \neq 0$

Sinais Polinômios

1º grau: $y = ax + b$

raiz: $x = \frac{-b}{a}$

$-\infty$ $-b/a$ $+\infty$

c/a   0   m/a

c/a = sinal contrário de a

m/a = mesmo sinal de a

2º grau: $y = ax^2 + bx + c$

$\Delta > 0$: raízes $x_1 < x_2$

$-\infty$ x_1 x_2 $+\infty$

y   m/a   0   c/a   0   m/a

$\Delta = 0$: raízes dupla x_0

$-\infty$ x_0 $+\infty$

y   m/a   0   m/a

$\Delta < 0$: $\nexists$ raiz real

$-\infty$ $+\infty$

y   m/a

/ /

<u>Módulo:</u> Se $a>0$:

(a) $|x| \leq a \Leftrightarrow -a \leq x \leq a$

(b) $|x| \geq a \Leftrightarrow x \geq a$ ou $x \leq -a$

<u>Exercícios</u>: Determinar sob forma de intervalo, o domínio de

1) $f(x) = \dfrac{1}{\ln(2x^2-x)}$

2) $f(x) = \sqrt{1 - \dfrac{3x}{2-x}}$

3) $f(x) = \ln\left(x - \dfrac{2}{x+1}\right)$

4) $f(x) = \sqrt{7 - |2x+1|}$

5) $f(x) = \sqrt{\dfrac{x^2+6x-7}{x^2-2x}}$ 5) $f(x) = \sqrt{\dfrac{x^2+7x+6}{x^2-2x}}$

6) $f(x) = \dfrac{\sqrt{x^2+6x-7}}{\sqrt{x^2-2x}}$ 6) $f(x) = \dfrac{\sqrt{x^2-7x+6}}{\sqrt{x^2-2x}}$

7) $f(x) = \sqrt{|3x-4| - 8}$

8) $f(x) = \sqrt{x(x-1)(x^2-9)}$

9) $f(x) = \dfrac{\ln(2 - |x+4|)}{\sqrt{x^2+3x}}$

10) $f(x) = \ln(6-x-x^2) + \sqrt{1 - \dfrac{2}{x+1}}$

tilibra

Respostas

1-) $f(x) = \dfrac{1}{\ln(2x^2-x)}$

$Df = \{x \in \mathbb{R} \mid 2x^2-x>0 \text{ e } \ln(2x^2-x) \neq 0\}$

$= \{x \in \mathbb{R} \mid \underbrace{2x^2-x>0}_{I} \text{ e } \underbrace{2x^2-x \neq 1}_{II}\}$

$2x^2-x>0$

$x = \dfrac{1 \pm \sqrt{1-0}}{4}$

$x_1 = \dfrac{1+1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

$x_2 = \dfrac{1-1}{4} = \dfrac{0}{4} = 0$

I: $-\infty \quad 0 \quad 1/2 \quad +\infty$

$+ \quad 0 \quad - \quad 0 \quad +$

$I =]-\infty, 0[\cup]0, +\infty[$?

$II: 2x^2-x \neq 1$

$2x^2-x=1$

$2x-x-1=0$

$x = \dfrac{1 \pm \sqrt{1+6}}{4}$ $\langle$ 1, $-1/2$

II: $-\infty \quad -1/2 \quad 1 \quad +\infty$

$II = \mathbb{R} - \{-\frac{1}{2}, 1\}$

I: $-\infty \quad 0 \quad 1/2 \quad +\infty$

II: $-\infty \quad -\frac{1}{2} \quad 1 \quad +\infty$

$Df =]-\infty, -\frac{1}{2}[\cup]-\frac{1}{2}, 0[\cup]1/2, 1[\cup]1, +\infty[$

ou

$Df = (]-\infty, 0[\cup]\frac{1}{2}, +\infty[) - \{-\frac{1}{2}, 1\}$

2-) $Df = \{ x \in \mathbb{R} \mid 1 - \frac{3x}{2-x} \geq 0 \}$

$$1 - \frac{3x}{2-x} = \frac{2-x-3x}{2-x} = \frac{2-4x}{2-x}$$

$N: \frac{1}{2}$; $D = 2$

$-\infty$		$1/2$		2		$+\infty$
N	+	0	−	−	−	
D	+	+	+	0	−	
÷	+	0	−	∄	+	

$Df =]-\infty, \frac{1}{2}] \cup]2, +\infty[$

3-) $f(x) = \ln\left(x - \frac{2}{x+1}\right)$

$Df = \{ x \in \mathbb{R} \mid x - \frac{2}{x+1} \neq 1 \}$

$x - \frac{2}{x+1} \neq 1$

$\frac{x^2+x-2}{x+1} \neq 1$

$= \{ x \in \mathbb{R} \mid x > 1 \text{ e } x < -2 \}$

$\boxed{R:]-2,-1[\cup]1, +\infty[}$

$x^2 + x - 2$

$x = \frac{-1 \pm \sqrt{1+8}}{2}$

$x_1 = \frac{-1+3}{2} = \frac{2}{2} = 1$

$x_2 = \frac{-1-3}{2} = \frac{-4}{2} = -2$

$-\infty$		-2		1		$+\infty$
2°	+	0	−	0	+	
1°	−	−	-1	+	+	
÷	−	0	0	0	+	

4-) $Df = \{ x \in \mathbb{R} \mid 7 - |2x+1| > 0 \}$

$$7 - |2x+1| \begin{cases} 7 - 2x - 1, & \text{se } x \geq 0 \\ 7 + 2x + 1, & \text{se } x < 0 \end{cases} = \begin{cases} 6 - 2x, & \text{se } x \geq -1/2 & \text{I} \\ 8 + 2x, & \text{se } x < 1/2 & \text{II} \end{cases}$$

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

$-\infty$	-4		3	$+\infty$
I	+ + + +	+	0	− −
II	− −	0	+ + + + + +	
÷	− −	∄	+ + +	0 − −

$Df =]-4, 3[$

$\boxed{R: = [-4, 3]}$

5.) $Df = \{ x \in \mathbb{R} \mid \frac{x^2+6x-7}{x^2-2x} \geq 0 \}$ (I), (II)

	$-\infty$		-7		-2		0		1		$+\infty$
I	+		0	-	-	-	0	+	+		
II	+	+	+	0	-	0	+	+	++		
÷	-	0	-	0	+	0	-	0	+		

$D = [-2, 0] \cup [1, +\infty]$

x^2+6x-7

$x = \frac{-6 \pm \sqrt{36+28}}{2}$

$x_1 = \frac{-6+8}{2} = \frac{2}{2} = 1$

$x_2 = \frac{-6-8}{2} = \frac{-14}{2} = -7$

x^2-2x

$x(x-2)$

$x = 0$

$x = -2$

4.- Solução do Professor

$7 - |2x+1| \geq 0 \iff -|2x+1| \geq -7 \iff |2x+1| \leq 7$

$\iff -7 \leq 2x+1 \leq 7$

$\iff -8 \leq 2x \leq 6$

$\iff -4 \leq x \leq 3$

5.) (NOVA) $Df = \{ x \in \mathbb{R} \mid \frac{x^2-7x+6}{x^2-2x} \geq 0 \}$

	$-\infty$	-2		0	1	2				6	$+\infty$
I	-	+	+	+	0	-	-	-	-	0	+
II	-	+	0	-	-	0	+	+	+	1	+
÷	+	0	-	~~+~~ +0	0	-				0	+

$Df =]-\infty, 0[\cup [1, 2[\cup [6, +\infty[$

$x = \frac{7 \pm \sqrt{49-24}}{2}$

$x_1 = \frac{7+5}{2} = \frac{12}{2} = 6$

$x_2 = \frac{7-5}{2} = \frac{2}{2} = 1$

$x(x-2) = 0$

$x = 0$

$x = +2$

6-) $Df = \{x \in \mathbb{R} \mid x^2+6x-7 \geq 0$ e $\sqrt{x^2-2x} > 0$

$-\infty$	-2	0	1	6	$+\infty$

I $+ + 0 - 0 + +$

II $+ 0 - 0 + +$

$\div + 0 - 0 + 0 - 0 + +$

$x^2-2x > 0$

$x(x-2)$

$x=0 \quad x=2$

$Df \;]-\infty, -2] \cup [0, 1] \cup [6, +\infty[$

7-) $Df = \{x \in \mathbb{R} \mid |3x-4|-8 \geq 0\}$

$|3x-4|-8 \geq 0$

$|3x-4| \geq 8$

$3x-4 \geq 8$

$3x \geq 12$

$x \geq 4$

$3x-4 \leq -8$

$3x \leq -4$

$x \leq \frac{-4}{3}$

$D =]-\infty, -\frac{4}{3}] \cup [4, +\infty[$

8-) $Df = \{x \in \mathbb{R} \mid x(x-1)(x^2-9) \geq 0\}$

$I = 0$, $II = 1$, $III = \pm 3$

$-\infty$	-3	0	1	3	$+\infty$
	0	0	0	0	

$x^2-9=0$

$x^2=9$

$x = \sqrt{9}$

$x = \pm 3$

$$3)\ \lim_{x\to 3}\left(\frac{\sqrt{x}-\sqrt{3}}{x-3}\right)=\lim_{x\to 3}\frac{(\sqrt{x}-\sqrt{3})}{(x-3)}\cdot\frac{(\sqrt{x}+\sqrt{3})}{(\sqrt{x}+\sqrt{3})}=\lim_{x\to 3}\frac{(x-3)}{(x-3)(\sqrt{x}+\sqrt{3})}=\lim_{x\to 3}\left(\frac{1}{\sqrt{x}+\sqrt{3}}\right)=\frac{1}{2\sqrt{3}}$$

multiplicação pelo conjugado

4) Função Contínua no ponto x_0

Uma função $f(x)$ é contínua no ponto x_0 se e somente se:

a) $\exists\ f(x_0)$

b) $\exists \lim_{x\to x_0} f(x)$

c) $\lim_{x\to x_0} f(x) = f(x_0)$

Exercícios

Calcular os limites

$$1)\ \lim_{x\to 3}\frac{x^2-9}{x-3}=\frac{(x-3)(x+3)}{(x-3}=\lim_{x\to 3}(x+3)=6$$

$a^3-a^3=(a-b)(a^2+ab+b^2)$

$x^3-1^3=(x-1)(x^2+x+1)$

x^3-x^2+x-1

$x^2(x-1)+(x-1)$

$(x-1)(x+1)$

$$2)\ \lim_{x\to 1}\frac{x^3-1}{x^3-x^2+x-1}\ \frac{\cancel{(x-1)}(x^2+x+1)}{(x^2+1)\cancel{(x-1)}}=\frac{x^2+x+1}{x^2+1}=\frac{3}{2}$$

$$3)\ \lim_{x\to 4}\frac{3-\sqrt{5+x}}{x-4}\ \lim_{x\to}\frac{(3-\sqrt{5+x})}{(x-4)}\cdot\frac{(3+\sqrt{5+x})}{(3+\sqrt{5+x})}=\frac{9-(5+x)}{(x-4)(3+\sqrt{5+x})}=\frac{9-5-x}{(x-4)(3+\sqrt{5+x})}=\frac{(4-x)}{(x-4)(3+\sqrt{5+x})}=\lim_{x\to 4}\frac{-1}{3+\sqrt{5+x}}=\frac{-1}{6}$$

R: -3 $\quad 4)\ \lim_{x\to 1}\left(\frac{x^2+x-2}{x^2-3x+2}\right)\ \frac{x(x+1)-2}{x(x-3)+2}=\frac{x+1}{x-3}$

$(x-3)(x-\quad)$

x^2-3x

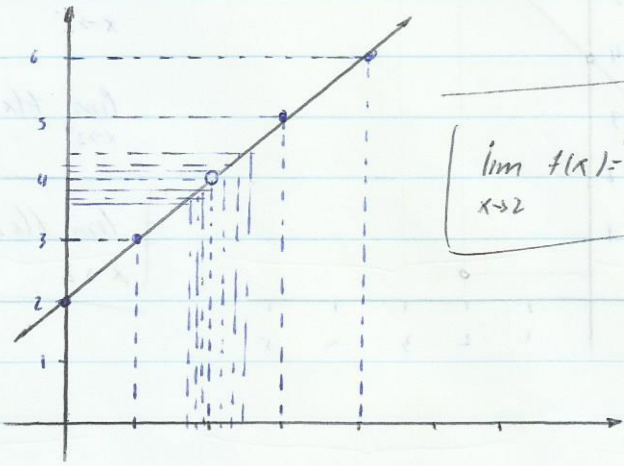
$x^2+x-2 \qquad x^2-3x+2$

$x(x+1)-2 \qquad x(x-3)+2$

Limites

Ex 1: Seja a função $f(x) = \dfrac{x^2-4}{x-2}$

x	$f(x)$
0	2
1	3
1,7	3,7
1,8	3,8
2 $\square$	$\square$ 4
2,1	4,1
2,2	4,2
3	5
4	6



$\lim\limits_{x \to 2} f(x) = 4$ $\quad$ $\nexists\, f(2)$

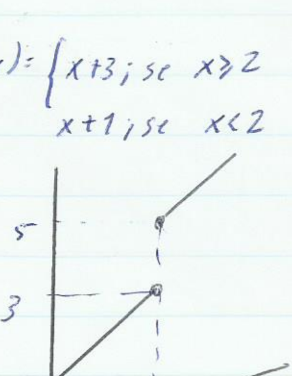
~~Quando~~ x aproxima de 2 c/ valores menores, f(x) se aproxima de 4

$\lim\limits_{x \to 2^-} f(x) = 4$

~~Quando~~ x se aproxima de 2 c/ valores maiores, f(x) se aproxima de 4

$\lim\limits_{x \to 2^+} f(x) = 4$

Ex 2: Seja $f(x) = \begin{cases} x+3; \text{ se } x \geq 2 \\ x+1; \text{ se } x < 2 \end{cases}$

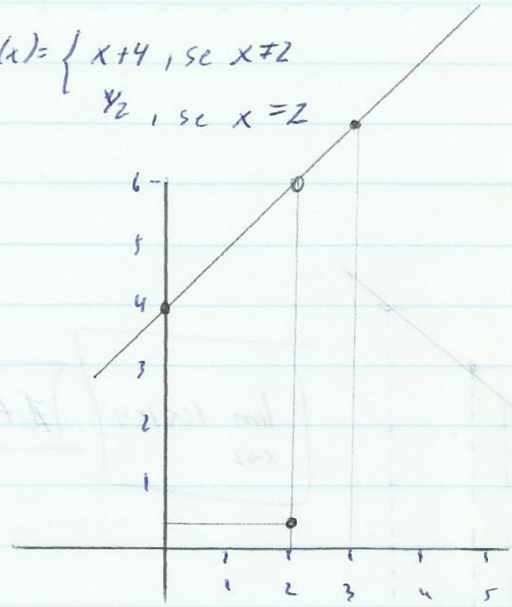


$\lim\limits_{x \to 2^-} f(x) = 3$

$\lim\limits_{x \to 2^+} f(x) = 5$

$\nexists \lim\limits_{x \to 2} f(x)$ $\quad$ $f(2) = 5$

Ex3. Seja $f(x) = \begin{cases} x+4, & \text{se } x \neq 2 \\ 1/2, & \text{se } x = 2 \end{cases}$

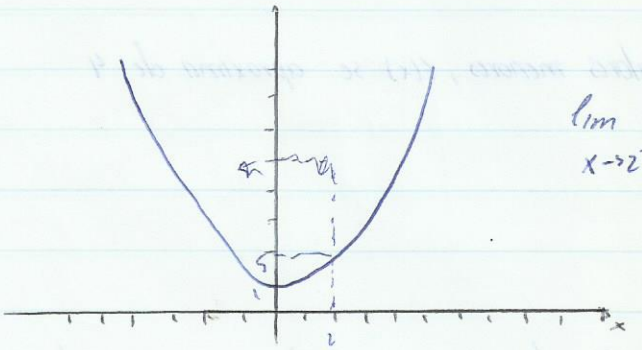


$\lim\limits_{x \to 2^-} f(x) = 6$

$\lim\limits_{x \to 2^+} f(x) = 6$

$\boxed{\lim\limits_{x \to 2} f(x) = 6}$ $\boxed{f(2) = \frac{1}{2}}$

Ex4. Seja $f(x) = x^2+1$



$\lim\limits_{x \to 2^-} f(x) = 5$ $\lim\limits_{x \to 2^+} f(x) = 5$

$\lim\limits_{x \to 2} f(x) = 5$ $\boxed{f(2) = 5}$

Prova: Calcular os limites

1) $\lim\limits_{x \to 2} (x^2+1) = 5$

Fatorar

2) $\lim\limits_{x \to 2} \left(\frac{x^2-4}{x-2}\right) = \lim\limits_{x \to} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim\limits_{x-2} (x+2) = 4$

11-) Sendo $f: [a, +\infty[\to \mathbb{R}$, $f(x) = x^2+x+1$

(a) Esboce o gráfico f e determine o menor valor possível de a de modo que f seja invertível em $[a, +\infty[$

(b) Determine $g(x) = f^{-1}(x)$ e esboce seu gráfico indicando domínio e imagem

12-) $f(x) = 2^x$

(a) f é invertível em $\mathbb{R}$? Justifique

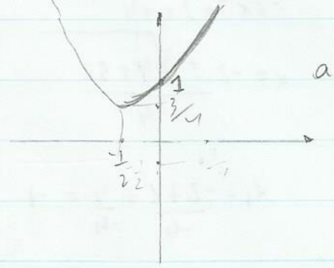
(b) determine $g(x) = f^{-1}(x)$ indicando domínio e imagem

13-) $f(x) = x|2-x| - x^2+4$, exprimir $f(x)$ sem o uso do símbolo de modo e esboçar seu gráfico indicando domínio e imagem

14-) I dim $f(x) = |x^2+2x|-3$

15-) Calcular $\lim_{x\to 1} \dfrac{\sqrt{x^3+5x-2} - 3x+1}{x^2-1}$

16-) Calcular $\lim_{x\to 3} \left(\dfrac{1}{x-3} - \dfrac{5}{x^2-x-6}\right)$

11-) 

$a = -1/2$ $\quad [a, +\infty[$

DEF. $f[g(x)] = x$, $\forall x \in D_g$

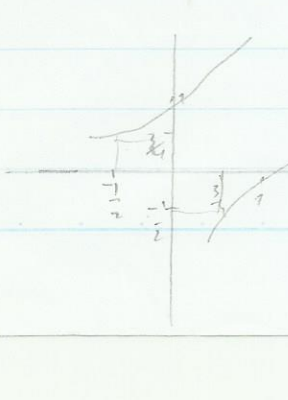
$[g(x)]^2 + g(x) + 1 = x$

$[g(x)]^2 + g(x) + 1 - x = 0$

$g(x) = \dfrac{-1 \pm \sqrt{1-4(1-x)}}{2}$

Daí: $f^{-1}(x) = g(x) = \dfrac{-1+\sqrt{4x-3}}{2}$

(a) $a = -1/2$; f é invertível em $[-1/2, +\infty[$ pois uma reta paralela a x, tem que possuir pelo 1 ponto no máximo



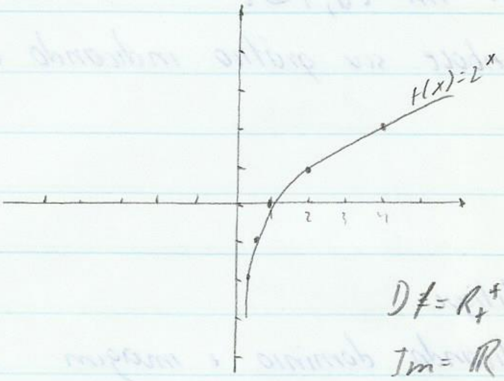
$f: [-1/2, +\infty[\to [\frac{3}{4}, +\infty[$

$g = f^{-1}: [\frac{3}{4}, +\infty[\to [-1/2, +\infty[$

12) $f(x) = 2^x$ $\quad ln_2^x$

É invertível em $\mathbb{R}$ pois toda reta paralela ao eixo x intercepta seu gráfico em, no máximo, um ponto

x	y
y	x
2	4
1	2
0	1
-1	1/2
-2	1/4



$Df = \mathbb{R}_+^*$ ou $]0, +\infty[$

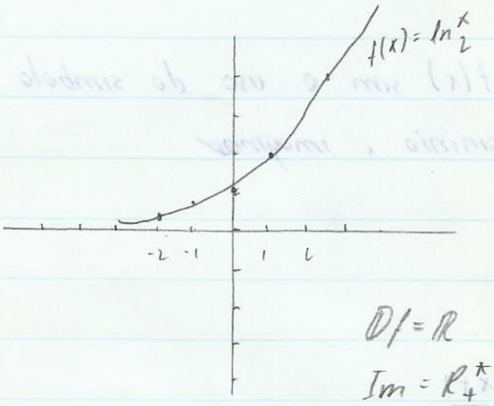
$Im = \mathbb{R}$

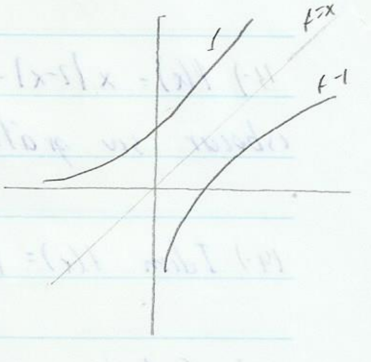
$2^{g(x)} = x$

$\ln 2^{g(x)} = \ln x$

$g(x) \ln 2 = \ln$

$g(x) = \dfrac{\ln x}{\ln 2} = \log_2^x$



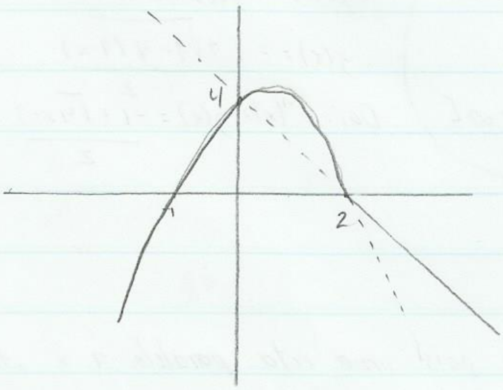


$Df = \mathbb{R}$

$Im = \mathbb{R}_+^*$ $\quad]0, +\infty[$

13) $f(x) = x|2-x| - x^2 + 4$

$$\begin{cases} 2x - x^2 - x^2 + 4 \\ -2x + x^2 - x^2 + 4 \end{cases} \begin{cases} -2x^2 + 2x + 4, & \text{se } x \leq 2 \\ -2x + 4, & \text{se } x > 2 \end{cases}$$



$-2x^2 + 2x + 4$

$x = \dfrac{-2 \pm \sqrt{4+32}}{-4}$

$x_1 = \dfrac{-2+6}{-4} = \dfrac{4}{-4} = -1$

$x_2 = \dfrac{-2-6}{-4} = \dfrac{-8}{-4} = 2$

14-) $f(x) = |x^2+2x| - 3$ $\begin{cases} x^2+2x-3, \text{ se } x \geq 0 \text{ ou } x \leq -2 \\ -x^2-2x-3, \text{ se } 0 < x < -2 \end{cases}$

x^2+2x-3

$x = \dfrac{-2 \pm \sqrt{4+12}}{2}$

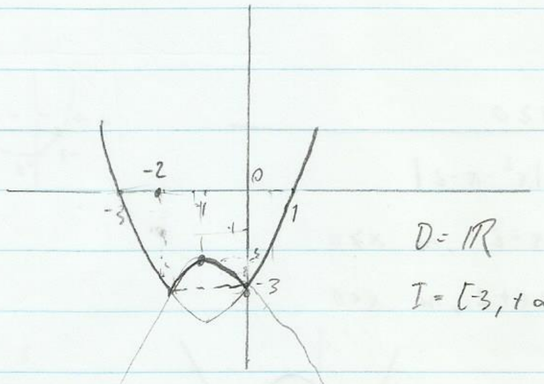
$x(x+2) = 0$

$x = 0$

$x = -2$

$x_1 = \dfrac{-2+4}{2} = \dfrac{2}{2} = 1$

$x_2 = \dfrac{-2-4}{2} = \dfrac{-6}{2} = -3$



$D = \mathbb{R}$

$I = [-3, +\infty[$ $\quad V = (-1, -2)$

15-) $\lim\limits_{x \to 1} \dfrac{\sqrt{x^3+5x-2} - 3x+1}{x^2-1}$

$\dfrac{\sqrt{x^3+5x-2} - (3x-1)}{x^2-1} \dfrac{[(\sqrt{x^3+5x-2}) + (3x-1)]}{[(\sqrt{x^3+5x-2} + (3x-1)]} = \dfrac{x^3+5x-2-(3x-1)^2}{x^2-1\,[\sqrt{x^3+5x-2} + (3x-1)]}$

$= \dfrac{x^3+5x-2-9x^2-6x+1}{(x^2-1)[\sqrt{x^3+5x-2} + (3x-1)]}$

/ /

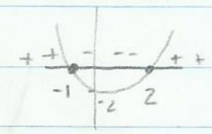
$f(x) = |x^2 - |x| - 2|$

$$\begin{cases} |x^2 - x - 2|, & \text{se } x \geq 0 \\ |x^2 + x - 2|, & \text{se } x < 0 \end{cases}$$

$x^2 - x - 2$

$x = \dfrac{1 \pm \sqrt{1+8}}{2}$ $\quad x_1 = \dfrac{1+3}{2} = \dfrac{4}{2} = 2$

$x_2 = \dfrac{1-3}{2} = \dfrac{-2}{2} = -1$



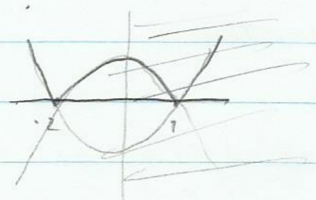
$x < 0$ $\qquad 0 \qquad$ $x \geq 0$

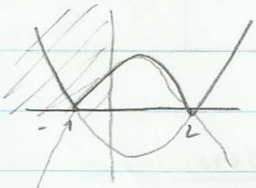
$f(x) = |x^2 + x - 2|$

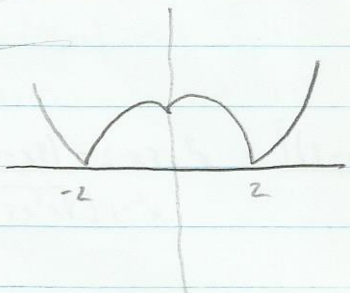
$$\begin{cases} x^2 + x - 2, & \text{se } x \geq 0 \\ -x^2 - x + 2, & \text{se } x < 0 \end{cases}$$

$f(x) = |x^2 - x - 2|$

$$\begin{cases} x^2 - x - 2, & \text{se } x \geq 0 \\ -x^2 + x + 2, & \text{se } x < 0 \end{cases}$$







Limites que contém o símbolo infinito ∞

Regras práticas

$K \pm \infty = \pm \infty$

$K \cdot (\pm\infty) = \begin{cases} +\infty & \text{se } K > 0 \\ -\infty & \text{se } K < 0 \end{cases}$

$\dfrac{K}{0} \stackrel{?}{=} \pm\infty$ Verificar os limites laterais

$\dfrac{K}{\pm\infty} = 0$

$(+\infty)^n = +\infty$

$(-\infty)^n = \begin{cases} +\infty, & \text{se } n \text{ par} \\ -\infty, & \text{se } n \text{ ímpar} \end{cases}$

$K^{+\infty} = \begin{cases} +\infty, & \text{se } K > 1 \\ 0, & \text{se } 0 \leq K < 1 \end{cases}$

Indeterminações

$\infty - \infty$

$0 \cdot \infty$

$\dfrac{0}{0}$

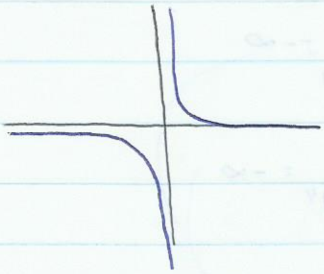
$\dfrac{\infty}{\infty}$

∞^0

0^0

1^∞

Exemplo 1 = $y = \frac{1}{x}$



$$\lim_{x \to \infty} \left(\frac{1}{x}\right) = \frac{1}{\infty} = 0$$

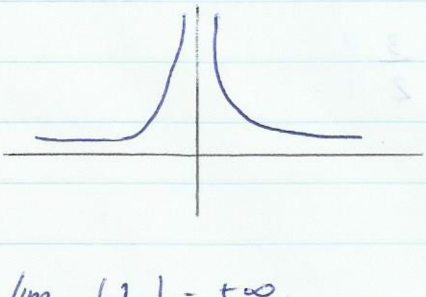
$$\lim_{x \to -\infty} \left(\frac{1}{x}\right) = \frac{1}{-\infty} = 0$$

$$\lim_{x \to 0} \left(\frac{1}{x}\right) = \frac{1}{0}$$ Não existe

$$\lim_{x \to 0^+} \left(\frac{1}{x}\right) = \frac{1}{0} = \infty$$

$$\lim_{x \to 0^-} \left(\frac{1}{x}\right) = -\infty$$

Exemplo 2 = $y = \frac{1}{x^2}$



$$\lim_{x \to 0} \left(\frac{1}{x^2}\right) = +\infty$$

$$\lim_{x \to 0^+} \left(\frac{1}{x^2}\right) = +\infty$$

$$\lim_{x \to 0^-} \left(\frac{1}{x^2}\right) = +\infty$$

Exemplo 3:

$$\lim_{x\to 2} \frac{5}{(x-2)^3} \quad \text{Não existe}$$

$$\lim_{x\to 2^+} \frac{5}{(x-2)^3} = +\infty$$

$$\lim_{x\to 2^-} \frac{5}{(x-2)^3} = -\infty$$

Exemplo 4:

$$\lim_{x\to 3} \frac{-7}{(x-3)^4} = -\infty$$

$$\lim_{x\to 3^+} \frac{-7}{(x-3)^4} = -\infty$$

$$\lim_{x\to 3^-} \frac{-7}{(x-3)^4} = -\infty$$

Exemplo 5 - Calcular

$$\lim_{x\to\infty} (x^2-x) = \infty^2 - \infty = \infty - \infty \ \text{(indeterminado)}$$

solução $\Rightarrow$ $\lim_{x\to\infty} (x^2-x) = x^2\left(1-\frac{1}{x}\right) = \infty^2\left(1-\frac{1}{\infty}\right) = \infty$

Exemplo 6

$$\lim_{x\to\infty} \frac{3x^2+5x}{2x^2+7} = \frac{3\infty^2+5\infty}{2\infty^2+7} = \frac{\infty+\infty}{\infty+7} = \frac{\infty}{\infty} \ \text{(Indeterminado)}$$

Solução $= \frac{\cancel{x^2}\left(3+\frac{5}{x}\right)}{\cancel{x^2}\left(2+\frac{7}{x^2}\right)} = \frac{3+\frac{5}{\infty}}{2+\frac{7}{\infty}} = \frac{3}{2}$

Exercícios: Calcular os limites

1) $\lim\limits_{x \to 1} \dfrac{x^3-1}{x^3-x^2+x-1}$ 2) $\lim\limits_{x \to \frac{1}{2}} \dfrac{4x^2+1}{2x^2+x-1}$ 3) $\lim\limits_{x \to 4} \dfrac{3-\sqrt{5+x}}{4-x}$

4) $\lim\limits_{x \to \infty} \dfrac{-1-2x^2}{(x-1)^2}$ 5) $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x^2-x} - \dfrac{1}{x^2} \right)$

1) $\lim\limits_{x \to 1} \dfrac{x^3-1}{x^3-x^2+x-1} = \lim\limits_{x \to 1} \dfrac{(x-1)(x^2+x+1)}{(x-1)(x^2+1)} = \dfrac{(x^2+x+1)}{x^2+1} = \dfrac{3}{2}$ $a^3-b^3 = (a-b)(a^2+ab+b^2)$

$x^3-1 = (x-1)(x^2+x+1)$

$x^2(x-1)+(x-1)$

$(x-1)(x^2+1)$

Briot-Ruffini

$\lim\limits_{x \to 1} \dfrac{x^3-1}{x^3-x^2+x-1} = \lim \dfrac{x^2+x+1}{x^2+1} = \dfrac{3}{2}$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ \hline 1 & 1 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{c|cccc} & 1 & -1 & 1 & -1 \\ \hline 1 & 1 & 0 & 1 & 0 \end{array}$$

2) $\lim\limits_{x \to 1/2} \left(\dfrac{4x^2-1}{2x^2+x-1} \right) = \dfrac{4x+2}{2x+2} = \dfrac{4}{3}$

$$\begin{array}{c|ccc} & 4 & 0 & -1 \\ \hline 1/2 & 4 & 2 & 0 \end{array} \qquad \begin{array}{c|ccc} & 2 & 1 & -1 \\ \hline 1/2 & 2 & 2 & 0 \end{array}$$

3) $\lim\limits_{x \to 4} \dfrac{3-\sqrt{5+x}}{4-x} = \dfrac{\overbrace{(3)-(\sqrt{5+x})}^{(a-b)}}{(4-x)} \cdot \dfrac{\overbrace{(3+\sqrt{5+x})}^{(a+b)}}{(3+\sqrt{5+x})} = \dfrac{\overbrace{9-(5+x)}^{a^2-b^2}}{(4-x)(3+\sqrt{5+x})} = \dfrac{\cancel{4-x}}{\cancel{(4-x)}(3+\sqrt{5+x})}$

$\lim\limits_{x \to 4} = \dfrac{1}{3+\sqrt{5+x}} = \dfrac{1}{6}$

4) $\lim\limits_{x \to \infty} \dfrac{-1-2x^2}{(x-1)^2} = \dfrac{-1-2x^2}{x^2-2x-1} = \dfrac{x^2\left(\frac{-1}{x^2}-2\right)}{x^2\left(\frac{-2}{x}-\frac{1}{x^2}\right)} = -$

17) $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2+x+1}-3x-2}{2x+1-\sqrt{x^2+7}}$

18) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2+x+1}-3x-2}{2x+1-\sqrt{x^2+7}}$

19) $\lim\limits_{x\to+\infty} \left(\sqrt{x^2+3x+1}-x\right)$

20) $\lim\limits_{x\to-\infty} \left(\sqrt{x^2+3x+1}-x\right)$

21) $\lim\limits_{x\to2} \left(\dfrac{1}{x-2}+\dfrac{3}{x^2-7x+10}\right)$

22) $\lim\limits_{x\to1} \dfrac{\sqrt{x+3}-2}{3-\sqrt{x^2+8}}$

23) $\lim\limits_{x\to0} \dfrac{\sqrt{5-\cos^2x}+\operatorname{sen}x-2}{x}$ 1

24) $\lim\limits_{x\to0} \dfrac{1-\cos x}{x^2}$ $\dfrac{1}{2}$

25) $\lim\limits_{x\to0} \dfrac{1-\cos 3x}{x\operatorname{sen}x}$ $\dfrac{9}{2}$

26) $\lim\limits_{x\to0} \dfrac{1-\sqrt{\cos 7x}}{x^2}$ $\dfrac{7}{4}$

27) $\lim\limits_{x\to0} \dfrac{\sqrt{1+2\operatorname{sen}3x}-\sqrt{1-5\operatorname{sen}2x}}{x}$ 8

15) $\lim\limits_{x\to1} \dfrac{\sqrt{x^3+5x-2}-3x+1}{x^2-1} = \lim\limits_{x\to1} \dfrac{\overbrace{\sqrt{x^3+5x-2}}^{A}-\overbrace{(3x-1)}^{B}}{x^2-1}$

Obs: conjugado $(A+B)$ é $(A-B)$
conjugado $(A-B)$ é $(A+B)$
e $(A+B)(A-B)=(A-B)(A+B)=A^2+B^2$

$= \lim\limits_{x\to1} \dfrac{\left[\sqrt{x^3+5x-2}-(3x-1)\right]\left[\sqrt{x^3+5x-2}+(3x-1)\right]}{(x^2-1)\left[\sqrt{x^3+5x-2}+(3x-1)\right]} =$

$= \lim\limits_{x\to1} \dfrac{(x^3+5x-2)-(3x-1)^2}{(x^2-1)\left[\sqrt{x^3+5x-2}+(3x-1)\right]} =$

$= \lim\limits_{x\to1} \dfrac{x^3+5x-2-(9x^2-6x+1)}{(x^2-1)\left[\sqrt{x^3+5x-2}+(3x-1)\right]} =$

$= \lim\limits_{x\to1} \dfrac{x^3-9x^2+11x-3}{(x^2-1)\left[\sqrt{x^3+5x-2}+(3x-1)\right]} = \left(\dfrac{0}{0}\right) =$

$$\begin{array}{l|l} x^3-9x^2+11x-3 & x-1 \\ \cline{2-2} -x^3+x^2 & x^2-8x+3 \\ \hline -8x^2+11x-3 & \\ +8x^2-8x & \\ \hline 3x-3 & \\ -3x+3 & \\ \hline 0 & \end{array}$$

$= \lim\limits_{x\to1} \dfrac{(x-1)(x^2-8x+3)}{(x-1)(x+1)\left[\sqrt{x^2+5x-2}+(3x-1)\right]} =$

$= \dfrac{-4}{2\cdot4} = \dfrac{-1}{2}$

/ /

OBS: 1º Limite fundamental

$$\lim_{x\to 0} \frac{\operatorname{sen} \alpha x}{x} = \alpha$$

17-) $\lim_{x\to+\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}-3x-2}{2x+1-\sqrt{x^2+7}}$

$= \frac{|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-3x-2}{2x+1-\sqrt{x^2+7}}$

$= \frac{x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-3-\frac{2}{x}\right)}{x\left(2+\frac{1}{x}-\sqrt{1+\frac{7}{x^2}}\right)}$

$\frac{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-3-\frac{2}{x}}{2+\frac{1}{x}-\sqrt{1+\frac{7}{x^2}}}$

$= \frac{\sqrt{1}-3}{2-1} = -2$

18-) $\lim_{x\to-\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}-3x-2}{2x+1-\sqrt{x^2\left(1+\frac{7}{x^2}\right)}}$

$= \frac{-x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+3+\frac{2}{x}\right)}{-x\left(-2-\frac{1}{x}-\sqrt{1+\frac{7}{x^2}}\right)}$

$= \frac{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+3+\frac{2}{x}}{-2-\frac{1}{x}-\sqrt{1+7/x^2}}$

$= \frac{\sqrt{1}+3}{-2-\sqrt{1}} = \frac{1+3}{-2-1} = -\frac{4}{3}$

19-) $\lim_{x\to+\infty} \frac{\sqrt{x^2+3x+1}-x \cdot \sqrt{x^2+3x+1}+x}{\sqrt{x^2+3x+1}+x} = \frac{x^2+3x+1-x^2}{\sqrt{x^2+3x+1}+x}$

$\frac{3x+1}{\sqrt{x^2+3x+1}+x} = \frac{x\left(3+\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}+1\right)} = \frac{3}{\sqrt{1}+1} = \frac{3}{2}$

20-) $\lim_{x\to-\infty} \left(\sqrt{x^2+3x-1}-x\right) \quad \frac{\sqrt{x^2+3x-1}-x \cdot \sqrt{x^2+3x-1}+x}{\sqrt{x^2+3x-1}+x}$

$\frac{x^2+3x-1-x^2}{\sqrt{x^2+3x-1}+x} = \frac{3x-1}{\sqrt{x^2\left(1+\frac{3}{x}-\frac{1}{x^2}\right)}+x} \quad \frac{\cancel{x}\left(3-\frac{1}{x}\right)}{\cancel{x}\left(-\sqrt{1+\frac{3}{x}-\frac{1}{x^2}}+1\right)} = \frac{3}{0}$

$+\infty \qquad = (+\infty-(-\infty)) = +\infty$

21-) $\lim_{x \to 2} \left(\frac{1}{x-2} - \frac{3}{x^2-7x+10} \right) =$

$$= \frac{x-5-3}{x^2-7x+10}$$

$$= \frac{x-8}{x^2-7x+10}$$

$$= \frac{(x-8)}{(x-2)(x-5)}$$

$$\begin{array}{l|l} x^2-7x+10 & x-2 \\ \hline -x^2+2x & x-5 \\ \hline -5x+10 & \\ +5x-10 & \\ \hline 0 & \end{array}$$

nova 21 $\left(\frac{1}{x-2} + \frac{3}{x^2-7x+10} \right)$

$$= \frac{(x-2)}{(x-2)(x-5)} = \frac{1}{(x-5)} = \frac{1}{2-5} = \frac{1}{-3}$$

$$\begin{array}{l|l} x^2-7x+10 & x-2 \\ \hline -x^2+2x & x-5 \\ \hline -5x+10 & \\ -5x-10 & \\ \hline 0 & \end{array}$$

$(x-5)(x-2)$

22-) $\lim_{x \to 1} \frac{\sqrt{x+3}-2}{3-\sqrt{x^2+8}} = \frac{\sqrt{x+3}-2 \cdot \sqrt{x+3}+2}{(3-\sqrt{x^2+8}).[\sqrt{x+3}+2]}$

$$= \frac{x+3-4}{(3-\sqrt{x^2+8}).(\sqrt{x+3}+2)} \cdot \frac{-\sqrt{x^2+8}-3}{(-\sqrt{x^2+8}-3)} = \frac{(-\sqrt{x^2+8}-3).(x-1)}{(x^2-1)(\sqrt{x+3}+2)}$$

$$= \frac{(-\sqrt{x^2+8}-3)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)(\sqrt{3+x}+2)}$$

$$= \frac{(-\sqrt{x^2+8}-3)}{(x+1)(\sqrt{3+x}+2)} = \frac{-6}{8} = \frac{-3}{4}$$

$\cos(A+B) = \cos A \cdot \cos B - \text{sen} A \, \text{sen} B$

$\text{sen}^2 x + \cos^2 x = 1$

$\text{sen}^2 x = 1 - \cos^2 x$

23) $\lim\limits_{x \to 0} \dfrac{\sqrt{5-\cos^2 x} + \text{sen}\, x - 2}{x}$

$= \dfrac{\sqrt{5-\cos^2 x} + (\text{sen}\, x - 2)}{x} \cdot \dfrac{\sqrt{5-\cos^2 x} - (\text{sen}\, x - 2)}{\sqrt{5-\cos^2 x} - (\text{sen}\, x - 2)}$

$= \dfrac{[5 - \cos^2 x - (\text{sen}^2 x - 4\,\text{sen}\, x + 4)]}{x\,[\sqrt{5-\cos^2 x} - (\text{sen}\, x - 2)]}$

$= \dfrac{5 - \cos^2 x - \text{sen}^2 x + 4\,\text{sen}\, x + 4}{x\,[\sqrt{5-\cos^2 x} - \text{sen}\, x + 2]}$

$= 9 \cdot \cos^2 x - \text{sen}^2 x + 4\,\text{sen}\, x$

24) $\lim\limits_{x \to 0} \dfrac{1-\cos x}{x^2} \cdot \dfrac{1+\cos x}{1+\cos x} = \dfrac{1-\cos x^2}{x^2(1+\cos x)} \to \dfrac{\text{sen}^2 x}{x^2(1+\cos x)} \left(\lim\limits_{x \to 0} \dfrac{\text{sen}\, x}{x}\right)\left(\lim\limits_{x \to 0} \dfrac{\text{sen}\, x}{x}\right)\left(\lim \dfrac{1}{1+\cos x}\right)$

$\lim\limits_{x \to 0} = \dfrac{1}{2}$

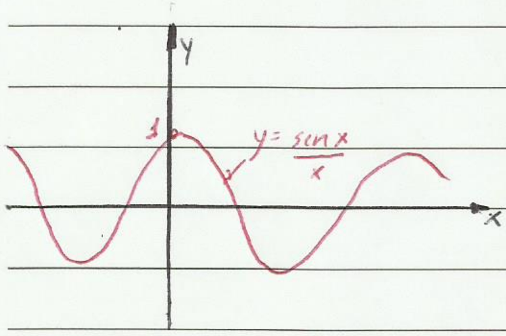
25) $\lim\limits_{x \to 0} \dfrac{1-\cos 3x}{x\, \text{sen}\, x}$

26) $\lim\limits_{x \to 0} \dfrac{1-\sqrt{\cos 7x}}{x^2} \cdot \dfrac{1+\sqrt{\cos 7x}}{1+\sqrt{\cos 7x}} = \dfrac{1-\cos 7x}{x^2(1+\sqrt{\cos 7x})}$

Limites fundamentais

① $\boxed{\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1}$

Gráfico

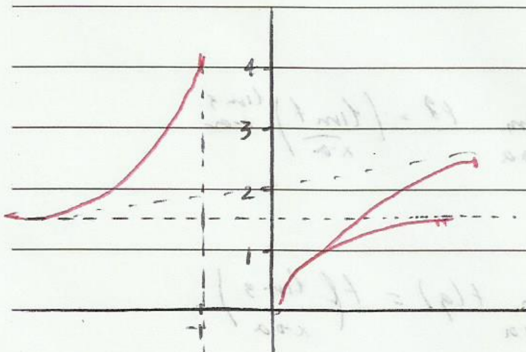


tabela

x	$\frac{\operatorname{sen} x}{x}$
0,1	0,9983416
0,01	0,9999833
0,001	0,9999968
0,0001	0,999999
⋮	
0	

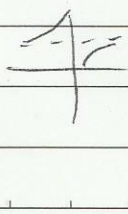
② $\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$

Gráfico



tabela

x	$\left(1 + \frac{1}{x}\right)^x$
1	2
2	2,25
10	2,59374
100	2,704813
1000	2,716983
10000	2,718149
⋮	
∞	$e = 2,71828$



DATA / /

$sen(2x) = 2 \cdot sen x \cdot cos x$

$sen(x+x) = sen x \cdot cos x + sen x \cdot cos x$

Exercícios: Calcule os limites

1) $\lim_{x\to 0} \frac{sen(2x)}{x}$

4) $\lim_{h\to 0} \frac{sen(x+h) - sen x}{h}$

2) $\lim_{x\to 0} \frac{tg x}{x}$

5) $\lim_{x\to a} \frac{tg x - tg a}{x-a}$

3) $\lim_{x\to 0} \frac{1-cos x}{x^2}$

Respostas

1) $\lim_{x\to 0} \frac{sen(2x)}{x} = \lim_{y\to 0} \frac{sen(y)}{\frac{y}{2}} = \lim_{y\to 0} \left(2 \cdot \frac{sen y}{y}\right) = \underbrace{\left(\lim_{y\to 0} 2\right)}_{2} \cdot \left(\lim_{y\to 0} \frac{sen y}{y}\right) = \boxed{2}$

Fazer $y = 2x$ $\begin{cases} x \to 0 \\ y \to 0 \end{cases}$

$\frac{y}{2} = x$

Propriedades

1) $\lim_{x\to a} K = K$

2) $\lim_{x\to a} (f \pm g) = \left(\lim_{x\to a} f\right) \pm \left(\lim_{x\to a} g\right)$

3) $\lim_{x\to a} (f \cdot g) = \left(\lim_{x\to a} f\right) \cdot \left(\lim_{x\to a} f\right)$

4) $\lim_{x\to a} \left(\frac{f}{g}\right) = \frac{\lim_{x\to a} f}{\lim_{x\to a} g}$

5) $\lim_{x\to a} f^g = \left(\lim_{x\to a} f\right)^{\lim_{x\to a} g}$

6) $\lim_{x\to a} f(g) = f\left(\lim_{x\to a} g\right)$

2) $\lim\limits_{x\to 0} \dfrac{tg\, x}{x} = \lim\limits_{x\to 0} \dfrac{\frac{sen\, x}{cos\, x}}{x} = \lim\limits_{x\to 0} \left(\dfrac{sen\, x}{cos\, x} \cdot \dfrac{1}{x} \right) = \lim\limits_{x\to 0} \left(\dfrac{sen\, x}{x} \cdot \dfrac{1}{cos\, x} \right)$

$= \underbrace{\left(\lim\limits_{x\to 0} \dfrac{sen\, x}{x} \right)}_{1} \cdot \underbrace{\left(\lim\limits_{x\to 0} \dfrac{1}{cos\, x} \right)}_{1} = \boxed{1}$

3) $\lim\limits_{x\to 0} \dfrac{(1-cos\, x)}{x^2} \cdot \dfrac{(1+cos\, x)}{(1+cos\, x)} = \lim\limits_{x\to 0} \dfrac{1-cos^2 x}{x^2(1+cos\, x)} = \lim \dfrac{sen^2 x}{x^2(1+cos\, x)}$

$\lim\limits_{x\to 0} \left(\dfrac{sen\, x}{x} \right) \cdot \left(\dfrac{sen\, x}{x} \right) \cdot \left(\dfrac{1}{1+cos\, x} \right)$

$\underbrace{\left(\lim\limits_{x\to 0} \dfrac{sen\, x}{x} \right)}_{1} \cdot \underbrace{\left(\lim\limits_{x\to 0} \dfrac{sen\, x}{x} \right)}_{1} \cdot \underbrace{\left(\lim\limits_{x\to 0} \dfrac{1}{(1+cos\, x)} \right)}_{1/2} = \boxed{\dfrac{1}{2}}$

4-) $\lim\limits_{h\to 0} \dfrac{sen(x+h) - sen\, x}{h} =$

$= \lim\limits_{h\to 0} \dfrac{2 \cdot sen\left(\frac{x+h-x}{2}\right) \cdot cos\left(\frac{x+h+x}{2}\right)}{h}$

$= \lim\limits_{h\to 0} \dfrac{sen\left(\frac{h}{2}\right) \cdot cos\left(\frac{2x+h}{2}\right)}{\frac{h}{2}}$

$= \underbrace{\left(\lim\limits_{h\to 0} \dfrac{sen\left(\frac{h}{2}\right)}{\frac{h}{2}} \right)}_{1} \cdot \underbrace{\left(\lim\limits_{h\to 0} cos\left(\dfrac{2x+h}{2}\right) \right)}_{cos\, x} = \boxed{cos\, x}$

Fórmula de transformação em produto

$sen\, p - sen\, q = 2 \cdot sen\left(\dfrac{p-q}{2}\right) \cdot cos\left(\dfrac{p+q}{2}\right)$

DATA / /

$$\operatorname{sen} p + \operatorname{sen} q = 2 \operatorname{sen}\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)$$

$$\operatorname{sen} p - \operatorname{sen} q = 2 \operatorname{sen}\left(\frac{p-q}{2}\right) \cdot \cos\left(\frac{p+q}{2}\right)$$

$$\cos p + \cos p = 2 \cos\left(\frac{p+q}{2}\right) \cdot \operatorname{sen}\left(\frac{p-q}{2}\right)$$

$$\cos p - \cos p = -2 \operatorname{sen}\left(\frac{p-q}{2}\right) \cdot \cos\left(\frac{p+q}{2}\right)$$

5-) $$\lim_{x\to a} \frac{tg\,x - Tg\,a}{x-a} = \lim_{x\to a} \frac{\left(\frac{\operatorname{sen} x}{\cos x}\right) - \left(\frac{\operatorname{sen} a}{\cos a}\right)}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{\operatorname{sen} x \cdot \cos a - \operatorname{sen} a \cdot \cos x}{\cos x \cdot \cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{\operatorname{sen}(x-a)}{\cos x \cdot \cos a}}{\frac{x-a}{1}}$$

$$= \underbrace{\left(\lim_{x\to a} \frac{\operatorname{sen}(x-a)}{(x-a)}\right)}_{1} \cdot \underbrace{\left(\lim_{x\to a} \frac{1}{\cos x \cdot \cos a}\right)}_{1/\cos^2 a} = \boxed{\frac{1}{\cos^2 a}}$$

Exercícios

6) $$\lim_{x\to 0} (1+x)^{\frac{1}{x}}$$

7) $$\lim_{x\to \infty} \left(1 - \frac{2}{x}\right)^x$$

8) $$\lim_{x\to \infty} \left(\frac{x}{x+1}\right)^x$$

DATA / /

Resolução

6) $\lim_{x \to 0} (1+x)^{\frac{1}{x}} = \lim_{y \to \pm\infty} \left(1+\frac{1}{y}\right)^{y} = \boxed{e}$

Fazer $x = \frac{1}{y}$ $\begin{cases} x \to 0 \\ y \to \pm\infty \end{cases}$

$y = \frac{1}{x}$

7) $\lim_{x \to \infty} \left(1 - \frac{2}{x}\right)^{x} = \lim_{x \to -\infty} \left(1+\frac{1}{y}\right)^{-2y} = \lim_{x \to -\infty} \left[\left(1+\frac{1}{y}\right)^{y}\right]^{-2} = \boxed{e^{-2}}$

$\frac{-2}{x} = \frac{1}{y}$ $\begin{cases} x \to \infty \\ y \to -\infty \end{cases}$

$x = -2y$

$y = \frac{x}{-2}$

8) $\lim_{x \to \infty} \left(\frac{x}{x+1}\right)^{x} = \lim_{x \to \infty} \left(\frac{x+1}{x}\right)^{-x} =$

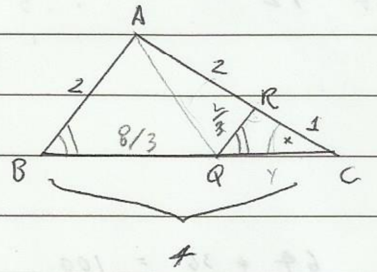
$= \lim_{x \to \infty} \left(1+\frac{1}{x}\right)^{-x}$

$= \lim_{x \to \infty} \left[\left(1+\frac{1}{x}\right)^{x}\right]^{-1} = \boxed{e^{-1}}$

DATA / /

28) Prove que $\frac{\cos x - \text{sen}x}{\cos x + \text{sen}x} + \frac{\cos x + \text{sen}x}{\cos x - \text{sen}x} = \frac{2}{\cos 2x}$

29)



$\frac{4}{Y} = \frac{3}{1}$

$3Y = 4$

$Y = \frac{4}{3}$

$4 - \frac{4}{3} = \frac{12-4}{3} = \frac{8}{3}$

RQ // AB

Determinar $\overline{AQ}$

30) Domínio $f(x) = \frac{\sqrt{x^2-2x-3}}{\ln(2-x^2)}$

$\cos(x+x) = \cos x . \cos x - \text{sen}x . \text{sen}x$

$(\cancel{2}\cos x^2 - \cancel{2}\text{sen}\, x^2)$

$\text{sen}(x+x) = \text{sen}x . \cos x - \cos x . \text{sen}x$

31) $\lim_{x \to 0} \frac{1-\cos 7x}{\sqrt{1+\text{sen}^2 x} - \cos x}$

32) Calcular $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ para $\begin{cases} (a)\ f(x) = x^2 \\ (b)\ f(x) = \frac{1}{x},\ x \neq 0 \end{cases}$

28-) $\frac{\cos^2 x - \cancel{2\cos x\, \text{sen}x} + \text{sen}x^2 + \cos^2 x + \cancel{2\cos x\, \text{sen}x} + \text{sen}^2 x}{\cos^2 x - \text{sen}^2 x}$

$\frac{2\cos^2 x + 2\,\text{sen}^2 x}{\cos^2 x - \text{sen}^2 x} = \frac{2}{\cos 2x}$

29-) $2^2 = 3^2 + 4^2 - 2.3.4.\cos x$

$4 = 9 + 16 - 24\cos x$

$4 = 25 - 24\cos x$

$-16 = -24\cos x$

$\cos x = \frac{-16}{-24} = \frac{2}{3}$

$\frac{2}{x} = \frac{3}{1}$

$3x = 2$

$x = \frac{2}{3}$

$3^2 = 2^2 + 4^2 + 2.2.4\cos x$

$9 = 4 + 16 - 16\cos x$

$9 = 20 - 16\cos x$

$-11 = -16\cos x$

$\cos x = \frac{11}{16}$. $\frac{1}{4}$

DATA / /

$1=\left(\frac{2}{3}\right)^2+x^2-2.\frac{2}{3}.1.\frac{11}{16}$

$1=\frac{4}{9}+x^2-\frac{44}{48}$ $\qquad \frac{44}{48}=\frac{22}{24}=\frac{11}{12}$

12 . 9	2
6 . 9	2
3 . 9	3
1 . 3	3

$x^2=\frac{44}{48}-\frac{4}{9}+1$

$x^2=\frac{23}{12}-\frac{4}{9} \Rightarrow \frac{69-16}{36}=\frac{53}{36}=$

$\frac{64}{9}+\frac{36}{9}=\frac{100}{9}$ $\qquad$ 32 16 $\quad \frac{11}{32} \quad \frac{3}{48} \quad \frac{32}{352}$

$x^2=\left(\frac{8}{3}\right)^2+2^2-2.\frac{8}{3}.2.\cos\frac{11}{16}$

$\frac{32}{3}.\frac{11}{16}=\frac{352}{48}=\frac{176}{24}=\frac{88}{12}=\frac{44}{6}$

$x^2=\frac{64}{9}+4-\frac{32}{3}.\cos\frac{11}{16}$ $\qquad \frac{22}{3}$

$x^2=\frac{100}{9}-\frac{22}{3}$ $\qquad$ 22 · 3 = 66

$x^2=\frac{100-66}{9}$ $\quad x=\frac{\sqrt{34}}{3}$

30- $f(x)=\frac{\sqrt{x^2-2x-3}}{\ln(2-x^2)}$

$x^2-2x-3\geq 0$ $\qquad \ln(2-x^2)>0$

$x=\frac{2\pm\sqrt{4+12}}{2}$ $\qquad 2-x^2=0$

$-x^2=-2$

$x=\sqrt{2}$

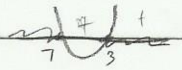
	$-\sqrt{2}$	-1	0	$\sqrt{2}$	3
I	+ + +	−	− −	−	+ +
II	− −	+ + +		− − −	

$]-\sqrt{2},-1]\cup]\sqrt{2},3]$

$x_1=\frac{2+4}{2}=\frac{6}{2}=3$

$x_2=\frac{2-4}{2}=\frac{-2}{2}=-1$ $\qquad 2-x^2=1$

$-x^2=-1$



$x^2=\sqrt{1}$

$x=1$

$sen^2 x + cos^2 x = 1$

DATA / /

31) $\lim_{x \to 0} \dfrac{1-\cos 7x}{\sqrt{1+sen^2 x} - \cos x} \cdot \dfrac{\sqrt{1+sen^2 x} + \cos x}{\sqrt{1+sen^2 x} + \cos x}$

$\lim_{x \to 0} \dfrac{(1-\cos 7x)(\sqrt{1+sen^2 x} + \cos x)}{1+sen^2 x - \cos^2 x} \dfrac{(1+\cos 7x)}{(1+\cos 7x)}$

$\lim_{x \to 0} \dfrac{(1-\cos^2 7x)(\sqrt{1+sen^2 x} + \cos x)}{1+sen^2 x - \cos^2 x\,(1+\cos 7x)}$

$\lim_{x \to 0} \dfrac{(sen^2 7x)(\sqrt{1+sen^2 x} + \cos x)}{2sen^2 x\,(1+\cos 7x)}$

$\lim_{x \to 0} \dfrac{(sen 7x)(sen 7x)(\sqrt{1+sen^2 x} + \cos x)}{2 sen x . sen x\,(1+\cos 7x)}$ / x (divide x)

$\lim_{x \to 0} \dfrac{7.7.2}{1.1.2} = \dfrac{49}{2}$

32) $\lim_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$ para $\begin{cases} (a)\ f(x) = x^2 \\ (b)\ f(x) = \dfrac{1}{x} \neq 0 \end{cases}$

$\boxed{\begin{array}{l} f(x) = x^2 \\ f(x+h) = (x+h)^2 \end{array}}$

$\lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$

$\lim_{h \to 0} \dfrac{x^2+2xh+h^2-x^2}{h}$ $\lim_{h \to 0} \dfrac{2xh+h^2}{h}$ $\dfrac{h(2x+h)}{h} =$

$\lim_{h \to 0} 2x+h$

$\lim_{h \to 0} 2x$ $\dfrac{f(x+h)-f(x)}{h}$

DATA / /

Exercícios (Prova 26/03/2009

2 1) Prove que $1 - tag \cdot cotg x = sen(x-y) \cdot cos x \cdot sec y$

2 2) Determine o domínio da função $f(x) = \frac{\sqrt{|x|-1}}{\sqrt{5-|x|}}$

2 3) Dadas as funções $f(x) = |x|+1$ e $g(x) = x^2+2x$ e a função $h(x) = (f \circ g)(x) = f(g(x))$, Determine:

a) A expressão de $h(x)$
b) $h(1)$
c) o gráfico de $h(x)$
d) o conjunto imagem de $h(x)$

4) Calcular os limites abaixo

1 a) $\lim_{x \to 0} \frac{\sqrt{1+sen^2 x} - cos x}{5 sen(2x)}$

b) $\lim_{x \to 2} \frac{(x^3 - x^2 - 4)}{(x^2-4) \cdot 3^x}$

c) $\lim_{x \to \infty} [\ln(3x^2-3x+1) - \ln(x^2+1)]$ [illegible]

d) $\lim_{x \to 2} \frac{sen(x-2)}{3x-6}$

DATA / /

Intersecção

2-) $f(x) = \dfrac{\sqrt{|x|-1}}{\sqrt{5+|x|}}$ $\qquad \begin{cases} 5+x & x \geq 5 \\ 5-x & x < 5 \end{cases}$

$|x|-1 \geq 0$ $\qquad 5+|x| \neq 0$ e

$|x| \geq 1$ $\qquad 5+|x| > 0$

$x \geq 1$ e $x \leq -1$ $\qquad |x| > -5$

$x \geq -5$ e $x \leq 5$

	-5	-1	1	5	
I	+	-		+	
II	-	+	+	+	-

$]-5,-1] \cup [1,5[$

3-) $f(x) = |x|+1$ $\qquad h(x) = (f \circ g)(x) = f(g(x))$

$g(x) = x^2+2x$

$f(g(x)) = |x^2+2x|+1$ $\qquad x^2+2x=0$

a) $\begin{cases} x^2+2x+1, & se\ x \geq -2 \text{ e } x \geq 0 \\ -x^2-2x+1, & se\ -2 \leq x \leq 0 \end{cases}$ $\qquad x(x+2)=0$

$x=0$ $\qquad$ + -2 - 0 +

$x=-2$

b) $-x^2-2x+1$

$-1-2+1 = -2$

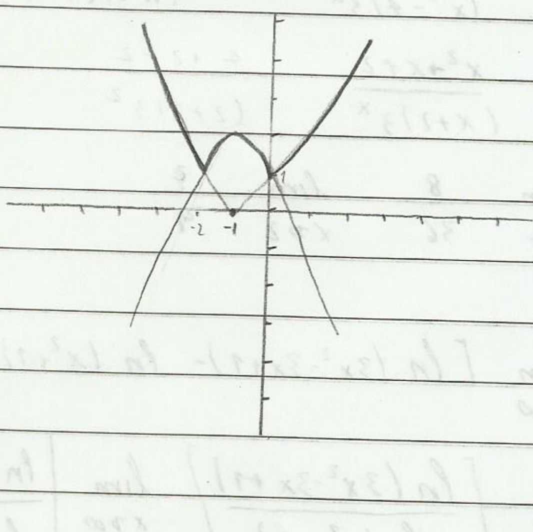
c) x^2+2x+1 $\qquad -x^2-2x+1$

$x = \dfrac{-2 \pm \sqrt{4-4}}{2}$ $\qquad x = \dfrac{2 \pm \sqrt{4+4}}{-2}$

$x = \dfrac{-2}{2} = -1$ $\qquad \Delta = 8$

$V_x = -\left(\dfrac{-2}{-2}\right) = -1$

$V_y = -\left(\dfrac{8}{-4}\right) = 2$



d) $[1,+\infty[$

DATA / /

$sen^2 x + cos^2 x = 1$ $\quad 2.3 + 4.5$

$sen(A+B) = (sen A . cos B) + (cos A . sen B)$

$(sen x . cos x) + (cos x . sen x)$

$2 sen x . cos x$

a) $\lim\limits_{x \to 0} \dfrac{\sqrt{1+sen^2 x} - cos x}{5 sen(2x)} \cdot \dfrac{\sqrt{1+sen^2 x} + cos x}{\sqrt{1+sen^2 x} + cos x}$

$\lim\limits_{x \to 0} \dfrac{(1+sen^2 x) - (cos^2 x)}{[5 sen(2x)][(\sqrt{1+sen^2 x}) + (cos x)]}$

$\lim\limits_{x \to 0} \dfrac{1 + sen^2 x - cos^2 x}{[5 sen(2x)][(\sqrt{1+sen^2 x}) + (cos x)]}$

$\lim\limits_{x \to 0} \dfrac{sen^2 x + sen^2 x}{[5 sen(2x)][(\sqrt{1+sen^2 x}) + (cos x)]} =$

$\lim\limits_{x \to 0} \dfrac{2 \frac{sen x}{x} . sen x}{5 \frac{sen(2x)}{x} [(\sqrt{1+sen^2 x} + cos x)]}$

$\lim\limits_{x \to 0} \dfrac{0}{5 . 2^0 [\sqrt{1+sen^2 x} + (cos x)]}$

$\lim\limits_{x \to 0} \dfrac{0}{20} = \lim\limits_{x \to 0} 0$

b) $\lim\limits_{x \to 2} \dfrac{(x^3 - x^2 - 4)}{(x^2 - 4) . 3^x}$ $\quad \lim\limits_{x \to 2} \dfrac{(x-2)(x^2+x+2)}{(x-2)(x+2) 3^x}$

	1	-1	0	-4
2	1	1	2	0

$\lim\limits_{x \to 2} \dfrac{x^2 + x + 2}{(x+2) 3^x}$ $\quad \dfrac{4 + 2 + 2}{(2+2) 3^2}$

$\lim\limits_{x \to 2} \dfrac{8}{36}$ $\quad \lim\limits_{x \to 2} \dfrac{2}{9}$

c) $\lim\limits_{x \to \infty} [\ln(3x^2 - 3x + 1) - \ln(x^2 + 1)]$

$\lim\limits_{x \to \infty} \left[\dfrac{\ln(3x^2 - 3x + 1)}{\ln(x^2 + 1)}\right]$ $\quad \lim\limits_{x \to \infty} \left[\dfrac{\ln[x^2(3 - \frac{3}{x} + \frac{1}{x^2})]}{\ln[x^2(1 + \frac{1}{x^2})]}\right]$

$\lim\limits_{x \to \infty} \ln \dfrac{3}{1}$ $\quad \lim\limits_{x \to \infty} \ln 3$

Derivadas

Seja I um intervalo aberto e $f: I \to \mathbb{R}$ dada por $y = f(x)$ e seja x_0 um ponto fixado de I

1. Derivada num ponto

Chama-se derivada de f em x_0 e se indica por $f(x_0)$ ao seguinte limite, caso exista e seja finito:

$$\boxed{f'(x_0) = \lim_{x \to 0} \frac{f(x) - f(x_0)}{x - x_0}}$$

Obs: fazendo, no limite, $x - x_0 = h$ ou $x = x_0 + h$, tem-se $x \to x_0 \iff h \to 0$. Daí:

$$\boxed{f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}}$$

Ex1 $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 + 3x$; calcular $f'(1)$

$$f'(1) = \lim_{x \to 1} \frac{(x^2 + 3x) - 4}{x - 1} = \lim_{x \to 1} \frac{x^2 + 3x - 4}{x - 1}$$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+4)}{\cancel{(x-1)}} = 5$$

Ex2 $f(x) = \frac{1}{x-1}$, $x_0 = 2$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{1}{(2+h)-1} - \frac{1}{2-1}}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{1}{1+h} - 1}{h} =$$

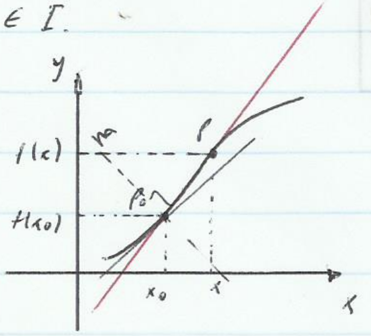
$$= \lim_{h\to 0} \frac{\frac{1-(1+h)}{(1+h)}}{h} =$$

$$= \lim_{h\to 0} \frac{-h}{h(1+h)} =$$

$$= \frac{-1}{1} = -1$$

2. Reta tangente e Reta Normal

Seja s a reta que une os pontos $P_0 = (x_0, f(x_0))$ e $P = (x, f(x))$, com $x \in I$.



É fácil notar que se $x \to x_0$ então $P \to P_0$ e a reta s se transforma em uma reta tangente ao gráfico f em $P_0 = (x_0, f(x_0))$. Daí, a definição:

Se f possui derivada no ponto x_0, então chama-se reta tangente ao gráfico f em x_0 ou no ponto $P_0 = (x_0, f(x_0))$ à reta que passa por $P_0 = (x_0, f(x_0))$ e cujo coeficiente angular é:

$$m_t = \lim_{x \to x_0} m_s = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$$

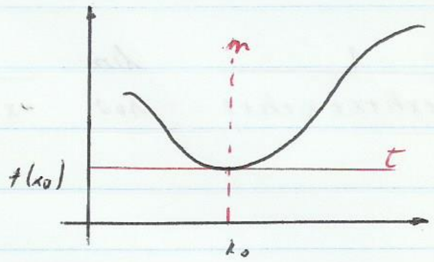
ou seja:

$$\boxed{t: \quad y - f(x_0) = f'(x_0) \cdot (x - x_0)}$$

Consequentemente, a equação da reta normal ao gráfico f em x_0 ou no ponto $P_0 = (x_0, f(x_0))$ é dada por:

$$n: \begin{cases} y - f(x_0) = -\dfrac{1}{f'(x_0)} (x - x_0), & \text{se } f'(x_0) \neq 0 \\ x = x_0, & \text{se } f'(x_0) = 0 \end{cases}$$

$f'(x_0) = 0$



3. Função Derivada

Se f possuir derivada em todo $x_0 \in I$, então f é derivável em I e a função derivada de f em I indicada por f' tem a seguinte expressão algébrica.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Ex1: Obter $f'(x)$, sendo $f(x) = \sqrt{x}$, com $x>0$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}-\sqrt{x})h} =$$

$$= \lim_{h\to 0} \frac{(x+h)-(x)}{(\sqrt{x+h}+\sqrt{x})h} = \lim_{h\to 0} \frac{\cancel{h}^{1}}{(\sqrt{x+h}+\sqrt{x}).\cancel{h}}$$

$$= \frac{1}{2\sqrt{x}}$$

$Ex_3 = f(x) = k$ (constante) $\quad 0$

$Ex_4 = f(x) = \sqrt{x^2+1} \quad x/\sqrt{x^2+1}$

$Ex_5 = f(x) = x^3 \quad 3x^2$

$Ex_6 = f(x) = \sin x \quad \cos x$

Ex2: $f(x) = \frac{x}{x+1}$, $x \neq -1$ $\quad \frac{1}{(x+1)^2}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{x+h}{(x+h)+1} - \frac{x}{x+1}}{h} = \lim_{h\to 0} \frac{\frac{(x+h)(x+1) - x(x+h+1)}{(x+h+1)(x+1)}}{h}$$

$$\lim_{h\to 0} \frac{(x^2+x+hx+h)-(x^2+xh+x)}{h(x+h+1)(x+1)} \quad \lim_{h\to 0} \frac{\cancel{x^2}+\cancel{x}+\cancel{hx}+h-\cancel{x^2}-\cancel{xh}-\cancel{x}}{h(x+h+1)(x+1)}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(x+h+1)(x+1)} \quad \lim_{h\to 0} \frac{1}{x^2+xh+x+x+h+1} \quad \lim_{h\to 0} \frac{1}{x^2+2x+1} \quad \lim_{h\to 0} \frac{1}{(x+1)^2}$$

Ex3 = $f(x) = k$ (constante)

$$f'(x) = \lim_{h\to 0} \frac{k-k}{h} = \lim_{h\to 0} \frac{0}{h} =$$

$$= \lim_{h\to 0} 0 = 0$$

Ext: $f(x) = \sqrt{x^2+1}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h}$$

$$\lim_{h\to 0} \frac{(\sqrt{(x+h)^2+1} - \sqrt{x^2+1})(\sqrt{(x+h)^2+1} + \sqrt{x^2+1})}{h(\sqrt{(x+h)^2+1} + \sqrt{x^2+1})}$$

$$\lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h(\sqrt{(x+h)^2+1} + \sqrt{x^2+1})} \qquad \lim_{h\to 0} \; x^2+1-x^2-1$$

$$\lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+1-x^2-1}{h(\sqrt{(x+h)^2+1} + \sqrt{x^2+1})}$$

$$\lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}(\sqrt{(x+h)^2+1} \; \sqrt{x^2+1})}$$

$$\lim_{h\to 0} \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} \qquad \lim_{h\to 0} \frac{x}{\sqrt{x^2+1}}$$

DATA / /

Derivadas (continuação)

* Função derivada

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

* derivada no ponto x_0

Obs.: $f'(x_0) = tg\,x$

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

Tabelas de derivadas

$f(x)$	$f'(x)$
K	0
x^n	$n \cdot x^{n-1}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$sen\,x$	$cos\,x$
$cos\,x$	$-sen\,x$
$tg\,x$	$sec^2 x$
e^x	e^x
a^x	$a^x \cdot \ln a$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \ln a}$

DATA / /

Regras de derivação

$F(x)$	$F'(x)$
$K.u$	$K.u'$
$u \pm v$	$u' \pm v'$
$u \cdot v$	$u'v + uv'$
$\frac{u}{v}$	$\frac{u'v - uv'}{v^2}$
Regra da cadeia	

Exercícios: Calcular as derivadas
Utilizando a tabela

1) $y = x^{2/3}$ $\quad y' = \frac{2}{3\sqrt[3]{x}}$

5) $y = \sqrt{x}\,(x^2+1)$ $\quad y' = \frac{5x^2+1}{2\sqrt{x}}$

9) $y = \frac{\operatorname{sen} x}{1+\cos x} \rightarrow y' = \frac{1}{1+\cos x}$

2) $y = \sqrt[5]{x^2}$ $\quad y' = \frac{2}{5\sqrt[5]{x^3}}$

6) $y = \frac{x+1}{\sqrt{x}}$ $\quad y' = \frac{x-1}{2x\sqrt{x}}$

10) $y = \frac{1-\operatorname{sen} x}{1-\cos x} \rightarrow y' = \frac{1-\cos x-\operatorname{sen} x}{(1-\cos x)^2}$

3) $y = \frac{5}{x^2}$ $\quad y' = \frac{-10}{x^3}$

7) $y = \frac{x^2-3x}{\sqrt[3]{x^2}}$ $\quad y' = \frac{4x-3}{3\sqrt[3]{x^2}}$

11) $y = \frac{\operatorname{tg} x - 1}{\sec x} \rightarrow y' = \frac{1+\operatorname{tg} x}{\sec x}$

4) $y = (x^3-1)(3-x^2)$
$y' = -5x^4 + 9x^2 + 2x$

8) $y = \frac{(1+2x)^2}{\sqrt{x}} \rightarrow y' = \frac{12x^2+4x-1}{2x\sqrt{x}}$

Respostas:

$\frac{2}{3} - 1 = \frac{2-3}{3} = \frac{-1}{3}$

1) $y = x^{2/3}$ $\quad y' = \frac{2}{3} x^{-\frac{1}{3}}$ ou $\frac{2}{3\sqrt[3]{x}}$

2) $y = \sqrt[5]{x^2}$ $\quad y' = x^{\frac{2}{5}}$ $\quad y' = \frac{2}{5} x^{-\frac{3}{5}}$ ou $\frac{2}{5\sqrt[5]{x^3}}$

3) $y = \frac{5}{x^2}$ $\quad y = 5 . x^{-2}$ $\quad -10x^{-3}$ ou $\frac{-10}{x^3}$

DATA / /

4) $y = (x^3 - 1)(3 - x^2)$

$y' = 3x^3 - x^5 - 3 + x^2$

$= -x^5 + 3x^3 + x^2 - 3$

$= -5x^4 + 9x^2 + 2x$

5) $y = \sqrt{x}\,(x^2+1)$

$y' = \frac{1}{2\sqrt{x}}(x^2+1) + \sqrt{x} \cdot 2x$ $\quad = \frac{4x^2 + x^2 + 1}{2\sqrt{x}}$

$= \frac{x^2}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} + 2x\sqrt{x}$

$= \frac{x^2+1}{2\sqrt{x}} + 2x\sqrt{x}$ $\quad = \frac{5x^2+1}{2\sqrt{x}}$

$= \frac{x^2 + 1 + 4x \cdot x}{2\sqrt{x}}$

6) $y = \frac{x+1}{\sqrt{x}}$ $\qquad y' = \frac{1 \cdot \sqrt{x} - (x+1)\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$

$= \frac{\sqrt{x} - \frac{x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}}{x}$

$= \frac{\sqrt{x} - \frac{x+1}{2\sqrt{x}}}{x}$

$= \frac{\frac{2x + x + 1}{2\sqrt{x}}}{x}$

$= \frac{x+1}{2x\sqrt{x}}$

DATA / /

7) $y = \dfrac{x^2-3x}{\sqrt[3]{x^2}}$

$\sqrt[3]{x^2} = x^{\frac{2}{3}} \quad \frac{2}{3\sqrt[3]{x}}$

$$y' = \frac{(2x-3)(\sqrt[3]{x^2}) - (x^2-3x)\left(\frac{2}{3\sqrt[3]{x}}\right)}{(\sqrt[3]{x^2})^2}$$

$(\sqrt[3]{x^2}).(\sqrt[3]{x} = x \quad \frac{1}{3}$

$\sqrt[3]{x^2}.\sqrt[3]{x^2}$

$\cancel{x^{\frac{2}{3}}} \cdot \cancel{x^{\frac{2}{3}}} = x^{\frac{4}{9}}$

$$= \frac{(2x-3)(\sqrt[3]{x^2}) - \left(\frac{2x^2-6x}{3\sqrt[3]{x}}\right)}{(\sqrt[3]{x^2})^2}$$

$$= \frac{\frac{(2x-3)3x - 2x^2+6x}{3\sqrt[3]{x}}}{(\sqrt[3]{x^2})^2} = \frac{\frac{4x^2-3x}{3\sqrt[3]{x}}}{(\sqrt[3]{x^2})^2}$$

$$= \frac{\frac{6x^2-9x-2x^2+6x}{3\sqrt[3]{x}}}{(\sqrt[3]{x^2})^2}$$

$$= \frac{4x^2-3x}{3\sqrt[3]{x}} : \frac{1}{(\sqrt[3]{x^2})(\sqrt[3]{x^2})}$$

$$= \frac{4x^2-3x}{3x(\sqrt[3]{x^2})}$$

$$= \frac{4x-3}{3(\sqrt[3]{x^2})}$$

6) $y = \text{arc sen}\left(\frac{x}{2}\right)$

$\frac{x}{2} = \frac{2 - x.0}{4} = \frac{2}{4} = \frac{1}{2}$

$y' = \frac{1}{2} : \sqrt{1 - \frac{x^2}{4}}$

$= \frac{1}{2} : \frac{\sqrt{4-x^2}}{2} = \frac{1}{\sqrt{4-x^2}}$

7) $y = \text{arctg}\left(\frac{x+a}{1-ax}\right) \rightarrow y' = \frac{-1}{1+x^2}$

8) $y = \ln\left(\frac{\cos x+1}{\cos x-1}\right) \rightarrow y' = \frac{-2}{\text{sen} x}$

9) $y = \ln(x - \sqrt{1+x^2}) \rightarrow y' = \frac{-1}{\sqrt{1+x^2}}$

7)

$f = \left(\frac{x+a}{1-ax}\right)$ $f' = \frac{1(1-ax) - (+a)(x+a)}{(1-ax)^2}$

$= \frac{(1-ax) - (ax+a^2)}{(1-ax)^2}$

$= \frac{1 - ax - ax + a^2}{1 - 2ax - a^2x^2}$

$= \frac{1 - 2ax + a^2}{1 - 2ax + a^2x^2} = \frac{1}{x^2}$

$y' = \frac{1}{x^2} : 1 + \left(\frac{x+a}{1-ax}\right)^2$

$y' = \frac{1}{x^2} : 1 + \left(\frac{x^2+2ax+a^2}{1-2ax+a^2x^2}\right)$

$y' = \frac{1}{x^2} : 1 - 2ax + a^2x^2 + x^2 + 2ax + a^2$

Regra da cadeia

Sejam as funções $y(u) = f(u)$ e $u(x) = g(x)$. A derivada da composição $y(x) = f(u(x))$ é dada por

$$\boxed{\frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}} \quad \text{ou} \quad \boxed{y'(x) = f'(u) \cdot u'(x)}$$

Ex: Calcular a derivada de $y = (\underbrace{x^3+2}_{u})^2 = u^2$

1ª Maneira (Desenvolver)

$y = x^6 + 4x^3 + 4$

$y' = 6x^5 + 12x^2$

2ª Maneira (Regra da cadeia)

$\frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$\frac{dy}{dx} = 2u \cdot 3x^2$

$\frac{dy}{dx} = 2(x^3+2) \cdot 3x^2$

$\frac{dy}{dx} = 6x^5 + 12x^2$

ou

$y = (\underbrace{x^3+2}_{u})^2$

$y' = 2(x^3+2)' \cdot u'$

$y' = 2(x^3+2) \cdot 3x^2$

$y' = 6x^5 + 12x^2$

Exercícios:

1) $y = (1-6x)^{\frac{2}{3}}$

2) $y = \left(\frac{2x+1}{3x-1}\right)^4$

3) $y = \sqrt{\frac{x+3}{1-x}}$

4) $y = (tg\, x)^4$

5) $y = arc\, tg\left(\frac{a}{x}\right)$ $\quad y' = \frac{-a}{a^2+x^2}$

6) $y = arc\, sen\left(\frac{x}{2}\right)$ $\quad y' = \frac{1}{\sqrt{4-x^2}}$

/ /

1) $y = (1-6x)^{\frac{2}{3}}$

$y' = 2/3\,(1-6x)^{\frac{-1}{3}} \cdot (-6)$

$y' = -4(1-6x)^{\frac{-1}{3}}$ ou $y' = \dfrac{-4}{\sqrt[3]{1-6x}}$

2) $y = \left(\dfrac{2x+1}{3x-1}\right)^4$

$y' = 4\left(\dfrac{2x+1}{3x-1}\right)^3 \cdot \dfrac{-5}{(3x-1)^2}$ $\quad d = \dfrac{2x+1}{3x-1}$ $\quad d' = \dfrac{2(3x-1)-(2x+1)3}{(3x-1)^2}$

$y' = \dfrac{-20}{(3x-1)^2} \cdot \left(\dfrac{2x+1}{3x-1}\right)^3$ $\quad = \dfrac{6x-2-6x-3}{(3x-1)^2}$

$y' = \dfrac{-20(2x+1)^3}{(3x-1)^5}$ $\quad = \dfrac{-5}{(3x-1)^2}$

3) $\sqrt{\dfrac{x+3}{1-x}}$

$y' = \dfrac{1}{2\sqrt{\dfrac{x+3}{1-x}}} \cdot \dfrac{4}{(1-x)^2}$ $\quad d = \dfrac{x+3}{1-x}$ $\quad d' = \dfrac{1(1-x)-(x+3)(-1)}{(1-x)^2}$

$y' = \dfrac{2}{\sqrt{\dfrac{x+3}{1-x}} \cdot (1-x)^2}$ $\quad y' = \dfrac{2\sqrt{1-x}}{\sqrt{x+3}\,(1-x)^2}$ $\quad d' = \dfrac{1-\cancel{x}+\cancel{x}+3}{(1-x)^2}$

$d' = \dfrac{4}{(1-x)^2}$

4) $(tg\,x)^4$

$y' = 4(tg\,x)^3 \cdot sec^2 x$

$t' \quad \dfrac{a}{x} = ax^{-1} \quad -ax^{-2} \quad \dfrac{-a}{x^2}$

ou

5) $y = arctg\left(\dfrac{a}{x}\right)$ $\quad y' = \dfrac{t'}{1+t^2} = \dfrac{-a}{x^2} : 1+\left(\dfrac{a^2}{x^2}\right)$ $\quad \dfrac{a}{x} = \dfrac{0.x - a.1}{x^2} = \dfrac{-a}{x^2}$

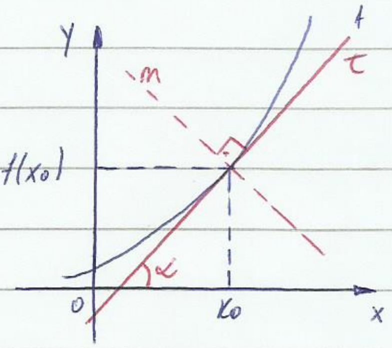
$\dfrac{-a}{x^2} : \dfrac{x^2+a^2}{x^2} = \dfrac{-ax^2}{x^2(x^2+a^2)} = \dfrac{-a}{x^2+a^2}$

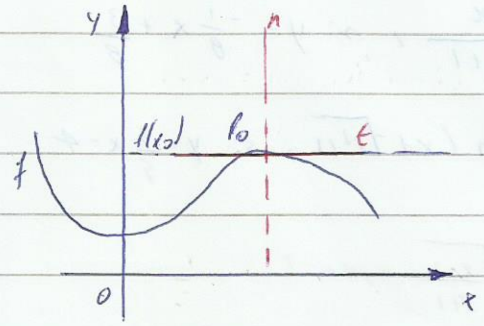
x^2

Exercícios: Reta Tangente e Normal

$t: \; y - f(x_0) = f'(x_0)(x - x_0)$

$$n: \begin{cases} y - f(x_0) = -\dfrac{1}{f'(x_0)} \cdot (x - x_0), \text{ se } f'(x_0) \neq 0 \\ \text{ou} \\ x = x_0, \text{ se } f'(x_0) = 0 \end{cases}$$





$f'(x_0) \neq 0$

$tg\,\alpha = f'(x_0)$

$f'(x_0) = 0$

$tg\,\alpha = 0$

Prova

(1) Eq. reta tangente e normal em x_0

(a) $f(x) = \sqrt{\dfrac{x-2}{x+1}}$, $x_0 = 3$

(b) $f(x) = 1 + \ln(1 + x^2)$, $x_0 = 0$

(c) $f(x) = \dfrac{x \arcsen(\cos x)}{e^x}$, $x_0 = \pi$

/ /

(2) Eq. reta tangente que contém A

(a) $f(x) = \ln x$, $A = (0,0)$

(b) $f(x) = \sqrt{5-x}$, $A = (9,0)$

(c) $f(x) = x^2+x+1$, $A = (0,-3)$

(3) Eq. reta tangente paralela a reta r

(a) $f(x) = \frac{x}{x^2+1}$, $r: y = -\frac{1}{8}x + \frac{3}{8}$

(b) $f(x) = \ln(x + \sqrt{x^2+1}$, $s: y = \frac{3}{4}x - 4$

(1) $f(x) = \sqrt{\frac{x-2}{x+1}}$, $x_0 = 3$

$$f'(x) = \frac{\left(\frac{x-2}{x+1}\right)'}{2\sqrt{\frac{x-2}{x+1}}} = \frac{\frac{(x-2)'(x+1)-(x-2)(x+1)'}{(x+1)^2}}{2\sqrt{\frac{x-2}{x+1}}}$$

$$= \frac{\frac{(1-0)(x+1)-(x-2)(1+0)}{(x+1)^2}}{2\sqrt{\frac{x-2}{x+1}}}$$

$$= \frac{\frac{3}{(x+1)^2}}{2\sqrt{\frac{x-2}{x+1}}} \quad \text{ou} \quad \frac{3}{2(x+1)^2\sqrt{\frac{x-2}{x+1}}}$$

$f(x_0) = f(3) = \frac{1}{2}$

$f'(x_0) = f'(3) = \frac{3}{16}$

$t: y - \frac{1}{2} = \frac{3}{16}(x-3)$

$y = \frac{3x}{16} - \frac{1}{16}$

$n: y - \frac{1}{2} = -\frac{16}{3}(x-3)$

$y = \frac{-16x}{3} + \frac{33}{2}$

(b) $f(x) = 1 + \ln(1+x^2)$, $x_0 = 0$

$= 1 + x^2 \quad = -\dfrac{2x}{(1+x^2)^2} \qquad f'(x) = -\dfrac{2x}{(1+x^2)^2}$

$= 2x$

$f(x_0) = f(0) = 1 + \ln(1+0^2) = 1$

$f'(x_0) = f(0) = \dfrac{-2.0}{(1+0^2)^2} = \dfrac{-0}{1} = 0$

$t: y - f(x_0) = f'(x_0)(x - x_0) \qquad n = \quad x = x_0$

$y - 1 = 0(x - 0) \qquad\qquad x = 1$

$y = 1$

(c) $f(x) = \dfrac{x \arcsin(\cos x)}{e^x}$, $x_0 = \dfrac{\pi}{2}$

$f'(x) = \dfrac{[x \operatorname{arc\,sen}(\cos x)]'(e^x) - (x \operatorname{arc\,sen}(\cos x))(e^x)'}{(e^x)^2}$

$f'(x) = \dfrac{\left[1 \cdot \operatorname{arcsen}(\cos x) + x \cdot \dfrac{-\operatorname{sen} x}{\sqrt{1-\cos^2 x}}\right] e^x - x \operatorname{arcsen}(\cos x)\, e^x}{(e^x)^2}$

$f'(x) = \dfrac{\operatorname{arcsen}(\cos x) - \dfrac{x \operatorname{sen} x}{\sqrt{1-\cos^2 x}} - x \operatorname{arcsen}(\cos x)}{e^x}$

$f(x_0) = f\left(\dfrac{\pi}{2}\right) = \dfrac{\frac{\pi}{2} \operatorname{arcsen} 0}{e^{\pi/2}} = 0$

$f'(x_0) = f'\left(\dfrac{\pi}{2}\right) = \quad -\dfrac{\frac{\pi}{2}}{e^{\frac{\pi}{2}}}$

2) (a) $f(x) = \ln x$, $A = (0,0)$

$f'(x) = \dfrac{1}{x}$ $t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - \ln x_0 = \dfrac{1}{x_0}(x - x_0)$

$A = (0,0)$

$0 - \ln x_0 = \dfrac{1}{x_0}(0 - x_0)$

$-\ln x_0 = \dfrac{-x_0}{x_0} \Rightarrow -\ln x_0 = -1$

$\ln x_0 = 1$, Portanto $x_0 = e$

Daí

$t: y - \ln e = \dfrac{1}{e}(x - e)$ ou $y = \dfrac{1}{e}(x - e) + \ln e$

$y = \dfrac{x}{e} - 1 + 1$ $y = \dfrac{x}{e}$

(b) $f(x) = \sqrt{5 - x}$, $A = (9,0)$

$f'(x) = \dfrac{-1}{2\sqrt{5-x}}$ $t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - \sqrt{5 - x_0} = \dfrac{-1}{2\sqrt{5 - x_0}}(x - x_0)$ $A = (9,0)$

$-\sqrt{5 - x_0} = \dfrac{-1}{2\sqrt{5 - x_0}}(9 - x_0)$

$-\sqrt{5 - x_0} = \dfrac{-9}{2\sqrt{5 - x_0}} + \dfrac{x_0}{2\sqrt{5 - x_0}}$ $t: y - f(x_0) = f'(x_0)(x - x_0)$

$-\sqrt{5 - x_0} = \dfrac{-9 + x_0}{2\sqrt{5 - x_0}}$ $y - 2 = \dfrac{-1}{4}(x - 1)$

$-2.(5 - x_0) = -9 + x_0$ $y = \dfrac{-x}{4} + \dfrac{1}{4} + 2$

$-10 + 2x_0 = -9 + x_0$

$x_0 = 1$ $y = \dfrac{-x}{4} + \dfrac{9}{4}$

c) $f(x) = x^2 + x + 1$, $A = 0,-3$

$f'(x) = 2x + 1$ $\quad$ $t:\ y - (x_0^2 + x_0 + 1) = 2x+1\,(x - x_0)$

$A = (0,-3)$

$-3 - (x_0^2 + x_0 + 1) = -x_0$

$-3 - x_0^2 - x_0 - 1 = -x_0$

$-x_0^2 - 4 = 0$

$-x_0^2 = 4$

$-x_0 = \sqrt{4}$

$+x_0 = \pm 2$

$y - ((-2)^2 - 2 + 1) = 2x+1(x-2)$

$y - 3 = 2x + x - 2$

$y - 3 = 3x - 2$

$y = 3x + 1$

Derivada implícita

Uma função do tipo $y = f(x)$ está na forma explícita onde x é a variável independente e y é a variável dependente

A expressão $3xy^5 + 4\,arctg\left(\frac{x}{y}\right) - \ln y = 0$ indica uma função na forma implícita e a derivada $y' = \frac{dy}{dx}$ pode ser obtida derivando-se ambos os membros de acordo com a regra da cadeia.

Exemplos:

Expressão	Derivada $y' = \frac{dy}{dx}$
x	1
y	y'
y^5	$5y^4y'$
$3x^2$	$6x$
$2xy^2$	$2 \cdot y^2 + 2x \cdot 2y \cdot y'$

Exercícios

Calcular a derivada $y' = \frac{dy}{dx}$ $\quad x^2 + y^2 = 25;\ (y > 0)$

isolar y

$y = \sqrt{25 - x^2}$

$y' = \dfrac{-2x}{2\sqrt{25 - x^2}}$

$y' = \dfrac{-x}{y}$

calcular y'

$2x + 2y \cdot y' = 0$

$2y \cdot y' = -2x$

$y' = \dfrac{-x}{y}$

/ /

Exercicios: calcular $y' = \frac{dy}{dx}$

1) $y^5 - 3y - 2x = 0$ $y' = \frac{2}{5y^4 - 3}$

5) $4xy^3 - x^2y + x^3 - 5x + 6 = 0$ $y' = \frac{5 - 3x^2 + 2xy - 4y^2}{1 - x^2 \cos y}$

2) $y^3 + xy + x^2 = 3$ $y' = -\frac{2x+y}{3y^2+x}$

6) $y = x^2 \operatorname{sen} y$

3) $y = \operatorname{sen}(x+y)$ $y' = \frac{\cos(x+y)}{1 - \cos(x+y)}$

4) $\sqrt{x} + \sqrt{y} = 1$ $y' = \sqrt{\frac{y}{x}}$

(1) $y^5 - 3y - 2x = 0$

$5y^4 \cdot y' - 3y' - 2 = 0$

$5y^4 y' - 3y' = 2$ $y'(5y^4 - 3) = 2$

$y' = \frac{2}{5y^4 - 3}$

(2) $y^3 + xy + x^2 = 3$

$x \cdot y = 1 \cdot y + x \cdot y'$

$y + xy'$

$3y^2 \cdot y' + y + xy' + 2x = 0$

$y'(3y^2 + x) = -2x - y$

$y' = \frac{-2x - y}{3y^2 + x}$ $\frac{-(2x+y)}{3y^2+x}$

(3) $y = \operatorname{sen}(x+y)$

$y' = y' \cdot \cos(x+y)$

$y' = (1+y') \cdot \cos(x+y)$

$y' = \cos(x+y) + y'(\cos(x+y))$

$y' = \frac{\cos(x+y)}{1 - \cos(x+y)}$

$y' - y'(\cos(x+y)) = \cos(x+y)$

$y'(1 - \cos(x+y)) = \cos(x+y)$

$\sqrt{\dfrac{y}{x}}$

4) $\sqrt{x} + \sqrt{y} = 1$ $\qquad \sqrt{y} = 1 - \sqrt{x}$

$$\frac{1}{2\sqrt{x}} + \frac{y'}{2\sqrt{y}} = 0$$

$$y' = -\frac{1}{2\sqrt{x}} \cdot 2\sqrt{y}$$

$$y' = -\frac{\sqrt{y}}{\sqrt{x}}$$

5) $4xy^3 - x^2y + x^3 - 5x + 6 = 0$

$$4y^3 + 4x \cdot 3y^2y' - (2xy + x^2y') + 3x^2 - 5 = 0$$

$$12xy^2y' - x^2y' = 5 - 3x^2 + 2xy - 4y^3$$

$$y'(12xy^2 - x^2) = 5 - 3x^2 + 2xy - 4y^3$$

$$y' = \frac{5 - 3x^2 + 2xy - 4y^3}{12xy^2 - x^2}$$

6) $y = x^2 \operatorname{sen} y$

$$y' = 2x \operatorname{sen} y + x^2 y' \cos y$$

$$y' - x^2y' \cos y = 2x \operatorname{sen} y$$

$$y'(1 - x^2 \cos y) = 2x \operatorname{sen} y$$

$$y' = \frac{2x \operatorname{sen} y}{1 - x^2 \cos y}$$

7) $x^2 + \sqrt{xy} = 7$ $\qquad y' = \dfrac{-4x\sqrt{xy} - y}{x}$

8) $\operatorname{sen}^2(3y) = x + y - 1$ $\qquad y' = \dfrac{1}{6\operatorname{sen}(3y) \cdot \cos(3y) - 1}$

9) $\cos^2(y^3) = x^2 - y$ $\qquad y' = \dfrac{2x}{1 - 6y^2 \cos(y^3) \operatorname{sen}(y^3)}$

10) $y = \operatorname{cossec}(xy)$ $\qquad y' = \dfrac{-y \operatorname{cotg}(xy) \cdot \operatorname{cossec}(xy)}{1 + x \operatorname{cotg}(xy) \cdot \operatorname{cossec}(xy)}$

(7) $x^2 + \sqrt{xy} = 7$

$xy = y + xy'$

$2x + \dfrac{y + xy'}{2\sqrt{xy}} = 0$

$\dfrac{y+xy'}{2\sqrt{xy}} = -2x$

$y + xy' = -2x(2\sqrt{xy})$

$xy' = -2x(2\sqrt{xy}) - y$

$y' = \dfrac{-4x\sqrt{xy} - y}{x}$

(8) $sen^2(3y) = x + y - 1$ $\qquad 3y = 3y'$

$sen(3y) . sen(3y) = x + y - 1$

$3y' cos(3y) . sen(3y) + sen(3y) . 3y' cos(3y) = 1 + y'$ $\qquad$ ou

$y'[3cos(3y) sen(3y) + sen(3y) . 3cos(3y)] = 1 + y'$ $\qquad$ fazer por

$y'[2(3cos(3y) sen(3y) - 1] = 1$ $\qquad [sen(3y)]^2$

$y' = \dfrac{1}{6cos(3y) . sen(3y) - 1}$

(9) $cos^2(y^3) = x^2 - y$

$[cos(y^3)]^2 = x^2 - y$

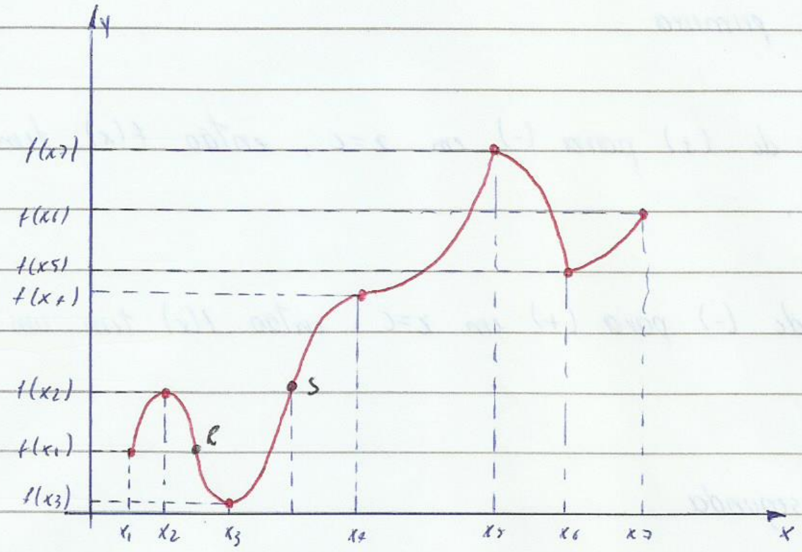
$2[cos(y^3)] . [-sen(y^3)] . 3y^2 y' = 2x - y'$

$[cos(y^3)] . [-sen(y^3)] . 6y^2 y' = 2x - y'$

$y'[cos(y^3) . -sen(y^3) . 6y^2 + 1] = 2x$

$y' = \dfrac{2x}{1 - sen(y^3) . cos(y^3) \, 6y^2}$

Estudo da variação das funções



x_2, x_3 e x_4 são pontos críticos

$f'(x_2) = f'(x_3) = f'(x_4) = 0$; $\nexists f'(x_1)$, $\nexists f'(x_5)$, $\nexists f'(x_6)$, $\nexists f'(x_7)$

$f(x_1)$, $f(x_3)$ e $f(x_6)$ são mínimos locais

$f(x_2)$, $f(x_5)$, e $f(x_7)$ são máximos locais

R, S, e T são pontos de inflexão

Propriedades e Teoremas

Seja $y = f(x)$ uma função contínua no intervalo $[a,b]$ e derivável em $]a,b[$ contendo o ponto c. Neste caso, temos:

* Se $y = f(x)$ tem um máximo ou mínimo local em $x = c$, então $f'(c) = 0$ ou $\nexists f'(c)$ (A recíproca não é verdadeira)

* Se $f'(x) > 0$ para todo $x \in]a,b[$, então $f(x)$ é crescente em $]a,b[$

* Se $f'(x) < 0$ para todo $x \in]a,b[$, então $f(x)$ é decrescente em $]a,b[$

Nas condições anteriores e sendo $f'(c)=0$ temos:

a) Teste da derivada primeira

* Se $f'(x)$ passa de $(+)$ para $(-)$ em $x=c$, então $f(x)$ tem um máximo em $x=c$

* Se $f'(x)$ passa de $(-)$ para $(+)$ em $x=c$, então $f(x)$ tem um mínimo em $x=c$

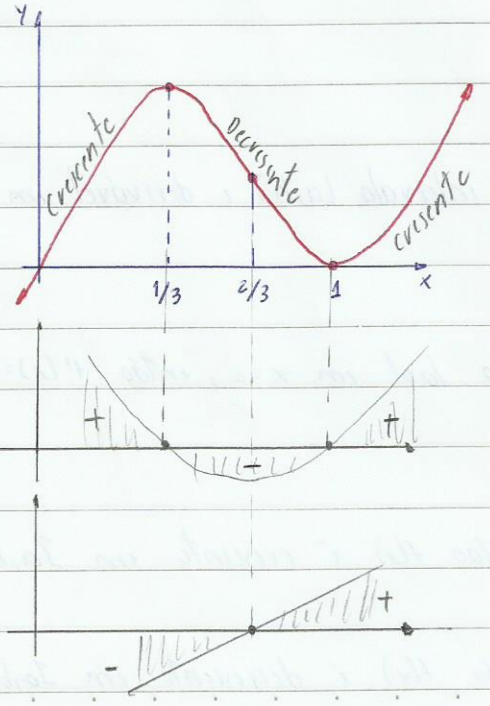
b) Teste da derivada segunda

* Se $f''(c)>0$, então $f(x)$ tem um mínimo em $x=c$

* Se $f''(c)=0$ e $f'''(c)\neq 0$, então $f(x)$ tem um ponto de inflexão em $x=c$

Ex: $f(x) = x^3 - 2x^2 + x \xrightarrow{\text{Derivada primeira}} f'(x) = 3x^2 - 4x + 1 = 0$

$\Delta = 4$ $\quad x_1 = 1/3$ MÁX $\quad x_2 = 1$ MIN



	$1/3$	1	
$f(x)$	↗	↘	↗
$f'(x)$	+	−	+

Sinal da deri primeira

$f''(x) = 6x - 4$

	$2/3$	
$f(x)$	∩	∪
$f''(x)$	−	+

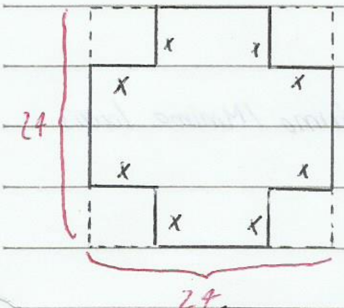
Concavidade de $f(x)$

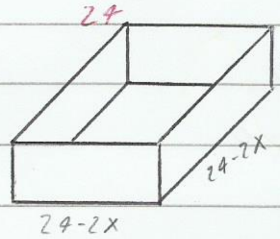
Sinal da der. segunda

$f''(x) = 6x - 4 = 0 \quad x = 2/3$ P. inflexão

$f'''(x) = 6 \neq 0$

Ex2: Com um quadrado de papelão de 24 cm de lado deseja-se construir uma caixa sem tampa cortando-se pequenos quadrados de lado x nos cantos conforme a figura. Calcule x para que a caixa tenha volume máximo.





Volume da caixa

$V(x) = (24-2x)^2 \cdot x$

$V(x) = (576 - 96x + 4x^2)x$

$V(x) = 576x - 96x^2 + 4x^3$

$V_{máx} = V' = 0$

$x = 4\ cm$

$V'(x) = 576 - 192x + 12x^2 = 0 \quad (\div 12)$

$V'(x) = x^2 - 16x + 48 = 0$

$\Delta = 64 \quad x_1 = 4 \quad x_2 = 12$

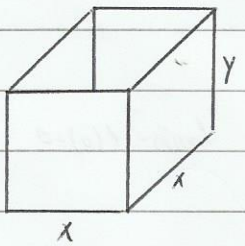
Teste da derivada segunda

$V''(x) = -192 + 24x$

$V''(4) = -96 < 0$ Ponto Máx

$V''(12) = 96 > 0$ Ponto Min

Ex3 Uma caixa aberta de base quadrada deve conter $32\ cm^3$. Determine as dimensões da caixa sabendo que o material gasto na sua construção deve ser o mínimo possível



Equação da área do material (mínima)

$A = 4xy + x^2$

$A = 4x \dfrac{32}{x^2} + x^2$

$A(x) = \dfrac{128}{x} + x^2$

$V = x^2 y = 32 \qquad y = \dfrac{32}{x^2}$

Teste da segunda derivada

$A'(x) = \dfrac{-128}{x^2} + 2x = 0$

$2x = \dfrac{128}{x^2}$

$x^3 = 64$

$x = 4$

$A''(x) = \dfrac{256}{x^3} + 2$

$A''(4) = \dfrac{256}{4^3} + 2 = 6 > 0$ Ponto Min.

$V = x^2 y = 32$

$y = 32/16 = 2$

Resposta: $x = 4\ cm$

$y = 2\ cm$

/ /

Gráfico de funções

Dada a função $y = f(x)$, pede-se

(a) Domínio e intersecção com os eixos

(b) Intervalos onde f é crescente / decrescente. Pontos de Máximo / Mínimo locais

(c) Intervalo de concavidade de f; pontos de inflexão

(d) Gráfico f indicando imagem (e) $\lim\limits_{x \to +\infty} f(x)$; $\lim\limits_{x \to -\infty} f(x)$

1) $y = f(x) = 2x^3 + 3x^2$ eixo x ($y=0$) eixo y ($x=0$)

(a) Domínio $= \mathbb{R}$ $x_1 = x_2 = 0$ $y = 2 \cdot 0^3 + 3 \cdot 0^2 = 0$

$2x^3 + 3x^2$ $x_3 = -3/2$

$x^2(2x+3) = 0$

(b) $y' = f'(x) = 6x^2 + 6x$ f é crescente em $]-\infty, -1[\cup]0, +\infty[$

$6x(x+1) = 0$ ou $]-\infty, -1] \cup [0, +\infty[$

$x_1 = 0$ $x_2 = -1$ f é decrescente em $]-1, 0[$

ou $[-1, 0]$

$-\infty$ -1 0 $+\infty$

f' + 0 − 0 +

f ↗ ↘ ↗

$x' = 0$ é pto min de f e $f_{min} = f(0) = 0$

$V_{min} = (0,0)$

$x'' = -1$ é o ponto max de f e

$f_{máx} = f(-1) = 2(-1)^3 + 3(-1)^2$

$= -2 + 3 = 1$ $V_{máx} = (-1, 1)$

(c) $y'' = f''(x) = 12x + 6$ f possui concavidade para cima em

$y'' = f''(x) = 0 \Rightarrow x = -1/2$ $]-\frac{1}{2}, +\infty[$ (ou $[-\frac{1}{2}, +\infty[$) e para baixo em

$-\infty$ $-1/2$ $+\infty$ $]-\infty, -1/2[$ (ou $]-\infty, -1/2]$)

f'' − 0 +

f ∩ ∪

$x_0 = -1/2$ é pto de inflexão

$f(x_0) = f(-1/2) = 2(-1/2)^3 + 3(-1/2)^2 = -\frac{2}{8} + 3/4 = -\frac{1}{4} + \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

$I = (\frac{-1}{2}, \frac{1}{2})$

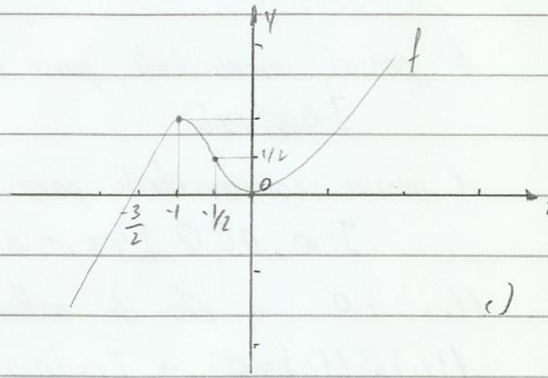
spiral

(d) $\lim_{x\to+\infty} (2x^3+3x^2) =$

$= \lim_{x\to+\infty} x^3\left(2+\frac{3}{x}\right) = +\infty.(2) = +\infty$

$\lim_{x\to-\infty} (2x^3+3x^2)$

$= \lim_{x\to-\infty} x^3\left(2+\frac{3}{x}\right) = -\infty . 2 = -\infty$



e) $Im f = \mathbb{R}$

2) $y = f(x) = 4x^3 - 3x^4$

3) $y = f(x) = \frac{1}{1+x^2}$

4) $y = f(x) = \frac{2x}{1+x^2}$

2) $y = f(x) = 4x^3 - 3x^4$

(a) $Df = \mathbb{R}$

eixo x $(y=0)$

$4x^3 - 3x^4 = 0$

$x^3(4-3x) = 0$

$x_1 = 0$

$x_2 = 4/3$

eixo y $(x=0)$

$4.0^3 - 3.0^4 = y$

$y = 0$

(b) $y' = f'(x) = 12x^2 - 12x^3$

$12x^2 - 12x^3 = 0$

$x^2(12 - 12x) = 0$

$x_1 = 0 \quad x_2 = 1$

	$-\infty$	0		1		$+\infty$
f'		0	+	0	−	
f			↗		↘	

f é crescente em $]-\infty, 1[$

f é decrescente em $]1, +\infty[$

$x' = 1$ é pto max de f

$f_{máx} = f(1) = 4 \cdot 1^3 - 3 \cdot 1^4 = 1$

$V_{max} = (1, 1)$

(c) $y'' = f''(x) = 24x - 36x^2$

$-36x^2 + 24x = 0$

$x(-36x + 24) = 0$

$x_1 = 0$

$x_2 = 2/3$

	$-\infty$		0		2/3		$+\infty$
f''		−	0	+	0	−	
f		∩		∪		∩	

f possui concavidade para cima em $]0, 2/3[$

f possui concavidade para baixo em $]-\infty, 0[\cup]2/3, +\infty[$

$f(x_1) = 0$ é pto de inflexão

$f(x_1) = f(0) = 0 \Rightarrow I_1 = (0,0)$

$f(x_2) = f\left(\frac{2}{3}\right) \quad 4 \cdot \left(\frac{2}{3}\right)^3 - 3\left(\frac{2}{3}\right)^4 = \frac{16}{27}$

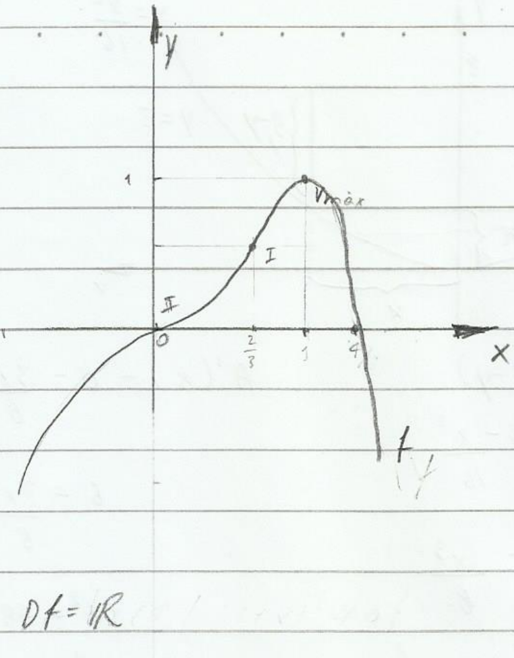
$I = \left(\frac{2}{3}, \frac{16}{27}\right)$

d) $\lim\limits_{x \to +\infty} (4x^3 - 3x^4)$

$= \lim\limits_{x \to +\infty} x^4\left(\frac{4}{x} - 3\right) = +\infty(-3) = -\infty$

$\lim\limits_{x \to -\infty} (4x^3 - 3x^4)$

$= \lim\limits_{x \to -\infty} x^4\left(\frac{4}{x} - 3\right) = +\infty(-3) = -\infty$



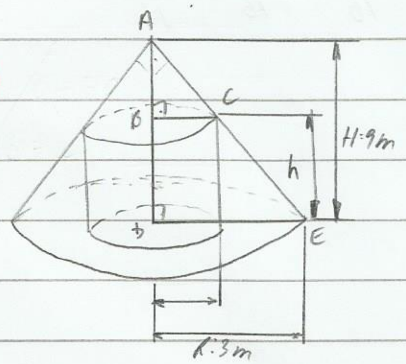
$Im f =]-\infty, 1]$

2) $y = f(x) = \frac{1}{1+x^2}$ $\quad D f = \mathbb{R}$

eixo x $(y=0)$

$\frac{1}{1+x^2} = 0$

4) 6)



Eq do volume

$V = \pi r^2 h$

$V = \pi r^2 (9-3r)$

$V(r) = 9\pi r^2 - 3\pi r^3$

$V'(r) = 18 r\pi - 9\pi r^2 = 0$

$9\pi r(2-r) = 0$

$r_1 = 0 \quad r_2 = 2$

$V''(r) = 18\pi - 18\pi r$

$V''(0) = 18\pi \quad V''(2) = -18\pi$

$\uparrow$ Vmin $\quad \uparrow$ Vmáx

$\frac{r}{3} = \frac{9-h}{9}$

$3(9-h) = 9r$

$27 - 3h = 9r$

$-3h = 9r - 27$

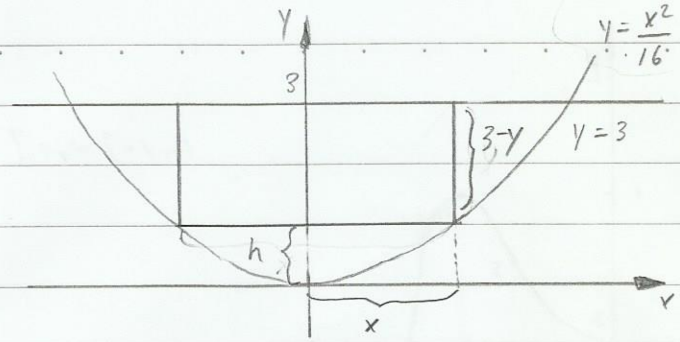
$h = \frac{9r-27}{-3} = 9 - 3r$

$h = 9-3r$, $r = 2$

$h = 3$

$r = 2 \quad h = 3$

8)



$$A = 2x(3-y)$$
$$= 2x\left(3 - \frac{x^2}{16}\right)$$
$$A(x) = 6x - \frac{x^3}{8}$$

$$A'(x) = 6 - \frac{3x^2}{8} = 0$$
$$6 = \frac{3x^2}{8}$$
$$3x^2 = 48$$
$$x^2 = 48/3$$
$$x = \sqrt{16} = \pm 4$$

4
-4

Teste da segunda derivada

$$A''(x) = \frac{-6x}{8} = \frac{-3x}{4} \; ; \; x = 4$$

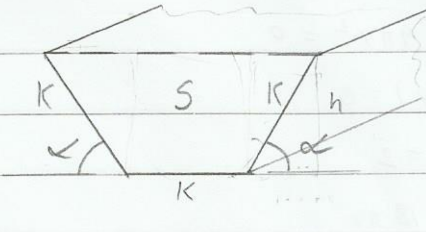
$$\frac{-3 \cdot 4}{4} = -3 \quad Vmáx$$

$$Y = \frac{x^2}{16} = \frac{4^2}{16} = \frac{16}{16} = 1$$

Base = $2x = 2 \cdot 4 = 8$ u.m

Altura = $3 - y = 3 - 1 = 2$ u.m

11)



$$A = \frac{(B+b)h}{2}$$
$$= \frac{(2K\cos\alpha + 2K)\,K\operatorname{sen}\alpha}{2}$$
$$= \frac{2K^2\operatorname{sen}\alpha\cos\alpha + 2K^2\operatorname{sen}\alpha}{2}$$
$$= \frac{2K^2\operatorname{sen}\alpha(\cos\alpha + 1)}{2}$$
$$= K^2(\operatorname{sen}\alpha\cos\alpha + \operatorname{sen}\alpha)$$
$$A' = K^2(\cos\alpha + \cos\alpha\cdot\cos\alpha + \operatorname{sen}\alpha\cdot -\operatorname{sen}\alpha)$$
$$= K^2(\cos\alpha + \cos^2\alpha - \operatorname{sen}^2\alpha) = 0$$

$\operatorname{sen}\alpha = \frac{h}{K}$ $\quad h = K\operatorname{sen}\alpha$

Continua na outra folha

$A' = K^2(\cos\alpha + \cos\alpha.\cos\alpha - \text{sen}\alpha\ \text{sen}\alpha) = 0$

$= \cos\alpha + \cos^2\alpha - \text{sen}^2\alpha = \dfrac{0}{K^2}$

$= \cos\alpha + \cos^2\alpha - 1 + \cos^2\alpha = 0$

$= \cos\alpha + 2\cos^2\alpha - 1 = 0$

$= 2\cos^2\alpha + \cos\alpha - 1 = 0 \Rightarrow 2t^2 + t - 1 = 0$

$\Delta = 1^2 - 4.(-1).2)$ $\quad x_1 = \cancel{-1}$

$\Delta = 9$ $\quad x_2 = 1/2$

$\cos\alpha = -1$ $\qquad \cos\alpha = 1/2$

$\cancel{\alpha} = 180$ $\qquad \alpha = 60°$

Vmim $\qquad$ Vmáx

Portanto o $\alpha = 60°$

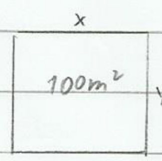
1) $a + b = 70$ $\quad a = 70 - b$ $\quad x' = 70 - 2b = 0$ $\quad a = 70 - b$

$a.b = x$ $\quad (70-b).b$ $\quad -2b = -70$ $\quad b = 35$

$x = 70b - b^2$ $\quad b = \dfrac{70}{2} = 35$ $\quad a = 70 - 35$

Teste da derivada da 2ª $\quad a = 35$

$x'' = -2$. (Máximo)

2) 

$xy = 100$ $\quad x = \dfrac{100}{y}$ $\quad f'(y) = \dfrac{-200}{y^2} + 2 = 0$

$2x + 2y = a$

$2\left(\dfrac{100}{y}\right) + 2y = a$ $\quad = \dfrac{-200 + 2y^2}{y^2} = 0$

$f(y) = \dfrac{200}{y} + 2y$

$= -200 + 2y^2 = 0$

teste $= f''(y) = \dfrac{400}{y^3} = 0$; $y = 10$ $\quad 2y^2 = 200$ $\quad x = \dfrac{100}{10}$

$y^2 = 100$

$\dfrac{400}{1000} = 0{,}4$ ou $-0{,}4$ $\quad y = \pm 10$ $\quad x = 10$

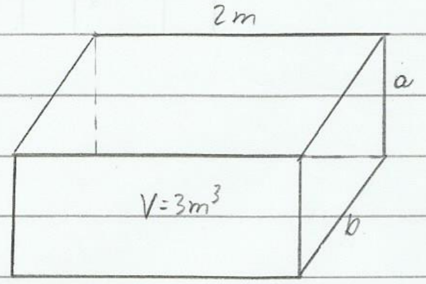
Mim $\quad$ Max $\quad y = 10_m$ $\; x = 10_m$

spiral® $-200\, y^{2-1}$

$12 \cdot 2b^{-1}$

3) Fazer em casa

5-)



$V = 2ab$

$3 = 2ab$

$M = 4a + 4b + 2ab$

$a = \frac{3}{2b}$

$M = 4 \cdot \frac{3}{2b} + 4b + 2 \cdot \frac{3}{2b} \cdot b$

$M = \frac{12}{2b} + 4b + \frac{6}{b} = \frac{12 + 8b^2 + 16}{2b}$

$M(b) = \frac{28 + 8b^2}{2b}$

$M(b) = \frac{14 + 4b^2}{b}$

$M'(b) = \frac{8b - 14(8b)}{b^2}$

$\begin{array}{r} 14 \\ \times 8 \\ \hline 112 \end{array}$

$= \frac{8b - 112b}{b^2} = 0$

$8b - 112b = 0$

$-112b = -8b$

$b = \frac{8b}{112} = \frac{4b}{56}$

4) $f(x) = \dfrac{2x}{1+x^2}$

(a) Domínio e intersecção com eixos

(b) Crescimento máximo / mínimo

(c) Concavidade, Inflexão

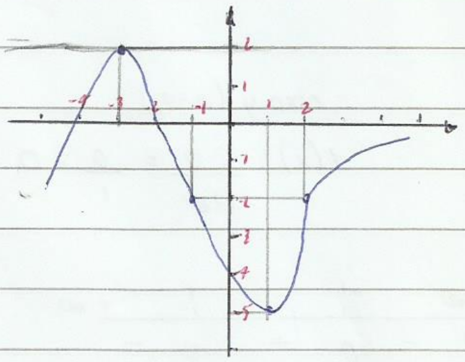
(d) $\lim\limits_{x \to +\infty} f(x)$ e $\lim\limits_{x \to -\infty} f(x)$

(e) Gráfico e Imagem

5) $f(x) = (x-3)\sqrt{x}$

(Realizar a - b - c - d - e

6) O gráfico abaixo representa o gráfico f de uma função derivável até pelo menos ordem 3 em $\mathbb{R}$



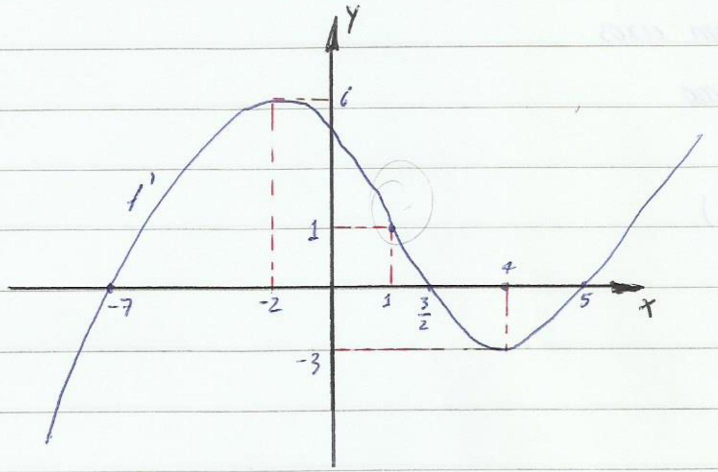
(a) Sinais e zeros de f'

(b) Sinais e zeros de f''

(c) Crescimento e máx/min de f'

(d) Eq. reta tangente ao gráfico f para $x_0 = -3$

7) O gráfico abaixo representa a derivada f' de uma função f derivável até pelo menos ordem 3 em $\mathbb{R}$



(a) Crescimento Máx. Mín de f

(b) Concavidade Inflexão de f

4) $f(x) = \frac{2x}{1+x^2}$

(a) $Df = \mathbb{R}$ eixo x $(y=0)$ $0 = \frac{2x}{1+x^2} = 0$ eixo y $(x=0)$ $f(x) = \frac{2.0}{1+0^2} = \frac{0}{1} = 0$

(b) $f'(x) = \frac{2(1+x^2) - 2x(2x)}{(1+x^2)^2}$

$= \frac{2+2x^2-4x^2}{(1+x^2)^2}$

$= \frac{2(1-x^2)}{(1+x^2)^2} = 0$

$2(1-x^2) = 0$

$-x^2 = -1$

$x^2 = 1$

$x = \pm 1$

	$-\infty$		-1		1		$+\infty$
f'		$-$	0	$+$	0	$-$	
f		$\searrow$		$\nearrow$		$\searrow$	

Máx $= (1, 1)$ Mín $(-1, -1)$

$\frac{2.1}{1+1^2} = \frac{2}{2} = 1$ $\frac{2.(-1)}{1+(-1)^2} = \frac{-2}{2} = -1$

Crescimento $]-1, 1[$

c) $f''(x) = \dfrac{[2(1-x^2)]'[(1+x^2)^2] - [2(1-x^2)][(1+x^2)^2]'}{[(1+x^2)^2]^2}$

$= \dfrac{2.(-2x)(1+x^2)^2 - 2(1-x^2).(1+x^2).2x}{[(1+x^2)]^2}$

$= \dfrac{2.-2x(1+x^2)-(1-x^2).4x}{(1+x^2)^3}$

$= \dfrac{(-2x-2x^3-4x+4x^3).2}{(1+x^2)^3}$

$= \dfrac{-6x+2x^3 . 2}{(1+x^2)^3}$

$= \dfrac{2x(x^2-3).2}{(1+x^2)^3} = \dfrac{4x(x^2-3)}{(1+x^2)^3}$

$\dfrac{4x(x^2-3)}{(1+x^2)^3} = 0$

$4x(x^2-3) = 0$

$x_1 = \pm\sqrt{3} \quad x_2 = 0$

	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		$+\infty$
f''		$-$	0	$+$	0	$-$	0	$+$	
f		$\cap$		$\cup$		$\cap$		$\cup$	

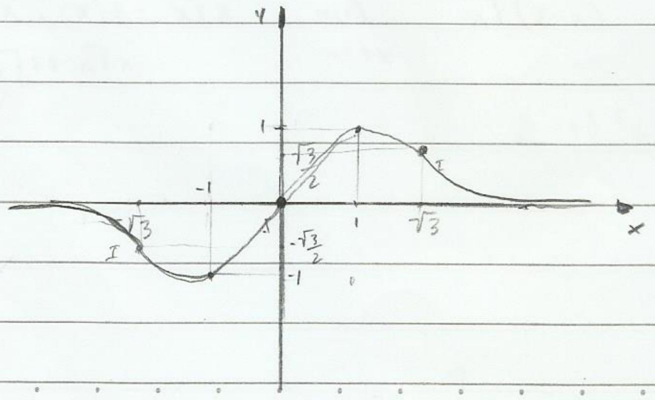
Concavidade $\cap \to]-\infty, -\sqrt{3}[\cup]0, \sqrt{3}[$

Concavidade $\cup \to]-\sqrt{3}, 0[\cup]\sqrt{3}, +\infty[$

$I_1 = (-\sqrt{3}, -\frac{\sqrt{3}}{2}) \quad I_2 = (0,0) \quad I_3 = (\sqrt{3}, \frac{\sqrt{3}}{2})$

d) $\lim\limits_{x\to+\infty} \dfrac{2x}{1+x^2} = \dfrac{2x}{x^2(\frac{1}{x^2}+1)} = \dfrac{2}{x(\frac{1}{x^2}+1)} = \dfrac{2}{+\infty} = 0$

$\lim\limits_{x\to-\infty} \dfrac{2x}{1+x^2} = \dfrac{2x}{x^2(\frac{1}{x^2}+1} = \dfrac{2}{x(\frac{1}{x^2}+1)} = \dfrac{2}{-\infty} = 0$



$Im f = [-1, 1]$

/ /

5) $f(x) = (x-3)\sqrt{x}$

$Df = \{x \in \mathbb{R} \mid x \geq 0\}$

eixo x $(y=0)$

$0 = (x-3)\sqrt{x}$

$0 = x-3$

$x = 3$

eixo y $(x=0)$

$f(x) = (x-3).\sqrt{x}$

$f(x) = (0-3).0$

$f(x) = 0$

b) $f'(x) = 1\sqrt{x} + (x-3)\frac{1}{2\sqrt{x}}$

$= \dfrac{2x + x - 3}{2\sqrt{x}}$

$= \dfrac{3x-3}{2\sqrt{x}} = 0$

$3x - 3 = 0$

$3x = 3$

$x = 1$

	$-\infty$	0		1		$+\infty$
f'			$-$	0	$+$	
f			$\searrow$		$\nearrow$	

Máx $= (0,0)$ Min $(1,-2)$

Crescimento $]1, +\infty[$

c) $f''(x) = 3(2\sqrt{x}) - (3x-3)\frac{1}{\sqrt{x}}$

$= \dfrac{6x - 3x + 3}{\sqrt{x}}$

$= \dfrac{\dfrac{3x+3}{\sqrt{x}}}{4x} = \dfrac{3x+3}{4x\sqrt{x}} = 0$

$x = -1$

	-1	0	$+\infty$
f''			

$\nexists$ Inflexão

d) $\lim\limits_{x \to +\infty} (x-3)\sqrt{x} = \lim\limits_{x \to +\infty} (x-3)\sqrt{x} \quad \lim\limits_{x \to +\infty} \dfrac{x\sqrt{x} - 3\sqrt{x} \cdot x\sqrt{x} + 3\sqrt{x}}{x\sqrt{x} + 3\sqrt{x}}$

$\lim\limits_{x \to +\infty} \dfrac{x^3 - 9x}{x\sqrt{x} + 3\sqrt{x}} = x^3\left(1 - \frac{9}{x^2}\right)$

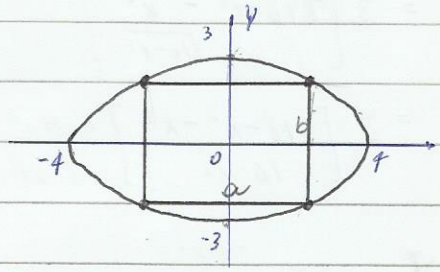
spiral

/ /

Otimização - Exercícios

1) Uma caixa retangular reta de base quadrada e sem tampa tem volume de 500 cm^3. Determinar suas dimensões de modo que a área lateral total (base + laterais) seja mínima

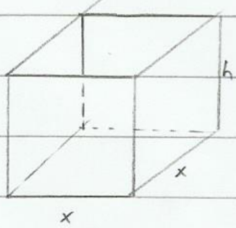
2) Um retângulo de lados paralelos aos eixos está inscrito na elipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ Determinar a área máxima



3) O eixo das abcissas (x) intercepta a parábola $y = 12 - 3x^2$ no ponto A e B e uma reta da forma $y = k$ $(0 < k < 12)$ intercepta aquela parábola nos pontos C e D. Determinar a área máxima do Trapézio ABCD.

4) Determinar o perímetro máximo de um triângulo retângulo de hipotenusa 5 m

1)



$V = x^2 h$

$x^2 h = 500$

$h = \frac{500}{x^2}$

$A_t = 4xh + x^2$

$= \frac{4x \cdot 500}{x^2} + x^2 \Rightarrow A(x) = \frac{2000}{x} + x^2$

$A'(x) = \frac{-2000}{x^2} + 2x = 0$

$-\infty \quad 10 \quad +\infty$

$A' \quad - \quad 0 \quad +$

A

$h = \frac{500}{100} = 5$

$-2000 + 2x^3 = 0$

$-1000 + x^3 = 0$

$x = \sqrt[3]{1000} = 10$

$x = 10cm$ e $h = 5cm$

$(1-x^2)^{\frac{1}{2}}$ $\frac{1}{2\sqrt{1-x^2}} \cdot -2x$

2) $\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow x = 4\sqrt{1-\frac{y^2}{9}} = \frac{4}{3}\sqrt{9-y^2}$

$y = 3\sqrt{1-\frac{x^2}{16}} = \frac{3}{4}\sqrt{16-x^2}$

$A = 4xy$

$A(x) = 4x \cdot \frac{3}{4}\sqrt{16-x^2}$

$= 3x\sqrt{16-x^2}$

$A'(x) = 3\sqrt{16-x^2} + 3x \cdot \frac{-2x}{2\sqrt{16-x^2}}$

$= 3\sqrt{16-x^2} - \frac{3x^2}{\sqrt{16-x^2}}$

$= 3\left[\sqrt{16-x^2} - \frac{x^2}{\sqrt{16-x^2}}\right]$

$= 3\left[\frac{16-x^2-x^2}{16-x^2}\right]$

$= \frac{3}{16-x^2}(16-2x^2) = 0$

$= 16-2x^2 = 0$

$-2x^2 = -16$

$x = \pm\sqrt{8}$

$-\infty$ $-\sqrt{8}$ $\sqrt{8}$ $+\infty$

A' $-$ 0 $+$ 0 $-$

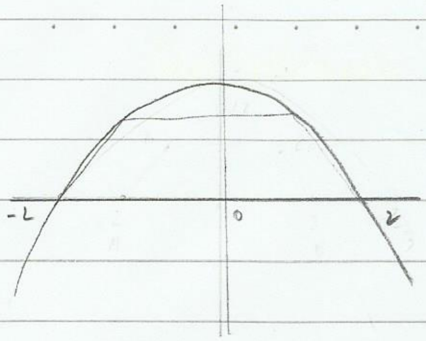
A $\searrow$ $\nearrow$ $\searrow$

Pmáx $\sqrt{8}$

$3\sqrt{8}\sqrt{16-8} = 3\sqrt{8}\sqrt{8} = 3.8 = 24$

Area Máxima $= 24$

3)



$A = (2+x) \cdot 2y \qquad y = 12 - 3x^2$

$A(x) = [(2+x) \cdot 2(12-3x^2)] \cancel{2}$

$A(x) = (2+x)(24-6x^2)$

$A(x) = 48 - 12x^2 + 24x - 6x^3$

$= -6x^3 - 12x^2 + 24x + 48$

$A'(x) = -18x^2 - 24x + 24 = 0$

$-6x^2 - 8x + 8 = 0$

$-3x^2 - 4x + 4 = 0$

$x = \frac{4 \pm \sqrt{16+48}}{-6}$

$x_1 = \frac{4+8}{-6} = -2$

$x_2 = \frac{4-8}{-6} = \frac{-4}{-6} = +\frac{2}{3}$

	$-\infty$		-2		$\frac{2}{3}$		$+\infty$
A'		$-$	0	$+$	0	$-$	
A		$\searrow$		$\nearrow$		$\searrow$	

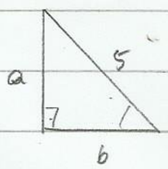
Pto Máx $= \frac{2}{3}$

$y = 12 - 3 \cdot \left(\frac{2}{3}\right)^2$

$y = \frac{96}{9} = \frac{32}{3}$

$A = \left(2 + \frac{2}{3}\right) \cdot \frac{32}{3} = \frac{8}{3} \cdot \frac{32}{3} = \frac{256}{9}$

4)



$P = a + b + 5$

$25 = a^2 + b^2$

$a^2 = b^2 - 25$

$a = \sqrt{b^2 - 25}$

$P = \sqrt{b^2-25} + b + 5$

$P'(b) = \frac{b}{\sqrt{b^2-25}} + 1 = 0$

$= b - \sqrt{b^2-25} = 0$

$b = \sqrt{b^2-25}$

$b = a$

$P = 2a + 5 \qquad 25 = 2a^2$

$P = 2\sqrt{12,5} + 5 \qquad a = \sqrt{12,5}$

ou

$P = 2 \cdot \frac{5}{\sqrt{2}} + 5$

$= \frac{10\sqrt{2}}{2} + 5$

$= 5\sqrt{2} + 5$ ou $5(\sqrt{2}+1)$

Taxas relacionadas

Nos problemas ~~que~~ onde a função $y = f(x)$ não é dada, podemos obter uma taxa de variação utilizando outras taxas, de acordo com a regra da cadeia.

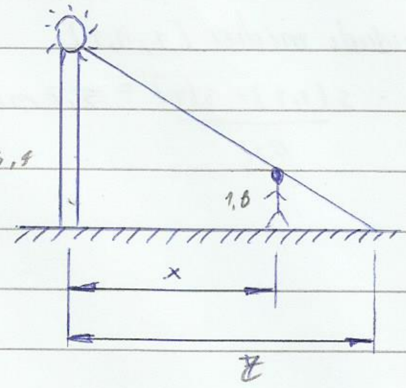
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \frac{dy}{dt} = \frac{dy}{dr} \cdot \frac{dr}{dm} \cdot \frac{dm}{dt}$$

$\frac{dr}{dt} = 2$

Ex. 3. Um incêndio se alastra em forma de círculo o raio do círculo aumenta à razão de 2m/min calcule a taxa de variação da área em relação ao tempo no instante em que o raio vale 20m.

$\left(\frac{dA}{dt}\right)_{20}$

EX4 Um homem com 1,8m caminha a 2,4 m/s afastando-se de uma lâmpada no topo de um poste com 5,4 m de altura. Com que velocidade a ponta de sua sombra se move?



$$\frac{5{,}4}{1{,}8} = \frac{z}{z-x}$$

Exemplo 3:

$$\frac{dA}{dt} = \frac{dr}{dt} \cdot \frac{dA}{dr} \qquad \frac{dA}{dr} \rightarrow A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = 2 \cdot 2\pi r$$

$$\frac{dA}{dt} = 4\pi r \qquad \left(\frac{dA}{dt}\right)_{t=20} = 80\pi \frac{m^2}{min}$$

spiral

/ /

Exemplo 4:

$$\frac{dx}{dt} = 2,4\ m/s \qquad \frac{dz}{dt} = ?$$

$$\frac{dz}{dt} = \underbrace{\frac{dx}{dt}}_{2,4} \cdot \frac{dz}{dx}$$

$$\frac{5,4}{1,8} = \frac{z}{z-x} \qquad \frac{3}{1} = \frac{z}{z-x}$$

?

$$\frac{dz}{dt} = 2,4 \cdot \frac{3}{2} = 3,6\ m/s$$

$z = 3(z-x)$

$z = 3z - 3x$

$2z - 3x = 0$

$$z = \frac{3x}{2} \qquad \frac{dz}{dx} = \frac{3}{2}$$

Lista Fecchio

1- $s(t) = 4t^2 + 3t$

Velocidade média $[1; 1,2]$

$$\frac{ds}{dt} = \frac{s(1,2) - s(1)}{0,2} = 6,8\ m/s$$

Velocidade média $[1; 1,1]$

$$\frac{ds}{dt} = \frac{s(1,1) - s(1)}{0,1} = 5,7\ m/s$$

Velocidade média $[1; 1,01]$

$$\frac{ds}{dt} = \frac{s(1,01) - s(1)}{0,01} =$$

Taxa de variação

Seja a função $y = f(x)$ definida no intervalo I contendo o ponto x_0

* Taxa de variação média de $y = f(x)$ no intervalo $[x_0; x_0 + \Delta x]$ é o número real dado por $\frac{\Delta y}{\Delta x} = f(x_0 + \Delta x)$ é o número real dado por $\frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$

* Taxa de variação instantânea (ou pontual) de $y = f(x)$ no instante (ou ponto) x_0 é o número real dado por

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Ex1: Seja a função $y = f(x) = x^2 + 2$. Determine a taxa de variação da função

a) No intervalo $[3; 3{,}5]$

b) No ponto $x_0 = 3$

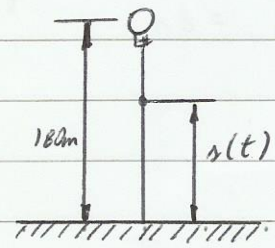
c) No ponto $x_0 = 3{,}5$

EX2: De um balão a 180m de altura, deixa-se cair um saco de areia a distância $s(t)$ do saco de areia até o solo após t segundos é dada por $s(t) = -5t^2 + 180$ metros. Qual é a taxa de variação da distância em relação ao tempo.

a) No intervalo $[1; 4]$?

b) No instante $t = 2$?

c) No instante em que o saco de areia atinge o solo



Exemplo 1:

a) $\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$ $\qquad x_0 = 3 \quad x_0 + \Delta x = 3,5$

$$\frac{\Delta y}{\Delta x} = \frac{f(3,5) - f(3)}{0,5} \Rightarrow \frac{\Delta y}{\Delta x} = \frac{14,25 - 11}{0,5} \Rightarrow \frac{\Delta y}{\Delta x} = \frac{3,25}{0,5} = \boxed{6,5}$$

b) $\frac{dy}{dx} = 2x \qquad \left(\frac{dy}{dx}\right)_{x_0 \to 3} = \boxed{6}$

c) $\frac{dy}{dx} = 2x \qquad \left(\frac{dy}{dx}\right)_{x_0 \to 3,5} = \boxed{7}$

Exemplo 2:

a) taxa de variação da média. $\qquad s(t) = -5t^2 + 180$

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \frac{f(4) - f(1)}{3} = \frac{100 - 175}{3} = \frac{-75}{3} = -25\,m/s$$

b) taxa de variação instantânea

$$\frac{ds}{dt} = -10t \qquad \left(\frac{ds}{dt}\right)_{t \to 2} = -20\,m/s$$

c) $-5t^2 + 180 = 0 \;;\; t \geq 0 \qquad \frac{ds}{dt} = -10t$

$-5t^2 = -180$

$t^2 = 180/5$

$t = \sqrt{36} = 6s \qquad \left(\frac{ds}{dt}\right)_{t \to 6} = -60\,m/s$

/ /

Regra L'Hôspital Se f e g são deriváveis, se $\lim_{x \to a} \frac{f(x)}{g(x)} = \left(\frac{0}{0}\right)$ ou $\left(\frac{\infty}{\infty}\right)$

então $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ com $a \in \mathbb{R}$ ou $a = +\infty$ ou $a = -\infty$

Calcular:

1) $\lim_{x \to \frac{\pi}{3}} \frac{2\cos x - 1}{3x - \pi}$

2) $\lim_{x \to 0} \frac{e^{x^2} - 1}{x}$

3) $\lim_{x \to 0} \frac{e^{tgx} - x - 1}{sen^2 5x}$ $\frac{1}{50}$

4) $\lim_{x \to -1} \frac{4x^5 + 5x^4 - 1}{3x^4 + 4x^3 + 1}$

5) $\lim_{x \to 0} \frac{1 - \sqrt[3]{\cos x}}{x^2}$

6) $\lim_{x \to 1} \left[\frac{x}{x-1} - \frac{1}{\ln x}\right]$

7) $\lim_{x \to +\infty} \frac{\ln(x^3 + x + 1)}{\ln(x^2 + x - 1)}$

8) $\lim_{x \to 0} \frac{1 - \sqrt{\cos x}}{x\, sen\, x}$

9) $\lim_{x \to 1} \frac{4\, arctg\, x - 1}{x - 1}$

10) $\lim_{x \to 1} \frac{\cos \frac{\pi x}{2}}{1 - \sqrt{x}}$

Respostas

1) $\lim_{x \to \frac{\pi}{3}} \frac{2\cos x - 1}{3x - \pi} = \frac{2 \cdot \frac{1}{2} - 1}{3 \cdot \left(\frac{\pi}{3}\right) - \pi} = \frac{0}{0} \Rightarrow \lim_{x \to \frac{\pi}{3}} \frac{-2\, sen\, x}{3} \quad \frac{-2 \cdot \frac{\sqrt{3}}{2}}{3} = -\frac{\sqrt{3}}{3}$

2) $\lim_{x \to 0} \frac{e^{x^2} - 1}{x} = \frac{e^0 - 1}{0} = \frac{0}{0} \Rightarrow \lim_{x \to 0} \frac{2x e^{x^2}}{1} = \frac{2 \cdot 0 e^{0^2}}{1} = \frac{0}{1} = 0$

3) $\lim_{x \to 0} \frac{e^{tgx} - x - 1}{sen^2 5x} \quad \frac{e^0 - 0 - 1}{0} = \frac{0}{0} \Rightarrow \frac{e^{tgx} \cdot \sec^2 x - 1}{2\, sen 5x \cos 5x\, 5} \Rightarrow \frac{0}{0} \;\Big|\; \underline{e^{tgx} \sec^2 x \cdot \sec^2 2x}$

4) $$\lim_{x\to -1} \frac{4x^5+5x^4-1}{3x^4+4x^3+1} \Rightarrow \frac{4(-1)^5+5(-1)^4-1}{3(-1)^4+4(-1)^3+1} = \frac{0}{0}$$

$$\lim_{x\to -1} \frac{20x^4+20x^3}{12x^3+12x^2} \Rightarrow \frac{20(-1)^4+20(-1)^3}{12(-1)^3+12(-1)^2} = \frac{0}{0} \Rightarrow \lim_{x\to -1} \frac{80x^3+60x^2}{36x^2+24x} = \frac{80(-1)^3+60(-1)^2}{36(-1)^2+24(-1)} = \frac{-5}{3}$$

5) $$\lim_{x\to 0} \frac{1-\sqrt[3]{\cos x}}{x^2} \Rightarrow \frac{1-\sqrt[3]{1}}{0^2} = \frac{0}{0} \Rightarrow \lim_{x\to 0} \frac{\operatorname{sen} x}{6x \cdot \sqrt[3]{\cos^2 x}} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sqrt[3]{\cos x} \cdot \cos x}{6\cos x + 4x \operatorname{sen} x} = \frac{1}{6}$$

6) $$\lim_{x\to 1} \left[\frac{x}{x-1} - \frac{1}{\ln x}\right] \Rightarrow \frac{1}{0} - \frac{1}{0} = \infty - \infty =$$

7) $$\lim_{x\to +\infty} \frac{\ln(x^3+x+1)}{\ln(x^2+x-1)} \Rightarrow \frac{\ln(+\infty^3+\infty+1)}{\ln(\infty^2+\infty-1)} = \frac{\infty}{\infty}$$

$$\lim_{x\to +\infty} \frac{3x^2+1}{x^3+x+1} : \frac{2x+1}{x^2+x-1} = \frac{x^2\left(3+\frac{1}{x^2}\right)}{x^3\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)} : \frac{x\left(2+\frac{1}{x^2}\right)}{x^2\left(1+\frac{1}{x}-\frac{1}{x^2}\right)} =$$

$$\lim_{x\to +\infty} \frac{3+0}{\infty(1+0+0)} : \frac{\infty(1+0+0)}{2+0} = \frac{3}{2}$$

8) $$\lim_{x\to 0} \frac{1-\sqrt{\cos x}}{x \operatorname{sen} x} \Rightarrow \frac{1-1}{0.0} = \frac{0}{0} \Rightarrow \lim_{x\to 0} \frac{\frac{\operatorname{sen} x}{2\sqrt{\cos x}}}{\operatorname{sen} x + x\cos x} = \frac{\operatorname{sen} x}{(2\sqrt{\cos x})(\operatorname{sen} x + x\cos x)} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\frac{\operatorname{sen} x}{x}}{\frac{2\sqrt{\cos x}(\operatorname{sen} x + x\cos x)}{x}} = \frac{1}{2\sqrt{\cos x}+2\sqrt{\cos x}} = \frac{1}{4}$$

Exercícios (Prova 6/12/06)

1) Derivar e simplificar $f(x) = x \,\text{arc sen}(3x) + \frac{1}{3}\sqrt{1-9x^2}$

1º Parcela

$$\text{arcsen}(3x) + x \cdot \frac{3}{\sqrt{1-9x^2}} = \frac{\sqrt{1-9x^2}\,\text{arcsen}(3x) + 3x}{\sqrt{1-9x^2}}$$

2º Parcela

$$\frac{1}{3} \cdot \frac{-18x}{2\sqrt{1-9x^2}} = \frac{-3x}{\sqrt{1-9x^2}}$$

$$f'(x) = \frac{\sqrt{1-9x^2}\,\text{arc sen}(3x) \cancel{+3x} \cancel{-3x}}{\sqrt{1-9x^2}} = \text{arcsen}(3x)$$

2) Calcular o limite $\lim\limits_{x \to 0} \frac{\text{sen}^2 x}{e^{3\text{sen}x} - 1 - 3\text{sen}x} = \frac{0}{0}$

$$\lim_{x \to 0} \frac{2\,\text{sen}x \cos x}{e^{3\text{sen}x} \cdot 3\cos x - 3\cos x} = \frac{0}{3 \cdot 3} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{2(\cos^2 x + \text{sen}x \cdot \text{sen}x)}{e^{3\text{sen}x} \cdot 9\cos^2 x - e^{3\text{sen}x} \cdot 3\text{sen}x + 3\text{sen}x} = \frac{2}{9}$$

$e^{3\text{sen}x} \cdot 3\cos x$

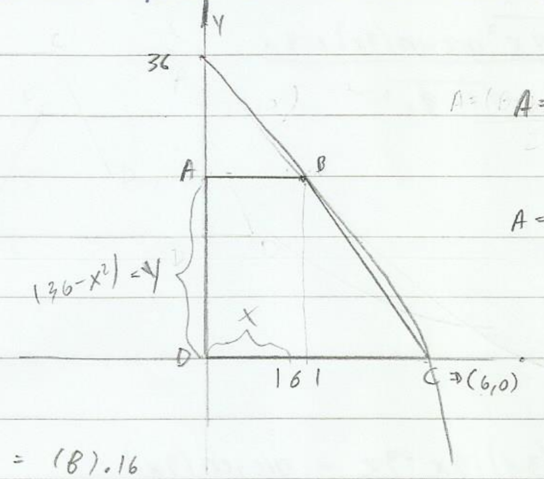
$= e^{3\text{sen}x} \cdot 3\cos x \cdot 3\cos x + e^{3\text{sen}x} \cdot 3 \cdot -\text{sen}x$

$= e^{3\text{sen}x} \cdot 9\cos^2 x - e^{3\text{sen}x} \cdot 3\text{sen}x$

$e^{3\text{sen}x}(9\cos^2 x - 3\text{sen}x)$

3) O Trapézio ABCD tem vértice A no eixo y, vértice B na parábola $y = 36 - x^2$, vértice C no ponto (6,0) e vértice D na origem. Sabendo que ele tem área máxima, determine:

a) as coordenadas do ponto B b) A Área do trapézio



$y = 36 - x^2$

$y(2) = 36 - 2^2$

$y(2) = 32$

$\underline{B = (2, 32)}$

$A = \dfrac{(x+6)(36-x^2)}{2}$

$A = \dfrac{36x - x^3 + 216 - 6x^2}{2}$

$A' = \frac{1}{2}(36 - 3x^2 - 12x) = 0$

$= -x^2 - 4x + 12 = 0$

$\cancel{x_1 = -6}$ $x_2 = 2$

$A'' = \frac{1}{2}(-6x - 12)$ $A''(2) = -12 < 0$

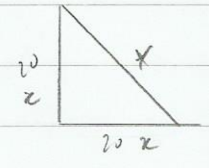
Pto Máximo

$A = \dfrac{(6+2) \cdot 32}{2} = (8) \cdot 16$

$\underline{A = 128}$

4) A área de um triângulo retângulo e isósceles cresce a uma taxa de $\sqrt{2}$ cm^2/min. Determinar a taxa de crescimento da hipotenusa no instante em que cada cateto mede 20cm

$\dfrac{dA}{dt} = \sqrt{2}$



$\dfrac{dy}{dt} = ?$

$\underbrace{\dfrac{dy}{dt}}_{?} = \underbrace{\dfrac{dA}{dt}}_{\sqrt{2}} \cdot \underbrace{\dfrac{dy}{dA}}_{x}$

$\dfrac{dy}{dt} = \sqrt{2} \cdot \dfrac{2}{y}$

$\dfrac{dy}{dt} = \dfrac{2\sqrt{2}}{y}$

$y^2 = 2x^2$ $x^2 = \dfrac{y^2}{2}$ $x = 20$ $y = 20\sqrt{2}$

$A = \dfrac{x^2}{2}$

$A = \dfrac{y^2}{4}$ $\dfrac{dA}{dy} = \dfrac{y}{2}$ $\dfrac{dy}{dA} = \dfrac{2}{y}$

$\dfrac{dy}{dt} = \dfrac{2\sqrt{2}}{20\sqrt{2}} = 0{,}1$ cm/min

f' + −

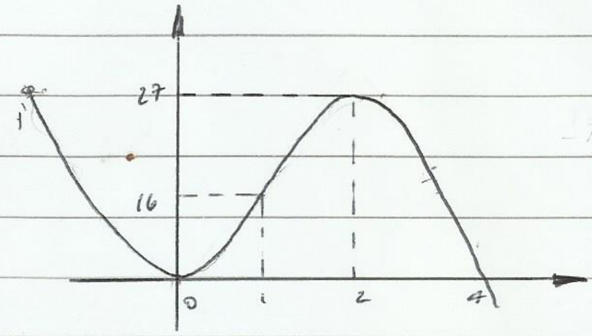
f ↗ ↘

/ /

5) Seja $f: \mathbb{R} \to \mathbb{R}$ de modo que sua derivada $f': \mathbb{R} \to \mathbb{R}$ tenha o gráfico abaixo determine

a) Intervalo(s) onde f é crescente

b) Intervalo(s) onde f tem concavidade para cima



	∅	0	1	2	3	4
f	↗	↗	↗	↗	↘	
f'	+	+	+	+	−	

	0	1	2	3	4
f	∩	∪	∪	∩	∩
f''	−	+	+	−	−

f é crescente $]-\infty, 4[$

f concavidade para cima $]0, 2[$

/ /

Prova 04/06/2009

1-) Calcular o limite $\lim_{x\to 0}\left[\frac{4(x-\operatorname{sen}x)}{\operatorname{sen}x}\right]$

$\lim_{x\to 0}\frac{4(1-\cos x)}{\cos x}$ $\frac{4(1-1)}{1}=\frac{0}{1}=0$

2a-) Derivar e simplificar $y=x^2[\operatorname{sen}(2\ln x)+\cos(2\ln x)]$

$a=\operatorname{sen}(2\ln x)$ $b=\cos(2\ln x)$

$a'=\cos(2\ln x).\frac{2}{x}$ $b'=-\operatorname{sen}(2\ln x).\frac{2}{x}$

$y'=2x[\operatorname{sen}(2\ln x)+\cos(2\ln x)]+x^2[\frac{2}{x}\cos(2\ln x)-\frac{2}{x}\operatorname{sen}(2\ln x)]$

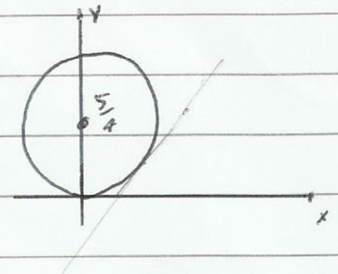
$y'=2x\operatorname{sen}(2\ln x)+2x\cos(2\ln x)+2x\cos(2\ln x)-2x\operatorname{sen}(2\ln x)$

$y'=4x\cos(2\ln x)$

2b-) Escreva a equação da reta ~~tangente~~ que passa pelo ponto $A=(1,2)$ e é tangente à circunferência $x^2+(y-\frac{5}{4})^2=\frac{25}{16}$

$x^2+y^2-\frac{5y}{2}+\frac{25}{16}=\frac{25}{16}\Rightarrow\frac{2y^2-5y}{2}=-x^2$

(III)



$2y^2-5y=-2x^2$

$y(2y-5)=-2x^2$

$y-2=\frac{-4}{3}(x-1)$

(I)

$y=\frac{-4}{3}x+\frac{4}{3}+2$

$\boxed{y=\frac{-4}{3}x+\frac{10}{3}}$

(II)

$2x+(y-\frac{5}{4}).2.y'=0$

$x^2+(y-\frac{5}{4})^2=\frac{25}{16}$

com $x=1$, por causa do ponto $A=(1,2)$

$y'=\frac{-2x}{2(y-5/4)}=\frac{-4x}{4y-5}$

temos:

$(y-\frac{5}{4})^2=\frac{25}{16}-1$

y' no ponto $(1,2)$

$\boxed{y'=\frac{-4\cdot 1}{8-5}=\frac{-4}{3}}$

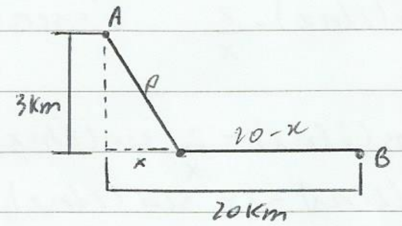
$(y-\frac{5}{4})^2=\frac{9}{16}$

$y=\frac{3}{4}+\frac{5}{4}=2$

spiral

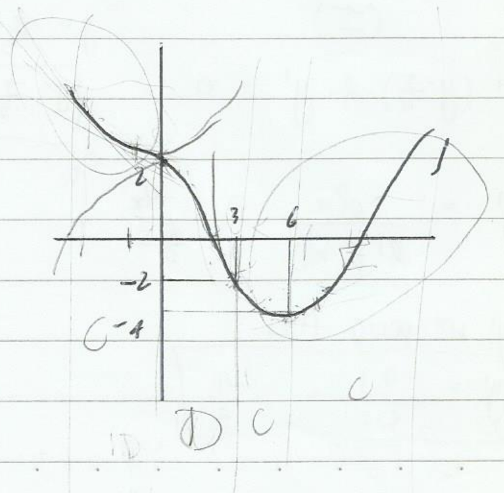
3) Dois carros partem de um mesmo ponto num mesmo instante. Um viaja para o sul a 60 Km/h o outro para Oeste a 25 Km/h. A taxa que está crescendo a distância entre duas horas depois?

4) Duas cidades separadas por um rio devem ser ligadas por uma ponte e uma estrada conforme a figura. O custo por Km da estrada é 4 milhões de reais e da ponte é 5 milhões de reais. Determine o comprimento da ponte e da estrada para que o custo total seja mínimo.

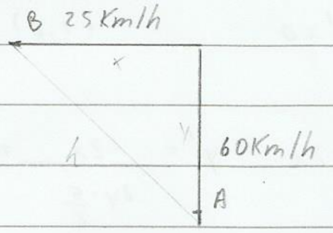


5) Seja uma função f(x) com f'(0)=0 dada pelo gráfico abaixo. Para que valores de x:

a) a derivada da ordem 1: f'(x) é positiva?
b) a derivada de ordem 2: f''(x) é negativa?
c) a derivada de ordem 1: f'(x) é crescente?
d) a derivada de ordem 2 f''(x) é nula?



3.-)



$\frac{dx}{dt} = 25$ Km/h $\quad \frac{dy}{dt} = 60$ Km/h

$h^2 = x^2 + y^2$

$2h\frac{dh}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$2h\frac{dh}{dt} = 2x \cdot 25 + 2y \cdot 60 \qquad h\frac{dh}{dt} = 25x + 60y$

$\frac{dh}{dt} = \frac{625 + 3600}{65} = \frac{4225}{65} = 65$ Km/h

4.-) $p^2 = 9 + x^2 \qquad C = 5\sqrt{9+x^2} + 4(20-x)$

$e = 20 - x \qquad C = 5\sqrt{9+x^2} + 80 - 4x$

$C' = 5 \cdot \frac{x}{\sqrt{9+x^2}} - 4 = 0 \Rightarrow \frac{5x - 4\sqrt{9+x^2}}{\sqrt{9+x^2}} = 0$

$5x - 4\sqrt{9+x^2} = 0$

$-4\sqrt{9+x^2} = -5x$

$16(9+x^2) = 25x^2$

$144 + 16x^2 = 25x^2$

$9x^2 = 144$

$x^2 = 16$

$x = 4$

$p = 5$ Km

$e = 16$ Km

5.-) $f' \Rightarrow + \quad | \qquad f'' \Rightarrow + \quad -$

$f \Rightarrow \nearrow \quad \searrow \qquad f \Rightarrow \cup \quad \cap$

a) $]6, +\infty[$

b) $]0, 3[$

c) $]-\infty, 2[\cup]3, +\infty[$

d) $x = 0$, $x = 3$

/ /

$\frac{4}{3} + 2 \quad \frac{10}{3}$

26) $x^2 + \left(y - \frac{5}{4}\right)^2 = \frac{25}{16}$ $\quad 2x + 2yy' - \frac{5}{4}y' = 0$ $\quad (1,2)$

$2x + \left(y - \frac{5}{4}\right) \cdot 2\, y' = 0$ $\quad y'\left(2y - \frac{5}{4}\right) = -2x$ $\quad y' = \frac{-2x}{2y - \frac{5}{2}} \Rightarrow \frac{-2}{4 - \frac{5}{2}} = \frac{-2}{\frac{3}{2}} = \frac{-4}{3}$

$y - f(x_0) = f'(x_0)(x - x_0)$

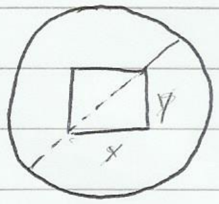
$y - 2 = \frac{-4}{3}(x - 1)$ $\quad y = \frac{-4}{3}x + \frac{10}{3}$

Exercícios

1) Escreva as eq. das retas tangente e normal à curva $f(x) = \frac{x^2}{x+6}$ no ponto de ~~d~~ abcissa $a=3$

2) Calcular o limite $\lim_{x \to 0} \left(\frac{1}{2(1-\cos x)} - \frac{1}{\sin^2 x} \right)$ $-\frac{1}{4}$

3) Uma peça circular contém um ressalto quadrado cujo diagonal é 1/5 do diâmetro quando aquecida seu raio aumenta a uma taxa de 2mm/t. Calcule a taxa de crescimento da área do quadrado no instante em que seu lado mede 5cm $40\sqrt{2}\ mm^2$



4) Um retângulo está inscrito numa região limitada pela parábola $y = \frac{x^2}{12}$ e pela reta $y=4$ tendo um dos lados sobre a reta e 2 vértices na parábola ache as dimensões do retângulo de área máxima $B=8$ $A = \frac{8}{3}$

5) A função Real $f(x) = \cancel{\frac{x^2-26}{2\sqrt{x}}}$ $x^2 - 26 \cdot \sqrt[3]{x}$ tem derivada primeira $f'(x) = \frac{7x^2-26}{3\sqrt[3]{x^2}}$

Determine:

a) Para seus valores de x a função $f(x)$ decrescente $]-2,2[$

b) As coordenadas dos pontos de máximo e mínimo $(-2, 24\sqrt[3]{2})_{max}$ $(2, -24\sqrt[3]{2})_{min}$

c) A derivada segunda $f''(x) = \frac{7}{4} \frac{4x^2+8}{x\sqrt[3]{x^2}}$

d) Para que valores de x a função $f(x)$ tem concavidade para baixo

$]-\infty, 0[$

1-) $f(x) = \frac{x^2}{x+6}$ $\quad x=3 \quad$ $f(3) = \frac{3^2}{3+6} = \frac{9}{9} = 1$

$f'(x) = \frac{2x(x+6)-x^2}{(x+6)^2} = \frac{2x^2+12x-x^2}{(x+6)^2} = \frac{x^2+12x}{(x+6)}$ $\quad f(3) = \frac{3^2+12.3}{(3+6)^2} = \frac{45}{81} =$

$\frac{15}{27} = \frac{5}{9}$

$t: \; y-1 = \frac{5}{9}(x-3)$

$y = \frac{5x}{9} - \frac{6}{9} = y = \frac{5x}{9} - \frac{2}{3}$

$n: \; y-1 = -\frac{9}{5}(x-3)$

$y = -\frac{9}{5}x + \frac{32}{5}$

2-) $\lim\limits_{x\to 0} \frac{sen^2x - 2(1-\cos^2 x)}{2(1-\cos x)sen^2 x}$ $\quad \frac{sen^2x - 2 + 2\cos^2 x}{2(1-\cos x)sen^2 x} = \frac{0-0}{0}$

$\lim\limits_{x\to 0} \frac{2sen x.\cos x + 4\cos x.-sen x}{(4\cos x.-sen x)sen^2 x + 2(1-\cos x).2sen x\cos x} = \frac{0}{0}$

$\lim\limits_{x\to 0} \frac{2\cos^2 x + 2sen x.sen^2 x - 4\cos^2 x}{(+4sen^2 x - 4\cos^2)sen^2 x + (4\cos x.-sen x).2sen x + \cos x + 4\cos x.sen x.(2sen x\cos x) + (2(1-\cos x)(2\cos^2 x. +}$

$sen^2 x - 2 + 2\cos^2 x$

$(2-2\cos x)sen^2 x$

$2sen x\cos x + 4\cos x\, sen x$

$2sen^3 x + (2-2\cos x)2sen x\cos x$

$2\cos^2 x + 2sen^2 x + 4sen^2 x + 4\cos^2 x =$

$6sen\,0 + (2-2\cos x) - 2\cos^2 x + 2sen -$

S T Q Q S S D

/ /

$\times \quad \dfrac{1-\operatorname{sen}(5x)}{1-\cos(5x)}$

Cálculo I
Derivada

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

1) $f'(x)$

$f(x) = \sqrt{x}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{2\sqrt{x}} \quad \text{ou} \quad \sqrt{x} = x^{\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

2) $f(x) = \dfrac{x}{x+1}$, $x \neq -1$

$$f'(x) = \lim_{h\to 0} \frac{\dfrac{x+h}{x+h+1} - \dfrac{x}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{(x+h)(x+1)-x(x+h+1)}{(x+h+1)(x+1)}}{h}$$

$$= \lim_{h\to 0} \frac{x^2+x+hx+h-x^2-xh-x}{h[(x+h+1)(x+1)]}$$

$$= \lim_{h\to 0} \frac{h}{h[(x+h+1)(x+1)]}$$

$$= \lim_{h\to 0} \frac{1}{x^2+x+x+1}$$

$$= \lim_{h\to 0} \frac{1}{x^2+2x+1}$$

ou

$$\frac{1(x+1)-x.1}{(x+1)^2}$$

$$\frac{1}{x^2+2x+1}$$

S T Q Q S S D

/ /

3) $f(x) = K \quad = 0$

4) $f(x) = \sqrt{x^2+1}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{(x+h)^2+1} - \sqrt{x^2+1}\right)\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2\cancel{+1}\cancel{-x^2}\cancel{-1}}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} = \lim_{h\to 0} \frac{x}{\sqrt{x^2+1}}$$

5) $f(x) = x^3$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-x^3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$= \lim_{h\to 0} 3x^2$$

PlastPark

S T Q Q S S D

/ /

$sen^2 x + cos^2 x = 1$

6) $f(x) = sen\,x$

$$f'(x) = \lim_{h \to 0} \frac{sen(x+h) - sen\,x}{h}$$

$$= \lim_{h \to 0} \frac{sen\,x \cos h + \cos x\, sen\,h - sen\,x}{h}$$

$$= \lim_{h \to 0} \frac{sen\,x(\cos h - 1) + \cos x\, sen\,h}{h}$$

$$= \lim_{h \to 0} (sen\,x)\frac{(\cos h - 1)}{h} + (\cos x)\frac{sen\,h}{h} \quad 1$$

$$= \lim_{h \to 0} \frac{(\cos h - 1)(\cos h + 1)}{h(\cos h + 1)}$$

$$= \lim_{h \to 0} \frac{\cos^2 h - 1}{h(\cos h + 1)}$$

$$= \lim_{h \to 0} \frac{-sen^2 h}{h(\cos h + 1)} \qquad \lim_{h \to 0} -\frac{sen\,h}{h} \cdot \frac{sen\,h}{(\cos h + 1)}$$

$$\lim_{h \to 0} -1.0 + \cos x$$

$$\lim_{h \to 0} \cos x$$

S T Q Q S S D

/ /

Calcular as derivadas

1) $y^{\frac{2}{3}}$ $\frac{2}{3}y^{-\frac{1}{3}}$ $\frac{2}{3}-1=\frac{2-3}{3}=-\frac{1}{3}$

$\frac{2}{3}\cdot\sqrt[3]{y^{-1}} = \frac{2}{3\sqrt[3]{y}}$

2) $y=\sqrt[5]{x^2}$ $y' = x^{\frac{2}{5}}$ $\frac{2}{5}-1=\frac{2-5}{5}=\frac{-3}{5}$

$= \frac{2}{5}x^{-\frac{3}{5}}$

$= \frac{2}{5}\sqrt[5]{x^{-3}}$

$= \frac{2}{5\sqrt[5]{x^3}}$

3) $y=\frac{5}{x^2}$ $y'=5x^{-2}$

$= -10x^{-3}$

$= \frac{-10}{x^3}$

4) $y=(x^3-1)(3-x^2)$ $y'=3x^3-x^5-3+x^2$

$= 9x^2-5x^4+2x$

$= -5x^4+9x^2+2x$

5) $y=\sqrt{x}\,(x^2+1)$ $y'=\frac{1}{2\sqrt{x}}(x^2+1)+\sqrt{x}\cdot 2x$

$= \frac{x^2}{2\sqrt{x}}+\frac{1}{2\sqrt{x}}+2x\sqrt{x}$

$= \frac{x^2+1+4x\cdot x}{2\sqrt{x}}$

$= \frac{5x^2+1}{2\sqrt{x}}$

STQQSSD

/ /

6) $y = \dfrac{x+1}{\sqrt{x}}$ $\quad y' = \dfrac{\sqrt{x} - (x+1)\frac{1}{2\sqrt{x}}}{x}$ $\quad y' = \dfrac{\frac{2x - x + 1}{2\sqrt{x}}}{x}$

$= \dfrac{\sqrt{x} - \frac{x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}}{x}$ $\quad = \dfrac{x+1}{2x\sqrt{x}}$

7) $y = \dfrac{x^2 - 3x}{\sqrt[3]{x^2}}$ $\quad y' = \dfrac{(2x-3)\sqrt[3]{x^2} - (x^2-3x)\frac{2}{3\sqrt[3]{x}}}{(\sqrt[3]{x^2})^2}$

$\sqrt[3]{x^2} = x^{\frac{2}{3}}$

$\frac{2}{3} \cdot \frac{3}{3} = \cdot$

$\cdot \ 2/3 \cdot x^{-\frac{1}{3}}$

$= \dfrac{2x\sqrt[3]{x^2} - 3\sqrt[3]{x^2} - \left(\frac{2x^2}{3\sqrt[3]{x}} - \frac{6x}{3\sqrt[3]{x}}\right)}{(\sqrt[3]{x^2})^2}$

$2/3\ \sqrt[3]{x}$

$\sqrt[3]{x^2} \cdot \sqrt[3]{x} =$

$= 2x.x - 3x - 2x^2 + 6x$

(Refazer)

8) $y = \dfrac{(1+2x)^2}{\sqrt{x}}$ $\quad \dfrac{(1+4x+4x^2)}{\sqrt{x}}$

$y' = \dfrac{(4+8x)\sqrt{x} - (1+4x+4x^2)\frac{1}{2\sqrt{x}}}{x}$

$= \dfrac{4\sqrt{x} + 8x\sqrt{x} - \left(\frac{1+4x+4x^2}{2\sqrt{x}}\right)}{x}$

$= \dfrac{8x + 16x^2 - 1 - 4x - 4x^2}{2x\sqrt{x}}$

$= \dfrac{12x^2 + 4x - 1}{2x\sqrt{x}}$

S T Q Q S S D

/ /

$sen^2 x + cos^2 x = 1$

$sen(A+A) = senA\,cosA + cosA\,senA$

$2 . senA\,cosA$

$\frac{1}{1+cosx}$

9) $y = \frac{senx}{1+cosx}$ $y' = \frac{cosx(1+cosx) - senx(-senx)}{(1+cosx)^2}$

10) $y = \frac{1-senx}{1-cosx}$ $y' = \frac{-cosx(1-cosx) - (1-senx)(senx)}{(1-cos\,x)^2}$

$y' = \frac{-cosx + cos^2 x - (senx - sen^2 x)}{(1-cosx)^2}$

$y' = \frac{-cosx + cos^2 x - senx + sen^2 x}{(1-cosx)^2}$

$y' = \frac{-cosx - senx + 1}{(1-cosx)^2}$

y'

11.) $y = \dfrac{tg\,x - 1}{\sec x}$ $\dfrac{u'v - uv'}{v^2}$

$$y' = \frac{(tg\,x-1)'(\sec x) - (tg\,x-1)(\sec x)'}{(\sec x)^2}$$

$$= \frac{\sec^2 x \cdot \sec x - (tg\,x-1)(\sec x \cdot tg\,x)}{\sec^2 x}$$

$$= \frac{\sec^2 x \cdot \sec x - \sec x\, tg^2 x + \sec x\, tg\,x}{\sec^2 x}$$

$\dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x}$ $= \dfrac{\sec x\,(\sec^2 x - tg^2 x + tg\,x)}{\sec^2 x}$

$tg^2 x + 1 = \sec^2 x$ $= \dfrac{1 + tg\,x}{\sec x}$

$\sec^2 x - tg^2 x = 1$

(12-) $y = (x^3-1)(3-x^2)$ $u'v + uv'$

$$y' = (x^3-1)'(3-x^2) + (x^3-1)(3-x^2)'$$

$$= 2x^2(3-x^2) + (x^3-1)\cdot -2x$$

$$= 6x^2 - 2x^4 - 2x^4 + 2x$$

$$= -4x^4 + 6x^2 + 2x$$

~~207~~) Exercício Novazzi

7) $y = \operatorname{arctag}\left(\frac{x+a}{1-ax}\right)$

$$f' = \frac{1(1-ax) - (x+a) \cdot -a}{(1-ax)^2}$$

$$= \frac{1-ax+ax+a^2}{(1-ax)^2}$$

$$= \frac{1+a^2}{(1-ax)^2} = \frac{1+a^2}{1-2ax+a^2x^2}$$

$$y' = \frac{1+a^2}{(1-ax)^2} : 1 + \frac{(x+a)^2}{(1-ax)^2}$$

$$= \frac{1+a^2}{(1-ax)^2} : \frac{(1-ax)^2+(x+a)^2}{(1-ax)^2}$$

$$= \frac{(1+a^2)(1-ax)^2}{(1-ax)^2\left[(1-ax)^2+(x+a)^2\right]}$$

$$= \frac{1+a^2}{(1-ax)^2+(x+a)^2}$$

$$= \frac{1+a^2}{1-\cancel{2ax}+a^2x^2+x^2+\cancel{2ax}+a^2}$$

$$= \frac{1+a^2}{1+a^2x^2+x^2+a^2}$$

$$\frac{-1}{1+x^2}$$

$$\ln\left(\frac{\cos x + 1}{\cos x - 1}\right)$$

spiral

S T Q Q S S D

/ /

Exercícios do Livro

248- $f(x) = 3x+2$ $\quad f'(x) = 3$

ou (Pela definição)

~~249~~- $f'(x) = \dfrac{[3(x+h)+2]-(3x+2)}{h}$

$= \dfrac{\cancel{3x}+3h+\cancel{2}-\cancel{3x}-\cancel{2}}{h}$

$= 3$

249- $f(x) = x^3$ $\quad f'(x) = 2x^2$

pela definição:

$f'(x) = \dfrac{(x+h)^3 - x^3}{h}$

$= \dfrac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$

$= \dfrac{h(3x^2+3xh+h^2)}{h}$

$= 3x^2$

250- $f(x) = x^2-x+1$ $\quad f'(x) = 2x-1$

Pela definição

$f'(x) = \lim\limits_{h\to 0} \dfrac{(x+h)^2-(x+h)+1-(x^2-x+1)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{x^2}+2xh+h^2\cancel{-x}-h\cancel{+1}\cancel{-x^2}\cancel{+x}\cancel{-1}}{h}$

$= \lim\limits_{h\to 0} \dfrac{h(2x+h-1)}{h}$

$= \lim\limits_{h\to 0} 2x-1$

PlastPark

251- $f(x) = \frac{x-1}{x+1}$; $x \neq -1$

Pela definição

$$f'(x) = \frac{(x+1)+(x-1)}{(x+1)^2}$$

$$= \frac{2x}{x^2+2x+1}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{x-h-1}{x+h+1} - \frac{x-1}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{(x-h-1)(x+1)-(x-1)(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{(x^2-xh-x+x-h-1)-(x^2+xh+x-x-h-1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}-xh-x+x-\cancel{h}-x-\cancel{x^2}-xh-x+x+\cancel{h}+\cancel{1}}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{-2xh}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{-2x}{x^2+2x+1}$$

Dúvida

252- $f(x) = \sqrt{x^2+1}$

$$f'(x) = \lim_{h\to 0} \frac{\left(\sqrt{(x+h)^2+1} - \sqrt{x^2+1}\right)\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+\cancel{1}-x^2-\cancel{1}}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h\left(\sqrt{(x+h)^2+1} + (\sqrt{x^2+1})\right)}$$

$$= \lim_{h\to 0} \frac{2x}{2\sqrt{x^2+1}}$$

$$= \lim_{h\to 0} \frac{x}{\sqrt{x^2+1}}$$

STQQSSD

/ /

253-) $f(x) = \dfrac{1}{x}$; $x \neq 0$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \qquad = \lim_{h \to 0} \frac{-1}{x^2}$$

$$= \lim_{h \to 0} \frac{\frac{x-(x+h)}{(x+h)x}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{(x^2+xh)h}$$

254-) $f(x) = \dfrac{1}{\sqrt{x}}$; $x > 0$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x}) - (\sqrt{x+h})}{[(\sqrt{x+h})(\sqrt{x})} \cdot \frac{(\sqrt{x}) + (\sqrt{x+h})}{(\sqrt{x}) + (\sqrt{x+h})] \cdot h}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{h[(x\sqrt{x+h}) + (\sqrt{x}(x+h))]}$$

$$= \lim_{h \to 0} \frac{-h}{h[(x\sqrt{x+h}) + \sqrt{x}(x+h)]}$$

$$= \lim_{h \to 0} \frac{-1}{2\sqrt{x}}$$

STQQSSD

/ /

255.) $f(x) = \frac{1}{x^2}$ $\quad -\frac{f'}{f^2} = \frac{-2x}{x^4}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{x^2 - (x^2+2xh+h^2)}{(x+h)^2 (x^2)}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{(x+h)^2 (x^2)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-2x-h)}{x^2 - 2xh - h.\cancel{h}.x^2} = \lim_{h \to 0} \frac{-2x}{x^4}$$

256) $f(x) = \frac{1}{1-x}$; $x \neq 1$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{1-(x+h)} - \frac{1}{1-x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1-x-(1-x-h)}{(1-x-h)(1-x)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1}-\cancel{x}-\cancel{1}+\cancel{x}+h}{[(1-x-h)(1-x)]h}$$

$$= \lim_{h \to 0} \frac{h}{h[(1-x-h)(1-x)]}$$

$$= \lim_{h \to 0} \frac{1}{(1-x)^2}$$

$$= \lim_{h \to 0} \frac{1}{1-2x+x^2}$$

PlastPark

257) $f(x) = \operatorname{sen} x$

$$f'(x) = \lim_{h \to 0} \frac{\operatorname{sen}(x+h) - \operatorname{sen} x}{h}$$

$$= \lim_{h \to 0} \frac{\operatorname{sen} x \cos h + \cos x \operatorname{sen} h - \operatorname{sen} x}{h}$$

$$= \lim_{h \to 0} \operatorname{sen} x \cos h + \cos x - \operatorname{sen} x$$

$$= \lim_{h \to 0} \operatorname{sen} x - \operatorname{sen} x + \cos x \quad = \lim_{h \to 0} \cos x$$

258) $f(x) = \cos x$

$$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \operatorname{sen} x \operatorname{sen} h - \cos x}{h}$$

$$= \lim_{h \to 0} \cos x \cos h - \operatorname{sen} x - \cos x$$

$$= \lim_{h \to 0} \cos x - \operatorname{sen} x - \cos x$$

$$= \lim_{h \to 0} -\operatorname{sen} x$$

259) $f(x) = \sqrt{1-x^2}$; $x \in]-1,1[$

$$f'(x) = \lim_{h \to 0} \frac{\left(\sqrt{1-(x+h)^2} - \sqrt{1-x^2}\right)\left(\sqrt{1-(x+h)^2} + \sqrt{1-x^2}\right)}{h\left(\sqrt{1-(x+h)^2} + \sqrt{1-x^2}\right)}$$

$$f'(x) = \lim_{h \to 0} \frac{1-(x+h)^2 - (1-x^2)}{h\left(\sqrt{1-(x+h)^2} + \sqrt{1-x^2}\right)}$$

$$f'(x) = \lim_{h \to 0} \frac{1 - (x^2+2xh+h^2) - 1 + x^2}{h\left(\sqrt{1-(x+h)^2} + \sqrt{1-x^2}\right)}$$

/ /

$$f'(x) = \lim_{h \to 0} \frac{1 - x^2 - 2xh - h^2 - 1 + x^2}{h(\sqrt{1-(x+h)^2} + \sqrt{1-x^2})}$$

$$f'(x) = \lim_{h \to 0} \frac{-2xh - h^2}{h(\sqrt{1-(x+h)^2} + \sqrt{1-x^2})}$$

$$f'(x) = \lim_{h \to 0} \frac{h(-2x-h)}{h(\sqrt{1-(x+h)^2} + \sqrt{1-x^2})}$$

$$= \lim_{h \to 0} \frac{-2x}{2\sqrt{1-x^2}} = \lim_{h \to 0} \frac{-x}{\sqrt{1-x^2}}$$

260-) $f(x) = \ln x \,;\ x > 0$

$$f'(x) = \lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h}$$

261-) $f(x) = sen(7x)$

$$f'(x) = \lim_{h \to 0} \frac{sen[7(x+h)] - sen(7x)}{h}$$

$$= \lim_{h \to 0} \frac{sen(7x + 7h) - sen\,7x}{h}$$

$$= \lim_{h \to 0} \frac{sen\,7x \cos 7h + \cos 7x \, sen\,7h - sen\,7x}{h}$$

$$= \lim_{h \to 0} sen\,7x \cos 7h + \cos 7x \,.\, 7 - sen\,7x$$

$$= \lim_{h \to 0} sen\,7x - sen\,7x + 7\cos 7x = \lim_{h \to 0} 7\cos 7x$$

(262-) $y = \frac{1-\operatorname{sen}x}{1+\operatorname{sen}x}$

$y' = \frac{u'v - uv'}{v^2}$

$= \frac{(1-\operatorname{sen}x)'(1+\operatorname{sen}x) - (1-\operatorname{sen}x)(1+\operatorname{sen}x)'}{(1+\operatorname{sen}x)^2}$

$= \frac{[(-\cos x)(1+\operatorname{sen}x)]-[(1-\operatorname{sen}x)\cos x]}{(1+\operatorname{sen}x)^2}$

$= \frac{(-\cos x - \cos x\operatorname{sen}x) - (\cos x - \operatorname{sen}x\cos x)}{(1+\operatorname{sen}x)^2}$

$= \frac{-\cos x - \cancel{\cos x\operatorname{sen}x} - \cos x + \cancel{\operatorname{sen}x\cos x}}{(1+\operatorname{sen}x)^2}$

$= \frac{-2\cos x}{(1+\operatorname{sen}x)^2}$

(263-) $y = 2^x + x^2 + \ln 2$

$y' = 2^x \ln 2 + 2x + 0$

$= 2^x \ln 2 + 2x$

(264-) $y = \frac{x^e}{e^x} + \sqrt{e}$

$y' = \frac{ex^{e-1}e^x - x^e e^x}{(e^x)^2}$

$= \frac{e^x(ex^{e-1} - x^e)}{(e^x)^2} \Rightarrow \frac{x^{e-1}(e-x)}{e^x}$

/ /

(265-) $y = -\frac{11}{(x-1)^2} - \frac{2}{x-1}$

$I = \frac{11}{(x-1)^2}$ $II = \frac{2}{x-1}$

$I' = 11.(x-1)^{-2}$ $II' = 2(x-1)^{-1}$

$= -22(x-1)^{-3}$ $= -2(x-1)^{-2}$

$= \frac{-22}{(x-1)^3}$ $= \frac{-2}{(x-1)^2}$

$y' = -\left[\frac{-22}{(x-1)^3}\right] - \left[\frac{-2}{(x-1)^2}\right]$

$= \frac{22}{(x-1)^3} + \frac{2}{(x-1)^2} = \frac{22+2(x-1)}{(x-1)^3}$

$= \frac{22+2x-2}{(x-1)^3} = \frac{20+2x}{(x-1)^3} = \frac{2(10+x)}{(x-1)^3}$

(266-) $y = \frac{2}{5}\sqrt{(x-1)^5} + \frac{2}{3}\sqrt{(x-1)^3}$

$a = \frac{2}{5} \cdot \frac{5\sqrt{(x-1)^3}}{2}$ $(x-1)^{\frac{5}{2}} = \frac{5}{2}(x-1)^{\frac{3}{2}} = \frac{5\sqrt{(x-1)^3}}{2}$

$= \sqrt{(x-1)^3}$

$b = \frac{2}{3} \frac{3\sqrt{x-1}}{2}$ $(x-1)^{\frac{3}{2}} = \frac{3}{2}(x-1)^{\frac{1}{2}}$ $\frac{3\sqrt{x-1}}{2}$

$= \sqrt{x-1}$ $y' = \sqrt{(x-1)^3} + \sqrt{x-1}$

$= \sqrt{(x-1)^2(x-1)} + \sqrt{x-1}$

$= \left[(x-1)\sqrt{(x-1)}\right] + \sqrt{(x-1)}$

$= \sqrt{x-1}\,(1 + x - 1)$

$= x\sqrt{x-1}$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

/ /

(268-) $y = \sqrt[4]{\frac{2x-1}{3-2x}}$ $\qquad y' = \frac{1}{4}\, t^{-\frac{3}{4}} \cdot t'$ $\qquad \frac{2(3-2x)-(2x-1)\cdot -2}{(3-2x)^2}$

$$y' = \frac{1}{\cancel{4}} \cdot \frac{1}{\sqrt[4]{\left(\frac{2x-1}{3-2x}\right)^3}} \cdot \frac{\cancel{4}}{(3-2x)^2} \qquad = \frac{6-4x+4x-2}{(3-2x)^2} = \frac{4}{(3-2x)^2}$$

$$= \frac{1}{\sqrt[4]{(2x-1)^3(3-2x)^{-3}} \cdot (3-2x)^2}$$

=

Dúvida

(269-) $y = \frac{x^6}{6(1-x^2)^3}$

$$y' = \frac{6x^5 \cdot 6(1-x^2)^3 - x^6 \cdot 18(1-x^2)^2 \cdot -2x}{([6(1-x^2)]^3)^2}$$

$$= \frac{36x^5(1-x^2)^3 + 36x \cdot x^6(1-x^2)^2}{([6(1-x^2)]^3)^2}$$

$$= \frac{36x^5(1-x^2)^3 + 36x^7(1-x^2)^2}{\{[6(1-x^2)]^3\}^2} = \frac{\cancel{36}[x^5(1-x^2)^3 + x^7(1-x^2)^2]}{\cancel{36}(1-x^2)^6}$$

$$= \frac{x^5(1-x^2)^3 + x(1-x^2)^2}{(1-x^2)^6} \Rightarrow \frac{(1-x^2)^2[x^5(1-x^2) + x^7]}{(1-x^2)^6}$$

$$= \frac{x^5(1-x^2) + x^7}{(1-x^2)^4} \Rightarrow \frac{x^5 - x^7 + x^7}{(1-x^2)^4} = \frac{x^5}{(1-x^2)^4}$$

/ /

(1270-) $y = \frac{1}{5} \operatorname{arctg} \frac{x}{5} + \operatorname{sen} \frac{\pi}{13}$

$a = \frac{x}{5}$

$a' = \frac{1.5 - x.0}{25} = \frac{5}{25} = \frac{1}{5}$

$a = x5^{-1}$

$a' = 1.5^{-1} + x.0 = 5^{-1} = \frac{1}{5}$

$y' = \frac{1}{5} \cdot \frac{\frac{1}{5}}{1 + \left(\frac{x}{5}\right)^2} = \frac{\frac{1}{5}}{5 + 5\left(\frac{x}{5}\right)^2}$

$= \frac{1}{5} : 5 + \frac{5x^2}{25} = \frac{1}{5} : \frac{125 + 5x^2}{25}$

$= \frac{1}{5} : \frac{25 + x^2}{5} = \frac{5}{125 + x^2} + \cos \frac{\pi}{13} . 0 = \frac{5}{125 + x^2}$

(1271-) $y = \frac{x^2}{2} \ln x - \frac{x^2}{4} + x(\ln x - 1)$

$a = \frac{x^2}{2}$ $\quad a' = \frac{2x.2 - x^2.0}{4} = \frac{4x}{4} = x$ $\quad u' = x \ln x + \frac{x^2}{2} . \frac{1}{x} = x \ln x + \frac{x^2}{2x}$

$b = \frac{2x.4 - x^2.0}{16} = \frac{8x}{16} = \frac{x}{2}$

$c = \ln x - 1 + x . \frac{1}{x} = \ln x$

$\frac{x^2}{2x} - \frac{x}{2} = \frac{x^2 - x^2}{2x} = \frac{0}{2x} = 0$

$y' = x \ln x + \frac{x^2}{2x} - \frac{x}{2} + \ln x$

$= x \ln x + \ln x$

$= \ln x (x + 1)$

=

/ /

(272.) $y = \sqrt{x(x+2)(x+3)}$

$y = \sqrt{(x^2+2x)(x+3)}$

$y = \sqrt{x^3+3x^2+2x^2+6x}$

$y = \sqrt{x^3+5x^2+6x}$

$y' = (x^3+5x^2+6x)^{\frac{1}{2}}$

$= \frac{1}{2} \cdot (x^3+5x^2+6x)^{-\frac{1}{2}} \cdot 3x^2+10x+6$

$= \frac{1}{2\sqrt{x^3+5x^2+6x}} \cdot 3x^2+10x+6$

$= \frac{3x^2+10x+6}{2\sqrt{x^3+5x^2+6x}}$.

(273.) $y = \frac{1}{7}\sqrt[5]{(1+x^5)^7} - \frac{1}{2}\sqrt[5]{(1+x^5)^2}$

$a = (1+x^5)^{\frac{7}{5}} = \frac{7}{5}(1+x^5)^{\frac{2}{5}} \cdot 5x^4 = \frac{35x^4\sqrt[5]{(1+x^5)^2}}{5}$

$= 7x^4\sqrt[5]{(1+x^5)^2} \cdot \frac{1}{7} = x^4\sqrt[5]{(1+x^5)^2}$

$b = (1+x^5)^{\frac{2}{5}} = \frac{2}{5}(1+x^5)^{-\frac{3}{5}} \cdot 5x^4 = \frac{2x^4}{\sqrt[5]{(1+x^5)^3}} \cdot \frac{1}{2}$

$= \frac{x^4}{\sqrt[5]{(1+x^5)^3}}$

=

$y' = x^4\sqrt[5]{(1+x^5)^2} - \frac{x^4}{\sqrt[5]{(1+x^5)^3}}$

$= \frac{x^4(1+x^5) - x^4}{\sqrt[5]{(1+x^5)^3}}$

$= \frac{\cancel{x^4} + x^9 \cancel{-x^4}}{\sqrt[5]{(1+x^5)^3}} =$ ou $x^9(1+x^5)^{-\frac{3}{5}}$

=

/ /

(274-) $y = \frac{1}{6} \ln \frac{x-3}{x+3}$ $b = \frac{x-3}{x+3}$ $b' = \frac{1.(x+3)-(x-3).1}{(x+3)^2}$

$a = \ln \frac{x-3}{x+3}$ $a' = \frac{6}{(x+3)^2} \div \frac{x-3}{x+3}$ $= \frac{x+3-x+3}{(x+3)^2} = \frac{6}{(x+3)^2}$

$' = \frac{6x+18}{(x+3)^2(x-3)} = \frac{6x+18}{(x+3)(x+3)(x-3)} = \frac{6x+18}{(x+3)(x^2-9)}$

$y' = \frac{1}{6} \cdot \frac{6x+18}{(x+3)(x^2-9)} = \frac{(x+3)1}{(x+3)(x^2-9)} = \frac{1}{x^2-9}$

(275-) $y = \frac{1}{2a} \ln \frac{x-a}{x+a}$; $a \neq 0$ e constante

$b = \frac{x-a}{x+a}$ $b' = \frac{1(x+a)-(x-a)1}{(x+a)^2} = \frac{2a}{(x+a)^2}$

$c = \ln \frac{x-a}{x+a}$ $c' = \frac{2a}{(x+a)^2} : \frac{x-a}{x+a} = \frac{2a(x+a)}{(x+a)^2(x-a)}$

$= \frac{2a(\cancel{x+a})}{(x+a)(x+a)(x-a)} = \frac{2a}{(x+a)(x-a)} = \frac{2a}{x^2-a^2}$

$y' = \frac{1}{\cancel{2a}} \cdot \frac{\cancel{2a}}{x^2-a^2} = \frac{1}{x^2-a^2}$

spiral

(276-) $y = e^{5x} \cdot \operatorname{sen}^2 \frac{x}{5}$ $\rightarrow$ $y' = e^{5x} \cdot 5 + e^{5x} \cdot 2 \operatorname{sen}\left(\frac{x}{5}\right) \cos \frac{x}{5} \cdot \frac{1}{5}$

$y' = 5 e^{5x} \cdot \operatorname{sen}^2 \frac{x}{5} + e^{5x} \cdot 2 \operatorname{sen} \frac{x}{5} \cos \frac{x}{5} \cdot \frac{1}{5}$

$= e^{5x} \operatorname{sen} \frac{x}{5} \left(5 \operatorname{sen} \frac{x}{5} + \frac{2}{5} \cos \frac{x}{5} \right)$

(277-) $y = \frac{1}{\sqrt{2}} \cdot \operatorname{arctg}(\sqrt{2}\, tg\, x)$

$a = \sqrt{2}\, tg\, x$

$a' = 0 \cdot tg\, x + \sqrt{2} \cdot \sec^2 x$

$= \sec^2 x \cdot \sqrt{2}$

$y' = u'v + uv'$

$v' = \frac{\sqrt{2} \sec^2 x}{1 + 2 tg^2 x}$

$(\operatorname{sen}^2 x + \cos^2 x = 1) / \cos^2 x)$

$tg^2 x + 1 = \sec^2 x$

$1 + cotg^2 x = cosec^2 x$

$\sec^2 x = \frac{1}{\cos^2 x}$

$y' = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2} \sec^2 x}{1 + 2 tg^2 x} = \frac{\sec^2 x}{1 + 2 tg^2 x}$

$= \frac{1}{\cos^2 x} : 1 + 2 \frac{\operatorname{sen}^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} : \frac{\cos^2 x + 2 \operatorname{sen}^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x + 2 \operatorname{sen}^2 x} = \frac{1}{1 + \operatorname{sen}^2 x}$

(278-) $y = 3^{\cot g \frac{1}{x}}$ $\qquad a = \cot g \frac{1}{x} \quad a' = \frac{-1}{x^2} \cdot -\operatorname{cosec}^2 \frac{1}{x}$

$y' = 3^{\cot g \frac{1}{x}} \cdot \ln 3 \cdot \frac{1}{x^2} \operatorname{cosec}^2 \frac{1}{x} \qquad = \frac{1}{x^2} \operatorname{cosec}^2 \frac{1}{x}$

(279-) $y = 18 \operatorname{arc\,sen} \frac{x}{6} + \frac{x}{2}\sqrt{36-x^2}$

$a = 18 \cdot \frac{\frac{1}{6}}{\sqrt{1-\left(\frac{x}{6}\right)^2}} = \frac{18}{6\sqrt{1-\frac{x^2}{36}}} = \frac{3}{\sqrt{1-\frac{x^2}{36}}} = \frac{3}{\sqrt{\frac{36-x^2}{36}}}$

$= \frac{3}{\frac{\sqrt{36-x^2}}{6}} = \frac{3}{6\sqrt{36-x^2}} = \frac{1}{2\sqrt{36-x^2}}$

$b\ \frac{x}{2} \Rightarrow \frac{1.2 - x.0}{4} = \frac{2}{4} = \frac{1}{2}$

$b' = \frac{1}{2} \cdot \sqrt{36-x^2} - \frac{x}{2} \cdot \frac{1}{2\sqrt{36-x^2}} \qquad (36-x^2)^{\frac{1}{2}} = \frac{1}{2}(36-x^2)^{-\frac{1}{2}}$

$= \frac{\sqrt{36-x^2}}{2} - \frac{x}{4\sqrt{36-x^2}} = \frac{2(36-x^2)-x}{4\sqrt{36-x^2}} \qquad \frac{1}{2\sqrt{36-x^2}}$

$y' = \frac{1}{2\sqrt{36-x^2}} + \frac{\sqrt{36-x^2}}{2} - \frac{x}{4\sqrt{36-x^2}}$

$= \frac{1}{2\sqrt{36-x^2}} + \frac{2(36-x^2)-x}{4\sqrt{36-x^2}} = \frac{2+72-2x^2-x}{4\sqrt{36-x^2}} = 74 -$

/ /

(279) $y = 18 \operatorname{arc\,sen} \underset{a}{\frac{x}{6}} + \underset{b}{\frac{x}{2}\sqrt{36-x^2}}$

$a' = 18 \cdot \frac{1}{6} : \sqrt{1-\frac{x^2}{36}} = 18 \cdot \frac{1}{6} : \frac{\sqrt{36-x^2}}{6} = \frac{18}{\sqrt{36-x^2}}$

$b' = \frac{\sqrt{36-x^2}}{2} + \frac{x}{2} \cdot \frac{-\cancel{2}x}{\cancel{2}\sqrt{36-x^2}} = \frac{\sqrt{36-x^2}}{2} + \left(\frac{-\cancel{2}x^2}{2\sqrt{36-x^2}}\right)$

$y' = \frac{18}{\sqrt{36-x^2}} + \frac{\sqrt{36-x^2}}{2} - \frac{x^2}{2\sqrt{36-x^2}}$

$= \frac{36 + 36 - x^2 - x^2}{2\sqrt{36-x^2}} = \frac{72-2x^2}{2\sqrt{36-x^2}} = \frac{\cancel{2}(36-x^2)}{\cancel{2}\sqrt{36-x^2}} = \frac{36-x^2}{\sqrt{36-x^2}}$

$= \underline{\sqrt{36-x^2}}$

(280) $y = \frac{x}{2}\sqrt{4-x^2} + 2\operatorname{arsen}\frac{x}{2}$

$a = \frac{\sqrt{4-x^2}}{2} + \frac{x}{2} \cdot \frac{-\cancel{2}x}{\cancel{2}\sqrt{4-x^2}} \Rightarrow \frac{\sqrt{4-x^2}}{2} + \frac{-x^2}{2\sqrt{4-x^2}}$

$b = 2 \cdot \frac{1}{2} : \sqrt{1-\frac{x^2}{4}} = \frac{2}{2\sqrt{4-x^2}}$

$y' = \frac{\sqrt{4-x^2}}{2} - \frac{x^2}{2\sqrt{4-x^2}} + \frac{2}{\sqrt{4-x^2}} = \frac{4-x^2-x^2+4}{2\sqrt{4-x^2}}$

$= \frac{\cancel{2}(4-x^2)}{\cancel{2}\sqrt{4-x^2}} = \frac{(\cancel{4-x^2})\sqrt{4-x^2}}{\cancel{4-x^2}} = \sqrt{4-x^2}$

spiral

(283-) $y = \frac{1}{3} \ln \frac{x^2 - 2x + 1}{x^2 + x + 1}$

Anexo

$$a = \frac{(2x-2)(x^2+x+1) - (x^2-2x+1)(2x+1)}{(x^2+x+1)^2}$$

$$= \frac{\cancel{2x^3} + \cancel{2x^2} + \cancel{2x} - \cancel{2x^2} - \cancel{2x} - 2 - \cancel{2x^3} + 4x^2 - \cancel{2x} - x^2 + \cancel{2x} - 1}{(x^2+x+1)^2}$$

$$= \frac{3x^2 - 3}{(x^2+x+1)^2}$$

$$y' = \frac{1}{3} \cdot \frac{3x^2-3}{(x^2+x+1)^2} \cdot \frac{(x^2+x+1)}{x^2-2x+1} = \frac{1}{3} \frac{3x^2-3}{x^2-2x+1} = \frac{\cancel{3}(x^2-1)}{\cancel{3}(x^2-2x+1)}$$

$$= = \frac{x^2-1}{x^2-2x+1} \cdot \frac{1}{x^2+x+1} =$$

(284-) $y = \frac{e^x}{2} (\operatorname{sen} x + \cos x)$

$$y' = \frac{2e^x}{4} \Rightarrow \frac{e^x}{\cancel{2}} (\operatorname{sen} x + \cos x) + \frac{e^x}{\cancel{2}} (\cos x - \operatorname{sen} x)$$

$$= \frac{e^x}{2} \left(\cancel{\operatorname{sen} x} + \cos x + \cos x - \cancel{\operatorname{sen} x} \right)$$

$$\frac{e^x \cdot 2\cos x}{2} = e^x \cos x$$

/ /

(285.) $y = \dfrac{x}{5\sqrt{5+x^2}}$ $\qquad \dfrac{5 \cdot 2x}{2\sqrt{5+x^2}} = \dfrac{5x}{\sqrt{5+x^2}}$

$$y' = \dfrac{5\sqrt{5+x^2} - x \cdot \dfrac{5x}{\sqrt{5+x^2}}}{(5\sqrt{5+x^2})^2} \Rightarrow \dfrac{\dfrac{5(5+x^2) - 5x^2}{(\sqrt{5+x^2})^2}}{(5\sqrt{5+x^2})^2} \Rightarrow$$

$$\Rightarrow \dfrac{25+5x^2-5x^2}{\sqrt{5+x^2}} \cdot \dfrac{1}{25(5+x^2)} = (5+x^2)^{-\frac{1}{2}} \cdot (5+x^2)^{-1}$$

$$= (5+x^2)^{-\frac{3}{2}}$$

(286.) $y = \text{arc sen}(1-x) + \sqrt{2x-x^2}$

$$y' = \dfrac{-1}{\sqrt{1-(1-x)^2}} + \dfrac{2-2x}{2\sqrt{2x-x^2}} \Rightarrow \dfrac{-1}{\sqrt{1-(1-x)^2}} + \dfrac{2(1-x)}{2\sqrt{2x-x^2}}$$

$$\Rightarrow \dfrac{-1}{\sqrt{1-(1-2x+x^2)}} + \dfrac{1-x}{\sqrt{2x-x^2}} = \dfrac{-1}{\sqrt{1-1+2x-x^2}} + \dfrac{1-x}{\sqrt{2x-x^2}}$$

$$\Rightarrow \dfrac{-1}{\sqrt{2x-x^2}} + \dfrac{1-x}{\sqrt{2x-x^2}} = \dfrac{-x}{\sqrt{2x-x^2}} \cdot \dfrac{\sqrt{2x-x^2}}{\sqrt{2x-x^2}} = \dfrac{-x\sqrt{2x-x^2}}{2x-x^2}$$

$$\Rightarrow \dfrac{-x\sqrt{2x-x^2}}{-x(x-2)} = \dfrac{\sqrt{2x-x^2}}{x-2}$$

(287.) $y = \dfrac{x^3+2x}{4} \cdot \sqrt{x^2+4} - 2\ln(x+\sqrt{x^2+4})$

$$a = \dfrac{(3x^2+2x)}{16} \cdot \sqrt{x^2+4} + \dfrac{x^3+2x}{4} \cdot \dfrac{x}{\sqrt{x^2+4}}$$

$$= \dfrac{(3x^2+2x)\sqrt{x^2+4}}{16} + \dfrac{x^4+2x^2}{4\sqrt{x^2+4}}$$

$$= \dfrac{4(3x^2+2x)(x^2+4) + 16(x^4+2x^2)}{64\sqrt{x^2+4}}$$

$$= \dfrac{(3x^4+12x^2+2x^3+8x) + 4(x^4+2x^2)}{16\sqrt{x^2+4}}$$

spiral

(288-) $y = \frac{1}{2} \cdot \operatorname{sen} x \cdot \sec^2 x + \frac{1}{2} \ln(\sec x + \operatorname{tg} x)$

$a = \cos x \sec^2 x + \operatorname{sen} x \cdot 2 \sec x \sec x \operatorname{tg} x$

$= \cos x \frac{1}{\cos^2 x} + \operatorname{sen} x \cdot 2 \frac{1}{\cos^2 x} \cdot \frac{\operatorname{sen} x}{\cos}$

$= \frac{1}{\cos x} + \frac{2 \operatorname{sen}^2 x}{\cos^3 x} = \frac{\cos^2 x + 2 \operatorname{sen}^2 x}{\cos^3 x} = \frac{1 + \operatorname{sen}^2 x}{\cos^3 x} \cdot \frac{1}{2}$

$b = \frac{\sec x \cdot \operatorname{tg} x + \sec^2 x}{\sec x + \operatorname{tg} x} = \frac{\sec x (\operatorname{tg} x + \sec^2 x)}{\sec x + \sec x - 1} = \frac{\sec x (\operatorname{tg} x + \sec^2 x)}{\sec x (}$

(289-) $y = \frac{x^3}{3} \operatorname{arc tg}(3x) - \frac{x^2}{18} + \frac{\ln(9x^2+1)}{162}$

$a = \frac{1}{3} \cdot 3x^2 \Rightarrow x^2 \operatorname{arctg}(3x) + \frac{x^3}{\cancel{3}} \cdot \frac{\cancel{3}}{1+9x^2} = x^2 \operatorname{arctg}(3x) + \frac{x^3}{1+9x^2}$

$b = \frac{-1}{18} \cdot 2x = \frac{-2x}{18} = \frac{-x}{9}$

$c = \frac{18x}{9x^2+1} \cdot \frac{1}{162} = \frac{9x}{9x^2+1} \cdot \frac{1}{81} = \frac{x}{9x^2+1} \cdot \frac{1}{9} = \frac{x}{(9x^2+1)9}$

$y' = x^2 \operatorname{arcTg}(3x) + \frac{x^3}{1+9x^2} - \frac{x}{9} + \frac{x}{(9x^2+1)9}$

$= \frac{x^2 \operatorname{arctg}(3x)(9x^2+1)9 + 9x^3 - x(9x^2+1) + x}{(9x^2+1)9}$

$= \frac{x^2 \operatorname{arctg}(3x)\cancel{(9x^2+1)9} + \cancel{9x^3} - \cancel{9x^3} - \cancel{x} + \cancel{x}}{\cancel{(9x^2+1)9}}$

$= x^2 \operatorname{artg}(3x)$

/ /

(293-) $y = \frac{2s+1}{4}\sqrt{s^2+s+1} + \frac{3}{8}\ln\left(2s+1+2\sqrt{s^2+s+1}\right)$

1.ª Parcela

$\frac{1}{4}\cdot\left(2\sqrt{x^2+x+1} + \frac{2x+1}{4}\cdot\frac{2x+1}{2\sqrt{x^2+x+1}}\right)$

$\frac{1}{4}\left(2\sqrt{x^2+x+1} + \frac{(2x+1)^2}{8\sqrt{x^2+x+1}}\right) \Rightarrow \frac{1}{4}\cdot\left(\frac{16(x^2+x+1)+(2x+1)^2}{16\quad 8\sqrt{x^2+x+1}}\right)$

2.ª Parcela

$\frac{3}{8}\;2+2\cdot\frac{2x+1}{2\sqrt{x^2+x+1}} = 2+\frac{2x+1}{\sqrt{x^2+x+1}} \Rightarrow \frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{x^2+x+1}} : 2x+1+2\sqrt{x^2+x+1}$

$\frac{2\sqrt{x^2+x+1}+2x+1}{2x\sqrt{x^2+x+1}+\sqrt{x^2+x+1}+2(x^2+x+1)} \Rightarrow \frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{x^2+x+1}\left(2x+1+2\sqrt{x^2+x+1}\right)}$

(295-) $y = -\frac{e^{-x}}{50}\left[7\cos(7x)+\mathrm{sen}(7x)\right]$

$y' = \frac{-1}{50}\left[-e^{-x}\cdot\left(7\cos(7x)+\mathrm{sen}(7x)\right)+e^{-x}\cdot\left(-49\,\mathrm{sen}(7x)+7\cos(7x)\right)\right]$

$= \frac{-1}{50}\left[e^{-x}\left[\cancel{-7\cos(7x)}-\mathrm{sen}(7x)+(-49\,\mathrm{sen}(7x)+\cancel{7\cos(7x)}\right]\right]$

$= \frac{+1}{50}\left(e^{-x}\left[+50\,\mathrm{sen}(7x)\right]\right]$

$= e^{-x}\,\mathrm{sen}(7x)$

(1296-) $y = 2\sqrt{e^x+1} + \ln \dfrac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}$

$$a = 2 \cdot \frac{e^x}{2\sqrt{e^x+1}} = \frac{e^{x-1}}{\sqrt{e^x+1}}$$

$$b = \frac{\dfrac{e^x}{2\sqrt{e^x+1}} \cdot (\sqrt{e^x+1}+1) - (\sqrt{e^x+1}-1) \cdot \dfrac{e^x}{2\sqrt{e^x+1}}}{(\sqrt{e^x+1}+1)^2}$$

$$= \frac{\dfrac{\cancel{e^x \cdot \sqrt{e^x+1}} + e^x - \cancel{e^x\sqrt{e^x+1}} + e^x}{2\sqrt{e^x+1}}}{(\sqrt{e^x+1}+1)^2}$$

$$= \frac{2e^x}{2\sqrt{e^x+1}} : (\sqrt{e^x+1}+1)^2 \qquad \frac{e^x}{(\sqrt{e^x+1})(\sqrt{e^x+1}+1)^2} : \frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}$$

$$= \frac{e^x}{(\sqrt{e^x+1})(e^x+1-1)} = \frac{1}{(\sqrt{e^x+1})}$$

$$y' = \frac{e^x}{\sqrt{e^x+1}} + \frac{1}{\sqrt{e^x+1}} = \frac{e^x+1}{\sqrt{e^x+1}} = \sqrt{e^x+1}$$

/ /

(297) - $y = \ln|1-3x|$; $x \neq \frac{1}{3}$

$\frac{-3}{1-3x} = \frac{3}{3x-1}$

(298.) $y = \ln|\sec x|$; $\cos x \neq 0$

$y' = \frac{\sec x . tg x}{\sec x} = tg x$

(299.) $y = \frac{1}{10} \ln\left|\frac{x-5}{x+5}\right|$; $x \neq \pm 5$

$y' = \frac{1}{10} . \left[\frac{x+5-(x-5)}{(x+5)^2} : \frac{x-5}{x+5} \right]$

$= \frac{1}{10} \left[\frac{10}{(x+5)^2} : \frac{x-5}{x+5} \right] \Rightarrow \frac{1}{\cancel{10}} \left[\frac{\cancel{10}}{(x+5)(x-5)} \right]$

$= \frac{1}{x^2+25}$

(300) $y = \frac{1}{9} \ln\left|\frac{x-1}{x+8}\right|$; $x \neq 1$, $x \neq -8$

$y' = \frac{1}{9} \left[\frac{x+8-(x-1)}{(x+8)^2} : \frac{x-1}{x+8} \right] \Rightarrow \frac{1}{9} \left[\frac{9}{(x+8)^2} : \frac{x-1}{(x+8)} \right]$

$= \frac{1}{\cancel{9}} \left[\frac{\cancel{9}}{(x+8)(x-1)} \right] = \frac{1}{x^2+7x-8}$

(301-) $y = \ln|1-\cos x|; \cos x \neq 1$

$$y' = \frac{\operatorname{sen} x}{1-\cos x} \cdot \frac{(1+\cos x)}{(1+\cos x)} = \frac{\operatorname{sen} x + \cos x \operatorname{sen} x}{1-\cos^2 x} = \frac{\operatorname{sen} x + \cos x \operatorname{sen} x}{\operatorname{sen}^2 x}$$

$$= \frac{\cancel{\operatorname{sen} x}(1+\cos x)}{\operatorname{sen}^{\cancel{2}} x} = \frac{1+\cos x}{\operatorname{sen} x}$$

(302-) $y = \ln|\sec x + \operatorname{tg} x|; \cos x \neq 0$

$$y' = \frac{\sec x . \operatorname{tg} x + \sec^2 x}{\sec x + \operatorname{tg} x} = \frac{\sec x (\operatorname{tg} x + \sec x)}{\cancel{\sec x + \operatorname{tg} x}} = \sec x$$

(303-) $y = \ln|e^x - \sqrt{1+e^{2x}}|$

$$y' = e^x - \frac{\cancel{2}e^{2x}}{\cancel{2}\sqrt{1+e^{2x}}} : e^x - \sqrt{1+e^{2x}}$$

$$= \frac{e^x\sqrt{1+e^{2x}} - e^{2x}}{\sqrt{1+e^{2x}}} : e^x - \sqrt{1+e^{2x}} \Rightarrow \frac{-e^x(e^x - \sqrt{1+e^{2x}})}{\sqrt{1+e^{2x}}} : e^x - \sqrt{1+e^x}$$

$$= -\frac{e^x}{\sqrt{1+e^{2x}}}$$

$D(f^g) = f^g\left(g' \ln f + g \cdot \frac{f'}{f}\right)$

(304-) $y = x^x \; ; \; x > 0$

$y' = \left(\ln x + \cancel{x} \frac{1}{\cancel{x}}\right) \cdot x^x = x^x (\ln x + 1)$

(305-) $y = (x^2+1)^{\operatorname{sen} x}$

$y' = (x^2+1)^{\operatorname{sen} x}\left(-\cos x \cdot \ln(x^2+1) + \operatorname{sen} x \cdot \frac{2x}{x^2+1}\right)$

$= (x^2+1)^{\operatorname{sen} x}\left(-\cos x \cdot \ln(x^2+1) + \frac{2x \operatorname{sen} x}{x^2+1}\right)$

(306-) $y = x^{\ln x} \; ; \; x > 0$

$y' = x^{\ln x}\left(\frac{1}{x} \cdot \ln x + \ln x \cdot \frac{1}{x}\right)$

$= x^{\ln x}\left(\frac{2 \ln x}{x}\right) = x^{\ln x} \cdot 2 \ln x \cdot x^{-1} = 2 \ln x \cdot x^{\ln x - 1}$

(307-) $y = (xe^x)^x \; ; \; x > 0$

$xe^x = e^x + x \cdot e^x$
$= e^x(1+x)$

$y' = (xe^x)^x\left(\ln(xe^x) + \cancel{x} \cdot \frac{e^x(1+x)}{\cancel{xe^x}}\right)$

$= (xe^x)^x \left(\ln(xe^x) \cdot (1+x)\right)$

(308.) $y=(1-x)^{x-1}$

$$y' = (1-x)^{x-1}\left[\ln(1-x)+(x-1)\cdot\frac{-1}{(1-x)}\right]$$

$$=(1-x)^{x-1}\left[\ln(1-x)+1\right]$$

(309). $y=(\ln x)^x$; $x>1$

$$y'=(\ln x)^x\left[\ln(\ln x)+x\cdot\frac{1}{x}:\ln x\right]$$

$$=(\ln x)^x\cdot\left[\ln(\ln x)+\frac{1}{\ln x}\right]=(\ln x)^x\left(\ln(\ln x)+(\ln x)^{-1}\right)$$

$$=(\ln x)^x\cdot\ln(\ln x)+(\ln x)^{x-1} \Rightarrow (\ln x)^{x-1}\left[1+\ln(\ln x).(\ln x)\right]$$

(310) $f(x)=\begin{cases} x^2, & \text{si } x\leq 2 \\ 2x, & \text{si } x>2 \end{cases}$

si $x<2$ $\quad f'(x)=2x$

si $x>2$ $\quad f'(x)=2$

si $x=2$ $\quad \lim\limits_{x\to x_0}\dfrac{f(x)-f(x_0)}{x-x_0} \Rightarrow \lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2} \quad \nexists$

$$\lim_{x\to 2^+}\frac{f(x)-f(2)}{x-2}=\frac{x^2-2}{x-2}\cdot\lim_{x\to 2^+}\frac{x\cdot x-2}{x-2}$$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

(318.) $\lim_{x \to 2} \frac{x^5 - 32}{x - 2}$ $\lim_{x \to 2} \frac{5x^4}{1}$ $f(x_0) = 2$ $5.2^4 = 80$

(319.) $\lim_{x \to 1} \frac{\sqrt{3x+1} - 2}{x - 1}$ $\lim_{x \to 1} \frac{3}{2\sqrt{3x+1}}$ $f(x_0) = 1$ $\frac{3}{2\sqrt{3.1+1}} = \frac{3}{4}$

(320.) $\lim_{x \to 5} \frac{\ln x - \ln 5}{x - 5} \Rightarrow \lim_{x \to 5} \frac{1}{x}$ $f(x_0) = \frac{1}{5}$

(321.) $\lim_{x \to \frac{\pi}{3}} \frac{2\cos x - 1}{3x - \pi} \Rightarrow \lim_{x \to \frac{\pi}{3}} \frac{2. - \operatorname{sen} x}{3}$ $f(x_0) = \frac{\pi}{3}$ $\frac{-2. \operatorname{sen} \frac{\pi}{3}}{3} = \frac{-2. \frac{\sqrt{3}}{2}}{3} = -\frac{\sqrt{3}}{3}$

(322.) $\lim_{x \to -1} (x^2+7)^{\frac{1}{3}}$ $\lim_{x \to -1} \frac{1}{3}(x^2+7)^{-\frac{2}{3}}$ $\lim_{x \to -1} \frac{1}{3\sqrt[3]{(x^2+7)^2}} = \frac{1}{3\sqrt[3]{((-1)^2+7)^2}}$ $.2x2 = \frac{-2}{12} = \frac{-1}{6}$

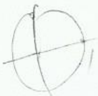
(323.) $\lim_{x \to 1} \frac{\operatorname{arc} tg x - \frac{\pi}{4}}{x - 1}$ $\lim_{x \to 1} \frac{1}{1 + x^2}$ $f(x_0) = 1$ R: $\frac{1}{2}$

(324.) $\lim_{x \to 2} \frac{e^x - e^2}{x - 2}$ $\lim_{x \to 2} e^x$ $f(x_0) = 2$ R: e^2

(325.) $\lim_{x \to 3} \frac{\ln(2x) - \ln 6}{x - 3}$ $\lim_{x \to 3} \frac{1}{x}$ $f(x_0) = 3$ R: $\frac{1}{3}$

spiral

/ /



(326-) $\lim\limits_{x\to a} \dfrac{\operatorname{sen}x - \operatorname{sen}a}{x-a}$ $\lim\limits_{x\to a} +\cos x$ $f(x_0)=a$ R: $+\cos a$

(327-) $\lim\limits_{x\to 0} \dfrac{\sec x - 1}{x}$ $\lim\limits_{x\to 0} \sec x \, tg\, x$ $f(x_0)=0$ R: 0

(328-) $\lim\limits_{x\to 64} \dfrac{\sqrt[6]{x}-2}{x-64}$ $\lim\limits_{x\to 64} \dfrac{1}{6\sqrt[6]{x^5}}$ $f(x_0)=64$ R: $\dfrac{1}{192}$

(329-) $\lim\limits_{x\to 0} \dfrac{\ln(\cos x)}{x}$ $\lim\limits_{x\to 0} \dfrac{-\operatorname{sen}x}{\cos x}$ $\lim\limits_{x\to 0} -tg\, x$ $f(x_0)=0$ R: 0

(330-) $\lim\limits_{x\to 0}$. 1 $\nexists$

(331-) $\lim\limits_{x\to \frac{1}{2}} \dfrac{6\cdot\frac{1}{\sqrt{1-x^2}}}{2} = \dfrac{6}{2\sqrt{1-x^2}} = \dfrac{6}{2\sqrt{1-\frac{1}{4}}} = \dfrac{6}{2\sqrt{\frac{4-1}{4}}} = \dfrac{6}{4\sqrt{3}} = \dfrac{3}{2\sqrt{3}}$

ou $\dfrac{6}{2\sqrt{\frac{3}{4}}} = \dfrac{6}{\sqrt{3}}\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$

(332) $\lim\limits_{x\to a} \dfrac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a} = \dfrac{1}{3\sqrt[3]{x^2}}$ $f(x_0)=a$ $= \dfrac{1}{3\sqrt[3]{a^2}}$

(333) $\lim\limits_{x\to e} \dfrac{x\ln x - e}{x-e} = \ln x + x\dfrac{1}{x} = \ln x$ $f(x_0)=e$ $\ln e = 1$

(334) $\lim\limits_{x\to 1} \dfrac{x^2\ln x}{x-1}$ $2x\ln x + x^2\dfrac{1}{x} = 2x\ln + x = x(2\ln x + 1)$

$f(x_0)1$ R= 1

(374-) $y = e^{\operatorname{sen} x}$, calcular y''

$y' = \cos x \cdot e^{\operatorname{sen} x}$

$y'' = -\operatorname{sen} x \cdot e^{\operatorname{sen} x} + \cos x \cdot e^{\operatorname{sen} x} \cdot \cos x$

$y'' = -\operatorname{sen} x \cdot e^{\operatorname{sen} x} + \cos^2 x \cdot e^{\operatorname{sen} x}$

$= e^{\operatorname{sen}} (\cos^2 x - \operatorname{sen} x)$

(375-) $f(x) = \dfrac{x}{x^2+1}$, $f''(x)$

$f'(x) = \dfrac{(x^2+1) - x(2x)}{(x^2+1)^2} = \dfrac{x^2+1-2x^2}{(x^2+1)^2} = \dfrac{-x^2+1}{(x^2+1)^2}$

$f''(x) = \dfrac{-2x(x^2+1)^2 - (-x^2+1)\,2(x^2+1) \cdot 2x}{[(x^2+1)^2]^2}$

$= \dfrac{-2x(x^2+1)^2 - 4x(-x^2+1)(x^2+1)}{[(x^2+1)^2]^2}$

$= \dfrac{\cancel{(x^2+1)}[-2x(x^2+1) - 4x(-x^2+1)]}{[(x^2+1)^2]^2}$

$= \dfrac{-2x[(x^2+1) + 2(-x^2+1)]}{(x^2+1)^3} = \dfrac{-2x(x^2+1-2x^2+2)}{(x^2+1)^3}$

$= \dfrac{-2x(-x^2+3)}{(x^2+1)^3}$ ou $\dfrac{2x(x^2-3)}{(x^2+1)^3}$

/ /

(376-) $y = x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2)$ $\quad y''' = ?$

$y' = \operatorname{arctg} x + x \cdot \frac{1}{1+x^2} = \operatorname{arctg} x + \frac{x}{1+x^2}$

$\frac{2x}{1+x^2} \cdot \frac{1}{2} = \frac{x}{1+x^2}$ $\qquad y' = \operatorname{arctg} x + \cancel{\frac{x}{1+x^2}} - \cancel{\frac{x}{1+x^2}}$

$y' = \operatorname{arctg} x$

$y'' = \frac{1}{1+x^2}$ $\qquad y''' = \frac{-2x}{(1+x^2)^2}$

(377-) $y = e^x(5\cos(3x) - 7\operatorname{sen}(3x))$ $\qquad E = y'' - 2y' + 10y$

y':

$e^x(5\cos(3x) - 7\operatorname{sen}(3x)) + e^x(15 - \operatorname{sen}(3x) - 21\cos(3x)$

$= e^x(5\cos(3x) - 7\operatorname{sen}(3x) - 15\operatorname{sen}(3x) - 21\cos(3x))$

$\to e^x(-16\cos(3x) - 22\operatorname{sen}(3x))$

y'': $e^x(-16\cos(3x) - 22\operatorname{sen}(3x)) + e^x(+48\operatorname{sen}(3x) - 66\cos(3x))$

$e^x(-16\cos(3x) - 22\operatorname{sen}(3x) + 48\operatorname{sen}(3x) - 66\cos(3x))$

$e^x(-82\cos(3x) + 26\operatorname{sen}(3x)$

$E = e^x(-82\cos(3x) + 26\operatorname{sen}(3x)) - 2e^x(-16\cos(3x) - 22\operatorname{sen}(3x) + 10e^x($
$5\cos(3x) - 7\operatorname{sen}(3x)) = 0 + 0 + 0 = 0$

(376-) $y = 5e^{-x} + 7e^{3x} - x + 2$ $\quad E = y''' - 2y'' - 3y'$

y':

$-5e^{-x} + 21e^{3x} - 1$

$E = \cancel{-5e^{-x}} + \cancel{189e^{3x}} \cancel{-10e^{-x}} \cancel{-126e^{3x}} + \cancel{15e^{-x}}$
$\cancel{-63e^{3x}} + 3$

y'':

$5e^{-x} + 63e^{3x}$

$E = 3$

y''':

$-5e^{-x} + 189e^{3x}$

(382-) $\operatorname{tg} y = xy$

$$\sec^2 y \cdot y' = y + xy'$$
$$\sec^2 y \cdot y' - xy' = y$$
$$y'(\sec^2 y - x) = y$$
$$y' = \frac{y}{\sec^2 y - x}$$

//

$$\sec^2 y \cdot y' = y + x \cdot y'$$
$$y'(\sec^2 y - x) = y$$
$$y' = \frac{y}{\sec^2 y - x}$$

(383-) $e^y y = e^{x+1}$

$$y'e^y \cdot y + e^y \cdot y' = e^{x+1}$$
$$y'(e^y \cdot y + e^y) = e^{x+1}$$
$$y' = \frac{e^{x+1}}{e^y \cdot y + e^y}$$

(335-) $\lim_{x \to 1} \dfrac{\sqrt{x+3}-2}{\sqrt{10-x}-3}$ $f' : \dfrac{1}{2\sqrt{x+3}} : \dfrac{-1}{2\sqrt{10-x}} = \dfrac{\sqrt{10-x}}{-\sqrt{x+3}}$

$f(x_0)=1$ $\sqrt{9} / -\sqrt{4} = 3/2 = -\dfrac{3}{2}$

(336-) $\lim_{x \to 2} \dfrac{\sqrt{x^2+x+3}-x-1}{x-2}$ $f' = \dfrac{2x+1}{2\sqrt{x^2+x+3}} - 1$ $f(x_0)=2$

$\dfrac{2.2.+1}{2\sqrt{2^2+2+3}} - 1 = \dfrac{-1}{6}$

(337-) $\lim_{x \to 1}$ $\dfrac{-1}{x^2} . \cos x$ $f(x_0)=1$ $R: -\cos x$

(338-) $\lim_{x \to 1}$ $\dfrac{(x-2)}{x^3} . \cos\left(\dfrac{1-x}{x^2}\right)$ $f(x_0)=1$ $R = -1$

(339-) $\lim_{x \to -1}$ $e^{x^2-x-2} . 2x-1+1$ $f(x_0)=1$ $R: -2$

$(-1)^2-(-1)-2)$

$1+1-2 = 0$

$3+2$

(340-) $\lim_{x \to 1}$ $3x^2e^{2x} + x^3e^{2x}.2 \Rightarrow e^{2x}(3x^2+2x^3)$ $R = 5e^2$

$f(x_0)=1$

(341) $\lim_{x \to \pi}$ $x . \cos x$ $f(x_0)=\pi$ $R: -\pi$

(394-) $f(x) = \sqrt{\frac{x-2}{x+1}}$ $\quad x_0 = 3$ $\quad f'(x) = \frac{(x+1)-(x-2)}{(x+1)^2} = \frac{3}{(x+1)^2}$

$= \frac{3}{(x+1)^2} : 2\sqrt{\frac{x-2}{x+1}}$

$= \frac{3}{16} : 1 = \frac{3}{16}$

t: $y - f(x_0) = f'(x_0)(x - x_0)$

$y - \frac{1}{2} = \frac{3}{16}(x-3)$

$y = \frac{3x}{16} - \frac{9}{16} + \frac{1}{2}$ $\quad y = \frac{3x-1}{16}$

n: $y - f(x_0) = -\frac{1}{f(x_0)}(x - x_0)$

$y - \frac{1}{2} = \frac{-16}{3}(x-3)$

$y = \frac{-16x}{3} + 16 + \frac{1}{2}$ $\quad y = \frac{-16x}{3} + \frac{33}{2}$ $\quad$ ou $\quad 6y + 32x - 99 = 0$

(395-) $f(x) = e^{x^2+x-2}$, $x_0 = 1$ $\quad f'(x) = e^{x^2+x-2} \cdot 2x+1$

$f'(x_0) = 3$

t: $y - f(x_0) = f'(x_0)(x - x_0)$

$y - 1 = 3(x-1)$

$y = 3x - 2$

n- $y - f(x_0) = -\frac{1}{f'(x_0)}(x - x_0)$

$y = \frac{-1}{3}(x-1)$

$y = -\frac{x}{3} + \frac{1}{3} + 1$ $\quad y = \frac{-x+1+3}{3}$ $\quad y = \frac{4-x}{3}$

/ /

(346-) $f(x) = (x^2+1)\ arctg\,x - x, \quad x_0 = 0$

$f'(x) = 2x \cdot arctg\,x + (x^2+1) \cdot \frac{1}{1+x^2} - 1$

$= 2x\ arctg\,x + \frac{x^2+1}{x^2+1} - 1$

t: $y - f(x_0) = f'(x_0)(x - x_0)$

$y - 0 = 0\ (x - 0)$

$y = 0$

n= $x = x_0$

$x = 0$

(347-) $f(x) = \frac{x}{x^2+1}, \quad x_0 = -1$

$f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2}$

$= \frac{-x^2+1}{(x^2+1)^2}$

t: $y - f(x_0) = f'(x_0)(x - x_0)$

$y - \frac{(-1)}{2} = 0\ (x+1)$

$y = \frac{-1}{2}$

n: $x = x_0$

$x = -1$

$y = -1$

(348-) $f(x) = \frac{x\ arc\ sen(\cos x)}{e^x}, \quad x_0 = 0$

$f'(x) = \frac{arc\,sen(\cos x) - x \cdot \frac{sen\,x}{\sqrt{1 - ar\,sen^2(\cos x)}}}{e^x}$

t: $y - f(x_0) = f'(x_0)(x - x_0)$

$y - 0 =$

/ /

(345-) $f(x) = e^{x^2+x-2}$, $x_0 = 1$

$f'(x) = e^{x^2+x-2} . 2x+1$

$f'(x_0) = 3$

$f(x_0) = 1$

$t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - 1 = 3(x - 1)$

$y = 3x - 2$

$n: y - f(x_0) = -\frac{1}{f'(x_0)}(x - x_0)$

$y - 1 = \frac{-1}{3}(x-1)$ $\quad y = \frac{-x}{3} + \frac{1}{3} + 1$ $\quad y = \frac{-x+4}{3}$

(346-) $f(x) = (x^2+1) \operatorname{arctg} x - x$; $x_0 = 0$

$f'(x) = 2x . \operatorname{arctg} x + \cancel{(x^2+1)} \frac{1}{\cancel{1+x^2}} \cancel{-1}$

$= 2x \operatorname{arctg} x$

$f(x_0) = 0$

$f'(x_0) = 0$

$t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - 0 = 0(x - x_0)$

$y = 0$

$n: x = x_0$

$x = 0$

spiral

$\sin^2 x + \cos^2 x = 1$

(347-) $f(x) = \dfrac{x}{x^2+1}$, $x_0 = -1$

$f'(x) = \dfrac{(x^2+1) - x(2x)}{(x^2+1)^2}$

$= \dfrac{1-x^2}{(x^2+1)^2}$

$f(x_0) = \dfrac{-1}{1+1} = \dfrac{-1}{2}$

$f'(x_0) = 0$

t: $y - f(x_0) = f'(x_0)(x - x_0)$

$y - \left(-\frac{1}{2}\right) = 0\,(x-(-1))$

$y = \dfrac{-1}{2}$

n: $f'(x_0) = 0$

$x = x_0$

$x = -1$

(348-) $f(x) = \dfrac{x \operatorname{arcsen}(\cos x)}{e^x}$, $x_0 = 0$

$f'(x) = \dfrac{\left(\underbrace{\operatorname{arcsen}(\cos x)}_{\frac{\pi}{2}} + x.\dfrac{-\operatorname{sen} x}{\sqrt{1-\cos^2 x}}\right).e^x - x\operatorname{arcsen}(\cos x).\,e^x}{(e^x)^2}$

$= \dfrac{(\operatorname{arcsen}(\cos x) + x.-1)e^x - x\operatorname{arcsen}(\cos x).\,e^x}{(e^x)^x}$

$= \dfrac{e^x(-x\operatorname{arcsen}(\cos x) - x\operatorname{arcsen}(\cos x))}{(e^x)^2}$

$= \dfrac{-2x\operatorname{arcsen}(\cos x)}{e^x}$

$f(x_0) = 0$

$f'(x_0) = \dfrac{\pi}{2}$

t: $y - 0 = \dfrac{\pi}{2}(x-0)$

$y = \dfrac{\pi x}{2}$

n: $y - 0 = \dfrac{-2}{\pi}(x-0)$

$y = \dfrac{-2x}{\pi}$

(349-) $f(x) = \ln x$, $x>0$ $A=(0,0)$ $f'(x) = \frac{1}{x}$ $f(x_0) = \ln x_0$

$f'(x_0) = \frac{1}{x_0}$

$t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - \ln x_0 = \frac{1}{x_0}(x - x_0) \Rightarrow 0 = \frac{x}{x_0} - 1 + \ln x_0$

$\frac{x}{x_0} + \ln x_0 = 1 \Rightarrow \ln x_0 = 1$

$x_0 = e$

$t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - 1 = \frac{x}{e} - 1$

$y = \frac{x}{e}$

(350-) $f(x) = \ln x$, $x>0$ $A = (0, 2\ln 2 - 1)$ $f(x_0) = \ln x_0$

$f'(x_0) = \frac{1}{x_0}$

$t: y - f(x_0) = f'(x_0)(x - x_0)$

$2\ln 2 - 1 - \ln x_0 = \frac{1}{x_0}(-x_0)$

$2\ln 2 - 1 - \ln x_0 = -1$

$2\ln 2 - \ln x_0 = 0$

$2\ln 2 = \ln x_0$

$\ln 2^2 = \ln x_0 \Rightarrow \ln 4 = \ln x_0$ $x_0 = 4$

$y - \ln 4 = \frac{1}{4}(x - 4)$

$y = \frac{x}{4} - 1 + \ln 4$ $y = \frac{x-4}{4} + \ln 4$

/ /

(1)

351-) $f(x) = \sqrt{5-x}$, $x \leq 5$ $A = (9,0)$ $f(x_0) = \sqrt{5-x_0}$

$f'(x_0) = \dfrac{-1}{2\sqrt{5-x_0}}$

$t: \ y - \sqrt{5-x_0} = \dfrac{-1}{2\sqrt{5-x_0}}(x - x_0)$

$-\sqrt{5-x_0} = \dfrac{-1}{2\sqrt{5-x_0}}(9-x_0) \Rightarrow \dfrac{x_0}{2\sqrt{5-x_0}} = \dfrac{-9}{\sqrt{5-x_0}} - \sqrt{5-x_0}$

$\dfrac{x_0}{2\sqrt{5-x_0}} = \dfrac{-9-(5-x_0)}{\sqrt{5-x_0}} \Rightarrow \dfrac{x}{2\sqrt{5-x_0}} = \dfrac{14+9x_0}{\sqrt{5-x_0}} \Rightarrow x_0 = (14+9x_0).2$

$x_0 = 28 + 18x_0 \Rightarrow 17x_0 = -28$

$x_0 =$

352-) $f(x) = x^2 + x + 1$ $A = (0,-3)$ $f(x_0) = x_0^2 + x_0 + 1$

$t: \ y - (x_0^2 + x_0 + 1) = (2x_0 + 1)(x - x_0)$ $f'(x_0) = 2x_0 + 1$

$-3 - (x_0^2 + x_0 + 1) = (2x_0+1)(-x_0)$

$-x_0^2 - \cancel{x_0} + 2x_0^2 + \cancel{x_0} = 4$

$x_0^2 = 4$

$x_0 = \pm 2$

$y - 7 = 5(x - 2)$ ou $y - 3 = -3(x+2)$

$y = 5x - 3$ $y = -3x - 3$

353-) $f(x) = \ln x \quad ; x > 0 \qquad s: y = \frac{x}{2} + 1 \qquad f(x_0) = \ln x_0$

$f'(x_0) = \frac{1}{x_0}$

Os coeficientes angular são iguais, portanto

$f'(x_0) = \frac{1}{2} \qquad f'(x_0) = \frac{1}{x_0} \qquad \frac{1}{2} = \frac{1}{x_0} \quad x_0 = 2$

$y - \ln 2 = \frac{1}{2}(x - 2) \qquad y = \frac{x}{2} - 1 + \ln 2$

(354-) $f(x) = x^2 + x + 1 \qquad s: y = 7 - x \qquad f(x_0) = x_0^2 + x_0 + 1$

$f'(x_0) = 2x_0 + 1$

$f'(x_0) = -1 \quad f'(x_0) = 2x_0 + 1 \qquad 2x_0 + 1 = -1$

$x_0 = -1$

$y - 1 = -1(x + 1)$

$y = -x$

(355) $f(x) = \frac{x}{x^2+1} \quad , \quad s: y = \frac{-1}{8}x + \frac{3}{8} \qquad f(x_0) = \frac{x_0}{x_0^2+1}$

$f'(x_0) = \frac{-1}{8} \qquad \frac{1 - x_0^2}{(x_0^2+1)^2} = \frac{-1}{8} \qquad f'(x_0) = \frac{1 - x_0^2}{(x_0^2+1)^2}$

$1 - (x_0^2+1)^2 = 8(1 - x_0^2)$

$-x_0^4 - 2x_0^2 - 1 = 8 - 8x_0^2$

$-x_0^4 + 6x_0^2 = 9 \qquad m = x^2$

$x_0^4 - 6x_0^2 + 9 = 0$

$m^2 - 6m + 9 = 0$

$x = \frac{6 \pm \sqrt{36 - 36}}{2}$

$x_1 = 3 \qquad m = x^2$

$x_2 = 3 \qquad 3 = x^2$

$x = \sqrt{3} \quad x = \pm\sqrt{3}$

$y - \frac{\sqrt{3}}{4} = \frac{-1}{8}(x - \sqrt{3})$

$y = -\frac{x}{8} + \frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4}$

$y = -\frac{x}{8} + \frac{3\sqrt{3}}{8} \qquad 8y = -x + 3\sqrt{3}$

spiral

(356-) $f(x) = \ln(x + \sqrt{1+x^2})$ s: $y = \frac{3}{5}x - 4$

$f(x_0) = \ln(x_0 + \sqrt{1+x_0^2})$

$f'(x_0) = \dfrac{1 + \dfrac{x_0}{\sqrt{1+x_0^2}}}{x + \sqrt{1+x_0^2}} = \dfrac{1}{\sqrt{1+x_0^2}}$

$f'(x_0) = \frac{3}{5}$ $\quad \frac{3}{5} = \frac{1}{\sqrt{1+x_0^2}}$

$3\sqrt{1+x_0^2} = 5$

$1 + x_0^2 = \left(\frac{5}{3}\right)^2$

$x_0^2 = \frac{25}{9} - 1$ $\quad x_0^2 = \frac{16}{9}$ $\quad x_0 = \pm\frac{4}{3}$

$y - \ln 3 = \frac{3}{5}\left(x - \frac{4}{3}\right)$ $\quad y - \ln\frac{1}{3} = \frac{3}{5}\left(x + \frac{4}{3}\right)$

$y = \frac{3}{5} - \frac{4}{3} + \ln 3$ $\quad y + \ln 3 = \frac{3}{5}x + \frac{4}{5}$ $\quad y = \frac{3}{5}x + \frac{4}{5} - \ln 3$

$y = \frac{3x}{5} - \frac{12}{25}(\ln 3^{-1}) = -\ln 3$

(357-) $f(x) = \ln x$ r: $y = -4x$

$f(x_0) = \ln x_0$

$f'(x_0) = \frac{1}{x_0}$

$f'(x_0)_r = -4$ $\quad f'(x_0)_t = \frac{1}{4}$ $\quad \frac{1}{4} = \frac{1}{x_0}$ $\quad x_0 = 4$

$y - \ln 4 = \frac{1}{4}(x - 4)$

$y = \frac{x}{4} - 1 + \ln 4$

/ /

(358-) $f(x) = x^2 + x + 1$ $\quad r: x + 7y - 2 = 0$ $\quad f(x_0) = x_0^2 + x_0 + 1$

$f'(x_0) = 2x_0 + 1$

$x + 7y - 2 = 0 \quad /(\div 7)$

$\frac{x}{7} + y - \frac{2}{7} = 0$

$y = \frac{2}{7} - \frac{x}{7} \quad y = \frac{2 - x}{7}$ $\quad f'(x_0)_r = \frac{-1}{7} \quad f'(x_0)_t = 7$

$2x_0 + 1 = 7$

$2x_0 = 6 \quad x_0 = 3$

$y - 13 = 7(x - 3)$

$y = 7x - 8$

(359-) $f(x) = (x-3)\sqrt{x}$, $x \geq 0$ $\quad r: 9y + 4x - 117 = 0$ $\quad f(x_0) = (x_0 - 3)\sqrt{x_0}$

$f'(x_0) = \frac{3x_0 - 3}{2\sqrt{x_0}}$

$9y + 4x - 117 = 0 \quad \div(9)$

$y + \frac{4x}{9} - \frac{117}{9} = 0$ $\quad f'(x_0)_r = -\frac{4}{9} \quad f'(x_0)_t = +\frac{9}{4}$

$+\frac{9}{4} = \frac{3(x_0 - 1)}{2\sqrt{x_0}} \Rightarrow 12(x_0 - 1) = 18\sqrt{x_0}$

$(x_0 - 1) = \frac{3}{2}\sqrt{x_0}$

$\frac{2}{3}(x_0 - 1) = \sqrt{x_0}^2$

$\left(\frac{2x_0 - 2}{3}\right)^2 = x_0 \Rightarrow \frac{4x_0^2 - 8x_0 + 4}{9} = x_0$

$4x_0^2 - 8x_0 + 4 = 9x_0$

$4x_0^2 - 17x_0 + 4 = 0$

$x_0 = 4, \frac{1}{4}$

$y - 2 = \frac{9}{4}(x - 4)$

$y = \frac{9x}{4} - 7$

(360-) $f(x) = \ln(x + \sqrt{x^2+3})$ $r: y = 3 - 2x$

$$f'(x) = \frac{1 + \frac{x}{\sqrt{x^2+3}}}{} = \frac{\sqrt{x^2+3} + x}{\sqrt{x^2+3}} ; \quad x + \sqrt{x^2+3} = \frac{1}{\sqrt{x^2+3}}$$

$$f'(x_0)_r = -2 \qquad f'(x_0)_t = \frac{1}{2} \qquad \frac{1}{2} = \frac{1}{\sqrt{x^2+3}} \Rightarrow \sqrt{x^2+3} = 2$$

$$x^2 + 3 = 4$$

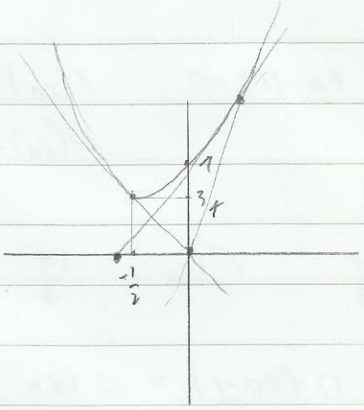
$$x = \pm 1$$

$$y - \ln 3 = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}x + \frac{1}{2}$$

$$y = \frac{x}{2} - \frac{1}{2} + \ln 3$$

(361-)



$\rightarrow y(x) = x^2 + x + 1$

$y'(x) = 2x + 1$

$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$y - 7 = 5(x - 2)$$

$$y = 5x - 3$$

$$x = \frac{3}{5} \qquad A = \left(\frac{3}{5}, 0\right)$$

(362-) $t:$ $\cancel{y} - f(x_0) = f'(x_0)(\cancel{x} - x_0)$ $P(1,3)$ $P_0\ (-1,1)$

$$-f(x_0) = f'(x_0) \cdot (-x_0)$$

$$-(x^2 + x + 1) = (2x + 1)(-x)$$

$$-x^2 - \cancel{x} - 1 = -2x^2 - \cancel{x}$$

$$x^2 = 1$$

$$x = \pm 1$$

(363-) $y - \frac{7}{9} = \frac{1}{3}\left(x + \frac{1}{3}\right)$

$f(x_0) = x_0^2 + x_0 + 1$

$f'(x_0) = 2x_0 + 1$

$y = \frac{x}{3} + \frac{1}{9} + \frac{7}{9}$

$0 = \frac{x}{3} + \frac{8}{9}$

$y = \frac{x}{3} + \frac{8}{9}$

$A = \left(\frac{-8}{3}, 0\right)$

$\frac{x}{3} = \frac{-8}{9} \quad x = \frac{-8}{3}$

(364-) $t: \ y - f(x_0) = f'(x_0)(x - x_0)$

$-y - (x^2 + x + 1) = (2x+1)(-1+x)$

$-1 - x^2 \cancel{-x} - 1 = -2x - 1 - 2x^2 \cancel{-x}$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0 \quad x = -2$

$P_1 = (0,1) \quad P_2 = (-2,3)$

(365-) Para que o alvo não seja atingido o $f'(x_0)$ = coeficiente angular tem que ser igual a $= 0$

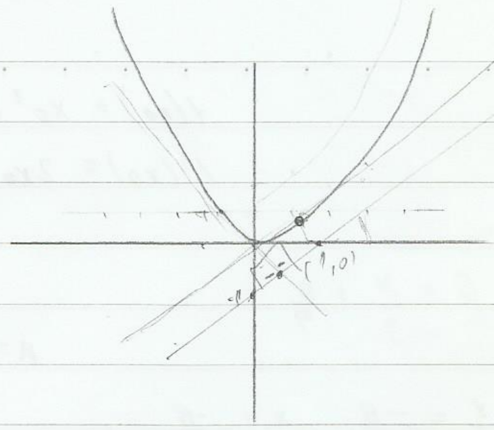
$0 = 2x + 1$

$x = \frac{-1}{2} \qquad y = \left(\frac{-1}{2}\right)^2 - \frac{1}{2} + 1$

$y = \frac{3}{4}$

$P = \left(\frac{-1}{2}, \frac{3}{4}\right)$

(367.)



$y = x^2$

$y = x-1$

$y' = 2x$

$t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - 1 = -2(x+1)$

$y = -2x - 2 + 1$

$y = -2x - 1$ $\quad -2x-1 = x-1 \quad y = x-1$

$-3x = 0 \quad y = 0-1$

$A = (0,-1) \quad y = -1$

(368.) $t: y - f(x_0) = f'(x_0)(x - x_0)$

$-x_0^2 = 2x_0(1 - x_0)$ $\quad (0,0)$ ou $(2,4)$

$-x_0^2 = 2x_0 - 2x_0^2$

$x_0^2 = 2x_0$

$x(x-2) = 0$

$x_0 = 0 \quad x_0 = 2$

(369.) $y = x_0^2 - 1 \quad \frac{dy}{dx} = 1 \quad 1 = 2x \quad \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$x = \frac{1}{2}$

$P = \left(\frac{1}{2}, \frac{1}{4}\right)$

/ /

(384-) $\sqrt{x^2+y^2} = \operatorname{arctg} \frac{y}{x}$

$$\frac{2x+2y.y'}{2\sqrt{x^2+y^2}} \Rightarrow \frac{2(x+y.y')}{2\sqrt{x^2+y^2}} \Rightarrow \frac{x+y.y'}{\sqrt{x^2+y^2}}$$

$$\frac{y}{x} \Rightarrow \frac{y'x-y}{x^2} \Rightarrow \frac{y'x-y}{x^2} : 1+\frac{y^2}{x^2} \Rightarrow \frac{y'x-y}{x^2} : \frac{x^2+y^2}{x^2}$$

$$\Rightarrow \frac{y'x-y}{x^2+y^2} = \frac{x+y.y'}{\sqrt{x^2+y^2}}$$

$$(y'x-y)\sqrt{x^2+y^2} = (x+y.y')(x^2+y^2)$$

$$y'x-y = \frac{(x+y.y')(x^2+y^2)}{\sqrt{x^2+y^2}}$$

$$y'x-y = (x+y.y')\sqrt{x^2+y^2}$$

$$y'x = x\sqrt{x^2+y^2} + y.y'\sqrt{x^2+y^2} + y$$

$$y'x - y.y'\sqrt{x^2+y^2} = x\sqrt{x^2+y^2} + y$$

$$y'(x - y\sqrt{x^2+y^2}) = x\sqrt{x^2+y^2} + y$$

$$y' = \frac{x\sqrt{x^2+y^2}+y}{x-y\sqrt{x^2+y^2}}$$

(385-) $\operatorname{arctg} \frac{x}{2} = \frac{1}{2} \ln(x^2+y^2)$

$$\frac{y-xy'}{y^2+x^2} = \frac{x+y.y'}{x^2+y^2}$$

$$(y-xy')(x^2+y^2) = (x+y.y')(y^2+x^2)$$

$$y-xy' = x+y.y'$$

$$-xy'-y.y' = x-y$$

$$y'(-x-y) = x-y$$

$$y' = \frac{x-y}{-x-y} \cdot -(1) = \frac{y-x}{y+x}$$

spiral®

(386.) $y^2 = x + \ln \frac{y}{x}$

$\frac{y}{x} \Rightarrow \frac{y'x - y}{x^2} : \frac{y}{x} \Rightarrow \frac{x(y'x-y)}{yx^2}$

$2y.y' = 1 + \frac{x(y'x-y)}{yx^2} \Rightarrow 2y.y' = \frac{yx^2 + y'x^2 - xy}{yx^2}$

$\Rightarrow 2y^2y'x^2 = yx^2 + y'x^2 - xy$

$2y^2x^2y' - y'x^2 = yx^2 - xy$

$y'(2y^2x^2 - x^2) = yx^2 - xy$

$\Rightarrow y' = \frac{yx^2 - xy}{2y^2x^2 - x^2}$

$= \frac{x(yx-y)}{x(2y^2x-x)} \Rightarrow \frac{y(x-1)}{x(2y^2-1)}$

(387.) $\sqrt{x^2+y^2} + \operatorname{arctg} \frac{x+y}{x-y} = 3$

$\Rightarrow \frac{(1+y')(x-y) - (x+y)(1-y')}{(x-y)^2}$

$\frac{2x + 2yy'}{2\sqrt{x^2+y^2}} \Rightarrow \frac{x + y.y'}{\sqrt{x^2+y^2}}$

$\Rightarrow \frac{2(-y + xy')}{(x-y)^2} : \frac{x+y}{x-y} \Rightarrow \frac{2(-y+xy')}{(x-y)(x+y)}$

$\frac{x + y.y'}{\sqrt{x^2+y^2}} + \frac{2(xy'-y)}{x^2-y^2} = 0$

$\Rightarrow \frac{(x+y.y')(x^2-y^2) + 2(xy'-y)\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2})\, x^2-y^2} = 0$

$x(x^2-y^2) + y.y'(x^2-y^2) + 2xy'(\sqrt{x^2+y^2}) - 2y\sqrt{x^2+y^2} = 0$

$y.y'(x^2-y^2) + 2xy'(\sqrt{x^2+y^2}) = 2y\sqrt{x^2+y^2} - x(x^2-y^2)$

$y'[y(x^2-y^2) + 2x\sqrt{x^2+y^2}] = 2y\sqrt{x^2+y^2} - x(x^2-y^2)$

$y' = \frac{2y\sqrt{x^2+y^2} - x(x^2-y^2)}{y(x^2-y^2) + 2x\sqrt{x^2+y^2}}$

Rever

(386-) $y^2 = x + \ln \frac{y}{x}$

$2y \cdot y' = 1 + \frac{y'x - y}{x^2} : \frac{y}{x}$ $\quad 2yy' = 1 + \frac{y'x - y}{xy}$ $\quad 2yy' = \frac{xy + y'x - y}{xy}$

$2yy'xy - y'x = xy - y$ $\quad \Rightarrow y' = \frac{y(x-1)}{x(2y^2-1)}$

$y'(2yxy - x) = xy - y$

(387-) $\ln\sqrt{x^2+y^2} + \operatorname{arctg} x \frac{x+y}{x-y} = 3$

$\frac{x+yy'}{\sqrt{x^2+y^2}} : \sqrt{x^2+y^2} = \frac{x+yy'}{x^2+y^2}$

$\frac{(1+y')(x-y) - (x+y)(1-y')}{(x-y)^2} : \frac{1}{1+\left(\frac{x+y}{x-y}\right)^2} \Rightarrow \frac{\frac{x + xy' - y - yy' - x - y + xy' + yy'}{(x-y)^2}}{\frac{(x-y)^2 + (x+y)^2}{(x-y)^2}}$

$\Rightarrow \frac{2(xy'-y)}{(x-y)^2} : \frac{(x-y)^2 + (x+y)^2}{(x-y)^2} \Rightarrow \frac{2(xy'-y)}{(x-y)^2 + (x+y)^2}$

$\frac{2xy' - 2y}{x^2 - 2xy + y^2 + x^2 + 2xy + y^2} \Rightarrow \frac{2(xy' - 0y)}{2(x^2+y^2)} \Rightarrow \frac{xy' - \cancel{y}y}{x^2+y^2}$

$\frac{xy'-2y}{x^2+y^2} + \frac{x+yy'}{x^2+y^2} = 0 \quad \frac{xy'-2y}{x^2+y^2} = -\frac{x+yy'}{x^2+y^2}$

$(xy'-2y)(x^2+y^2) = -(x+yy')(x^2+y^2)$

$(xy'-2y) = -(x+yy')$

$xy' - 2y = -x - yy'$

$xy' + yy' = -x + 2y$

$y'(x+y) = -x + 2y$

$y' = \frac{2y-x}{y+x}$

(388-) $1 + xy + \ln\left(e^{-xy} + e^{xy}\right) = 0$

$$\underbrace{y + xy' +}_{\mathrm{I}} \underbrace{\frac{e^{-xy}(-y-xy') + e^{xy}(y+xy')}{e^{-xy}+e^{xy}}}_{0} = 0$$

$y + xy' = 0$

$xy' = -y \qquad y' = \frac{-y}{x}$

(389-) $y = \ln \frac{x-y}{x+y}$

$$y' = \frac{(1-y)(x+y) - (x-y)(1+y')}{(x+y)^2} : \frac{x-y}{\cancel{x+y}}$$

$$y' = \frac{(1-y')(x+y) - (x-y)(1+y')}{x^2+y^2} \Rightarrow y' = \frac{x + y - xy' - yy' - x + y - xy' + yy'}{x^2+y^2}$$

$$y' = \frac{2(y-xy')}{x^2+y^2} \Rightarrow y' = \frac{2y-2xy'}{x^2+y^2} \Rightarrow y'(x^2+y^2) = 2y - 2xy'$$

$y'x^2 + y'y^2 + 2xy' = 2y$

$y'(x^2+y^2+2x) = 2y$

$$y' = \frac{2y}{x^2+2x\cancel{y}+y^2}$$

/ /

(395-) C: $\sqrt[3]{x^2+y^2} + x \ln(y-1) - y = 0 \quad P=(2,2)$

1ª Parcela $(x^2+y^2)^{\frac{1}{3}} \Rightarrow \frac{1}{3}(x^2+y^2)^{-\frac{2}{3}} \cdot 2x + 2y \cdot y' \Rightarrow \frac{2x+2yy'}{3\sqrt[3]{(x^2+y^2)^2}}$

2ª Parcela $x \ln(y-1) \Rightarrow \ln(y-1) + \frac{y'}{y-1} \Rightarrow \frac{(y-1)\ln(y-1)+y'}{y-1}$

$$\frac{2x+2yy'}{3\sqrt[3]{(x^2+y^2)^2}} + \frac{(y-1)\ln(y-1)+y'}{y-1} - y' = 0$$

$$\frac{2x+2yy'}{12} + (y-1)\ln(y-1) + \cancel{y'} - \cancel{y'} = 0$$

$$\frac{x+yy'}{6} + y' = 0 \quad x + yy' + 6y' = 0$$

$$y'(y+6) = -x$$

$$y' = \frac{-x}{y+6} \Rightarrow y' = \frac{-2}{2+6} = \frac{-1}{4}$$

t: $y - f(x_0) = f'(x_0)(x - x_0)$

$$y - 2 = -\frac{1}{4}(x-2)$$

$$y = -\frac{x}{4} + \frac{1}{2} + 2$$

$$y = -\frac{x}{4} + \frac{5}{2}$$

ou

$$y = \frac{-x}{4} + \frac{10}{4} \quad \text{ou} \quad y = \frac{-x+10}{4}$$

spiral

/ /

(397-) f é Crescente $]-\infty,-3[\cup]1,+\infty[$

f é Decrescente $]-3,1[$

Pto Máximo $(-3,4)$

Pto Mínimo $(1,-5)$

(398-) concava para cima é $]-2,3[$

concava para baixo é $]-\infty,-2[\cup]3,+\infty[$

(399-) $\lim_{x\to+\infty} 0$ $\lim_{x\to-\infty} +\infty$

(400-) f' é positiva $]-\infty,-3[\cup]1,+\infty[$

f' é negativa $]-3,1[$

P_1 é $(-3,4)$ tal que $f'(-3)=0$

P_2 é $(1,-5)$ tal que $f'(1)=0$

(401-) f'' é positiva $]-2,3[$

f'' é negativa $]-\infty,-2[\cup]3,+\infty[$

I_1 é $(-2,\frac{3}{2})$ é tal que $f''(-2)=0$

I_2 é $(3,-2)$ é tal que $f''(3)=0$

(402-) crescimento de f' $]-2,3[$

decrescimento de f' $]-\infty,-2[\cup]3,+\infty[$

Pto Máximo $=(3,-2)$

Pto Mínimo $=(-2,\frac{3}{2})$

(403-) $y=4$ no ponto $x_1=-3$

$y=-5$ no ponto $x_2=1$

spiral

(388.) $1 + xy + \ln(e^{-xy} + e^{xy}) = 5$

$$y + xy' + \frac{-y'e^{-xy} + y'e^{xy}}{e^{-xy} + e^{xy}} = 0$$

$$y + xy' + \frac{y'(-e^{-xy} + e^{xy})}{e^{-xy} + e^{xy}} = 0$$

(389.) $y = \ln \frac{x-y}{x+y}$ $\qquad \frac{x-y}{x+y} \Rightarrow \frac{(1-y')(x+y)-(x-y)(1+y')}{(x+y)^2}$

$$y' = \frac{2(xy'-y)}{x^2+y^2} \qquad \Rightarrow \frac{2(xy'-y)}{(x+y)^2} : \frac{x-y}{x+y} \quad \text{ou} \quad \frac{2(xy'-y)}{(x+y)(x-y)}$$

$$= \frac{2xy'-2y}{x^2+y^2} \Rightarrow y'(x^2+y^2) - 2(y'x) = -2y$$

$$y'(x^2+y^2-2x) = -2y$$

$$y' = \frac{-2y}{x^2-2x+y^2}$$

ou

$$y'(x+y)(x-y) = 2xy' - 2y$$

$$y'[(x+y)(x-y) - 2x] = -2y$$

$$y' = \frac{-2y}{(x+y)(x-y)-2x} \Rightarrow \frac{-2y}{x^2-y^2-2x} \Rightarrow \frac{-2y}{x^2-2x-y^2} \quad \text{ou} \quad \frac{2y}{-x^2+2y+y^2}$$

(390.) $C: x^2+y^2-6x-10y-31=0 \qquad P_0=(2,-3)$

$y^2-10y=-x^2+6x+31 \qquad 2y.y'-10y=-2x+6$

$y(y-10)=-x^2+6x+31 \qquad y'y(2y'-10)=-2x+6$

$t: \; y-f(x_0)=f'(x_0)(x-x_0) \qquad y'=\left(\frac{-2x+6}{y}+10\right)\div 2$

$y+3=\frac{-14}{3}(x-2)$

$\frac{2}{-3}+10=\frac{2}{3}+\frac{30}{3}=\frac{-26}{3}$

$y=\frac{-14x}{3}+\frac{28}{3}-3 \qquad \frac{-28}{3}:2 \quad \frac{-28}{6} \quad \frac{-14}{3}$

$y=\frac{-14x}{3}+\frac{19}{3}$

(390.) $x^2+y^2-6x-10y-31=0$

$2x+2yy'-6-10y'=0$

$y'(2y-10)=6-2x$

$y'=\dfrac{6-2x}{2y-10}$ $\quad y'=\dfrac{6-2.2}{2.(-3)-10}=\dfrac{2}{-16}=-\dfrac{1}{8}$

$t: \ y-f(x_0)=f'(x_0)(x-x_0)$

$y+3=-\dfrac{1}{8}(x-2)$

$y=-\dfrac{x}{8}-\dfrac{11}{4}$

(391.) $C: \ x^3+y^3-2xy-5=0 \quad P_0=(1,2)$

$3x^2+3y^2y'-2(y+xy')=0$

$y'(3y^2-2x)=2y-3x^2$

$y'=\dfrac{2y-3x^2}{3y^2-2x} \Rightarrow y'=\dfrac{2.2-3.1^2}{3.2^2-2.1}=\dfrac{1}{10}$

$t: \ y-f(x_0)=f'(x_0)(x-x_0)$

$y-2=\dfrac{1}{10}(x-1)$

$y=\dfrac{x}{10}+\dfrac{19}{10} \qquad y=\dfrac{x+19}{10}$

(393.) $C: \ y^2 e^{xy}+x^3(1-y)+1=0 \quad P_0=(-1,0)$

$2yy'e^{xy}+y^2e^{xy}(y+xy')+3x^2(1-y)+x^3 \cdot y'=0$

$y'(2ye^{xy}+y^2e^{xy}x-x^3)=-y^3e^{xy}-3x^2(1-y)$

$y'=\dfrac{-y^3e^{xy}-3x^2+3x^2y}{2ye^{xy}+y^2e^{xy}x-x^3} \Rightarrow y'=\dfrac{-3}{1}=-3$

$t: \ y-f(x_0)=f'(x_0)(x-x_0)$

$y-0=-3(x+1) \qquad y=-3x-3$

/ /

(392-) $C: y\ln x + x\ln y - 2e = 0 \quad P = (e,e)$

$y'\ln x + y\frac{1}{x} + \ln y + \frac{x}{y}y' = 0$

$y'\left(\ln x + \frac{x}{y}\right) = -\frac{y}{x} - \ln y \Rightarrow y'\left(\ln x + \frac{x}{y}\right) = \frac{-y - x\ln y}{x}$

$y' = \frac{-y - x\ln y}{x} : \frac{y\ln x + x}{y} \Rightarrow y' = \frac{y(-y - x\ln y)}{x(y\ln x + x)}$

$\frac{e(-e-e)}{e(e+e)} \Rightarrow \frac{e.(-2e)}{e(2e)} = -1$

$t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - e = -1(x - e)$

$y = -x + 2e$

(394-) $C: \sqrt{x^2+y^2} - x^2 - y^2 + xy + 8 = 0 \quad P = (3,4)$

$\frac{x + yy'}{\sqrt{x^2+y^2}} - 2x - 2yy' + y + xy' = 0 \Rightarrow \frac{x + yy'}{5} - 2x - 2yy' + y + xy' = 0$

$x + yy' - 10x - 10yy' + 5y + 5xy' = 0$

$y'(y - 10y + 5x) = 10x - x - 5y$

$y' = \frac{9x - 5y}{-9y + 5x} \quad y' = \frac{9.3 - 5.4}{-9.4 + 5.3} \Rightarrow \frac{7}{-21} = -\frac{1}{3}$

$t: y - f(x_0) = f'(x_0)(x - x_0)$

$y - 4 = -\frac{1}{3}(x - 3)$

$y = -\frac{x}{3} + 5$

16

/ /

$f(x) = \frac{2x}{x^2+1}$ 1º Determinar Dominio e Intersecção com os eixos

$D_f = \mathbb{R}$

eixo x $(y=0)$ eixo y $(x=0)$

$\frac{2x}{x^2+1} = 0 \quad \underline{x = 0}$ $\underline{y = 0}$

2º Determinar cresc/decresc. e Máx/Min

$$f'(x) = \frac{2(x^2+1) - 2x.2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2}$$

$2-2x^2=0 \quad 2x^2=2 \quad \underline{x_1=1} \quad \underline{x_2=-1}$

	$-\infty$		-1		1		$+\infty$
f'		$-$	0	$+$	0	$-$	
f		$\searrow$		$\nearrow$		$\searrow$	

Cresc. = $]-1,1[$

Decresc = $]-\infty,-1[\cup]1,+\infty[$

Máx = $(1,1)$

Min = $(-1,-1)$

3º Concavidade Inflexão

$$f'' = \frac{-4x(x^2+1)^2 - (2-2x^2)2(x^2+1).2x}{[(x^2+1)^2]^2}$$

$$= \frac{\cancel{(x^2+1)}[-4x(x^2+1) - 4x(2-2x^2)]}{(x^2+1)^4}$$

$$= \frac{-4x(x^2+1) - 4x(2-2x^2)}{(x^2+1)^3} = \frac{4x(-x^2-1-2+2x^2)}{(x^2+1)^3} = \frac{4x(x^2-3)}{(x^2+1)^3}$$

teste da derivada da segunda = ——

$4x(x^2-3)=0 \quad x_1=0 \quad x_2=+\sqrt{3} \quad x_3=-\sqrt{3}$

	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		$+\infty$
f''		$-$	0	$+$	0	$-$	0	$+$	
f		$\cap$		$\cup$		$\cap$		$\cup$	

$I_1 = (-\sqrt{3}, \frac{\sqrt{3}}{2})$

$I_2 = (0, 0)$

$I_3 = (\sqrt{3}, \frac{\sqrt{3}}{2})$

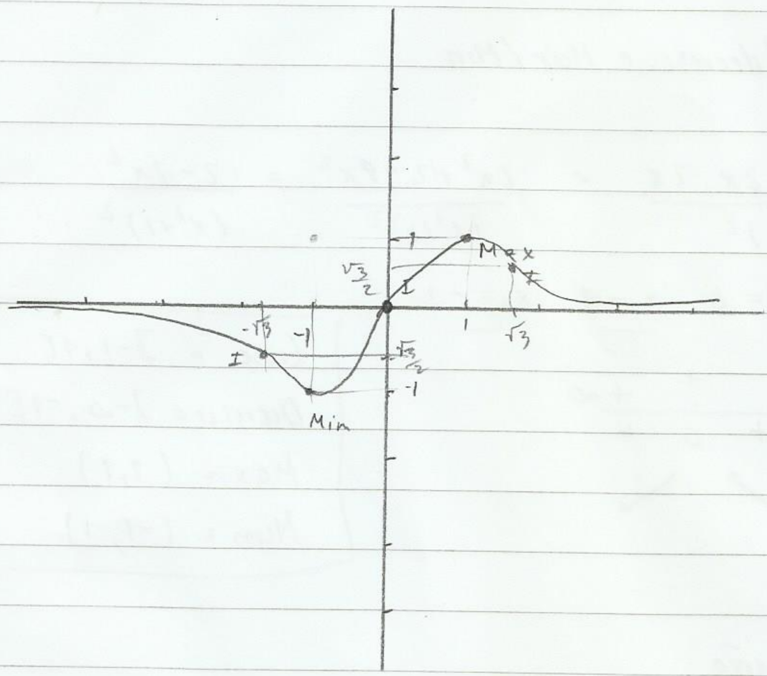
$\cup =]-\sqrt{3},0[\cup]\sqrt{3},+\infty[$

$\cap =]-\infty,-\sqrt{3}[\cup]0,\sqrt{3}[$

spiral

$$\lim_{x \to +\infty} \frac{2x}{x^2+1} \quad \frac{2}{2x} = \frac{0}{x} = \frac{0}{+\infty} = 0$$

$$\lim_{x \to -\infty} \frac{2x}{x^2+1} \quad \frac{2}{2x} = \frac{0}{-\infty} = 0$$



Introdução

Crescé Decres.

1º Derivar a função
2º Igualar a zero
3º Estudar o sinal

Estudar o crescimento $f(x) = \frac{x}{x^2+1}$

$$f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$\frac{1-x^2}{(x^2+1)^2} = 0 \quad 1-x^2=0 \quad -x^2=-1 \quad x^2=1 \quad x=\pm 1$

	$-\infty$		-1		1		$+\infty$
f'		$-$	0	$+$	0	$-$	
f		↘		↗		↘	

f é crescente $]-1, 1[$
f é decrescente $]-\infty, -1[\cup]1, +\infty[$

Máximo e Mínimo

$f' < 0 \Rightarrow$ Pto Mim $\quad f'' < 0$ Pto Máx
$f' > 0 \cdot$ Pto Máx $\quad f'' > 0$ Pto Min

$\frac{76}{27}$

$f(x) = x^3 + x^2 - x + 3$
$f'(x) = 3x^2 + 2x - 1$
$x_1 = -1 \quad x_2 = \frac{1}{3}$

	$-\infty$		-1		$1/3$		$+\infty$
f'		$+$	0	$-$	0	$+$	
f		↗		↘		↗	

$f''(x) = 6x + 2 \quad = -4 < 0$ Máx (-1) $\quad \cancel{-1} + \cancel{1} + 1 + 3 = 4$
$6x + 2 \quad = 4 > 0$ Mim $(\frac{1}{3})$
Máx $= (-1, 4)$ $\quad$ Mim $(\frac{1}{3})$

$\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \frac{1}{3} + 3$

$\frac{1}{27} + \frac{1}{9} - \frac{1}{3} + 3$

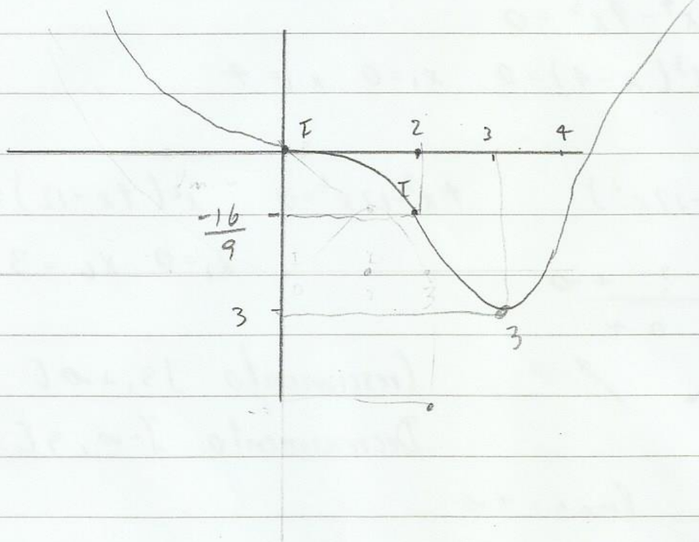
$\frac{1 + 3 - 9 + 81}{27} \quad \frac{76}{27}$

		$-\frac{1}{3}$	
f''	$-$	0	$+$
	∩		∪

$27 \div 3 = 9$

$$\lim_{x \to +\infty} \frac{1}{9}(x^4 - 4x^3) \quad \lim_{x \to +\infty} \frac{x^4 - 4x^3}{9} \quad \frac{x^4\left(1 - \frac{4}{x}\right)}{9} = x^4 \cdot \underline{+\infty}$$

Os dois da $+\infty$



∪ - con para cima

∩ conc para baixo

(397-) f é crescente em: $]-\infty,-3[\,\cup\,]1,+\infty[$

f é decrescente em: $]-3,1[$

Ponto máximo = $(-3,4)$

Ponto mínimo = $(1,-5)$

(398-) Concava para cima $\Rightarrow \cup \Rightarrow]-2,3[$

Concava para baixo $\Rightarrow \cap \Rightarrow]-\infty,-2[\,\cup\,]3,+\infty[$

(399-) $\lim_{x\to+\infty} f(x) = 0$ $\lim_{x\to-\infty} f(x) = -\infty$

?

(400-) f' é positivo $]-\infty,-3[\,\cup\,]1,+\infty[$

f' é negativo $]-3,1[$

$f'(x_0)=0 \Rightarrow (-3,4)$ e $(1,-5)$

(401-) f''

Duvida

(403-) $y - f(x_0) = f'(x_0)(x-x_0)$ $y = -5$

$y = 4$

/ /

(408-) $f(x) = -\frac{x^4}{4} + x^3 - x^2$

Df = ℝ   eixo x (y=0)

$-\frac{x^4}{4} + x^3 - x^2 = 0$ (x4) $-x^4 + 4x^3 - 4x^2 = 0$ $-x^2(-x^2 + 4x - 4) = 0$

$x_1 = 0$ v $x_1 = 2$

eixo y (x=0)

$-\frac{0^4}{4} + 0^3 - 0^2 = 0$

$f'(x) = \frac{1}{4} \cdot -4x^3 \Rightarrow -x^3 + 3x^2 - 2x \Rightarrow x(-x^2 + 3x - 2) = 0$

$x_1 = 0$ $x_2 = 1$ $x_2 = 2$

	$-\infty$		0		1		2		$+\infty$
x_1		−	0	+		+		+	
x_2, x_3		−		−	0	+	0	−	
		+	0	−	0	+	0	−	
f		↗		↘		↗		↘	

Crescimento: $]-\infty, 0[\cup]1, 2[$

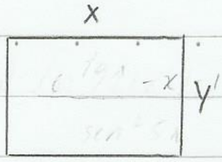
Decrescimento: $]0, 1[\cup]2, +\infty[$

Pto Máx: (0,0) e ~~(2,0)~~

Pto Mín: $(1, \frac{-1}{4})$

$f''(x) = -3x^2 + 6x - 2$ $-3x^2 + 6x - 2 = 0$

spiral

(426-) 

$P = 2x + y$ $\quad$ $x.y = ?$

$100 = 2x + y$

$y = 100 - 2x$

$A(x) = x.y$

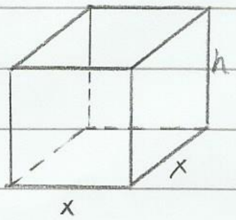
$A(x) = x(100 - 2x)$ $\quad$ $A(x) = 100x - 2x^2$ $\quad$ $100 - 4x = 0$ $\quad$ $y = 100 - 2x$

$A'(x) = 100 - 4x$ $\quad$ $-4x = -100$ $\quad$ $y = 50$

$x = 25$

$R = (50 \times 25)\,m$

(427-)

(426-) 

$V = 384$ $\quad$ $x^2h = 384$ $\quad$ $x = \sqrt{\frac{384}{h}}$

$V = x^2h$ $\quad$ $h = \frac{384}{x^2}$

$C = 3x^2 + 8xh$ $\quad$ $C'(x) = 6x + \frac{-3072.2}{x^3}$

$C = 3x^2 + \frac{3072}{x^2}$

$= 6x - \frac{6142}{x^3} = 0$

$6x^4 - 6142 = 0$

$x_1 = -1{,}127$

$y_2 = -1{,}00$

Profissor

(406-) $f(x) = -\frac{x^4}{4} + x^3 - x^2$

$Df = \mathbb{R}$

eixo x $(y=0)$

$-x^4 + 4x^3 - 4x^2 = 0$

$x^2(-x^2 + 4x - 4x) = 0$

$x_1 = 0 \quad x_2 = 2$

eixo y $(x=0)$

$y = \frac{-0^4}{4} + 0^3 - 0^2$

$y = 0$

$f'(x) = -x^3 + 3x^2 - 2x = 0$

$x(-x^2 + 3x - 2) = 0$

$\Delta = 9 - 8 = 1$

$x_1 = 0$

$x_2 = 1$

$x_3 = 2$

	$-\infty$		0		1		2		$+\infty$
f'		+	0	−	0	+	0	−	
f		↗		↘		↗		↘	

f é crescentem em $]-\infty, 0[\cup]1, 2[$

f é decrescente em $]0, 1[\cup]2, +\infty[$

$f'' = -3x^2 + 6x - 2 = 0$ $\quad f''(0) = -3.0^2 + 6.0 - 2 = -2$ Pto Máximo

$f''(2) = -3.2^2 + 6.2 - 2 = -2$ Pto Máximo $\quad f''(1) = -3.1 + 6.1 - 2 = 1$ Pto mínimo

$-3x^2 + 6x - 2 = 0$

$\Delta = 36 - 24 = 12$ $V = (1, 1)$

$x_1 = \frac{-6 + 3\sqrt{2}}{6} = \frac{-1 + \sqrt{2}}{3}$ $\quad x_2 = \frac{-6 - 3\sqrt{2}}{6} = \frac{-1 - \sqrt{2}}{3}$

	$-\infty$		$\frac{-1-\sqrt{2}}{3}$		$\frac{-1+\sqrt{2}}{3}$		$+\infty$
f''		−	0	+	0	−	
f		∩		∪		∩	

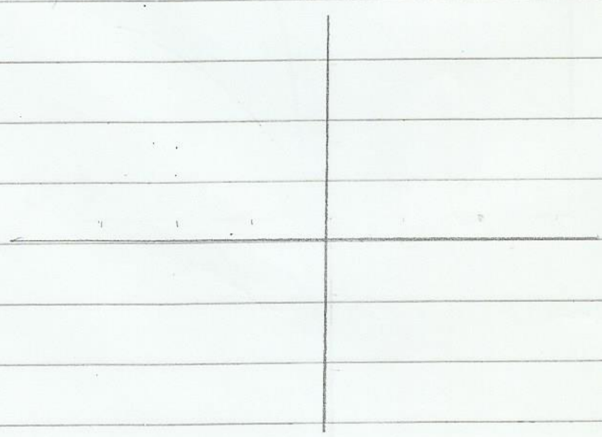
Concavidade voltada para cima $]\frac{-1-\sqrt{2}}{3}, \frac{-1+\sqrt{2}}{3}[$

Concavidade voltada para baixo $]-\infty, \frac{-1-\sqrt{3}}{3}[\cup]\frac{-1+\sqrt{2}}{3}, +\infty[$

$PM_1 = (0, 0)$

$PM_2 = (2, 0)$

$P_m = (1, -\frac{1}{4})$



$\lim_{x \to +\infty} + \infty$

$\lim_{x \to -\infty} + \infty$

spiral

(409) $f(x) = \ln(1+x^2)$ $\quad D_f = \mathbb{R} - \{0\}$ $\quad$ Crescimento $]0,+\infty[$

eixo x ($y=0$) $\quad$ eixo y ($x=0$) $\quad$ Decrescimento $]-\infty,0[$

$x=0$ $\quad$ $y = \ln 1 \Rightarrow y = 0$ $\quad$ PV Min $= (0,0)$

$f'(x) = \frac{2x}{1+x^2} = 0 \quad x=0$

	$-\infty$		0		$+\infty$
f'		$-$	0	$+$	
f		↘		↗	

$f''(x) = \frac{2(1+x^2)-2x.2x}{(1+x^2)^2} = \frac{2+2x^2-4x^2}{(1+x^2)^2} = \frac{2-2x^2}{(1+x^2)^2} = 0 \quad x_1 = 1 \quad x_2 = -1$

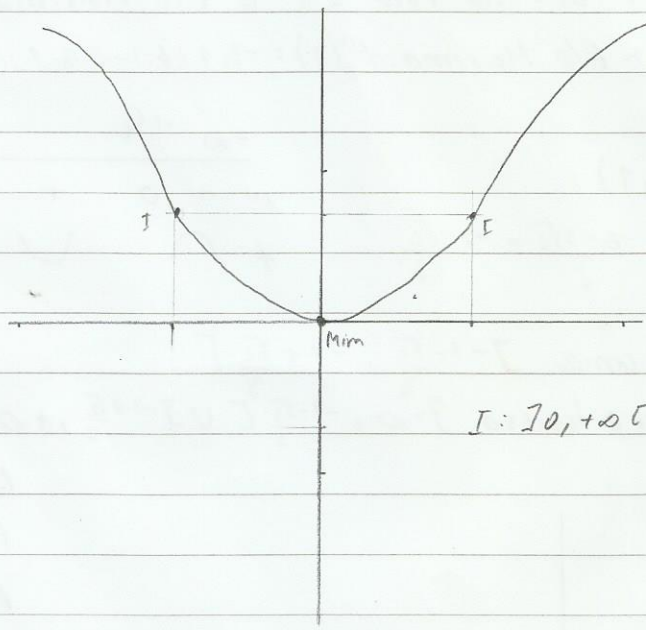
	$-\infty$		-1		1		$+\infty$
f''		$-$	0	$+$	0	$-$	
f		$\cap$		$\cup$		$\cap$	

f tem conc. para cima $]-1,1[$

f tem conc para baixo $]-\infty,-1[\cup]1,+\infty[$

$I_1 = (-1, \ln 2) \quad I_2 = (1, \ln 2)$

$\lim_{x \to +\infty} \ln(1+x^2) = +\infty \quad \lim_{x \to -\infty} \ln(1+x^2) = +\infty$



$I:]0,+\infty[$

(910.) $f(x) = (x-3)\sqrt{x}$ $Df = \{x \in \mathbb{R} \mid x \geq 0\}$

eixo x $(y=0)$

$0 = x\sqrt{x} - 3\sqrt{x}$ $(\div \sqrt{x})$

$0 = x - 3$ $x = 3$

eixo y $(x=0)$

$y = 0 - 3\sqrt{0}$

$y = 0$

$f'(x) = \sqrt{x} + \frac{(x-3)}{2\sqrt{x}} = \frac{2x + x - 3}{2\sqrt{x}}$

$\frac{3x-3}{2\sqrt{x}} = 0$ $x = 1$

	$-\infty$		1		$+\infty$
f'		$-$	0	$+$	
f		↘		↗	

Crescimento $]1, +\infty[$ Decrescimento $]-\infty, 1[$

Vmin $(1, -2)$

$f''(x) = \frac{\frac{6\sqrt{x} - (3x-3)}{\sqrt{x}}}{4x} = \frac{6x - 3x + 3}{4x\sqrt{x}} = 0$ $3x + 3 = 0$ $x = -1$

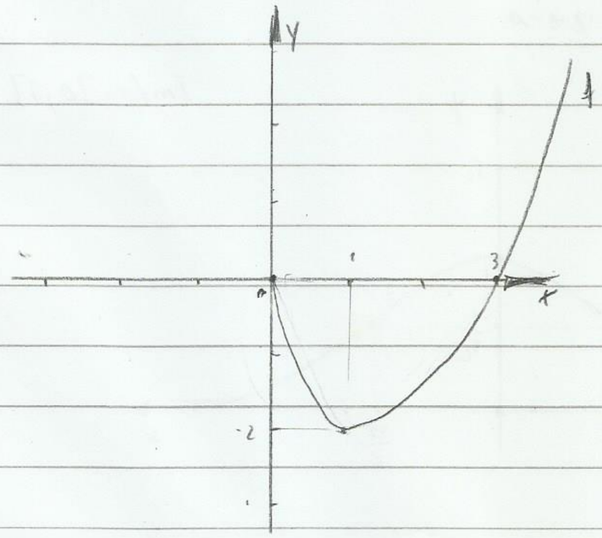
	$-\infty$		-1		$+\infty$
f''		$-$	0	$+$	
f		$\cap$		$\cup$	

Concavidade para cima $]-1, +\infty[$

Concavidade para baixo $]-\infty, -1[$

$I = \nexists$

$\lim_{x \to +\infty} (x-3)\sqrt{x} = +\infty$ $\lim_{x \to -\infty} (x-3)\sqrt{x} = \nexists$



Im f $]-2, +\infty[$

/ /   1,6
6,06

(411) $f(x) = e^{-x^2}$ $\qquad D_f = \mathbb{R}$

eixo x $(y=0)$ $\qquad$ eixo y $(x=0)$

$0 = e^{-x^2}$ $\quad x = \nexists$ $\qquad y = e^0$ $\quad y = 1$

$f'(x) = -2x\,e^{-x^2} = 0$ $\qquad x = 0$

	$-\infty$		0		$+\infty$
f'		$+$	0	$-$	
		$\nearrow$		$\searrow$	

crescente em $]-\infty, 0[$; decrescente em $]0, +\infty[$

$V_{max} = (0, 1)$

$f''(x) = -2\,e^{-x^2} + -2x \cdot -2x \cdot e^{-x^2} = -2e^{-x^2} + 4x^2e^{-x^2} = 0$ $\qquad e^{-x^2}(-2 + 4x^2) = 0$

$-2 + 4x^2 = 0$ $\quad 4x^2 = 2$ $\quad x = \sqrt{\frac{1}{2}}$ $\quad x = \frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

	$-\infty$		$-\frac{\sqrt{2}}{2}$		$\frac{\sqrt{2}}{2}$		$+\infty$
f''		$+$	0	$-$	0	$+$	
f		$\cup$		$\cap$		$\cup$	

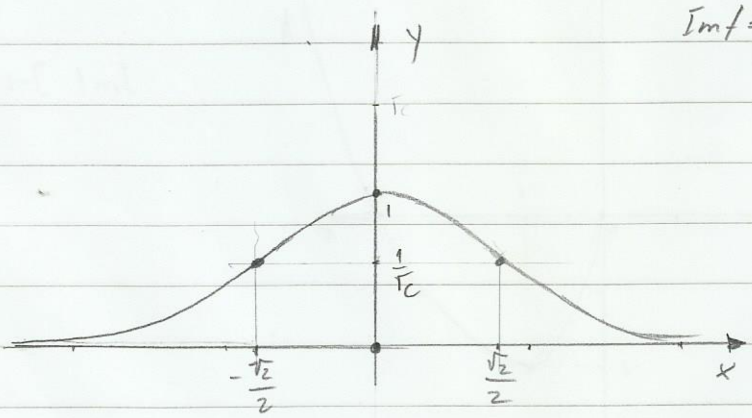
Concavidade para cima $= \left]-\infty, -\frac{\sqrt{2}}{2}\right[\cup \left]\frac{\sqrt{2}}{2}, +\infty\right[$

Concavidade para baixo $= \left]-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right[$

$I = \left(-\frac{\sqrt{2}}{2}, \frac{1}{\sqrt{e}}\right)$ $\qquad I = \left(\frac{\sqrt{2}}{2}, \frac{1}{\sqrt{e}}\right)$

$\lim_{x \to +\infty} e^{-x^2} = e^{-\infty} = 0$ $\qquad \lim_{x \to -\infty} e^{-x^2} = e^{-\infty} = 0$

$Im_f =]0, 1]$



/ /

(912-) $f(x) = x + \frac{1}{x}$ $\quad D_f = \{x \in \mathbb{R} \mid x \neq 0\}$ ou $x \in \mathbb{R}^*$

eixo x $(y=0)$

$0 = x + \frac{1}{x} \Rightarrow \frac{x^2+1}{x} = 0 \quad x = \nexists$

eixo y $(x=0)$

$y = 0 + \frac{1}{0} \quad y = \nexists$

$f'(x) = 1 - \frac{1}{x^2} \Rightarrow \frac{x^2-1}{x^2} = 0 \quad x_1 = 1 \quad x_2 = -1$

	$-\infty$		-1		1		$+\infty$
f'		+	0	−	0	+	
f		↗		↘		↗	

crescente = $]-\infty, -1[\cup]1, +\infty[$ $\quad$ decrescente = $]-1, 1[$

$V_{max} = (-1, -2)$ $\quad V_{min} = (1, 2)$

$f''(x) = \frac{2x}{x^4} = 0 \quad x = 0$

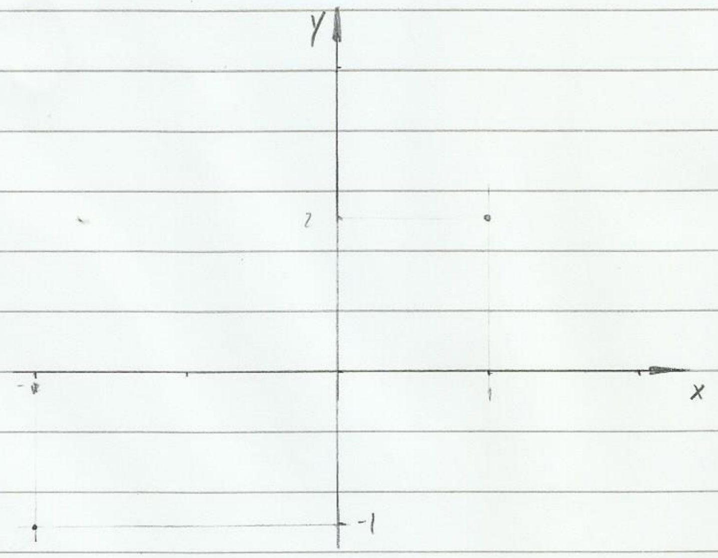
	$-\infty$		0		$+\infty$
f''		−	0	+	
f		∩		∪	

Concavidade para cima $]0, +\infty[$

Concavidade para baixo $]-\infty, 0[$

$I = (0, \nexists)$

$\lim_{x \to +\infty} x + \frac{1}{x} \quad +\infty + \frac{1}{\infty} = +\infty \qquad \lim_{x \to -\infty} x + \frac{1}{x} = -\infty + \frac{1}{-\infty} = -\infty$



$f''(-1) = \frac{2 \cdot (-1)}{(-1)^4} = \frac{-2}{1} = -2 \qquad f''(1) = \frac{2 \cdot 1}{1} = 2$

spiral®

(413) - $f(x) = \frac{2x+1}{3-x}$ $D_f = \{x \in \mathbb{R} \mid x \neq 3\}$

eixo x (y=0)

$\frac{2x+1}{3-x} = 0$ $2x+1 = 0$

$x = \frac{-1}{2}$

eixo y (x=0)

$\frac{1}{3} = y$

$f'(x) = \frac{2(3-x) - (2x+1) \cdot -1}{(3-x)^2}$

$= \frac{6-2x+2x+1}{(3-x)^2} = \frac{7}{(3-x)^2}$